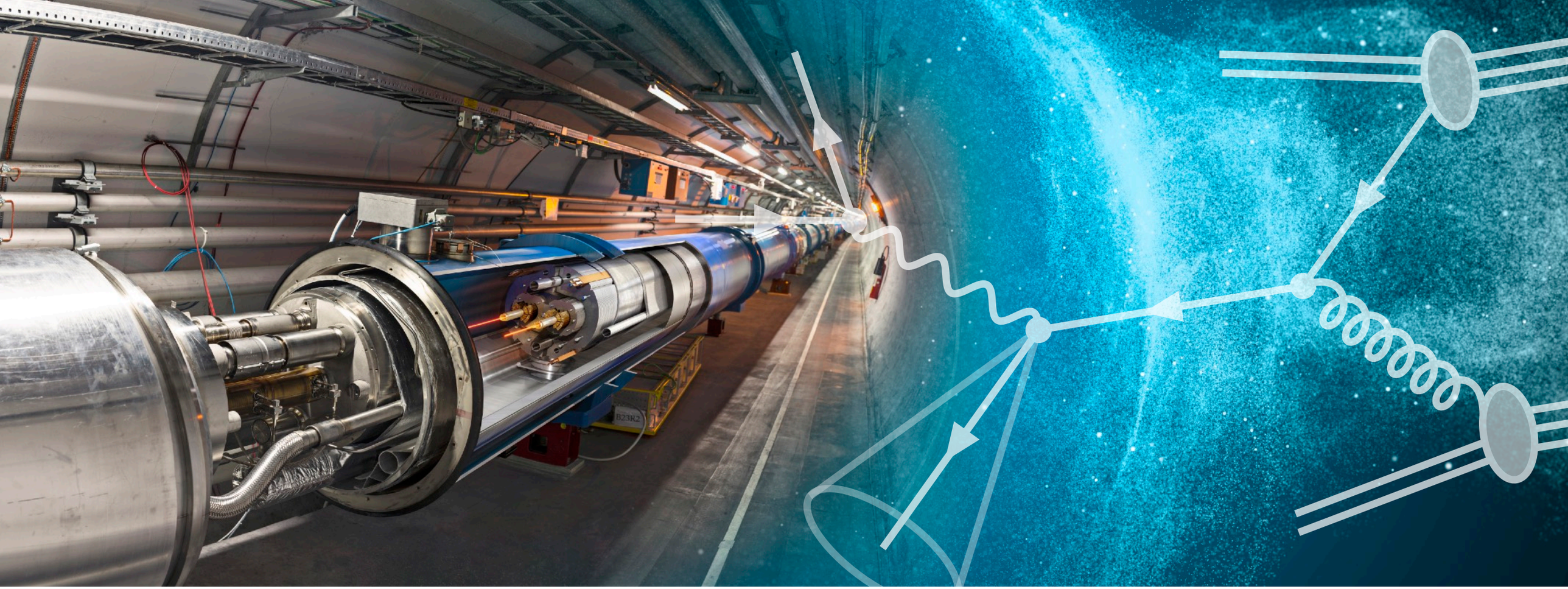


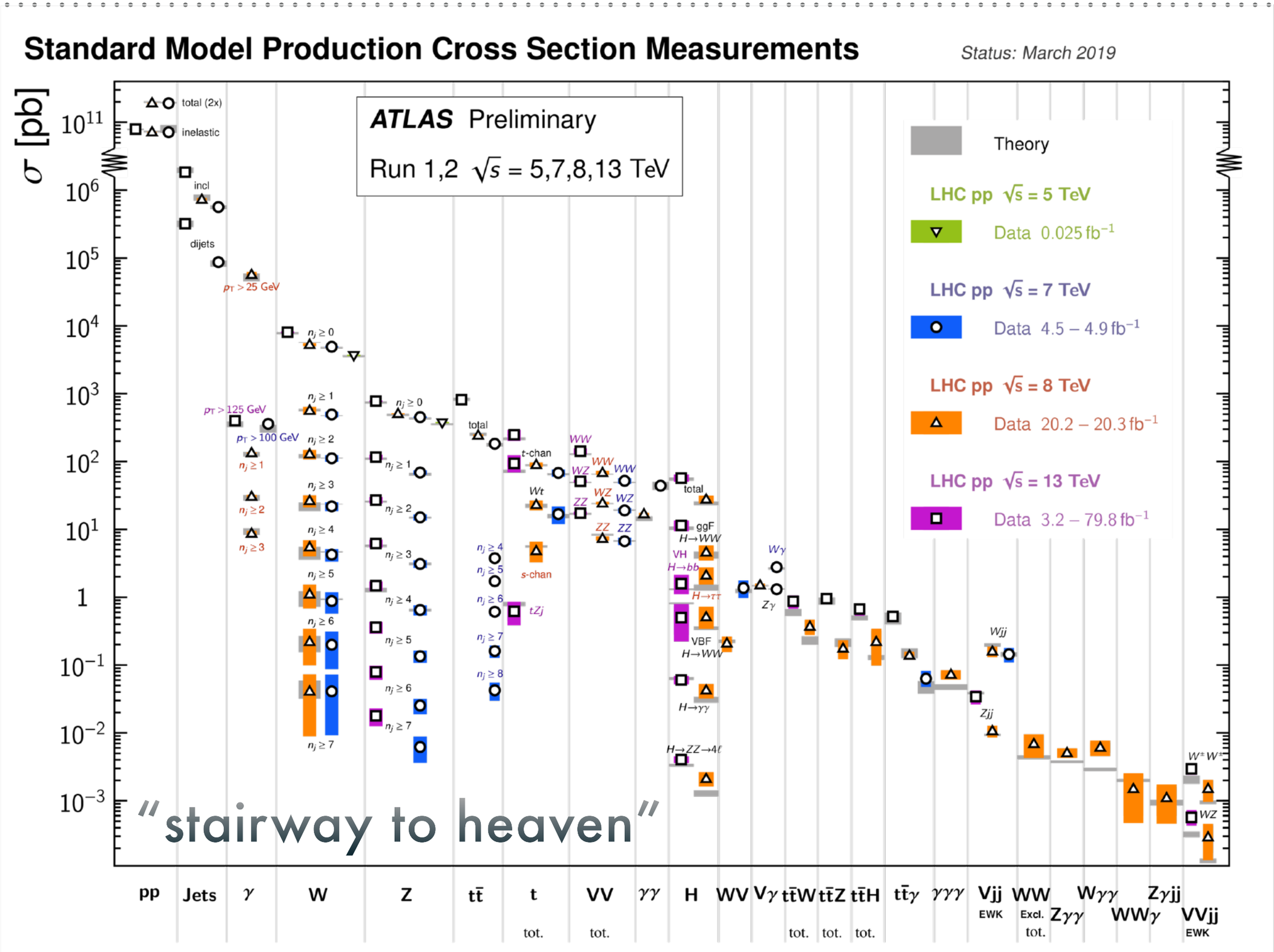
HIGH-PRECISION PREDICTIONS FOR THE LHC

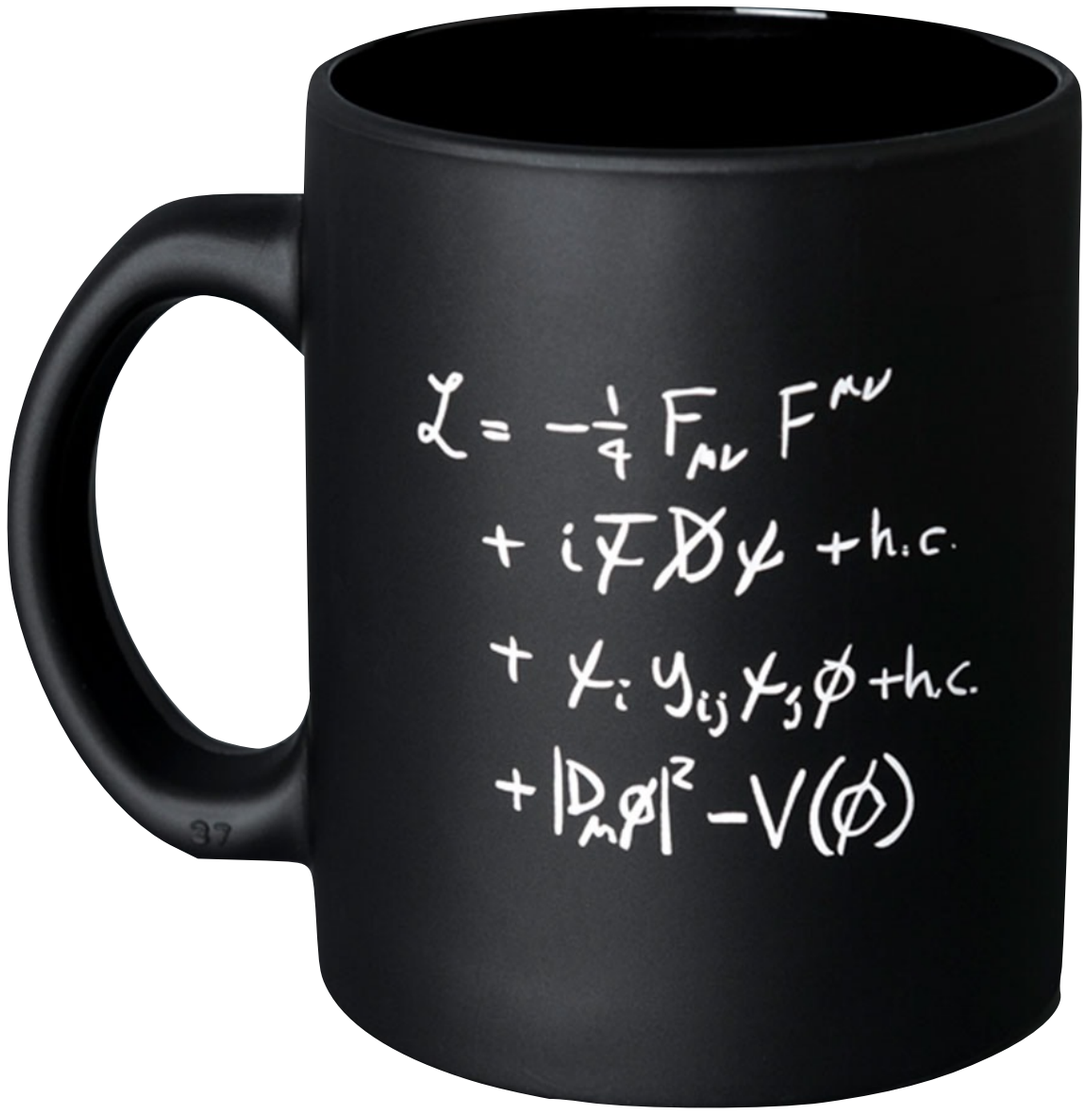
Standard Candles and the Higgs to lighten the path to discoveries

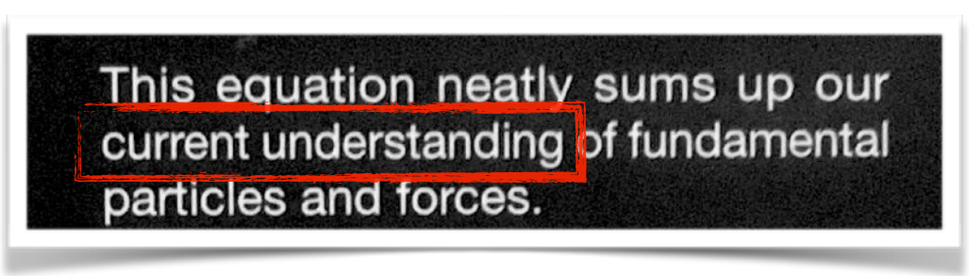
Alexander Huss

NIKHEF Theory Seminar — Amsterdam, 31st October 2019

A Remarkable Success Story...



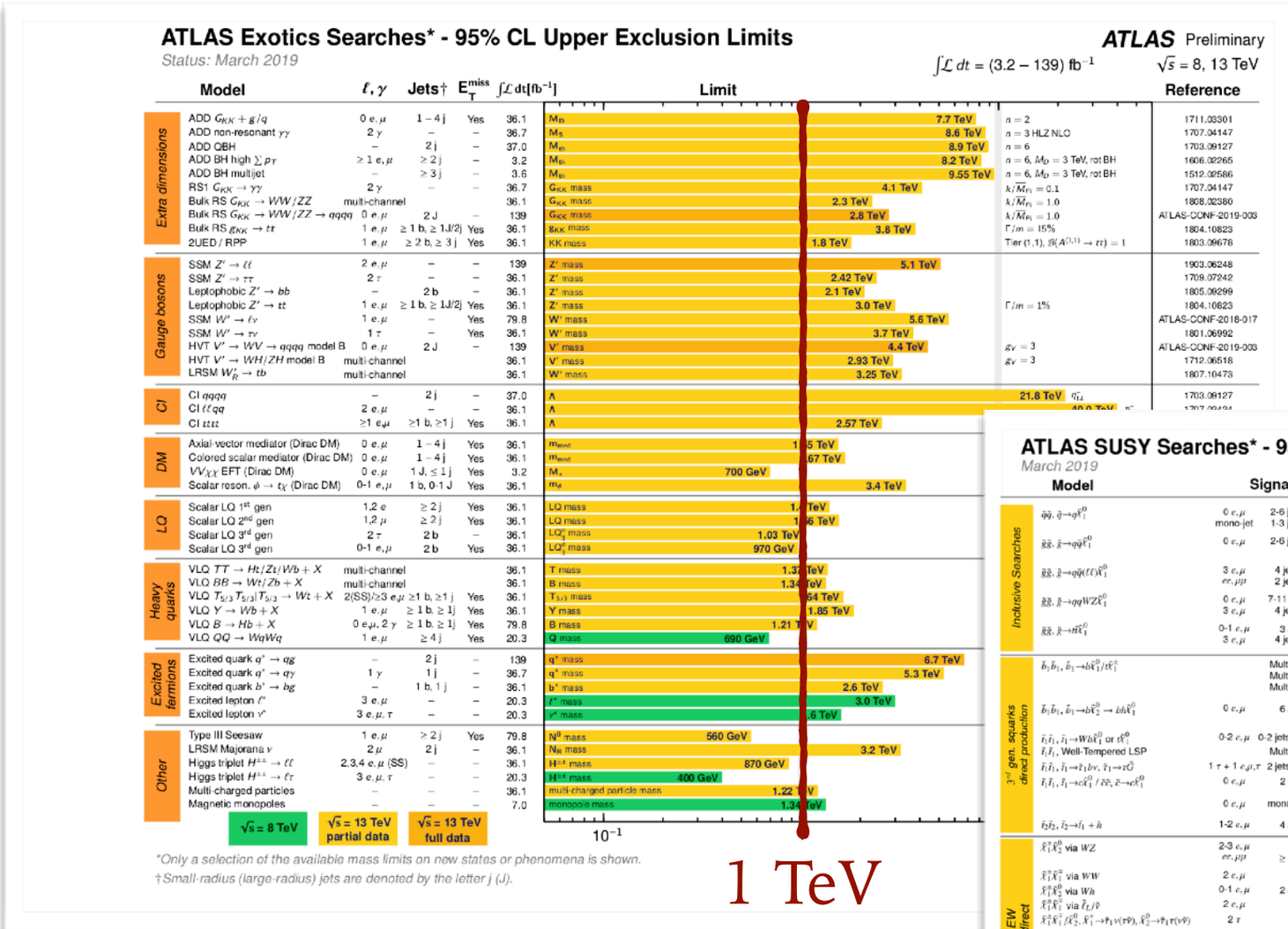




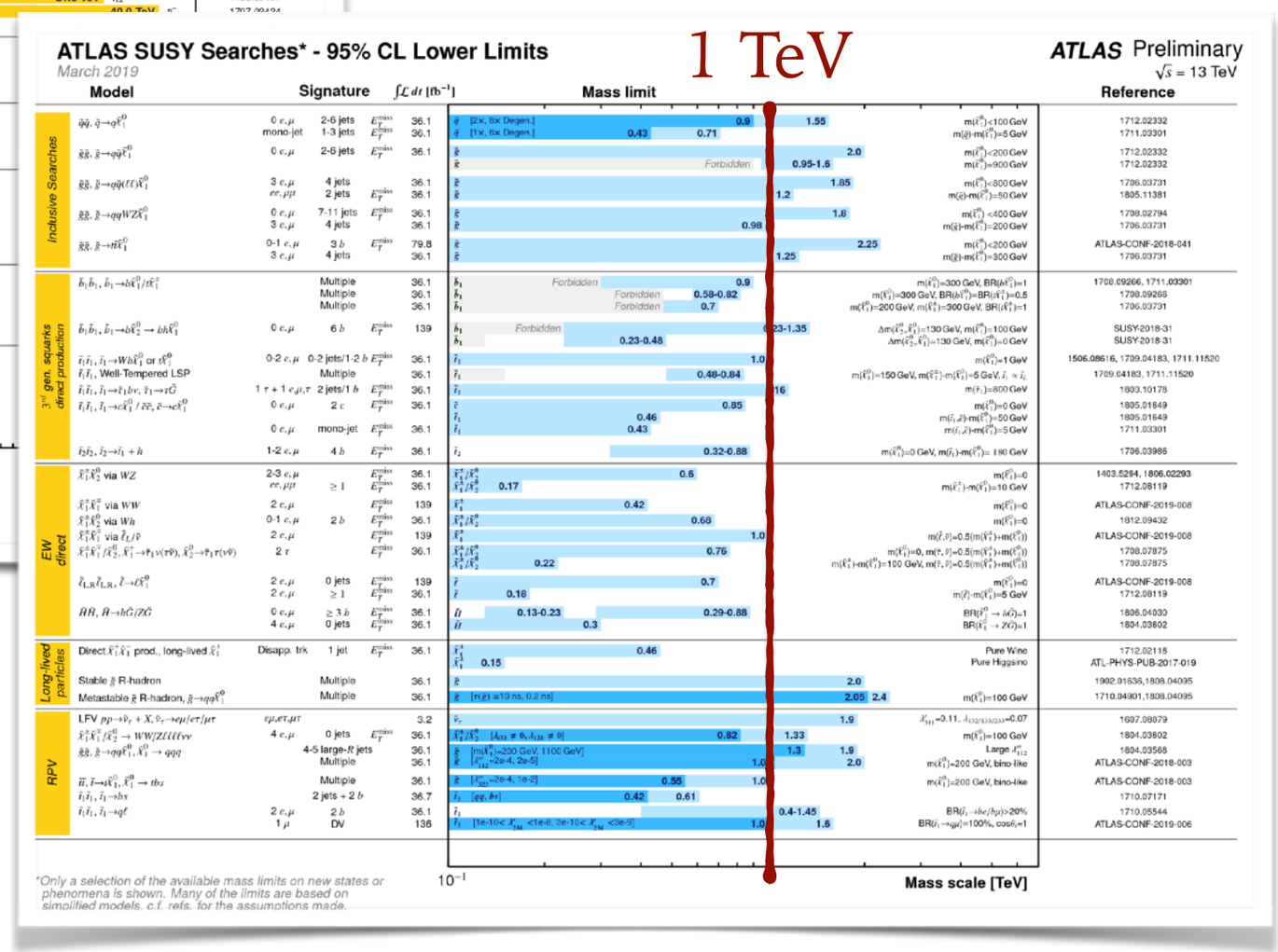
...But not the Full Story

- origin of dark matter
- hierarchy problem
- matter anti-matter asymmetry
- hierarchy of scales (generations)
- unification with gravity
- ...
- what is the Higgs potential?
- establish the Yukawa's Y_{ij}
- ...

New Physics — Direct Searches



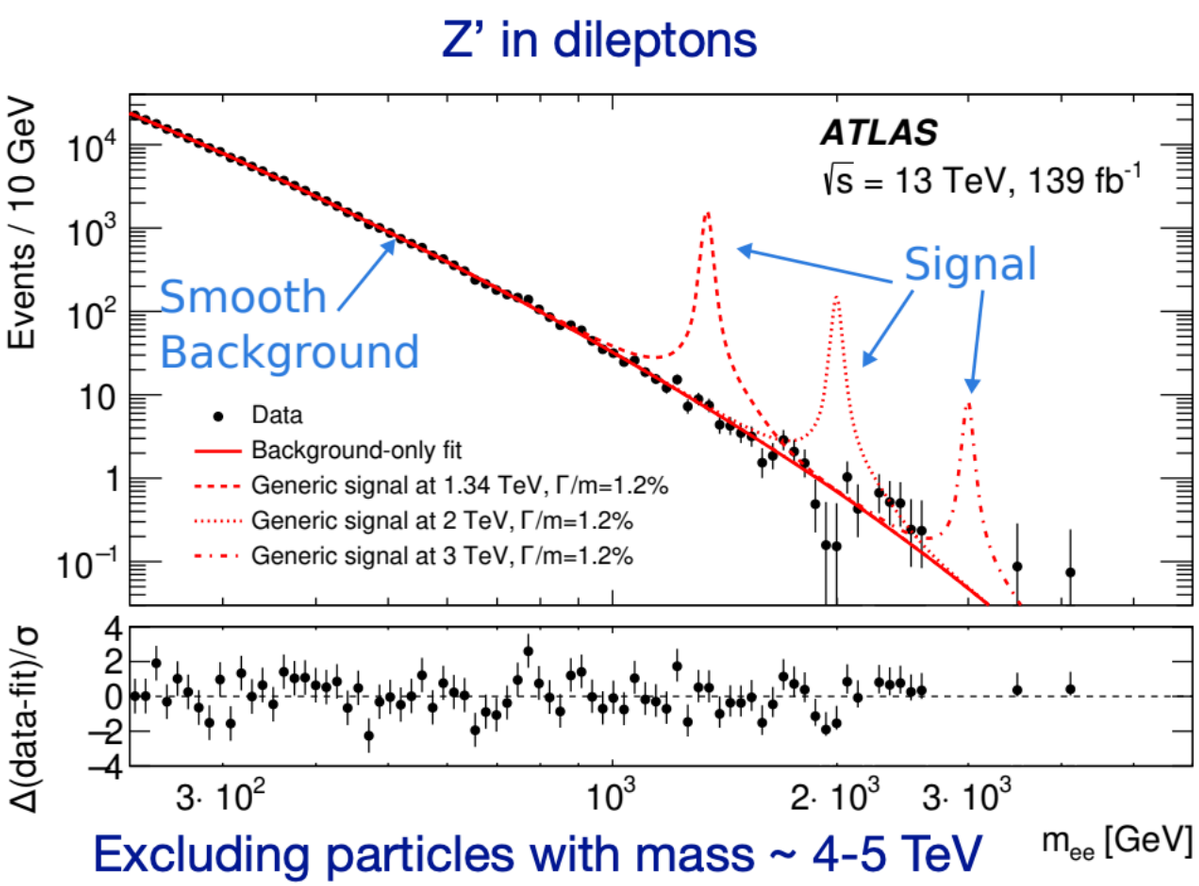
No striking signals so far!



Exotics

SUSY

New Physics — Hiding in Small & Subtle Effects?

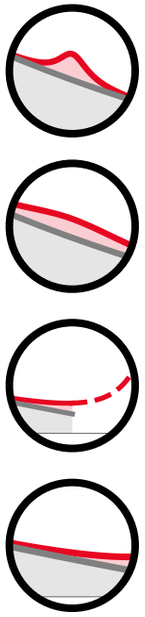


"bump hunting"

➡ *little to no theory input needed*

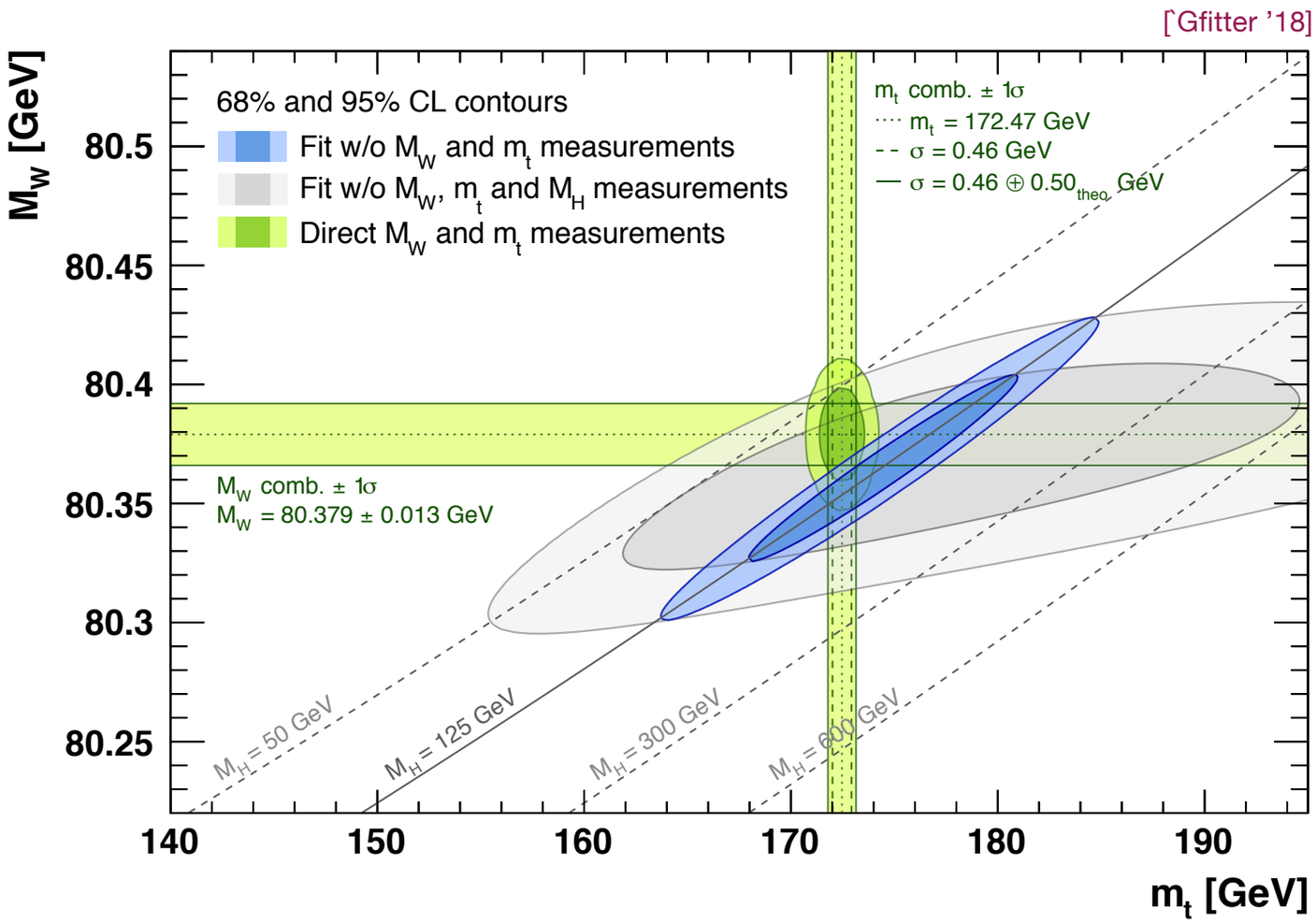
What if?

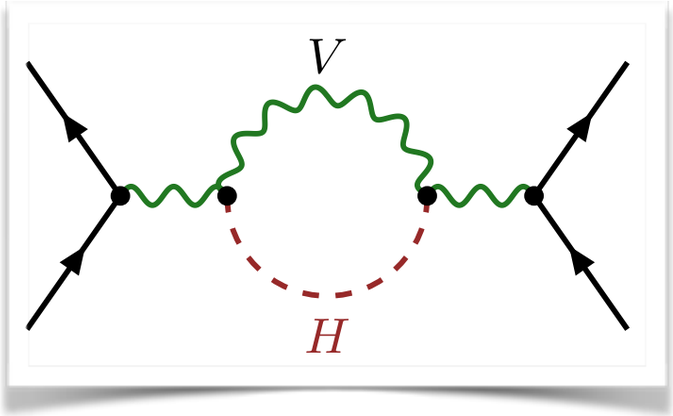
- *interaction weak*
- *wide resonance*
- *too heavy*
- *shape distortion*
- *challenging signature*
- …



requires solid understanding and control of SM backgrounds

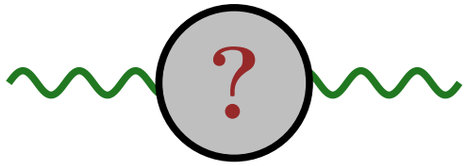
Precision Measurements & Indirect searches





- constrained system

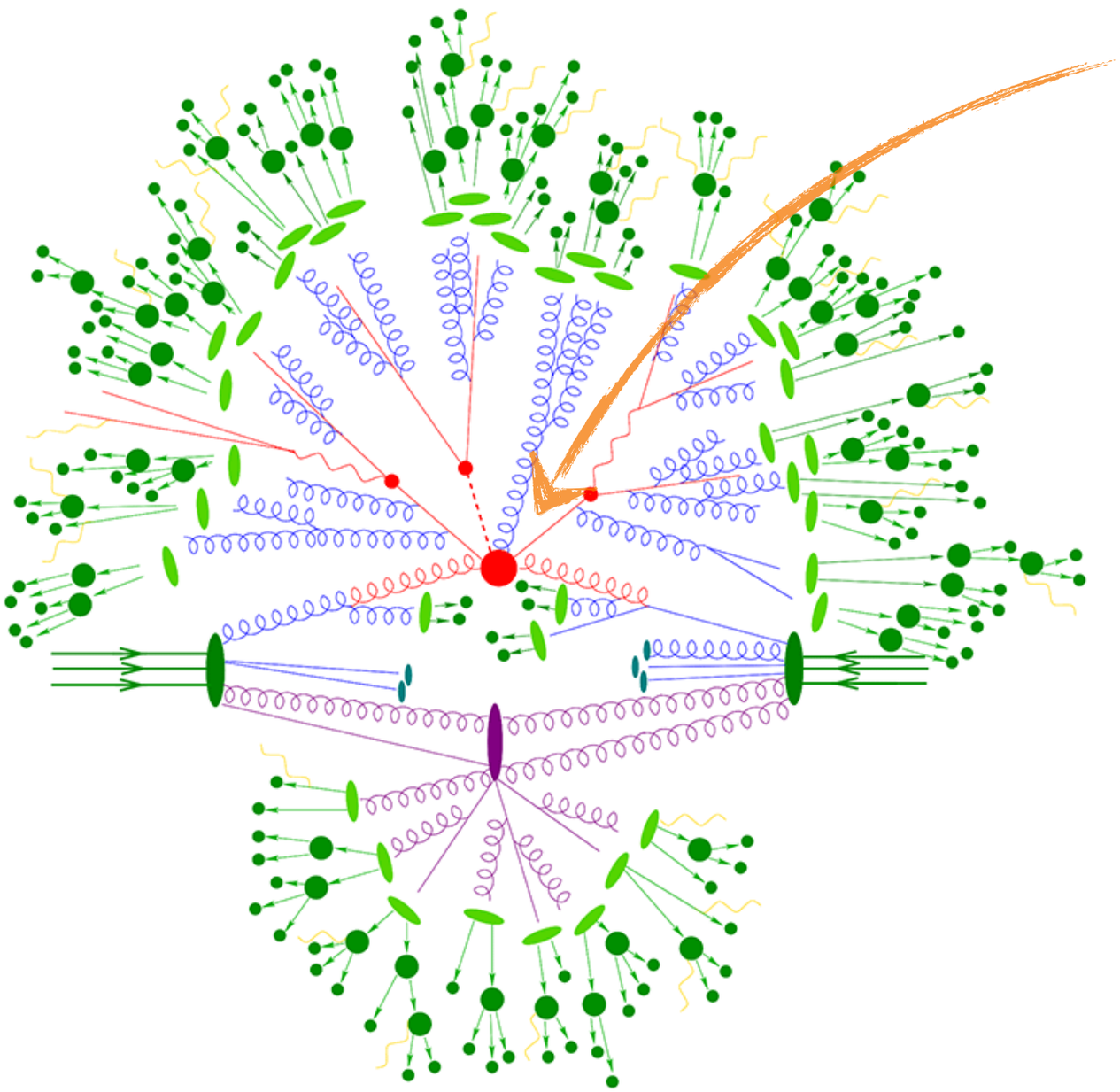
➡ self consistent?

