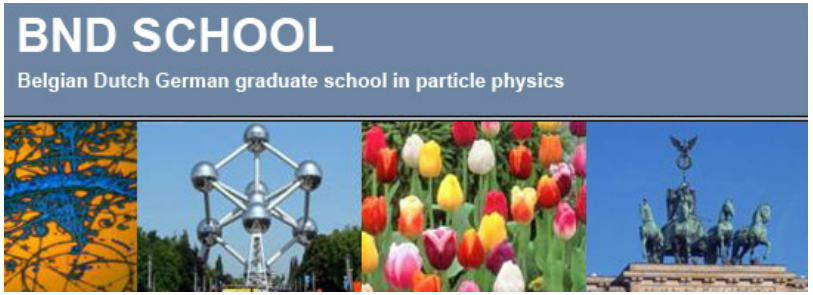
CERN



Accelerators

Part 2 of 3: Lattice, Longitudinal Motion, Limitations

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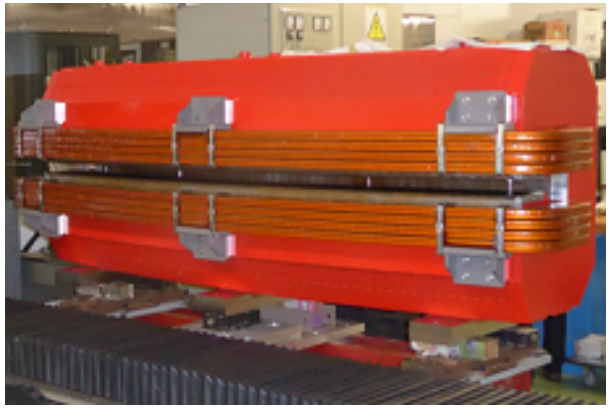
Topics

- A Brief Recap and Transverse Optics
- Longitudinal Motion
- Main Diagnostics Tools
- Possible Limitations

A brief recap and then we continue on transverse optics

Magnetic Element & Rigidity

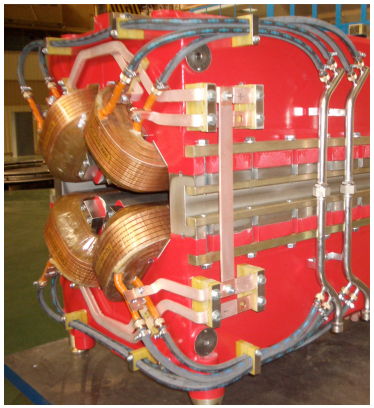
Dipole magnets



$$\theta = \frac{LB}{(B\rho)}$$

$$k = \frac{K}{B\rho}[m^{-2}]$$

Quadrupole magnets



$$F = q\vec{v} \times \vec{B} = \frac{mv^2}{\rho}$$

$$B\rho[\mathrm{Tm}] = \frac{mv}{q} = \frac{p[\mathrm{GeV/c}]}{q}$$

$$B\rho = 3.3356\, p$$

- Increasing the energy requires increasing the magnetic field with Bρ to maintain radius and same focusing
- The magnets are arranged in cell, such as a FODO lattice

Hill's Equation

- Hill's equation describes the horizontal and vertical **betatron oscillations**

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

Position:

$$x(s) = \sqrt{\varepsilon\beta_s}\cos(\varphi(s) + \varphi)$$

Angle:

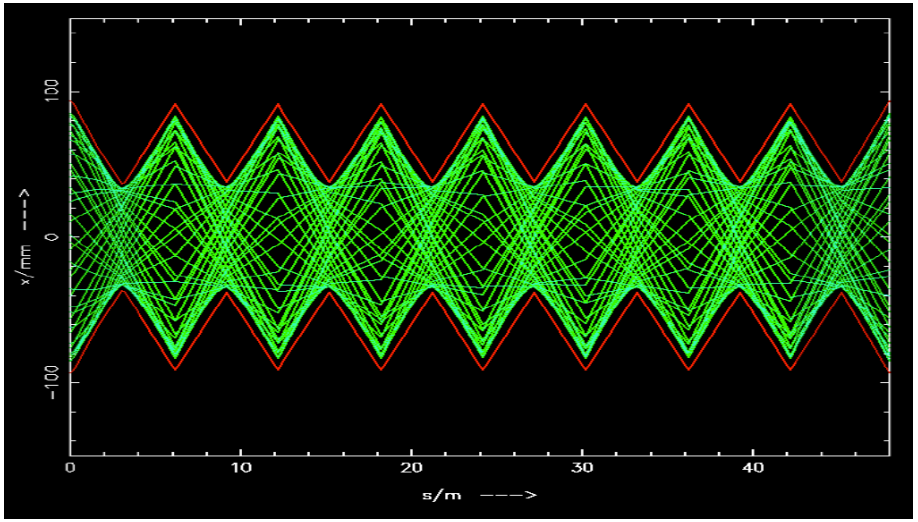
$$x' = -\alpha\sqrt{\varepsilon/\beta}\cos(\varphi) - \sqrt{\varepsilon/\beta}\sin(\varphi)\varphi$$

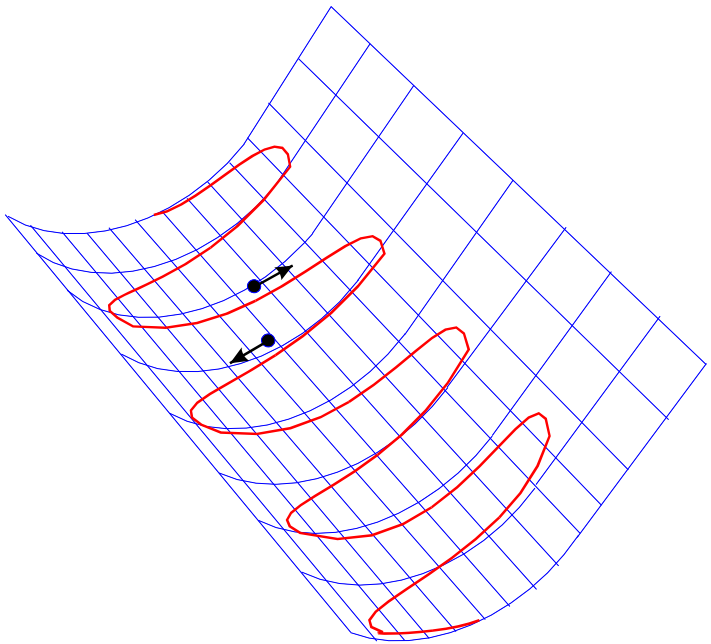
- ε and φ are constants determined by the **initial conditions**
- β**(s)** is the periodic **envelope function** given by the lattice configuration

$$Q_{x/y} = \frac{1}{2\pi}\int_0^{2\pi}\frac{ds}{\beta_{x/y}(s)}$$

- $\mathbf{Q_x}$ and $\mathbf{Q_y}$ are the **horizontal and vertical tunes**: the number of oscillations per turn around the machine

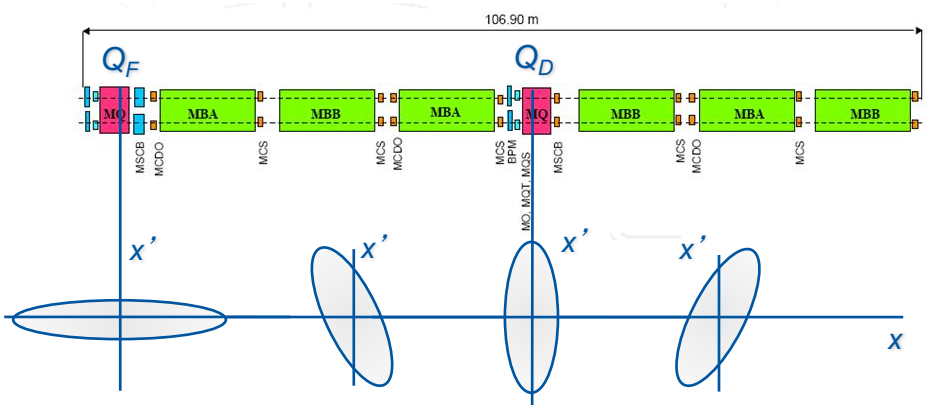
Betatron Oscillations & Envelope





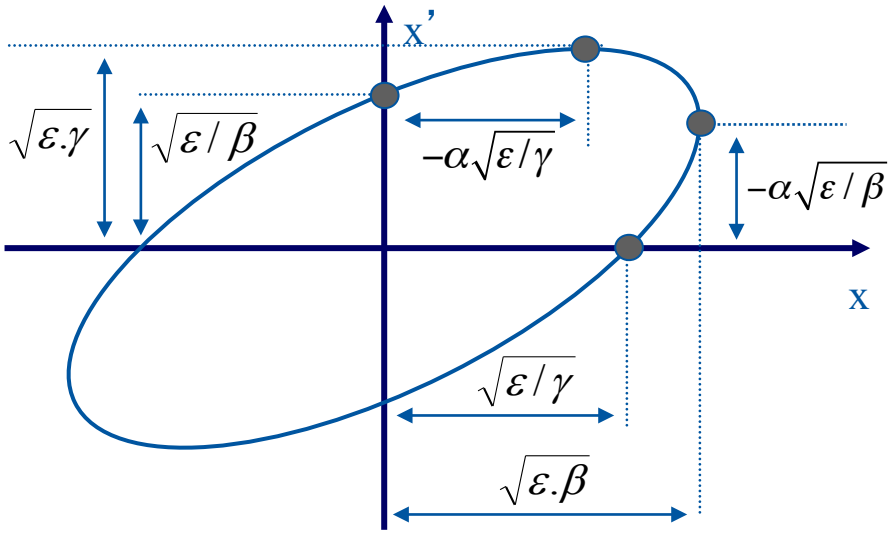
- The β function is the envelope function within which all particles oscillate
- The shape of the β function is determined by the lattice

FODO Lattice & Phase Space



- Calculating a single FODO Lattice and repeat it several time
- Make adaptations where you have insertion devices e.g. experiment, injection, extraction etc.

- Horizontal and vertical phase space
- Q_h = 3.5 means 3.5 horizontal betatron oscillations per turn around the machine, hence 3.5 turns on the phase space ellipse
- Each particle, depending on it's initial conditions will turn on it's own ellipse in phase space



Let’s continue….

Momentum Compaction Factor

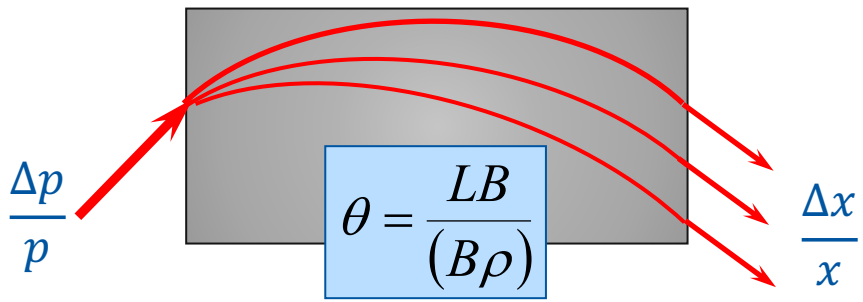
- The change in orbit length for particles with different momentum than the average momentum
- This is expressed as the momentum compaction factor, $\boldsymbol{\alpha}_p$, where:

$$\frac{\Delta r}{r} = \alpha_p \frac{\Delta p}{p}$$

- $\boldsymbol{\alpha}_p$ expresses the change in the radius of the closed orbit for a particle as a a function of the its momentum

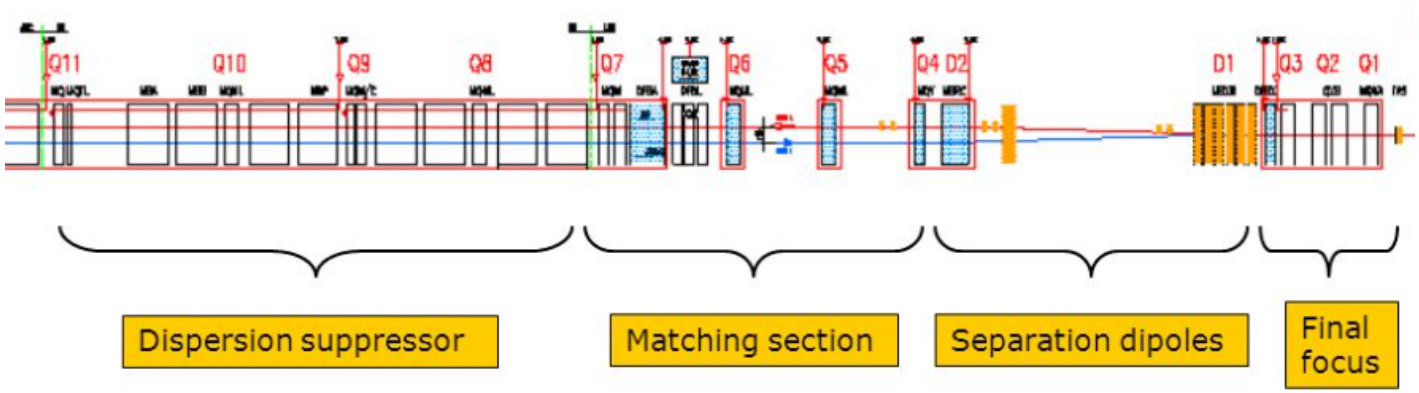
Dispersion

- A particle beam has a momentum spread that in a homogenous dipole field will translate in a beam position spread at the exit of a magnet



$$\frac{\Delta x}{x} = D(s)\frac{\Delta p}{p}$$

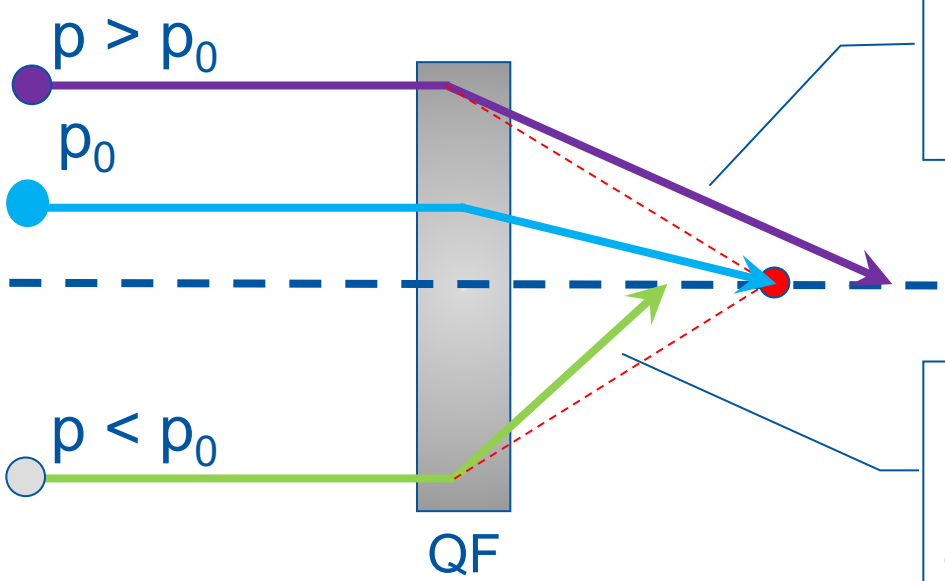
- The beam will have a finite horizontal size due to it's momentum spread, unless we install and dispersion suppressor to create dispersion free regions e.g. for experiments



Chromaticity

- The chromaticity relates the tune spread of the transverse motion with the momentum spread in the beam.

$$\frac{\Delta Q_{h/v}}{Q_{h/v}} = \xi_{h/v} \frac{\Delta p}{p}$$



A particle with a higher momentum as the central momentum will be deviated less in the quadrupole and will have a lower betatron tune

A particle with a lower momentum as the central momentum will be deviated more in the quadrupole and will have a higher betatron tune

Q1: How to Measure Chromaticity

- Looking at the formula for Chromaticity, could you think about how to measure the actual chromaticity in you accelerator ?

$$\frac{\Delta Q_{h/v}}{Q_{h/v}} = \xi_{h/v} \frac{\Delta p}{p}$$

- What beam parameter would you change ?
  - Any idea how ?

- What beam parameter would you observe ?
  - Any idea how ?

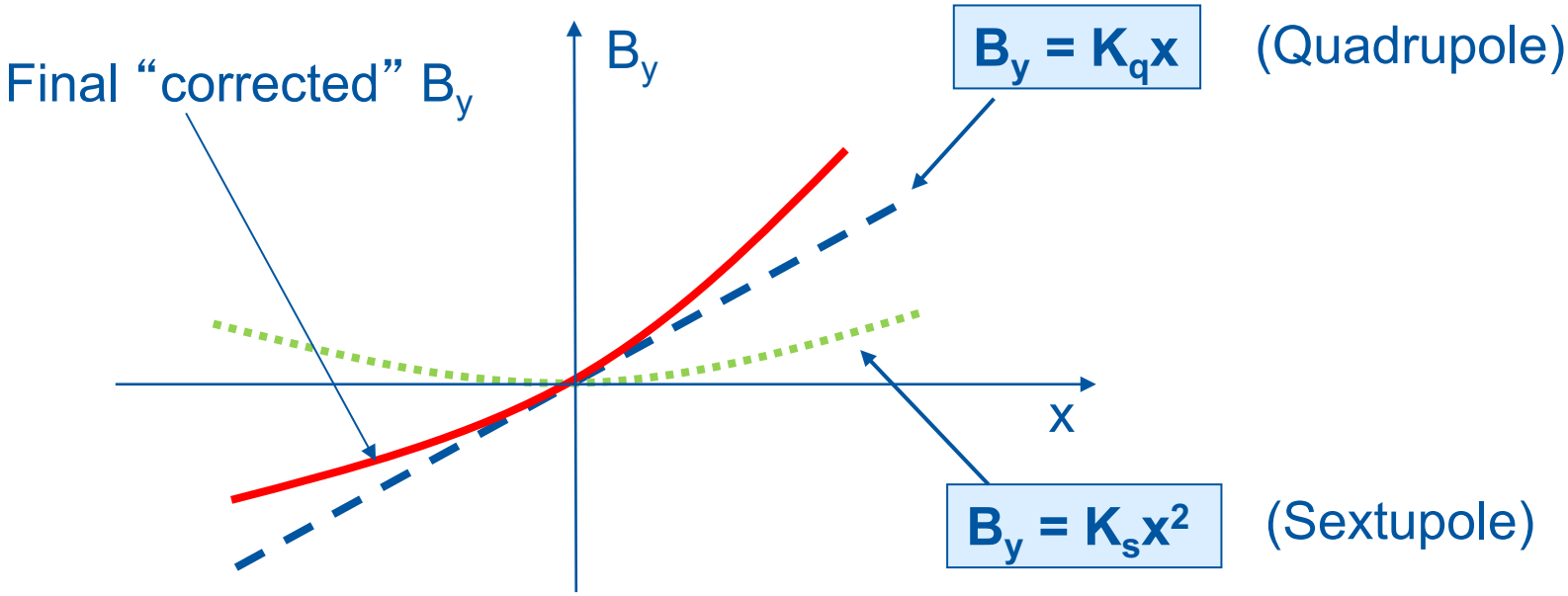
Q1: How to Measure Chromaticity

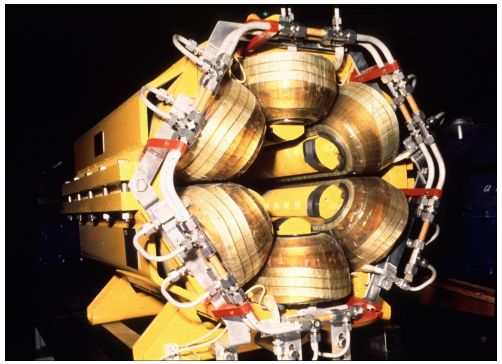
- Looking at the formula for Chromaticity, could you think about how to measure the actual chromaticity in you accelerator ?

