Tracking – 1
Basics and Reconstruction

Jochen Kaminski
University of Bonn

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Overview Course

Lecture 1: Tracking - Basics and Reconstruction
  Monday

Lecture 2: Detector Basic Principle
  Tuesday

Lecture 3: Gaseous Detectors
  Tuesday

Lecture 4: Semiconductor Detectors
  Wednesday
Overview Course

Lecture 1: Tracking - Basics and Reconstruction
  Tracking detectors in HEP
  Multiple scattering
  Resolution Limitation
  Track Reconstruction
  Example of tracking detectors: ILD

Lecture 2: Detector Basic Principle

Lecture 3: Gaseous Detectors

Lecture 4: Semiconductor Detectors
Tracking Systems in 'Old Days'

At the beginning of particle physics, the techniques for observing particles were developed together with the physical ideas about them. Most techniques were based on optically observing their paths and recording them by photographs.

A first tracking detector was the cloud chamber invented by Wilson in 1910: Particles traverse supersaturated water/alcohol vapor. The e−/ion pairs in the track act as condensation nuclei.
Bubble Chambers

The bubble chamber was invented by D. Glaser in 1952 (NP 1960). Particles traverse a superheated liquid and the deposited energy creates bubbles → Photos are made.

Discovery of Neutral Currents

Discovery of the $\Omega^{-}$ in 1964
Spark Chamber and Emulsions

Invented by M. Blau (1930s), particles initiate a chemical reaction that blackens the emulsion.

Discovery of the muon Neutrino in 1962 (M. Schwartz)

C. Powell, Discovery of muon and pion 1947

https://physics.aps.org/articles/v8/75
Detectors in High Energy Physics

Today's detectors are read out completely electronically → reconstruction and analysis can be done by computers.

2 main layouts of HEP experiments
1.) Fixed target experiments, e.g. Compass, (LHCb) Because of the boost, particles go predominantly in one direction, only a small part of the solid angle has to be covered.

2.) Collider experiments, e.g. CMS, ATLAS Particles are created at rest and decay products go in all directions → $4\pi$ coverage necessary
Detectors in High Energy Physics

Key:
- **Muon**
- **Electron**
- **Charged Hadron (e.g. Pion)**
- **Neutral Hadron (e.g. Neutron)**
- **Photon**

Transverse slice through CMS

Schematic diagram showing the layout of detectors:
- **Silicon Tracker**
- **Electromagnetic Calorimeter**
- **Hadron Calorimeter**
- **Superconducting Solenoid**
- **Iron return yoke interspersed with Muon chambers**

D. Stenson, CRD, February 2014

universität bonn
General Requirements for a Tracking System

Some general statements can be made on general requirements of a tracking system. Obviously, experiment specific requirements will often surpass these.

- Detect charged particles with high efficiency
- Precise measurement of particle track (direction, origin, etc)
- Momentum measurement
- Determining the sign of the charge, possibly also the charge itself
- Particle identification for example by $dE/dx$
- Study impact parameter, that is the distance of the track to the expected point of collision
- Robust measurements in environments with many tracks
- Operation for a long time, in particular also able to stand a lot of radiation without aging
- Cheap 😊
A straight line in the 3dimensional room is described by 6 parameters.

\[ \bar{x}(t) = \bar{x}_0 + \bar{p} \cdot t \]

However, a straight track is described by 5 parameters. (If \( t \) is replaced special geometries can be chosen, so that one more parameter is not needed.

The choice of parameters depends on the track model and is mostly driven by optimizing the reconstruction algorithm and computing time.

Often the point of closest approach to the nominal interaction point and two angles are used.
Charged Particles in a Magnetic Field

Free particles moving with constant velocity will be forced on a circular track by a magnetic field

$$\vec{F}_{\text{Lorentz}} = \vec{F}_{\text{centrif}}$$

$$q \left( \vec{v} \times \vec{B} \right) = \gamma m \frac{v^2}{R} \hat{e}_{\perp, \vec{v}}$$

In scalar notation:

$$qvB = \gamma m \frac{v^2}{R}$$

$$\Rightarrow qBR = \gamma mv = p$$

Taking a look at the units: $[q] = e$ $[B] = \frac{Vs}{m^2}$ $[R] = m$

$$[qBR] = e \frac{Vs}{m^2} m = eVs \frac{s}{m} c = 3 \cdot 10^8 \frac{m}{s} eV \frac{3 \cdot 10^8}{c} = 0.3 \frac{\text{GeV}}{c}$$

$$p \left( \frac{\text{GeV}}{c} \right) = 0.3 \ B(T) \ R(m)$$
In HEP experiments three field configurations are used:

1) Dipole fields  
(for fixed target experiments)

2) Solenoidal fields  
(for collider experiments)