➡ 

m/GeV	measured	fit value
m_t	172.47 ± 0.68	176.4 ± 2.1
M_H	125.1 ± 0.2	90^{+21}_{-18}
M_W	80.379 ± 0.013	80.354 ± 0.007

precision theory
for
"standard candles"

HIGH-PRECISION THEORY PREDICTIONS!

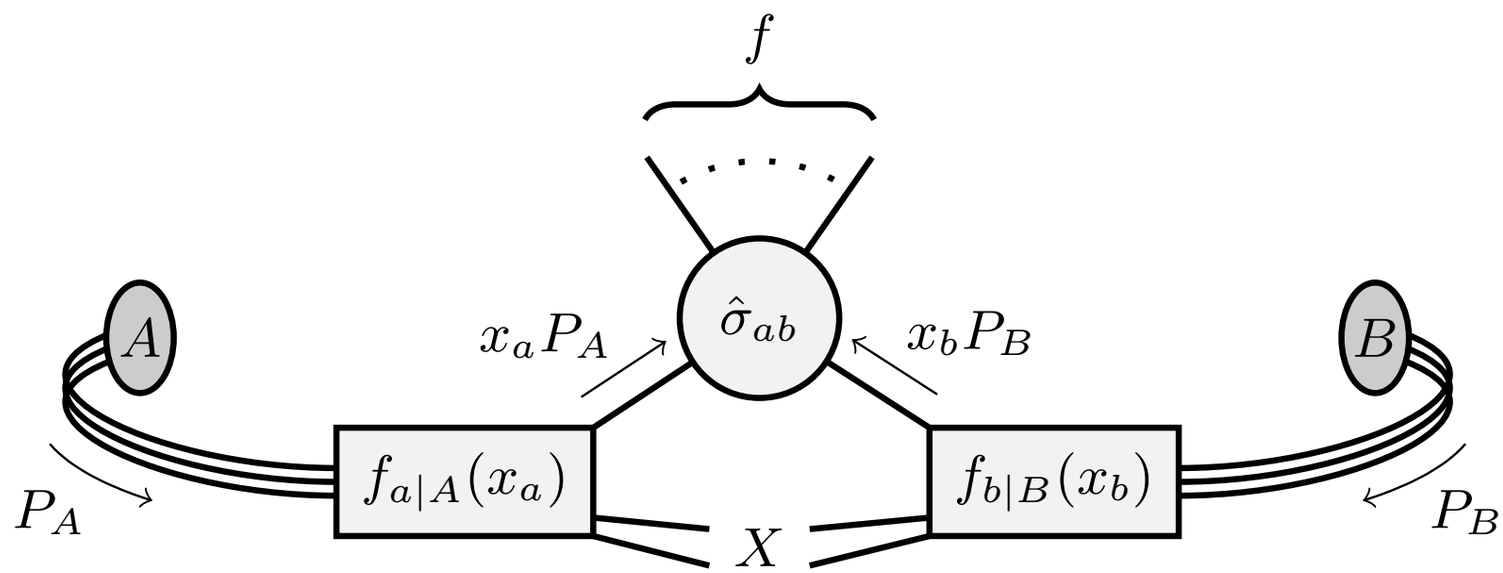


- (HL-)LHC — per-cent level!
- **FOCUS** — clean processes with high momentum transfer
  - *perturbative QCD*
- with $\alpha_s \sim 0.1$
  - *NLO* $\sim \mathcal{O}(10\%)$, *NNLO* $\sim \mathcal{O}(1\%)$
  - *exceptions: Higgs, new channels, …*
- predictions as close as possible to the experiment
  - *fiducial cross sections & differential distributions*

The Plan.

1. *Precision Predictions for the LHC*
   - *The Antenna Subtraction Formalism*
2. *Hard QCD Probes*
   - *Jets & Photon Production at NNLO*
3. *Differential Higgs Production*
   - *The Projection-to-Born Method*

Theory Predictions for the LHC



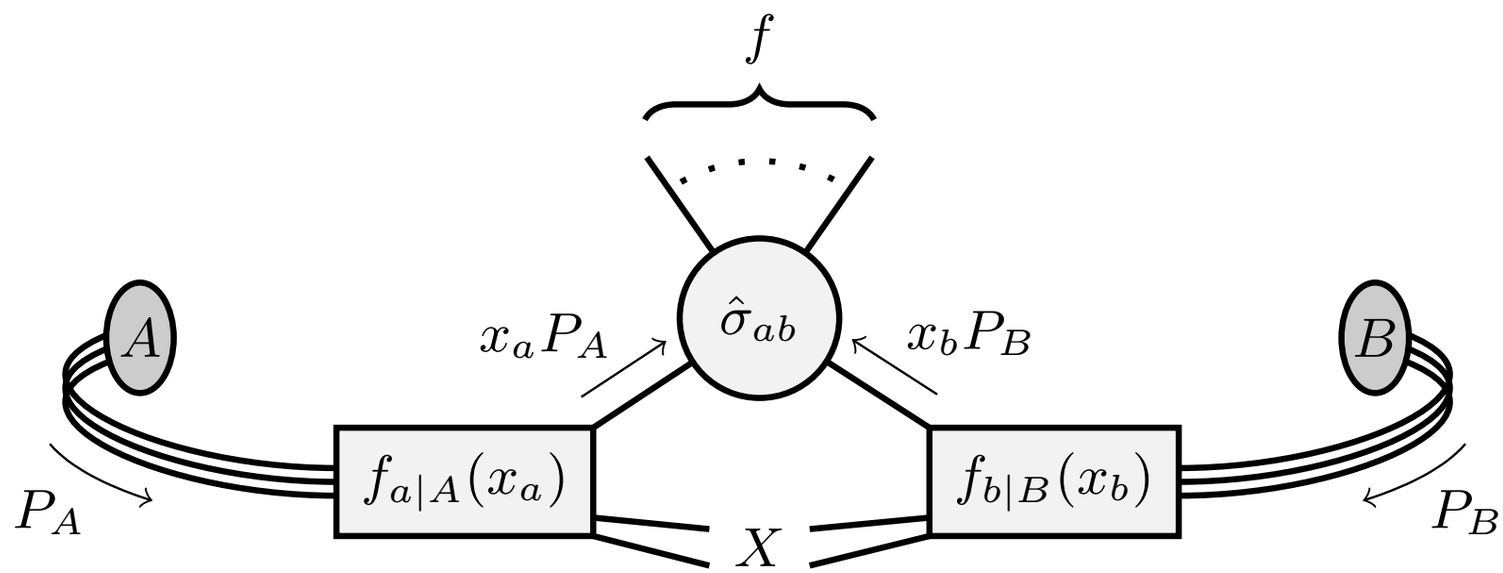
$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \; f_{a|A}(x_a)\; f_{b|B}(x_b)\; \hat{\sigma}_{ab}(x_a, x_b)\; \left(1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\right)$$

parton distribution functions
(non-perturbative, universal)

hard scattering
(perturbation theory)

non-perturbative effects
(power suppressed)
ultimately, limiting factor?

Theory Predictions for the LHC



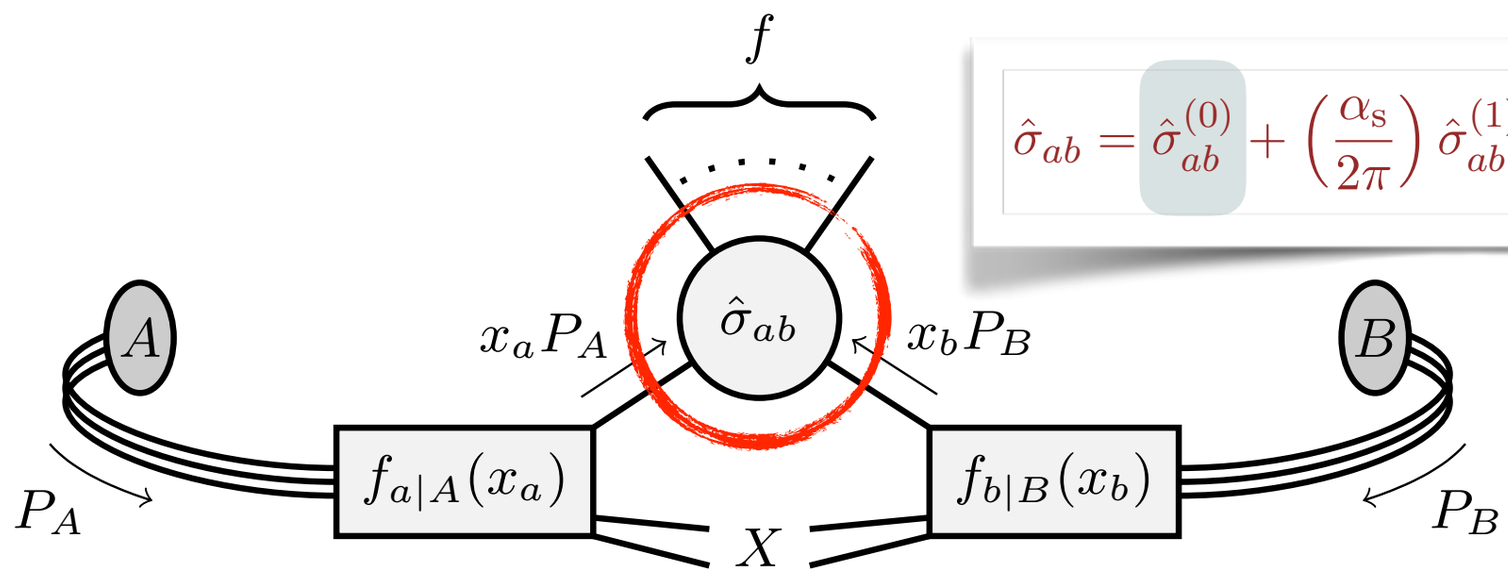
$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \, f_{a|A}(x_a)\, f_{b|B}(x_b)\, \hat{\sigma}_{ab}(x_a, x_b)\, \left(1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\right)$$

parton distribution functions
(in principle, improvable)
few % at LHC

hard scattering
(systematically improvable)
aim for few % level!

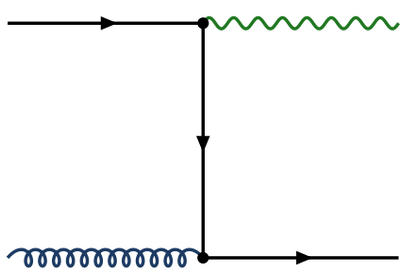
non-perturbative effects
(no good understanding)
$\sim$ few %?

Hard Scattering — Perturbation Theory



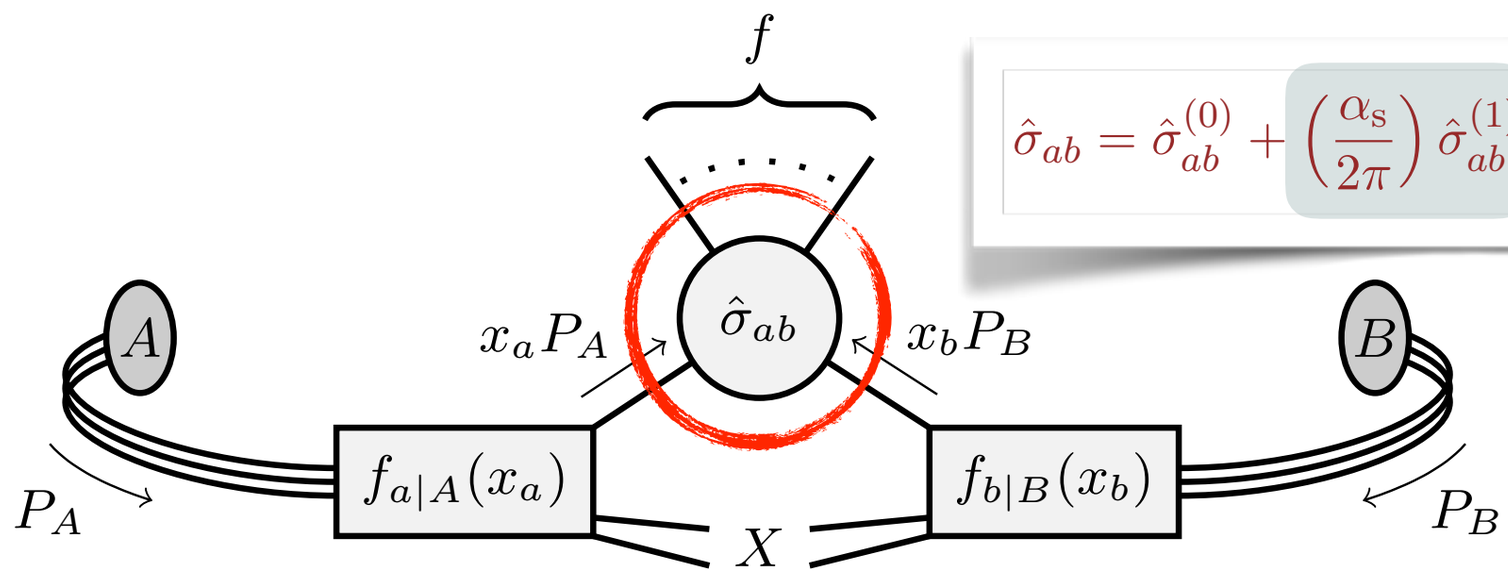
$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}_{ab}^{(2)} + \ldots$$

leading order (LO)



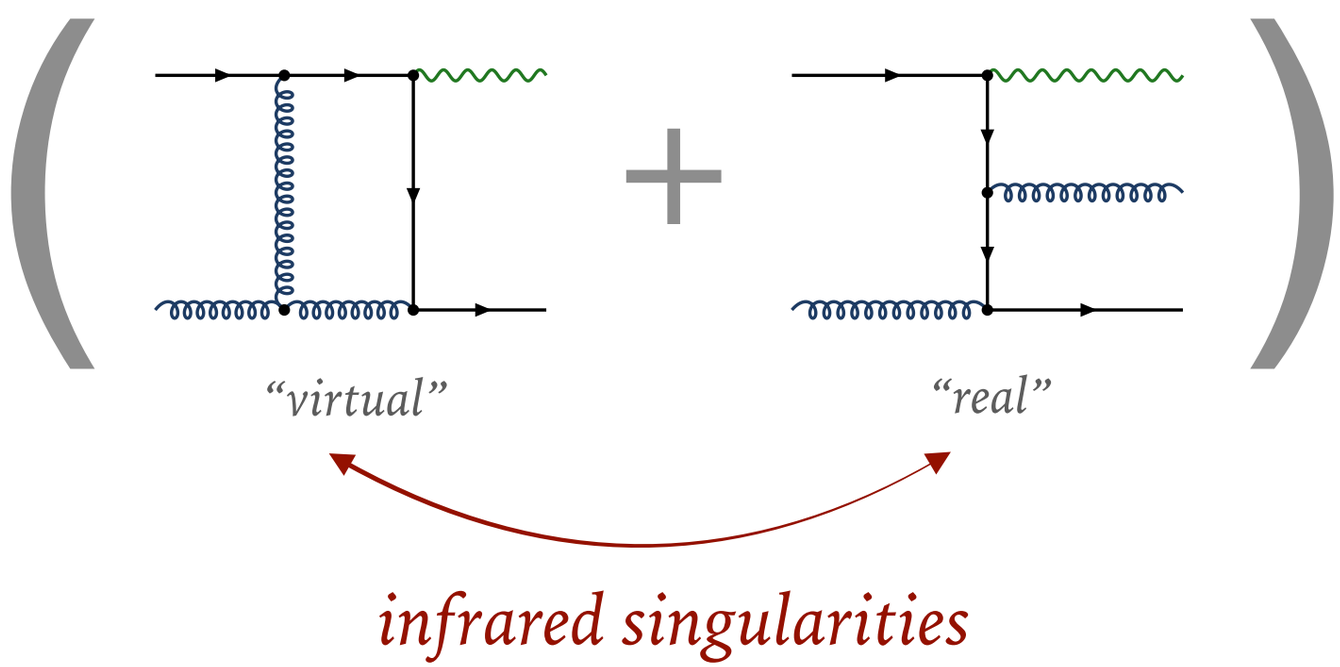
"tree level"

Hard Scattering — Perturbation Theory

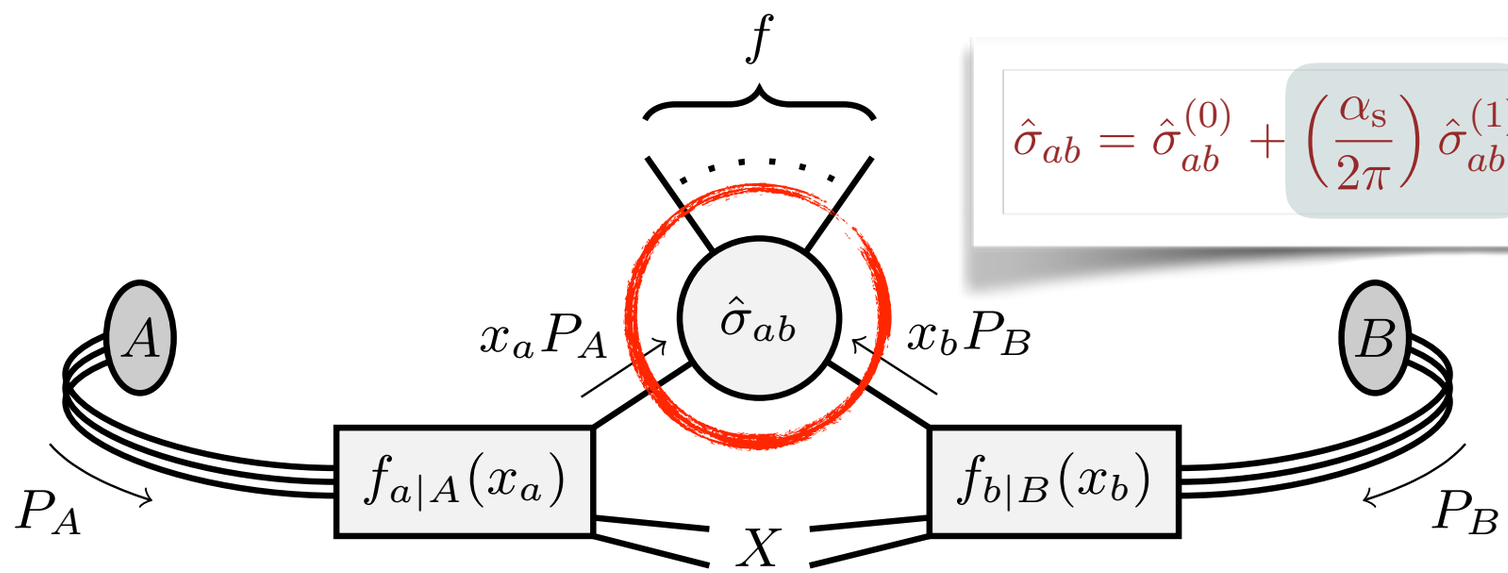


$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \ldots$$

next-to-leading order (NLO)

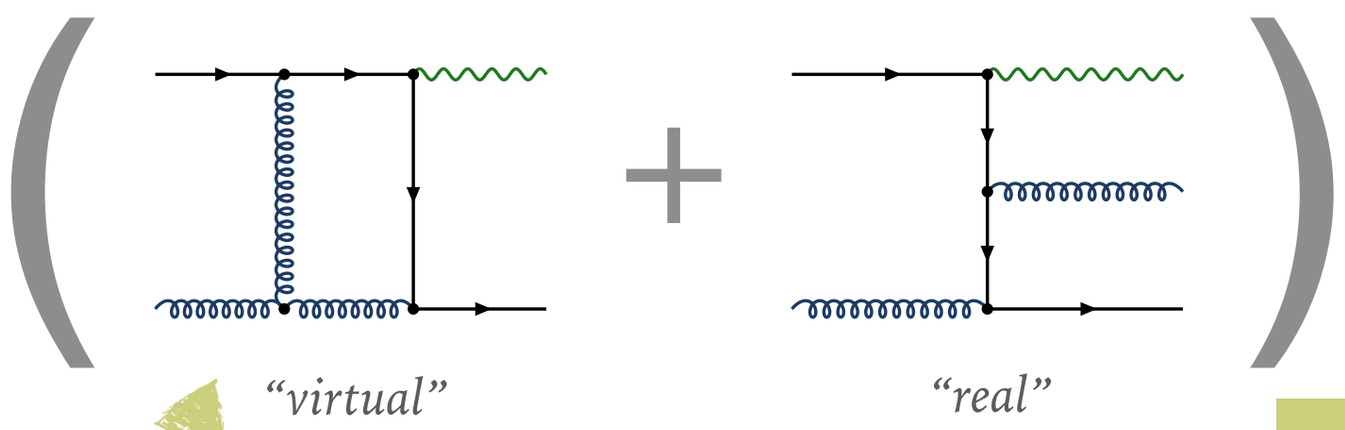


Hard Scattering — Perturbation Theory



$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \ldots$$

next-to-leading order (NLO)

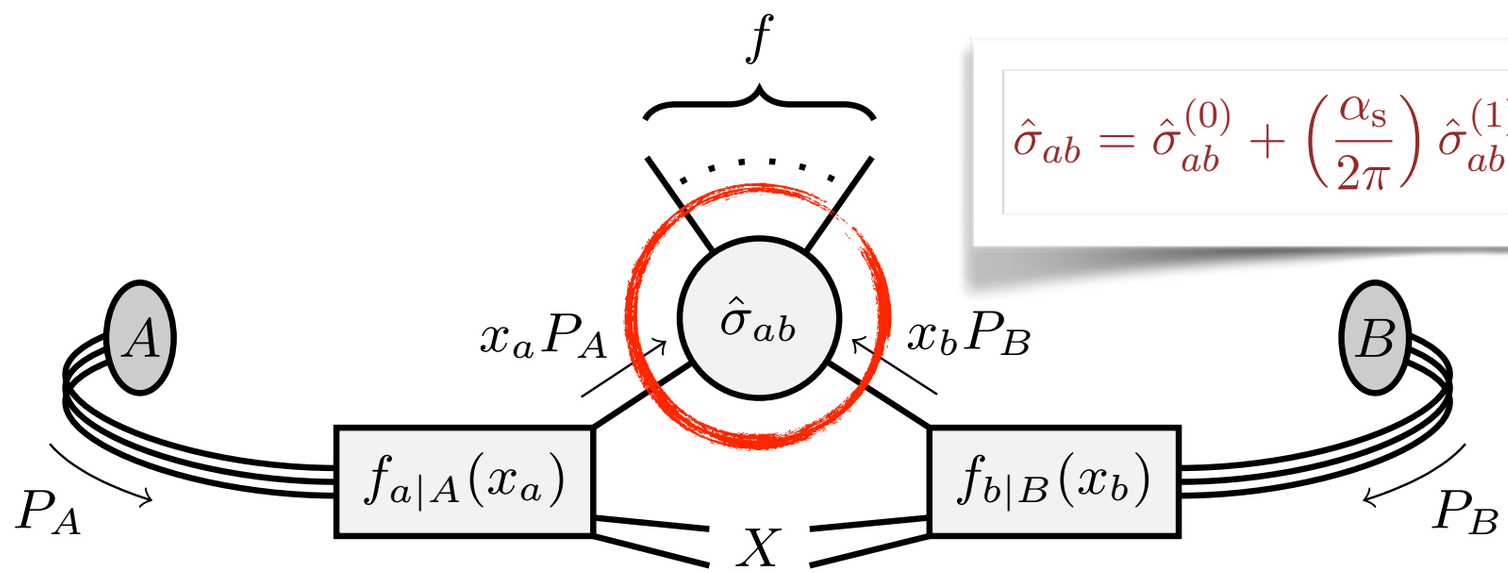


one-loop amplitudes
(all master integrals known, well understood: log, Li_2)

infrared singularities

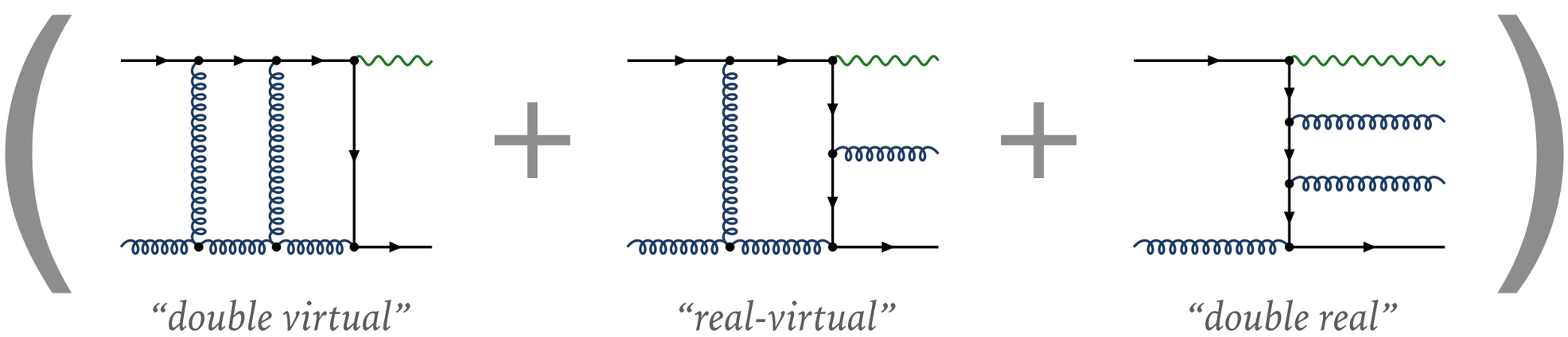
IR subtraction
(conceptually solved: CS, FKS, …)

Hard Scattering — Perturbation Theory

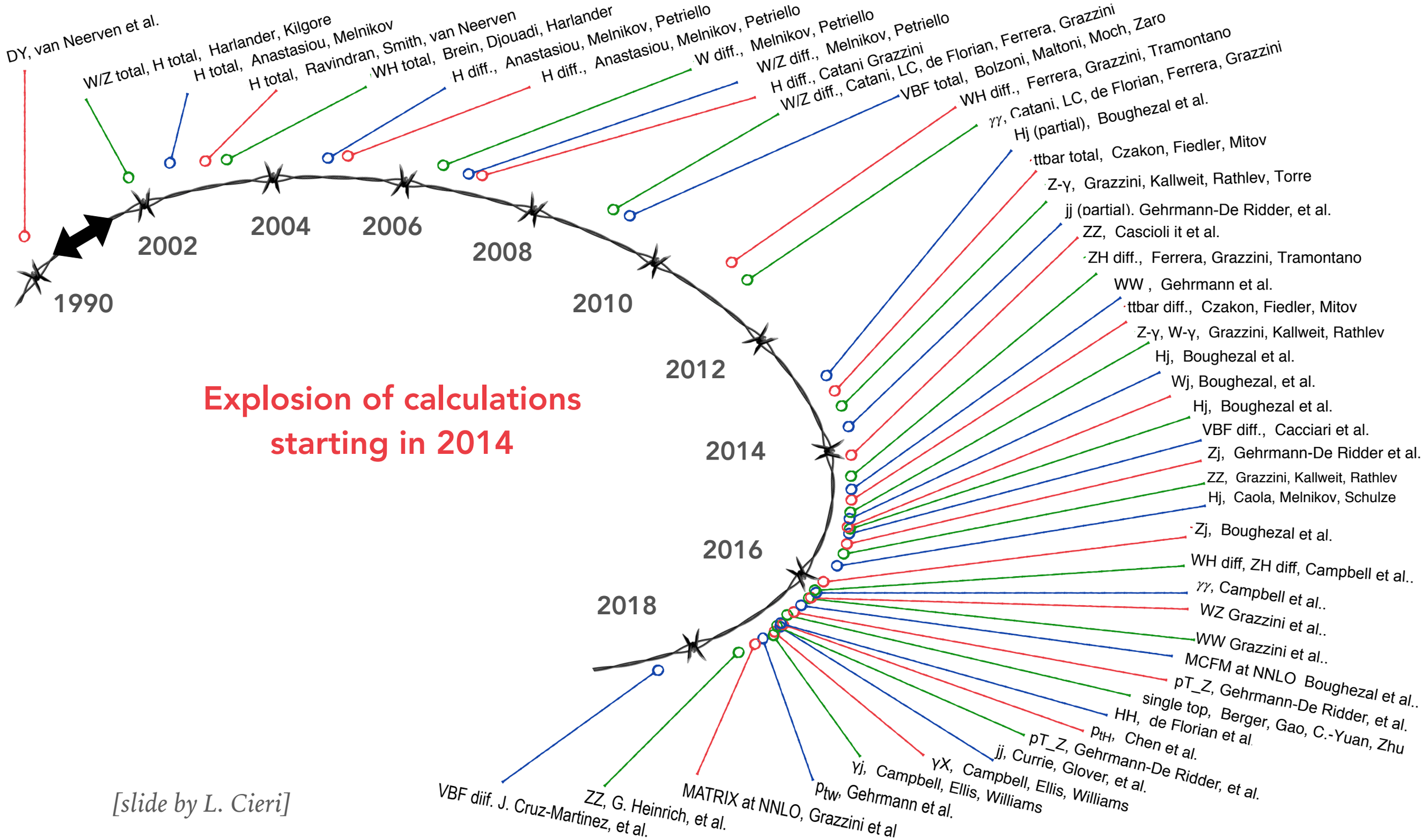


$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \ldots$$

next-to-next-to-leading order (NNLO)



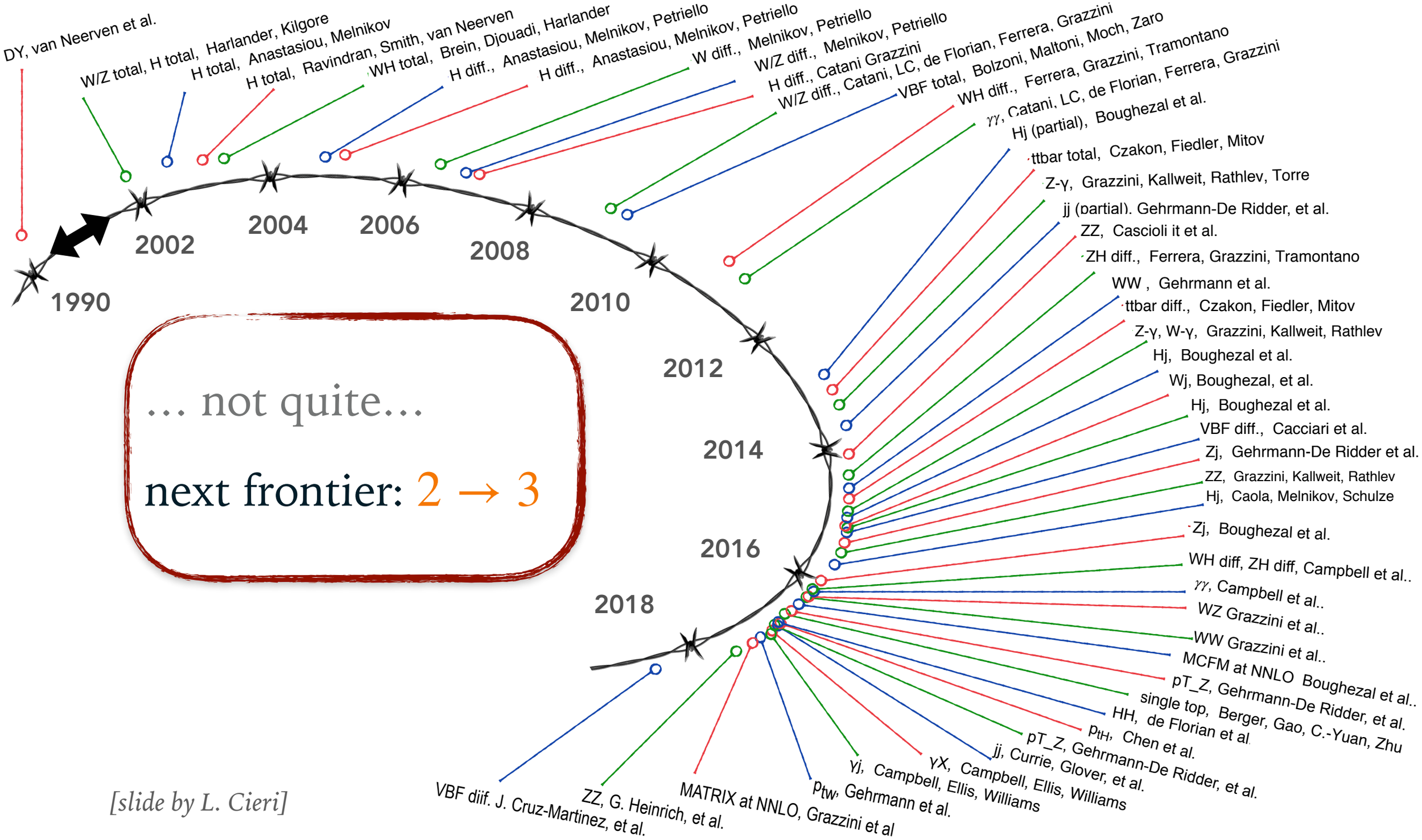
Timeline for NNLO @ Hadron Colliders



[slide by L. Cieri]

The "NNLO Revolution"?