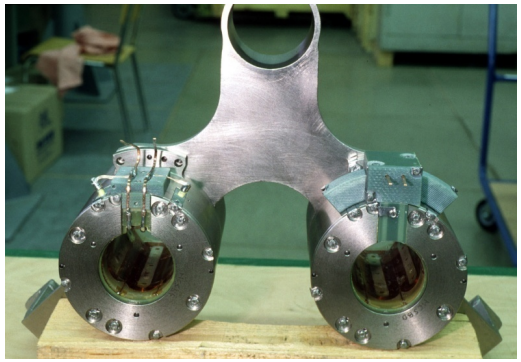
$$\frac{\Delta Q_{h/v}}{Q_{h/v}} = \xi_{h/v} \frac{\Delta p}{p}$$

- What beam parameter would you change ?
- Change the average momentum of the beam and you beam will move coherently as a single particle with a different momentum
  - Any idea how ?
  - Add an offset to the RF system to slightly increase the beam momentum at a constant magnetic field
- What beam parameter would you observe ?
- You would need to observe the change in beam tune for a change in beam momentum
  - Any idea how ?
  - Measuring the beam position over many turns and make an FFT that will show the change in frequency

Chromaticity Correction







$$\frac{\Delta Q}{Q} = \underbrace{\frac{1}{4\pi} l\beta(s) \frac{d^2 B_y}{dx^2} \frac{D(s)}{(B\rho)Q}}_{\text{Chromaticity Control through sextupoles}} \frac{\Delta p}{p}$$

Longitudinal Motion

Motion in the Longitudinal Plane

- What happens when particle momentum increases in a constant magnetic field?
  - Travel faster (initially)
  - Follow a longer orbit
- Hence a momentum change influence on the revolution frequency

$$\frac{df}{f} = \frac{dv}{v} - \frac{dr}{r}$$

- From the momentum compaction factor we have: $\frac{\Delta r}{r} = \alpha_p \frac{\Delta p}{p}$
- Therefore: $\frac{df}{f} = \frac{dv}{v} - \alpha_p \frac{dp}{p}$

Revolution Frequency - Momentum

$$\frac{df}{f} = \frac{dv}{v} - \alpha_p \frac{dp}{p}$$

$$\frac{dv}{v} = \frac{d\beta}{\beta} \Leftrightarrow \beta = \frac{v}{c}$$

From the relativity theory: $p = \frac{E_0 \beta \gamma}{c}$

We can get: $\frac{dv}{v} = \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}$

Resulting in : $\frac{df}{f} = \left[\frac{1}{\gamma^2} - \alpha_p\right]\frac{dp}{p}$

Transition

$$\frac{df}{f} = \left[\frac{1}{\gamma^2} - \alpha_p\right]\frac{dp}{p}$$

- Low momentum (β << 1 & γ is small) → $\frac{1}{\gamma^2} > \alpha_p$
- High momentum ($\beta \approx 1$ & γ >> 1) → $\frac{1}{\gamma^2} < \alpha_p$
- Transition momentum → $\frac{1}{\gamma^2} = \alpha_p$

Frequency Slip Factor

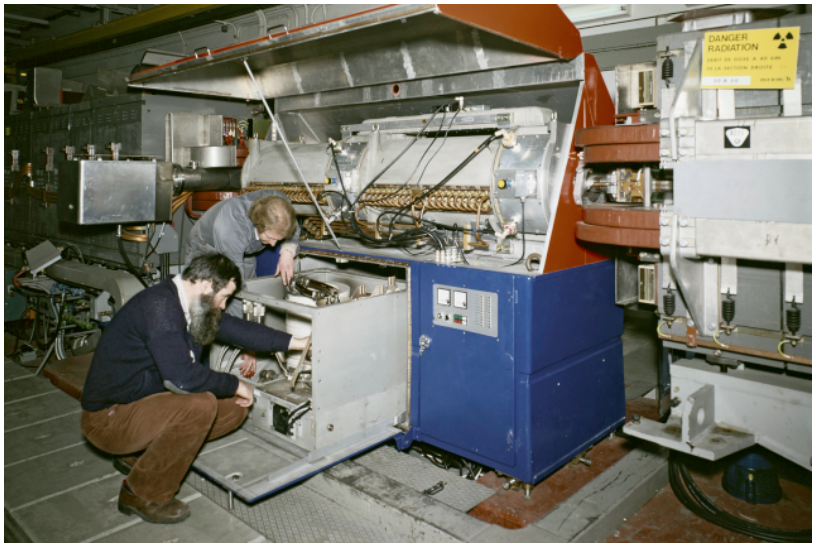
$$\frac{df}{f} = \left[\frac{1}{\gamma^2} - \alpha_p\right]\frac{dp}{p} = \left[\frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}\right]\frac{dp}{p} = \eta\frac{dp}{p}$$

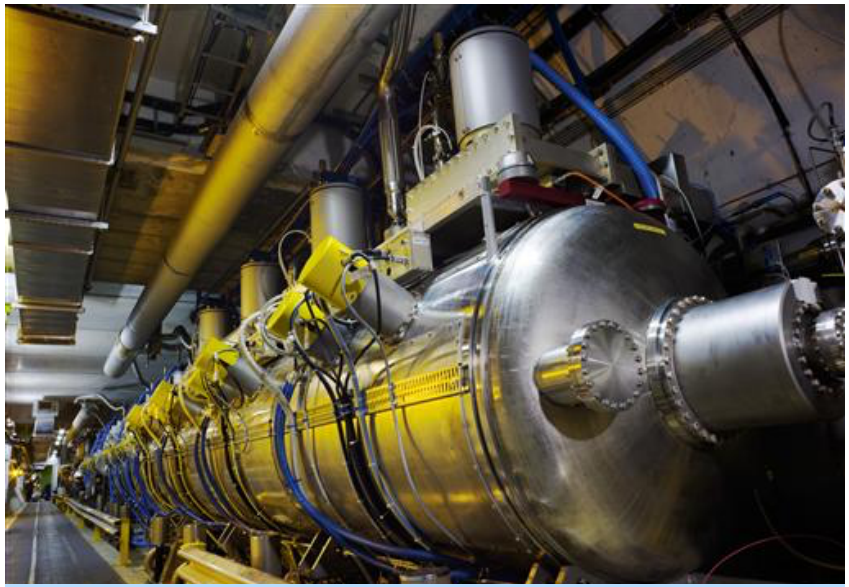
- Below transition: $\frac{1}{\gamma^2} > \alpha_p \quad \Longrightarrow \quad \eta > 0$
- Transition: $\frac{1}{\gamma^2} = \alpha_p \quad \Longrightarrow \quad \eta = 0$
- Above transition: $\frac{1}{\gamma^2} < \alpha_p \quad \Longrightarrow \quad \eta < 0$

- Transition is very important in hadron machines
  - CERN PS: γ_{tr} is at ~ 6 GeV/c (injecting at 2.12 GeV/c → below)
  - LHC : γ_{tr} is at ~ 55 GeV/c (injecting at 450 GeV/c → above)
- Transition does not exist in lepton machines, why …..?

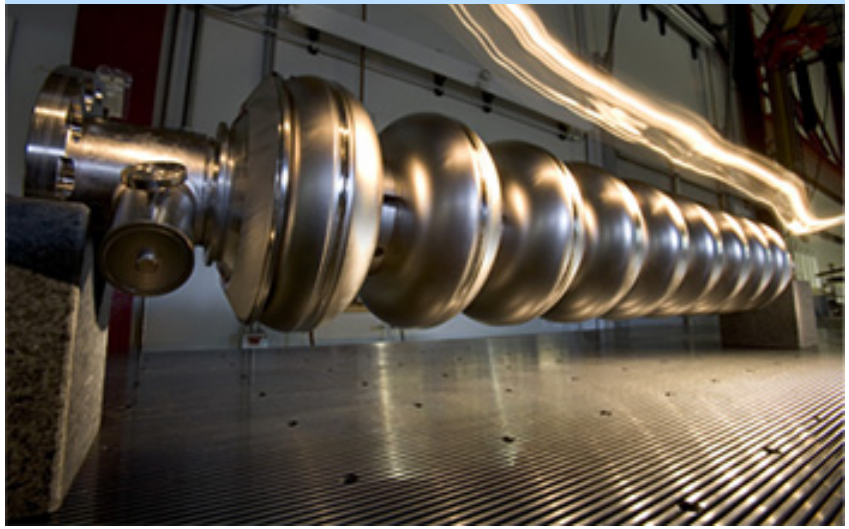
RF Cavities

Variable frequency cavity
(CERN – PS)



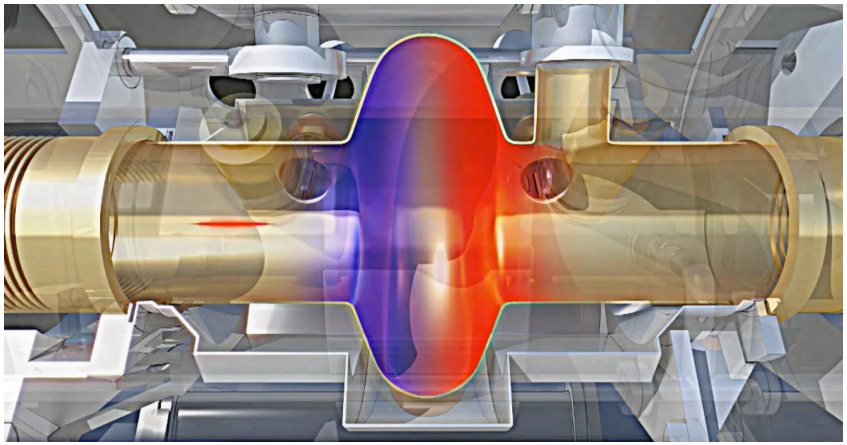


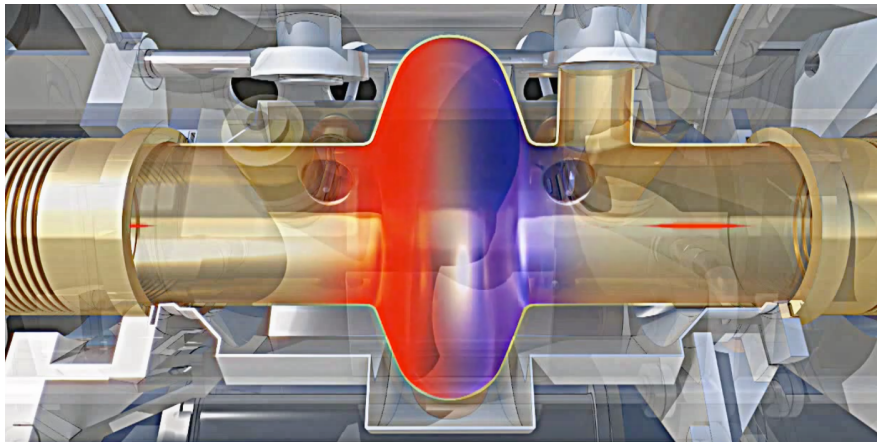
Super conducting fixed frequency cavity
(LHC)



RF Cavity

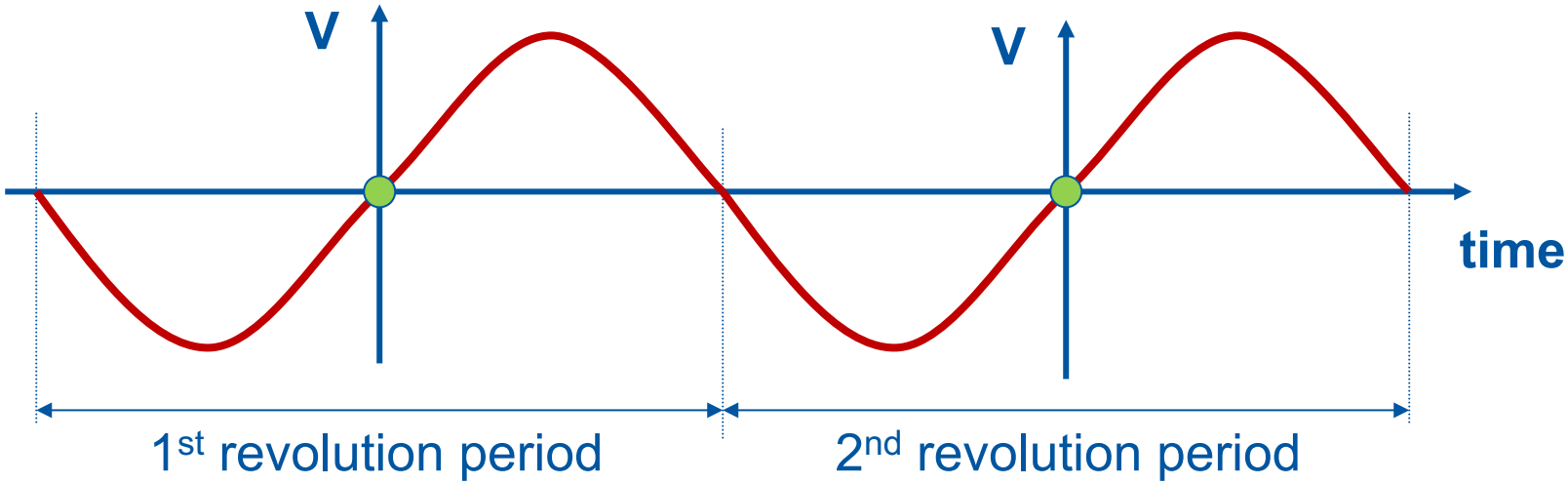
- Charged particles are accelerated by a longitudinal electric field
- The electric field needs to alternate with the revolution frequency





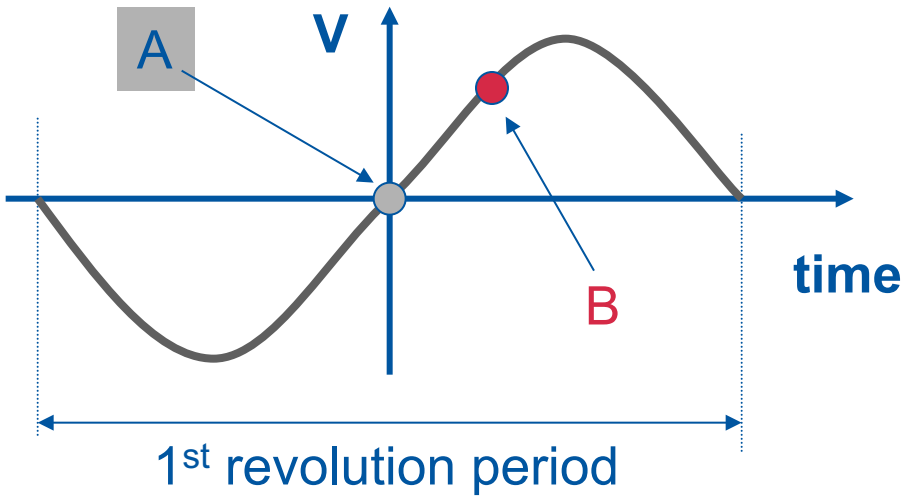
Low Momentum Particle Motion

- Lets see what a low energy particle does with this oscillating voltage in the cavity

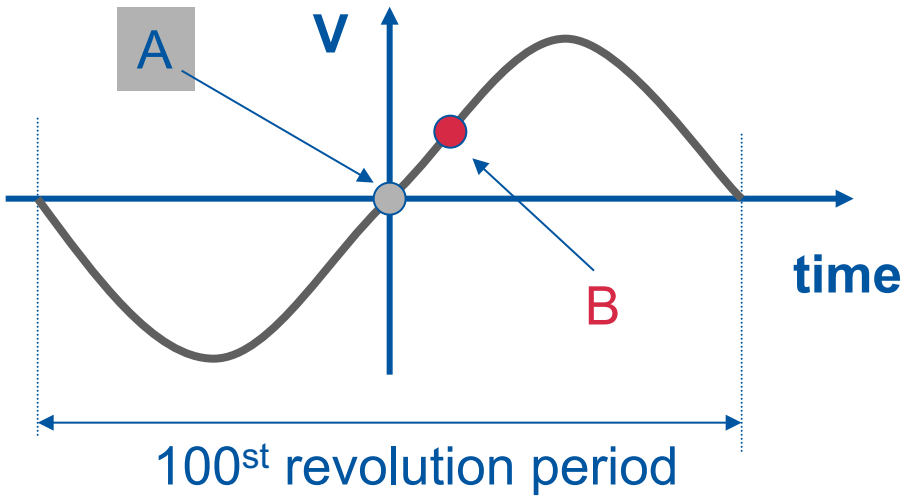


- Lets see what a low energy particle does with this oscillating voltage in the cavity

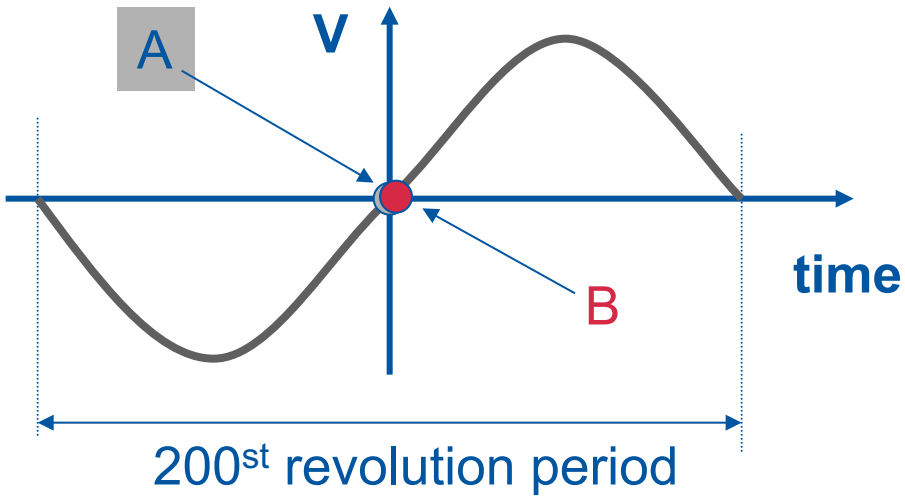
Longitudinal Motion Below Transition



….after many turns…



….after many turns…



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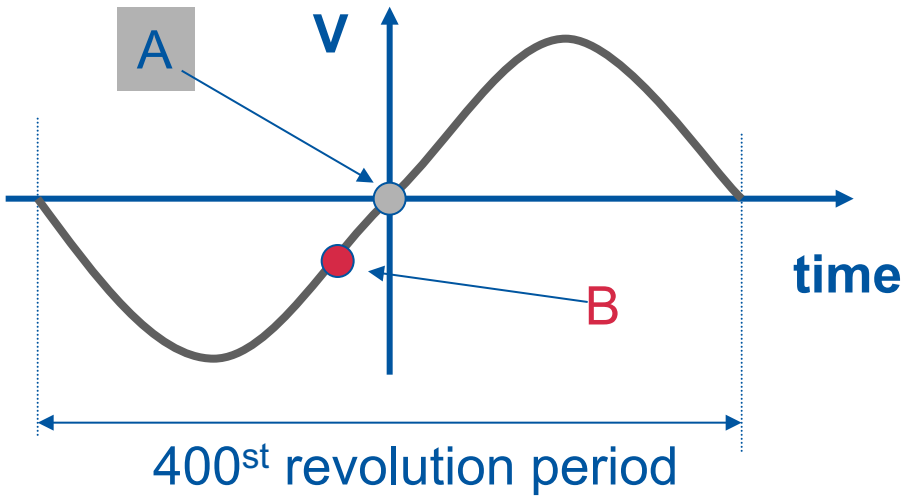

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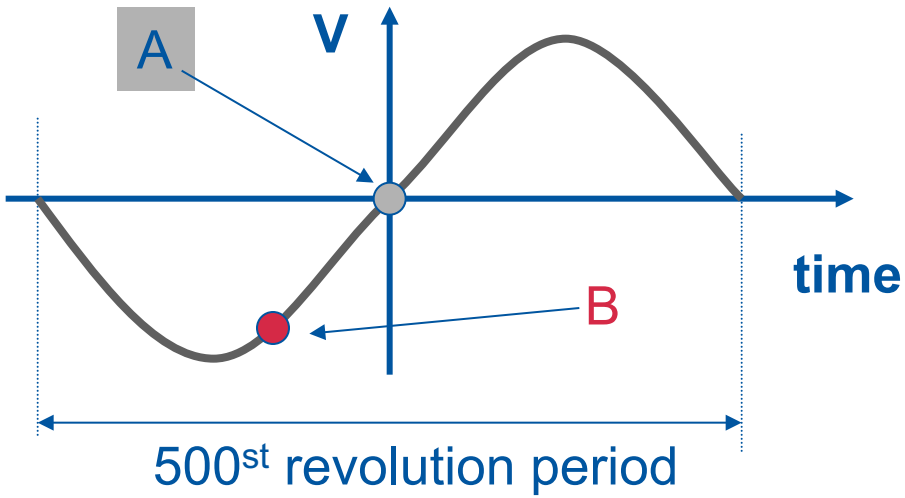

