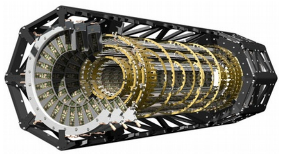
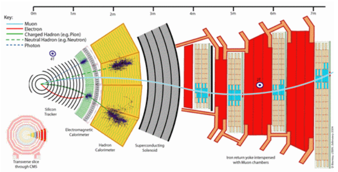
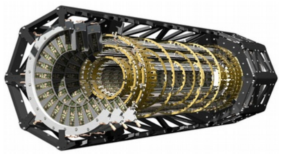




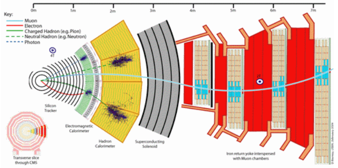
Tracking – 1
Basics and Reconstruction

Jochen Kaminski
University of Bonn

BND – summer school
Callantsoog, Netherlands
4.-6.9.2017



Overview Course



Lecture 1: Tracking - Basics and Reconstruction
  Monday

Lecture 2: Detector Basic Principle
  Tuesday

Lecture 3: Gaseous Detectors
  Tuesday

Lecture 4: Semiconductor Detectors
  Wednesday

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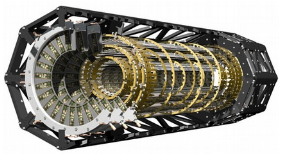
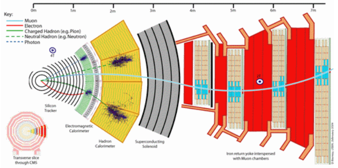


Overview Course



Lecture 1: Tracking - Basics and Reconstruction
- Tracking detectors in HEP
- Multiple scattering
- Resolution Limitation
- Track Reconstruction
- Example of tracking detectors: ILD

Lecture 2: Detector Basic Principle

Lecture 3: Gaseous Detectors

Lecture 4: Semiconductor Detectors

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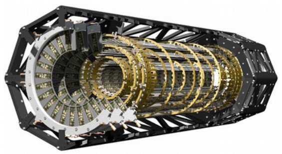
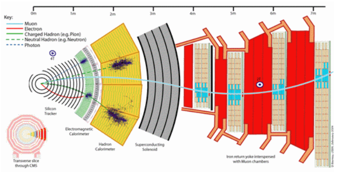
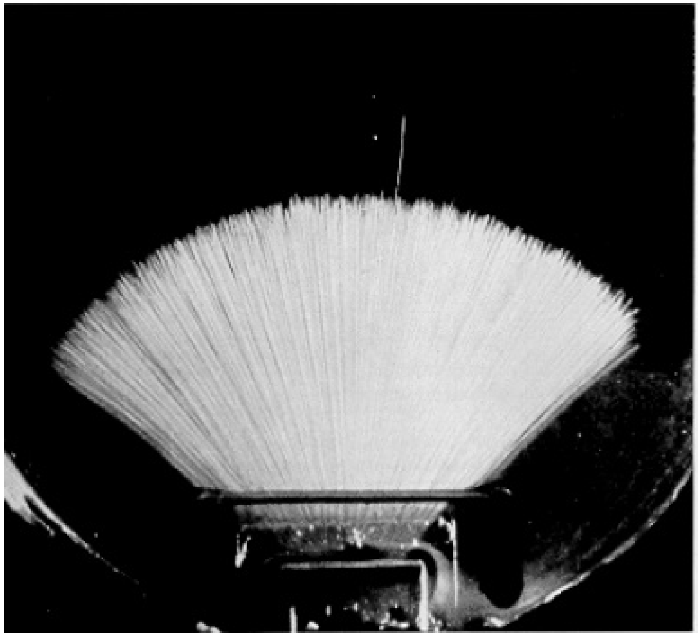


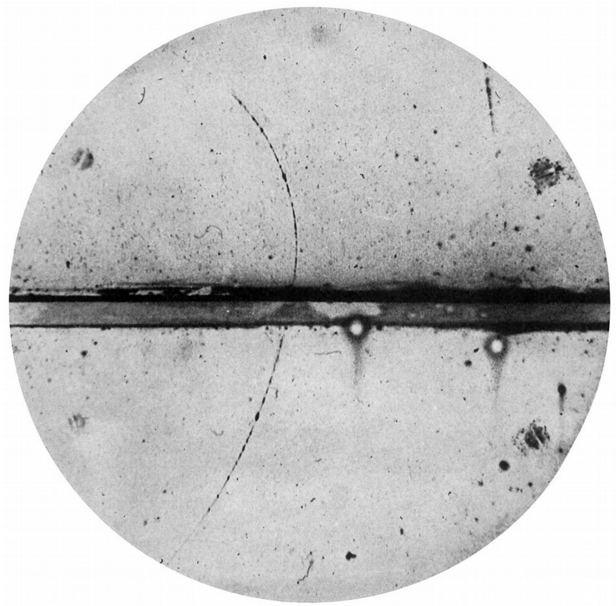
Tracking Systems in 'Old Days'



At the beginning of particle physics, the techniques for observing particles were developed together with the physical ideas about them.
Most techniques were based on optically observing their paths and recording them by photographs.

A first tracking detector was the cloud chamber invented by Wilson in 1910: Particles traverse supersaturated water/alcohol vapor. The e^-/ion pairs in the track act as condensation nuclei.





https://en.wikipedia.org/wiki/Positron

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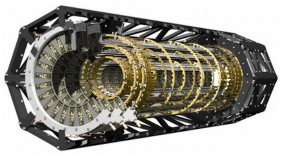
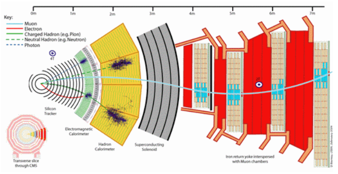
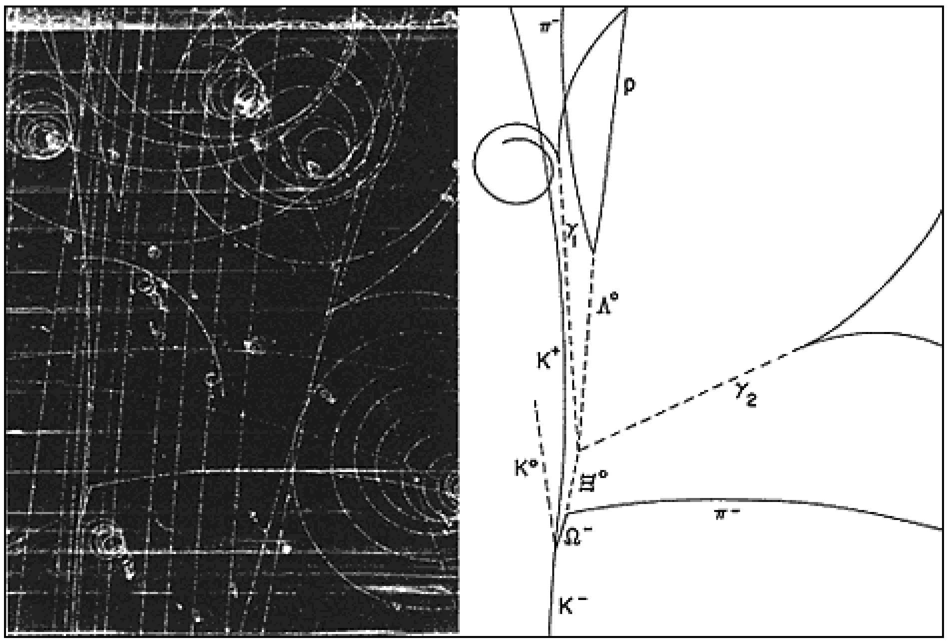


Bubble Chambers

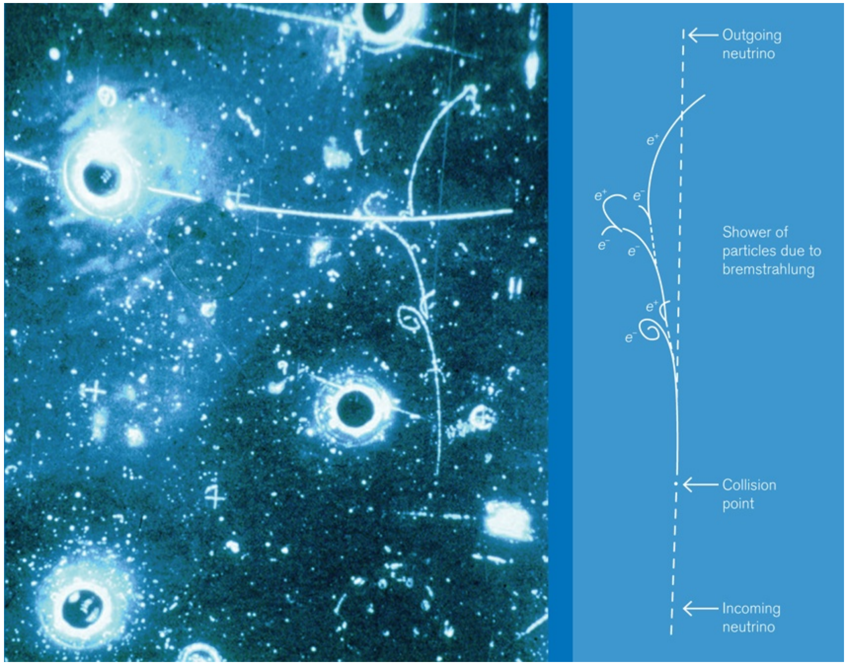


The bubble chamber was invented by D. Glaser in 1952 (NP 1960). Particles traverse a superheated liquid and the deposited energy creates bubbles → Photos are made.

http://www.hep.fsu.edu/~wahl/satmorn/history/Omega-minus.asp.htm



Discovery of the Ω^- in 1964



Discovery of Neutral Currents

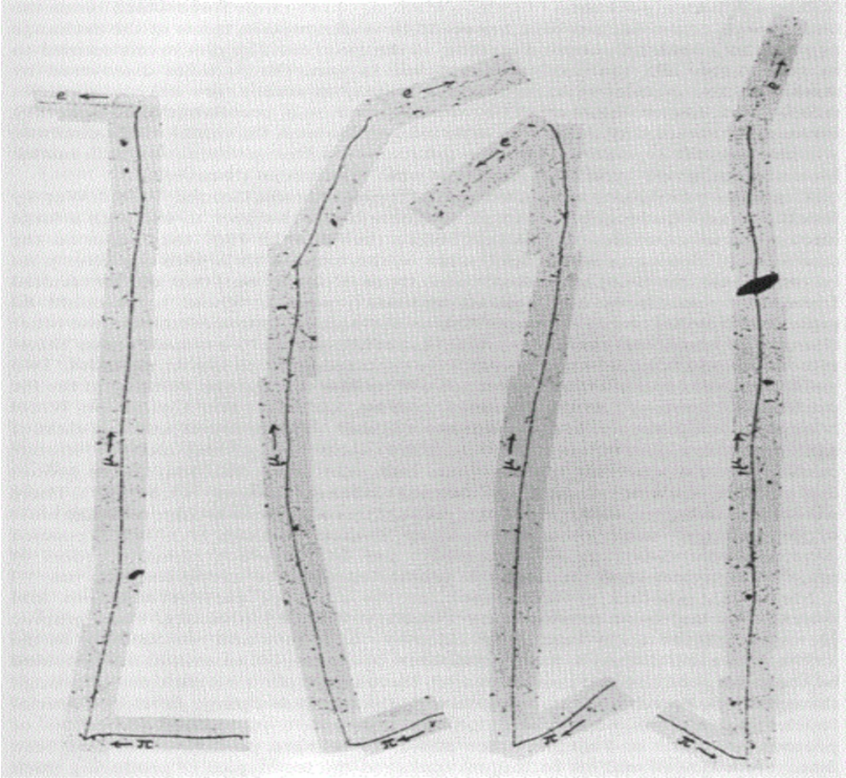
http://www.symmetrymagazine.org/article/august-2009/weak-neutral-current

Spark Chamber and Emulsions

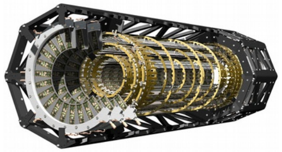
Invented by M. Blau (1930s), particles initiate a chemical reaction that blackens the emulsion.

https://physics.aps.org/articles/v8/75

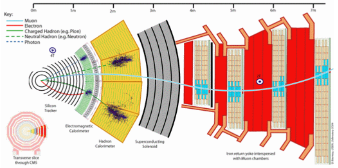
Discovery of the muon Neutrino in 1962 (M. Schwartz)



C. Powell, Discovery of muon and pion 1947

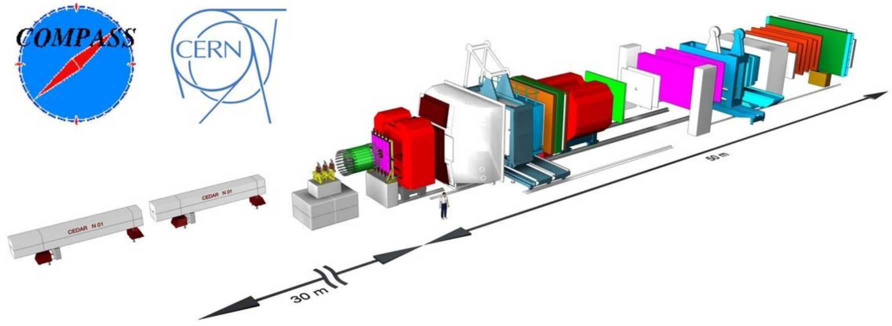


Detectors in High Energy Physics



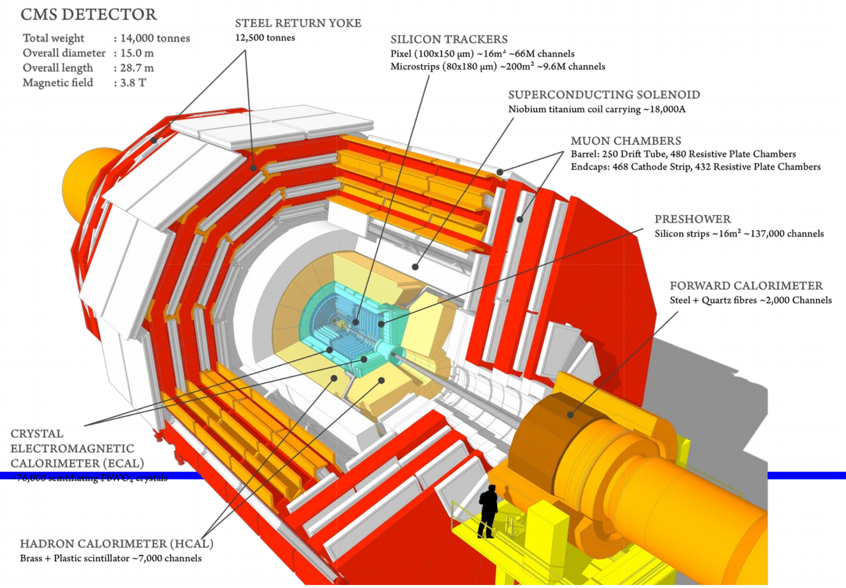
Today's detectors are read out completely electronically
→ reconstruction and analysis can be done by computers.

2 main layouts of HEP experiments
1.) Fixed target experiments,
e.g. Compass, (LHCb)
Because of the boost, particles go predominantly in one direction, only a small part of the solid angle has to be covered.



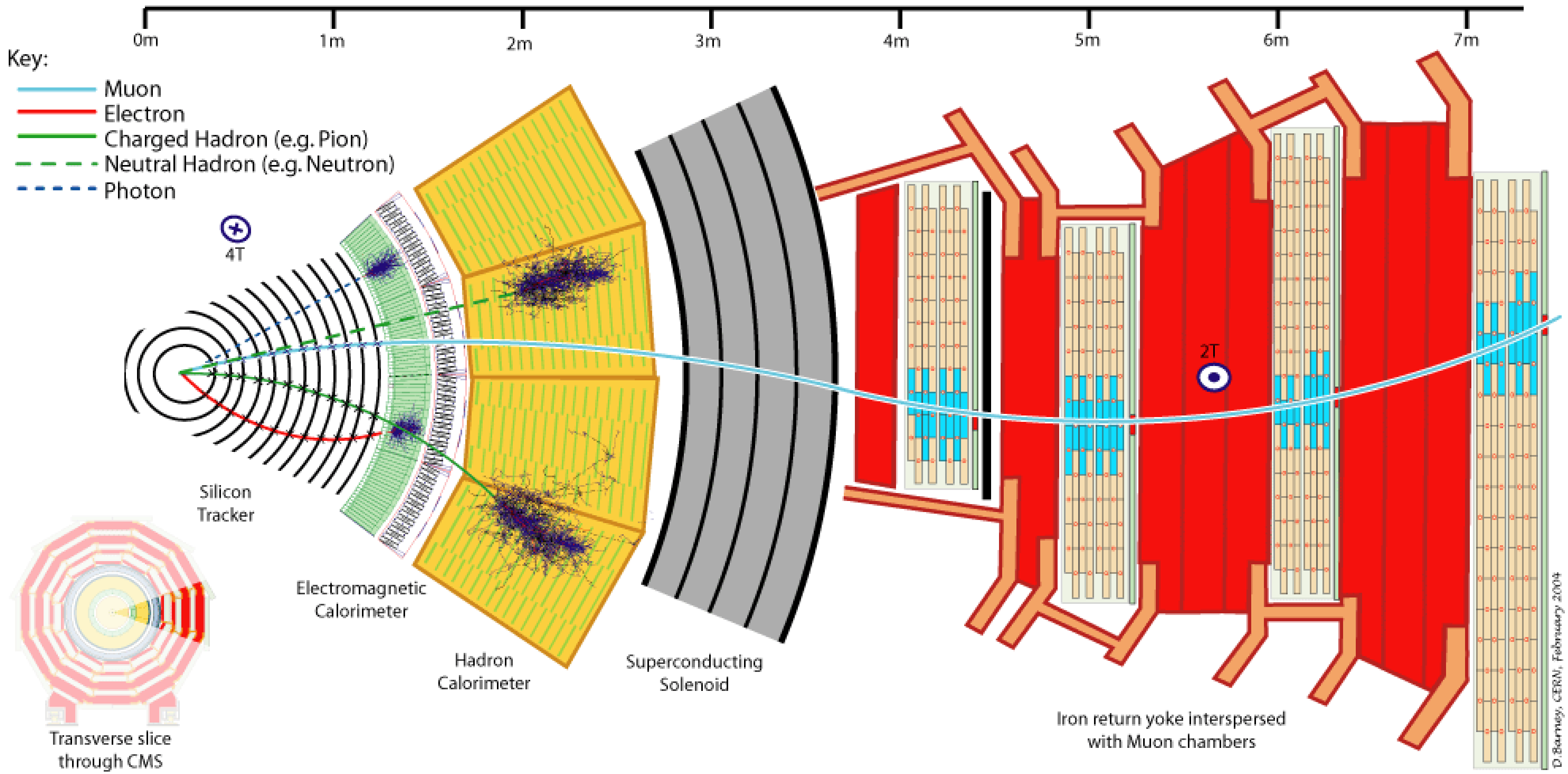
https://www.e18.ph.tum.de/research/high-energy-physics-with-hadrons-at-cern-compass/

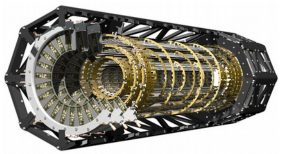
2.) Collider experiments,
e.g. CMS, ATLAS
Particles are created at rest and decay products go in all directions
→ 4π coverage necessary



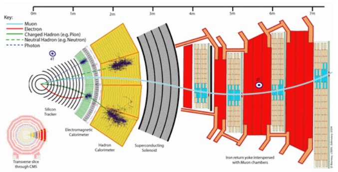
https://cms.cern/detector

Detectors in High Energy Physics





General Requirements for a Tracking System



Some general statements can be made on general requirements of a tracking system.
Obviously, experiment specific requirements will often surpass these.