~~Timeline for NNLO @ Hadron Colliders~~



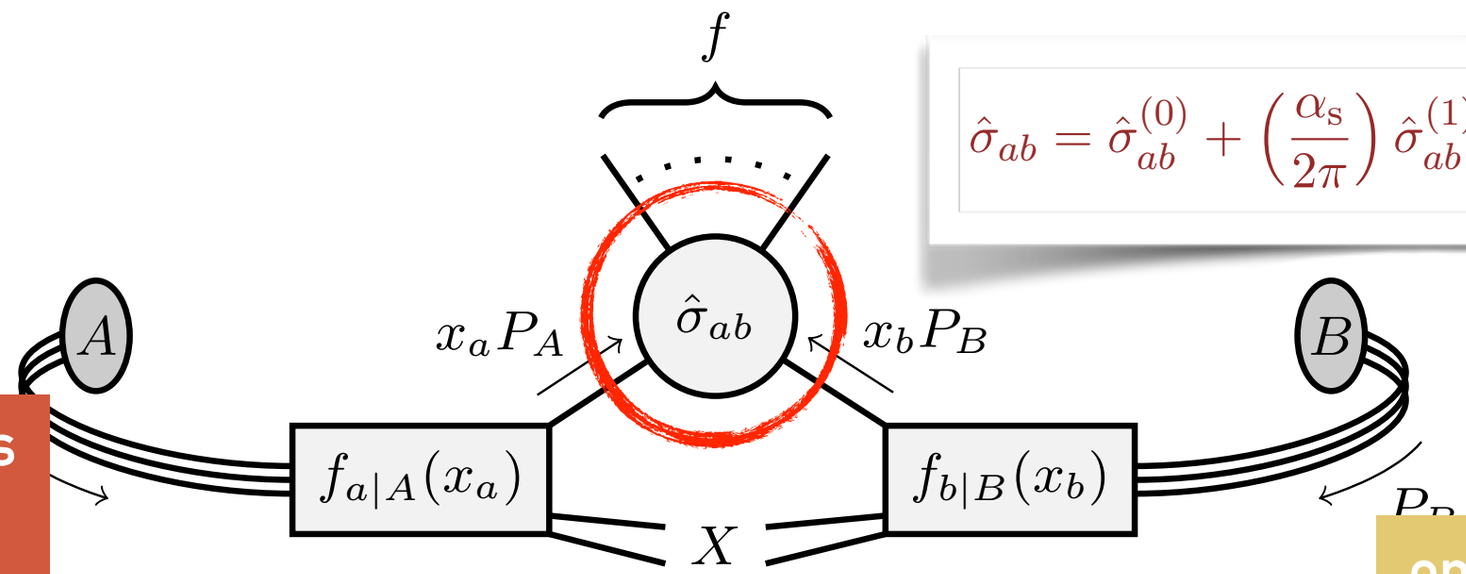
… not quite…

next frontier: 2 → 3

[slide by L. Cieri]

NNLO — Bottle Necks

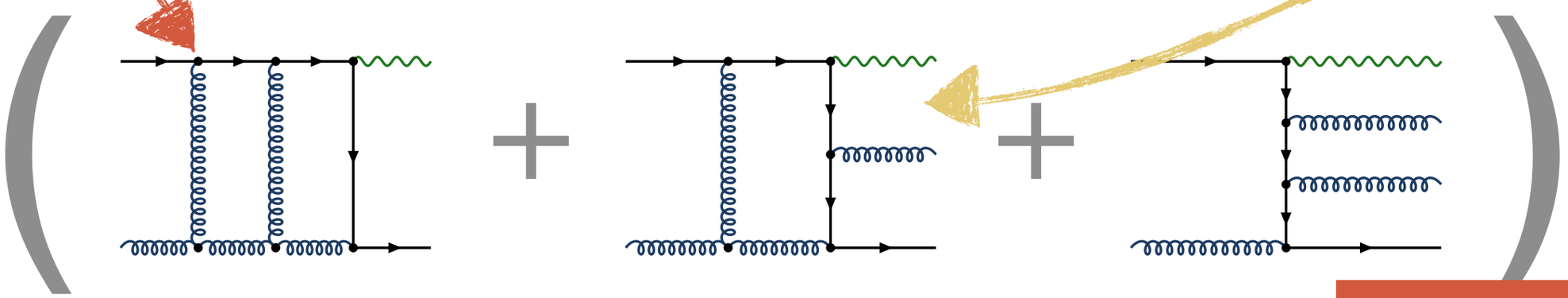
$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \ldots$$



two-loop amplitudes
(new class of functions, combinatoric & algebraic complexity)

one-loop amplitudes
(evaluation in singular & unstable regions)

next-to-next-to-leading order (NNLO)



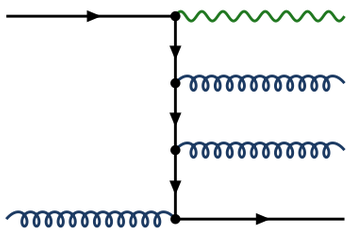
"double virtual" *"real-virtual"* *"double re*

IR subtraction
(involved IR structure, numerical stability, construction)

infrared singularities

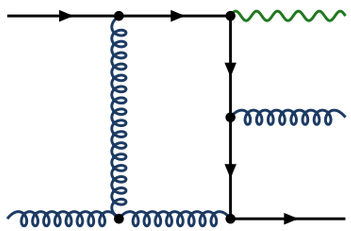
Anatomy of NNLO Calculations

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} \mathrm{d}\sigma^{\mathrm{RR}}_{\text{NNLO}}$$



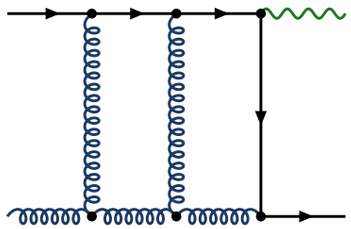
- single-unresolved
- double-unresolved

$$+ \int_{\Phi_{Z+2}} \mathrm{d}\sigma^{\mathrm{RV}}_{\text{NNLO}}$$



- single-unresolved
- $1/\epsilon^2$, $1/\epsilon$

$$+ \int_{\Phi_{Z+1}} \mathrm{d}\sigma^{\mathrm{VV}}_{\text{NNLO}}$$



- $1/\epsilon^4$, $1/\epsilon^3$, $1/\epsilon^2$, $1/\epsilon$

$\sum$ finite (Kinoshita–Lee–Nauenberg & factorization)

Non-trivial cancellation of infrared singularities

NNLO using Subtraction

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} \left(d\sigma^{RR}_{\text{NNLO}} - d\sigma^{S}_{\text{NNLO}} \right)$$

$$+ \int_{\Phi_{Z+2}} \left(d\sigma^{RV}_{\text{NNLO}} - d\sigma^{T}_{\text{NNLO}} \right)$$

$$+ \int_{\Phi_{Z+1}} \left(d\sigma^{VV}_{\text{NNLO}} - d\sigma^{U}_{\text{NNLO}} \right)$$

$\sum$ finite $- 0$

- $d\sigma^{S}_{\text{NNLO}}$, $d\sigma^{T}_{\text{NNLO}}$: mimic $d\sigma^{RR}_{\text{NNLO}}$, $d\sigma^{RV}_{\text{NNLO}}$ in unresolved limits
- $d\sigma^{T}_{\text{NNLO}}$, $d\sigma^{U}_{\text{NNLO}}$: *analytic* cancellation of poles in $d\sigma^{RV}_{\text{NNLO}}$, $d\sigma^{VV}_{\text{NNLO}}$

$\Rightarrow$ each line suitable for numerical evaluation in $D = 4$

Antenna Factorization

- antenna formalism operates on *colour-ordered* amplitudes
- exploit universal factorisation properties in IR limits

$$\underbrace{|\mathcal{A}^0_{m+1}(\ldots,i,j,k,\ldots)|^2}_{\text{colour-ordered amplitude}} \xrightarrow{j\ \text{unresolved}} \underbrace{X^0_3(i,j,k)}_{\text{antenna function}} \quad \underbrace{|\mathcal{A}^0_m(\ldots,\widetilde{I},\widetilde{K},\ldots)|^2}_{\text{reduced ME}}$$

$+$ mapping

$\{p_i, p_j, p_k\} \to \{\widetilde{p}_I, \widetilde{p}_K\}$

- captures *multiple limits* and smoothly interpolates between them*

limit	$X^0_3(i,j,k)$	mapping
$p_j \to 0$	$\dfrac{2s_{ik}}{s_{ij}s_{jk}}$	$\widetilde{p}_I \to p_i,\ \ \widetilde{p}_K \to p_k$
$p_j \parallel p_i$	$\dfrac{1}{s_{ij}}P_{ij}(z)$	$\widetilde{p}_I \to (p_i + p_j),\ \ \widetilde{p}_K \to p_k$
$p_j \parallel p_k$	$\dfrac{1}{s_{jk}}P_{kj}(z)$	$\widetilde{p}_I \to p_i,\ \ \widetilde{p}_K \to (p_j + p_k)$

* c.f. dipoles: $X^0_3(i,j,k) \sim \mathcal{D}_{ij,k} + \mathcal{D}_{kj,i}$

Antenna Factorization

- antenna formalism operates on *colour-ordered* amplitudes
- exploit universal factorisation properties in IR limits

$$\underbrace{|\mathcal{A}^0_{m+2}(\ldots,i,j,k,l,\ldots)|^2}_{\text{colour-ordered amplitude}} \xrightarrow{j\ \&\ k\ \text{unresolved}} \underbrace{\boxed{X^0_4(i,j,k,l)}}_{\text{antenna function}} \underbrace{|\mathcal{A}^0_m(\ldots,\widetilde{I},\widetilde{L},\ldots)|^2}_{\text{reduced ME}}$$

$+$ mapping
$\{p_i, p_j, p_k, p_l\} \to \{\widetilde{p}_I, \widetilde{p}_L\}$

- captures *multiple limits* and smoothly interpolates between them*

limit	$X^0_3(i,j,k)$
$p_j \to 0$	$\dfrac{2s_{ik}}{s_{ij}s_{jk}}$
$p_j \parallel p_i$	$\dfrac{1}{s_{ij}} P_{ij}(z)$
$p_j \parallel p_k$	$\dfrac{1}{s_{jk}} P_{kj}(z)$

- double soft: $j,k \to 0$
- triple-collinear:
  $(i \parallel j \parallel k)$ & $(j \parallel k \parallel l)$
- double collinear: $(i \parallel j), (k \parallel l)$
- soft–collinear:
  $(i \parallel j), k \to 0$ & $(k \parallel l), j \to 0$
- single-unresolved

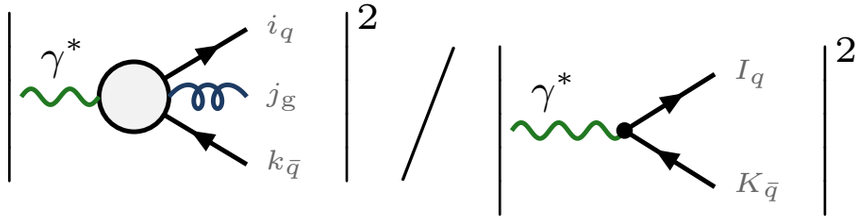
Antenna Subtraction — Building Blocks

- $X(\ldots)$ based on physical matrix elements $\quad X = \overbrace{A,\ B,\ C}^{q\bar{q}}, \overbrace{D,\ E}^{qg}, \overbrace{F,\ G,\ H}^{gg}$

$$X_3^0(i,j,k) = \frac{|\mathcal{A}_3^0(i,j,k)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2}, \qquad X_4^0(i,j,k,l) = \frac{|\mathcal{A}_4^0(i,j,k,l)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{L})|^2},$$

$$X_3^1(i,j,k) = \frac{|\mathcal{A}_3^1(i,j,k)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2} - X_3^0(i,j,k)\frac{|\mathcal{A}_2^1(\widetilde{I},\widetilde{K})|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2},$$

$A_3^0(i_q, j_g, k_{\bar{q}}) =$



- integrating the antennae $\longleftrightarrow$ phase-space factorization

$$\begin{aligned} &\mathrm{d}\Phi_{m+1}(\ldots, p_i, p_j, p_k, \ldots) \\ &\qquad = \mathrm{d}\Phi_m(\ldots, \widetilde{p}_I, \widetilde{p}_K, \ldots)\, \mathrm{d}\Phi_{X_{ijk}}(p_i, p_j, p_k; \widetilde{p}_I + \widetilde{p}_K) \end{aligned}$$

$$\mathcal{X}_3^{0,1}(i,j,k) = \int \mathrm{d}\Phi_{X_{ijk}} X_3^{0,1}(i,j,k), \quad \mathcal{X}_4^0(i,j,k,l) = \int \mathrm{d}\Phi_{X_{ijkl}} X_4^0(i,j,k,l)$$

Antenna Subtraction — Building Blocks

$q\bar{q}$ qg gg

All building blocks known!

X_3^0, X_4^0, X_3^1 and integrated counterparts $\mathcal{X}_3^0$, $\mathcal{X}_4^0$, $\mathcal{X}_3^1$

$\forall$ configurations relevant at hadron colliders:

$\hookrightarrow$ final–final ..... $\mathrm{e^+e^-}$

[Gehrmann-De Ridder, Gehrmann, Glover '05]

$\hookrightarrow$ initial–final ..... $\mathrm{e^+p}$

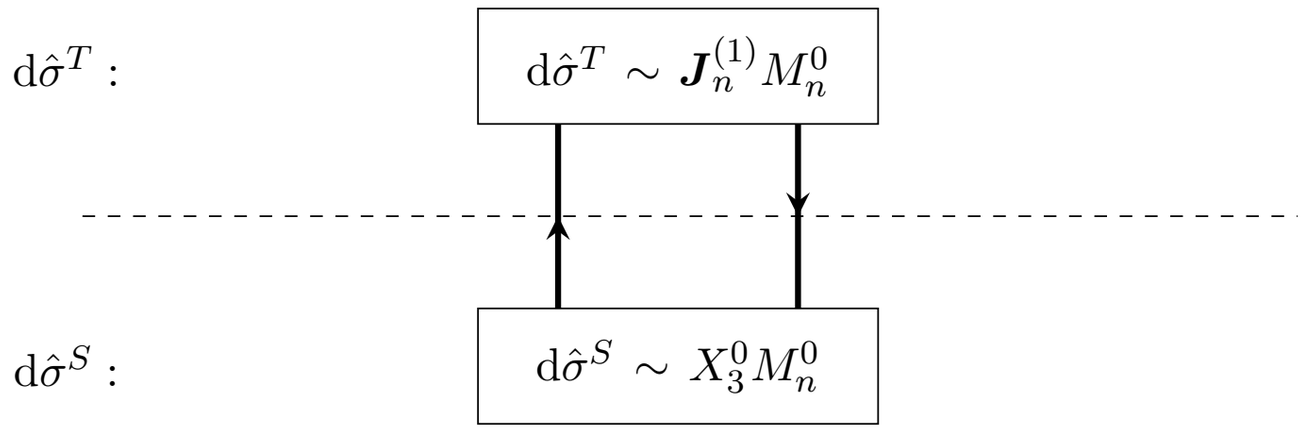
[Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, Maitre '06,'09,'12]

$\hookrightarrow$ initial–initial ..... pp

[Boughezal, Daleo, Gehrmann-De Ridder, Gehrmann, Maitre, et al. '10,'11,'12]

$$\mathcal{X}_3^{0,1}(i,j,k) = \int \mathrm{d}\Phi_{X_{ijk}} X_3^{0,1}(i,j,k), \quad \mathcal{X}_4^0(i,j,k,l) = \int \mathrm{d}\Phi_{X_{ijkl}} X_4^0(i,j,k,l)$$

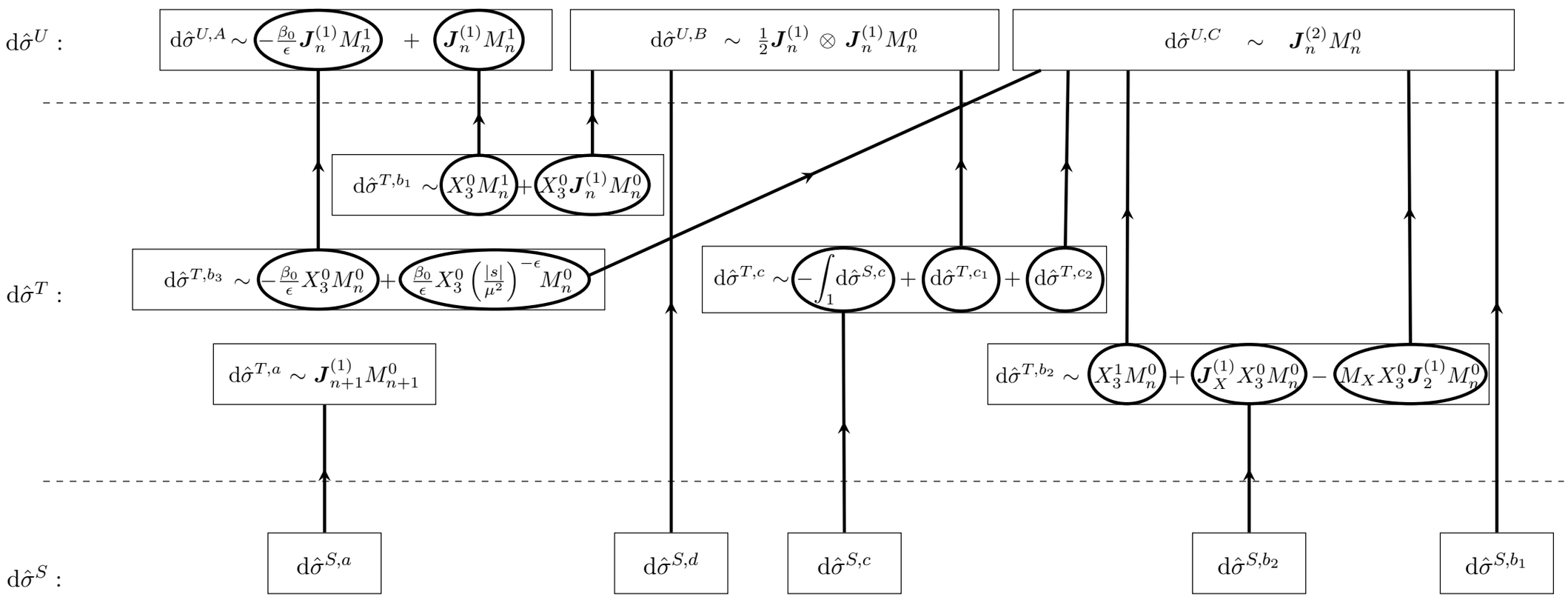
Antenna Subtraction @ NLO — $q\bar{q} \to \mathrm{ggZ}$



$$
\begin{aligned}
&\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{Z+1jet}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{Z+1jet}} \right\} \\
&= \int \mathrm{d}\Phi_{\mathrm{Z+2}} \Big\{ \left| \mathcal{A}_4^0(1_q, 3_{\mathrm{g}}, 4_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z}) \right|^2 \mathcal{J}(\Phi_{\mathrm{Z+2}}) \\
&\qquad - d_3^0(1_q, 3_{\mathrm{g}}, 4_{\mathrm{g}}) \left| \mathcal{A}_3^0(\widetilde{1}_q, \widetilde{(34)}_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z}) \right|^2 \mathcal{J}(\widetilde{\Phi}_{\mathrm{Z+1}}) \\
&\qquad - d_3^0(2_{\bar{q}}, 4_{\mathrm{g}}, 3_{\mathrm{g}}) \left| \mathcal{A}_3^0(1_q, \widetilde{(34)}_{\mathrm{g}}, \widetilde{2}_{\bar{q}}, \mathrm{Z}) \right|^2 \mathcal{J}(\widetilde{\Phi}_{\mathrm{Z+1}}) \Big\} + (3 \leftrightarrow 4) \\
&\int \left\{ \mathrm{d}\sigma^{\mathrm{V}}_{\mathrm{Z+1jet}} - \mathrm{d}\sigma^{\mathrm{T}}_{\mathrm{Z+1jet}} \right\} \\
&= \int \mathrm{d}\Phi_{\mathrm{Z+1}} \Big\{ \left| \mathcal{A}_3^1(1_q, 3_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z}) \right|^2 \\
&\qquad + \frac{1}{2} \left[\mathcal{D}_3^0(s_{13}) + \mathcal{D}_3^0(s_{23}) \right] \left| \mathcal{A}_3^0(1_q, 3_{\mathrm{g}}, 2_{\bar{q}}, \mathrm{Z}) \right|^2 \Big\} \mathcal{J}(\Phi_{\mathrm{Z+1}})
\end{aligned}
$$

Antenna Subtraction @ NNLO

[J. Currie , E.W.N. Glover, S. Wells '13]



- double real: $d\sigma^{\mathrm{S}} \sim X_3^0 \, |\mathcal{A}_{m+1}^0|^2, \quad X_4^0 \, |\mathcal{A}_m^0|^2, \quad X_3^0 \, X_3^0 \, |\mathcal{A}_m^0|^2$
- real–virtual: $d\sigma^{\mathrm{T}} \sim \mathcal{X}_3^0 \, |\mathcal{A}_{m+1}^0|^2, \quad X_3^0 \, |\mathcal{A}_m^1|^2, \quad X_3^1 \, |\mathcal{A}_m^0|^2$
- double virtual: $d\sigma^{\mathrm{U}} =$ (collect rest) $\sim \mathcal{X} \, |\mathcal{A}_m^{0,1}|^2$

Antenna Subtraction — Checks of the Calculation

Analytic pole cancellation

- Poles $\left(\mathrm{d}\sigma^{\mathrm{RV}} - \mathrm{d}\sigma^{\mathrm{T}}\right) = 0$
- Poles $\left(\mathrm{d}\sigma^{\mathrm{VV}} - \mathrm{d}\sigma^{\mathrm{U}}\right) = 0$

DimReg: $D = 4 - 2\epsilon$

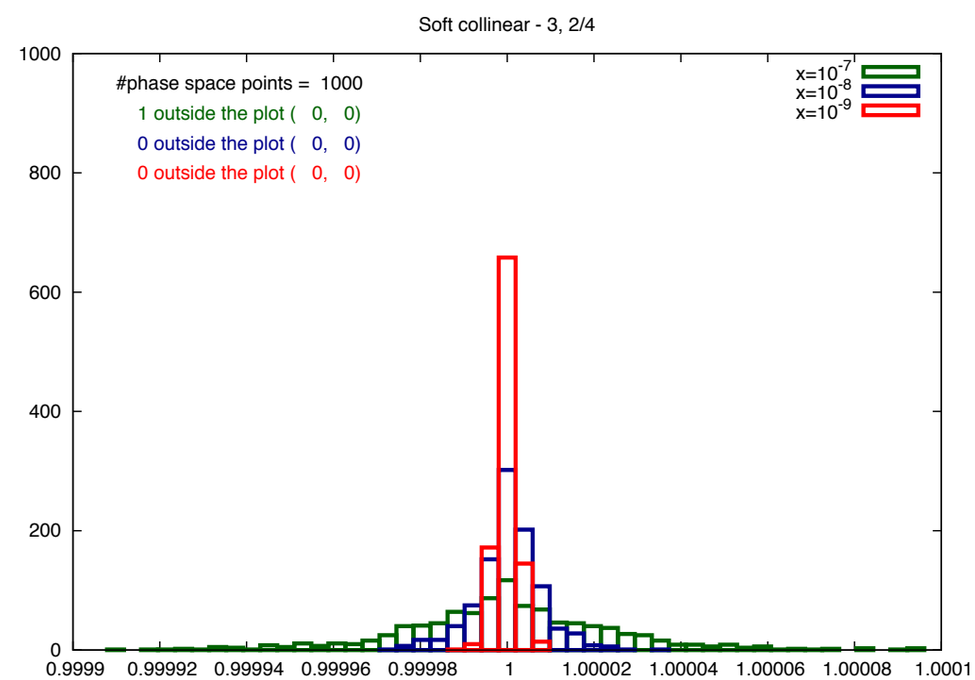
```
09:26:35  ...maple/process/Z
$ form autoqgB1g2ZgtoqU.frm
FORM 4.1 (Mar 13 2014) 64-bits
    #-

   poles = 0;

  6.58 sec out of 6.64 sec
```

Unresolved limits

- $\mathrm{d}\sigma^{\mathrm{S}} \rightarrow \mathrm{d}\sigma^{\mathrm{RR}}$ (single- & double-unresolved)
- $\mathrm{d}\sigma^{\mathrm{T}} \rightarrow \mathrm{d}\sigma^{\mathrm{RV}}$ (single-unresolved)

bin the ratio: $\mathrm{d}\sigma^{\mathrm{S}}/\mathrm{d}\sigma^{\mathrm{RR}} \xrightarrow{\text{unresolved}} 1$

$q\,\bar{q} \rightarrow \mathrm{Z} + \mathrm{g}_3\ \mathrm{g}_4\ \mathrm{g}_5$ (g_3 soft & $\mathrm{g}_4 \parallel \bar{q}$)



(approach singular limit: $x_i = 10^{-7},\ 10^{-8},\ 10^{-9}$)

```
   _   ___   ____    ____      _____________
  / |/ / |/ / /  / __ \__ / / __/_   __/
 /    /    / /__/ /_/ / // / _/  / /
/_/|_/_/|_/____/\____/\___/___/ /_/
```

X. Chen, J. Cruz-Martinez, J. Currie, R. Gauld, A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, M. Höfer, AH, I. Majer, J. Mo, T. Morgan, J. Niehues, J. Pires, D. Walker, J. Whitehead

Processes computed using the antenna subtraction method

- $\mathrm{pp} \to V$ @ NNLO
- $\mathrm{pp} \to V + j$ @ NNLO
  - $\hookrightarrow V \to \ell\bar{\ell}$ $(V = \mathrm{Z}/\gamma^*, \mathrm{W}^\pm)$
- $\mathrm{pp} \to$ jets (inc. jets, 2j) @ NNLO
- $\mathrm{pp} \to \gamma + j$ @ NNLO
- $\mathrm{ep} \to 1j$ @ $\mathrm{N^3LO}$
- $\mathrm{ep} \to 2j$ @ NNLO
- $\mathrm{e^+e^-} \to 3$ jets @ NNLO
- $\mathrm{pp} \to \mathrm{H}$ (ggH) @ $\mathrm{N^3LO}$
- $\mathrm{pp} \to \mathrm{H} + j$ (ggH) @ NNLO
- $\mathrm{pp} \to \mathrm{H} + 2j$ (VBF) @ NNLO
  - $\hookrightarrow \mathrm{H} \to \gamma\gamma,\ \tau\tau,\ V\gamma,\ VV$
- $\mathrm{pp} \to V\mathrm{H}$ @ NNLO
  - $\hookrightarrow \mathrm{H} \to \mathrm{bb}$
- ...

```
  _   ___   ____    ____      _____________
 / |/ / |/ / /  / __ \__ / / __/_   __/
/    /    / /__/ /_/ / // / _/  / /
/_/|_/_/|_/____/\____/\___/___/ /_/
```

X. Chen, J. Cruz-Martinez, J. Currie, R. Gauld, A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, M. Höfer, AH, I. Majer, J. Mo, T. Morgan, J. Niehues, J. Pires, D. Walker, J. Whitehead

Processes computed using the antenna subtr[illegible] m[illegible]

- $\mathrm{pp} \to V$ @ NNLO
- $\mathrm{pp} \to V + j$ @ [illegible]
  - $\hookrightarrow V \to \ell\bar{\ell}$ (V[illegible]
- $\mathrm{pp} \to$ jets [illegible]
- $\mathrm{pp} \to \gamma +$ [illegible]
- ep [illegible]
- ep [illegible] @ NNLO
- $\mathrm{e^+e^-}$ [illegible] @ NNLO

[illegible] N^{3}LO
[illegible] @ NNLO
[illegible] (VBF) @ NNLO
[illegible] $\to \gamma\gamma,\ \tau\tau,\ V\gamma,\ VV$
$\mathrm{pp} \to V\mathrm{H}$ @ NNLO
$\hookrightarrow \mathrm{H} \to \mathrm{bb}$
- ...