3) Toroidal fields  
(for muon system of collider experiment)
The field strength and the radius of the magnets are given by the physics requirements of the experiments. In particular the accelerator maximum energy gives a first guess on $BR^2$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Laboratory</th>
<th>$B$</th>
<th>Radius</th>
<th>Length</th>
<th>Energy</th>
<th>$X/X_0$</th>
<th>$E/M$</th>
<th>Energy</th>
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<tr>
<td>TOPAZ*</td>
<td>KEK</td>
<td>1.2</td>
<td>1.45</td>
<td>5.4</td>
<td>20</td>
<td>0.70</td>
<td>4.3</td>
<td>50-64 GeV (e+/e-)</td>
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<td>CDF*</td>
<td>Tsukuba/Fermi</td>
<td>1.5</td>
<td>1.5</td>
<td>5.07</td>
<td>30</td>
<td>0.84</td>
<td>5.4</td>
<td>1.96 TeV (p/p)</td>
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<td>1.75</td>
<td>5.64</td>
<td>12</td>
<td>0.52</td>
<td>2.8</td>
<td>50-64 GeV (e+/e-)</td>
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<td>AMY*</td>
<td>KEK</td>
<td>3</td>
<td>1.29</td>
<td>3</td>
<td>40</td>
<td>†</td>
<td>†</td>
<td>3.5-12 GeV (e+/e-)</td>
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<tr>
<td>CLEO-II*</td>
<td>Cornell</td>
<td>1.5</td>
<td>1.55</td>
<td>3.8</td>
<td>25</td>
<td>2.5</td>
<td>3.7</td>
<td>920/27.5 GeV (p/\bar{e})</td>
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<tr>
<td>ALEPH*</td>
<td>Saclay/CERN</td>
<td>1.5</td>
<td>2.75</td>
<td>7.0</td>
<td>130</td>
<td>2.0</td>
<td>5.5</td>
<td>90-216 GeV (e+/e-)</td>
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<td>DELPHI*</td>
<td>RAL/CERN</td>
<td>1.2</td>
<td>2.8</td>
<td>7.4</td>
<td>109</td>
<td>1.7</td>
<td>4.2</td>
<td>9/3.1 GeV (e+/e-)</td>
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<tr>
<td>ZEUS**</td>
<td>INFN/DESY</td>
<td>1.8</td>
<td>1.5</td>
<td>2.85</td>
<td>11</td>
<td>0.9</td>
<td>5.5</td>
<td>7/4 GeV (e+/e-)</td>
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<td>H1*</td>
<td>RAL/DESY</td>
<td>1.2</td>
<td>2.8</td>
<td>5.75</td>
<td>120</td>
<td>1.8</td>
<td>4.8</td>
<td>4.6 GeV (e+/e-)</td>
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<tr>
<td>BaBar*</td>
<td>INFN/SLAC</td>
<td>1.5</td>
<td>1.5</td>
<td>3.46</td>
<td>27</td>
<td>†</td>
<td>†</td>
<td>13 TeV (p/p)</td>
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<tr>
<td>D0*</td>
<td>Fermi</td>
<td>2.0</td>
<td>0.6</td>
<td>2.73</td>
<td>5.6</td>
<td>0.9</td>
<td>3.7</td>
<td>90-500 GeV (e+/e-)</td>
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<tr>
<td>BELLE*</td>
<td>KEK</td>
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<td>1.8</td>
<td>4</td>
<td>42</td>
<td>†</td>
<td>†</td>
<td>1-3 TeV (e+/e-)</td>
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<td>BES-III</td>
<td>IHEP</td>
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<td>1.475</td>
<td>3.5</td>
<td>9.5</td>
<td>†</td>
<td>†</td>
<td>100 TeV (p/p)</td>
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<td>ATLAS-CS</td>
<td>ATLAS/CERN</td>
<td>2.0</td>
<td>1.25</td>
<td>5.3</td>
<td>38</td>
<td>0.66</td>
<td>7.0</td>
<td></td>
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<tr>
<td>ATLAS-BT</td>
<td>ATLAS/CERN</td>
<td>1</td>
<td>4.7-9.75</td>
<td>26</td>
<td>1080</td>
<td>(Toroid)†</td>
<td>†</td>
<td></td>
</tr>
<tr>
<td>ATLAS-ET</td>
<td>ATLAS/CERN</td>
<td>1</td>
<td>0.825-5.35</td>
<td>5</td>
<td>2 × 250</td>
<td>(Toroid)†</td>
<td>†</td>
<td></td>
</tr>
<tr>
<td>CMS</td>
<td>CMS/CERN</td>
<td>4</td>
<td>6</td>
<td>12.5</td>
<td>2600</td>
<td>†</td>
<td>12</td>
<td></td>
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<tr>
<td>SiD**</td>
<td>ILC</td>
<td>5</td>
<td>2.9</td>
<td>5.6</td>
<td>1560</td>
<td>†</td>
<td>†</td>
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<tr>
<td>ILD**</td>
<td>ILC</td>
<td>4</td>
<td>3.8</td>
<td>7.5</td>
<td>2300</td>
<td>†</td>
<td>†</td>
<td></td>
</tr>
<tr>
<td>SiD**</td>
<td>CLIC</td>
<td>5</td>
<td>2.8</td>
<td>6.2</td>
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<tr>
<td>ILD**</td>
<td>CLIC</td>
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<td>6</td>
<td>6</td>
<td>23</td>
<td>54000</td>
<td>†</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>
Track Parameters in B-fields (I)