- Detect charged particles with high efficiency
- Precise measurement of particle track (direction, origin, etc)
- Momentum measurement
- Determining the sign of the charge, possibly also the charge itself
- Particle identification for example by dE/dx
- Study impact parameter, that is the distance of the track to the expected point of collision
- Robust measurements in environments with many tracks
- Operation for a long time, in particular also able to stand a lot of radiation without aging
- Cheap

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Track Parameters

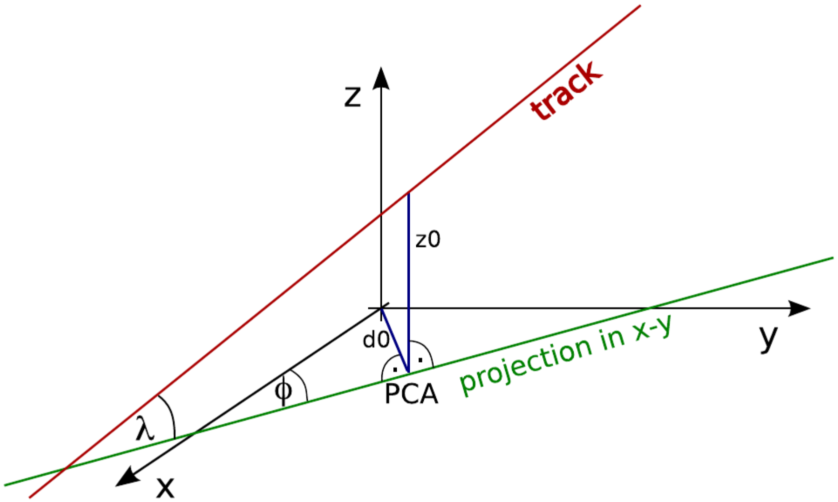
A straight line in the 3dimensional room is described by 6 parameters.

$$\vec{x}(t) = \vec{x}_0 + \vec{p} \cdot t$$

However, a straight track is described by 5 parameters.
(If t is replaced special geometries can be chosen, so that one more parameter is not needed.

The choice of parameters depends on the track model and is mostly driven by optimizing the reconstruction algorithm and computing time.

Often the point of closest approach to the nominal interaction point and two angles are used.



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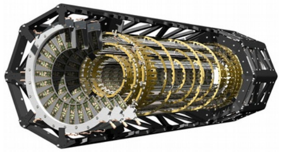
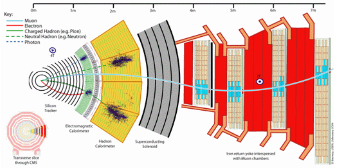


Charged Particles in a Magnetic Field



Free particles moving with constant velocity will be forced on a circular track by a magnetic field

$$\vec{F}_{Lorentz} \stackrel{!}{=} \vec{F}_{centrif}$$

$$q\left(\vec{v}\times\vec{B}\right) = \gamma m \frac{\vec{v}^2}{R}\hat{e}_{\perp,\vec{v}}$$

In scalar notation:

$$qvB = \gamma m \frac{v^2}{R}$$

$$\Rightarrow qBR = \gamma m v = p$$

Taking a look at the units: $[q] = e \quad [B] = \frac{\mathrm{V\,s}}{\mathrm{m}^2} \quad [R] = \mathrm{m}$

$$[qBR] = e\frac{\mathrm{V\,s}}{\mathrm{m}^2}\mathrm{m} = eV\frac{\mathrm{s}}{\mathrm{m}} \stackrel{c=3\cdot 10^8\frac{\mathrm{m}}{\mathrm{s}}}{=} eV\frac{3\cdot 10^8}{c} = 0.3\ \frac{\mathrm{GeV}}{\mathrm{c}}$$

$$p\left(\frac{\mathrm{GeV}}{\mathrm{c}}\right) = 0.3\ B(\mathrm{T})\ R(\mathrm{m})$$

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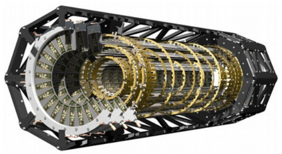
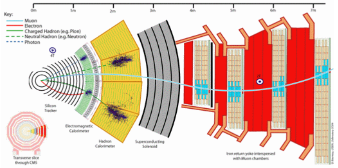
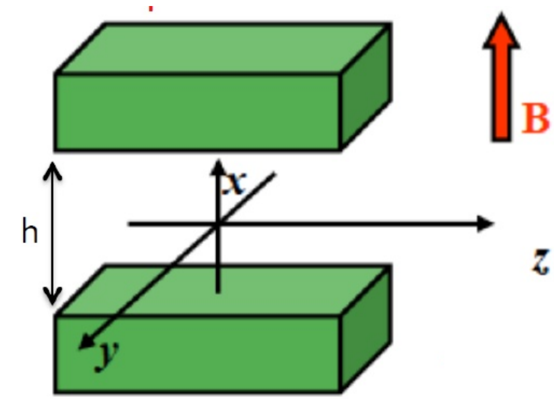


Magnet Field Configurations



In HEP experiments three field configurations are used:

1) Dipole fields
   (for fixed target
   experiments)



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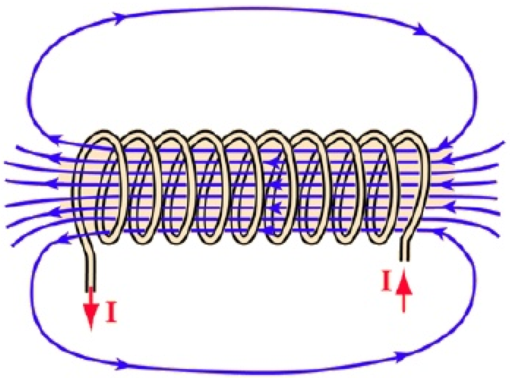
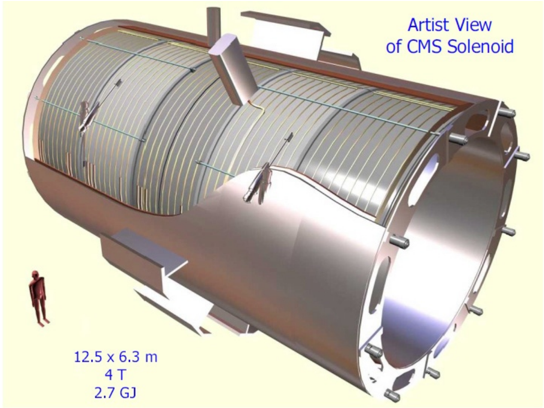
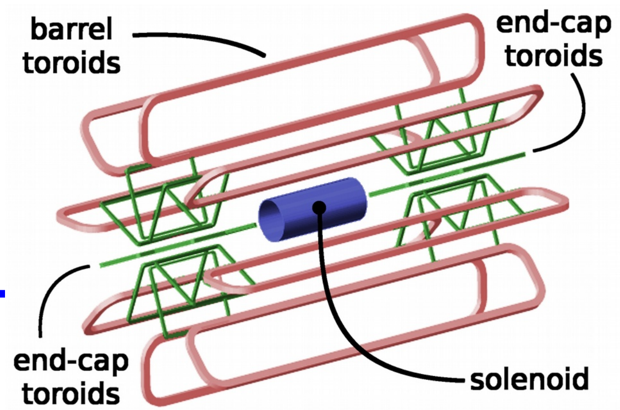
2) Solenoidal fields
   (for collider experiments)





http://newsline.linearcollider.org/2011/05/05/one-hundred-years-of-superconductivity/cms-solenoid-magnet/

3) Toroidal fields
   (for muon system of
   collider experiment)



http://www.jetgoodson.com/images/thesisImages/magnetSystems.png

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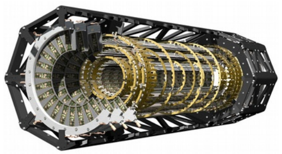
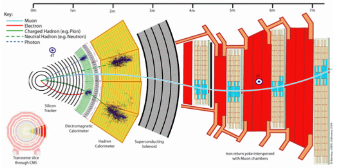


Dipole Magnets in HEP



The field strength and the radius of the magnets are given by the physics requirements of the experiments. In particular the accelerator maximum energy gives a first guess on BR^2

Experiment	Laboratory	B [T]	Radius [m]	Length [m]	Energy [MJ]	X/X_0	E/M [kJ/kg]	
TOPAZ*	KEK	1.2	1.45	5.4	20	0.70	4.3	50-64 GeV (e+/e-)
CDF*	Tsukuba/Fermi	1.5	1.5	5.07	30	0.84	5.4	1.96 TeV ($p/\bar{p}$)
VENUS*	KEK	0.75	1.75	5.64	12	0.52	2.8	50-64 GeV (e+/e-)
AMY*	KEK	3	1.29	3	40	†		
CLEO-II*	Cornell	1.5	1.55	3.8	25	2.5	3.7	3.5-12 GeV (e+/e-)
ALEPH*	Saclay/CERN	1.5	2.75	7.0	130	2.0	5.5	90-216 GeV (e+/e-)
DELPHI*	RAL/CERN	1.2	2.8	7.4	109	1.7	4.2	
ZEUS*	INFN/DESY	1.8	1.5	2.85	11	0.9	5.5	920/27.5 GeV ($p/\bar{e}$)
H1*	RAL/DESY	1.2	2.8	5.75	120	1.8	4.8	
BaBar*	INFN/SLAC	1.5	1.5	3.46	27	†	3.6	9/3.1 GeV (e+/e-)
D0*	Fermi	2.0	0.6	2.73	5.6	0.9	3.7	1.96 TeV ($p/\bar{p}$)
BELLE*	KEK	1.5	1.8	4	42	†	5.3	7/4 GeV (e+/e-)
BES-III	IHEP	1.0	1.475	3.5	9.5	†	2.6	4.6 GeV (e+/e-)
ATLAS-CS	ATLAS/CERN	2.0	1.25	5.3	38	0.66	7.0	13 TeV (p/p)
ATLAS-BT	ATLAS/CERN	1	4.7–9.75	26	1080	(Toroid)†		
ATLAS-ET	ATLAS/CERN	1	0.825–5.35	5	2×250	(Toroid)†		
CMS	CMS/CERN	4	6	12.5	2600	†	12	
SiD**	ILC	5	2.9	5.6	1560	†	12	90-500 GeV (e+/e-)
ILD**	ILC	4	3.8	7.5	2300	†	13	
SiD**	CLIC	5	2.8	6.2	2300	†	14	1-3 TeV (e+/e-)
ILD**	CLIC	4	3.8	7.9	2300	†		
FCC**		6	6	23	54000	†	12	100 TeV (p/p)

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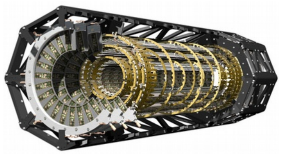
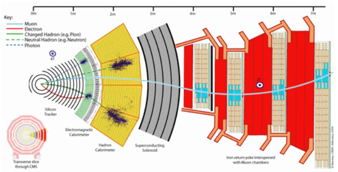


Track Parameters in B-fields (I)



In magnetic fields, the particle path can be derived starting from the Lorentz force:

$$\vec{F}_{Lorentz} = m\dot{\vec{v}} = q\left(\vec{v} \times \vec{B}\right)$$

Assuming a homogenous B field, which can be directed along the z axis allows to solve for *v*:

$$\vec{v}(t) = \begin{pmatrix} v_T \cos\left(\eta\omega_B t + \psi_0\right) \\ -v_T \sin\left(\eta\omega_B t + \psi_0\right) \\ v_z \end{pmatrix}$$

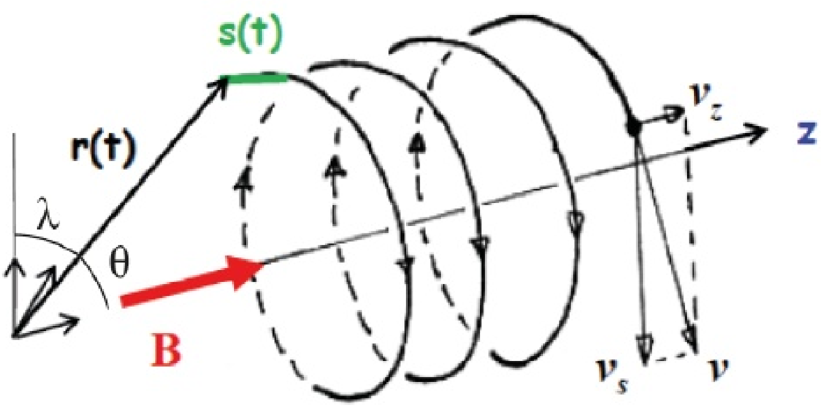
with $\omega_B = |q|B/\gamma m$, $\eta = q/|q|$, and $v_T = \sqrt{v_x^2 + v_y^2}$

Integration gives

$$\vec{x}(t) = \begin{pmatrix} x_0 + \frac{v_T}{\eta\omega_B} \sin\left(\eta\omega_B t + \psi_0\right) \\ y_0 + \frac{v_T}{\eta\omega_B} \cos\left(\eta\omega_B t + \psi_0\right) \\ z_0 + v_z t \end{pmatrix}$$

=> Helix trajectory

with 6 parameters x_0, y_0, v_T, η, ω_B, ψ_0



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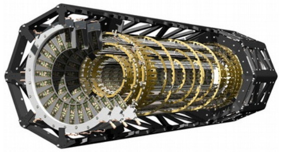
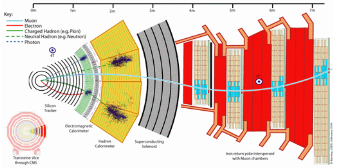


Track Parameters in B-fields (II)

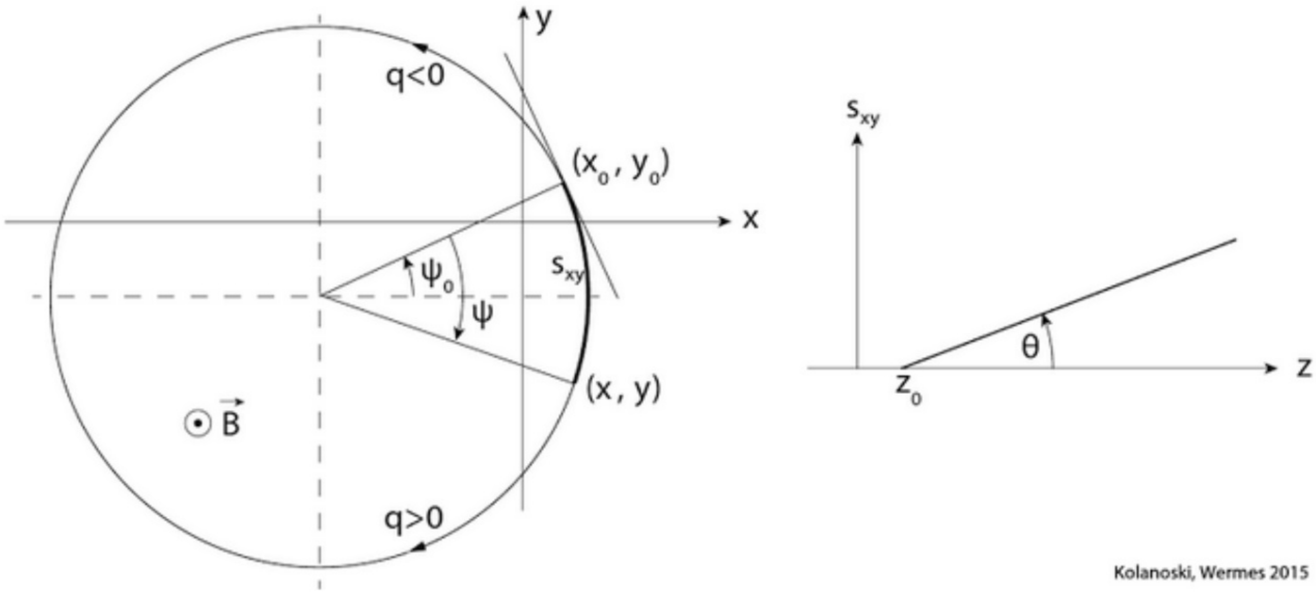


Eliminating t, introducing the radius $R=v_T/\omega_B$ an rearranging gives

$$\vec{x}(t) = \begin{pmatrix} x_0 + R\left(\cos\left(\psi_0 - \eta\psi\right) - \cos\psi_0\right) \\ y_0 + R\left(\sin\left(\psi_0 - \eta\psi\right) - \sin\psi_0\right) \\ z_0 + \frac{R\psi}{\tan\theta} \end{pmatrix}$$

with parameters x_0, y_0, z_0, R, ψ_0, θ and η.