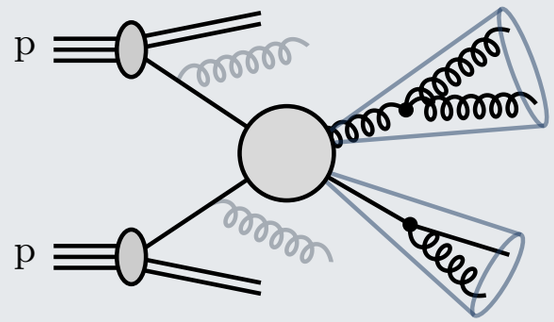
NNLO subtraction set up for

p p → "colour neutral" + 0,1,2 jets

The Plan.

1. *Precision Predictions for the LHC*
   - *The Antenna Subtraction Formalism*
2. *Hard QCD Probes*
   - *Jets & Photon Production at NNLO*
3. *Differential Higgs Production*
   - *The Projection-to-Born Method*

Jet Production at the LHC

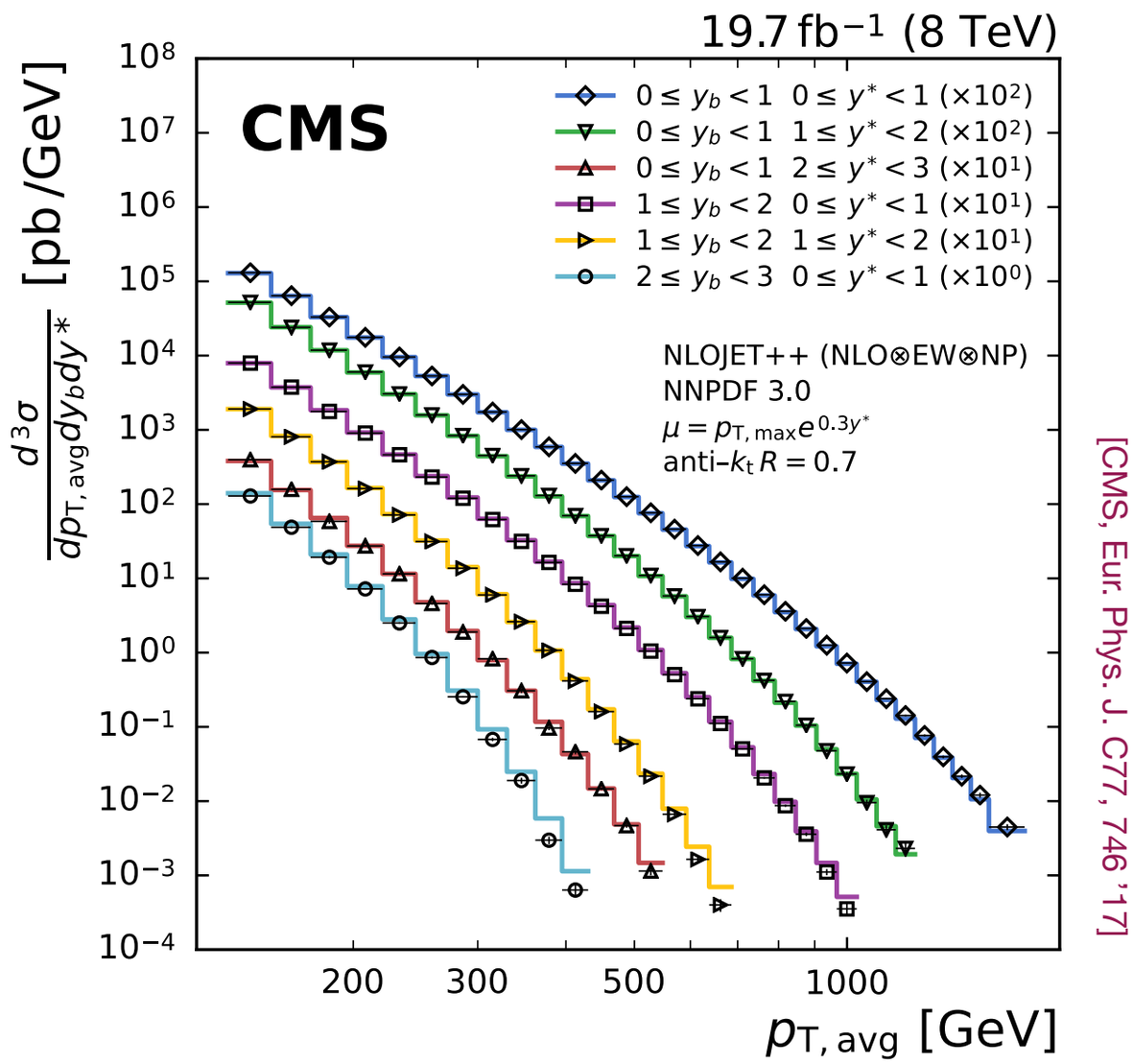


$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{jet(s)} + X$

- jets produced in abundance
- precise measurements $(p_{\mathrm{T},j} \gtrsim 20\ \mathrm{GeV})$
- wide kinematic range accessible

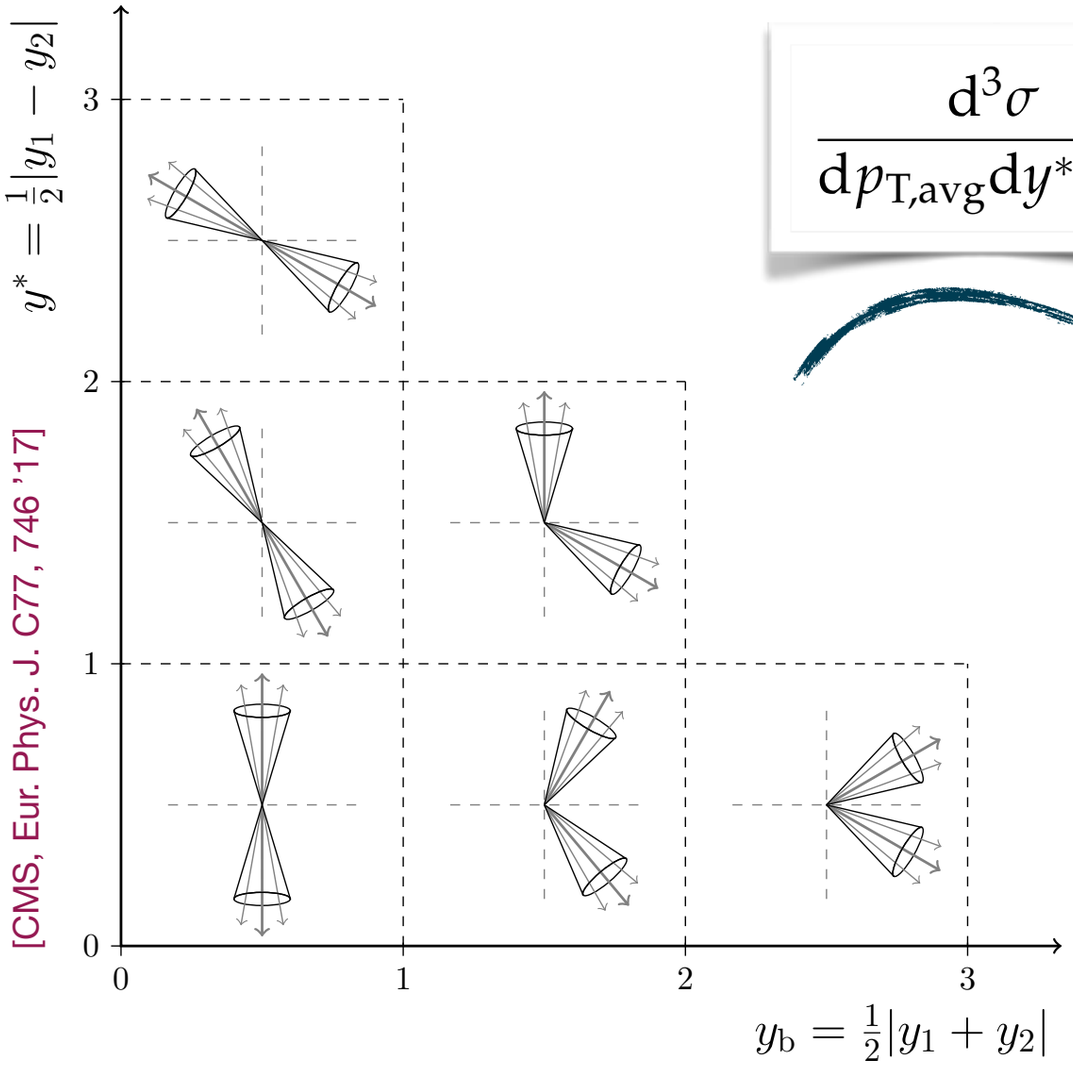
- test perturbative QCD
  ↪ study scale choices
- constrain PDFs
  ↪ sensitive to *gluon*
  ↪ probe wide x-range
- $\alpha_{\mathrm{s}}(M_{\mathrm{Z}})$ and *running*
- search for BSM physics

high-precision predictions mandatory!



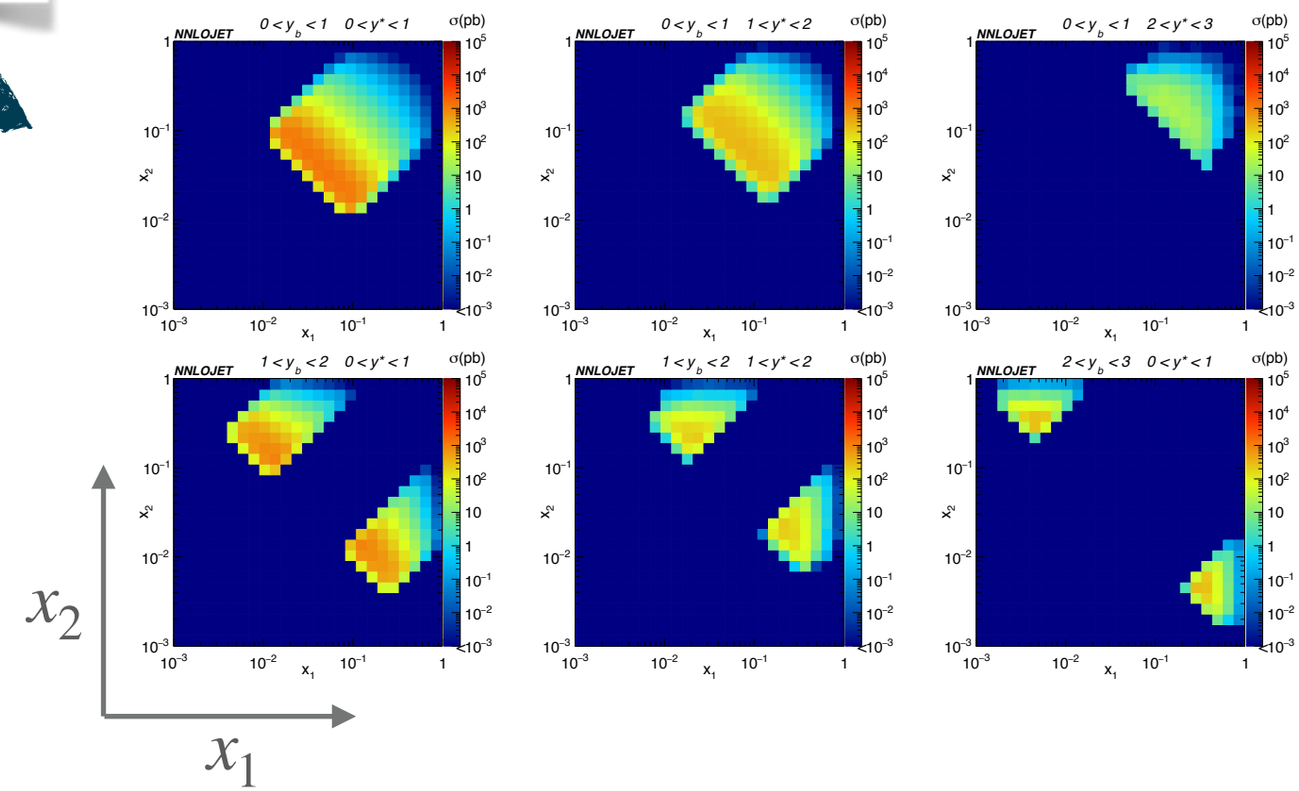
[CMS, Eur. Phys. J. C77, 746 '17]

Triple-Differential Cross Section



$$\frac{d^3\sigma}{dp_{T,avg}\,dy^*\,dy_b}$$

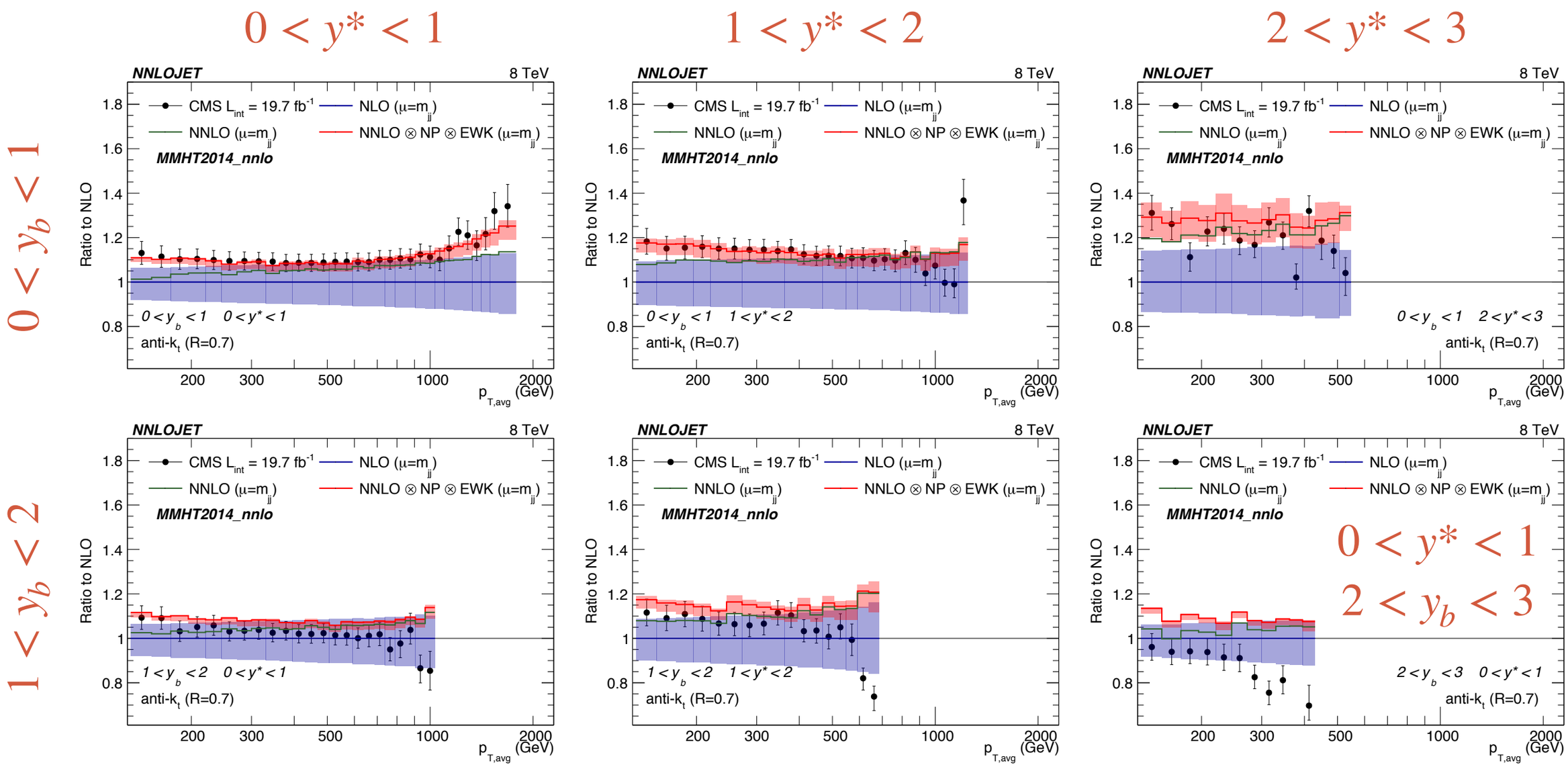
$$x_{1,2} = \frac{2p_{T,avg}}{\sqrt{s}} e^{\pm y_b} \cosh(y^*)$$



- study different kinematic regimes
- disentangle momentum fractions x_1 & x_2

Triple-Differential Cross Section @ NNLO

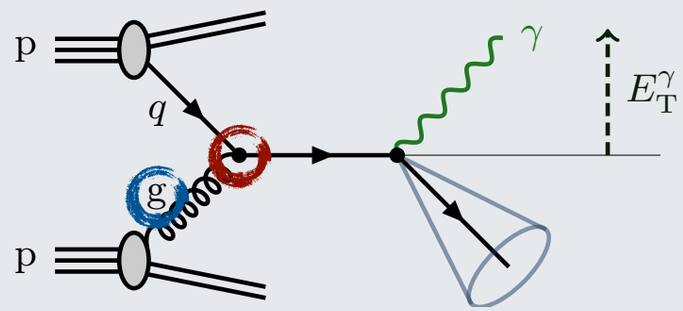
[Gehrmann-De Ridder, Gehrmann, Glover, AH, Pires '19]



NLO NNLO NNLO ⊗ NP ⊗ EWK

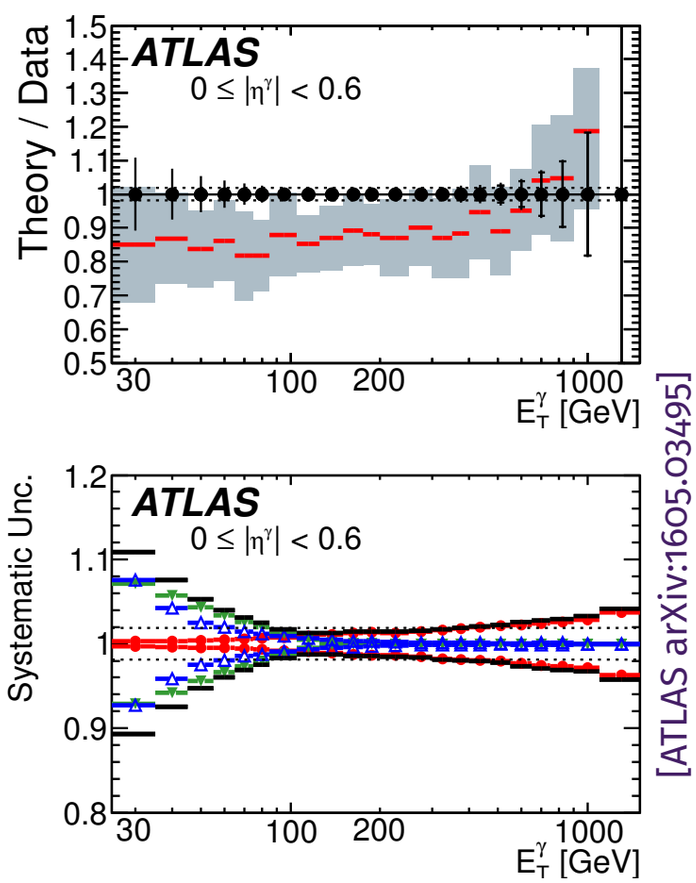
improved description of data & reduced uncertainties!

Photon & Photon+Jet Production



$\mathrm{p\,p} \to \gamma + X$

- highest-rate electroweak process @ LHC
- photon as probe of hard scattering
  $\rightsquigarrow$ sensitivity to α_s, gluon PDF



[ATLAS arXiv:1605.03495]

JetPhox (NLO QCD)
[Catani, Fontannaz, Guillet, Pilon '02]
$\hookrightarrow$ tension between theory vs. data
$\hookrightarrow$ large scale uncertainties: $\sim \pm 10\%$

- experimental uncertainties $\lesssim \pm 3\text{–}5\%$
  $\hookrightarrow$ smaller than NLO theory
  $\Rightarrow$ NNLO QCD needed!

Photon Isolation

Suppress contamination from secondary photons (e.g. $\pi^0 \to \gamma\gamma$)

$\rightsquigarrow$ *isolation cuts:* restrict hadronic activity in $R = \sqrt{\Delta\eta^2 + \Delta\varphi^2}$

Fixed cone isolation

can choose simple linear dependence:

$$E_{\mathrm{T}}^{\text{had.}}(R) < E_{\mathrm{T}}^{\max} = \epsilon E_{\mathrm{T}}^{\gamma} + E_{\mathrm{T}}^{\text{thresh.}}$$

✓ used in experiments

✗ sensitivity to fragmentation

Dynamic cone isolation [Frixione '98]

smoothly get rid of collinear radiation:

$$E_{\mathrm{T}}^{\text{had.}}(r) < \epsilon E_{\mathrm{T}}^{\gamma} \left(\frac{1 - \cos r}{1 - \cos R} \right)^n \quad \forall r < R$$

✓ eliminates fragmentation part

✗ no direct analogue in experiment

Mismatch: experiment vs. theory

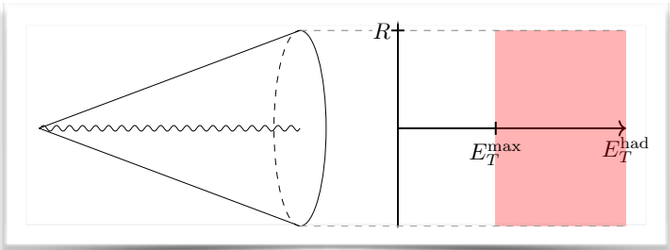
- "tight enough" isolation: $\sim$ few % [Les Houches '13 '15]

But: experiment & NNLO theory: $\lesssim 5\%$

$\hookrightarrow$ percent-level phenomenology a reality!

Can we do better?

Photon Isolation cont.



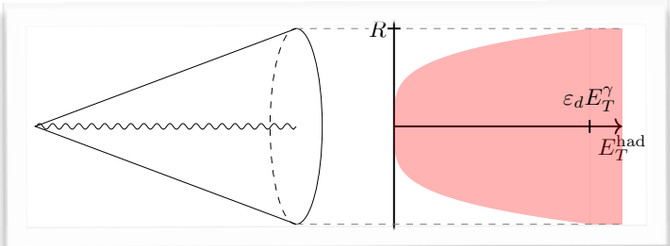
Fixed cone isolation

can choose simple linear dependence:

$$E_{\mathrm{T}}^{\text{had.}}(R) < E_{\mathrm{T}}^{\max} = \epsilon E_{\mathrm{T}}^{\gamma} + E_{\mathrm{T}}^{\text{thresh.}}$$

✓ used in experiments

✗ sensitivity to fragmentation



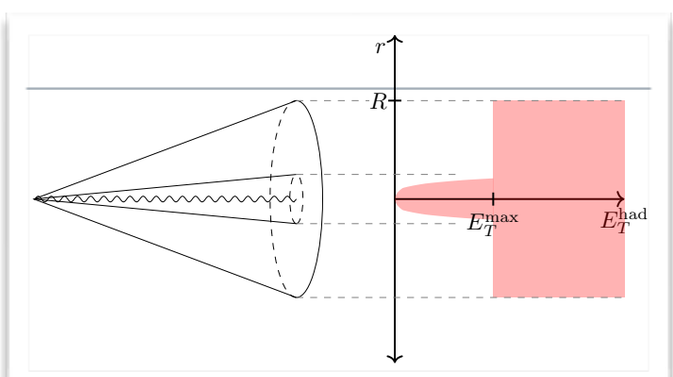
Dynamic cone isolation [Frixione '98]

smoothly get rid of collinear radiation:

$$E_{\mathrm{T}}^{\text{had.}}(r) < \epsilon E_{\mathrm{T}}^{\gamma} \left(\frac{1-\cos r}{1-\cos R} \right)^{n} \quad \forall r < R$$

✓ eliminates fragmentation part

✗ no direct analogue in experiment



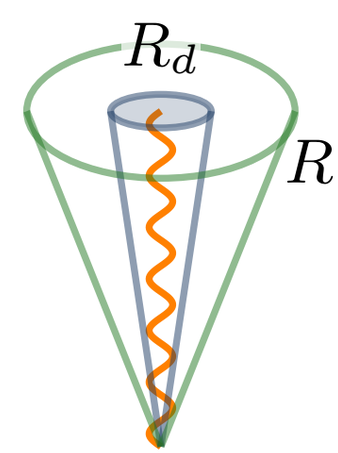
Hybrid cone isolation [Siegert '17]

1. *narrow* dynamic cone $R_d < R$ (0.1)
2. *wider* fixed cone R (0.4)

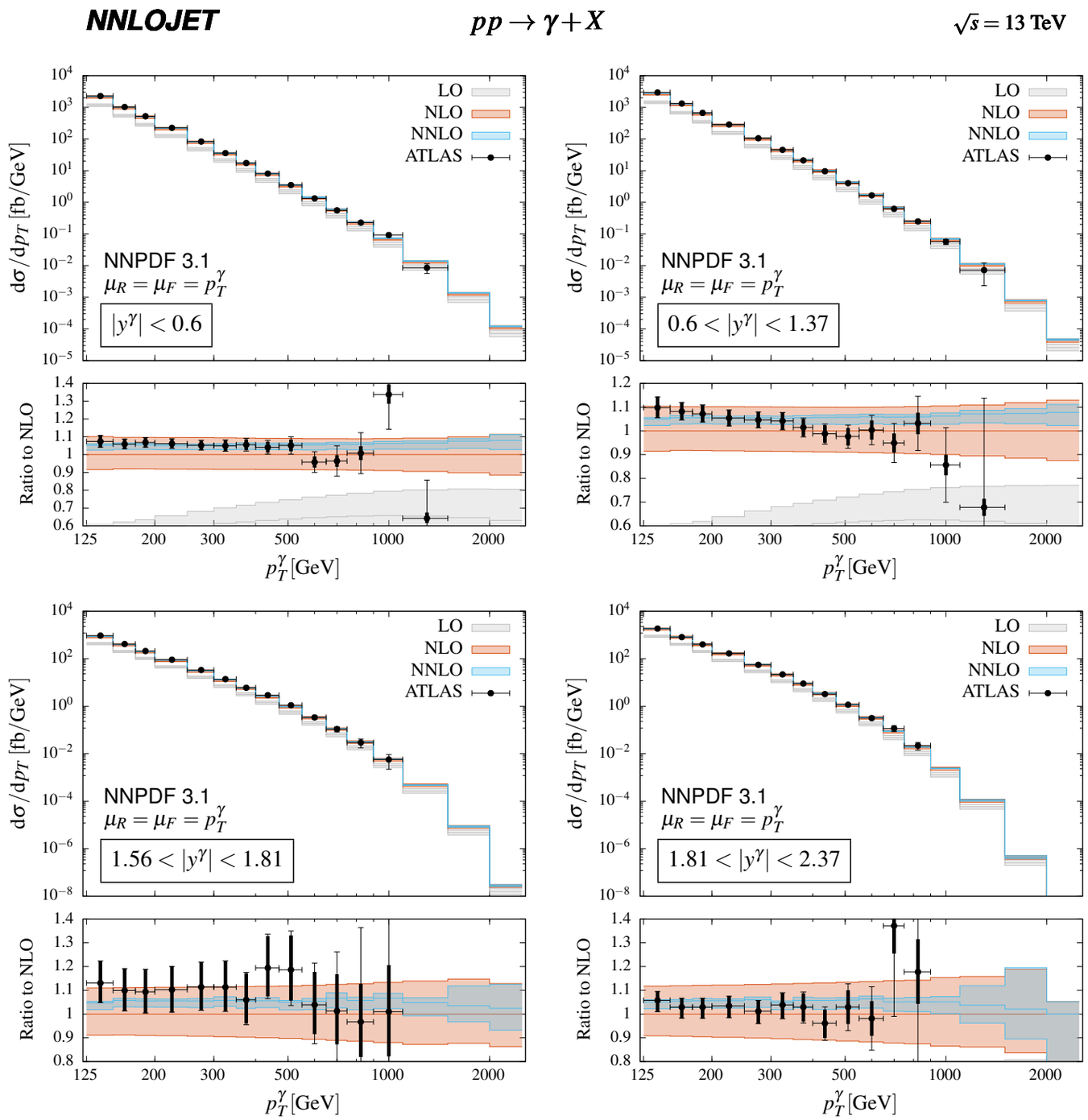
✓ eliminates fragmentation part

✓ reduces mismatch to experiment

✓ *correct* R dependence



Photon Production @ 13 TeV



[Chen, Gehrmann, Glover, Höfer, AH '19]

hybrid isolation

NLO ($\rightsquigarrow 1$)

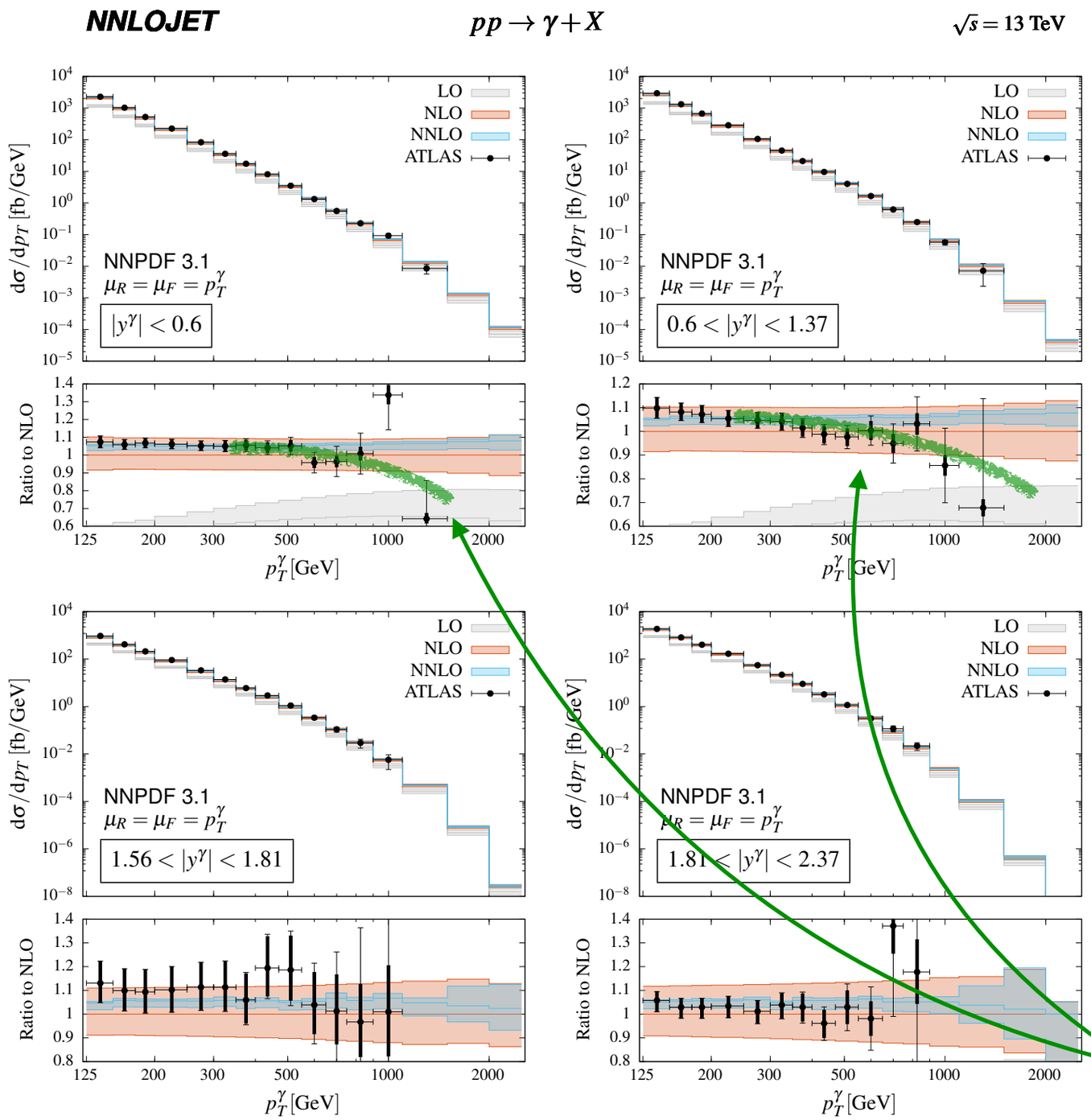
- $+40\%$ corrections
- $\pm 10\%$ uncertainties

NNLO

- $\sim 5\%$ corrections
- *shape distortions*
- $\lesssim 5\%$ uncertainties

- previous NNLO calculation
  τ_N [Campbell, Ellis, Williams '17]
  (dynamical cone isol.)