In magnetic fields, the particle path can be derived starting from the Lorentz force:

$$\vec{F}_{\text{Lorentz}} = m \vec{\dot{v}} = q \left( \vec{v} \times \vec{B} \right)$$

Assuming a homogenous B field, which can be directed along the z axis allows to solve for v:

$$\vec{v}(t) = \begin{pmatrix} v_T \cos (\eta \omega_B t + \psi_0) \\ -v_T \sin (\eta \omega_B t + \psi_0) \\ v_z \end{pmatrix}$$

with $$\omega_B = |q|B/\gamma m$$, $$\eta = q/|q|$$, and $$v_T = \sqrt{v_x^2 + v_y^2}$$

Integration gives

$$\vec{x}(t) = \begin{pmatrix} x_0 + \frac{v_T}{\eta \omega_B} \sin (\eta \omega_B t + \psi_0) \\ y_0 + \frac{v_T}{\eta \omega_B} \cos (\eta \omega_B t + \psi_0) \\ z_0 + v_z t \end{pmatrix}$$

=> Helix trajectory

with 6 parameters $$x_0$$, $$y_0$$, $$v_T$$, $$\eta$$, $$\omega_B$$, $$\psi_0$$
Track Parameters in B-fields (II)

Eliminating $t$, introducing the radius $R=\nu_\perp/\omega_B$ and rearranging gives

$$\bar{x}(t) = \left( \begin{array}{c} x_0 + R (\cos (\psi_0 - \eta \psi) - \cos \psi_0) \\ y_0 + R (\sin (\psi_0 - \eta \psi) - \sin \psi_0) \\ z_0 + \frac{R \psi}{\tan \theta} \end{array} \right)$$

with parameters $x_0$, $y_0$, $z_0$, $R$, $\psi$, $\theta$ and $\eta$.

Often also $\kappa = \eta/R$ and $d_0 = \sqrt{x_0^2 + y_0^2}$ is used, because the start point of the helix can not be measured (only 5 parameters).

1) Curvature $\kappa$
2) Angle between $x$-axis and the vector origin-PCA
3) Shortest distance of helix to origin in $r$-$\phi$: $d_0$
4) Angle $\theta$ of track to $z$-axis at PCA
5) Intersection of track with $z$-axis: $z_0$
Multiple Scattering
Multiple Scattering(I)

Particles are deviated in the electrical field of the nuclei. This Coulomb scattering gives small, random deviations of the original path. It is given by the Rutherford cross section

\[
\frac{d\sigma}{d\Omega}_{\text{Rutherford}} = z^2 Z^2 \alpha^2 \frac{1}{\beta^2 p^2} \frac{1}{4 \sin^4 \frac{\theta}{2}}
\]

In a sufficiently thick material the resulting scattering angle \( \theta \) is Gaussian distributed (central limit theorem), but there are tails because of stronger scatterings → Moliere-theory

Assuming only the Gaussian part, one is usually not interested in the spatial distribution, but only in the projection in one plane (e.g. \( r-\phi \) for solenoidal B-fields)

Usually not the 3D angle \( \theta_{\text{space}} \) is interesting, but only the projection in one plane:

\[
\theta_0 = \theta_{\text{plane}}^{\text{rms}} = \frac{1}{\sqrt{2}} \theta_{\text{space}}^{\text{rms}}
\]
Multiple Scattering (II)

The Gaussian distribution of $\theta_{\text{plane}}$ can be written as

$$f(\theta_{\text{plane}})d\theta_{\text{plane}} = \frac{1}{\sqrt{2\pi}\theta_0} e^{-\frac{\theta_{\text{plane}}^2}{2\theta_0^2}} d\theta_{\text{plane}}$$

where $\theta_0$ is on standard deviation and can be approximated by

$$\theta_0 = \frac{13.6 \text{ MeV}}{p\beta c} z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln \frac{x}{X_0}\right)$$