Often also $\kappa=\eta/R$ and $d_0 = \sqrt{x_0^2+y_0^2}$ is used, because the start point of the helix can not be measured (only 5 parameters).



1) Curvature κ
2) Angle between x-axis and the vector origin-PCA
3) Shortest distance of helix to origin in r-ϕ: d_0
4) Angle θ of track to z-axis at PCA
5) Intersection of track with z-axis: z_0

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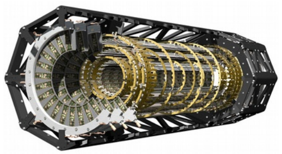
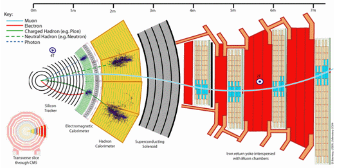
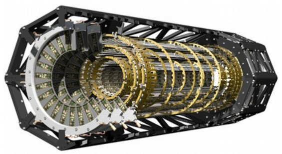
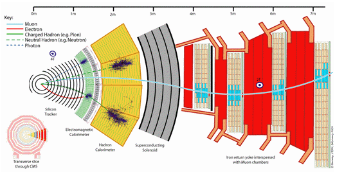




Multiple Scattering

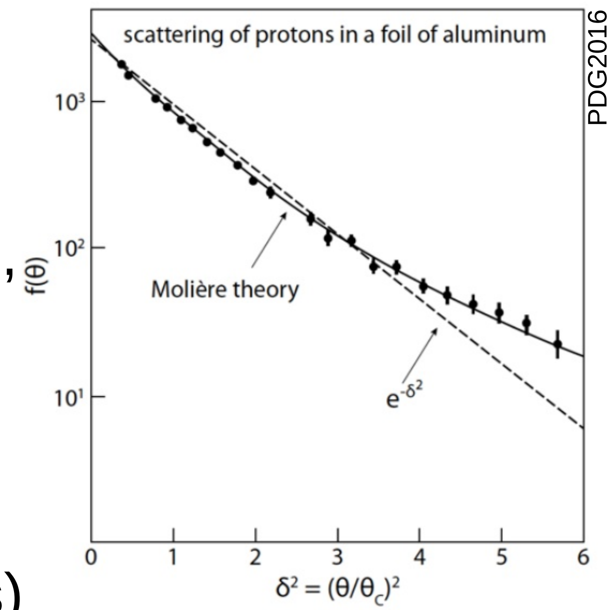


Multiple Scattering(I)



Particles are deviated in the electrical field of the nuclei.
This Coulomb scattering gives small, random deviations of the original path.
It is given by the Rutherford cross section

$$\left.\frac{d\sigma}{d\Omega}\right|_{Rutherford} = z^2 Z^2 \alpha^2 \hbar^2 \frac{1}{\beta^2 p^2} \frac{1}{4\sin^4\frac{\theta}{2}}$$



In a sufficiently thick material the resulting scattering angle θ is Gaussian distributed (central limit theorem), but there are tails because of stronger scatterings

→ Moliere-theory

Assuming only the Gaussian part, one is usually not interested in the spatial distribution, but only in the projection in one plane (e.g. r-ϕ for solenoidal B-fields)

Usually not the 3D angle θ_{space} is interesting, but only the projection in one plane:

$$\theta_0 = \theta^{rms}_{plane} = \frac{1}{\sqrt{2}}\theta^{rms}_{space}$$

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Multiple Scattering (II)

The Gaussian distribution of θ_{plane} can be written as

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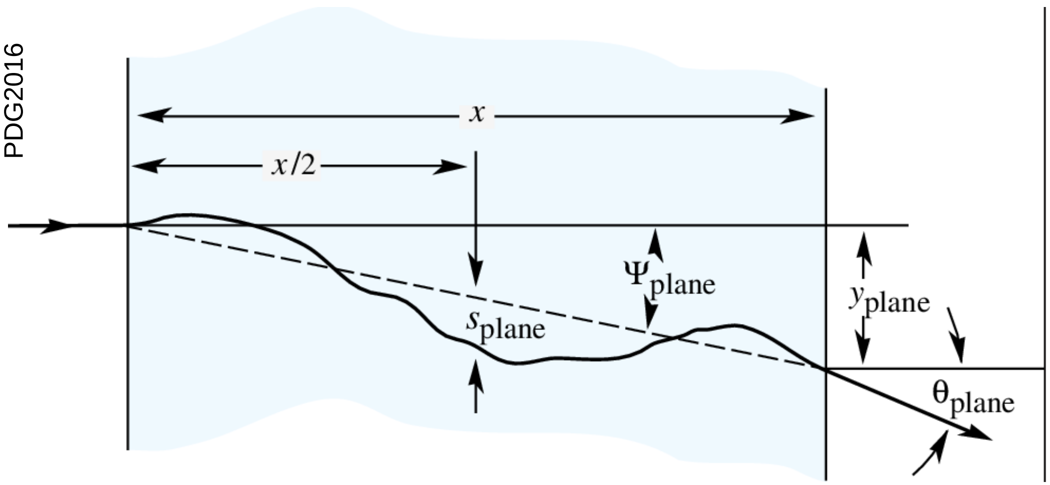
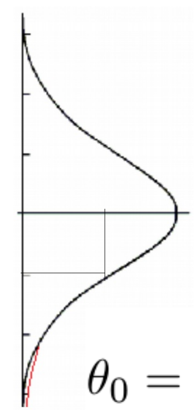


$$f(\theta_{plane})d\theta_{plane} = \frac{1}{\sqrt{2\pi}\theta_0} e^{-\frac{\theta^2_{plane}}{2\theta_0^2}} d\theta_{plane}$$



where θ_0 is on standard deviation and can be approximated by

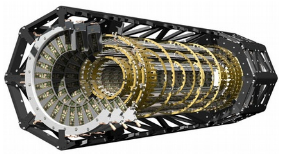
$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{p\beta c} z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln \frac{x}{X_0}\right)$$

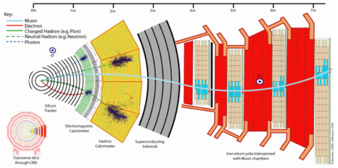
Depending on the problem under study, the following parameters might be of interest:

$$\psi^{\mathrm{rms}}_{\mathrm{plane}} = \frac{1}{\sqrt{3}} \theta^{\mathrm{rms}}_{\mathrm{plane}} = \frac{1}{\sqrt{3}} \theta_0 \ ,$$

$$y^{\mathrm{rms}}_{\mathrm{plane}} = \frac{1}{\sqrt{3}} x\, \theta^{\mathrm{rms}}_{\mathrm{plane}} = \frac{1}{\sqrt{3}} x\, \theta_0 \ ,$$

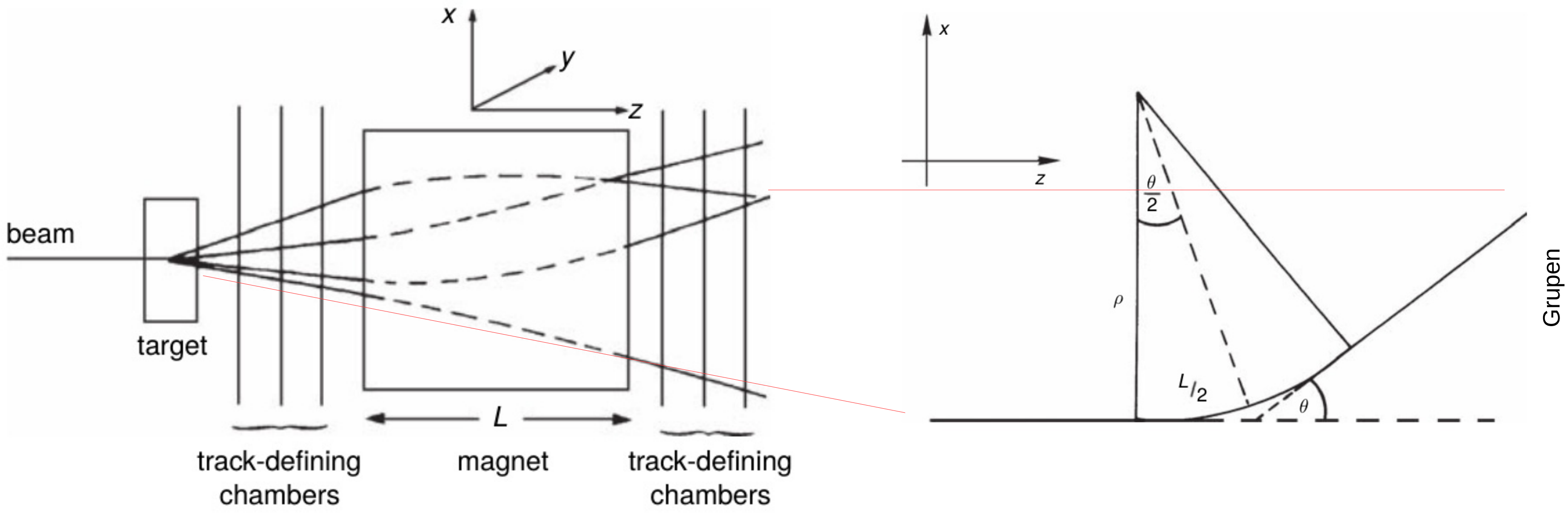
$$s^{\mathrm{rms}}_{\mathrm{plane}} = \frac{1}{4\sqrt{3}} x\, \theta^{\mathrm{rms}}_{\mathrm{plane}} = \frac{1}{4\sqrt{3}} x\, \theta_0$$





Resolution Limitation

Simple Case of Fixed Target (I)



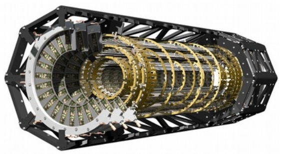
In the most simple case, there are two tracking detectors before the magnet and two after the magnet placed at the same distance from each other.

If B is well know, only θ is of interest.

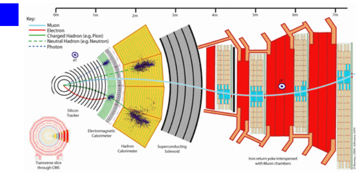
$$\theta = \frac{L}{R} = \frac{L}{p} eB$$

The additional transverse momentum gives

$$\Delta p_x = p \cdot \sin\theta \approx p\theta = LeB \qquad \rightarrow p = \frac{LeB}{\theta} \qquad \rightarrow dp = \frac{LeB}{\theta^2} d\theta = \frac{p}{\theta} d\theta$$

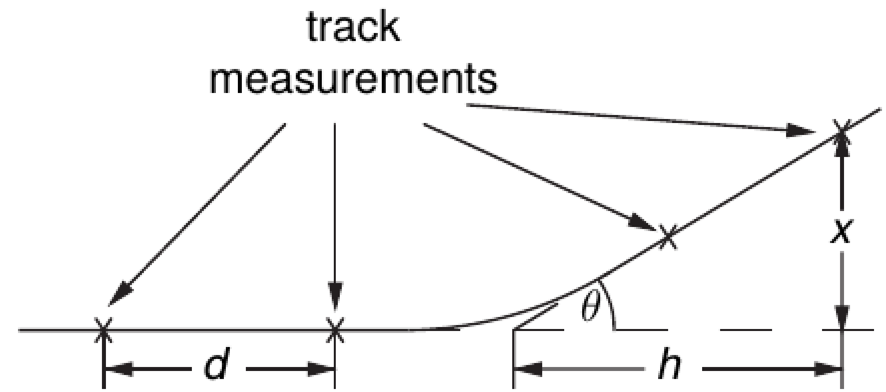


Simple Case of Fixed Target (II)



$$\frac{\sigma_p}{p} = \frac{\sigma_\theta}{\theta}$$

Assuming that all the 4 detectors have the same spatial resolution *σ(x)* the error on *θ* is given by



Grupen

$$\sigma^2(\theta) \sim \sum_{i=1}^{4} \sigma_i^2(x) = 4\sigma^2(x) \quad \rightarrow \quad \sigma(\theta) \sim 2\sigma^2(x)$$

Giving a final expression for the relative error caused by detector uncertainties.

$$\left.\frac{\sigma(p)}{p}\right|^{det} = \frac{2\sigma(x)/h}{eBL/p} = \frac{2\sigma(x)}{h} \cdot \frac{p}{\Delta p_x}$$

=> Important: Error on momentum is proportional to p^2

Multiple scattering gives an additional contribution to trans. momentum Δp_x.

$$\Delta p_x^{MS} = p \cdot \sin\theta_{rms} \approx p\theta_{rms} = \frac{19.2 \text{ MeV}}{\sqrt{2}c}\sqrt{\frac{L}{X_0}}$$

The relative error of MS to the transv. momentum given by the magnet is

$$\left.\frac{\sigma(p)}{p}\right|^{MS} = \frac{\Delta p_x^{MS}}{\Delta p_x} = \frac{19.2 \text{ MeV}}{\sqrt{2}LeBc}\sqrt{\frac{L}{X_0}}$$

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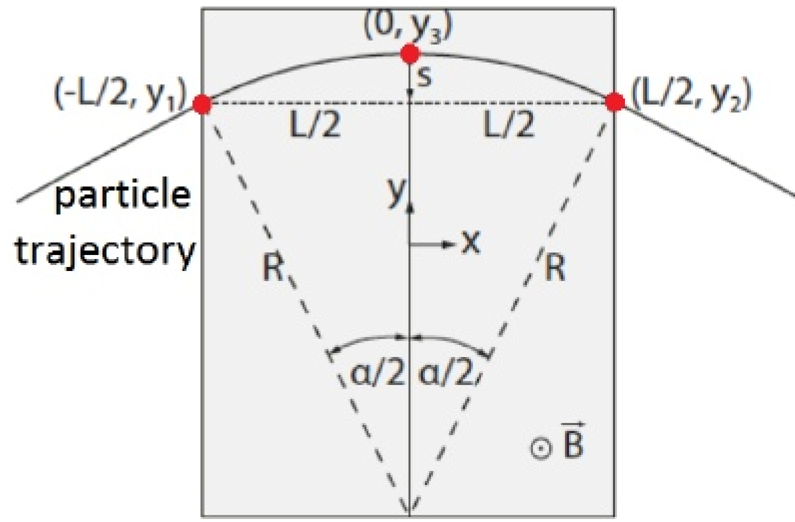
Simple Case of Fixed Target (III)

Alternative approach: The sagitta method
→ Detectors in the magnetic field



Kolanoski, Wermes 2015

$$\frac{R-s}{R} = 1 - \frac{s}{R} = \cos\frac{\alpha}{2} \approx 1 - \frac{\alpha^2}{8} \quad \rightarrow \quad \frac{s}{R} = \frac{\alpha^2}{8}$$

$$\frac{L}{2R} = \sin\frac{\alpha}{2} \approx \frac{\alpha}{2} \quad \rightarrow \quad \alpha = \frac{L}{R}$$

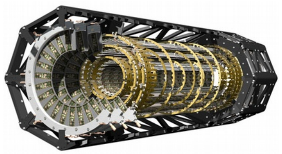
Putting it together gives

$$s = \frac{R\alpha^2}{8} = \frac{RL^2}{8R^2} = \frac{1}{8}\frac{L^2}{R} = \frac{1}{8}L^2\kappa$$

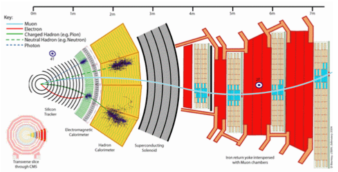
For three detectors: $s = y_3 - \frac{y_1+y_2}{2} \quad \rightarrow \quad \sigma(s) = \sqrt{\sigma_3(x)^2 + \frac{\sigma_1(x)^2+\sigma_2(x)^2}{4}} = \sqrt{\frac{3}{2}}\sigma(x)$

Error on the curvature $\sigma(\kappa) = \frac{8}{L^2}\sigma(s) = \frac{8}{L^2}\sqrt{\frac{3}{2}}\sigma(x) = \frac{\sqrt{96}}{L^2}\sigma(x)$

More than 3 detectors $\sigma(\kappa) = \frac{\sigma(x)}{L^2}\sqrt{\frac{720(N-1)^3}{(N-2)N(N+1)(N+2)}} \overset{N\gtrsim 10}{\approx} \frac{\sigma(x)}{L^2}\sqrt{\frac{720}{N+4}}$



Simple Case of Fixed Target (IV)



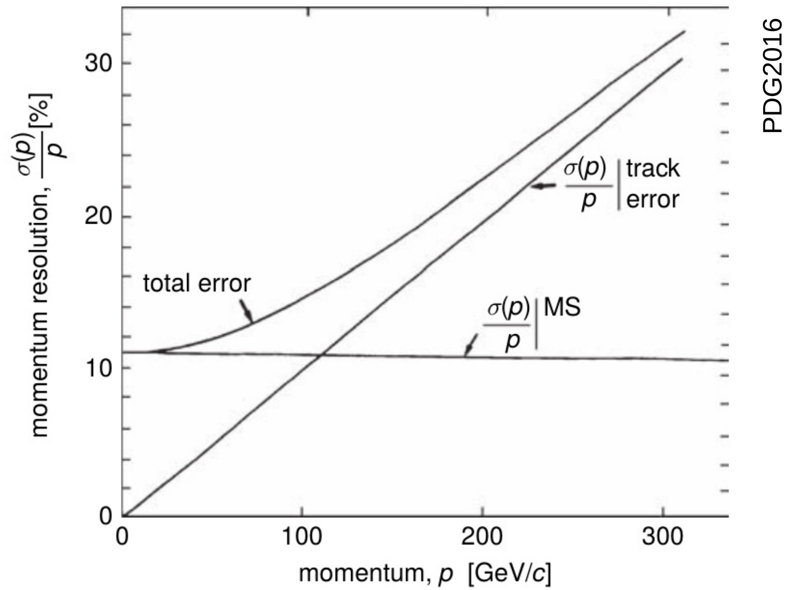
The error on the momentum is $\sigma(p_x) = \frac{p_x^2}{|q|B}\sigma(\kappa) \approx \frac{p_x^2}{0.3|z|B}\sigma(\kappa)$

and the momentum resolution because of limited detector resolution is given by the Gluckstern equation:

$$\left.\frac{\sigma(p_x)}{p_x}\right|^{det} = \frac{p_x}{0.3|z|}\frac{\sigma(x)}{BL^2}\sqrt{\frac{720}{N+4}}$$