Photon Production @ 13 TeV



[Chen, Gehrmann, Glover, Höfer, AH '19]

hybrid isolation

NLO ($\rightsquigarrow 1$)

- $+40\%$ corrections
- $\pm 10\%$ uncertainties

NNLO

- $\sim 5\%$ corrections
- *shape distortions*
- $\lesssim 5\%$ uncertainties

- previous NNLO calculation
  τ_N [Campbell, Ellis, Williams '17]
  (dynamical cone isol.)

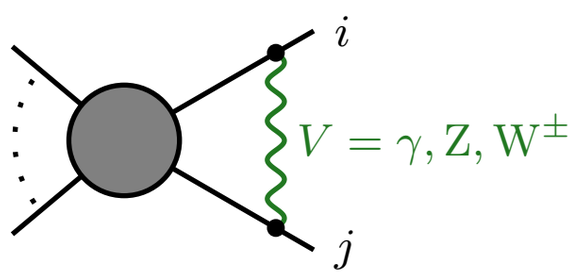
small systematic trend in the tails?

Electroweak Interactions

- generic size: $O(\alpha) \sim O(\alpha_s^2)$
- systematic enhancements possible:

Sudakov Logarithms

(kinematic tails)

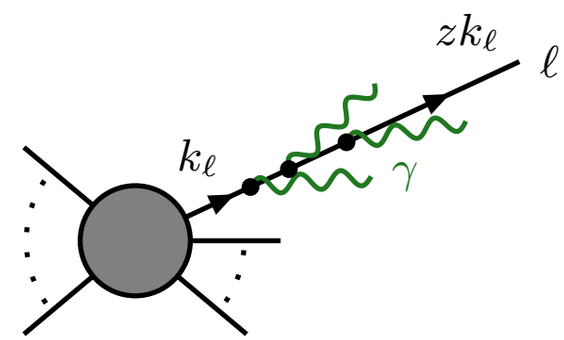


$$\sim \ln^2\left(\frac{s_{ij}}{M_{\mathrm{W}}^2}\right) + \text{sub-leading (collinear)}$$

O(10-20%) corrections!

Final-State Radiation

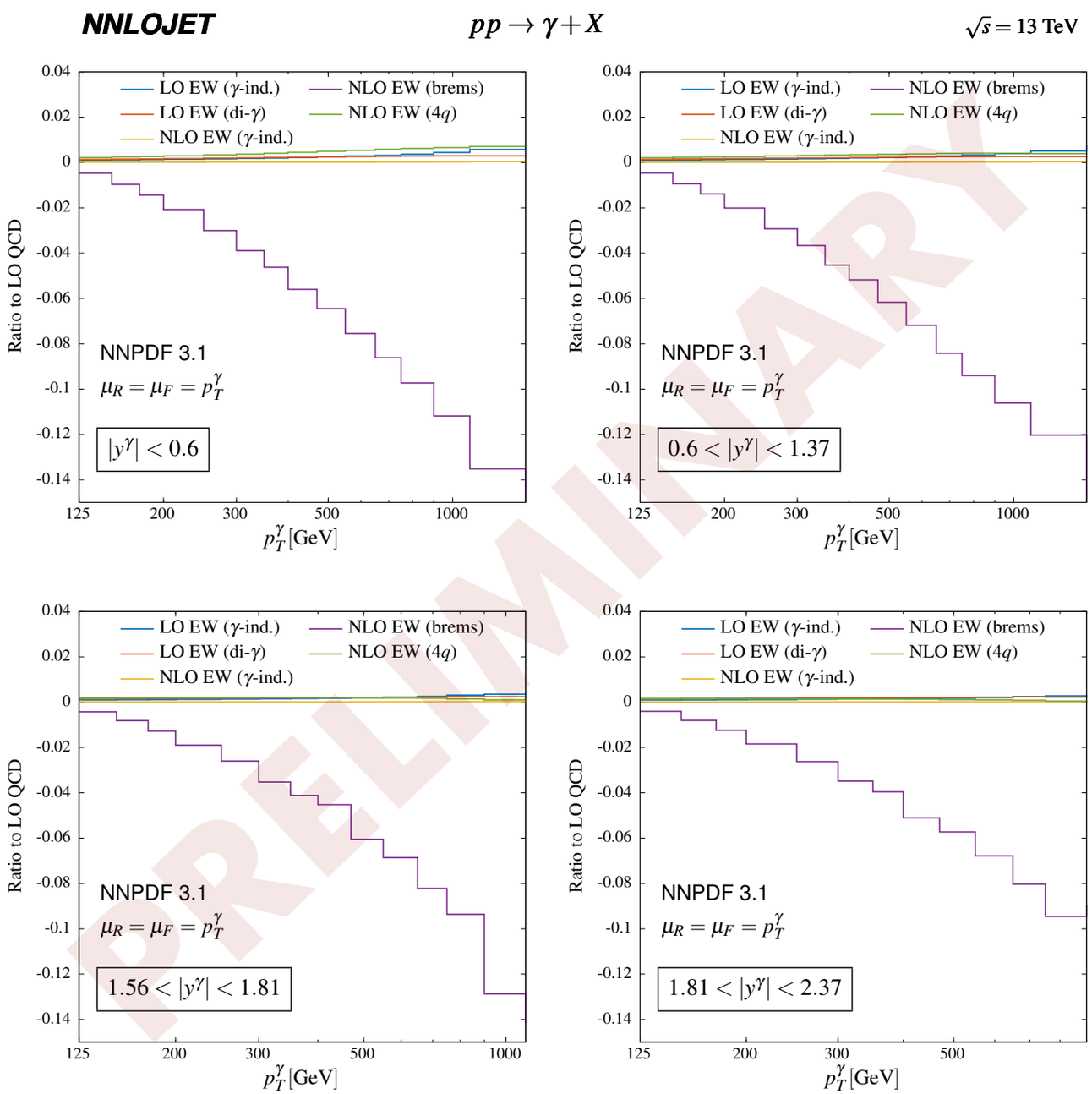
(resonances, shoulders, …)



$$\rightsquigarrow \alpha^n \ln^n\left(\frac{Q^2}{m_\ell^2}\right)$$

O(10-100%) corrections!

EW Corrections using Antennae



- *dipole subtraction:*

$$\sum_i \sum_{j \neq i} \mathscr{D}_{ik,j} \otimes |\mathscr{M}(\ldots, \tilde{i}, \tilde{j}, \ldots)|^2$$

- *antenna subtraction:**

$$\sum_i \sum_{j<i} A_3^0(i,k,j) \otimes |\mathscr{M}(\ldots, \tilde{i}, \tilde{j}, \ldots)|^2$$

$\Rightarrow$ *reduction in # of terms by* $\times 2$*!*

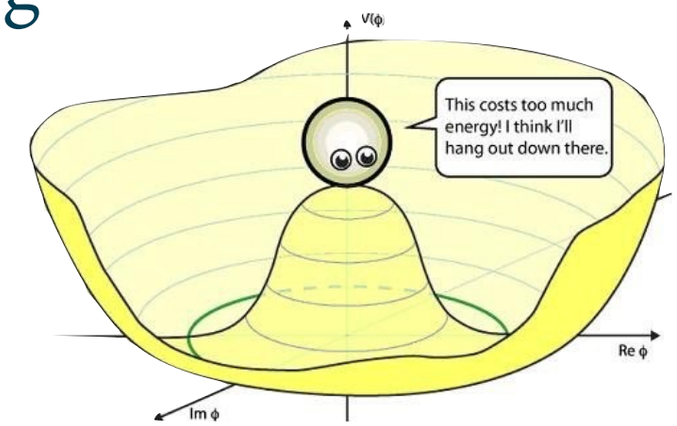
* *fully algorithmic & general*

The Plan.

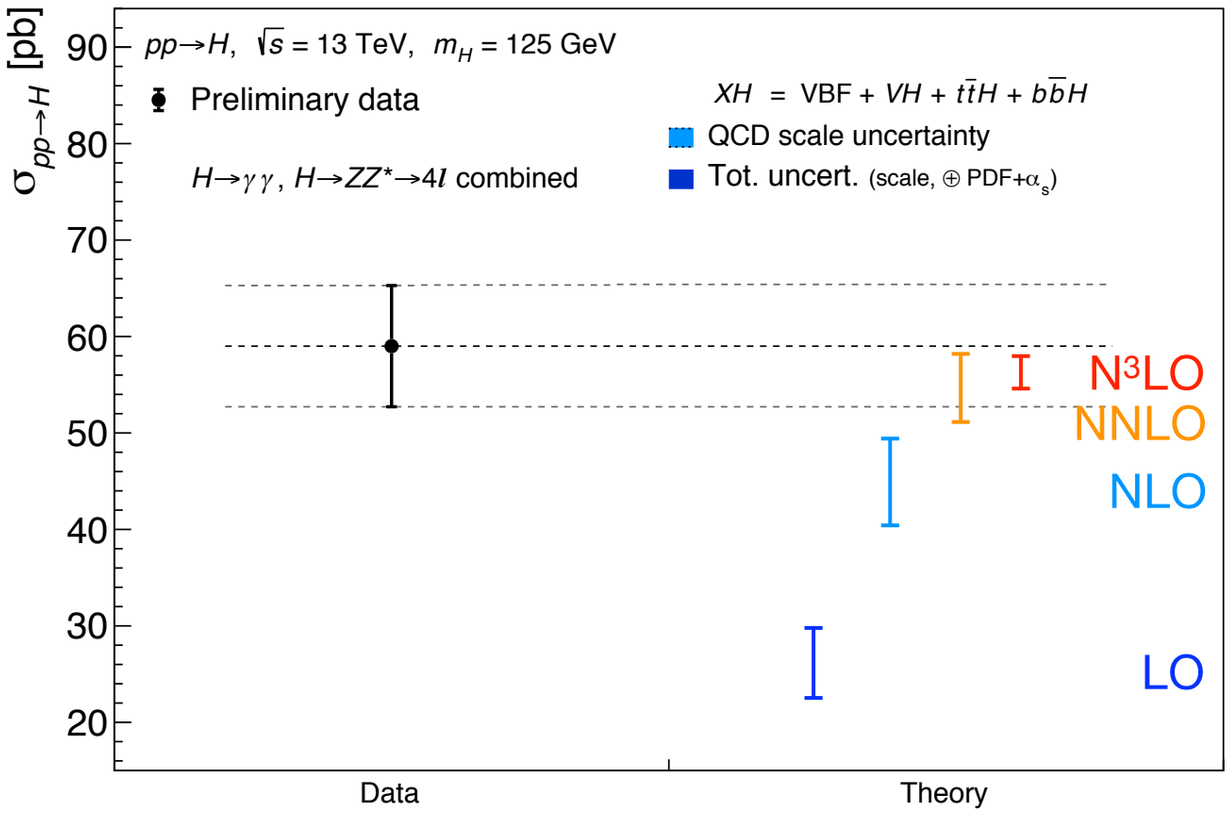
1. *Precision Predictions for the LHC*
   - *The Antenna Subtraction Formalism*
2. *Hard QCD Probes*
   - *Jets & Photon Production at NNLO*
3. *Differential Higgs Production*
   - *The Projection-to-Born Method*

The Higgs Boson

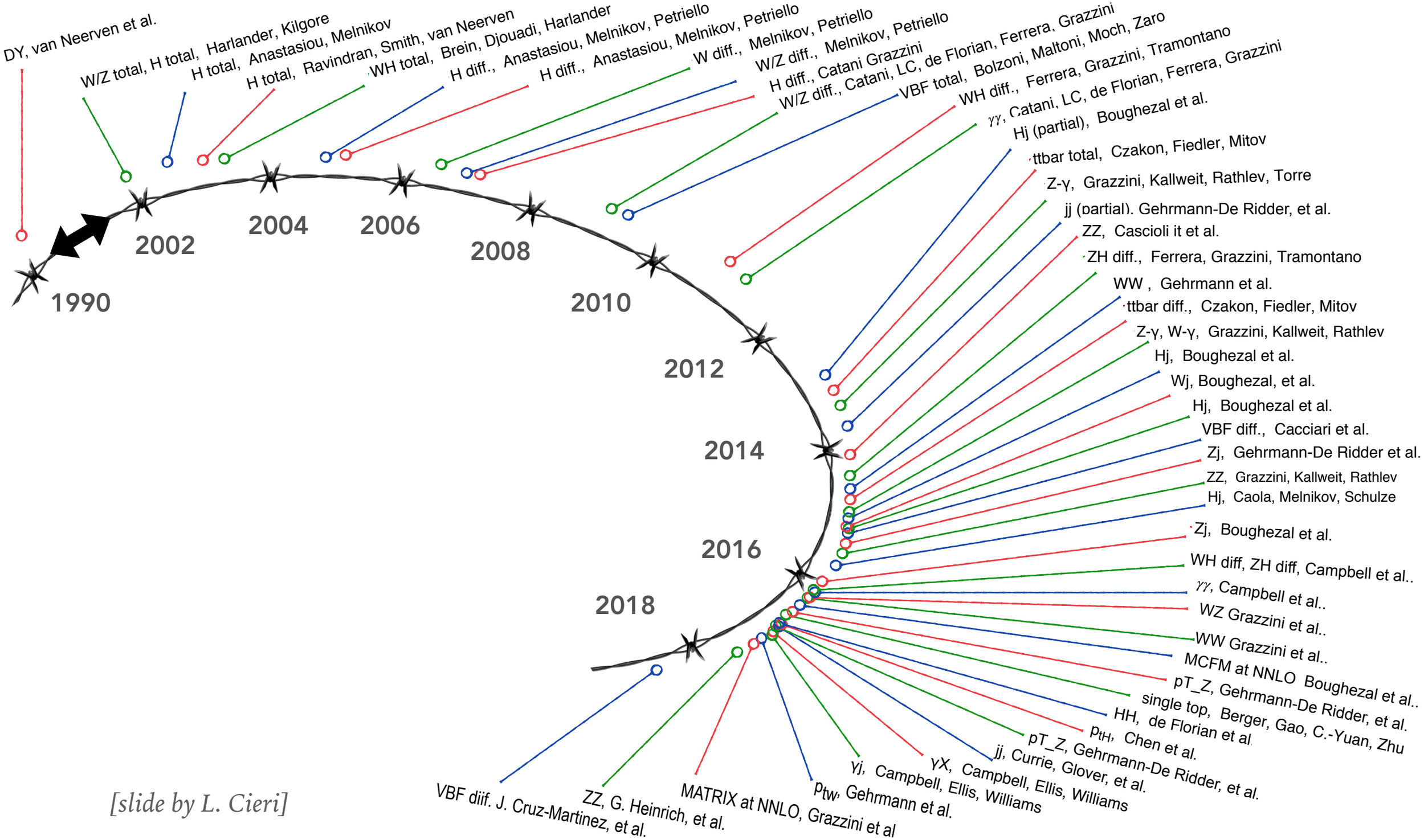
- experimental era of Higgs physics just starting
  - scrutinise all properties
  - couplings/interactions
  - probe the potential



- notoriously bad perturbative convergence (need N³LO)

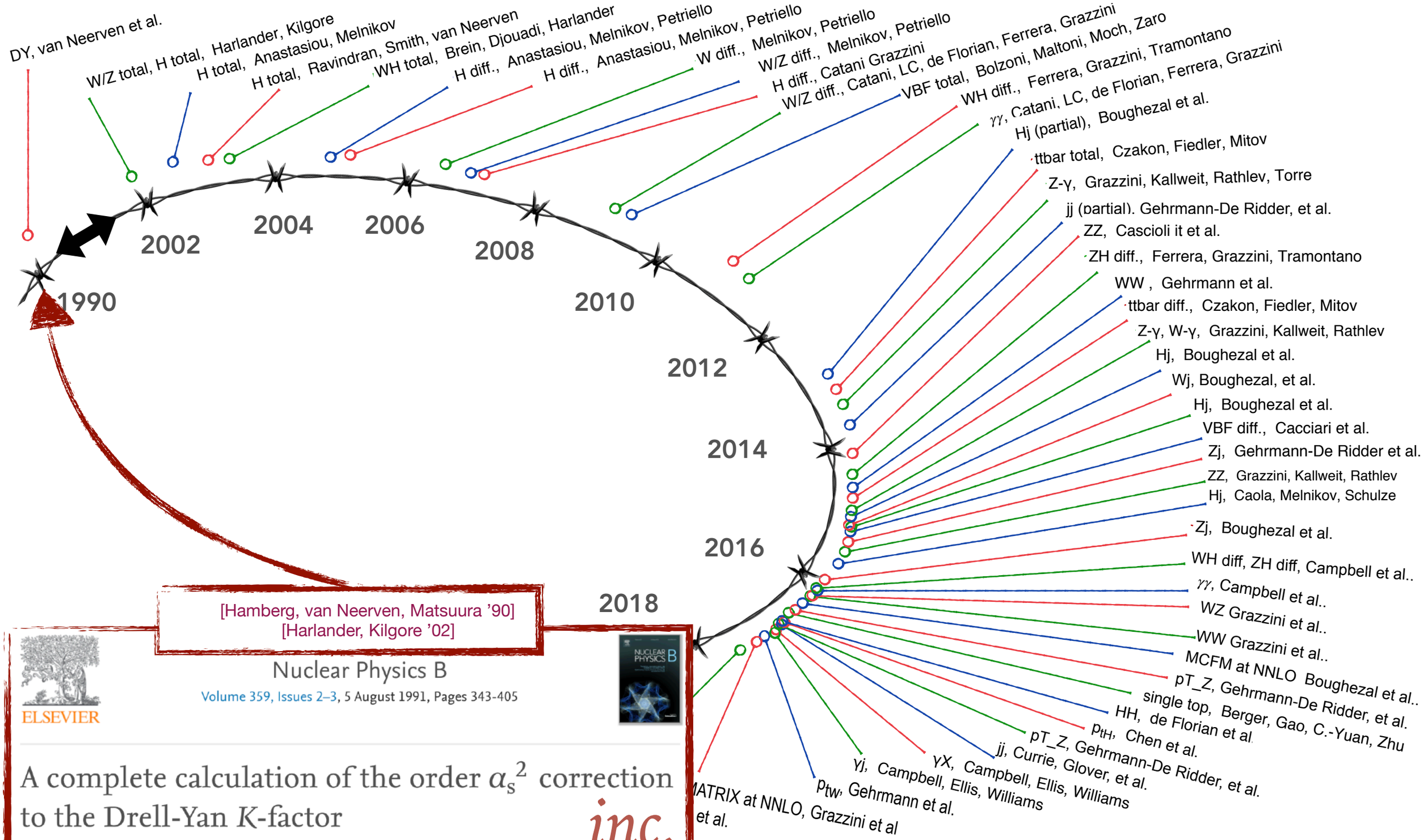


Drell–Yan @ NNLO: When?

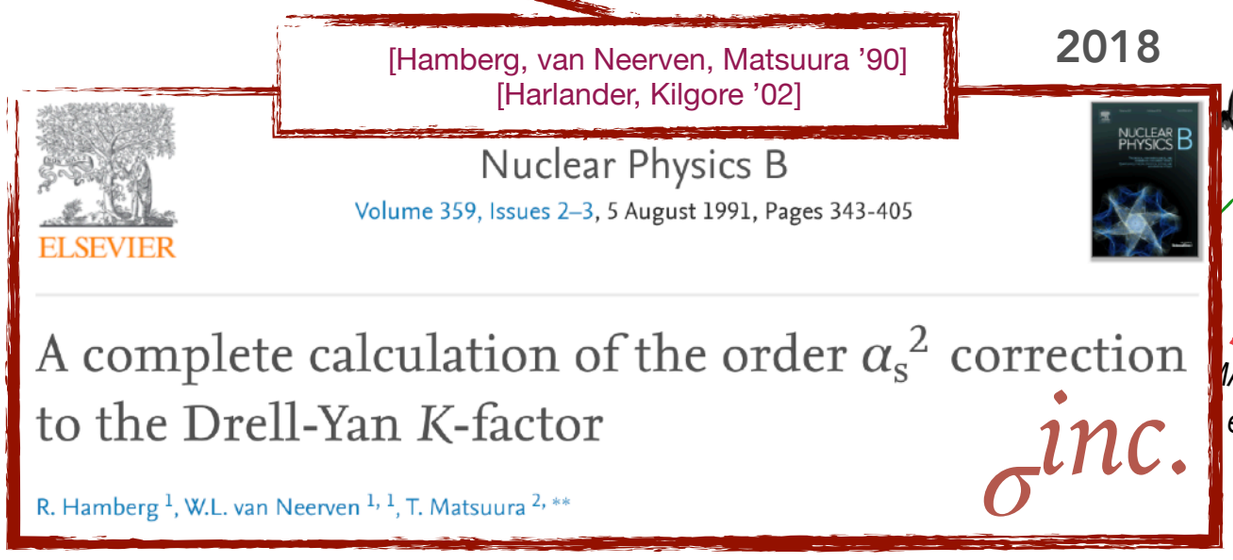


[slide by L. Cieri]

Drell–Yan @ NNLO: When?

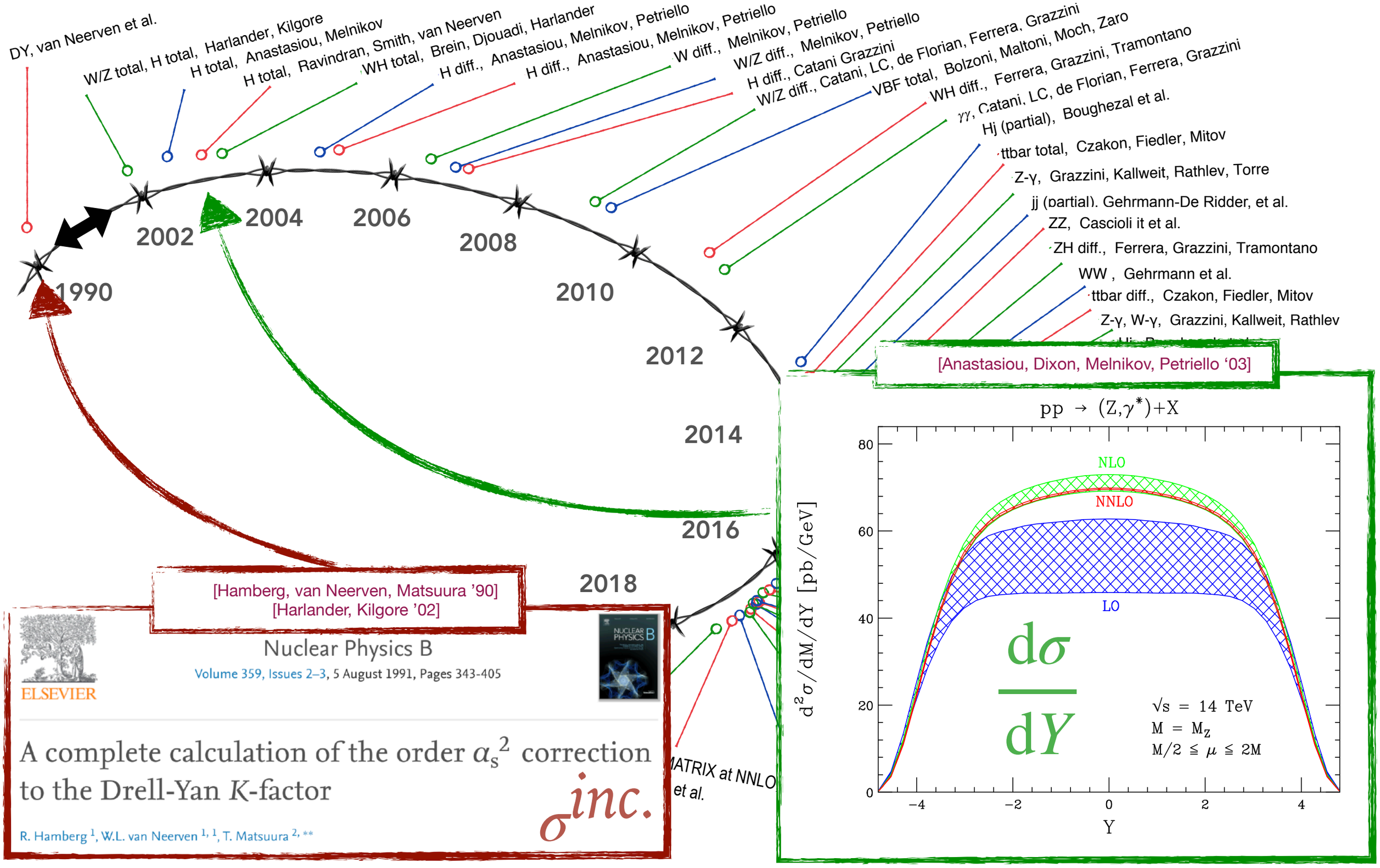


[Hamberg, van Neerven, Matsuura '90]
[Harlander, Kilgore '02]

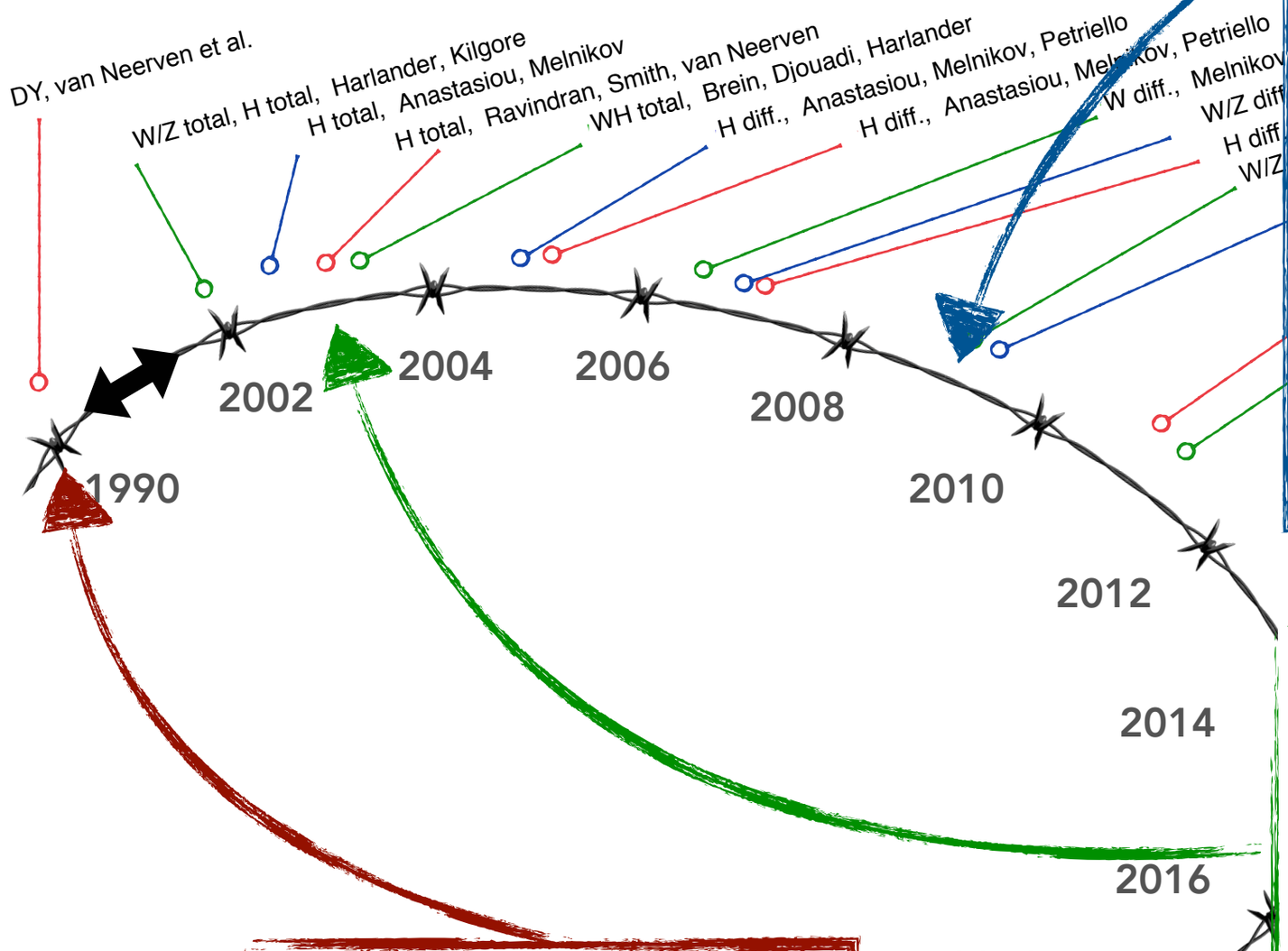


$\sigma^{inc.}$

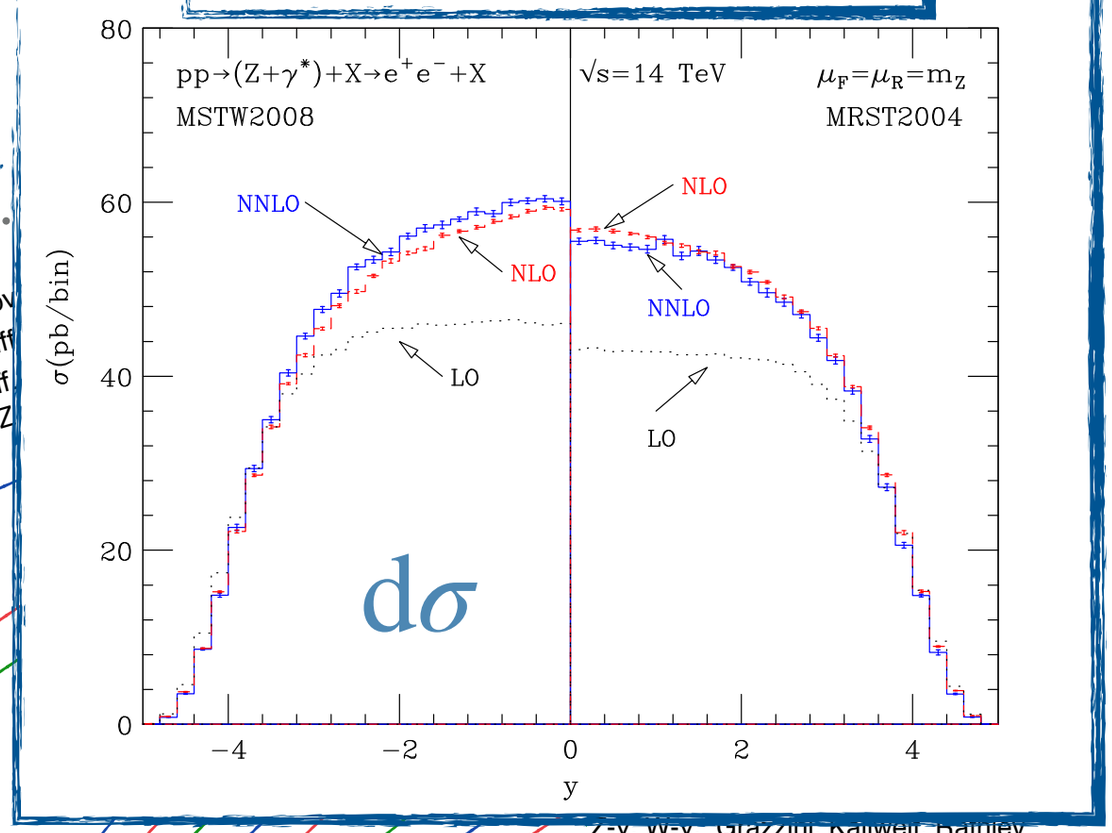
Drell–Yan @ NNLO: When?