Depending on the problem under study, the following parameters might be of interest:

$$\psi_{\text{rms}} = \frac{1}{\sqrt{3}} \theta_{\text{rms}} = \frac{1}{\sqrt{3}} \theta_0,$$

$$y_{\text{rms}} = \frac{1}{\sqrt{3}} x \theta_{\text{rms}} = \frac{1}{\sqrt{3}} x \theta_0,$$

$$s_{\text{rms}} = \frac{1}{4\sqrt{3}} x \theta_{\text{rms}} = \frac{1}{4\sqrt{3}} x \theta_0.$$
Resolution Limitation
In the most simple case, there are two tracking detectors before the magnet and two after the magnet placed at the same distance from each other. If $B$ is well known, only $\theta$ is of interest.

$$\theta = \frac{L}{R} = \frac{L}{p} eB$$

The additional transverse momentum gives

$$\Delta p_x = p \cdot \sin \theta \approx p \theta = LeB \quad \rightarrow p = \frac{LeB}{\theta} \quad \rightarrow dp = \frac{LeB}{\theta^2} d\theta = \frac{p}{\theta} d\theta$$
Simple Case of Fixed Target (II)

\[ \frac{\sigma_p}{p} = \frac{\sigma_\theta}{\theta} \]

Assuming that all the 4 detectors have the same spatial resolution \( \sigma(x) \) the error on \( \theta \) is given by

\[ \sigma^2(\theta) \sim \sum_{i=1}^{4} \sigma_i^2(x) = 4\sigma^2(x) \quad \rightarrow \quad \sigma(\theta) \sim 2\sigma^2(x) \]

Giving a final expression for the relative error caused by detector uncertainties.

\[ \left. \frac{\sigma(p)}{p} \right|^{det} = \frac{2\sigma(x)/h}{eBL/p} = \frac{2\sigma(x)}{h} \cdot \frac{p}{\Delta p_x} \quad \Rightarrow \text{Important: Error on momentum is proportional to } p^2 \]

Multiple scattering gives an additional contribution to trans. momentum \( \Delta p_x \).

\[ \Delta p_x^{MS} = p \cdot \sin \theta_{rms} \approx p\theta_{rms} = \frac{19.2 \text{ MeV}}{\sqrt{2c}} \sqrt{\frac{L}{X_0}} \]

The relative error of MS to the transv. momentum given by the magnet is

\[ \left. \frac{\sigma(p)}{p} \right|^{MS} = \frac{\Delta p_x^{MS}}{\Delta p_x} = \frac{19.2 \text{ MeV}}{\sqrt{2LeBc}} \sqrt{\frac{L}{X_0}} \]
Simple Case of Fixed Target (III)

Alternative approach: The sagitta method

\[
\frac{R - s}{R} = 1 - \frac{s}{R} = \cos \frac{\alpha}{2} \approx 1 - \frac{\alpha^2}{8} \quad \Rightarrow \quad \frac{s}{R} = \frac{\alpha^2}{8}
\]

\[
\frac{L}{2R} = \sin \frac{\alpha}{2} \approx \frac{\alpha}{2} \quad \Rightarrow \quad \alpha = \frac{L}{R}
\]

Putting it together gives

\[
s = \frac{R\alpha^2}{8} = \frac{RL^2}{8R^2} = \frac{1}{8} \frac{L^2}{R} = \frac{1}{8} L^2 \kappa
\]

For three detectors:

\[
s = y_3 - \frac{y_1 + y_2}{2} \quad \Rightarrow \quad \sigma(s) = \sqrt{\sigma_3(x)^2 + \frac{\sigma_1(x)^2 + \sigma_2(x)^2}{4}} = \sqrt{\frac{3}{2}} \sigma(x)
\]

Error on the curvature

\[
\sigma(\kappa) = \frac{8}{L^2} \sigma(s) = \frac{8}{L^2} \sqrt{\frac{3}{2}} \sigma(x) = \frac{\sqrt{96}}{L^2} \sigma(x)
\]

More than 3 detectors

\[
\sigma(\kappa) = \frac{\sigma(x)}{L^2} \sqrt{\frac{720(N-1)^3}{(N-2)N(N+1)(N+2)}} \quad N \geq 10 \quad \Rightarrow \quad \frac{\sigma(x)}{L^2} \sqrt{\frac{720}{N+4}}
\]
Simple Case of Fixed Target (IV)

The error on the momentum is

\[ \sigma(p_x) = \frac{p_x^2}{|q|B} \sigma(\kappa) \approx \frac{p_x^2}{0.3|z|B} \sigma(\kappa) \]

and the momentum resolution because of limited detector resolution is given by the Gluckstern equation:

\[ \left( \frac{\sigma(p_x)}{p_x} \right)^{det} = \frac{p_x}{0.3|z|B L^2} \sqrt{\frac{720}{N + 4}} \]

In the total momentum resolution is given by the quadratic sum of the two components:

\[ \left( \frac{\sigma(p_x)}{p_x} \right)^{tot} = \sqrt{\left( \left( \frac{\sigma(p_x)}{p_x} \right)^{det} \right)^2 + \left( \frac{\sigma(p_x)}{p_x} \right)^{MS}^2} \]
The track model has also 5 parameters $d_0$, $z_0$, $κ$, $ψ_0$ and $θ$.