In the total momentum resolution is given by the quadratic sum of the two components:

$$\left.\frac{\sigma(p_x)}{p_x}\right|^{tot} = \sqrt{\left(\left.\frac{\sigma(p_x)}{p_x}\right|^{det}\right)^2 + \left(\left.\frac{\sigma(p_x)}{p_x}\right|^{MS}\right)^2}$$



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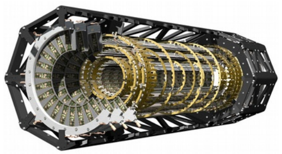
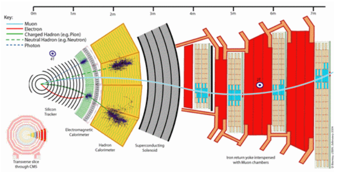
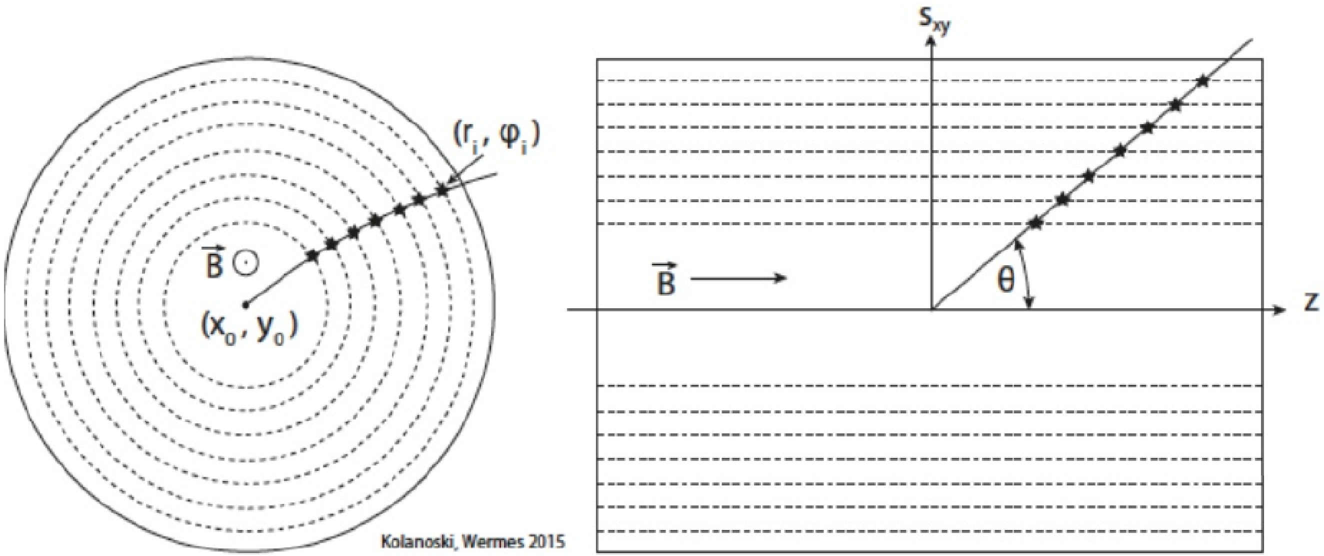


Solenoidal Setup



The track model has also 5 parameters d_0, z_0, κ, ψ_0 *and* θ.

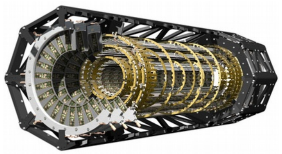
The calculation for *dp/p* is completely analog to the sagitta method



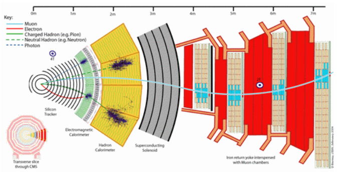
$$\left.\frac{\sigma(p_T)}{p_T}\right|^{det} = \frac{p_T}{0.3|z|}\frac{\sigma(x)}{BR^2}\sqrt{\frac{720}{N+4}}$$

What do we learn from the equation for the design of a HEP experiment?

1) Momentum resolution improves quadratically with radius
2) Momentum resolution improves linearly with the B-field
3) Detectors with better spatial resolution helps
4) Number of track points also improves the result
5) σ(p)/p increases with p → more difficult to measure high energetic tracks

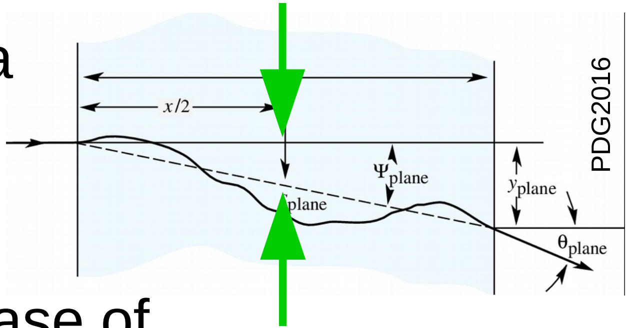


Multiple Scattering in the Sagitta Method



The scattering adds another component to the sagitta

$$\langle s_x \rangle = \frac{1}{4\sqrt{3}} L\theta_0 \approx \sigma(s)$$



This contributes to the error of the curvature. In the case of three equidistant detectors this is:

$$\sigma_\kappa = \frac{8}{L^2}\sigma(s) = \frac{8}{L^2}\frac{1}{4\sqrt{3}}L\theta_0 = \sqrt{\frac{4}{3}}\frac{\theta_0}{L} = \frac{0.0136\ \mathrm{GeV}}{p\beta c L} z \sqrt{\frac{L}{X_0 \sin\theta_0}}\sqrt{1.33}$$

For more detectors the error of κ changes only slightly

$$\sigma_\kappa = \frac{\theta_0}{L}\sqrt{1.43} = \frac{0.0136\ \mathrm{GeV}}{p\beta c L} z \sqrt{\frac{L}{X_0 \sin\theta_0}}\sqrt{1.43}$$

Giving a final result of

$$\left.\left(\frac{\sigma_{p_T}}{p_T}\right)\right|^{MS} = \frac{0.054}{LB\beta}\sqrt{\frac{L}{X_0 \sin\theta_0}}$$

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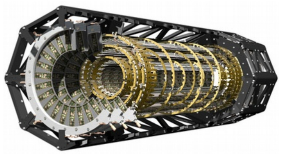
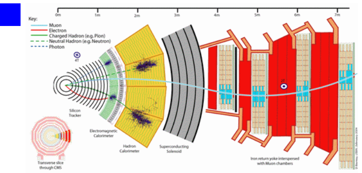


Example ATLAS Tracking Naive Approximation



<u>Pixel detector:</u> N = 3 layers,

Spatial resolution σ_x = 12 µm

Radius R = 5 cm – 12 cm

<u>SemiConductor Tracker (SCT):</u>

N = 4 layers,

Spatial resolution σ_x = 16 µm

Radius R = 30 cm – 51 cm

<u>Transition Radiation Tracker (TRT):</u>

N = 36 layers,

Spatial resolution σ_x = 170 µm

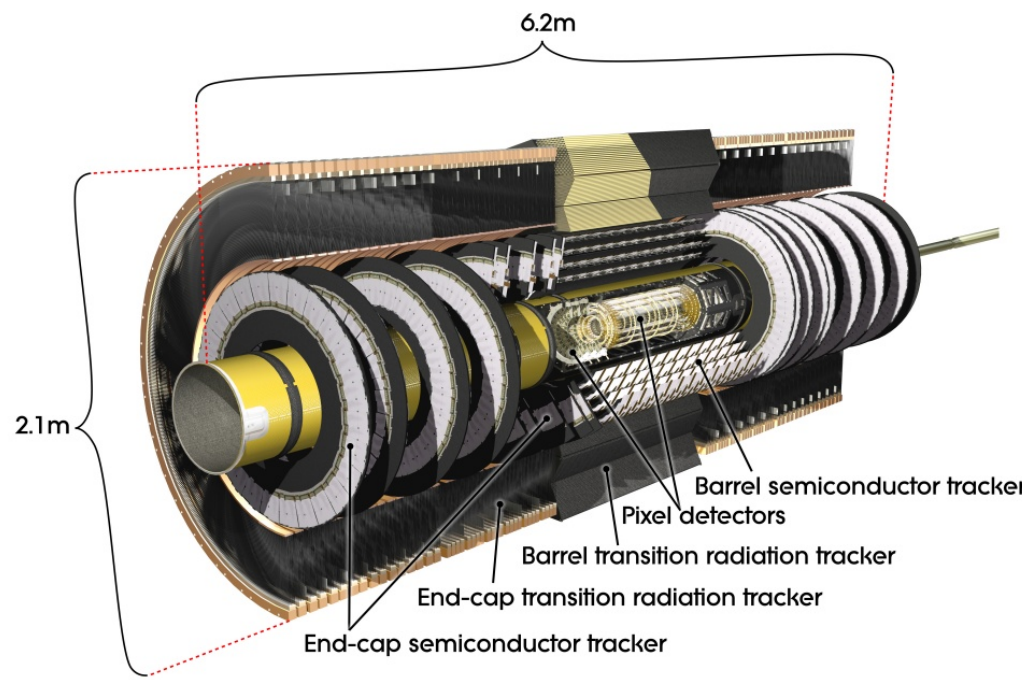
Radius R = 55 cm – 108 cm

=> Averaging for better σ_x

N = 1, σ_x = 28 µm, R = 82 cm

<u>Total Tracking System:</u> N = 8 layers,

σ_x = 17 µm, R = 5 cm – 80 cm → L = 75 cm



https://cds.cern.ch/images/CERN-GE-0803014-01

The momentum resolution because of detector resolution is:

$$\left.\frac{\sigma_{p_T}}{p_T}\right|^{det} = \frac{p_T\sigma_x}{0.3|z|BL^2}\sqrt{\frac{720}{N+4}} =$$

$$= \frac{p_T\cdot 17\cdot 10^{-6}\ \mathrm{m}}{0.3\cdot 1\cdot 2\ \mathrm{T}\cdot(0.75\ \mathrm{m})^2}\sqrt{\frac{720}{8+4}} = 0.0004\cdot p_T$$

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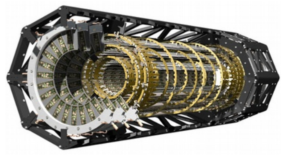
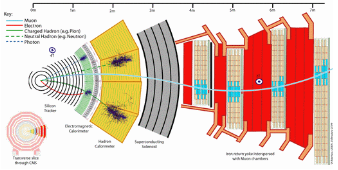


Example ATLAS Tracking Published Performance

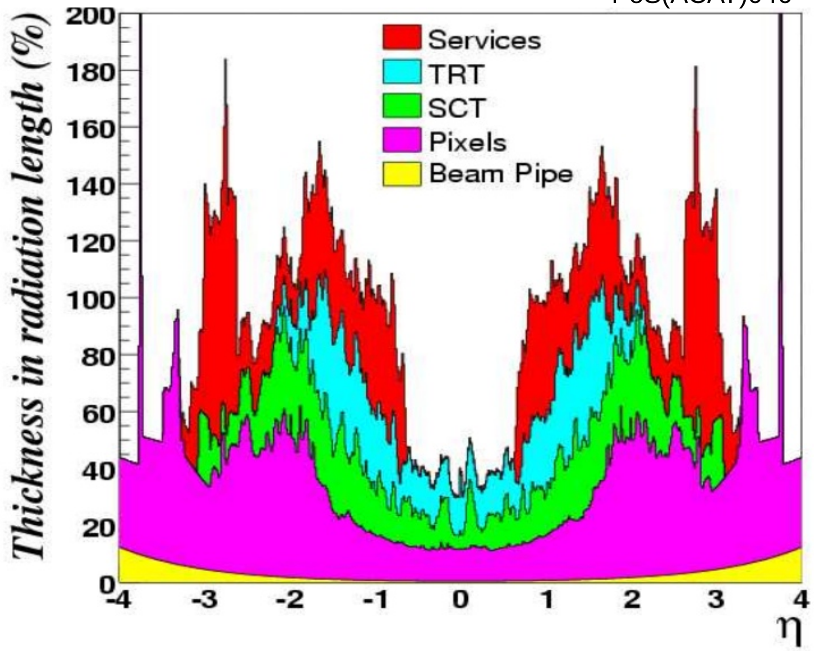


The estimate of $\left.\frac{\sigma_{p_T}}{p_T}\right|^{det} = 0.0004 \cdot p_T$ fits well with the official detector

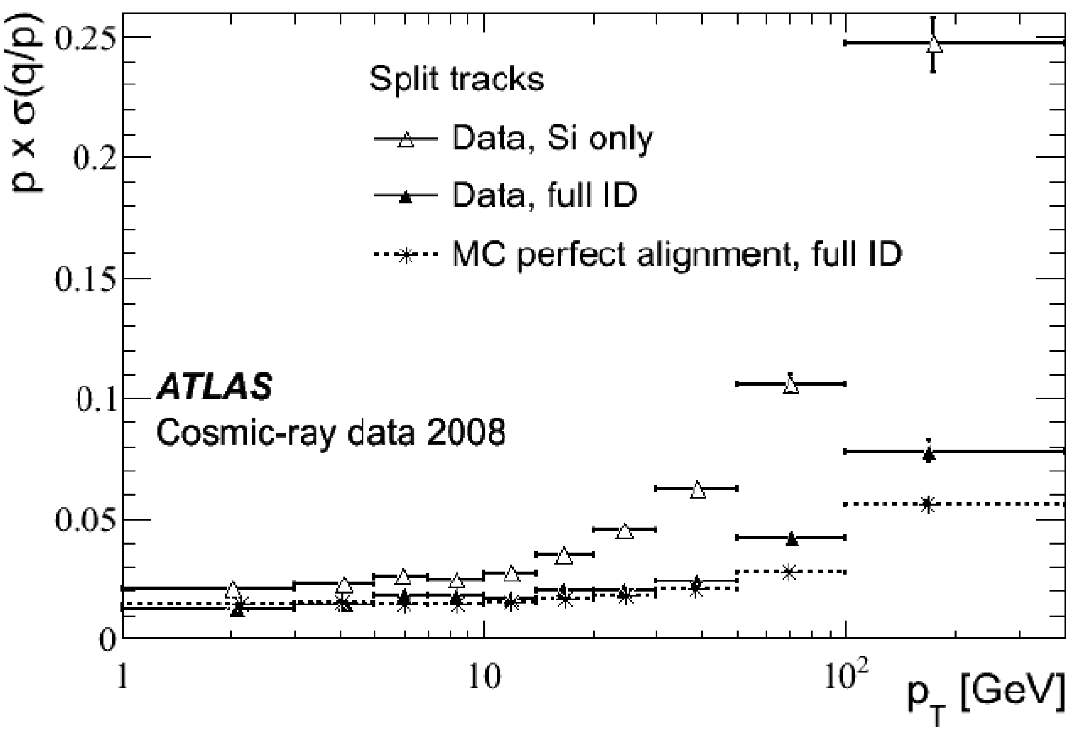
requirement of σ_{pT} /p_T = 0.05% p_T (GeV/c) + 1%

The additional 1% is given because of the material budget of the inner detectors:

PoS(ACAT)046



arXiv:1004.5293v2



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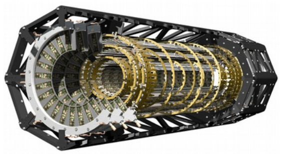
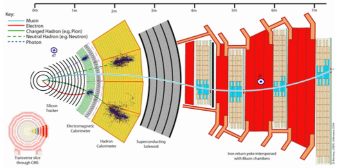
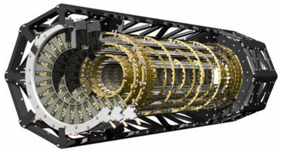
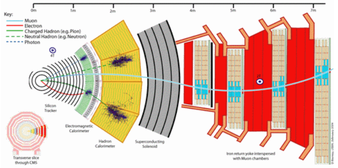




Vertex Reconstruction



Vertex Reconstruction



Particles produced in the collision decay fast, but they can fly a certain distance *l* before decaying:

$$l = \beta\gamma c\ \tau$$

Particle	m (GeV/c^2)	τ (10^{-12}s)	$l(p_T = 10\,\mathrm{GeV})$
τ^+	1.776	0.290	500 µm
D^0	1.865	0.410	700 µm
D^+	1.869	1.040	1700 µm
Λ_c^+	2.286	0.200	300 µm
B_S^0	5.367	1.512	800 µm
B^+	5.279	1.641	900 µm
Λ_b	5.619	1.425	800 µm