Drell–Yan @ NNLO: When?

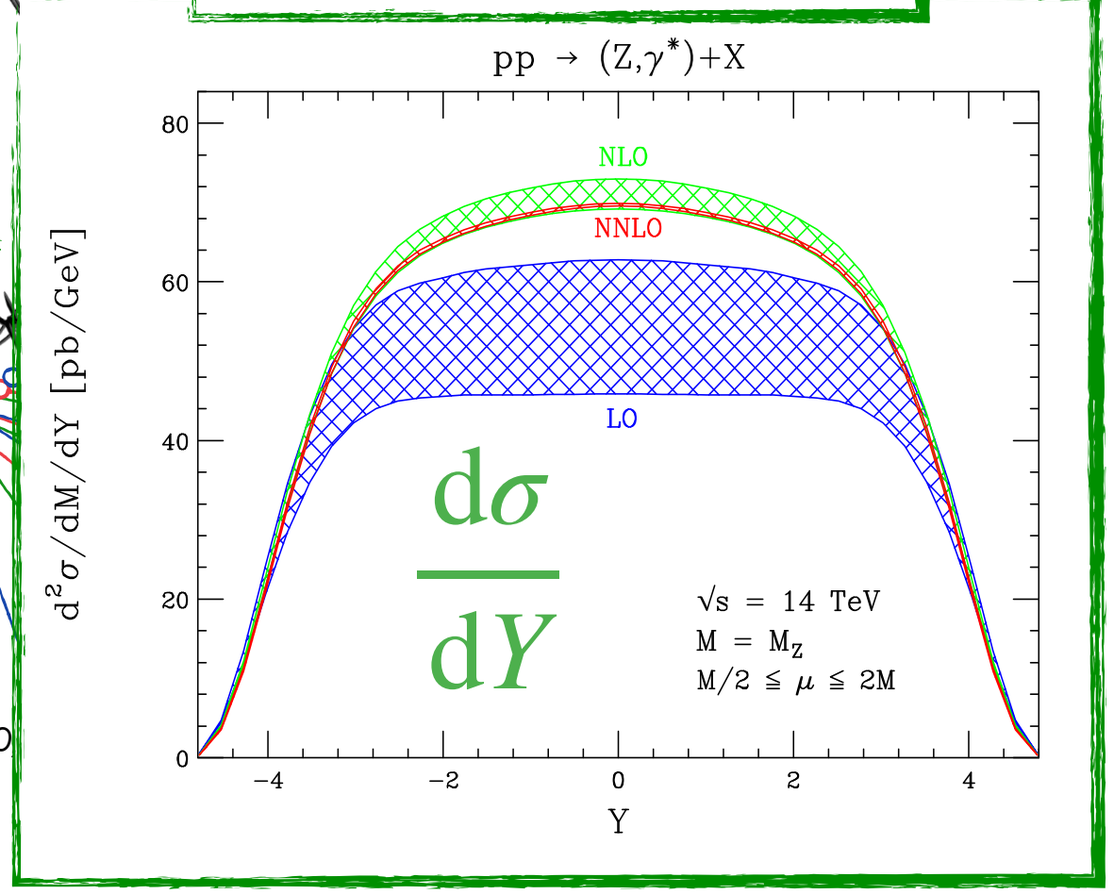


[Catani, Cieri, Ferrera, de Florian, Grazzini '09]

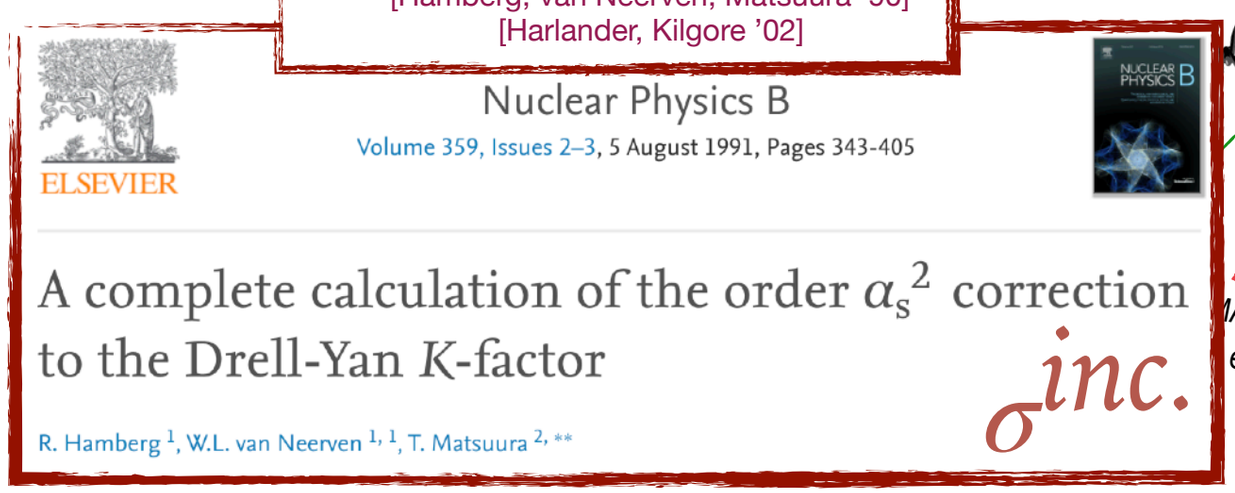


Z-γ, W-γ, Grazzini, Kallweit, Rathlev

[Anastasiou, Dixon, Melnikov, Petriello '03]



[Hamberg, van Neerven, Matsuura '90]
[Harlander, Kilgore '02]



ATRIX at NNLO
et al.

LHC — Going Fully Differential @ N³LO

inclusive

$$\sigma_{\text{tot}}^{\text{N}^3\text{LO}} = 48.68\ \text{pb}^{+2.07\ \text{pb}}_{-3.16\ \text{pb}}$$

✓ analytic integration over full phase space

✗ no information on final state

[Anastasiou et al. ’15] [Mistlberger ’18]

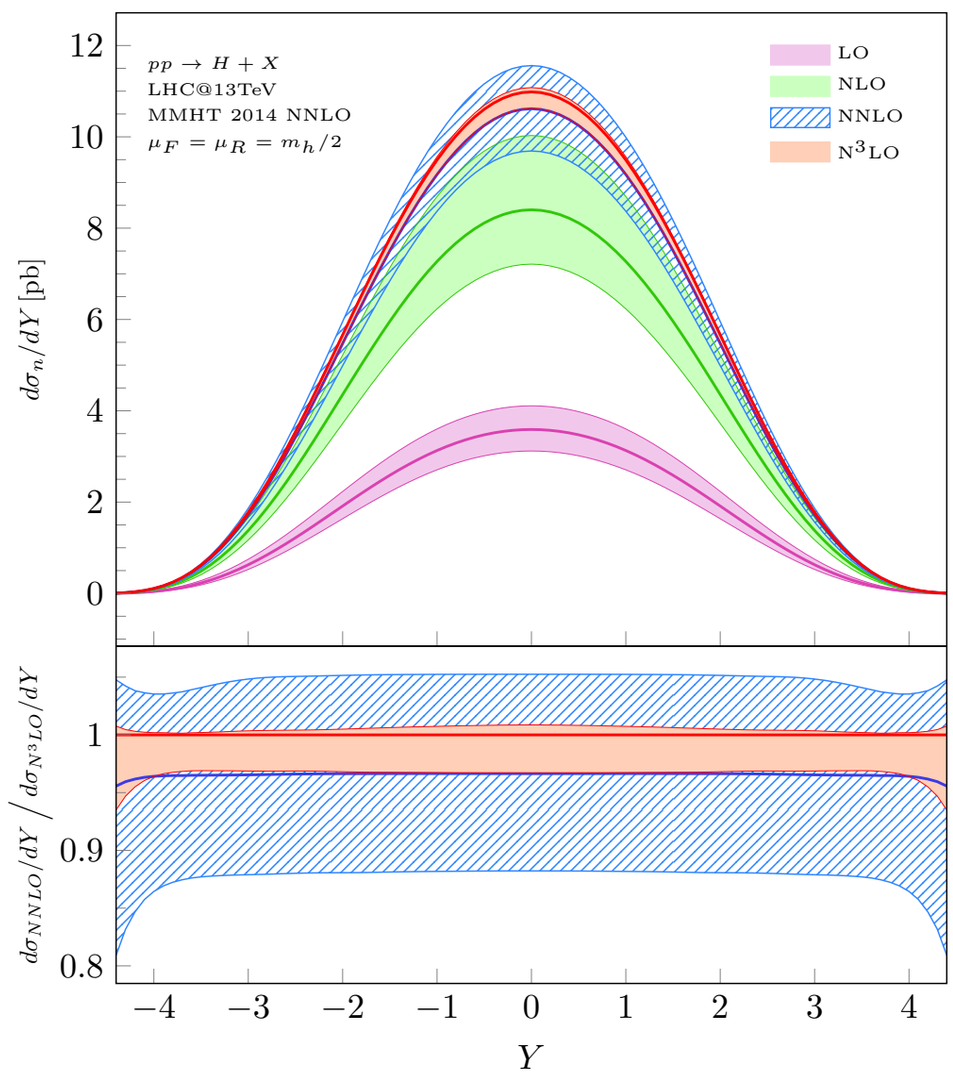
LHC — N³LO

$\sigma_{\text{tot}}^{\text{N}^3\text{LO}}$

✔ analytic i

✘ no inform

y_{H} differential



✔ analytic integration over QCD emissions

✘ partial information on final state

- only y_{H} ⇝ no decay kinematics
- no information on final-state partons

[Dulat, Mistlberger, Pelloni '18]

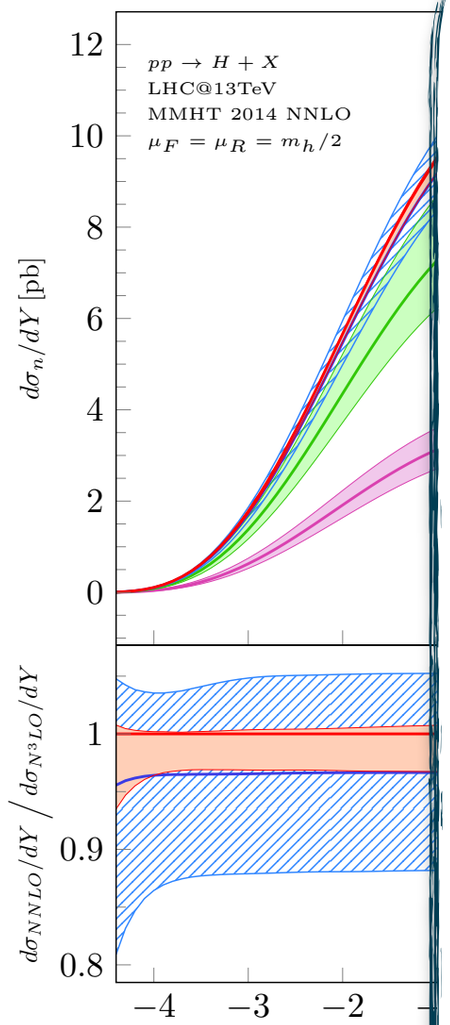
LHC –

$\sigma_{tot}^{N^3LO}$

✔ analytic i

✘ no inform

y_H differen

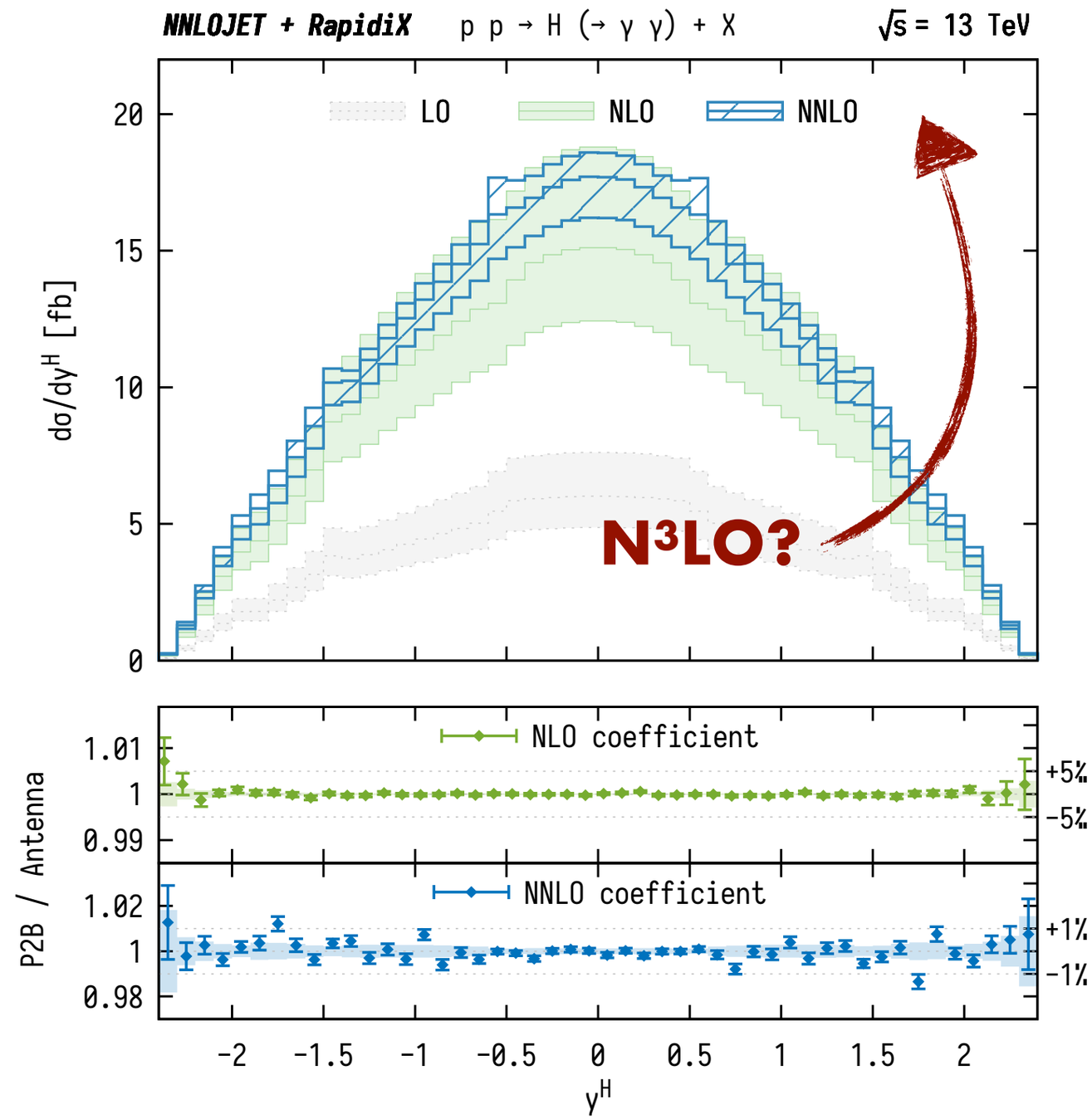


✔ analytic integratio

✘ partial informatio

- ➤ only y_H ⇝ r
- ➤ no informat

fully differential



✔ numerical integration of phase space

✔ complete final-state information (decay, isol., …)

[Chen, Dulat, Gehrmann, Glover, AH, Mistlberger, Pelloni (to appear)]

The Projection-to-Born Method

[Cacciari et al. '15]

$$\frac{\mathrm{d}\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}} = \frac{\mathrm{d}\sigma_{F,\,\mathrm{inc.}}^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}_B} + \left\{ \frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}} - \frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}}\bigg|_{\mathcal{O}\to\mathcal{O}_B} \right\}$$

observables projected to Born
fully local counter term

- start: inclusive calculation
  - differential in Born variables
- supplement fully differential information:
  - difference of a "+jet" calculation at one order lower

Higgs @ N³LO using Projection-to-Born

H + jet

@ $N^{n-1}LO$

Projection-to-Born

$\oplus$

$d\sigma/dy_H$

@ N^nLO

$=$

H fully differential @ N^nLO

?

* *Born variables: (Y, M²)*

Higgs @ N³LO using Projection-to-Born

H + jet @ $N^{n-1}LO$

Projection-to-Born

$\oplus$

$d\sigma/dy_H$ @ N^nLO

$=$ H fully differential @ N^nLO ?

* *Born variables: (Y, M²)*

- real-emission phase space: $d\Phi_{H+n}$

$$p_a + p_b \to p_H + k_1 + k_2 + \ldots + k_n$$

- projection to Born: $d\widetilde{\Phi}_H$

$$\tilde{p}_a + \tilde{p}_b \to \tilde{p}_H \qquad (\tilde{p}_a = \xi_a p_a, \;\; \tilde{p}_b = \xi_b p_b)$$

on-shell: $\tilde{p}_H^2 \equiv p_H^2 = M_H^2 \quad \Rightarrow \quad \xi_a\,\xi_b = \dfrac{2p_a p_b - 2(p_a+p_b)k_{1\ldots n} + k_{1\ldots n}^2}{2p_a p_b}$

rapidity: $\tilde{y}_H \equiv y_H \quad \Rightarrow \quad \xi_a/\xi_b = \dfrac{2p_b p_H}{2p_a p_H}$

$\hookrightarrow$ decay products: $p_H \to p_1 + \ldots + p_m \qquad (p_i^\mu \to \tilde{p}_i^\mu = \Lambda^\mu{}_\nu\, p_i^\nu)$

$$\Lambda^\mu{}_\nu(p_H, \tilde{p}_H) = g^\mu{}_\nu - \frac{2(p_H+\tilde{p}_H)^\mu (p_H+\tilde{p}_H)_\nu}{(p_H+\tilde{p}_H)^2} + \frac{2\tilde{p}_H^\mu p_{H,\nu}}{p_H^2}$$

Higgs @ N³LO using Projection-to-Born

H + jet

@ $N^{n-1}LO$

Projection-to-Born

$\oplus$

$d\sigma/dy_H$

@ N^nLO

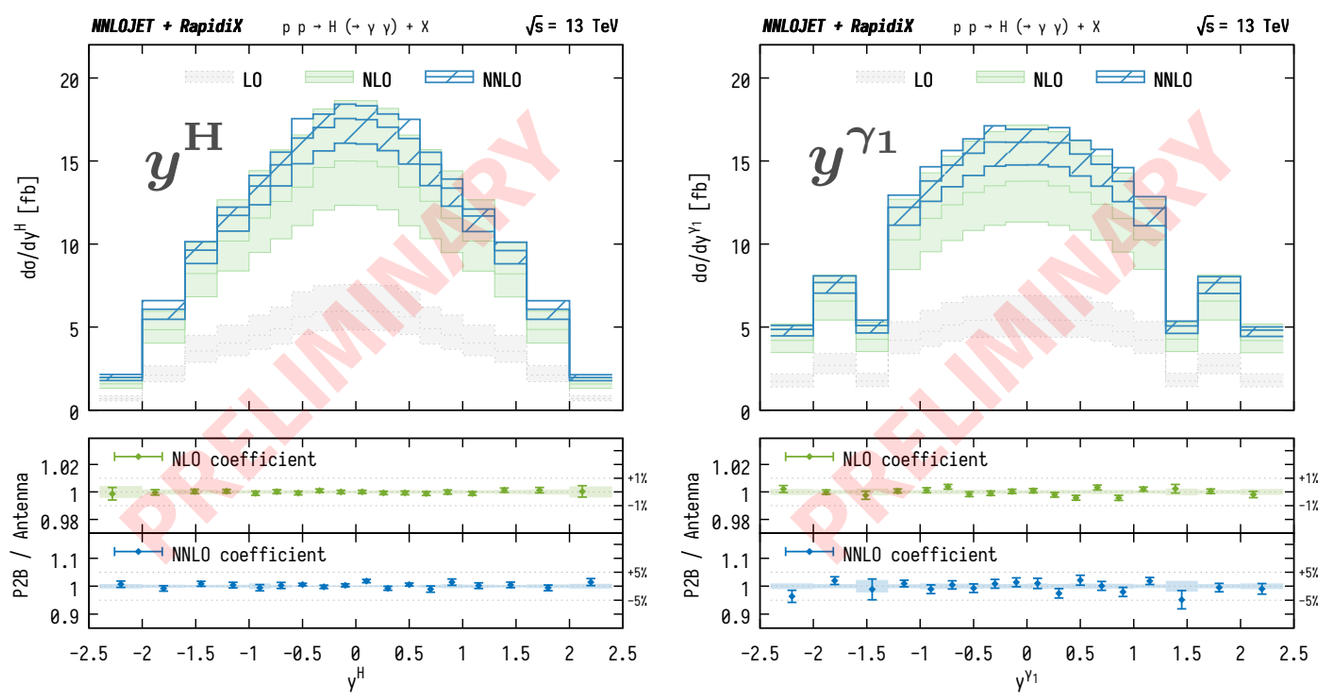
$=$

H fully differential @ N^nLO ?

* *Born variables: (Y, M²)*

Validated up to NNLO!

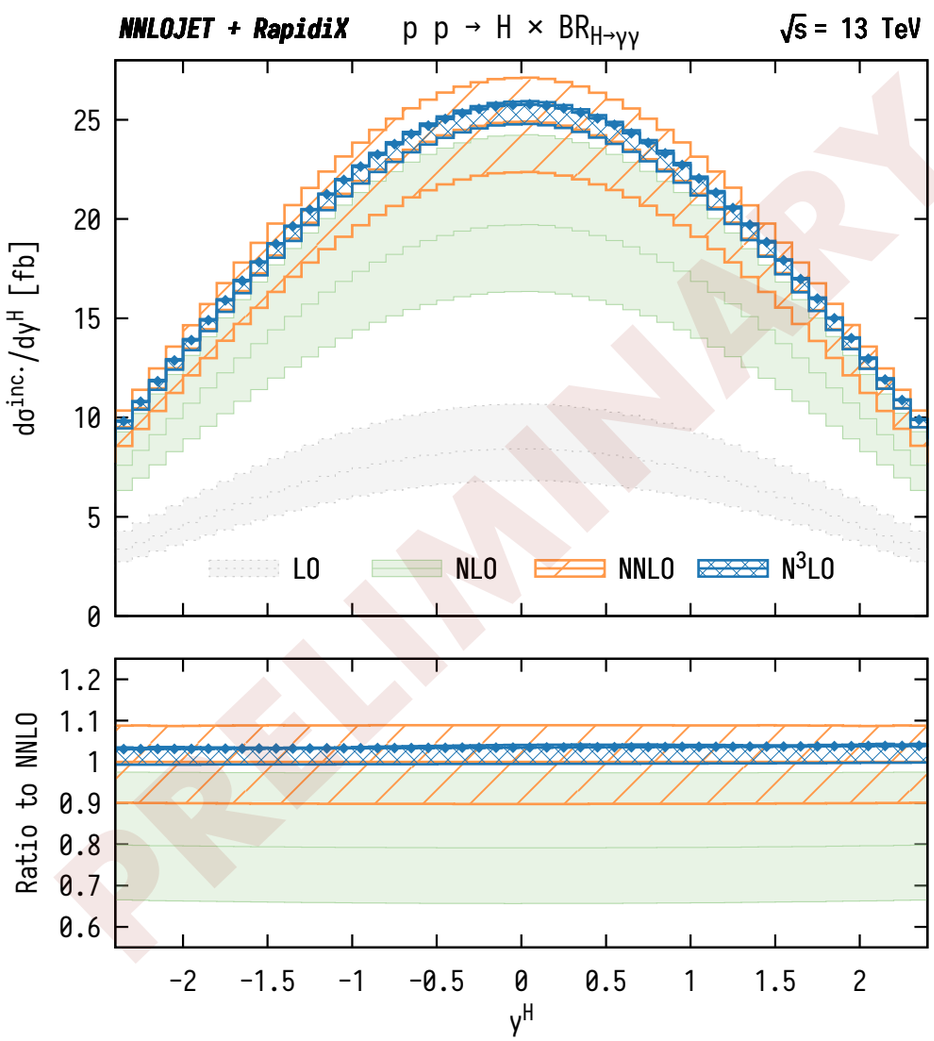
[Chen, Dulat, Gehrmann, Glover, AH, Mistlberger, Pelloni]



Higgs @ N³LO using Projection-to-Born $d\sigma/dY^H$

[Chen, Dulat, Gehrmann, Glover, AH, Mistlberger, Pelloni (to appear)]

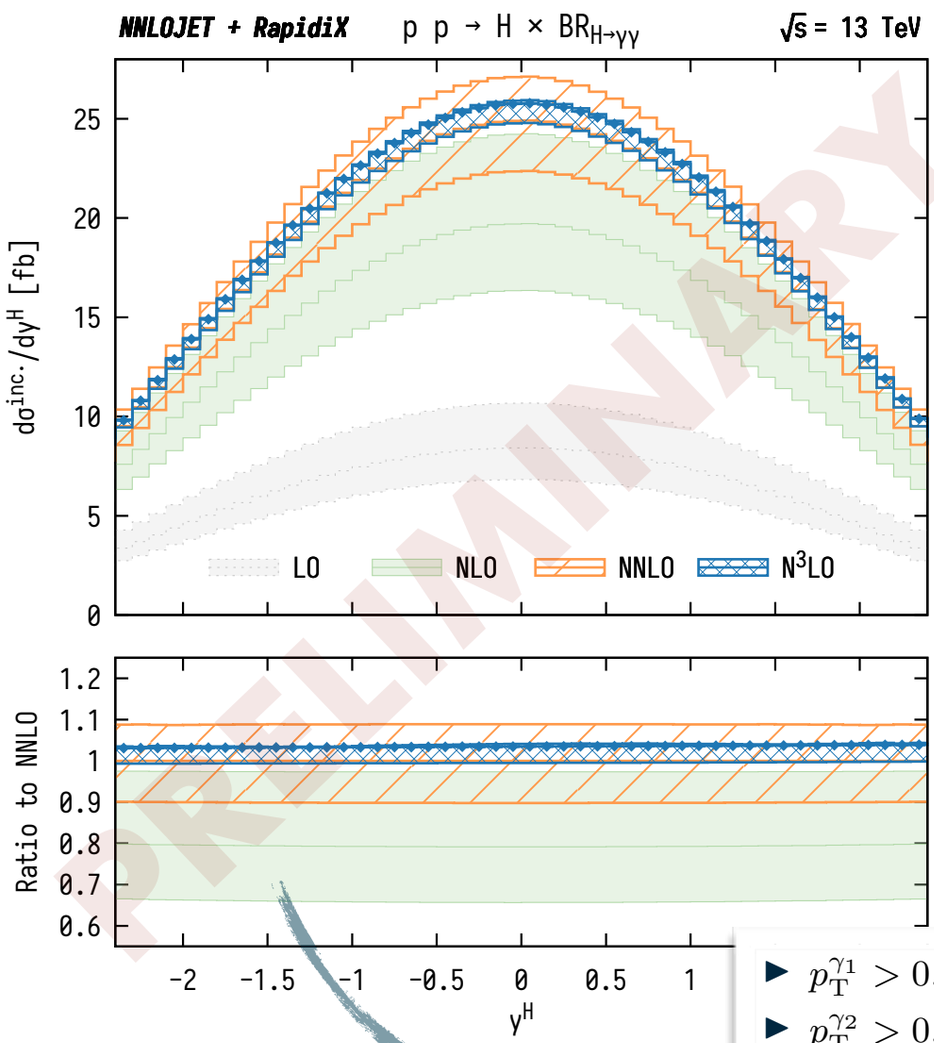
Inclusive



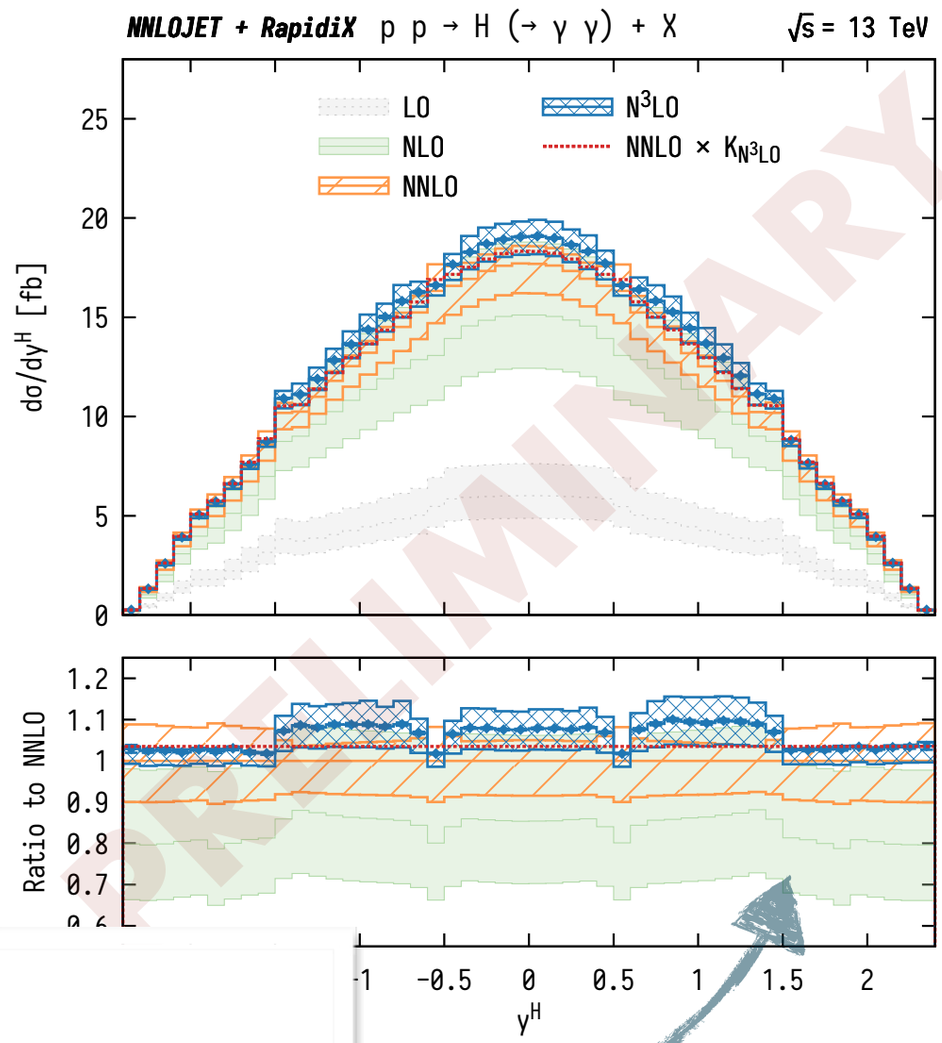
Higgs @ N³LO using Projection-to-Born $d\sigma/dY^H$