The calculation for $dp/p$ is completely analog to the sagitta method

$$\frac{σ(p_T)}{p_T} \bigg|^{det} = \frac{p_T}{0.3|z|} \frac{σ(x)}{BR^2} \sqrt{\frac{720}{N + 4}}$$

What do we learn from the equation for the design of a HEP experiment?
1) Momentum resolution improves quadratically with radius
2) Momentum resolution improves linearly with the B-field
3) Detectors with better spatial resolution helps
4) Number of track points also improves the result
5) $σ(p)/p$ increases with $p →$ more difficult to measure high energetic tracks
Multiple Scattering in the Sagitta Method

The scattering adds another component to the sagitta

\[ \langle s_x \rangle = \frac{1}{4\sqrt{3}} L \theta_0 \approx \sigma(s) \]

This contributes to the error of the curvature. In the case of three equidistant detectors this is:

\[ \sigma_\kappa = \frac{8}{L^2} \sigma(s) = \frac{8}{L^2} \frac{1}{4\sqrt{3}} L \theta_0 = \sqrt{\frac{4 \theta_0}{3}} \frac{0.0136 \text{ GeV}}{p_\beta c L} z \sqrt{\frac{L}{X_0 \sin \theta_0}} \sqrt{1.33} \]

For more detectors the error of \( \kappa \) changes only slightly

\[ \sigma_\kappa = \frac{\theta_0}{L} \sqrt{1.43} = \frac{0.0136 \text{ GeV}}{p_\beta c L} z \sqrt{\frac{L}{X_0 \sin \theta_0}} \sqrt{1.43} \]

Giving a final result of

\[ \left( \frac{\sigma_{p_T}}{p_T} \right)_{\overline{MS}} = \frac{0.054}{L B \beta} \sqrt{\frac{L}{X_0 \sin \theta_0}} \]
Example ATLAS Tracking
Naive Approximation

Pixel detector: $N = 3$ layers,
Spatial resolution $\sigma_x = 12 \mu m$
Radius $R = 5 \text{ cm} - 12 \text{ cm}$

SemiConductor Tracker (SCT):
$N = 4$ layers,
Spatial resolution $\sigma_x = 16 \mu m$
Radius $R = 30 \text{ cm} - 51 \text{ cm}$

Transition Radiation Tracker (TRT):
$N = 36$ layers,
Spatial resolution $\sigma_x = 170 \mu m$
Radius $R = 55 \text{ cm} - 108 \text{ cm}$

$=>$ Averaging for better $\sigma_x$
$N = 1, \sigma_x = 28 \mu m, R = 82 \text{ cm}$

Total Tracking System: $N = 8$ layers,
$\sigma_x = 17 \mu m, R = 5 \text{ cm} - 80 \text{ cm} \rightarrow L = 75 \text{ cm}$

The momentum resolution because of detector resolution is:

$$\frac{\sigma_{p_T}}{p_T} \bigg|^{det} = \frac{p_T \sigma_x}{0.3 z |B| L^2 \sqrt{N + 4}} = \frac{p_T \cdot 17 \cdot 10^{-6} \text{ m}}{0.3 \cdot 1 \cdot 2 \text{ T} \cdot (0.75 \text{ m})^2 \sqrt{8 + 4}} = 0.0004 \cdot p_T$$
Example ATLAS Tracking
Published Performance

The estimate of \( \frac{\sigma_{p_T}}{p_T} \bigg|^{det} = 0.0004 \cdot p_T \) fits well with the official detector requirement of \( \sigma_{p_T} / p_T = 0.05\% \ p_T \) (GeV/c) + 1%

The additional 1% is given because of the material budget of the inner detectors:
Vertex Reconstruction
Vertex Reconstruction

Particles produced in the collision decay fast, but they can fly a certain distance $l$ before decaying:

$$l = \beta \gamma c \tau$$

<table>
<thead>
<tr>
<th>Particle</th>
<th>$m$ (GeV$/c^2$)</th>
<th>$\tau$ ($10^{-12}s$)</th>
<th>$l(p_T = 10 \text{ GeV})$</th>
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<tbody>
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<td>$\tau^+$</td>
<td>1.776</td>
<td>0.290</td>
<td>500 $\mu$m</td>
</tr>
<tr>
<td>$D^0$</td>
<td>1.865</td>
<td>0.410</td>
<td>700 $\mu$m</td>
</tr>
<tr>
<td>$D^+$</td>
<td>1.869</td>
<td>1.040</td>
<td>1700 $\mu$m</td>
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<tr>
<td>$\Lambda^+_c$</td>
<td>2.286</td>
<td>0.200</td>
<td>300 $\mu$m</td>
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<tr>
<td>$B^0_S$</td>
<td>5.367</td>
<td>1.512</td>
<td>800 $\mu$m</td>
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<tr>
<td>$B^+$</td>
<td>5.279</td>
<td>1.641</td>
<td>900 $\mu$m</td>
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<tr>
<td>$\Lambda_b$</td>
<td>5.619</td>
<td>1.425</td>
<td>800 $\mu$m</td>
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</table>
Example of Decay Chain