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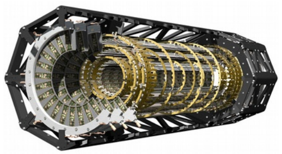
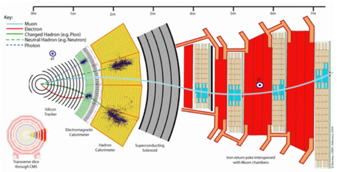
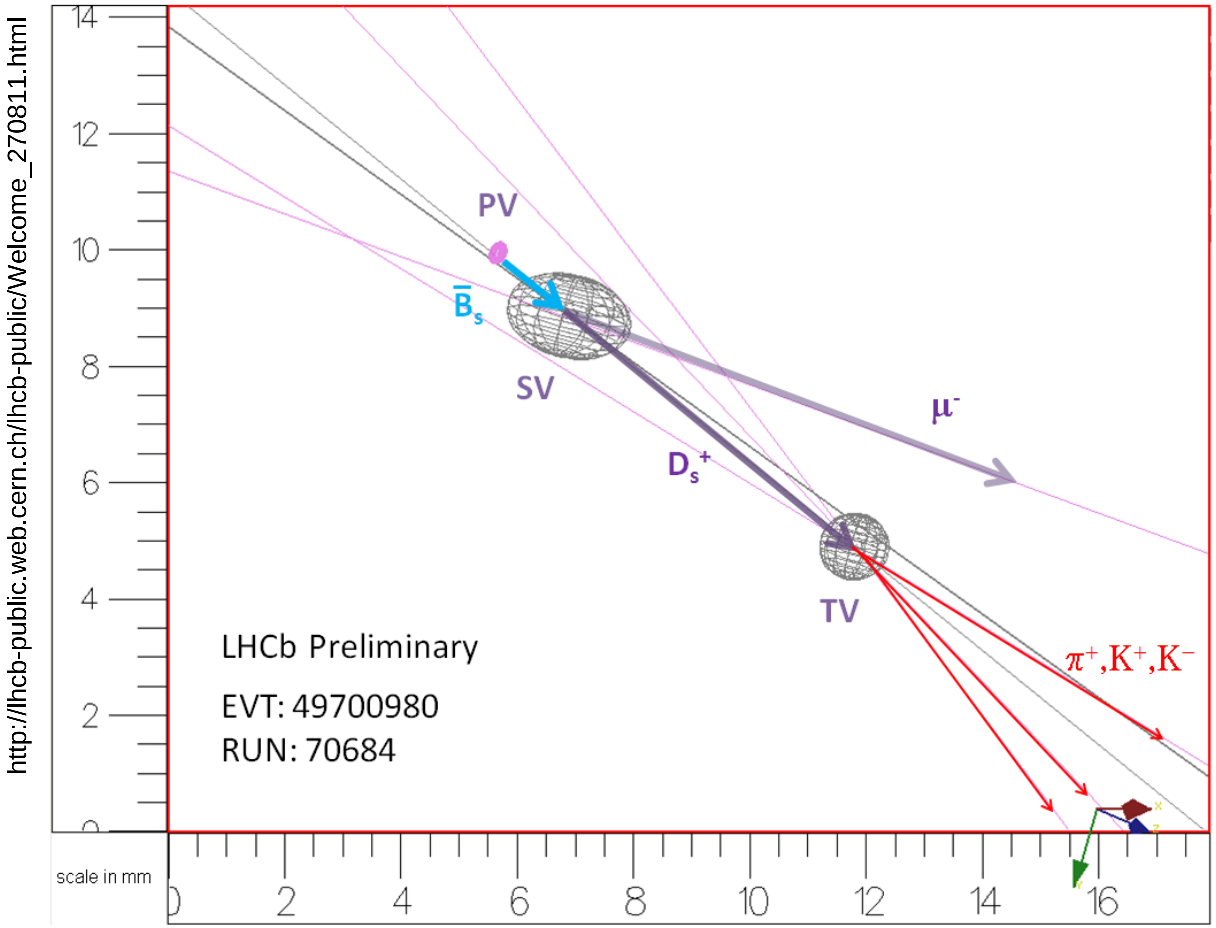


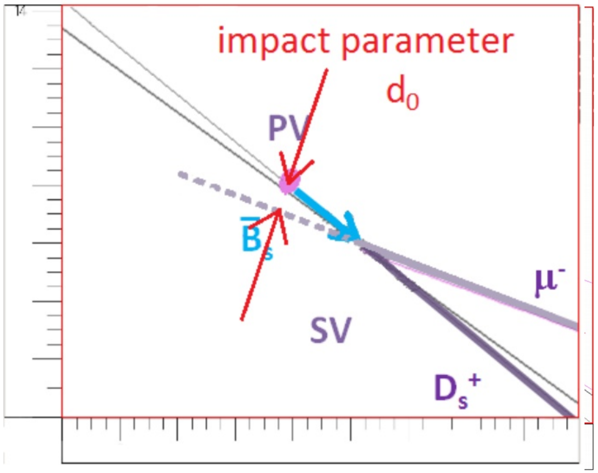
Example of Decay Chain



Identifying the primary vertex (PV), secondary vertex (SV) and tertiary vertex (TV) is important to understand the decay chain of the particle and to reconstruct its mass and life time completely.



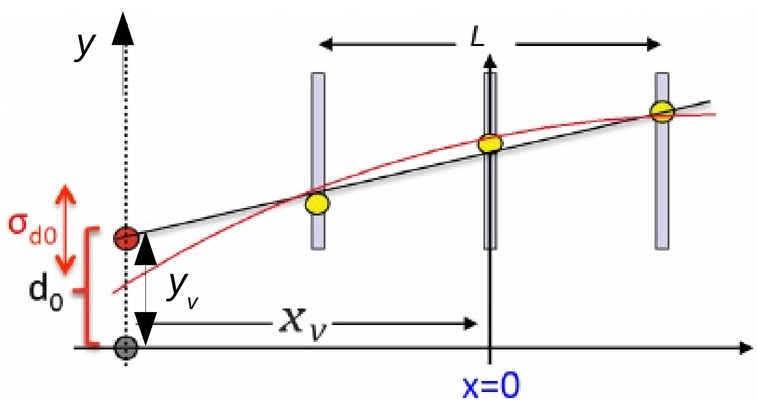
Impact Parameter



Impact parameter d_0 quantifies the mismatch to the primary vertex (PV)
The impact parameter significance $S_{d0} = d_0/\sigma_{d0}$ serves as selection criterion to distinguish tracks from primary or secondary vertices.

The track as measured by the tracking detectors has to be extrapolated to the vertices. This can be done with

- Linear extrapolation $y = a + bx$ (no magnetic field)
- Quadratic extrapolation $y = a + bx + cx^2$ (good approximation for helix in B)



Errors on the vertex position y_v is given for linear approx. by $\sigma_{d_0} = \sigma_y = \sqrt{\sigma_a^2 + x_v^2\sigma_c^2}$

or for quadratic approx. by

$$\sigma_{d_0} = \sigma_y = \sqrt{\sigma_a^2 + x_v^2\sigma_c^2 + \frac{1}{4}x_V^4\sigma_c^2 + x_v^2\sigma_{ac}}$$

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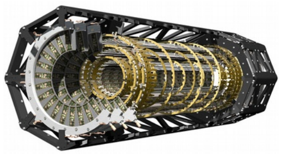
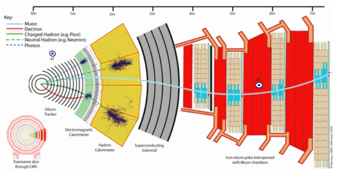
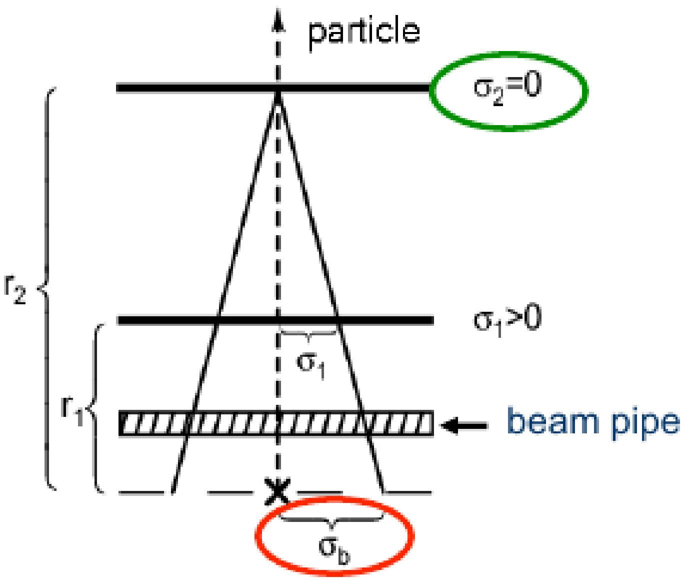
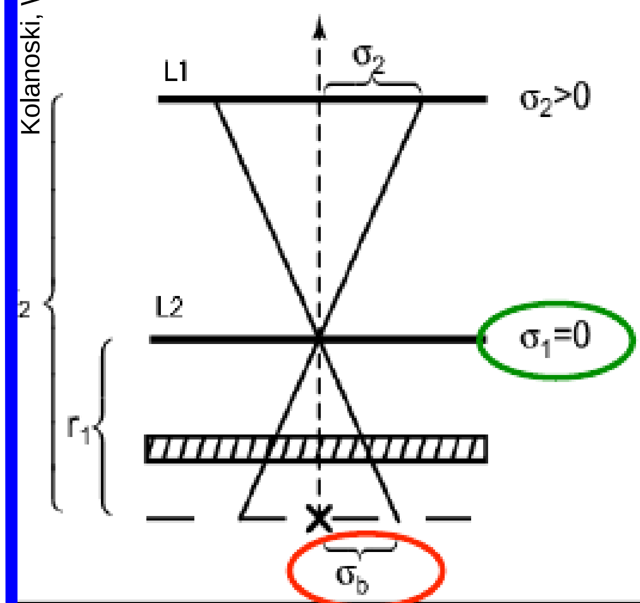


Simple Picture







Kolanoski, Wermes 2015

The theorem of intersection lines gives for the special cases for the impact parameter errors:

1.) $\sigma_1 > 0$, $\sigma_2 = 0$: $\sigma_{d_0} = \sigma_b = \dfrac{r_2}{r_2 - r_1}\sigma_1$

2.) $\sigma_1 > 0$, $\sigma_2 = 0$: $\sigma_{d_0} = \sigma_b = \dfrac{r_1}{r_2 - r_1}\sigma_2$

The complete error is then given by:

$$\sigma_{d_0} = \sqrt{\left(\frac{r_2}{r_2 - r_1}\sigma_1\right)^2 + \left(\frac{r_1}{r_2 - r_1}\sigma_2\right)^2 + \left(\sigma_{d_0}^{MS}\right)^2}$$

with $\sigma_{d_0}^{MS} = \theta_0 r_1$

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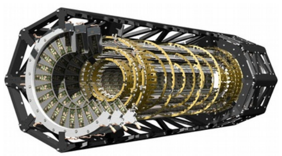
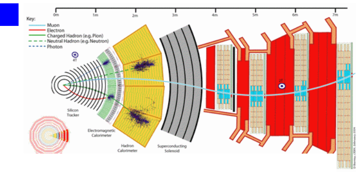


Discussion



For 2 layers with $\sigma_1 \neq \sigma_2$ at r_1 and r_2 $\sigma_{d_0} = \sqrt{\left(\frac{r_2}{r_2 - r_1}\sigma_1\right)^2 + \left(\frac{r_1}{r_2 - r_1}\sigma_2\right)^2 + \left(\sigma_{d_0}^{MS}\right)^2}$

For more than 2 layers with equal spatial resolutions $\sigma_1 = \sigma_2 = \sigma_N = \sigma_x$

$$\sigma_{d_0} = \sqrt{\sigma_a + x_V^2 \sigma_b^2} = \frac{\sigma_x}{\sqrt{N}}\sqrt{1 + \frac{12(N-1)}{N+1}\left(\frac{x_0}{L}\right)^2} = \frac{\sigma_x}{\sqrt{N}}\sqrt{1 + \frac{12(N-1)}{N+1}\left(\frac{\frac{r_N + r_1}{2}}{r_N - r_1}\right)^2}$$

From these formulas we can deduce the following rules for vertex detector optimization:

1) First layer needs best resolution

(factor $r_2/(r_2-r_1)$ is larger than for other layers)

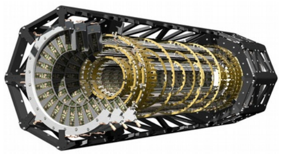
2) Large lever arm (r_2-r_1)

3) Place first layer as close as possible to vertex (r_1 small)

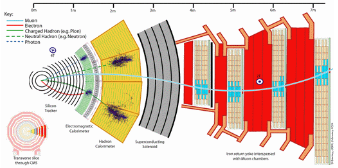
4) Larger number of layers improves result with $\sqrt{N}$

For extrapolations with curvature:

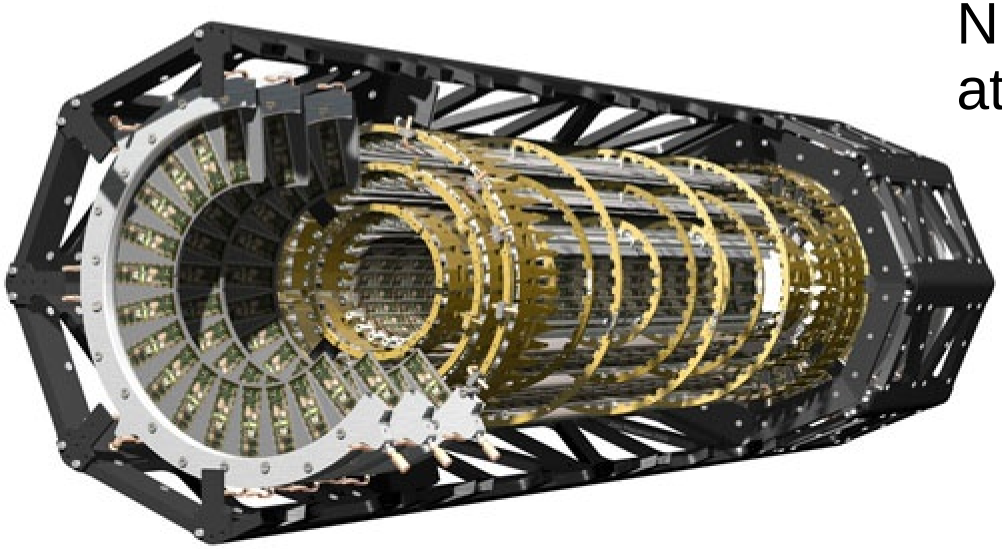
$$\sigma_{d_0} = \frac{\sigma_x}{\sqrt{N}}\sqrt{1 + \frac{12(N-1)}{N+1}\left(\frac{x_0}{L}\right)^2 + \frac{180(N-1)^3}{(N-2)(N+1)(N+2)}\left(\frac{x_0}{L}\right)^4 + \frac{30N^2}{(N-2)(N+2)}\left(\frac{x_0}{L}\right)^2}$$



ATLAS Vertex Detector



http://atlasexperiment.org/pixel-detector.html



Number of layer: N = 3

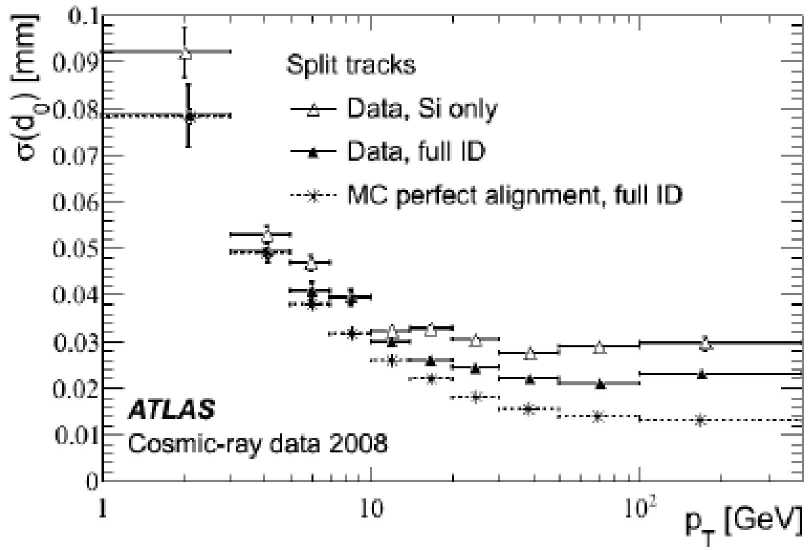
at $r_1 = 4.7$ cm, $r_2 = 9.1$ cm, $r_3 = 13.5$ cm

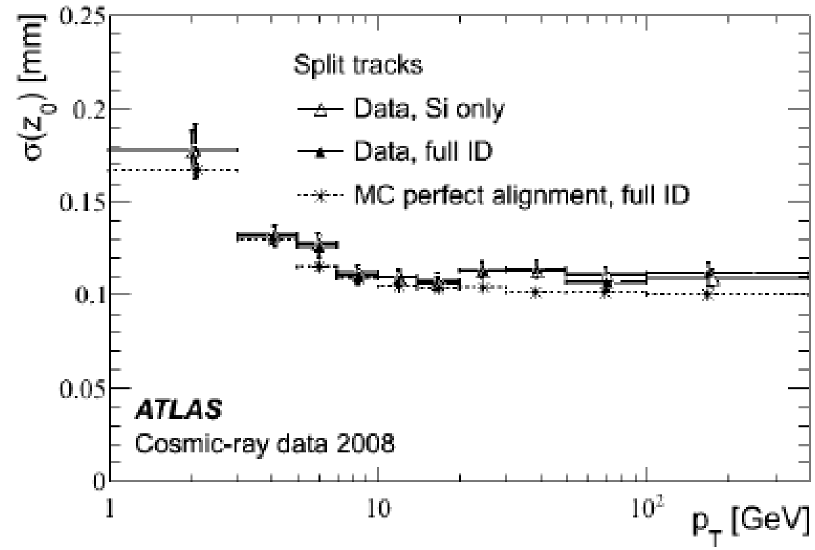
Putting in the numbers, one gets:

$\sigma_{do} = 15.7$ µm (linear extrapolation)

$\sigma_{do} = 45.5$ µm (quadr. extrapolation)

arXiv:1004.5293v2





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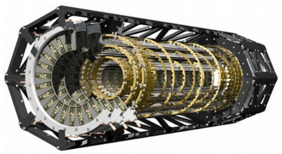

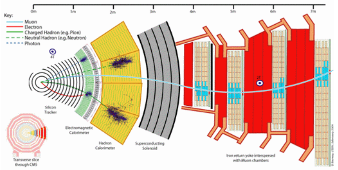

Track Reconstruction

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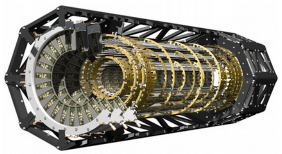
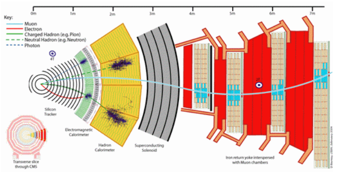


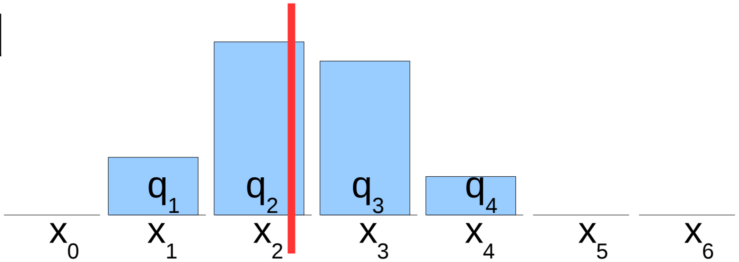
General Comments



Raw data usually have to be treated before any further steps can be done.