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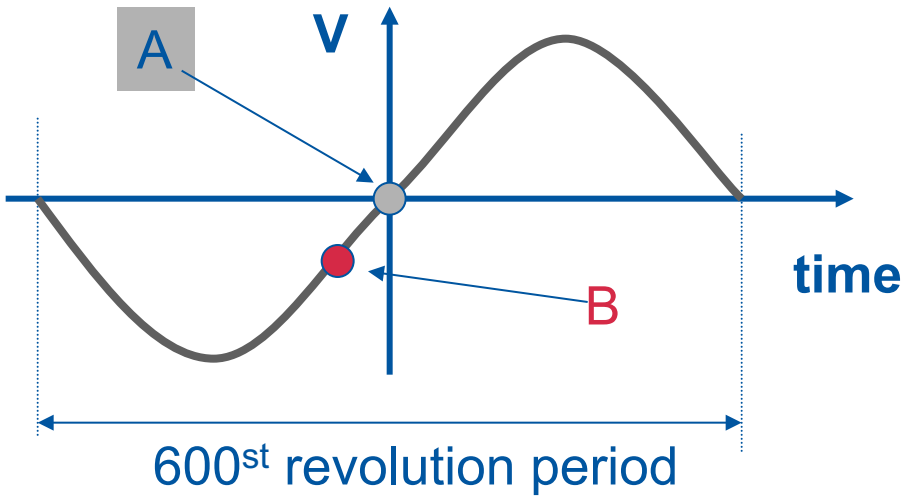

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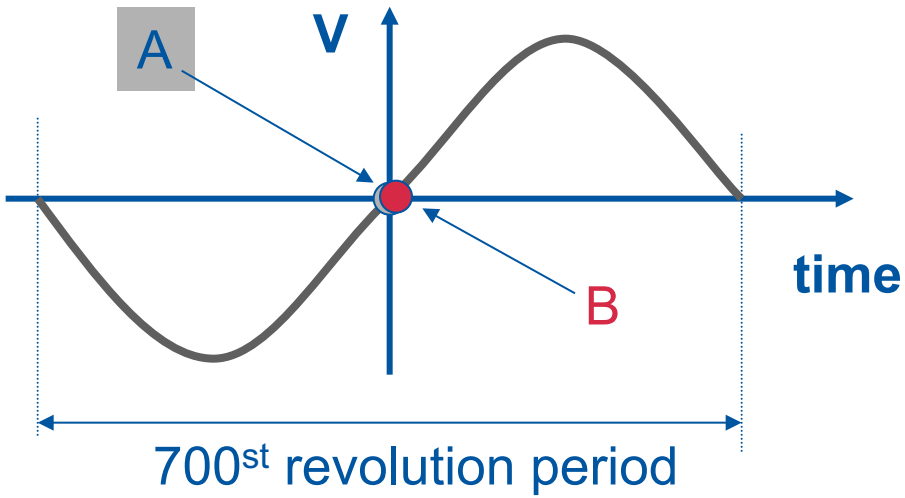

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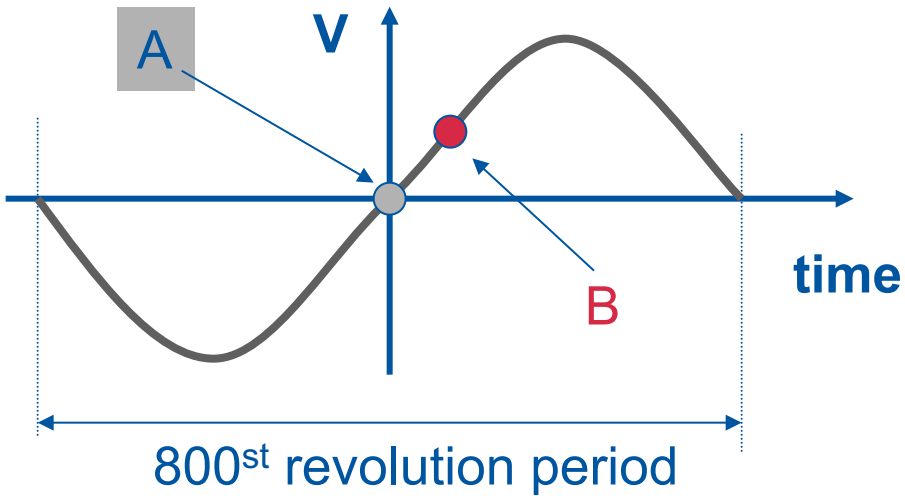
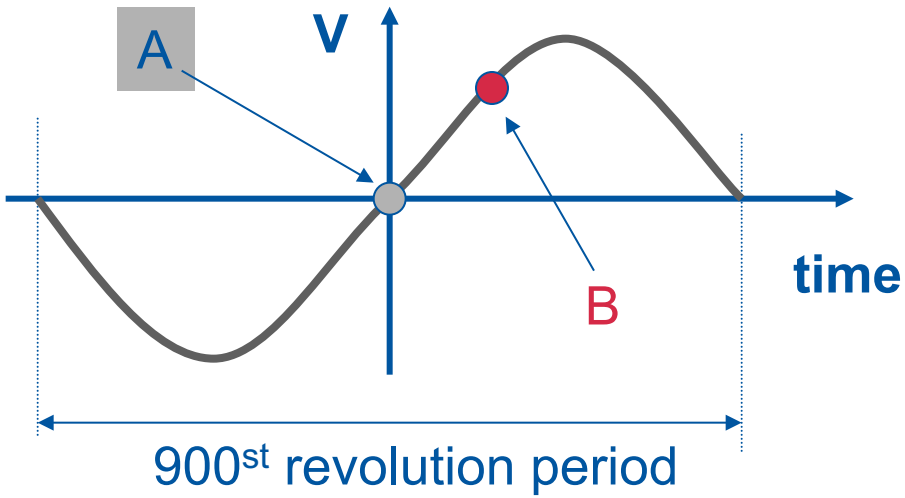


….after many turns…



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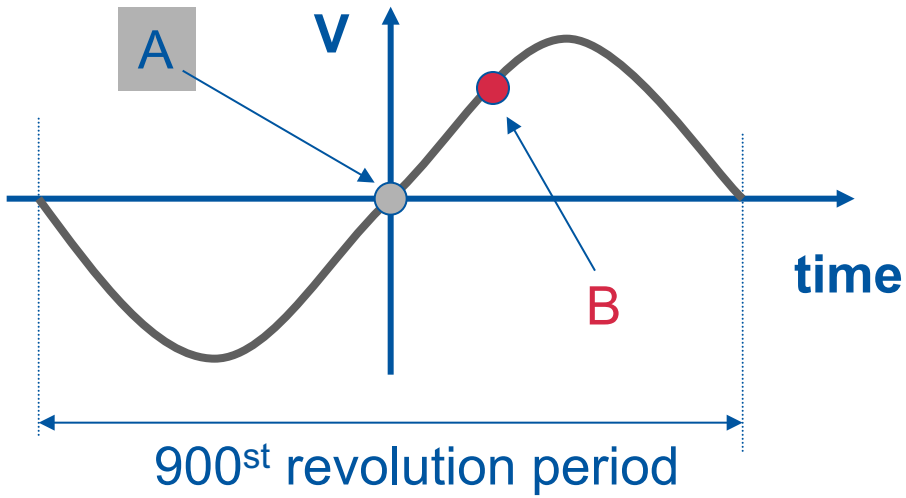
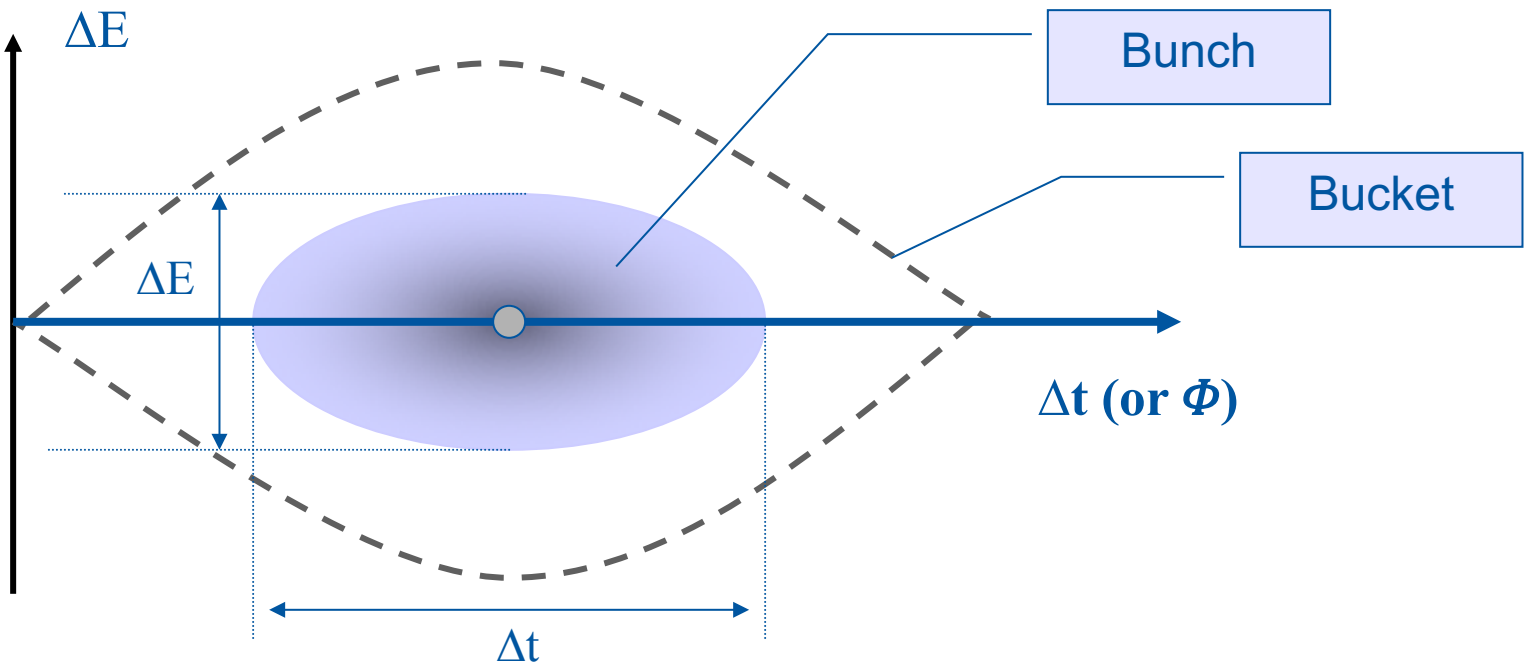


- Particle B has made 1 full oscillation around particle A
- The amplitude depends on the initial phase

- This are Synchrotron Oscillations
- Phase Stability: “off-momentum” particles are contained

Stationary Bunch & Bucket



- Bucket area = **<u>longitudinal Acceptance</u>** [eVs]
- Bunch area = **<u>longitudinal beam emittance</u>** = **$\pi.\Delta E.\Delta t/4$ [eVs]**

What About Beyond Transition

- Until now we have seen how things look like below transition

Higher energy ⇨ faster orbit ⇨ higher F_{rev} ⇨ next time particle will be **earlier**.

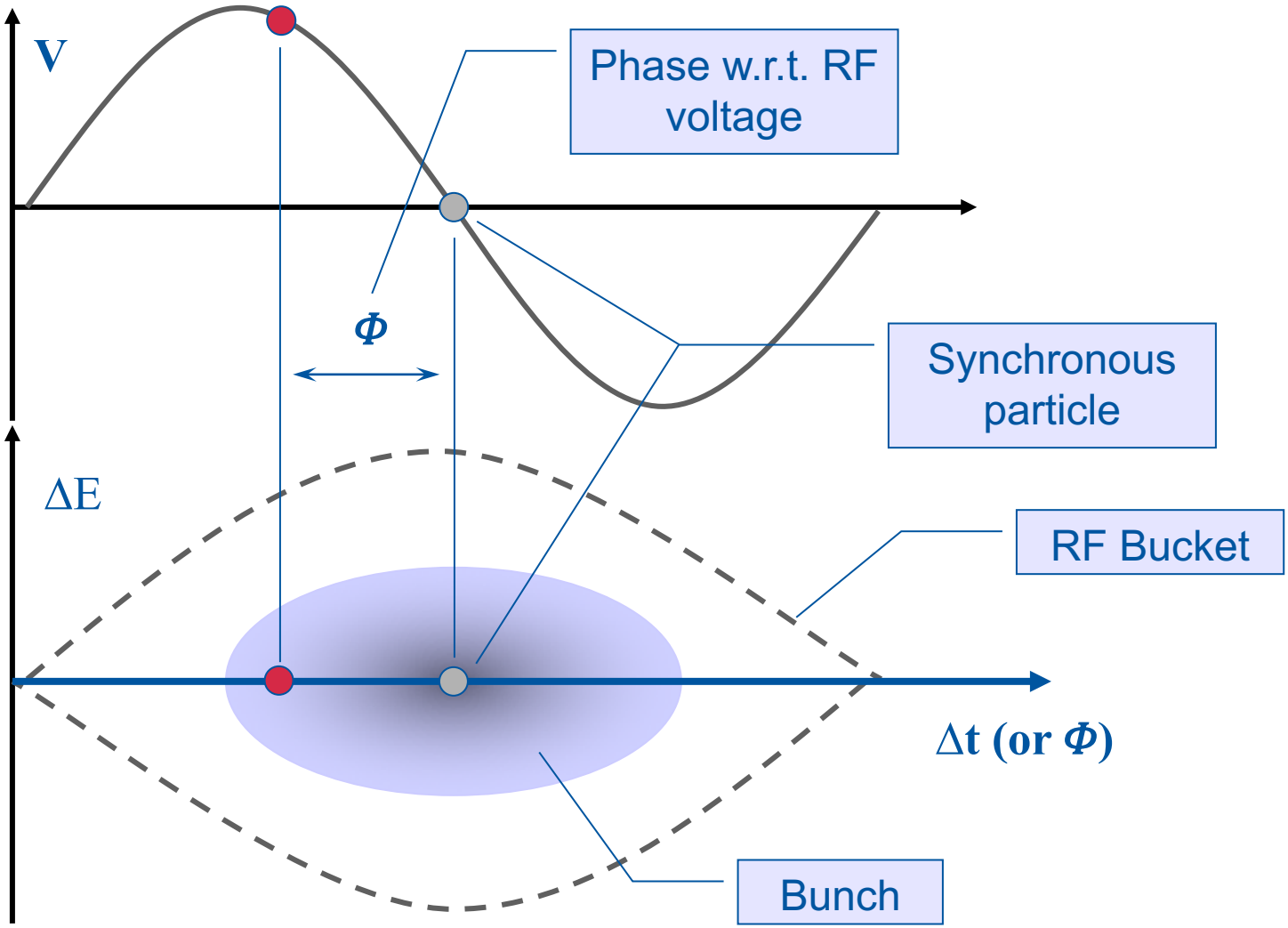
Lower energy ⇨ slower orbit ⇨ lower F_{rev} ⇨ next time particle will be **later**.

- What will happen above transition ?

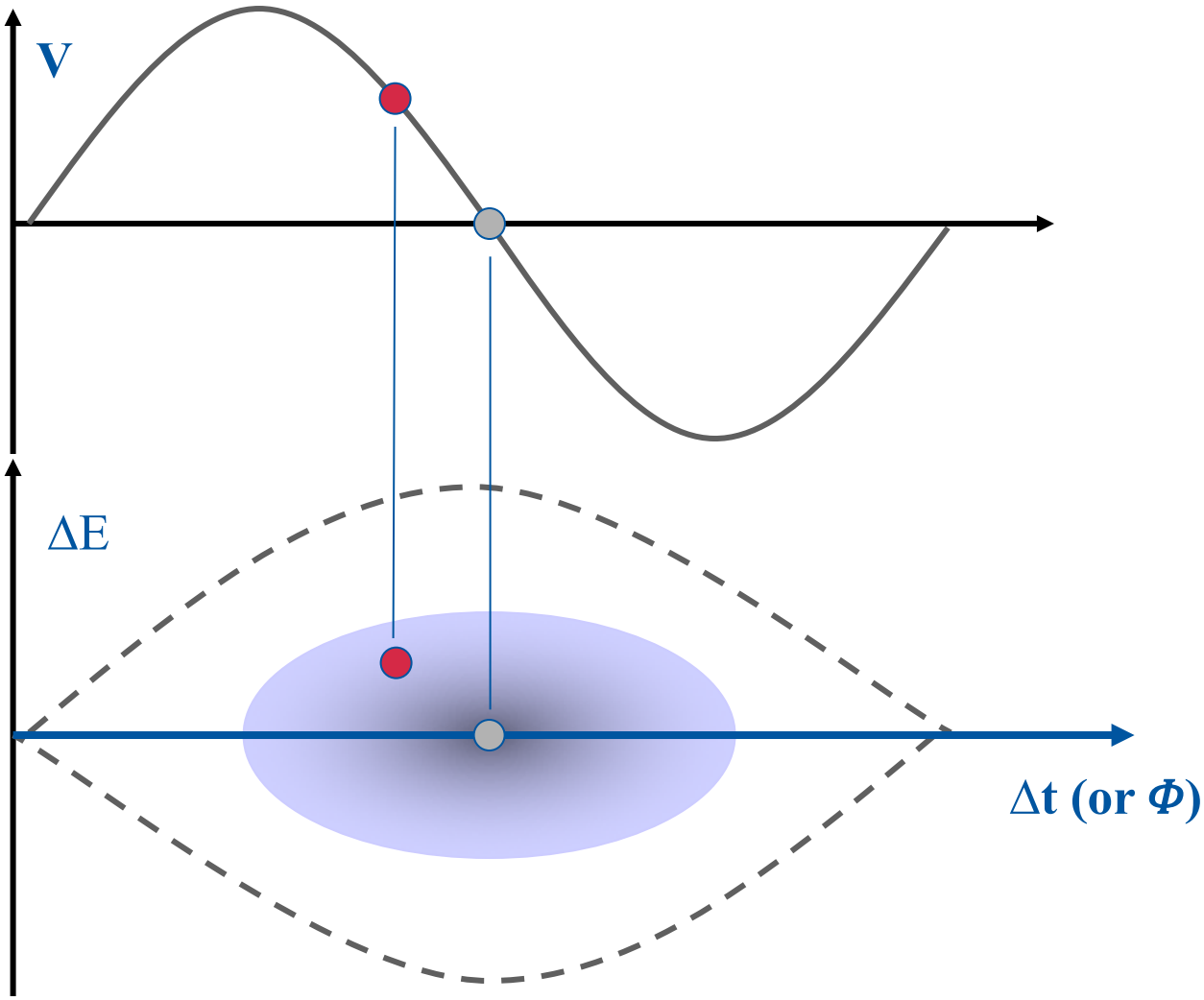
Higher energy ⇨ longer orbit ⇨ lower F_{rev} ⇨ next time particle will be **later**.

Lower energy ⇨ shorter orbit ⇨ higher F_{rev} ⇨ next time particle will be **earlier**.