Identifying the primary vertex (PV), secondary vertex (SV) and tertiary vertex (TV) is important to understand the decay chain of the particle and to reconstruct its mass and life time completely.
Impact Parameter

Impact parameter $d_0$ quantifies the mismatch to the primary vertex (PV).
The impact parameter significance $S_{d_0} = d_0 / \sigma_{d_0}$ serves as selection criterion to distinguish tracks from primary or secondary vertices.

The track as measured by the tracking detectors has to be extrapolated to the vertices. This can be done with

- Linear extrapolation $y = a + bx$ (no magnetic field)
- Quadratic extrapolation $y = a + bx + cx^2$ (good approximation for helix in B)

Errors on the vertex position $y_v$ is given for linear approx. by

$$\sigma_{d_0} = \sigma_y = \sqrt{\sigma_a^2 + x_v^2\sigma_c^2}$$

or for quadratic approx. by

$$\sigma_{d_0} = \sigma_y = \sqrt{\sigma_a^2 + x_v^2\sigma_c^2 + \frac{1}{4}x_V^4\sigma_{ac}^2}$$
The theorem of intersection lines gives for the special cases for the impact parameter errors:

1.) $\sigma_1 > 0$, $\sigma_2 = 0$: $\sigma_{d_0} = \sigma_b = \frac{r_2}{r_2 - r_1} \sigma_1$

2.) $\sigma_1 > 0$, $\sigma_2 = 0$: $\sigma_{d_0} = \sigma_b = \frac{r_1}{r_2 - r_1} \sigma_2$

The complete error is then given by:

$$\sigma_{d_0} = \sqrt{\left(\frac{r_2}{r_2 - r_1} \sigma_1\right)^2 + \left(\frac{r_1}{r_2 - r_1} \sigma_2\right)^2 + \left(\sigma_{d_0}^{MS}\right)^2}$$

with $\sigma_{d_0}^{MS} = \theta_0 r_1$
Discussion

For 2 layers with $\sigma_1 \neq \sigma_2$ at $r_1$ and $r_2$, $\sigma_{d0} = \sqrt{\left(\frac{r_2}{r_2 - r_1}\sigma_1\right)^2 + \left(\frac{r_1}{r_2 - r_1}\sigma_2\right)^2 + (\sigma_{d0}^{MS})^2}$

For more than 2 layers with equal spatial resolutions $\sigma_1 = \sigma_2 = \sigma_N = \sigma_x$

$\sigma_{d0} = \sqrt{\sigma_a + x_V^2\sigma_b^2} = \frac{\sigma_x}{\sqrt{N}}\sqrt{1 + \frac{12(N - 1)}{N + 1} \left(\frac{x_0}{L}\right)^2} = \frac{\sigma_x}{\sqrt{N}}\sqrt{1 + \frac{12(N - 1)}{N + 1} \left(\frac{r_N + r_1}{2} \left(\frac{r_N - r_1}{r_N - r_1}\right)^2\right)^2}$

From these formulas we can deduce the following rules for vertex detector optimization:

1) First layer needs best resolution
   (factor $r_2/(r_2 - r_1)$ is larger than for other layers)

2) Large lever arm ($r_2 - r_1$)

3) Place first layer as close as possible to vertex ($r_1$ small)

4) Larger number of layers improves result with $\sqrt{N}$

For extrapolations with curvature:

$\sigma_{d0} = \frac{\sigma_x}{\sqrt{N}}\sqrt{1 + \frac{12(N - 1)}{N + 1} \left(\frac{x_0}{L}\right)^2 + \frac{180(N - 1)^3}{(N - 2)(N + 1)(N + 2)} \left(\frac{x_0}{L}\right)^4 + \frac{30N^2}{(N - 2)(N + 2)} \left(\frac{x_0}{L}\right)^2}$
ATLAS Vertex Detector

Number of layer: \( N = 3 \) at \( r_1 = 4.7 \text{ cm} \), \( r_2 = 9.1 \text{ cm} \), \( r_3 = 13.5 \text{ cm} \)

Putting in the numbers, one gets:

\[ \sigma_{do} = 15.7 \mu m \text{ (linear extrapolation)} \]

\[ \sigma_{do} = 45.5 \mu m \text{ (quadr. extrapolation)} \]
Track Reconstruction
General Comments

Raw data usually have to be treated before any further steps can be done.