- Calibration results have to applied to raw data
  Electronics channels may have different amplifications/capacitors etc.
  → to compare the hits on channels, calibration is important
  → possibly also conversion from ADC units to absolute charge (in e^-)
- Determine time information and other properties from the time development of the signal on one channel.
- Reconstruct a cluster from neighboring channels.
  Often the center of gravity algorithm is used

$$x = \frac{\sum_{i_1}^{N} q_i x_i}{\sum_{i_1}^{N} q_i}$$



- Based on information from previous alignment studies, the reconstructed clusters may have to be shifted to the correct position.
- During track finding hits will be combined to form a track
- Track fitting gives the final parameters and errors of the track.

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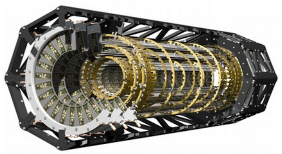
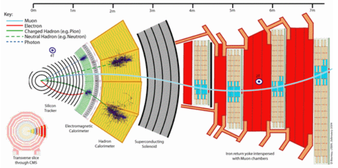


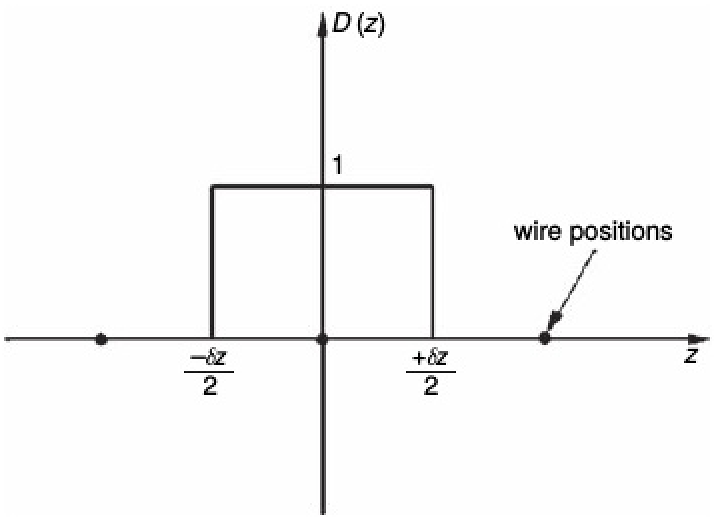
Calculation of Spatial Resolution



Spatial resolution of a uniformly distributed signal with pads of the width *d*.

We first need a normalized distribution of the particle (particle density function):

$$\int_{-\frac{d}{2}}^{\frac{d}{2}} f(x)\,dx = 1 \qquad \Rightarrow \qquad f(x) = \frac{1}{d}$$



Grupen

Now we calculate the variance of the position measurement:

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \int_{-\frac{d}{2}}^{\frac{d}{2}} x^2 f(x)\,dx - \left(\int_{-\frac{d}{2}}^{\frac{d}{2}} x f(x)\,dx \right)^2 = \frac{1}{d}\int_{-\frac{d}{2}}^{\frac{d}{2}} x^2\,dx - \left(\frac{1}{d}\int_{-\frac{d}{2}}^{\frac{d}{2}} x\,dx \right)^2 =$$

$$= \frac{1}{d}\,\frac{x^3}{3}\Big|_{-\frac{d}{2}}^{\frac{d}{2}} - \frac{1}{d}\,\frac{x^2}{2}\Big|_{-\frac{d}{2}}^{\frac{d}{2}} = \frac{1}{d}\cdot 2 \cdot \frac{1}{3}\,\frac{d^3}{8} - 0 = \frac{d^2}{12}$$

$$\sigma_x = \frac{d}{\sqrt{12}} \approx \frac{d}{3.5}$$

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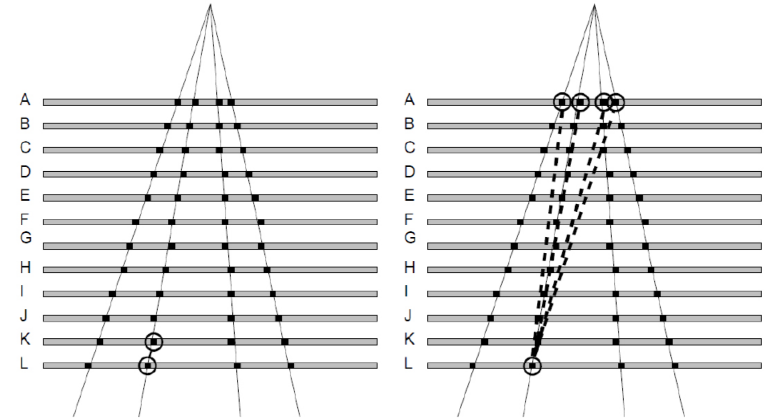
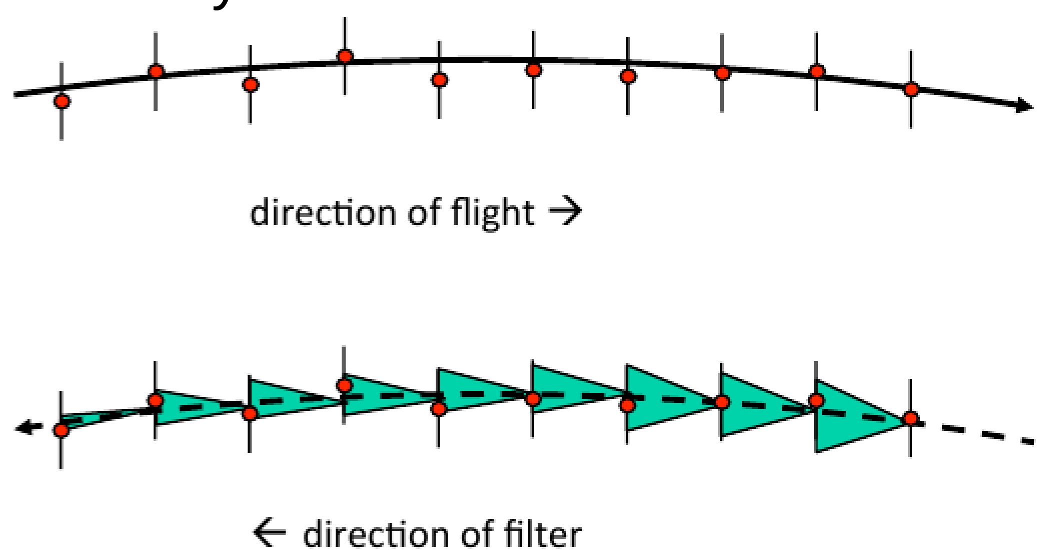
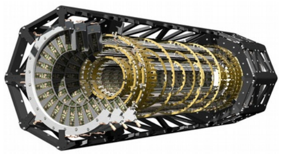
KF for Track Finding

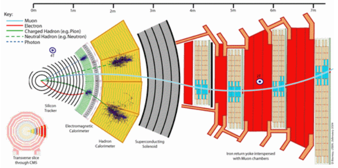


1) <u>Find a seed track:</u> This is often a combinatorial approach using the 2-3 outermost layers and an inner layers of measurement. The outside is chose, because tracks are separated and possible combinations are reduced.
2) <u>Make a track estimate:</u> Based on the currently available track points and their errors, a prediction is made for hits in the next detector layer.
3) <u>Looking for hit:</u> If hit is found, add it to track and update the track prediction for the next layer.





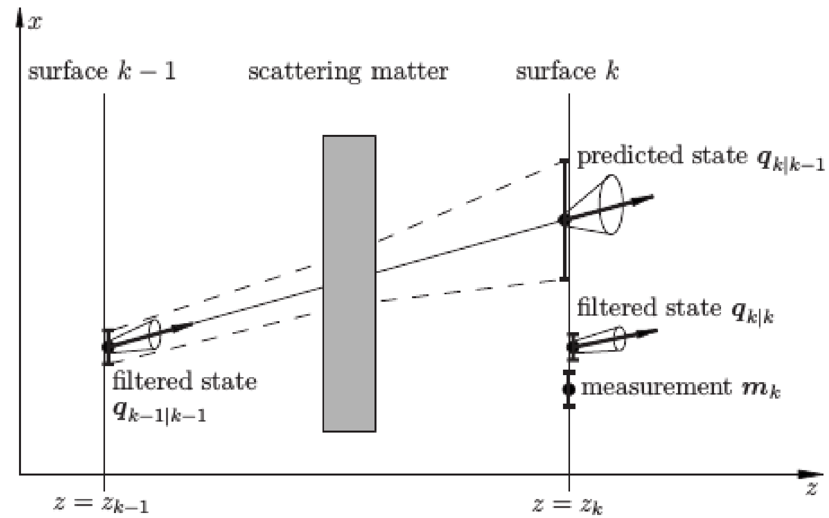
Advantages of the KF



It is easy to take detector effects into account

- Energy loss
- Multiple Scattering
- Bremsstrahlung

can be accommodated by recognizing the effect and correcting for it in the next step.



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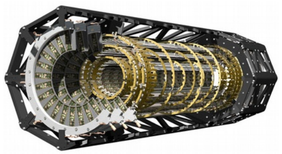
A filter can be built, that stores the intermediate results of the filter → outliers such as noise or hits from close tracks can be identified and removed afterward.

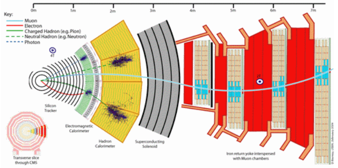
The best parameter estimate can be given at any point.

Because of the flexibility, the algorithm is widely used for tracking objects in real time:

Radar tracking, sonar ranging, satellite orbit computation, automated driving



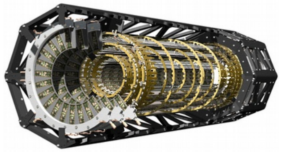
Hough Transformation



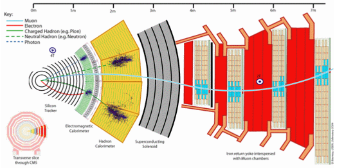
Robust global algorithm to find geometrical forms such as lines or circles in a random background. Invented by Paul Hough in 1962.

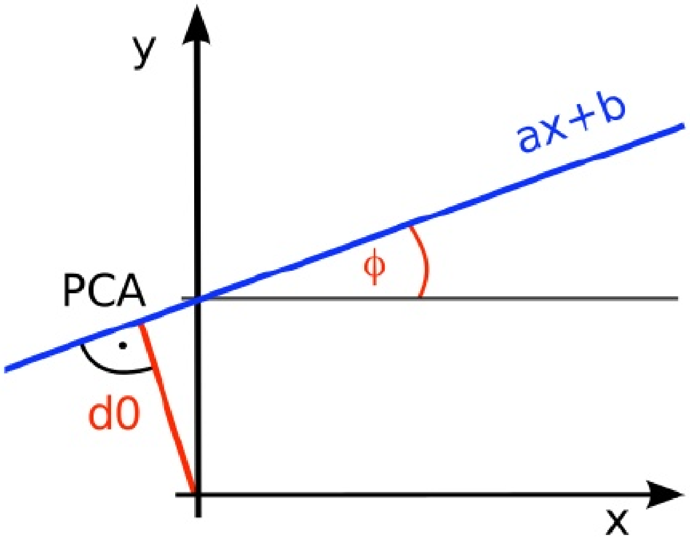
1) Space points are transformed according to the track model into the parameter space.
2) Space points correspond to lines in the parameter space.
3) Find point with most line crossings in the parameters space.
   → point gives good estimate for track parameter estimates

Often used in pattern recognitions software for picture.
e.g. in astronomy



HT in 2 Dimensions





Track model is $y = a + bx$.

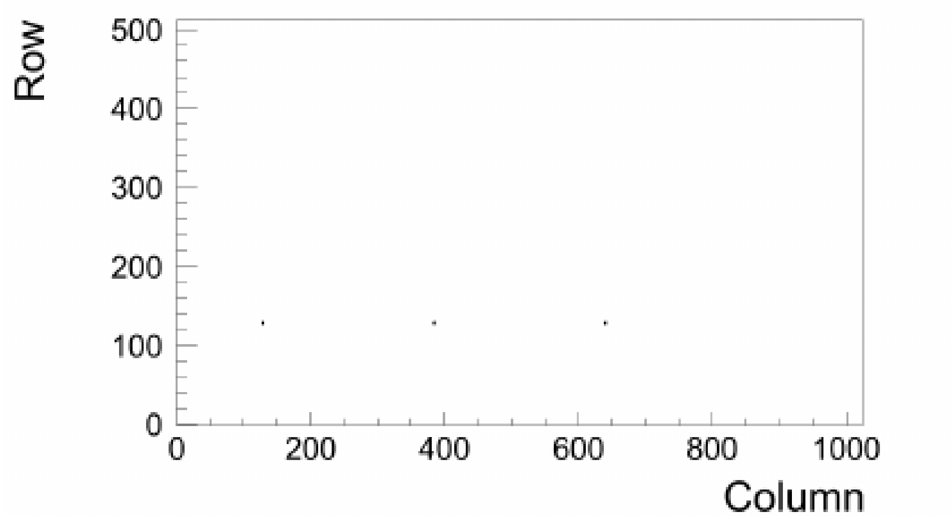
But it is better to use the parameters d_0 and ϕ.

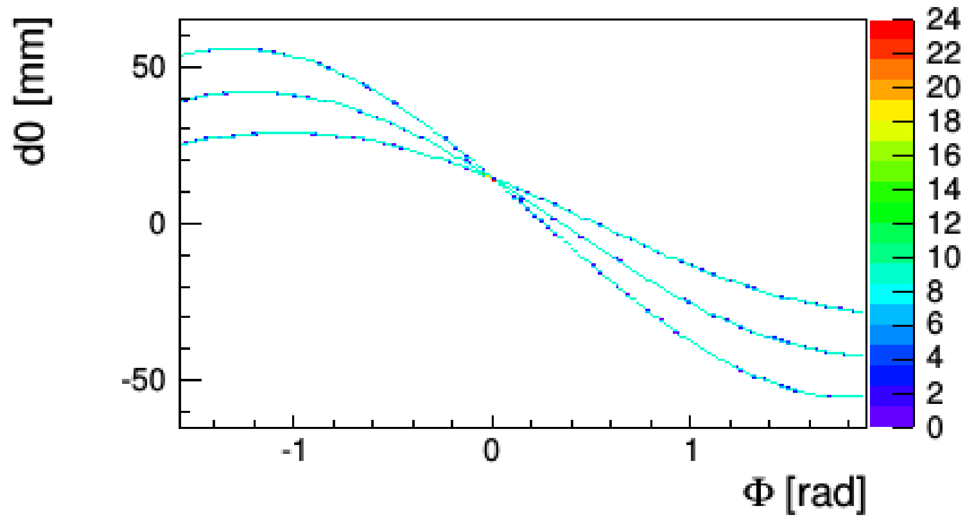
The transformation is:

$a = \tan\phi$ and $b = d_0/\cos\phi$

The track becomes then in the parameter space

$d_0 = y \cdot \cos\phi - x \cdot \sin\phi$





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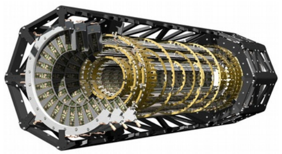
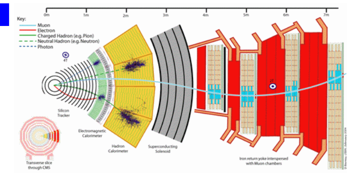
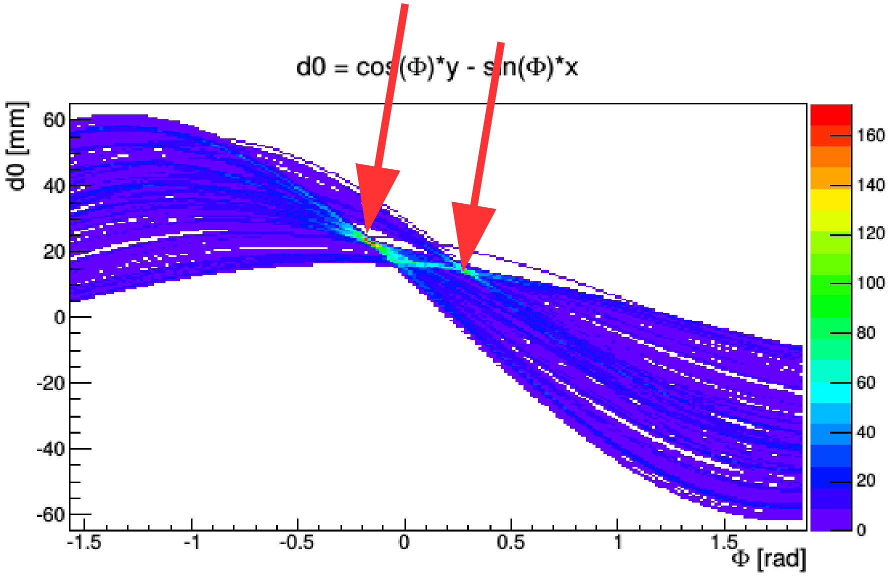
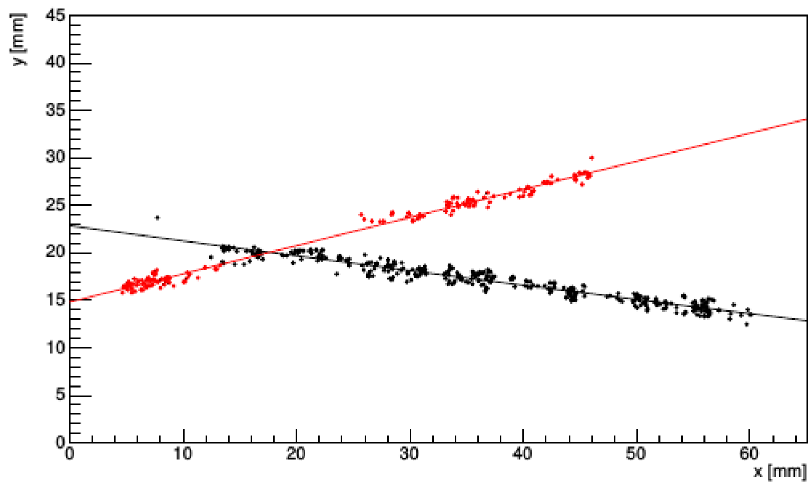


HT for Several Tracks







- Parameter space is a histogram
- Add the value 1 to all bins, which the lines touch
- Search for bin with highest entries
- Accept all lines in an ellipse given by the errors of the space point measurement.
- Combine these lines/space points to one track
- Refit the space points to get a better parameter estimate (only as good as histogram binning)
- Remove all entries of the space points from the histogram
- Restart with searching bin with most entries.