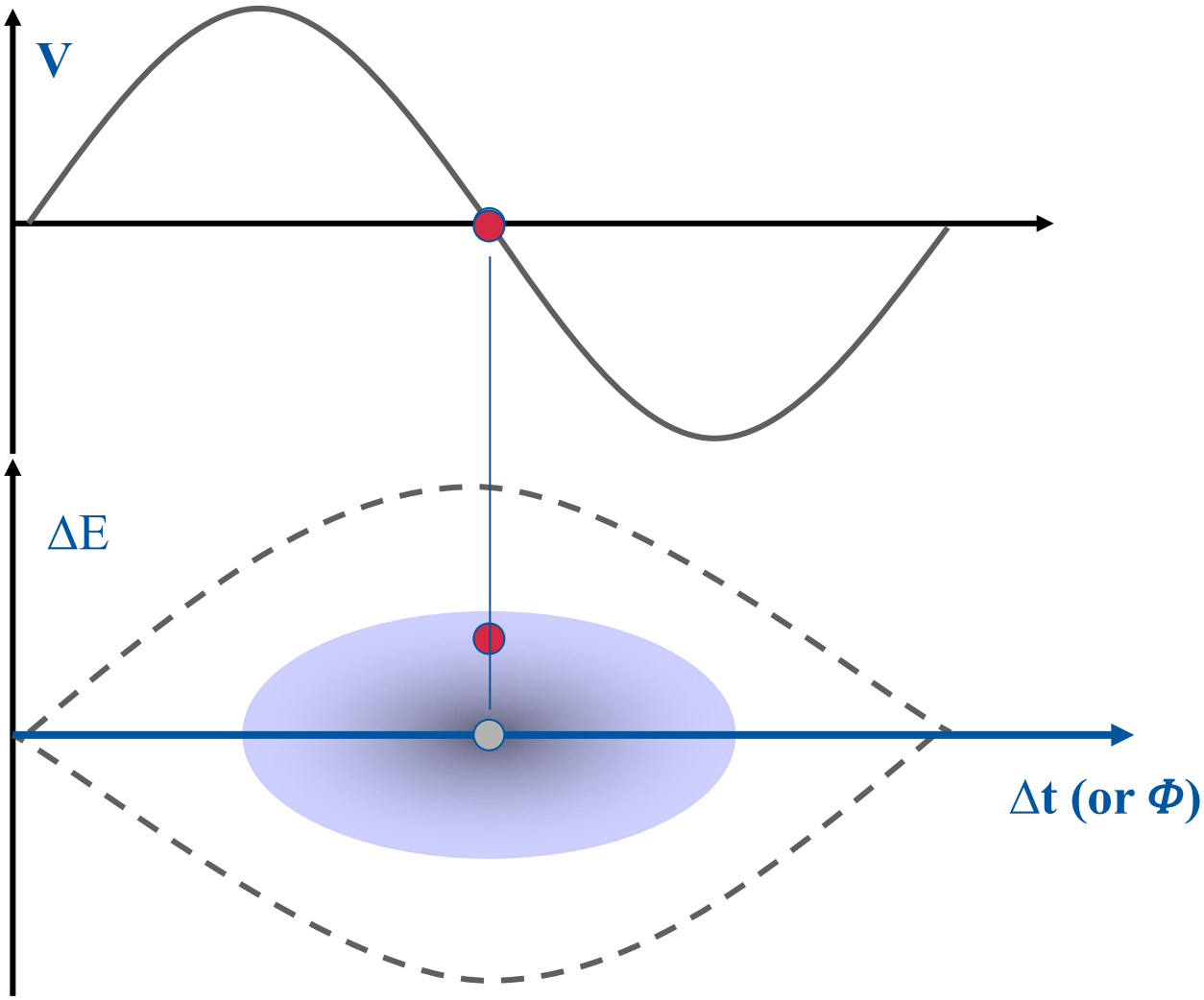
Longitudinal Motion Beyond Transition



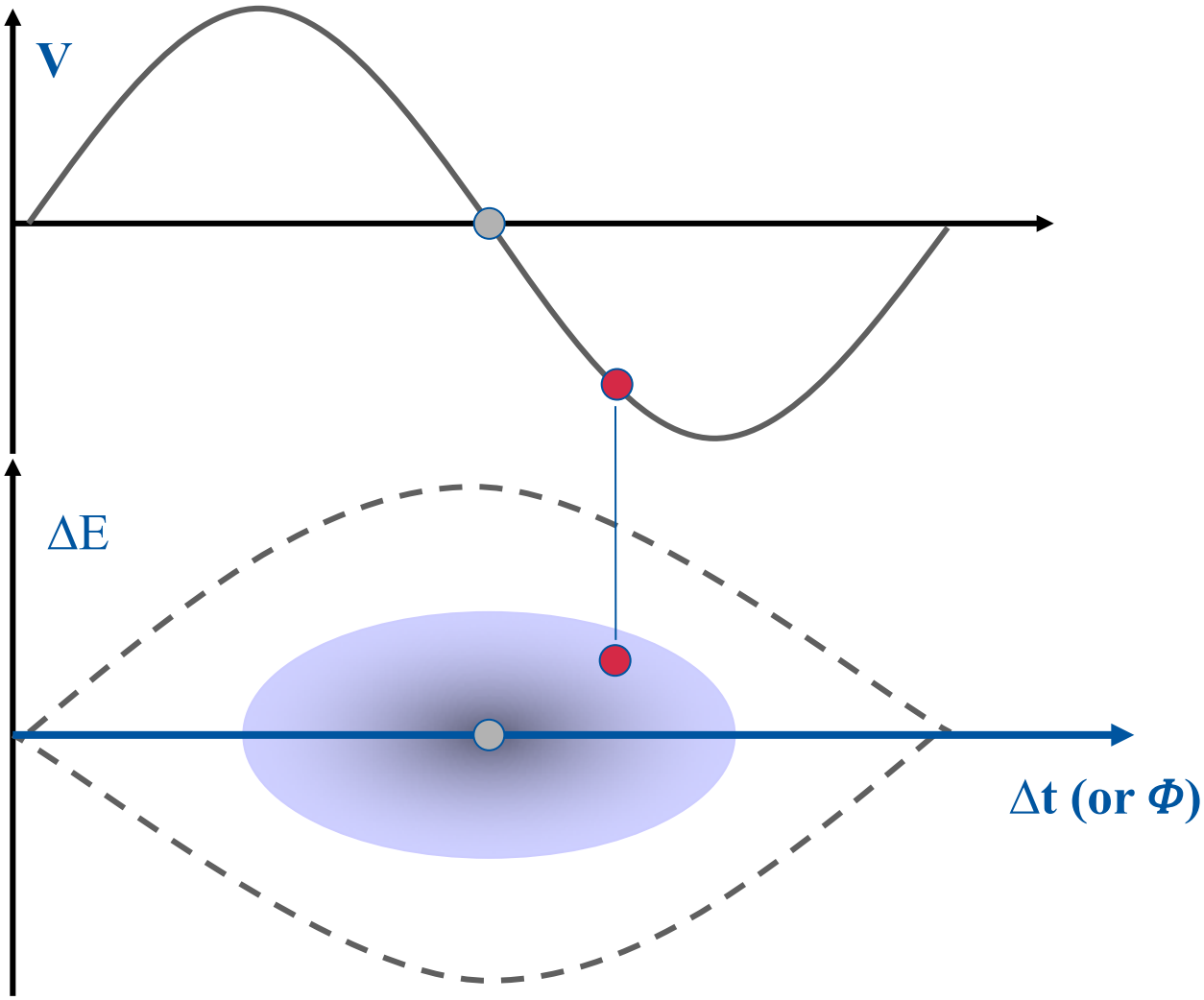
Longitudinal Motion Beyond Transition



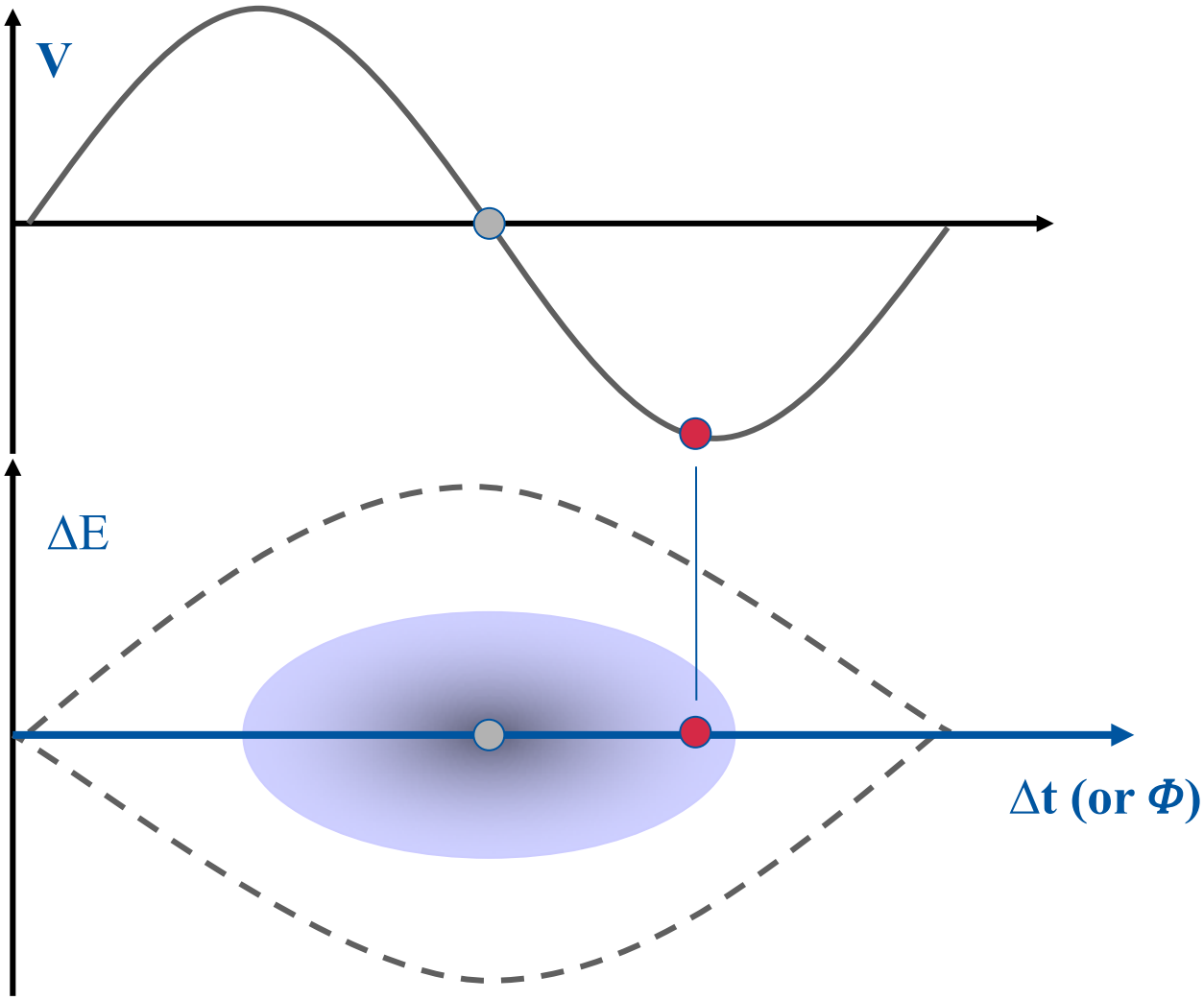
Longitudinal Motion Beyond Transition



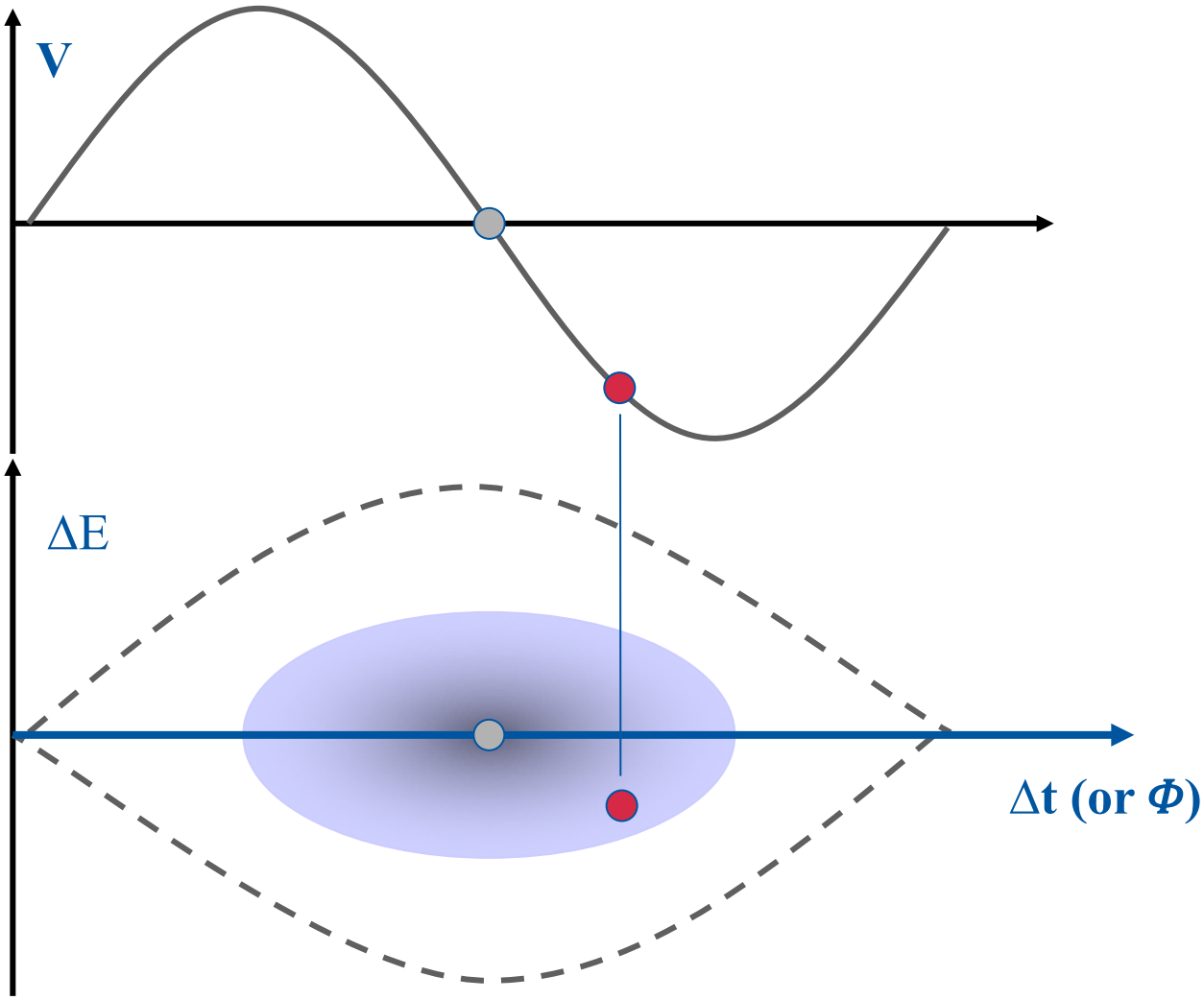
Longitudinal Motion Beyond Transition



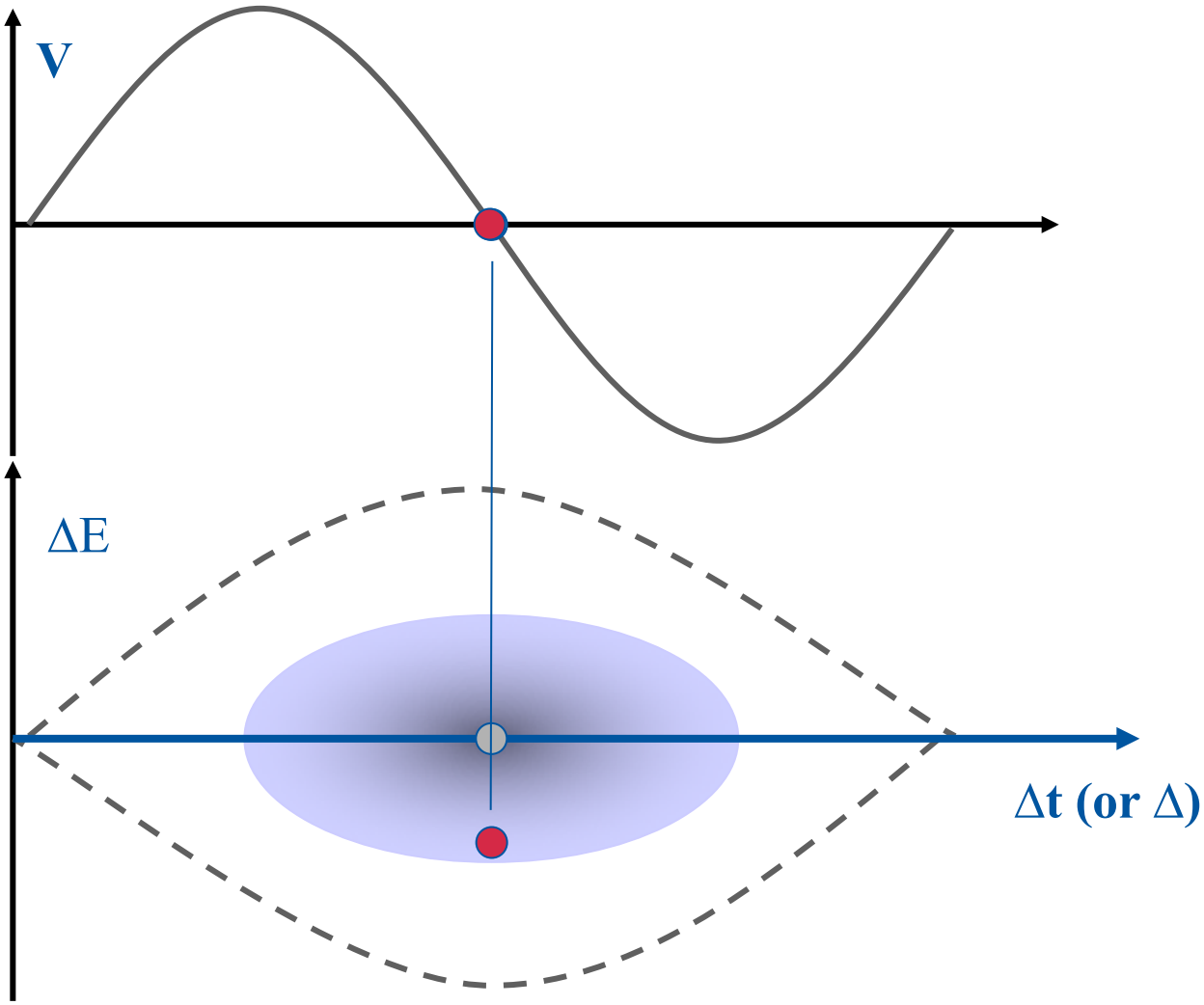
Longitudinal Motion Beyond Transition



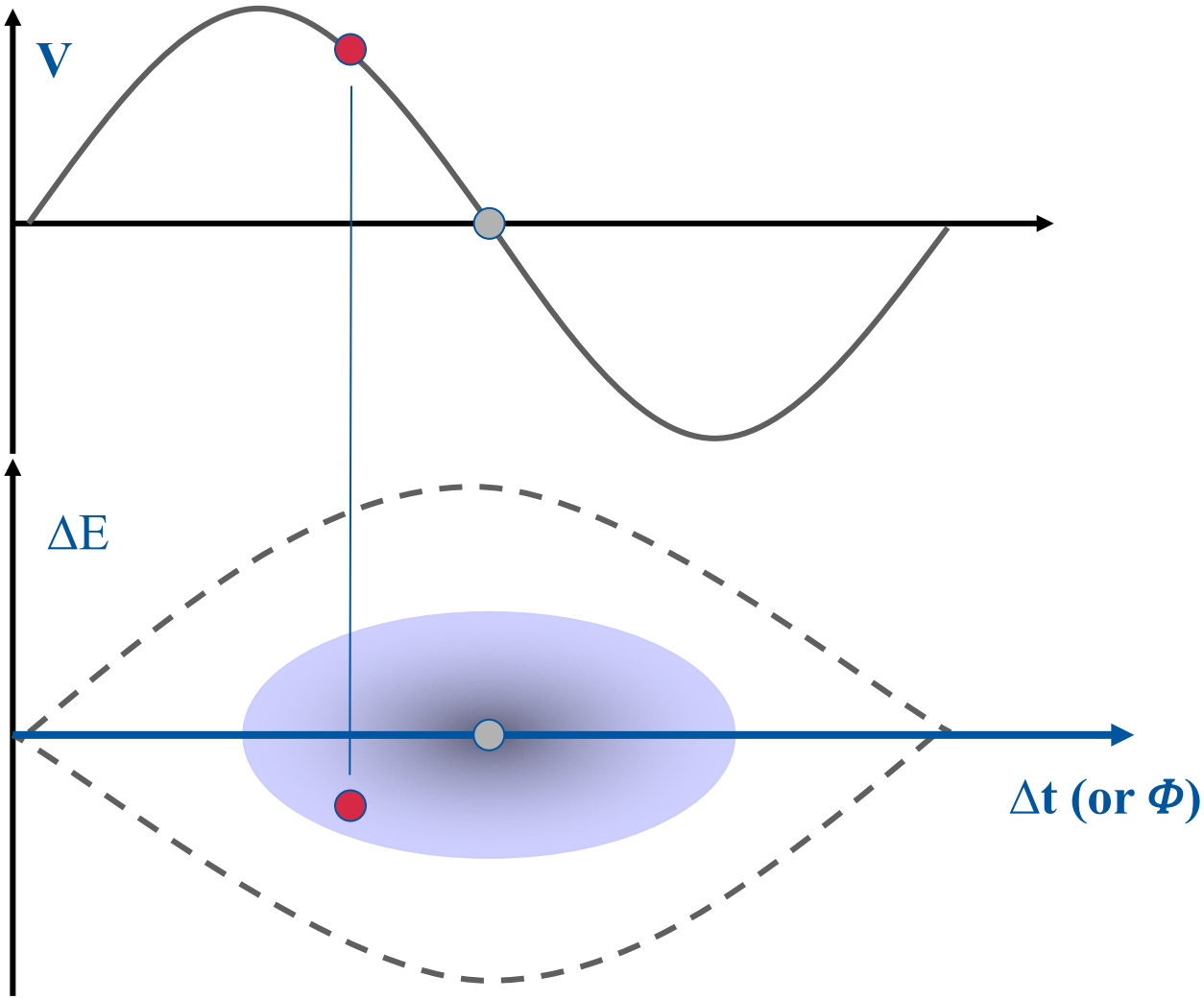
Longitudinal Motion Beyond Transition



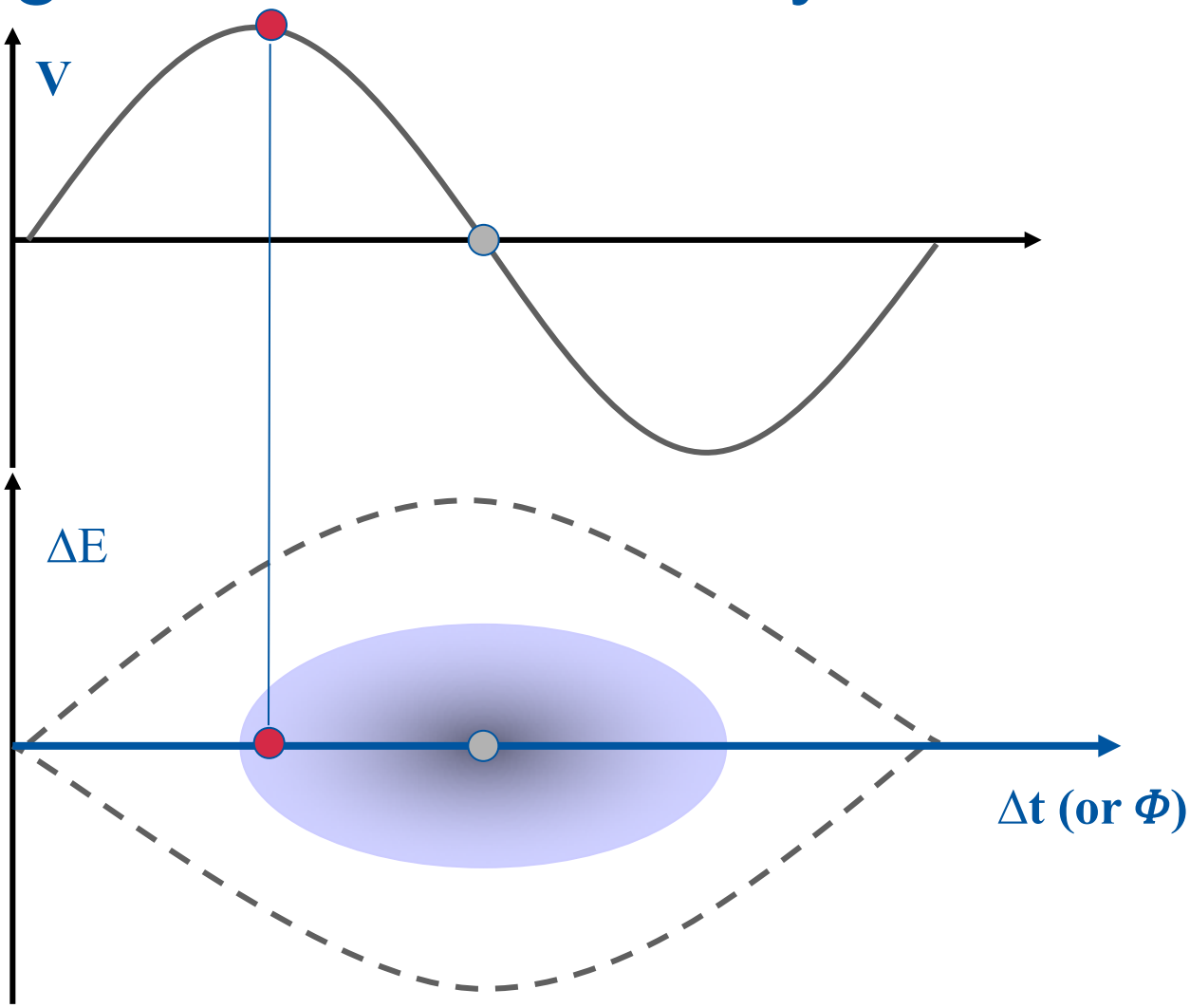
Longitudinal Motion Beyond Transition



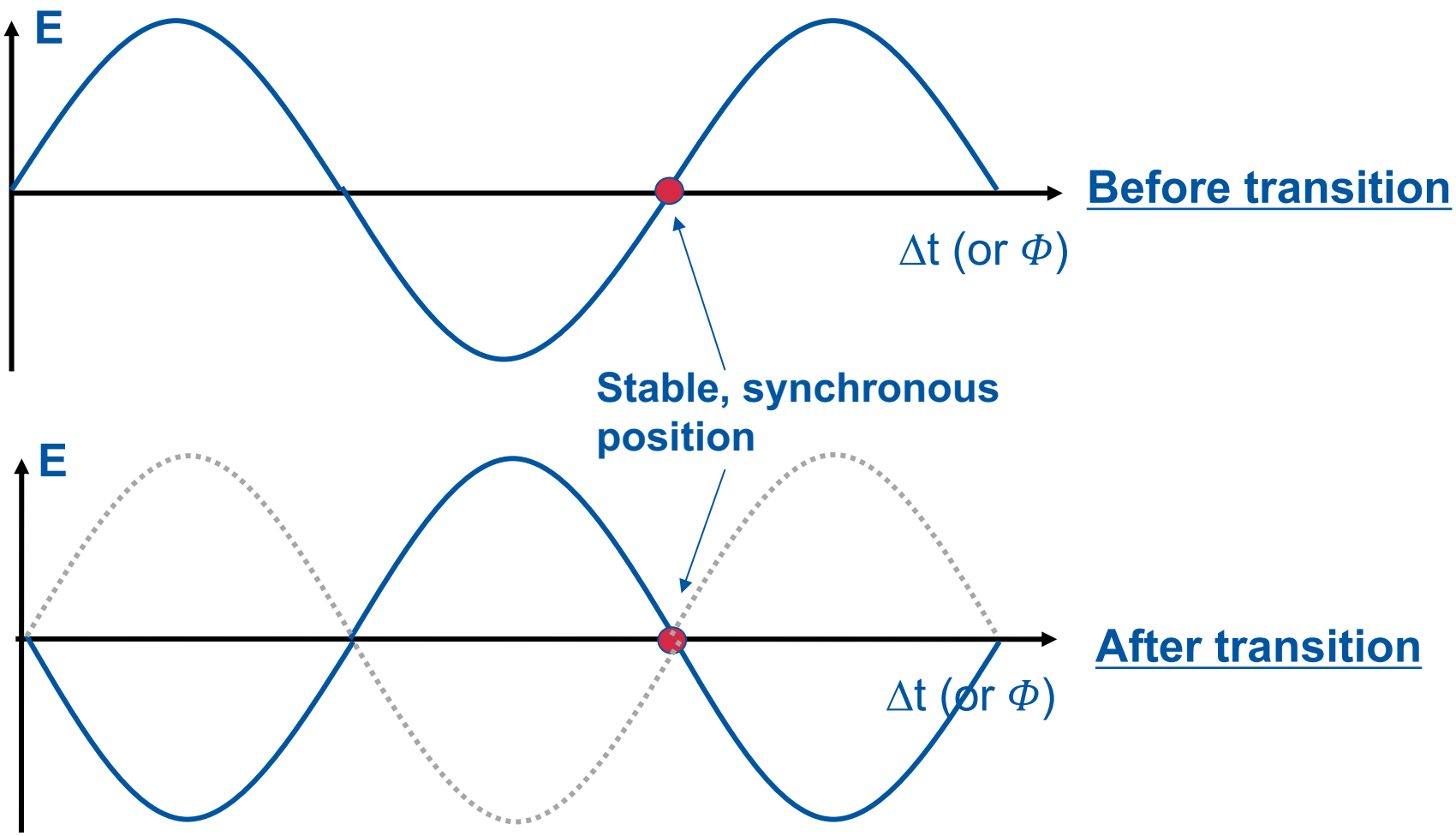
Longitudinal Motion Beyond Transition



Longitudinal Motion Beyond Transition



Before & Beyond Transition



Synchrotron Oscillation

- On each turn the phase, Φ, of a particle w.r.t. the RF waveform changes due to the synchrotron oscillations.

$$\frac{d\phi}{dt} = 2\pi h \Delta f_{rev}$$

Change in revolution frequency

Harmonic number

- We know that $$\frac{df_{rev}}{f_{rev}} = -\eta \frac{dE}{E}$$

- Combining this with the above $$\therefore \frac{d\phi}{dt} = \frac{-2\pi h \eta}{E} \cdot dE \cdot f_{rev}$$

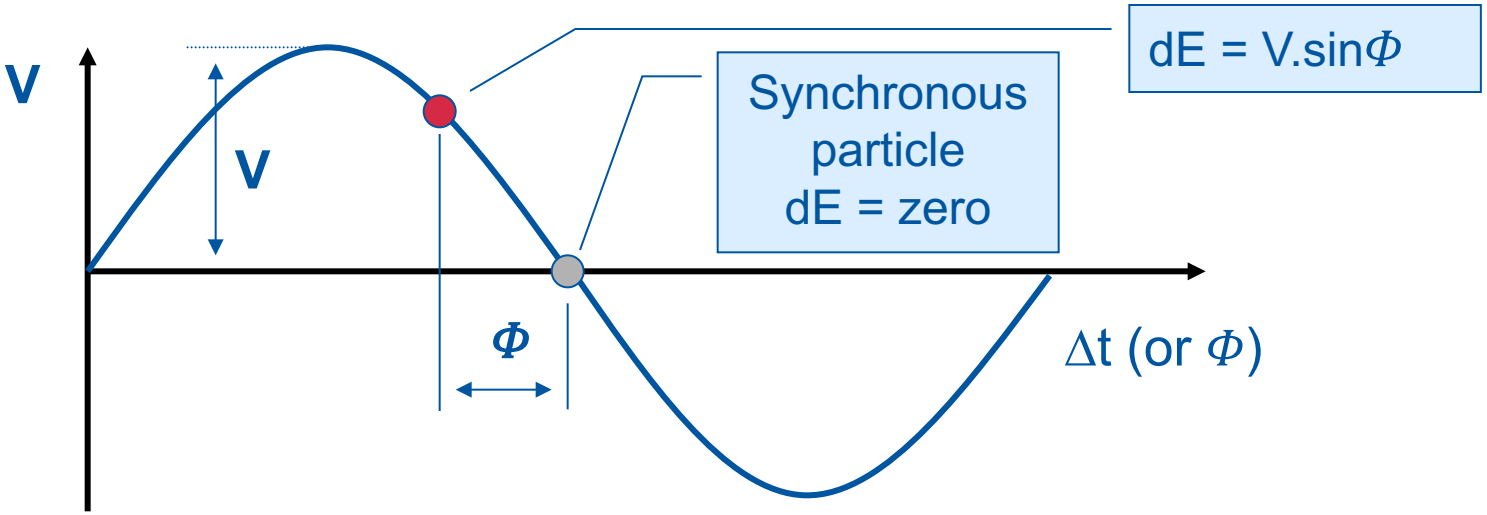
- This can be written as:

$$\frac{d^2\phi}{dt^2} = \frac{-2\pi h \eta}{E} \cdot f_{rev} \cdot \frac{dE}{dt}$$

Synchrotron Oscillation

- So, we have: $$\frac{d^2\phi}{dt^2} = \frac{-2\pi h \eta}{E} \cdot f_{rev} \cdot \frac{dE}{dt}$$

- Where dE is just the energy gain or loss due to the RF system during each turn



Synchrotron Oscillation

$$\frac{d^2\phi}{dt^2} = \frac{-2\pi h\eta}{E} \cdot f_{rev} \cdot \frac{dE}{dt}$$

and

$$dE = V\sin\phi \longrightarrow \frac{dE}{dt} = f_{rev} V \sin\phi$$

$$\frac{d^2\phi}{dt^2} = \frac{-2\pi h\eta}{E} \cdot f_{rev}{}^2 \cdot V.\sin\phi$$

- If Φ is small then **sinΦ=Φ**

$$\frac{d^2\phi}{dt^2} + \left(\frac{2\pi h\eta}{E} \cdot f_{rev}{}^2 \cdot V\right)\phi = 0$$

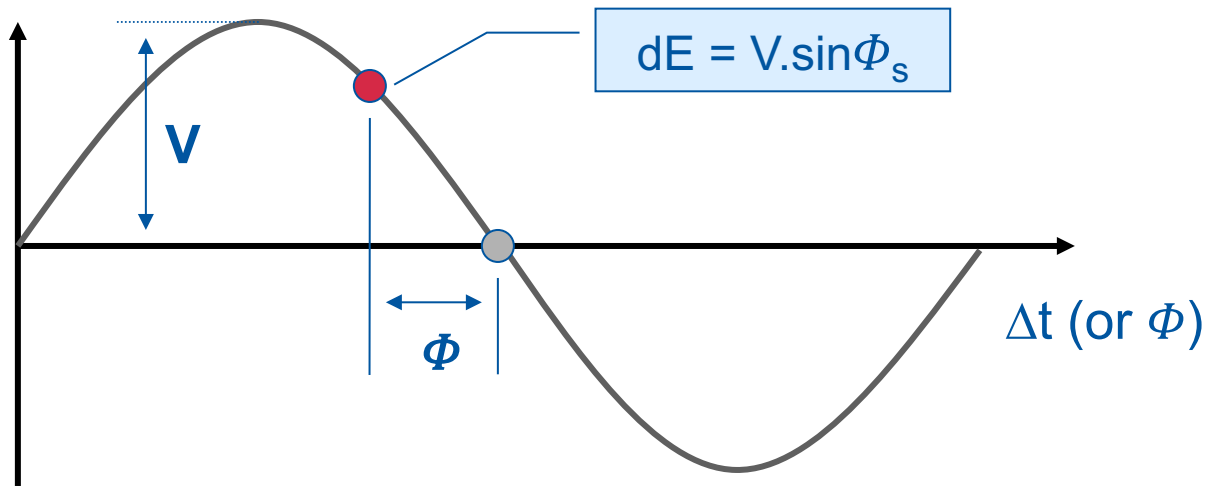
- This is a SHM where the synchrotron oscillation frequency is given by:

$$\left(\sqrt{\frac{2\pi \boldsymbol{h}\eta \boldsymbol{V}}{\boldsymbol{E}}}\right) \cdot \boldsymbol{f}_{rev}$$

Synchrotron tune Qs

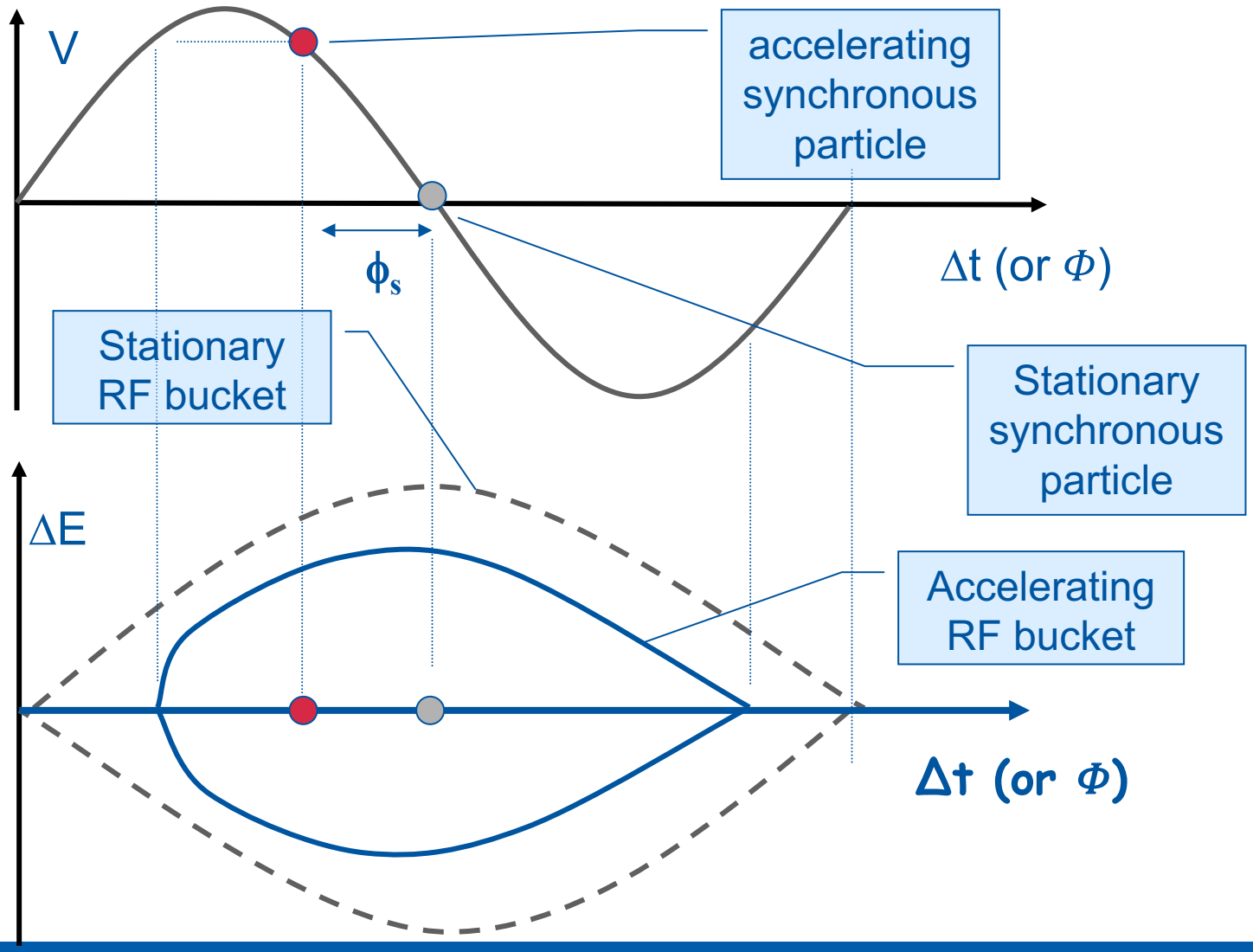
Acceleration

- Increase the magnetic field slightly on each turn.
- The particles will follow a shorter orbit. ($\mathbf{f_{rev} < f_{synch}}$)
- Beyond transition, early arrival in the cavity causes a gain in energy each turn.