- Calibration results have to be applied to raw data
  - Electronics channels may have different amplifications/capacitors etc.
    → to compare the hits on channels, calibration is important
    → possibly also conversion from ADC units to absolute charge (in $e^-$)
- Determine time information and other properties from the time development of the signal on one channel.
- Reconstruct a cluster from neighboring channels.
  - Often the center of gravity algorithm is used

$$x = \frac{\sum_{i=1}^{N} q_i x_i}{\sum_{i=1}^{N} q_i}$$

- Based on information from previous alignment studies, the reconstructed clusters may have to be shifted to the correct position.
- During track finding hits will be combined to form a track
- Track fitting gives the final parameters and errors of the track.
Calculation of Spatial Resolution

Spatial resolution of a uniformly distributed signal with pads of the width $d$.

We first need a normalized distribution of the particle (particle density function):

$$\int_{-\frac{d}{2}}^{\frac{d}{2}} f(x) \, dx = 1 \quad \Rightarrow \quad f(x) = \frac{1}{d}$$

Now we calculate the variance of the position measurement:

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \int_{-\frac{d}{2}}^{\frac{d}{2}} x^2 f(x) \, dx - \left( \int_{-\frac{d}{2}}^{\frac{d}{2}} x f(x) \, dx \right)^2 = \frac{1}{d} \int_{-\frac{d}{2}}^{\frac{d}{2}} x^2 \, dx - \left( \frac{1}{d} \int_{-\frac{d}{2}}^{\frac{d}{2}} x \, dx \right)^2 =$$

$$= \frac{1}{d} \cdot \frac{x^3}{3} \bigg|_{-\frac{d}{2}}^{\frac{d}{2}} - \frac{1}{d} \cdot \frac{x^2}{2} \bigg|_{-\frac{d}{2}}^{\frac{d}{2}} = \frac{1}{d} \cdot 2 \cdot \frac{d^3}{38} - 0 = \frac{d^2}{12}$$

$$\sigma_x = \frac{d}{\sqrt{12}} \approx \frac{d}{3.5}$$
KF for Track Finding

1) **Find a seed track:** This is often a combinatorial approach using the 2-3 outermost layers and an inner layers of measurement. The outside is chosen, because tracks are separated and possible combinations are reduced.

2) **Make a track estimate:** Based on the currently available track points and their errors, a prediction is made for hits in the next detector layer.

3) **Looking for hit:** If hit is found, add it to track and update the track prediction for the next layer.

direction of flight →

← direction of filter
Advantages of the KF

It is easy to take detector effects into account
- Energy loss
- Multiple Scattering
- Bremsstrahlung

can be accommodated by recognizing the effect and correcting for it in the next step.

A filter can be built, that stores the intermediate results of the filter
→ outliers such as noise or hits from close tracks can be identified and removed afterward.

The best parameter estimate can be given at any point.

Because of the flexibility, the algorithm is widely used for tracking objects in real time:
Radar tracking, sonar ranging, satellite orbit computation, automated driving
Hough Transformation

Robust global algorithm to find geometrical forms such as lines or circles in a random background. Invented by Paul Hough in 1962.

1) Space points are transformed according to the track model into the parameter space.
2) Space points correspond to lines in the parameter space.
3) Find point with most line crossings in the parameters space.
   → point gives good estimate for track parameter estimates

Often used in pattern recognitions software for picture.
   e.g. in astronomy
HT in 2 Dimensions

Track model is $y = a + bx$.

But it is better to use the parameters $d_0$ and $\phi$. The transformation is:

$$a = \tan \phi \quad \text{and} \quad b = \frac{d_0}{\cos \phi}$$

The track becomes then in the parameter space

$$d_0 = y \cdot \cos \phi - x \cdot \sin \phi$$
HT for Several Tracks

- Parameter space is a histogram
- Add the value 1 to all bins, which the lines touch
- Search for bin with highest entries
- Accept all lines in an ellipse given by the errors of the space point measurement.
- Combine these lines/space points to one track
- Refit the space points to get a better parameter estimate (only as good as histogram binning)
- Remove all entries of the space points from the histogram
- Restart with searching bin with most entries.
Experiment ILD at ILC
International Linear Collider

Linear $e^+e^-$ collider with $\sqrt{s} = 90/250/350/500/1000$ GeV:

Point-like particles collide and annihilate. Energy, momentum and all quantum numbers are known. High degree of polarization ($e^-\sim80\%, e^+\sim30\%$)

Bunch structure: Damping takes 0.2 s, then bunches are collided

1312 bunches
Experiments

Two detectors are used alternatively with a push-pull device.