[Chen, Dulat, Gehrmann, Glover, AH, Mistlberger, Pelloni (to appear)]

Inclusive



Fully Differential

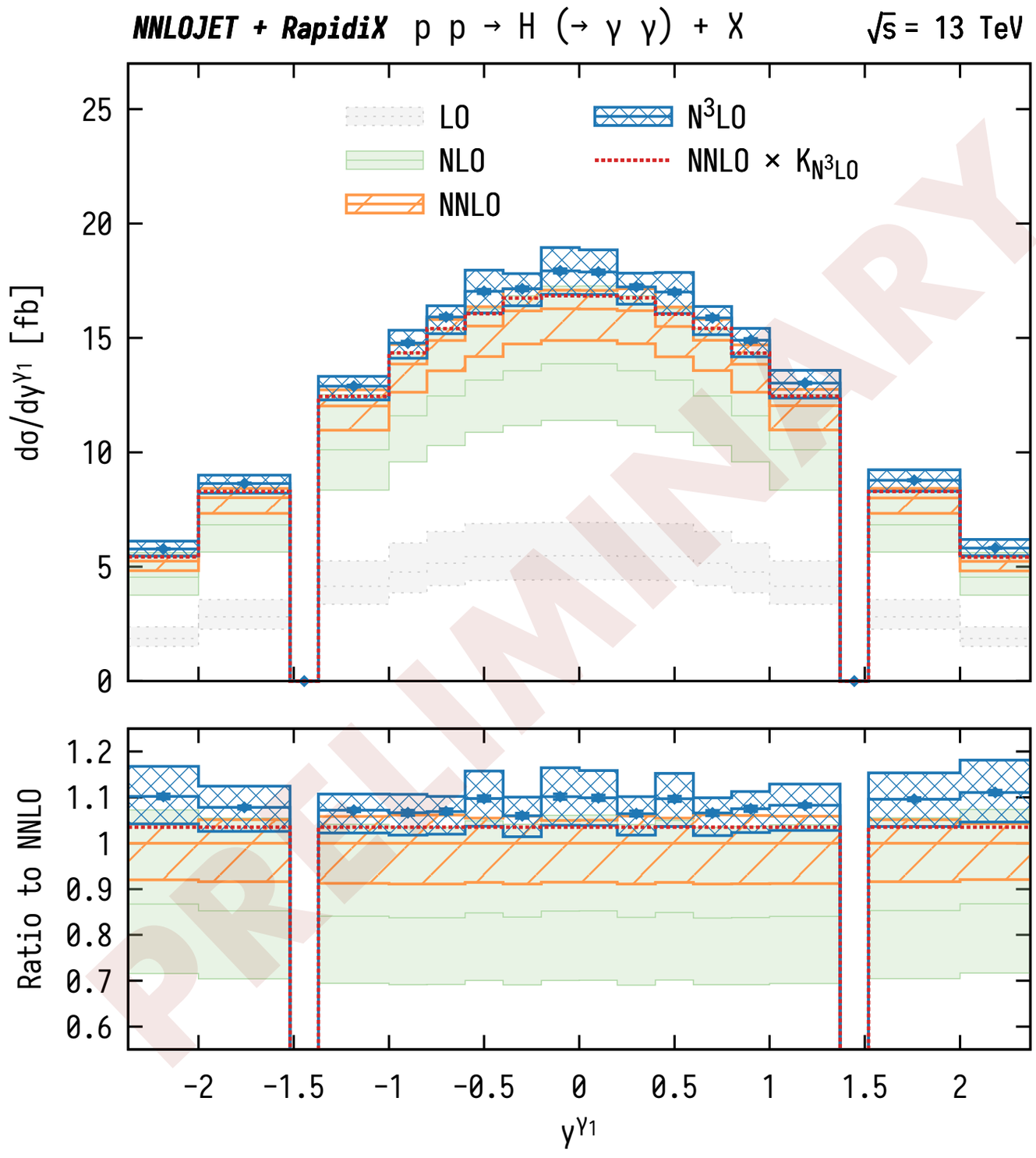


- $p_T^{\gamma_1} > 0.35 \cdot m_{\gamma\gamma}$
- $p_T^{\gamma_2} > 0.25 \cdot m_{\gamma\gamma}$
- $|y^\gamma| < 2.37$
- reject $1.37 < |y^\gamma| < 1.52$ (barrel-endcap)
- *photon isolation* in $\Delta R < 0.2$

  $\hookrightarrow \sum_{\Delta R_{i\gamma}<0.2} p_{T,i} < 0.05 \cdot E_T^\gamma$

Higgs @ N³LO using Projection-to-Born $d\sigma/dy^{\gamma_1}$

[Chen, Dulat, Gehrmann, Glover, AH, Mistlberger, Pelloni (to appear)]



Projection-to-Born — an "Antenna" View

Consider the real-emission subtraction in the antenna subtraction formalism
for $\mathrm{H} + 0\mathrm{jet}$ (@ LC):

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$
$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \texttt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\Phi_{\mathrm{H+1}})$$
$$- F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\ \texttt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

Projection-to-Born — an "Antenna" View

Consider the real-emission subtraction in the antenna subtraction formalism
for $\mathrm{H} + 0\mathrm{jet}$ (@ LC):

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$
$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\Phi_{\mathrm{H+1}})$$
$$- F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\ \mathtt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

Antennae = ratios of *physical* Matrix Elements:

$$F_3^0(i_{\mathrm{g}}, j_{\mathrm{g}}, k_{\mathrm{g}}) \equiv \frac{\mathtt{A3g0H}(i_{\mathrm{g}}, j_{\mathrm{g}}, k_{\mathrm{g}}, \mathrm{H})}{\mathtt{A2g0H}(\tilde{i}_{\mathrm{g}}, \tilde{k}_{\mathrm{g}}, \mathrm{H})}$$

Projection-to-Born — an "Antenna" View

Consider the real-emission subtraction in the antenna subtraction formalism
for $\mathrm{H} + 0\mathrm{jet}$ (@ LC):

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \texttt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\Phi_{\mathrm{H+1}})$$

$$- F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\ \texttt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}}\ \texttt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H}) \left\{ \mathcal{J}(\Phi_{\mathrm{H+1}}) - \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \right\}$$

$\Rightarrow$ Simple processes where antenna $\simeq$ real-emission Matrix Element
$\leftrightsquigarrow$ Projection-to-Born

Similarly at NNLO: X_4^0 & $X_3^0 \times X_3^0$ are "projections" of RR ME & NLO(+jet) subtraction term.

$\mathrm{d}\sigma_{\mathrm{N^3LO}}/\mathrm{d}y_{\mathrm{H}}$ $\simeq$ integrated antenna: $\mathcal{X}_5^0$, $\mathcal{X}_4^1$, $\mathcal{X}_3^2$

Conclusions & Outlook

- **LHC** — remarkable opportunity to study high-energy physics
  - search for new physics & probe the Higgs sector
  - precision measurements using "standard candles"

$\Rightarrow$ high-precision predictions essential!

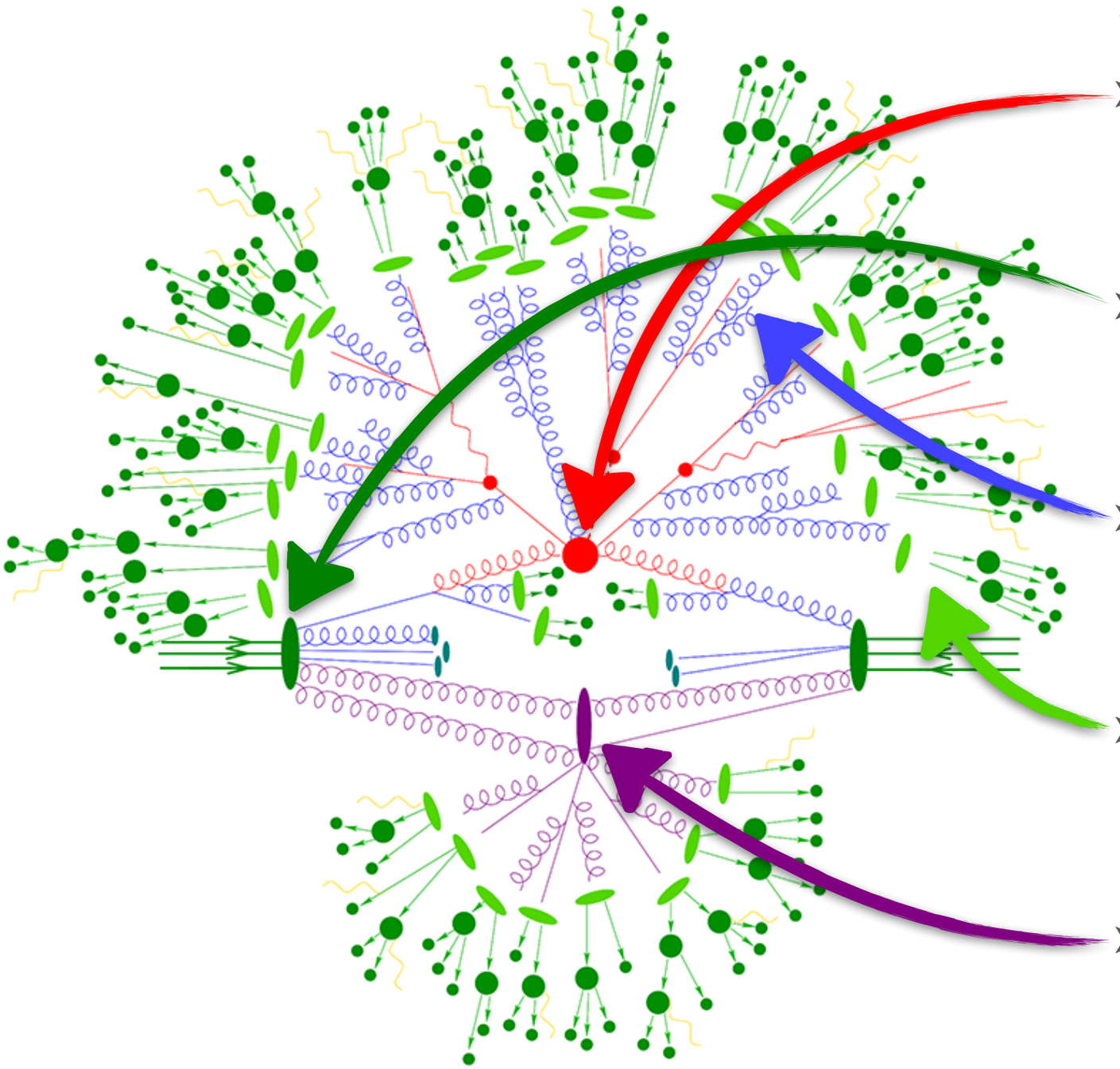
- **Antenna Subtraction @ NNLO:** $pp \to$ "colour neutral" $+\,0, 1, 2$ jets
  - reduced uncertainties & often resolves tension to data
  - next frontier: $2 \to 3$ processes
- **Antenna Subtraction @ NLO EW:** ($\rightsquigarrow$ $\times\, 2$ less terms than with dipoles)
  - promising first step towards NNLO mixed QCD—EW subtraction
- **exploration of the N³LO frontier:** $pp \to$ "colour neutral"
  - Projection-to-Born $\simeq$ Antennae

➡ **precision phenomenology** using these calculations only started!

Thank you!

BACKUP.

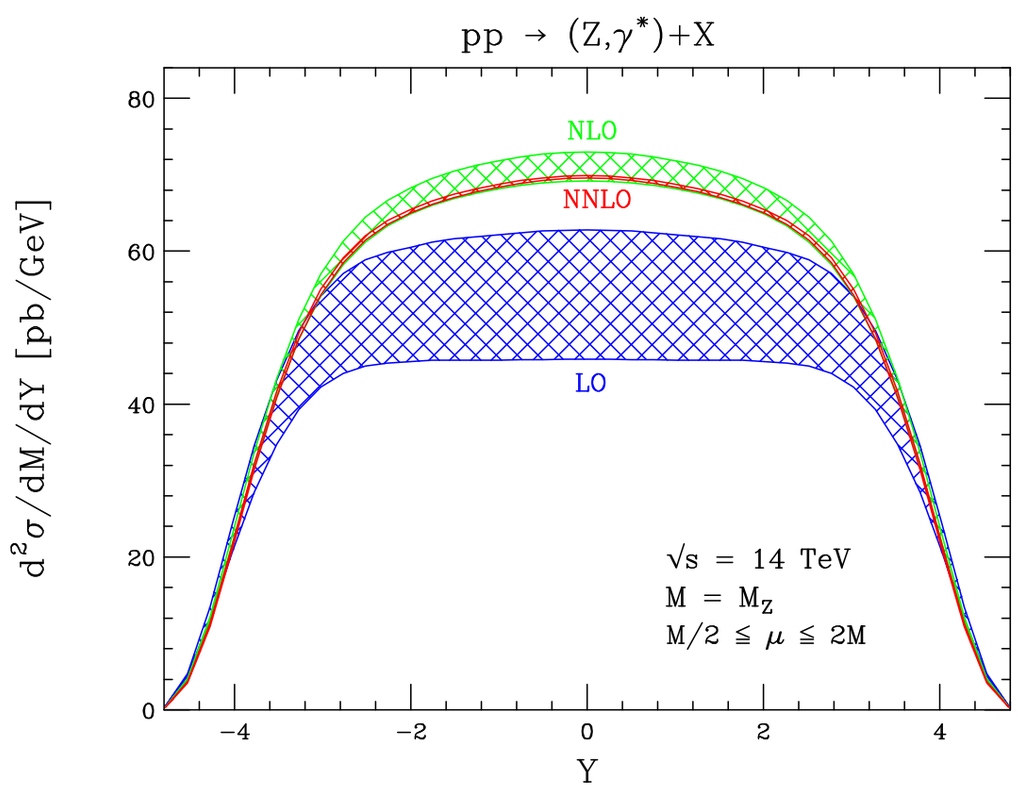
Predictions @ LHC



- Hard subprocess
- PDFs
- Parton shower
- Hadronization + decays
- Underlying event
- Jets, substructure, res.

Why Higher Orders?

- high-precision mandatory
  ↪ processes with large K-factors (H)
  ↪ "standard candles" (jets, V, t, ...)
- reduction of scale uncertainties
  ↪ variation of μ_{R} & μ_{F}

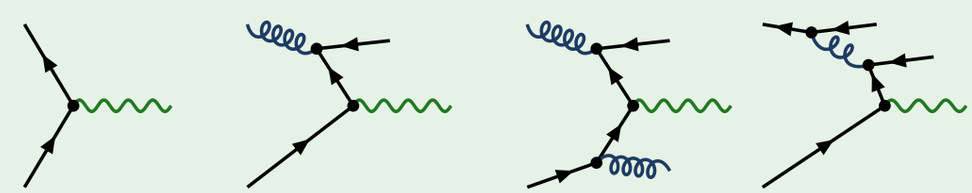


[Anastasiou, Dixon, Melnikov, Petriello '04]

Jet clustering

- better modelling of jet algorithm between theory & experiment

Initial-state radiation



LO	NLO	NNLO
$q\bar{q}$	$\oplus\ qg,\ \bar{q}g$	$\oplus\ gg,\ qq',\ q\bar{q}',\ \bar{q}\bar{q}'$

- opening up of all channels
- more complicated p_{T} recoil

Subtraction methods — Cancel ∞'s

- Remarkable progress in the development of methods to perform NNLO computations!

(not an exhaustive list)	local subtraction	analytic	pp collisions	final-state jet(s)
Antenna	✗ (local after rot^n)	✓	✓	✓
CoLorFul	✓	✓	✗	✓
q_T-Subtr.	✗	✓	✓	✗ (only t)
Stripper / nested soft–coll.	✓	✗ / ✓	✓	✓
N-jettiness	✗	✓	✓	✓ ($\leq$ 1 jet so far)

- Projection-to-Born, Local Analytic Sectors, Geometric, …

** more painful with massless particles*

What about Angular Terms?!

- Antenna subtraction: $X_n^l \, |\mathcal{A}_m|^2 \quad \leftrightarrow \quad$ spin averaged!
- angular terms in gluon splittings:

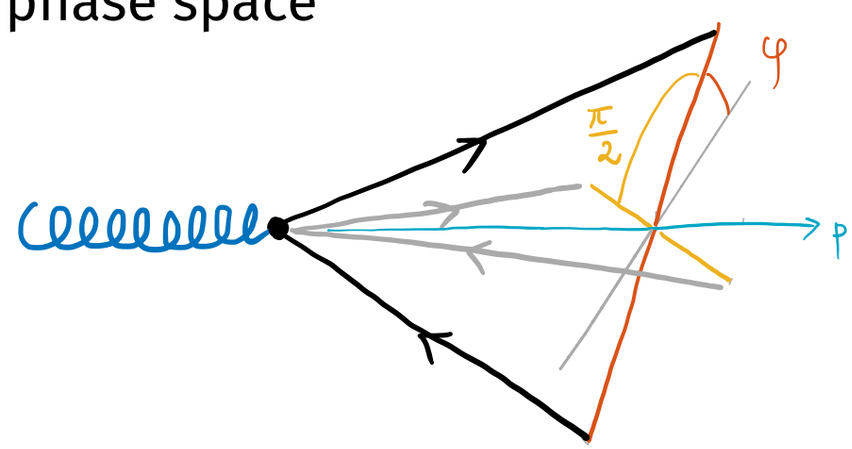
$$P_{g\to q\bar{q}} = \frac{2}{s_{ij}} \left[-g^{\mu\nu} + 4z(1-z) \, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right]$$

$\hookrightarrow$ subtraction non-local in these limits!
$\hookrightarrow$ vanish upon azimuthal-angle (φ) average ($\Rightarrow$ do not enter $\mathcal{X}$)

sol. 1: supplement angular terms in the subtraction
sol. 2: exploit φ dependence & average in the phase space

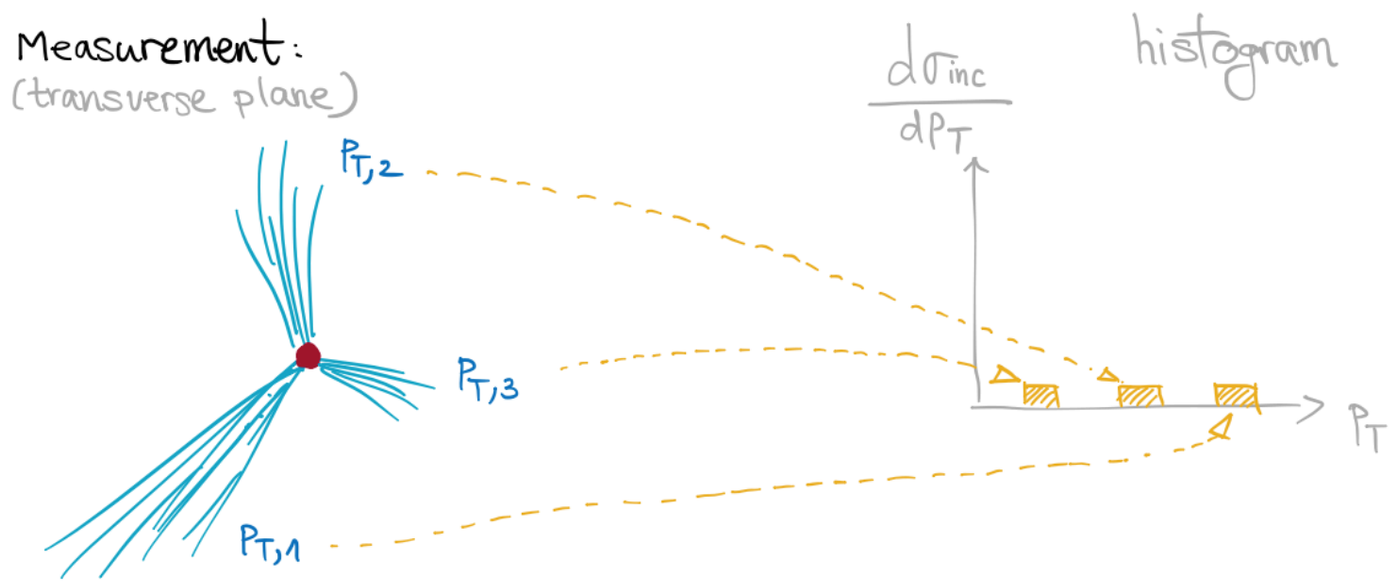
$$\mathcal{A}_\mu^* \, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \, \mathcal{A}_\nu \sim \cos(2\varphi + \varphi_0)$$

$\Rightarrow$ add φ & $(\varphi + \pi/2)$!



$$\vec{r} \longrightarrow \text{PS}_{\text{gen.}} \longrightarrow \begin{bmatrix} \{p_i, & p_j, & \ldots\} \\ \{p_i', & p_j', & \ldots\} \end{bmatrix} \xrightarrow{(i\|j)} \begin{bmatrix} \{p_i^{\varphi}, & p_j^{\varphi}, & \ldots\} \\ \{p_i^{\varphi+\pi/2}, & p_j^{\varphi+\pi/2}, & \ldots\} \end{bmatrix}$$

Inclusive Jet Production

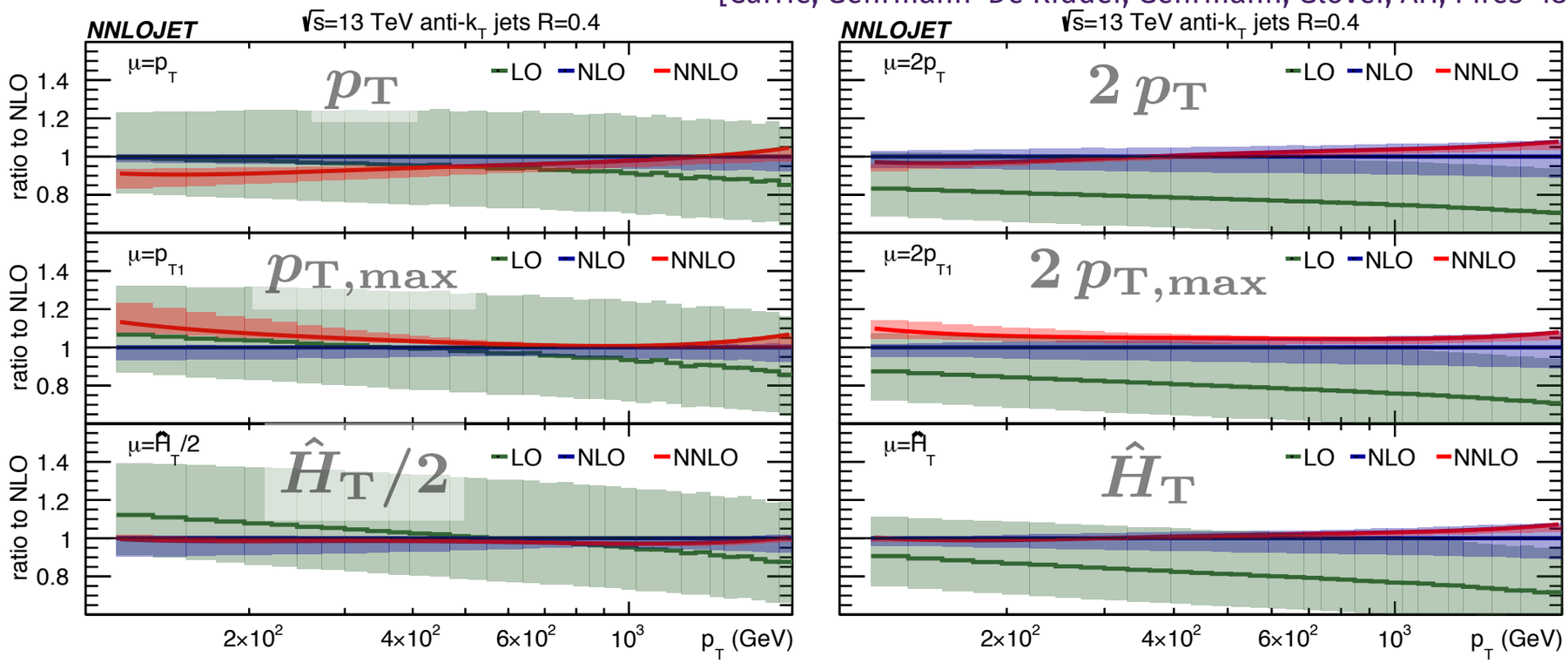


$$\left\{ \begin{array}{c} n \text{ reconstructed jets} \\ \text{in the event} \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} n \text{ binnings to} \\ \text{the histogram} \end{array} \right\} \Rightarrow \sum_{\text{bins}} \frac{\mathrm{d}\sigma_{\text{inc}}}{\mathrm{d}p_{\mathrm{T}}} \neq \sigma_{\text{tot}}$$

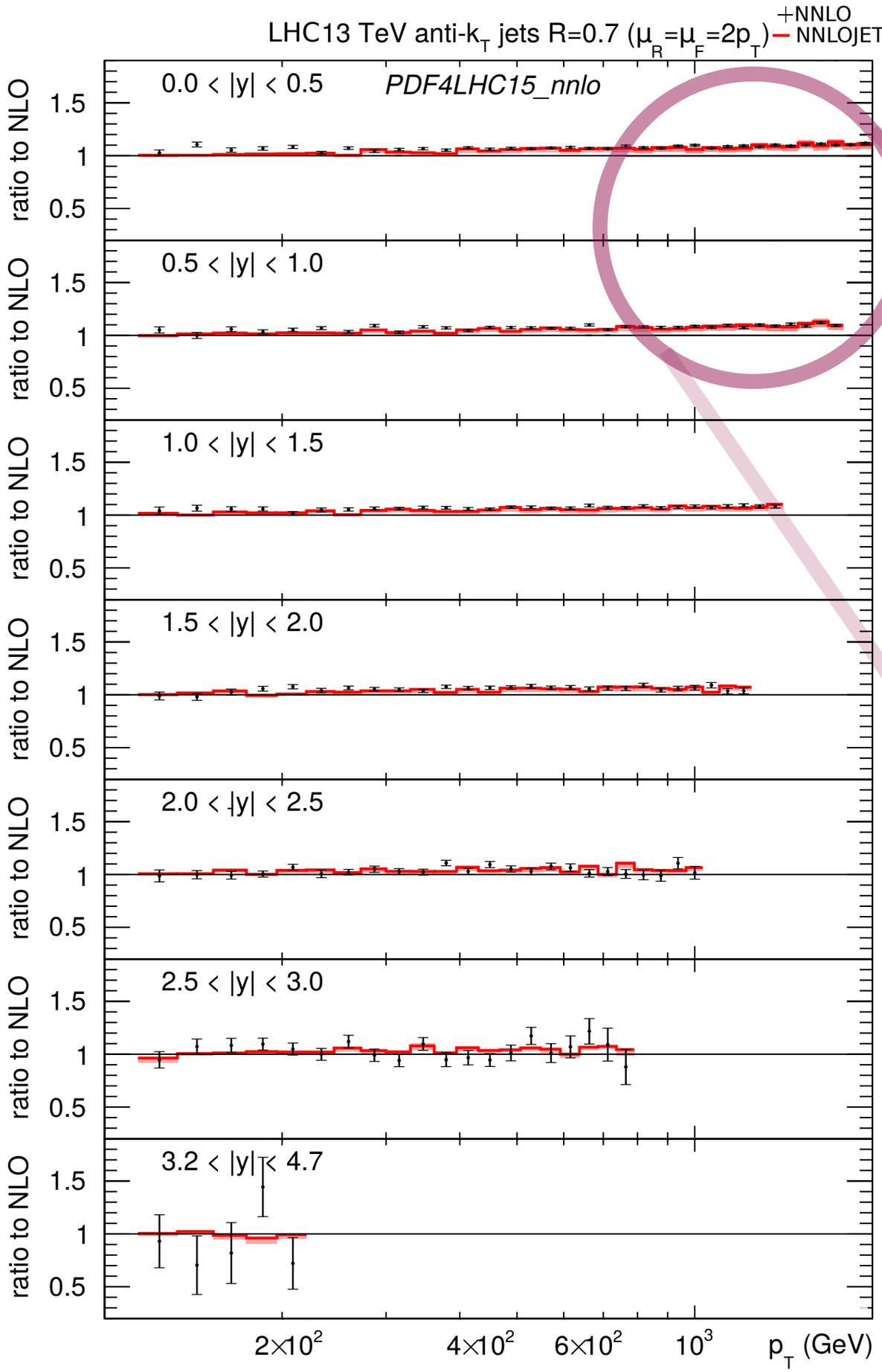
scale choices	$\leftrightsquigarrow$	binning of *individual jets* vs. *events*
▶ "global" scales (event):	$p_{\mathrm{T,max}}$,	$\langle p_{\mathrm{T}} \rangle$, ...
▶ "local" scales (jet):	p_{T}, ...	

Inclusive Jet Production — Scale Choices (R=0.4)

[Currie, Gehrmann–De Ridder, Gehrmann, Glover, AH, Pires '18]



- *most common* choice: $\mu = p_{\mathrm{T}}$ & $\mu = p_{\mathrm{T,max}}$
  ↪ worst perturbative behaviour
- harder scales preferred: $\mu = 2\,p_{\mathrm{T}}$ & $\mu = \hat{H}_{\mathrm{T}}$
  ↪ show good properties
- origin: infrared sensitivity of the inclusive-jet observable
  ↪ driven by 2nd leading jet distribution p_{T}^{j2} (very small @ NLO)
  ↪ mismatch between real & virtual corrections (alleviated with larger R)

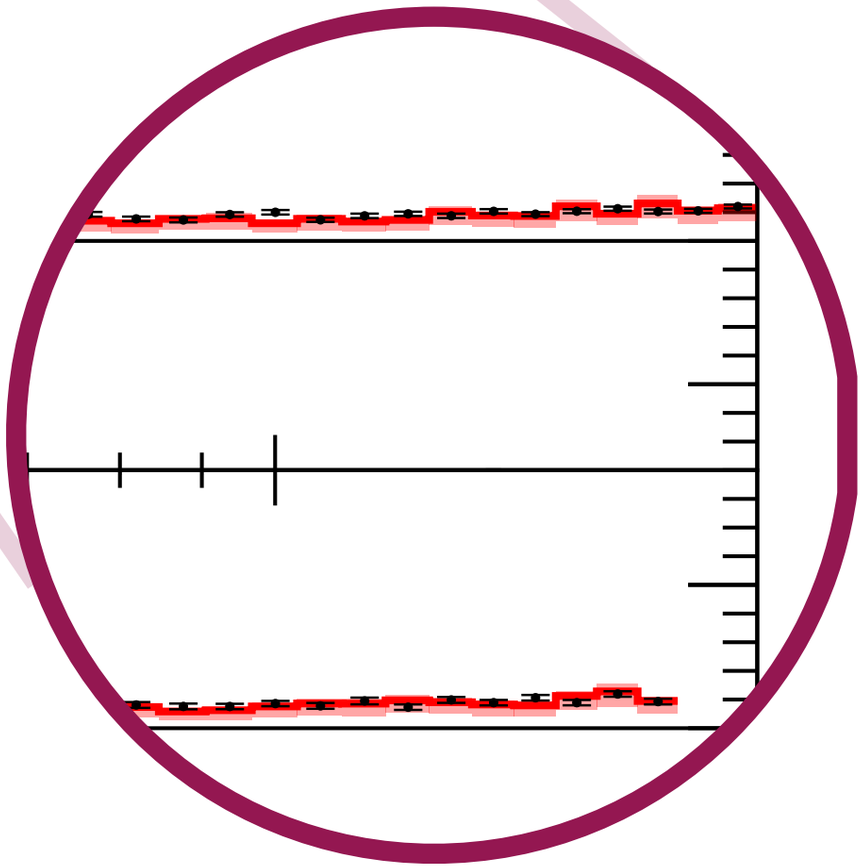


Two Calculations!