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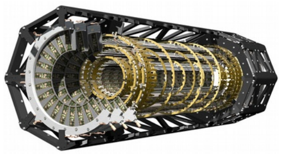
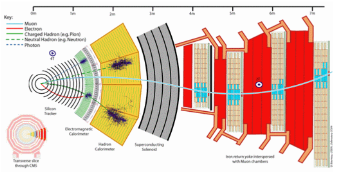
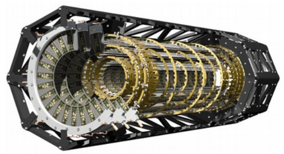
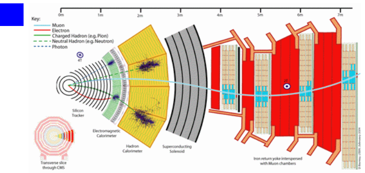




Experiment ILD at ILC

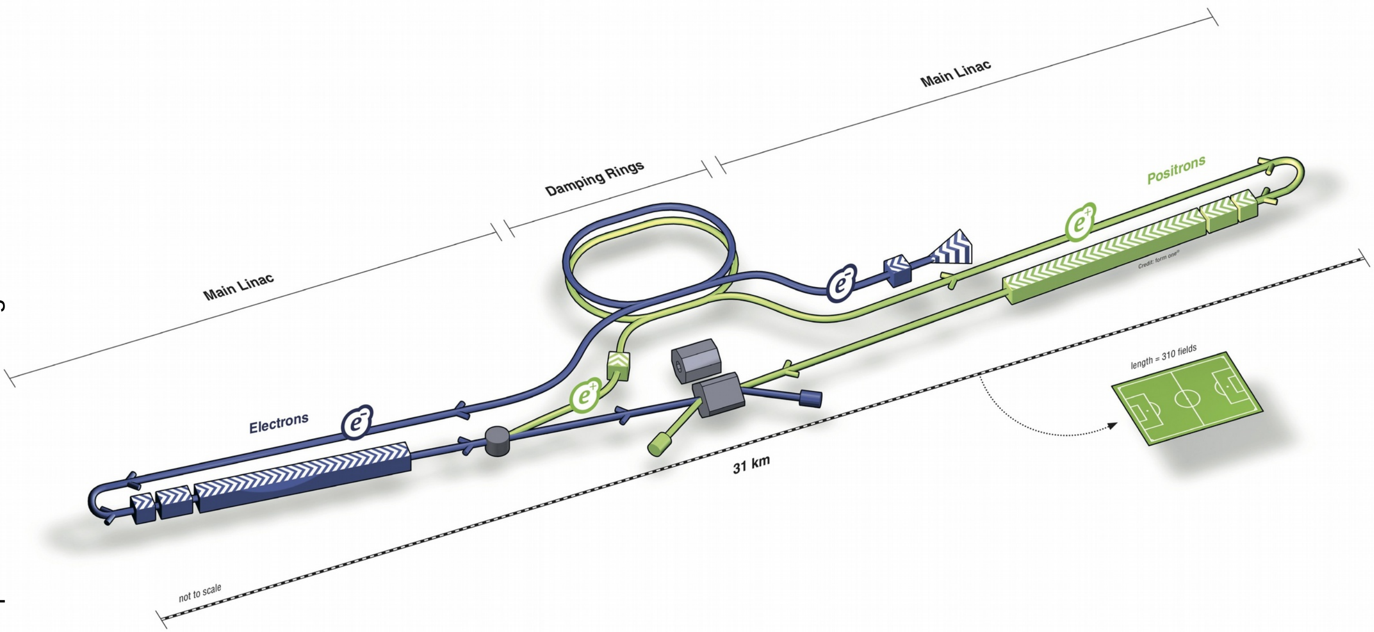


International Linear Collider



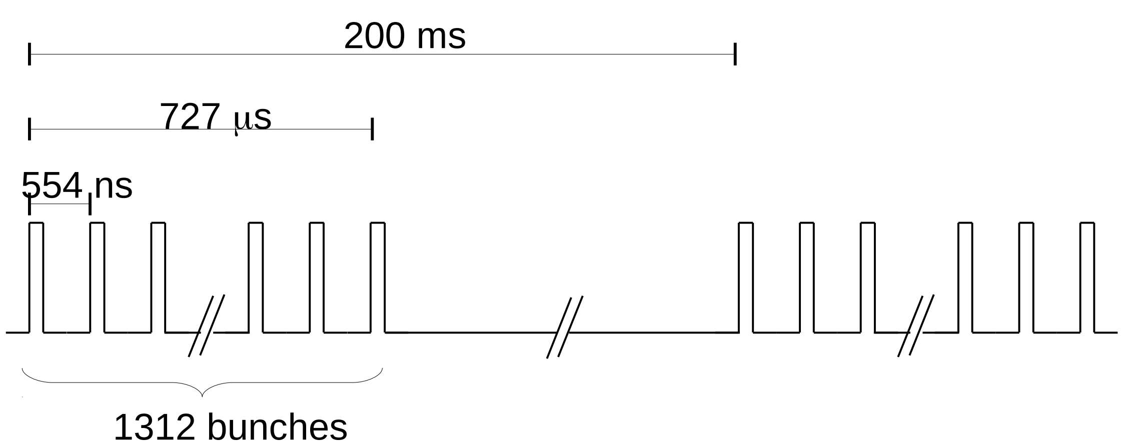
Linear e^+e^- collider with $\sqrt{s}$ = 90/250/350/500/1000 GeV:

https://www.linearcollider.org/



Point-like particles collide and annihilate. Energy, momentum and all quantum numbers are known. High degree of polarization (e^-~80%, e^+~30%)

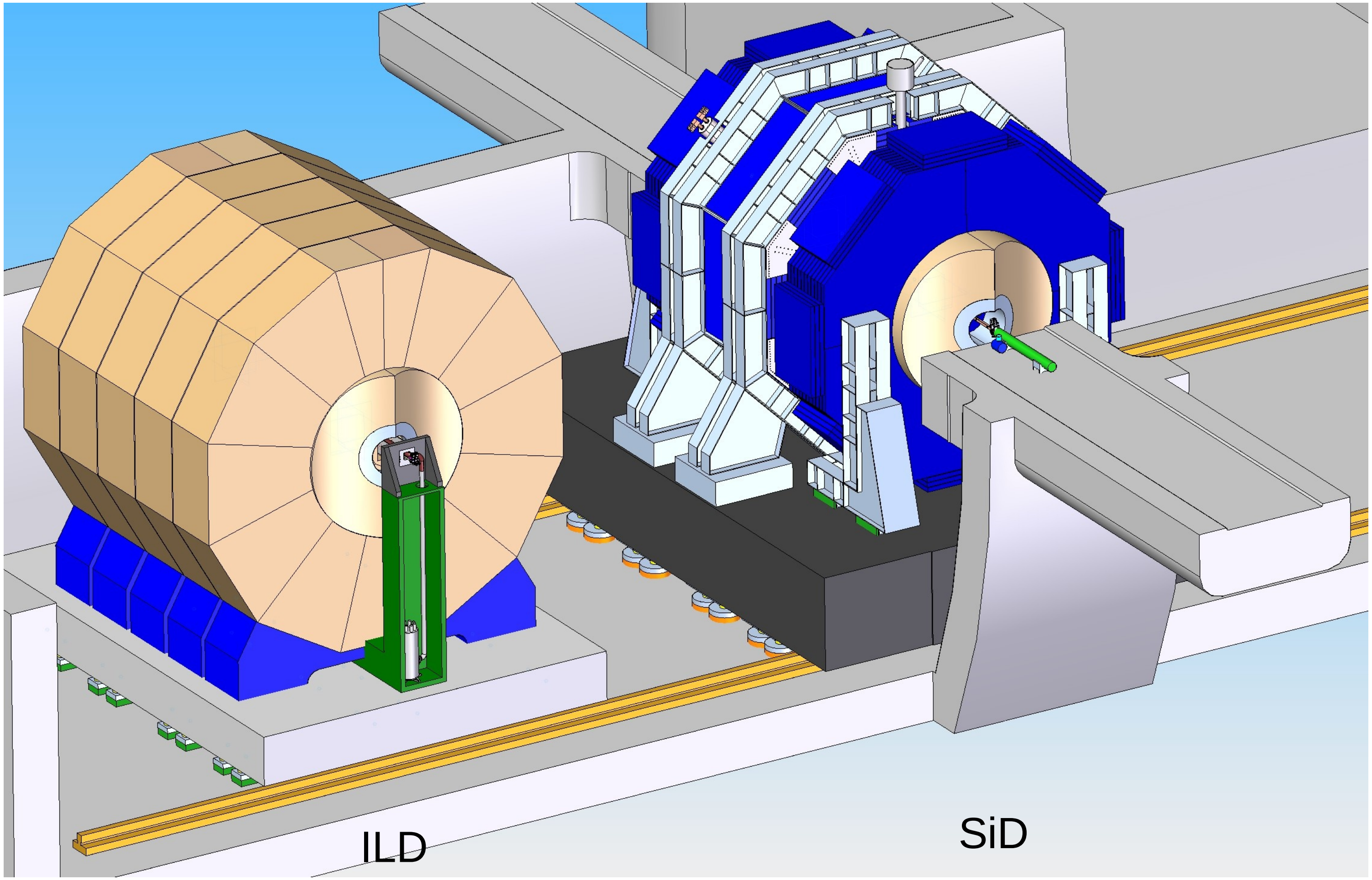
Bunch structure:
Damping takes 0.2 s,
then bunches are collided



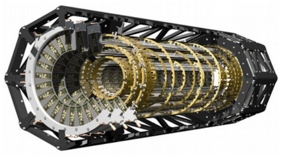
Experiments

Two detectors are used alternatively with a push-pull device.

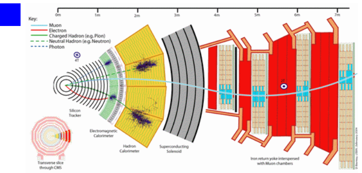
https://www.linearcollider.org/



Both detectors are based on the same design idea and differ in the implementation.

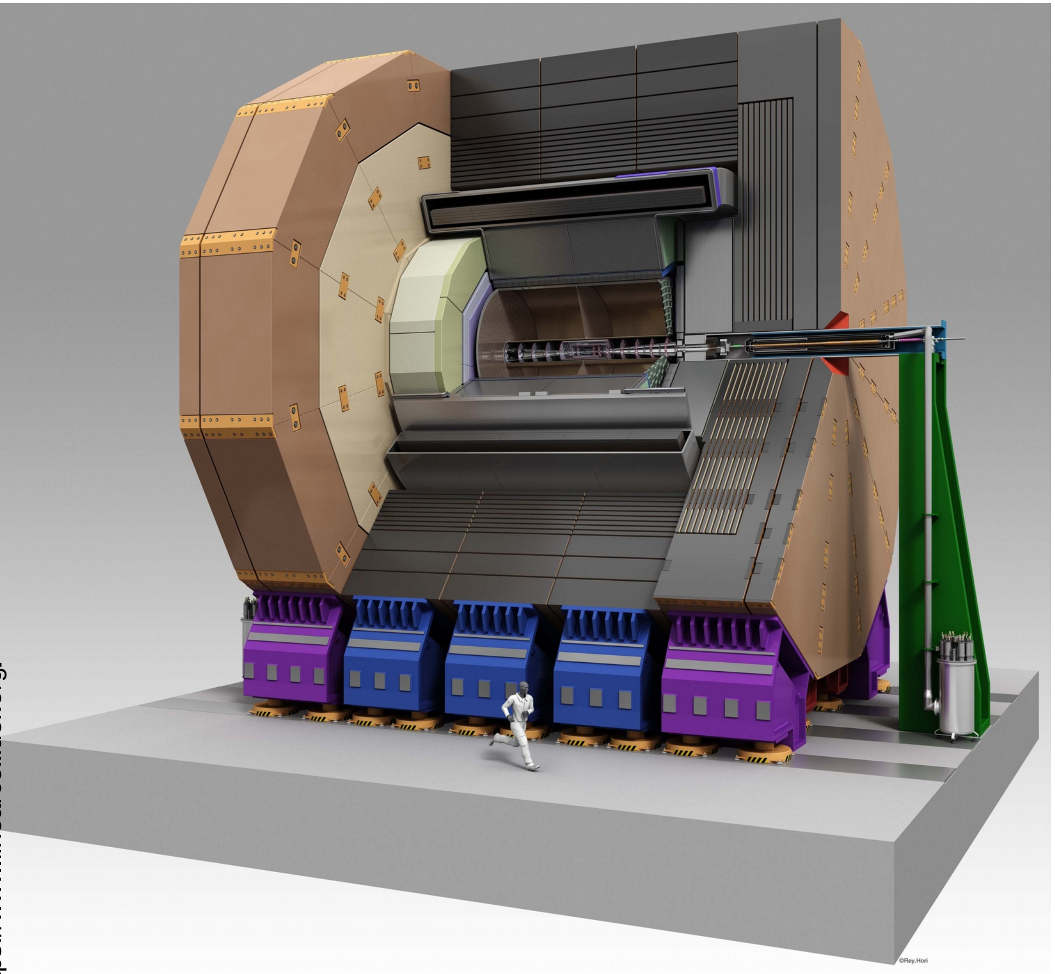


ILD



Detectors are standard multipurpose detectors of HEP

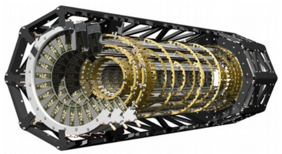
Similar to LEP detectors, but with much more advanced technologies



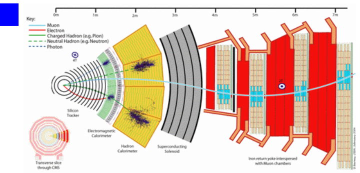
https://www.linearcollider.org/

- 6 layers of silicon pixel detectors
- 2 layers of silicon strip detector
- TPC
- 1 layer of silicon strip detector
- Electromagnetic calorimeter
- Hadronic calorimeter
- Coil for a B=3.5 T solenoidal field
- Instrumented return yoke

Detector has been laid out and optimized for
the particle flow concept



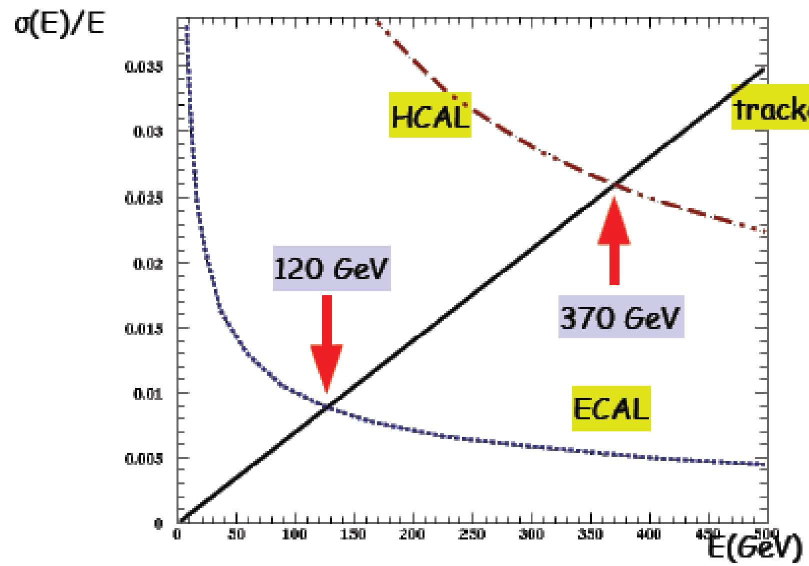
Particle Flow Concept

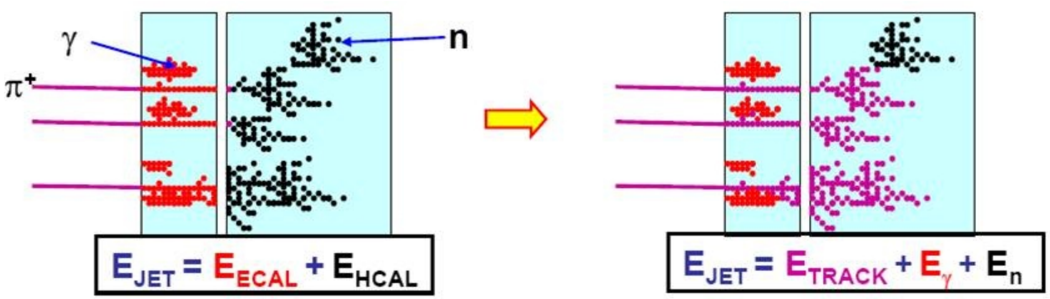


A limiting factor in multi-jet event reconstruction is the poor energy resolution of hadronic jets because of the traditional calorimetric approach:
Measure all components of jet energy in ECAL/HCAL !

But typical hadronic jet contains:
- 64 % charged hadrons (mainly $\pi^{\pm}$)
- 25 % photons (mainly from $\pi \rightarrow \gamma\gamma$)
- 11 % neutral hadrons (mainly K_L and n)

Best energy resolution for these particles can be obtained in different sub-detectors.