- We change the phase of the cavity such that the new synchronous particle is at Φ_s and therefore always sees an accelerating voltage
- $V_s = V\sin\Phi_s = V\Gamma$ = energy gain/turn = dE

Accelerating Bucket



Accelerating Bucket

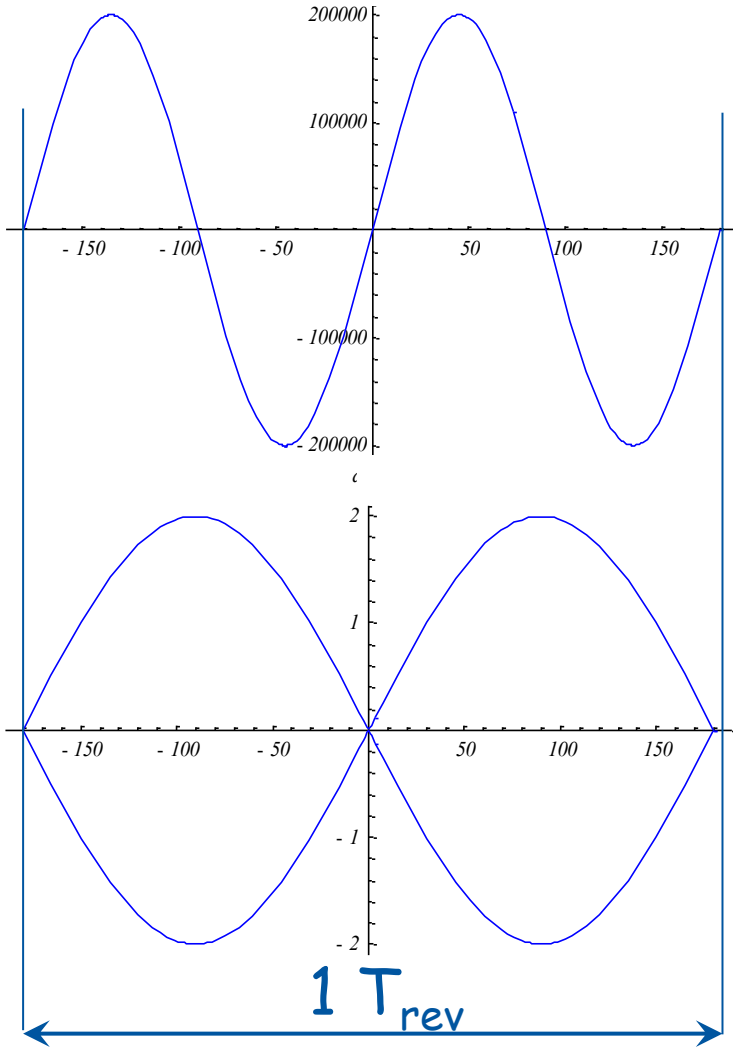
- The modification of the RF bucket reduces the acceptance
- The faster we accelerate (increasing sin Φ_s) the smaller the acceptance
- Faster acceleration also modifies the synchrotron tune.
- For a stationary bucket (Φs = 0) we had:

$$\left(\sqrt{\frac{2\pi h \eta}{E}}\right) \cdot f_{rev}$$

- For a moving bucket (Φs ≠ 0) this becomes:

$$\left(\sqrt{\frac{2\pi h \eta}{E}}\right) \cdot f_{rev} \cos\phi_s$$

Harmonics & Buckets



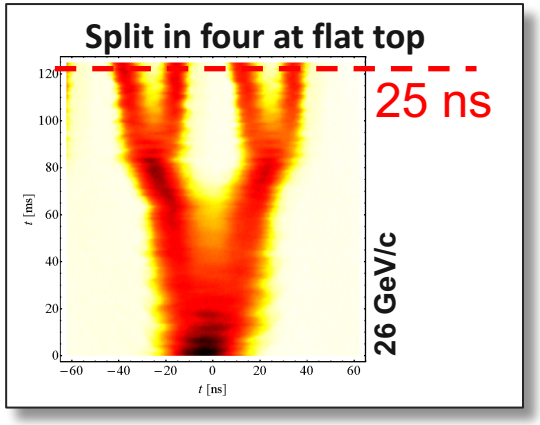
$$f_{rf} = h \times f_{rev}$$

Variable for $\beta < 1$

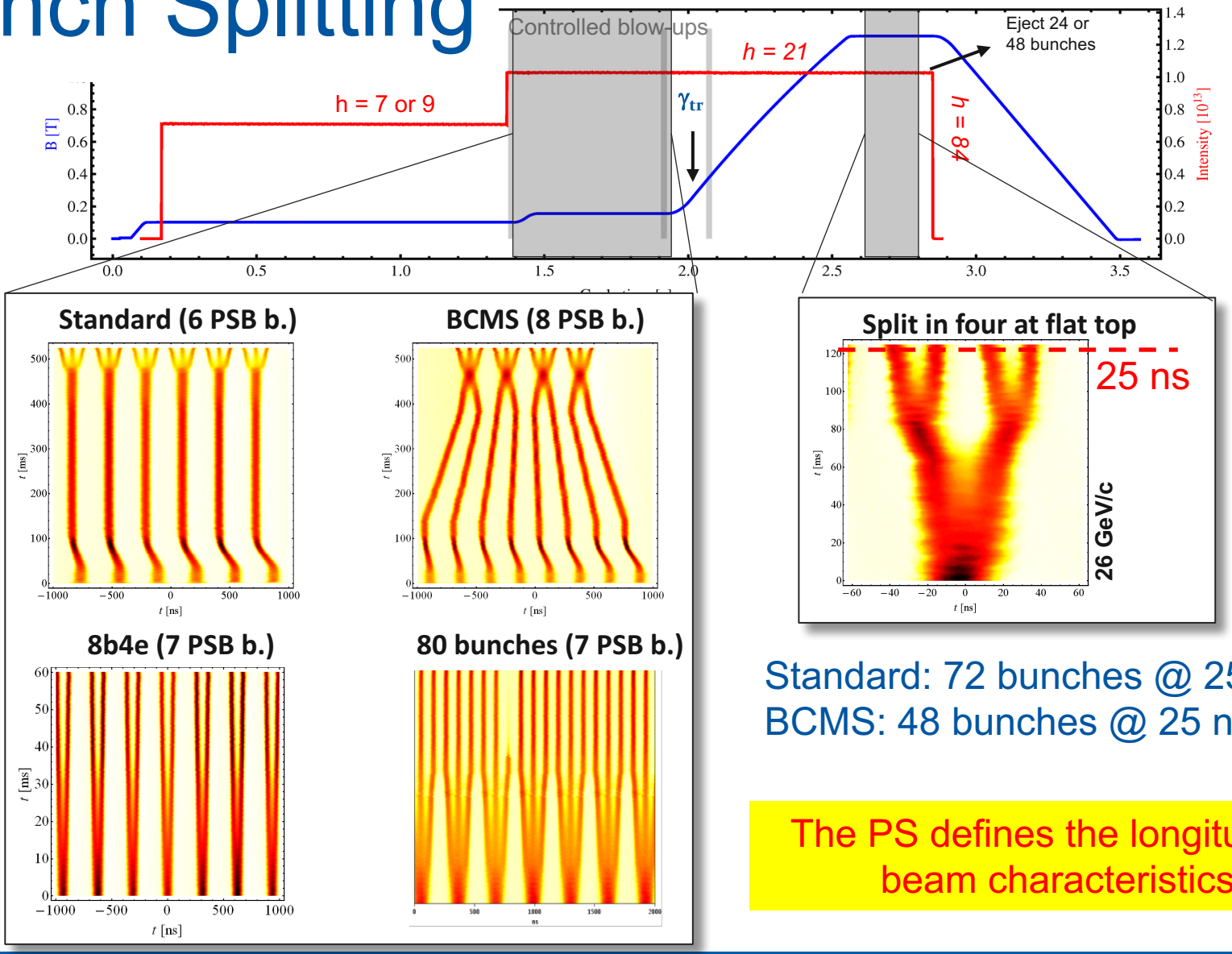
Frequency of cavity voltage

Harmonic number

- We will have: **h buckets**
- Doing this dynamically, we can perform bunch splitting



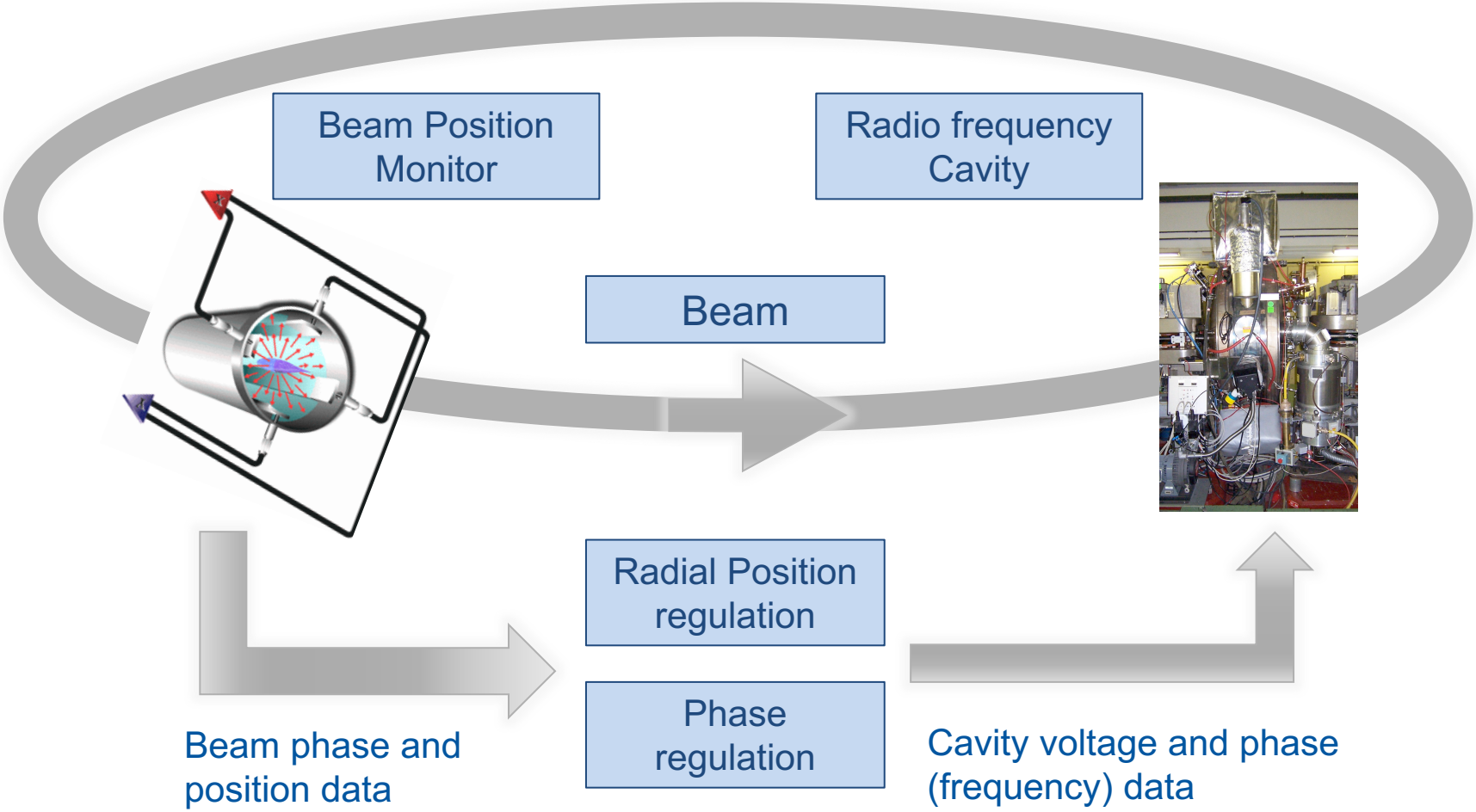
Bunch Splitting



Standard: 72 bunches @ 25 ns
BCMS: 48 bunches @ 25 ns

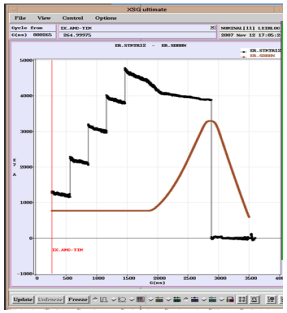
The PS defines the longitudinal beam characteristics

RF Beam Control



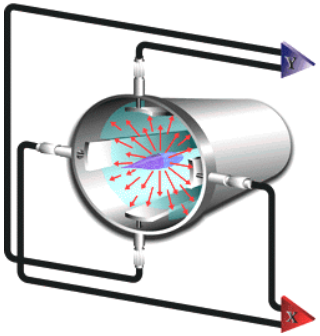
Main Diagnostics Tools

Beam Current & Position



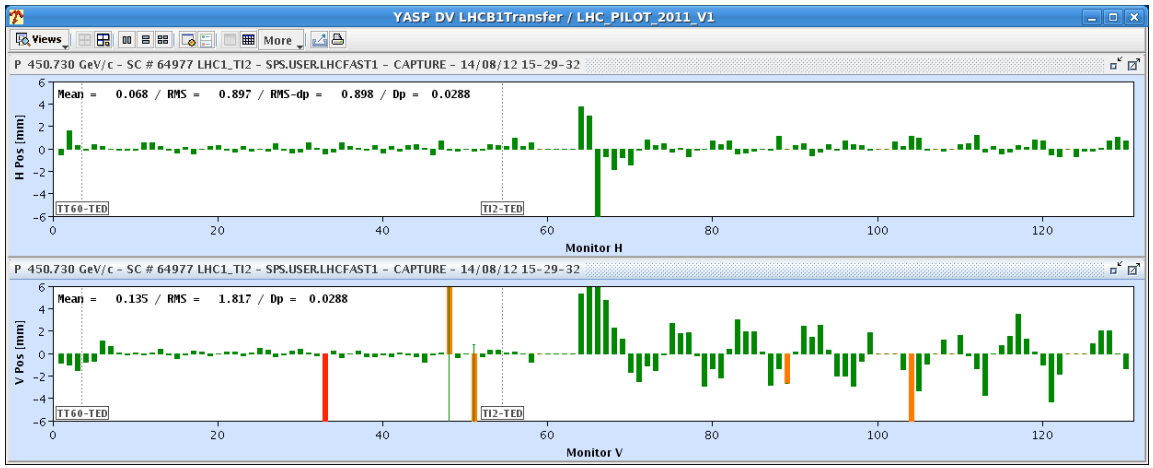
Beam intensity or current measurement:

- Working as classical transformer
- The beam acts as a primary winding



Beam position/orbit measurement:

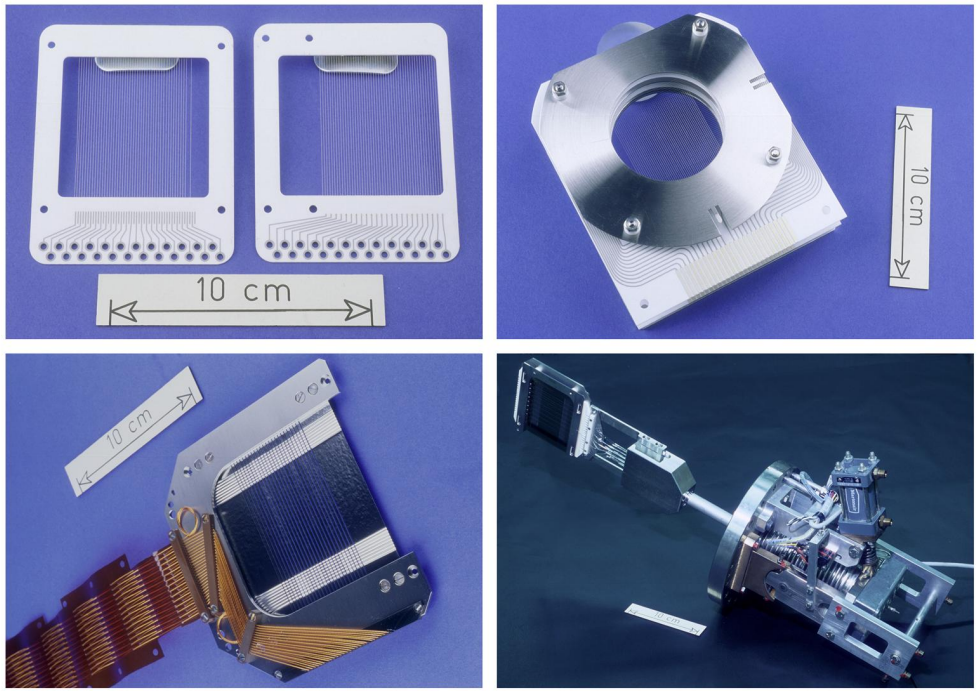
$$x = a \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} \equiv a \cdot \frac{\Delta}{\Sigma}$$

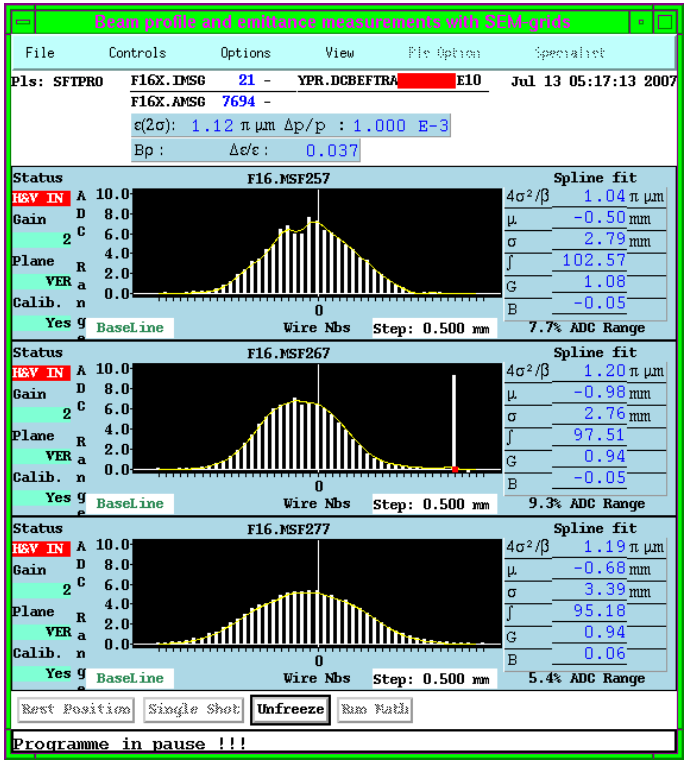


Correcting orbit using automated beam steering

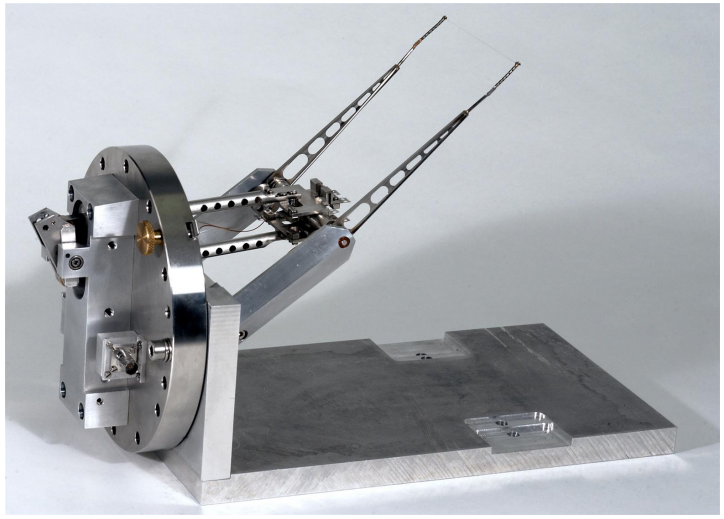
Transverse Beam Profile Monitor

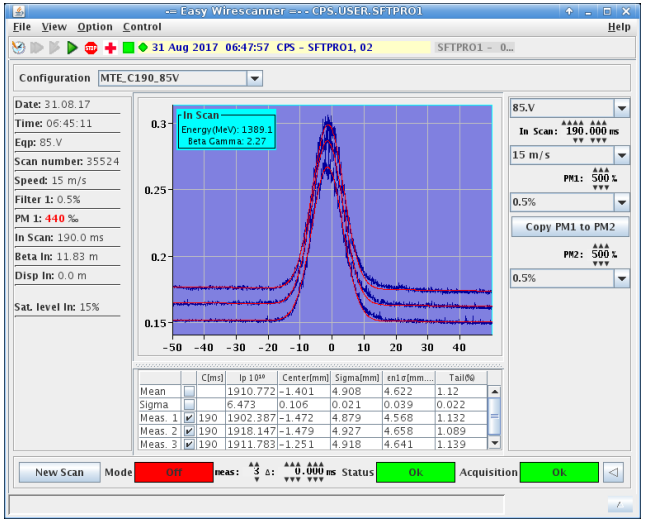
- Transverse beam profile/size measurement:
  - Secondary EMission Grids (SEM-Grid)
  - Based on integration of induced current