Both detectors are based on the same design idea and differ in the implementation.
Detectors are standard multipurpose detectors of HEP
Similar to LEP detectors, but with much more advanced technologies

- 6 layers of silicon pixel detectors
- 2 layers of silicon strip detector
- TPC
- 1 layer of silicon strip detector
- Electromagnetic calorimeter
- Hadronic calorimeter
- Coil for a B=3.5 T solenoidal field
- Instrumented return yoke

Detector has been laid out and optimized for
the particle flow concept
A limiting factor in multi-jet event reconstruction is the poor energy resolution of hadronic jets because of the traditional calorimetric approach: Measure all components of jet energy in ECAL/HCAL!

But typical hadronic jet contains:
- 64 % charged hadrons (mainly $\pi^\pm$)
- 25 % photons (mainly from $\pi \rightarrow \gamma \gamma$)
- 11 % neutral hadrons (mainly $K_L$ and n)

Best energy resolution for these particles can be obtained in different sub-detectors.

To realize the concept, one needs an unprecedented detector and an excellent reconstruction algorithm:
Tracks measured in central detector → but energy should not be counted twice → disentangle calorimeter measurements from neutral/charged particles.
Requirements on Tracking Detectors

Need a very robust, efficient tracking, which can identify and separate tracks even in high track densities.

Need calorimeters with a very high granularity: cell sizes of 3*3 cm² or even 1*1 cm² are discussed.
Requirements on Tracking System

Requirements are driven by benchmark processes, in the case of ILD – TPC the most stringent measurement is the Higgs-recoil measurement:

\[ \frac{\delta p_t}{p_t^2} = a \oplus \frac{b}{(p_t \sin \theta)} \]

- **ILD RDR**
  - \( a = 2.0 \times 10^{-5} \)
  - \( b = 1.0 \times 10^{-3} \)
  - \( \Delta M_h = 103 \text{ MeV} \)

- **ILD RDR**
  - \( a = 1.0 \times 10^{-5} \)
  - \( b = 1.0 \times 10^{-3} \)
  - \( \Delta M_h = 85 \text{ MeV} \)

- **ILD RDR**
  - \( a = 4.0 \times 10^{-5} \)
  - \( b = 1.0 \times 10^{-3} \)
  - \( \Delta M_h = 153 \text{ MeV} \)

- **ILD RDR**
  - \( a = 8.0 \times 10^{-5} \)
  - \( b = 1.0 \times 10^{-3} \)
  - \( \Delta M_h = 273 \text{ MeV} \)
Requirements of Tracking System

Design goals

Detector design

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Design Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>coverage</td>
<td>hermetic above $\theta \sim 10^\circ$</td>
</tr>
<tr>
<td>momentum resolution $\delta(1/p_T)$</td>
<td>$\sim 2 - 5 \times 10^{-5} \text{ GeV}/c$</td>
</tr>
<tr>
<td>material budget</td>
<td>$\sim 0.10 - 0.15X_0$ in central region</td>
</tr>
<tr>
<td></td>
<td>$\sim 0.20 - 0.25X_0$ in endcap region</td>
</tr>
<tr>
<td>hit efficiency</td>
<td>$&gt; 99%$</td>
</tr>
<tr>
<td>background tolerance</td>
<td>Full efficiency at 10× expected occupancy</td>
</tr>
</tbody>
</table>

**Barrel system**

<table>
<thead>
<tr>
<th>System</th>
<th>R(in)</th>
<th>R(out)</th>
<th>z</th>
<th>comments</th>
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<tbody>
<tr>
<td>VTX</td>
<td>16</td>
<td>60</td>
<td>125</td>
<td>3 double layers</td>
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<tr>
<td></td>
<td></td>
<td></td>
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<td>layer 1:</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td>$\sigma &lt; 3\mu m$</td>
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<td>layer 2:</td>
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<td>$\sigma &lt; 6\mu m$</td>
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<td>layer 3-6</td>
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<td></td>
<td>$\sigma &lt; 4\mu m$</td>
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<td>Silicon</td>
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<td>Silicon pixel sensors,</td>
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<td>SIT</td>
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<td>300</td>
<td>644</td>
<td>2 silicon strip layers</td>
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<td>$\sigma = 7\mu m$</td>
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<tr>
<td>SET</td>
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<td>2300</td>
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<td>2 silicon strip layers</td>
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<td>$\sigma = 7\mu m$</td>
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<tr>
<td>TPC</td>
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<td>2350</td>
<td>MPGD readout</td>
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<td>$1 \times 6\text{mm}^2$ pads</td>
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<td></td>
<td></td>
<td></td>
<td>$\sigma = 60\mu m$ at zero drift</td>
</tr>
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</table>

Simulation results

$$\sigma_{1/p_T} = a \oplus b/(p_T \sin \theta), \text{ with } a = 2 \times 10^{-5} \text{ GeV}^{-1} \text{ and } b = 1 \times 10^{-3}.$$


4.) ILC, https://www.linearcollider.org/

5.) Various web pages for pictures

6.) lecture notes of Prof. N. Wermes