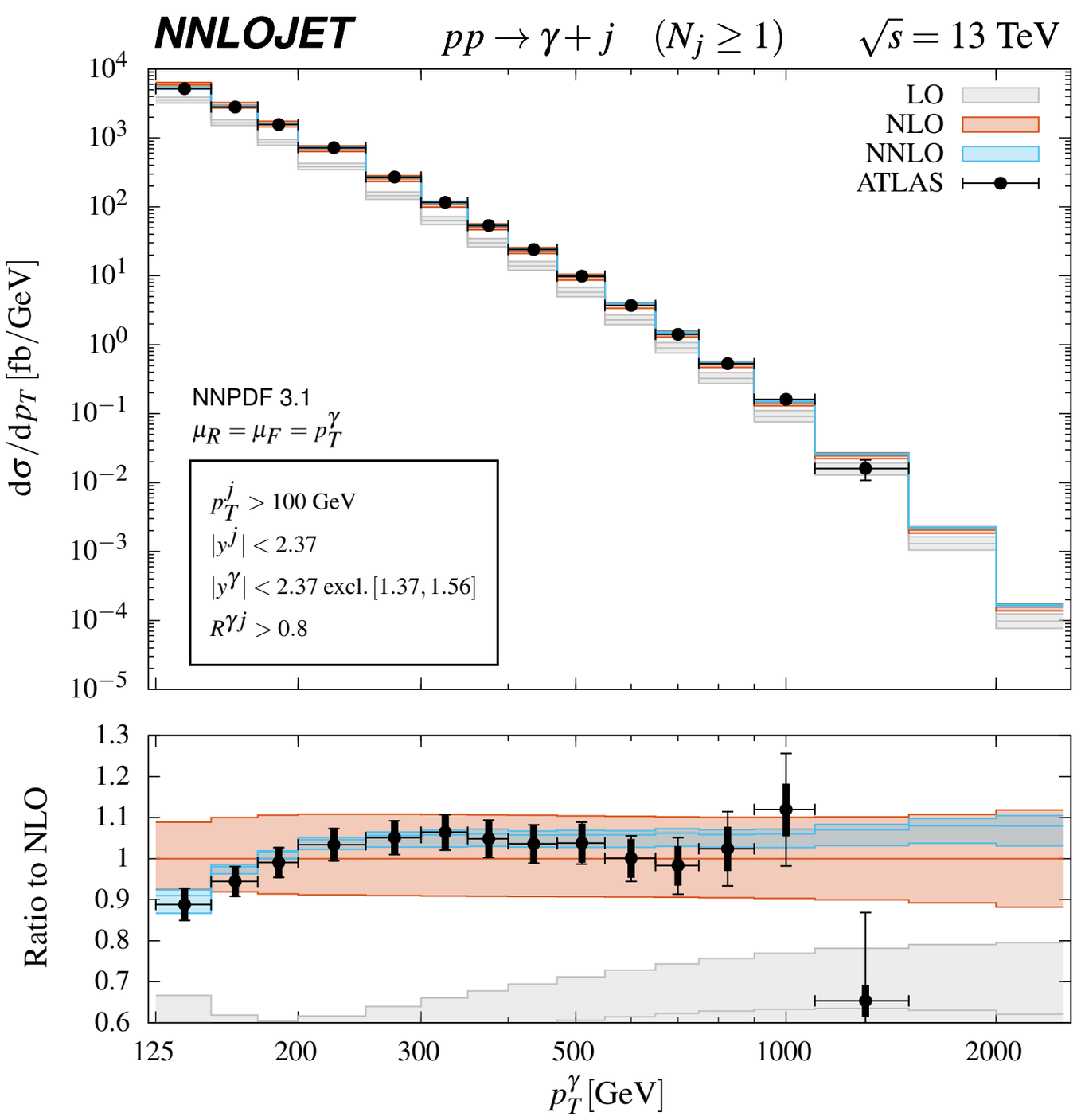
NNLOJET [Currie, Glover, Pires '16]

+ STRIPPER [Czakon, van Hameren, Mitov, Poncelet '19]

- excellent agreement
- sub-leading colour negligible (missing in NNLOJET)



Photon + Jet @ 13 TeV



[Chen, Gehrmann, Glover, Höfer, AH '19]

hybrid isolation

NLO ($\rightsquigarrow 1$)

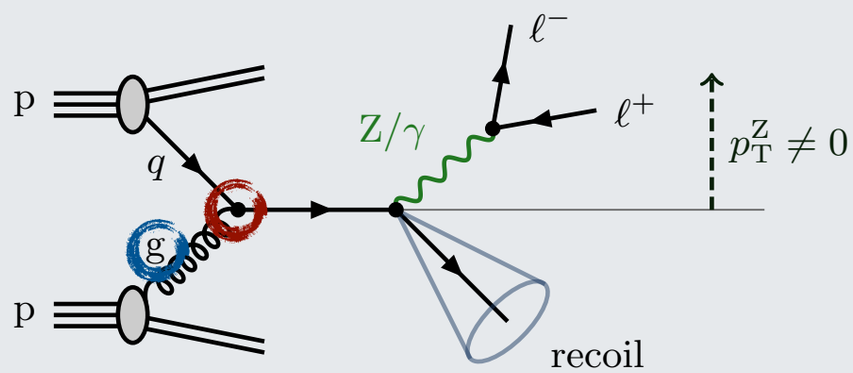
- $+40\%$ corrections
- $\pm 10\%$ uncertainties

NNLO

- $\sim 5\%$ corrections
- *shape distortions*
- $\lesssim 5\%$ uncertainties

- previous NNLO calculation
  τ_N [Campbell, Ellis, Williams '17]
  (dynamical cone isol.)

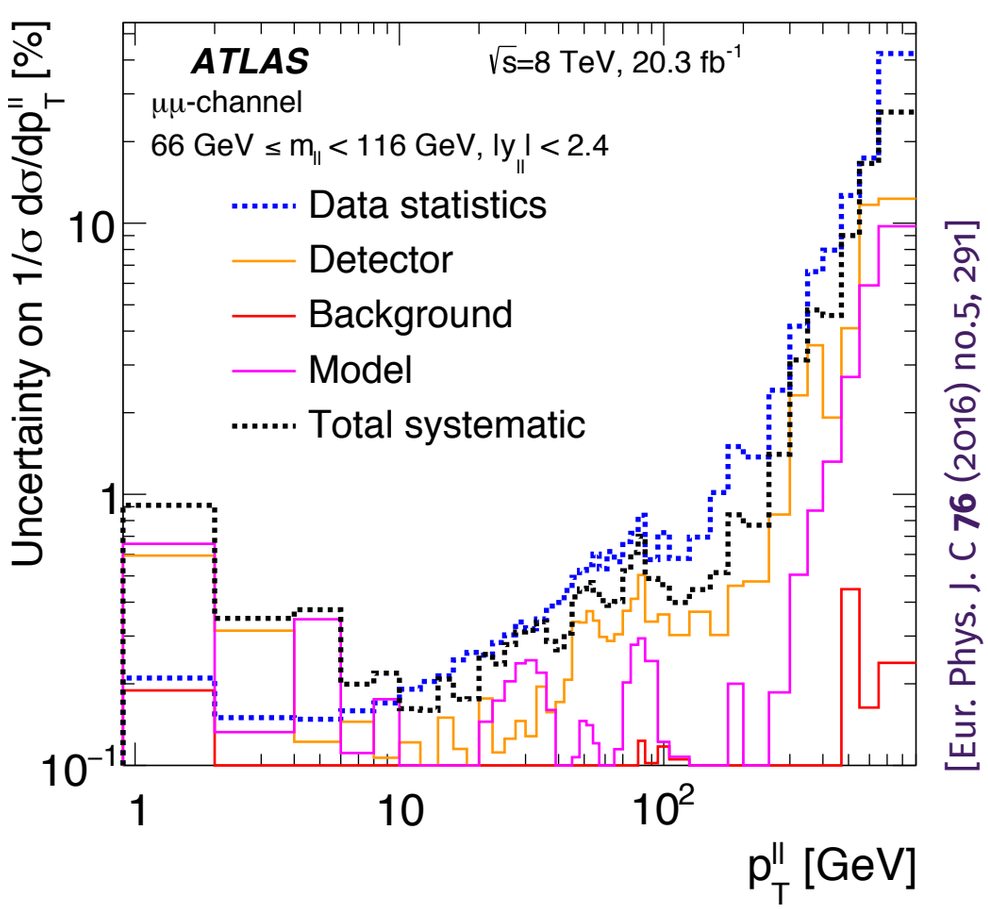
Towards per-cent Phenomenology



$$\mathrm{p\,p} \to \mathrm{Z}/\gamma^* + X \to \ell^-\ell^+ + X$$

- large cross section
- clean leptonic signature

recoil ⇝ sensitivity to α_s, gluon PDF

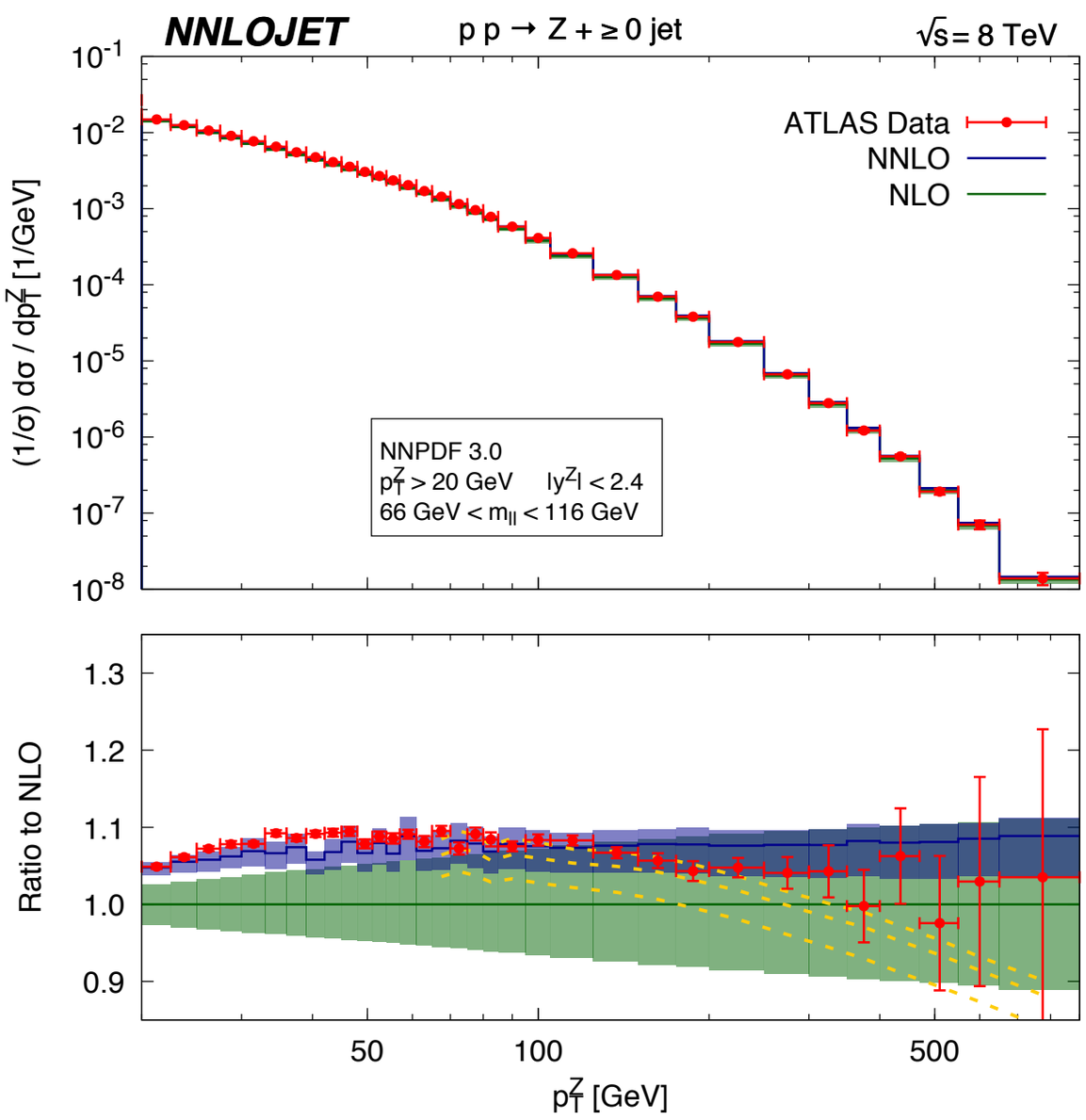


[Eur. Phys. J. C **76** (2016) no.5, 291]

- only reconstruct ℓ^+, ℓ^-
  ⇝ sub-% accuracy!
- important constraints in PDF fits
  [Boughezal et al. '17]
- probe various theory aspects:

very low p_T	non-pert. effects
low p_T	resummation
interm. p_T	fixed order
high p_T	EW Sudakov logs

Inclusive P_T Spectrum



[Gehrmann–De Ridder, Gehrmann, Glover, AH, Morgan '16]

$$\frac{1}{\sigma} \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}p_{\mathrm{T}}^{\mathrm{Z}}}$$

- removes luminosity error ($\sim 3\%$)

NLO

undershoots data by 5–10%

NNLO

significant improvement
in Data vs. Theory comparison

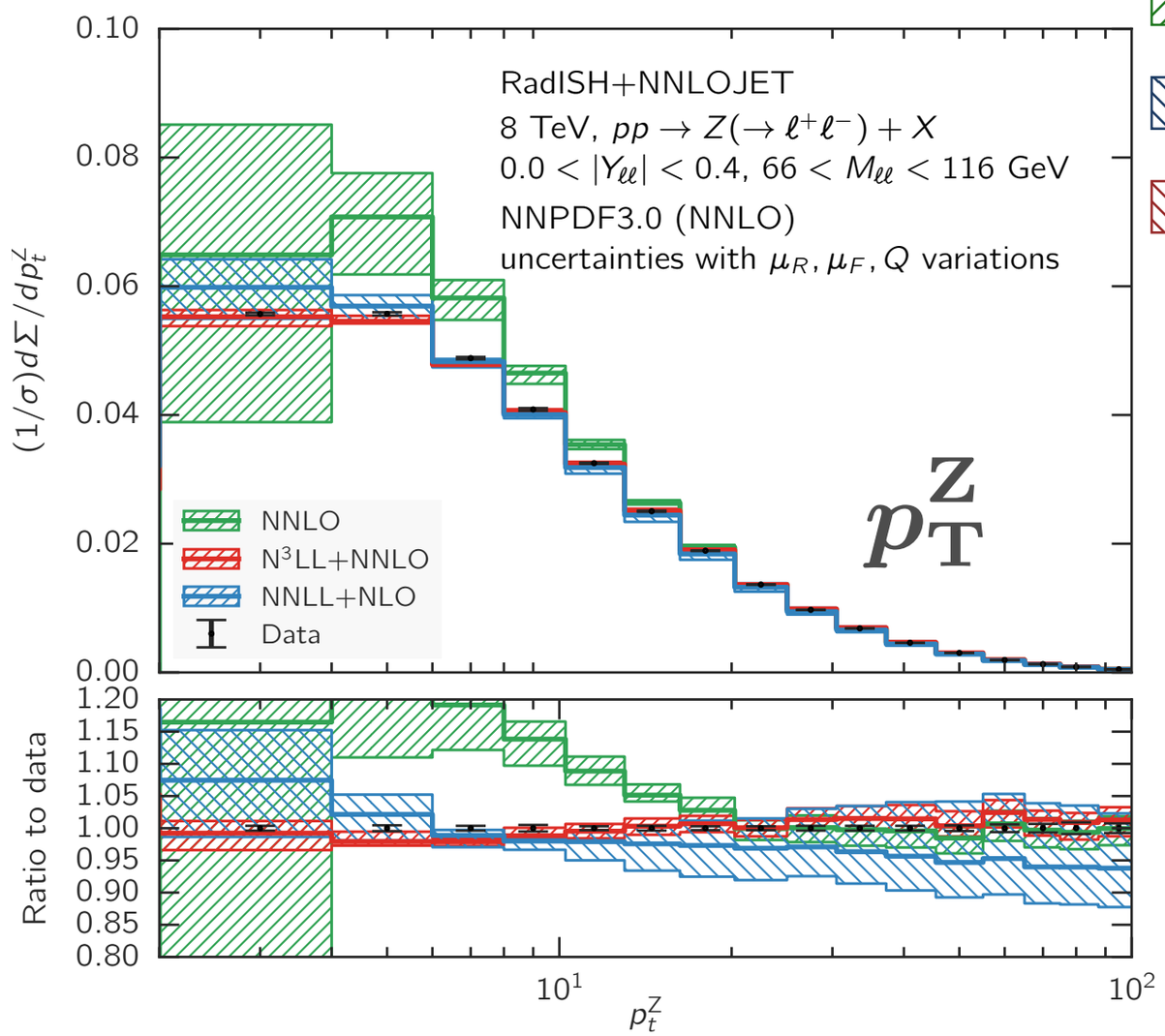
+ EW corrections: - - - -

[Denner, Dittmaier, Kasprzik, Mück '11]

$\Rightarrow$ large impact in the high-p_{T} tail
$\sim -20\%$ for $p_{\mathrm{T}}^{\mathrm{Z}} \sim 900\ \mathrm{GeV}$
(Sudakov logatithms)

Fixed Order + Resummation — NNLO + N³LL

[Bizoń, Chen, Gehrmann-De Ridder, Gehrmann, Glover, AH, Monni, Re, Rottoli, Torrielli '18]



NNLO fixed order: $\mathcal{O}(\alpha_s^3)$

NNLL+NLO ($\leftrightarrow$ DYRes)

N³LL+NNLO

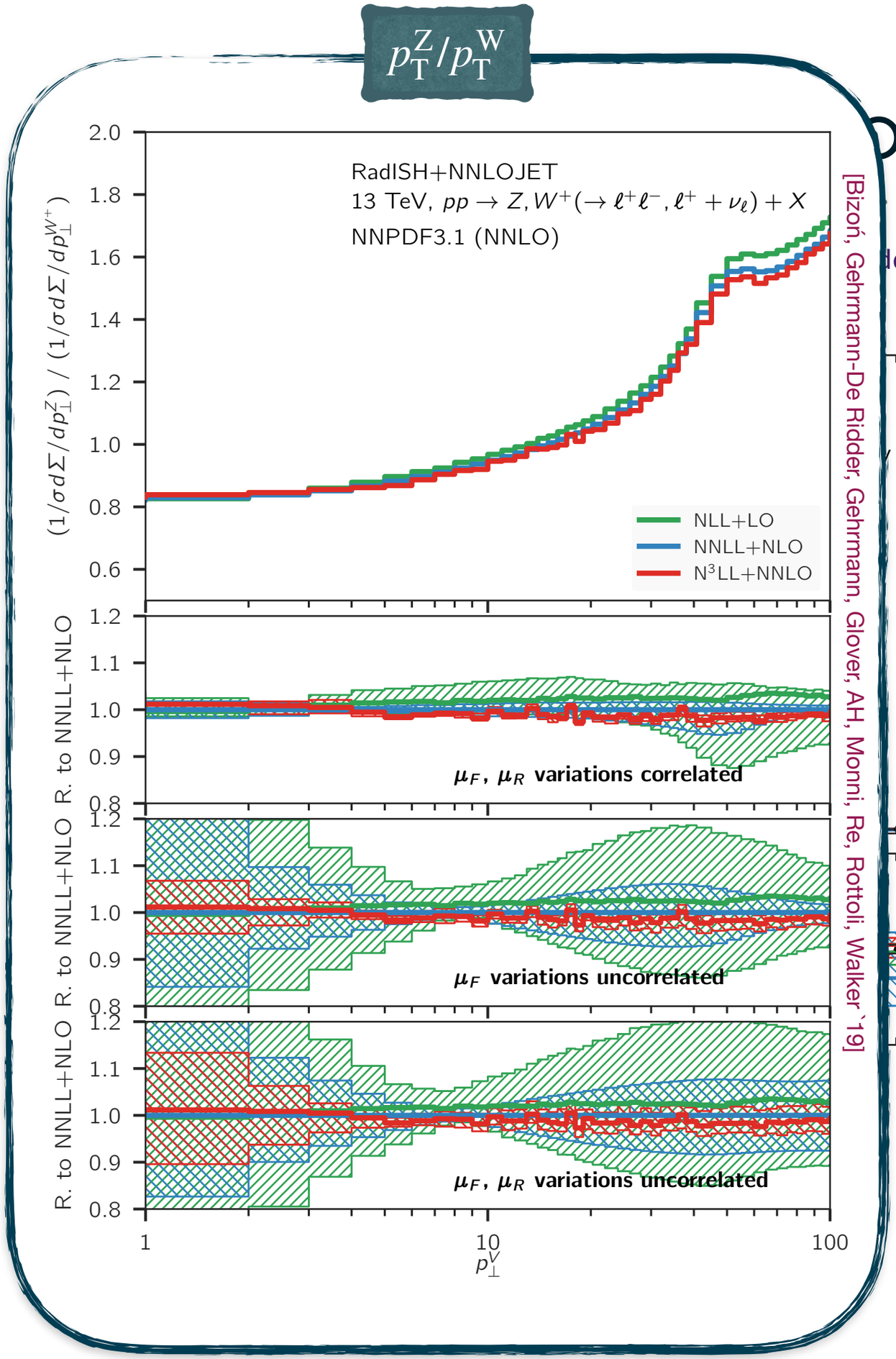
Data (ATLAS)

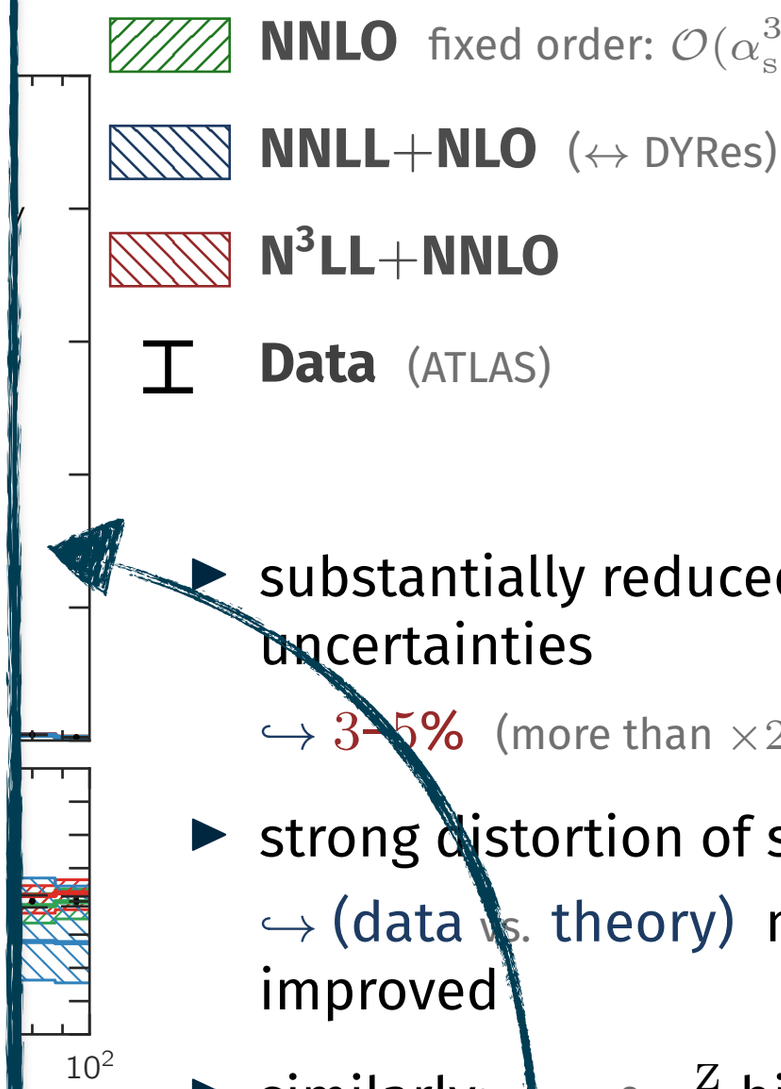
- substantially reduced uncertainties
  $\hookrightarrow$ 3–5% (more than $\times 2$)
- strong distortion of shape
  $\hookrightarrow$ (data vs. theory) much improved
- similarly: $m_{\ell\ell}$ & y^{Z} bins, ϕ_η^*

also: $p_{\mathrm{T}}^{\mathrm{W}}$ & $p_{\mathrm{T}}^{\mathrm{W}}/p_{\mathrm{T}}^{\mathrm{Z}}$ (for M_{W})

ON — NNLO + N³LL

der, Gehrmann, Glover, AH, Monni, Re, Rottoli, Torrielli '18]





- substantially reduced uncertainties
  $\hookrightarrow$ 3–5% (more than $\times 2$)
- strong distortion of shape
  $\hookrightarrow$ (data vs. theory) much improved
- similarly: $m_{\ell\ell}$ & y^Z bins, ϕ_η^*

also: p_T^W & p_T^W/p_T^Z (for M_W)

DIS_{1J} @ N^3LO using Projection-to-Born

DIS 2 jet @ NNLO

[Currie, Gehrmann, Niehues '16]
[Currie, Gehrmann, AH, Niehues '17]
CC: [Niehues, Walker '18]

Projection-to-Born

$\oplus$

[Cacciari, et al. '15]

DIS structure function @ N^3LO

[Moch, Vermaseren, Vogt '05]

$=$

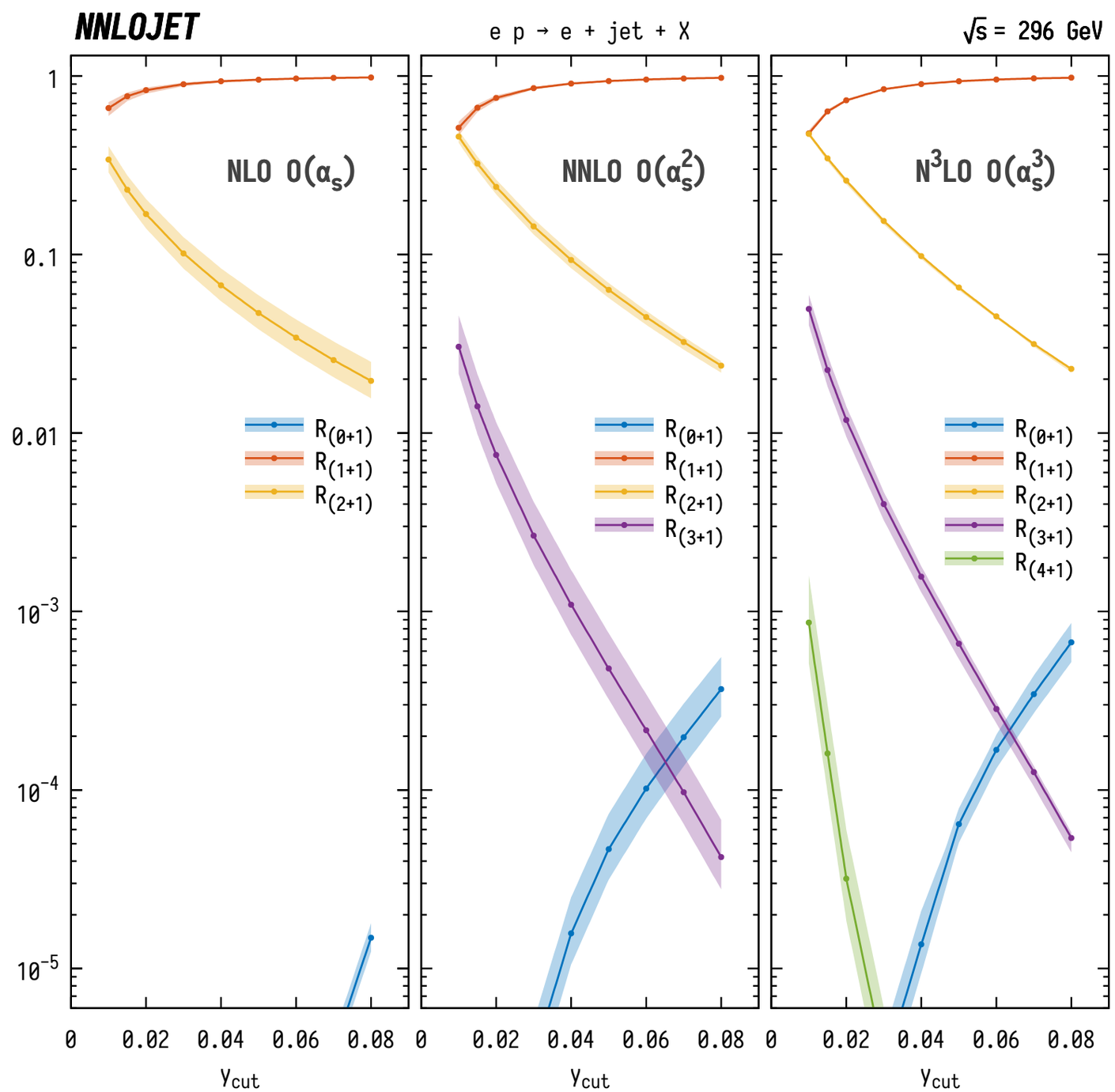
DIS fully differential @ N^3LO

[Currie, Gehrmann, Glover, AH, Niehues, Vogt. '18]
CC: [Gehrmann, Glover, AH, Niehues, Walker, Vogt '18]

** Born variables: (x, Q^2)*

Jet Rates (Neutral-current DIS_{1J})

[Currie, Gehrmann, Glover, AH, Niehues, Vogt. '18]



Jet rates:

$$R_{(n+1)} = N_{(n+1)}/N_{\text{tot}}$$

Jade algorithm

$\hookrightarrow$ cluster partons if:

$$\frac{2E_iE_j(1-\cos\theta_{ij})}{W^2} < y_{\text{cut}}$$

Higgs @ N³LO using Projection-to-Born Acceptances

[Chen, Dulat, Gehrmann, Glover, AH, Mistlberger, Pelloni (to appear)]