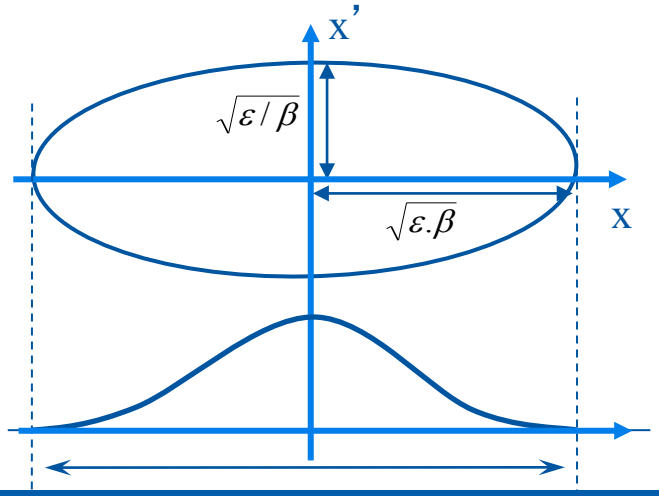


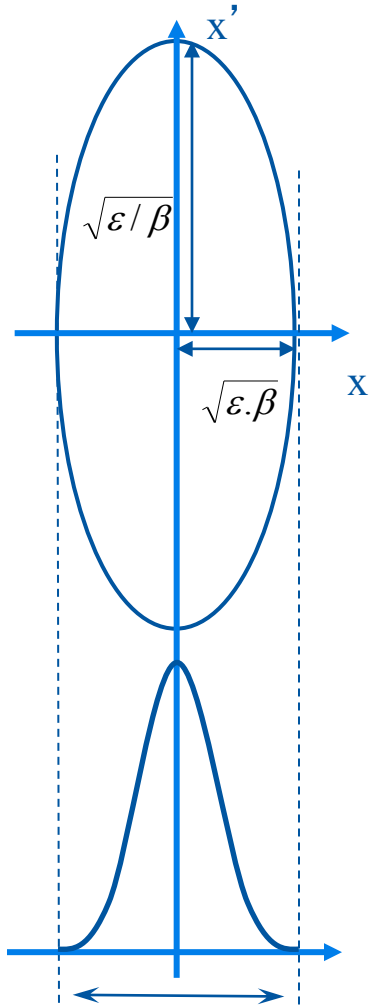


Transverse Beam Profile Measurement





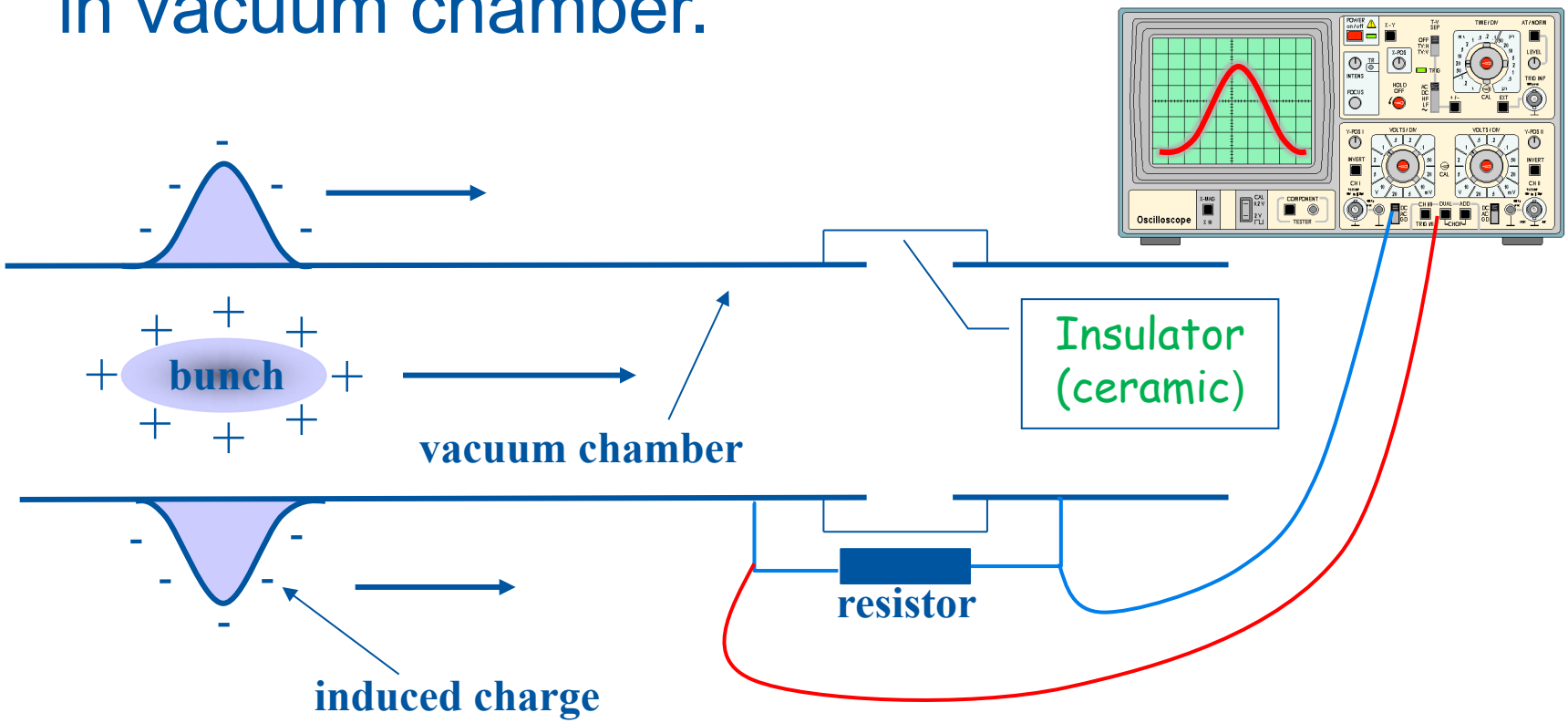




- (Fast) wire scanner
- Uses photo multipliers to measure scintillator light produced by secondary particles

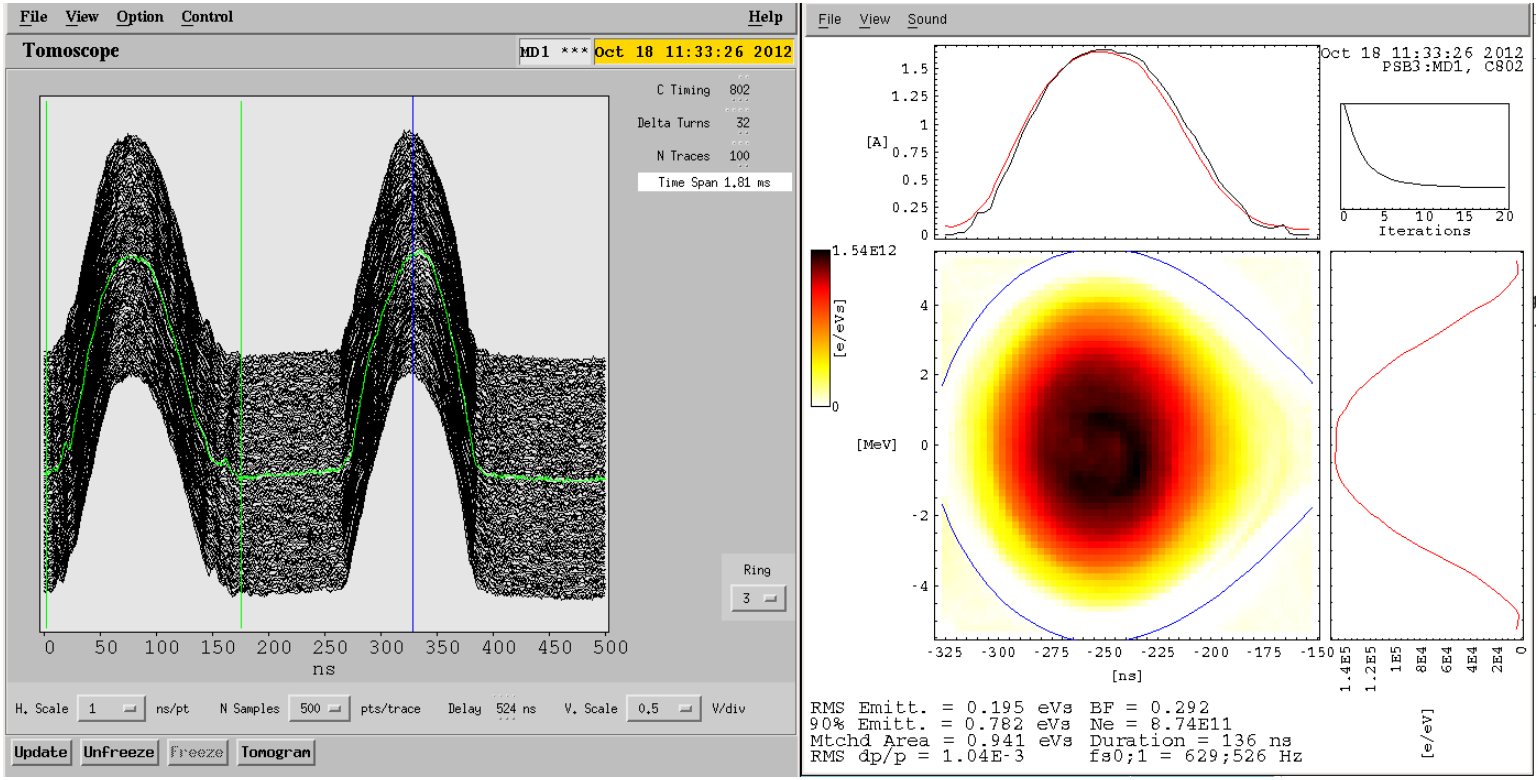
Wall Current Monitor

- A circulating bunch creates an image current in vacuum chamber.



- The induced image current is the same size but has the opposite sign to the bunch current.

Longitudinal TomoScope

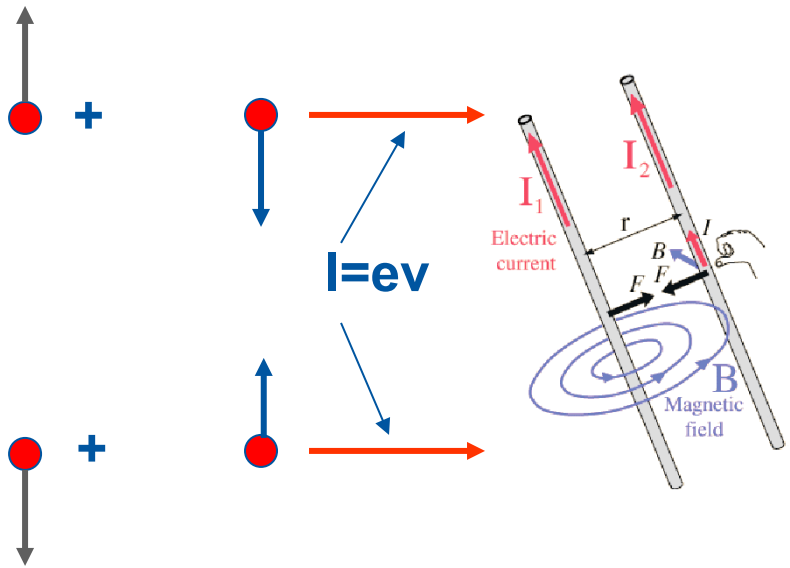


- Make use of the synchrotron motion that turns the “patient” in the Wall Current monitor

Possible Limitations

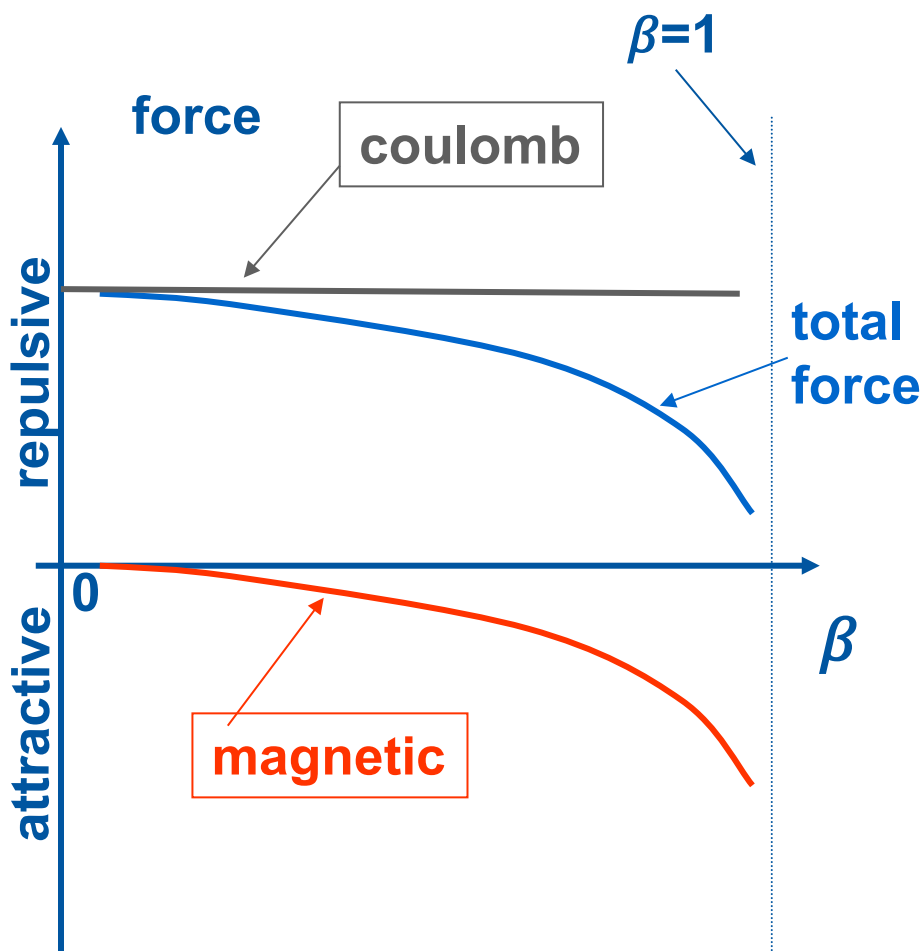
Space Charge

- Between two charged particles in a beam we have different forces:



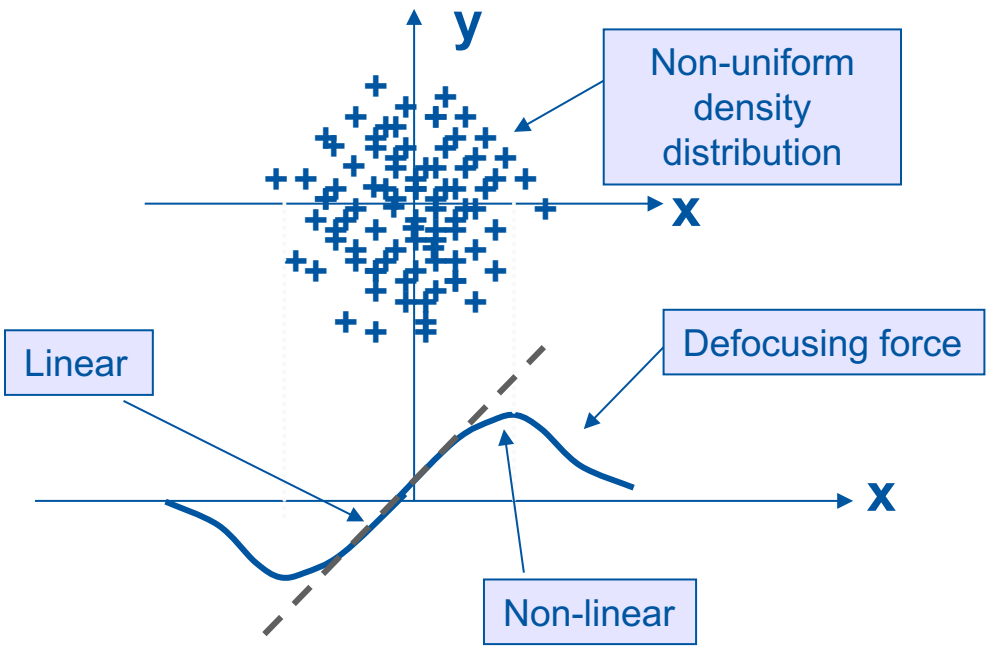
Coulomb repulsion

Magnetic attraction



Space Charge

- At low energies, which means β<<1, the force is mainly repulsive ⇒ defocusing
- It is zero at the centre of the beam and maximum at the edge of the beam



Laslett Tune Shift

- For the non-uniform beam distribution, this non-linear defocusing means the ΔQ is a function of x (transverse position)
- This leads to a spread of tune shift across the beam
- This tune shift is called the 'LASLETT tune shift'

$$\Delta Q_{h,v} \approx -\frac{r_0 N}{4\pi\varepsilon_{h,v}\beta^2\gamma^3}$$

Beam intensity

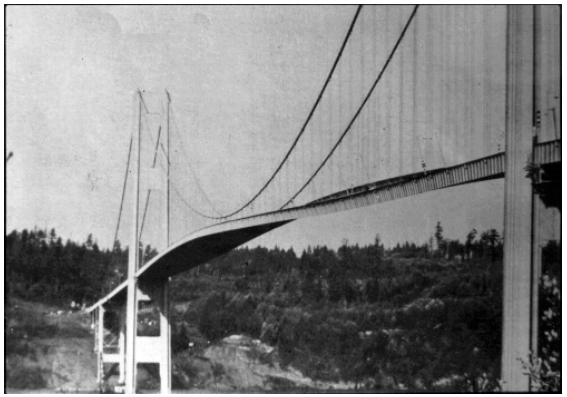
Relativistic parameters

Transverse emittance

- This tune spread cannot be corrected and does get very large at high intensity and low momentum

Imperfections & Resonances

- Machines and elements cannot be built and aligned with infinite perfection

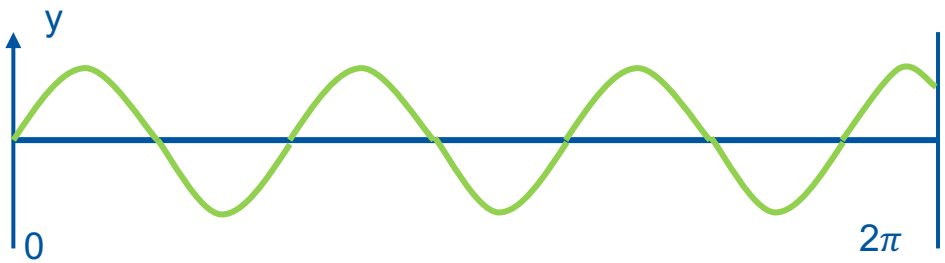


- Same phase and frequency for driving force and the system can cause resonances and be destructive

- We have to ask ourselves:
  - What will happen to the betatron oscillations due to the different field errors.
  - Therefore we need to consider errors in dipoles, quadrupoles, sextupoles, etc…

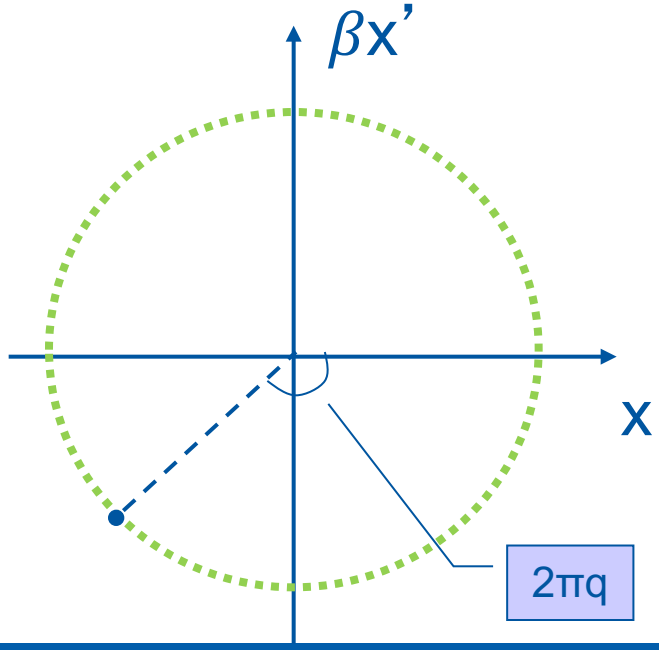
Phase Space & Betatron Tune

- If we unfold a trajectory of a particle that makes one turn in our machine with a tune of Q = 3.333, we get:



- This is the same as going 3.333 time around on the circle in phase space
- The net result is 0.333 times around the circular trajectory in the normalised phase space
- q is the fractional part of Q
- So here Q= 3.333 and q = 0.333

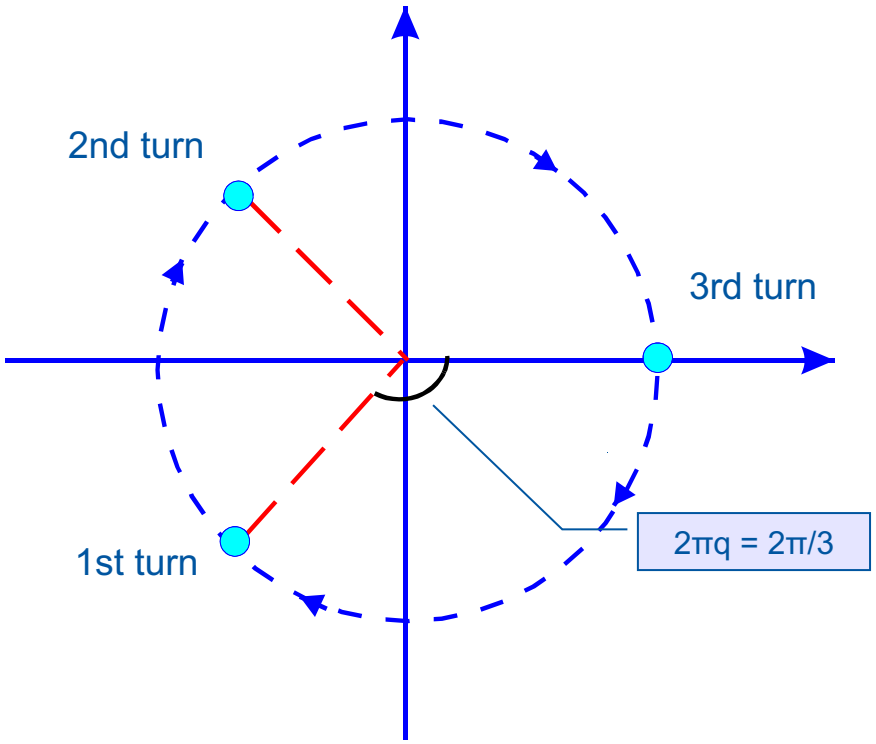
Normalised phase space



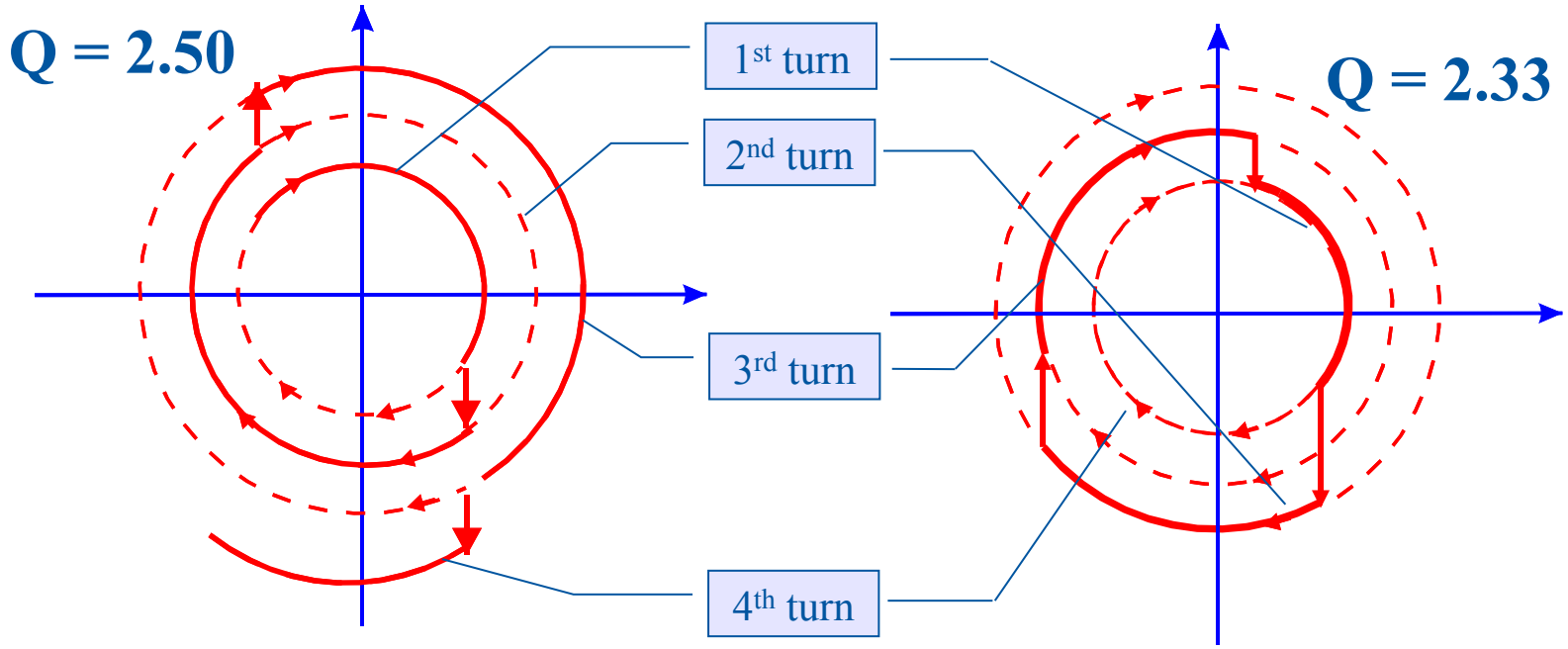
Resonance

- If the phase advance per turn is 120º then the betatron oscillation will repeat itself after 3 turns.
- This could correspond to Q = 3.333 or 3Q = 10
- But also Q = 2.333 or 3Q = 7
- The order of a resonance is defined as:

$$n \times Q = integer$$



Quadrupole (defl. ∝ position)



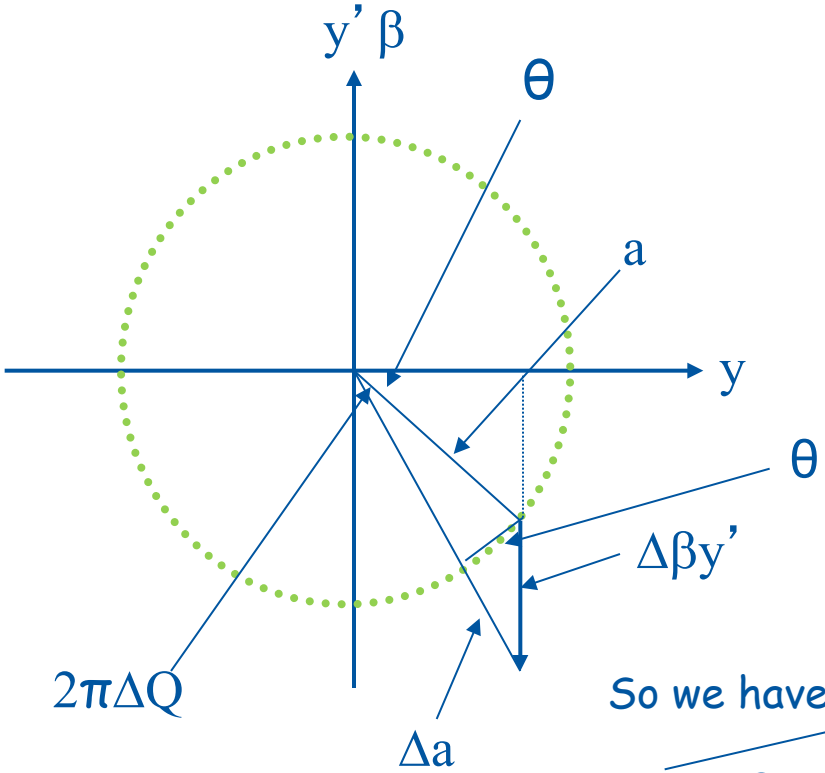
- For **Q = 2.50**: Oscillation induced by the **quadrupole kick** grows on each turn and the particle is lost

  (2nd order resonance 2Q = 5)

- For **Q = 2.33**: Oscillation is cancelled out **every third turn**, and therefore the particle **motion is stable**.

A more rigorous approach (1)

- Let us try to find a **mathematical expression** for the **amplitude growth** in the case of a **quadrupole** error:



2πQ = phase angle over 1 turn = θ
Δβy' = kick
a = old amplitude
Δa = change in amplitude
2πΔQ = change in phase

y does not change at the kick

$$y = a\cos(\theta)$$

In a quadrupole $\Delta y' = lky$

So we have:

Only if 2πΔQ is small

$$\Delta a = \beta \Delta y' \sin(\theta) = l\beta \sin(\theta)\, a\, k \cos(\theta)$$

A more rigorous approach (1)

- So we have: $\Delta a = l \cdot \beta \cdot \sin(\theta)\ a \cdot k \cdot \cos(\theta)$

$$\therefore \frac{\Delta a}{a} = \frac{\ell \beta k}{2} \sin(2\theta)$$

Sin(θ)Cos(θ) = 1/2 Sin (2θ)

- Each turn θ advances by 2πQ
- On the nth turn θ = θ + 2nπQ
- Over many turns: $\frac{\Delta a}{a} = \frac{\ell \beta k}{2} \sum_{n=1}^{\infty} \sin\left(\underline{2(\theta + 2n\pi Q)}\right)$

This term will be 'zero' as it decomposes in Sin and Cos terms and will give a series of + and – that cancel out in all cases where the fractional tune q ≠ 0.5

- So, for q = 0.5 the phase term, 2(θ + 2nπQ) is constant:

$$\sum_{n=1}^{\infty} \sin\left(2(\theta + 2n\pi Q)\right) = \infty$$

and thus: $\frac{\Delta a}{a} = \infty$

- So, resonance for q = 0.5

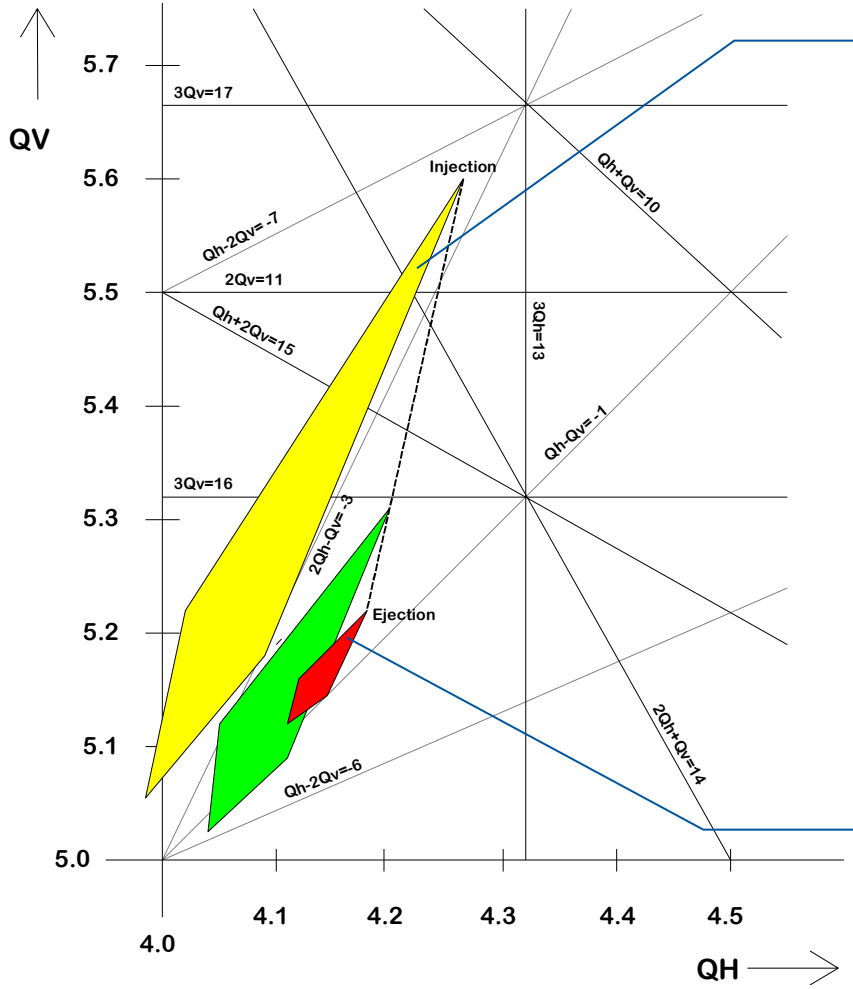
Resonances & Multipoles

- **Quadrupoles** excite **2nd** order resonances (q = 0.5)
- **Sextupoles** excite **1st** and **3rd** order resonances (q = 0.0 & q = 0.33)
- **Octupoles** excite **2nd** and **4th** order resonances (q = 0.25 & q = 0.5)

- This is true for small amplitude particles and low strength excitations

- However, for stronger excitations higher order resonance's can be excited which can be highly non-linear

Resonance & Tune Diagram

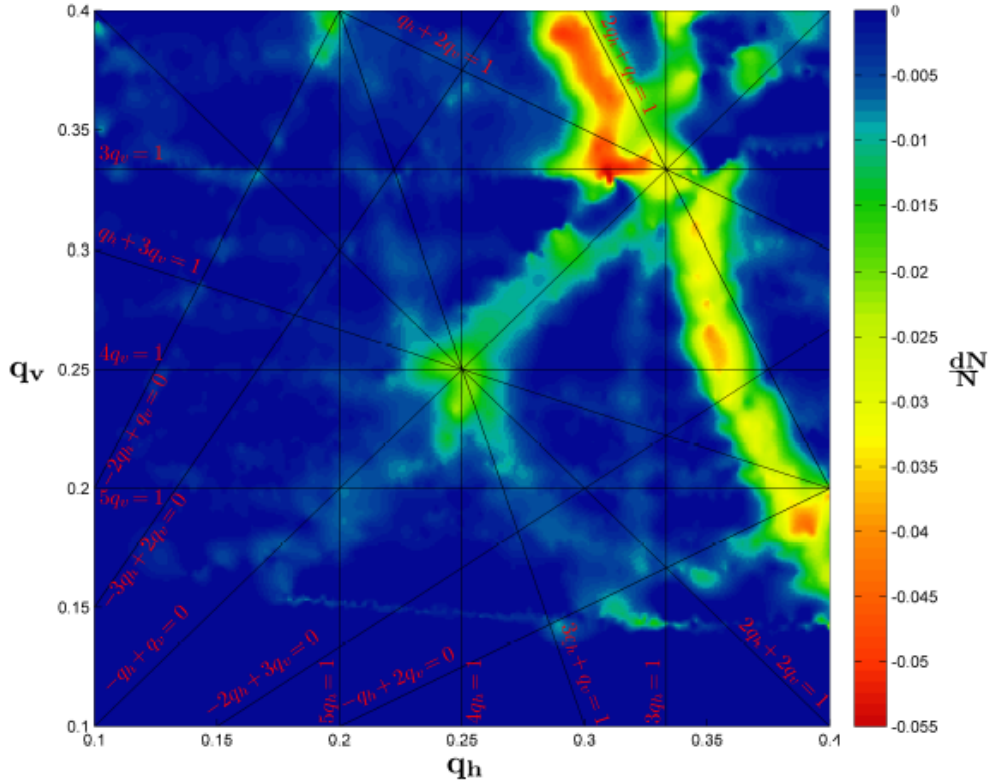


injection

During acceleration we change the horizontal and vertical tune to a place where the beam is the least influenced by resonances.

ejection

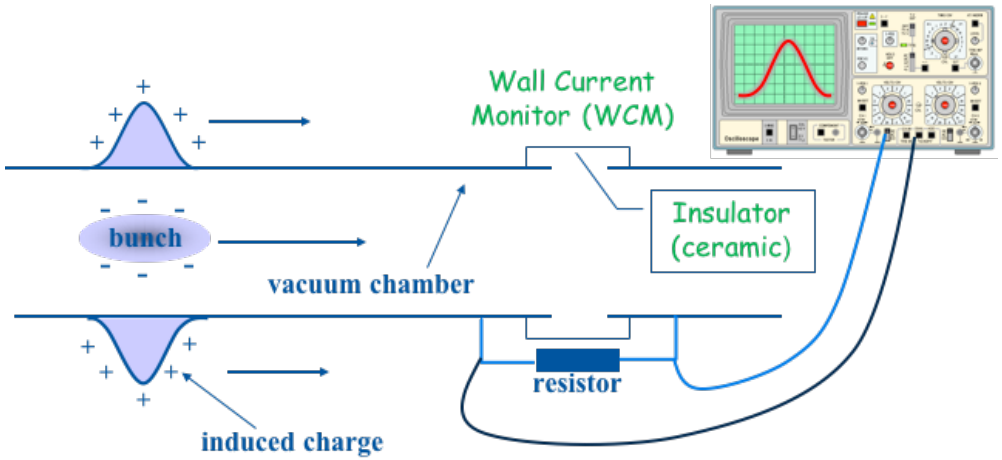
A Measured Tune Diagram

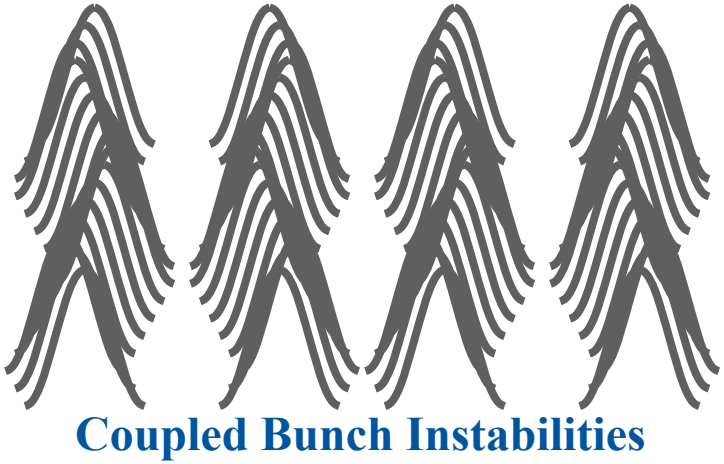


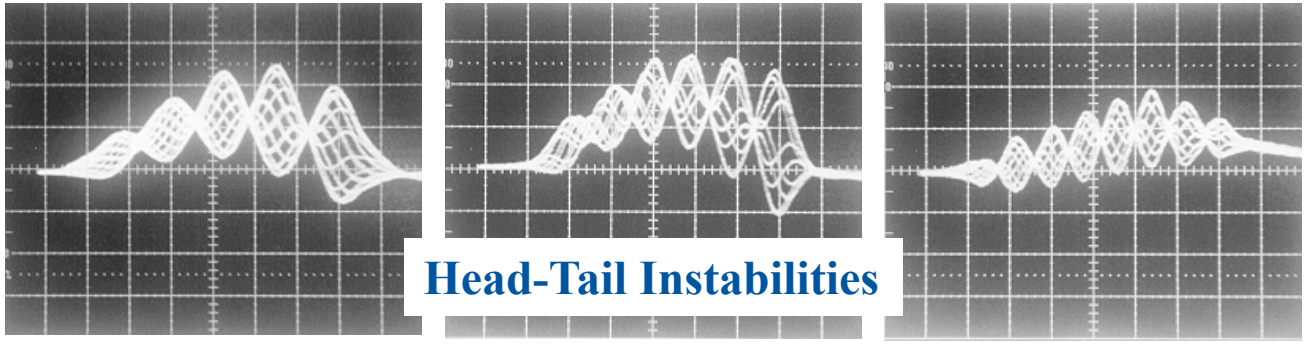
- Move a large emittance low intensity beam around in this tune diagram and measure the beam losses

Collective Effects

- Induced currents in the vacuum chamber (impedance) can result in electric and magnetic fields acting back on the bunch or beam







Cures for Collective Effects

- Ensure a spread in betratron/synchrotron frequencies
  - Increase Chromaticity
  - Apply Octupole magnets (Landau Damping)
- Reduce impedance of your machine
- Avoid higher harmonic modes in cavities
- Apply transverse and longitudinal feedback systems

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