To realize the concept, one needs an unprecedented detector and an excellent reconstruction algorithm:
Tracks measured in central detector
→ but energy should not be counted twice
→ disentangle calorimeter measure-ments from neutral/charged particles

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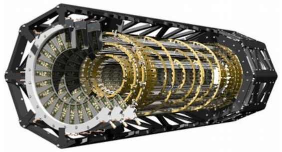
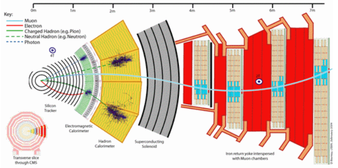
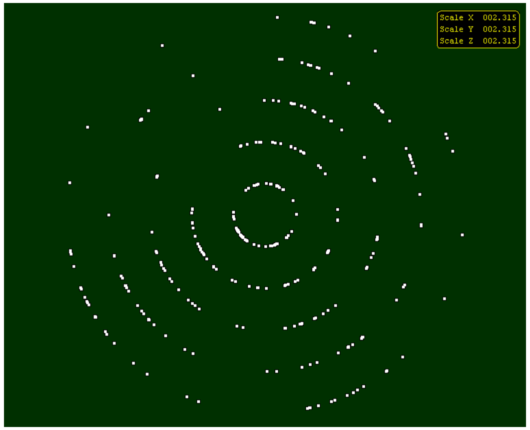
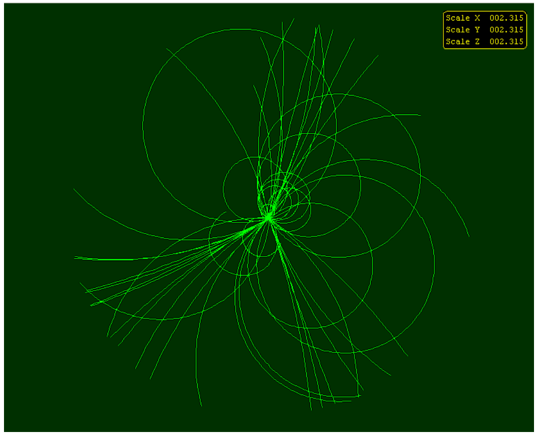




Requirements on Tracking Detectors

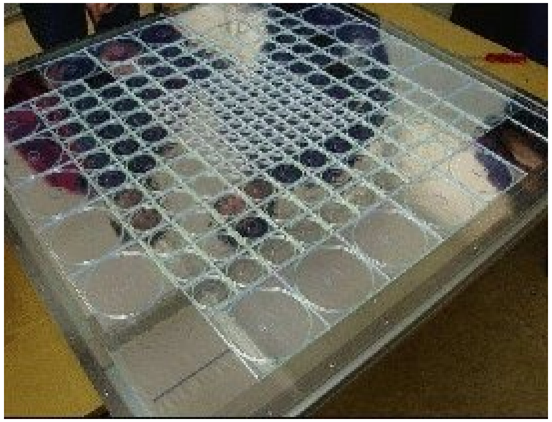
Need a very robust, efficient tracking, which can identify and separate tracks even in high track densities.

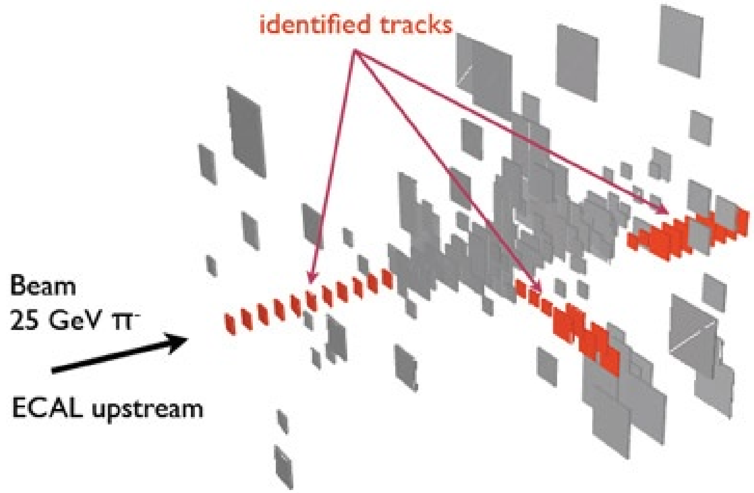


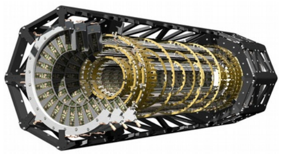


→ TPC

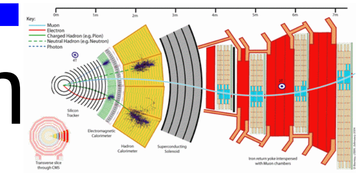
Need calorimeters with a very high granularity: cell sizes of 3*3 cm² or even 1*1 cm² are discussed.



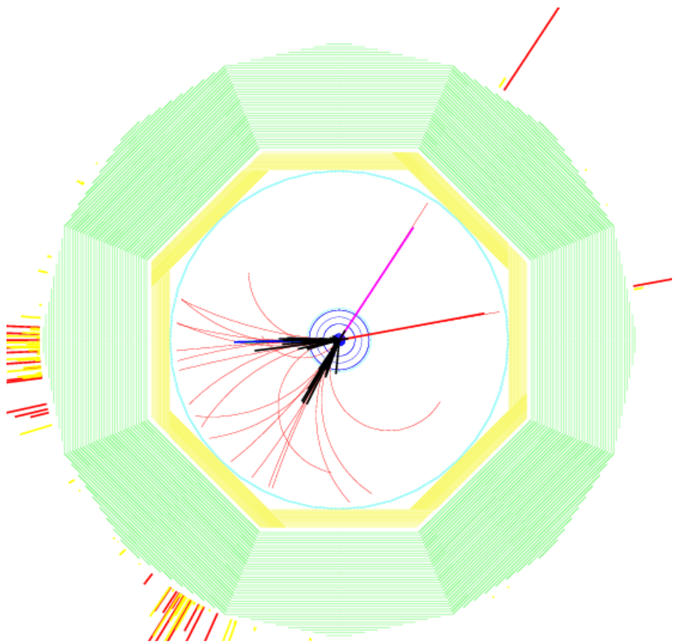


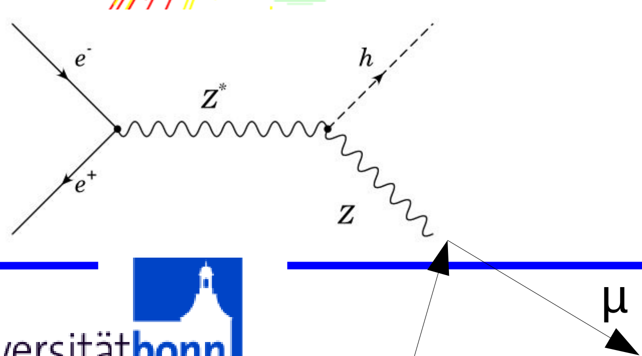


Requirements on Tracking System

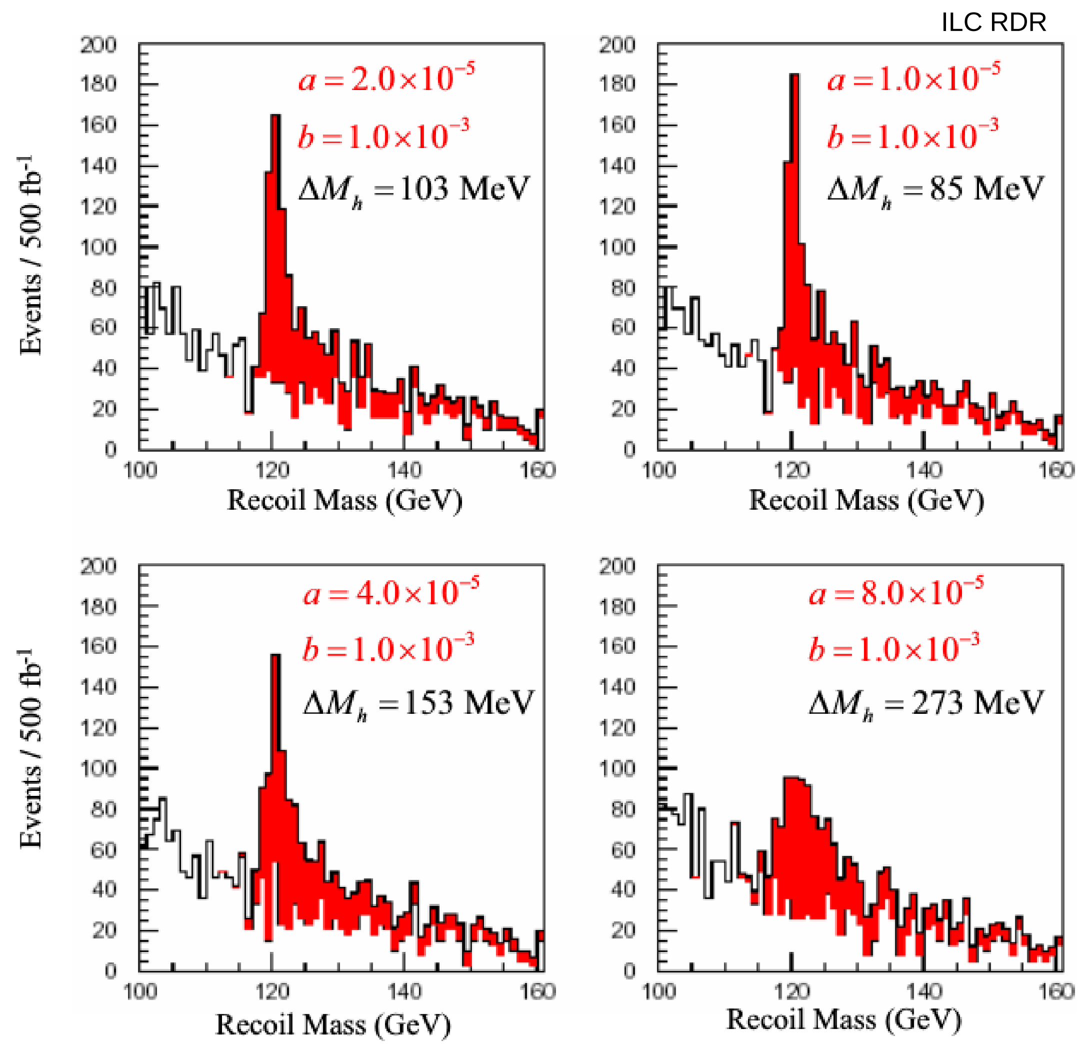


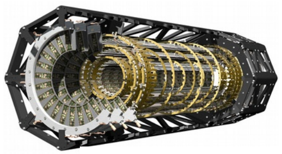
Requirements are driven by benchmark processes, in the case of ILD – TPC the most stringent measurement is the Higgs-recoil measurement:



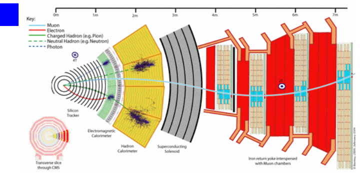


$$\delta p_t/p_t^2 = a \oplus b/(p_t \sin\theta)$$





Requirements of Tracking System



ILC DBD V4

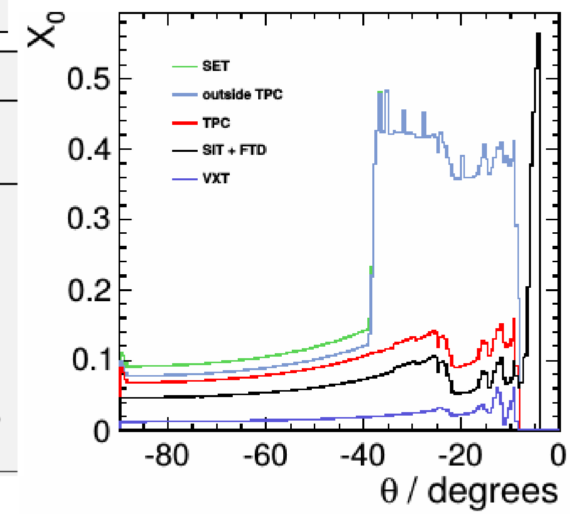
Design goals

Table II-3.2
Performance goals for the main tracker.

Parameter	Design Goal
coverage	hermetic above $\theta \sim 10°$
momentum resolution $\delta(1/p_T)$	$\sim 2-5 \times 10^{-5}$/GeV/c
material budget	$\sim 0.10 - 0.15X_0$ in central region
	$\sim 0.20 - 0.25X_0$ in endcap region
hit efficiency	$> 99\%$
background tolerance	Full efficiency at $10\times$ expected occupancy

Detector design

Barrel system						
System	R(in)	R(out) [mm]	z	comments		
VTX	16	60	125	3 double layers	Silicon pixel sensors,	
				layer 1:	layer 2:	layer 3-6
				$\sigma < 3\mu m$	$\sigma < 6\mu m$	$\sigma < 4\mu m$
Silicon						
- SIT	153	300	644	2 silicon strip layers	$\sigma = 7\mu m$	
- SET	1811		2300	2 silicon strip layers	$\sigma = 7\mu m$	
- TPC	330	1808	2350	MPGD readout	$1 \times 6\mathrm{mm}^2$ pads	$\sigma = 60\mu m$ at zero drift



Simulation results

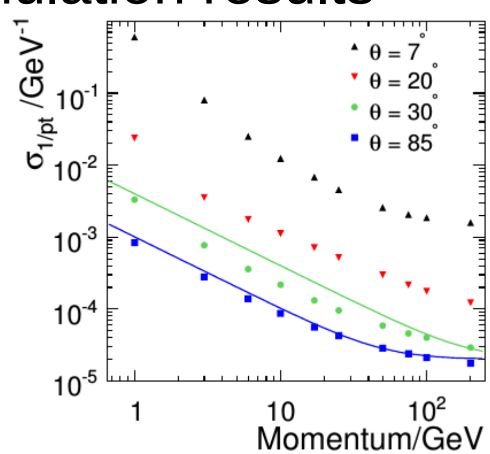
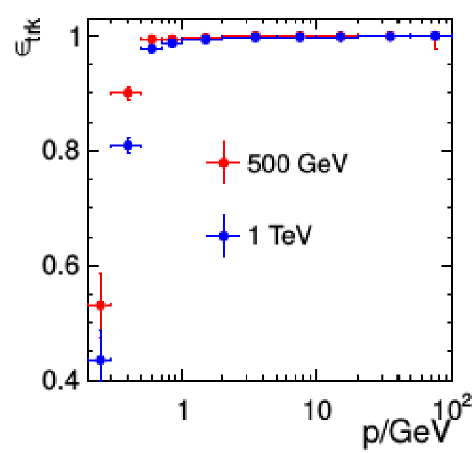


Figure III-1.5
Left: Momentum resolution as a function of the transverse momentum of particles, for tracks with different polar angles. Also shown is the theoretical expectation. Right: Flavour tagging performance for $Z \to q\bar{q}$ samples at different energies.



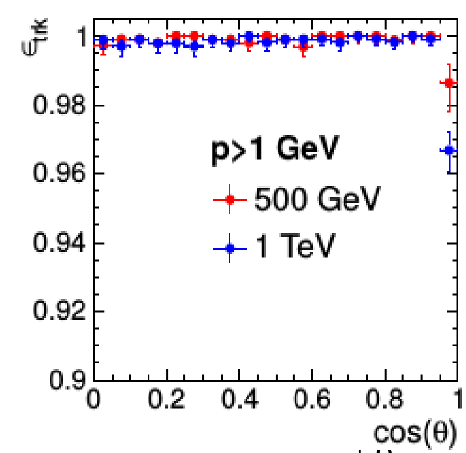


Figure III-6.2
Tracking Efficiency for $t\bar{t} \to 6$ jets at 500GeV and 1 TeV plotted against (left) momentum and (right) $\cos\theta$.

$$\sigma_{1/p_T} = a \oplus b/(p_T \sin\theta), \text{ with } a = 2\times10^{-5}\ \mathrm{GeV}^{-1} \text{ and } b = 1\times10^{-3}.$$

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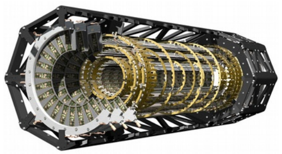
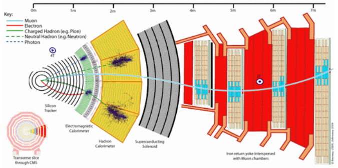


Literature



1.) Hermann Kolanoski, Norbert Wermes, Teilchendetektoren, Springer ISBN 13: 9783662453490

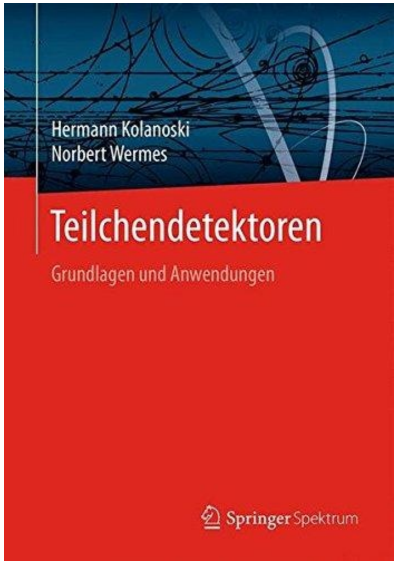
2.) Claus Grupen, Boris Shwartz, Particle Detectors, Cambridge, ISBN-13: 978-0521840064

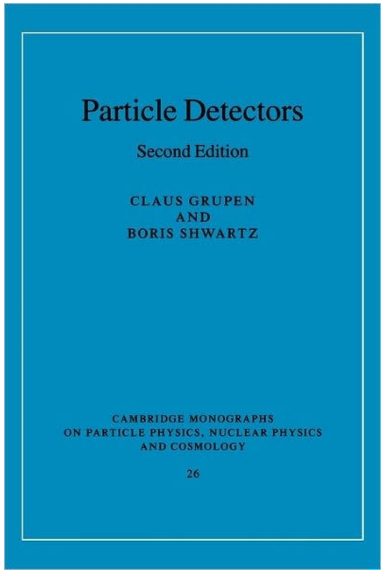
3.) PDG, The review of particle physics, http://pdg.lbl.gov/

4.) ILC, https://www.linearcollider.org/

5.) Various web pages for pictures

6.) lecture notes of Prof. N. Wermes





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