Quark-gluon plasma and thermodynamics of QCD

Alexei Bazavov
HotQCD collaboration
Michigan State University

HotQCD collaboration

- A. Bazavov (MSU)
- H.-T. Ding (CCNU)
- P. Hegde (IIS Bangalore)
- O. Kaczmarek (Bielefeld)
- F. Karsch (Bielfeld/BNL)
- N. Karthik (BNL)
- E. Laermann (Bielefeld)
- A. Lahiri (Bielefeld)
- R. Larsen (BNL)
- S.-T. Li (CCNU)
- S. Mukherjee (BNL)
- H. Ohno (Tsukuba)
- P. Petreczky (BNL)
- H. Sandmeyer (Bielefeld)
- C. Schmidt (Bielefeld)
- S. Sharma (IMS Chennai)
- P. Steinbrecher (BNL)

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Outline

- QCD Thermodynamics in theory and experiment
- Theoretical methodology
- Improved actions and continuum limit
- Criticality in QCD
- Deconfinement
- QCD equation of state
- Outlook

Units of measure

- Convenient unit of energy for subatomic physics is

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

- For this talk: $1 \text{ MeV} = 10^6 \text{ eV}$
- Typical length scale: $1 \text{ fm} = 10^{-15} \text{ m}$
- Natural units $\hbar = c = 1$

$$1 = \hbar c \simeq 200 \text{ MeV} \times 1 \text{ fm}$$

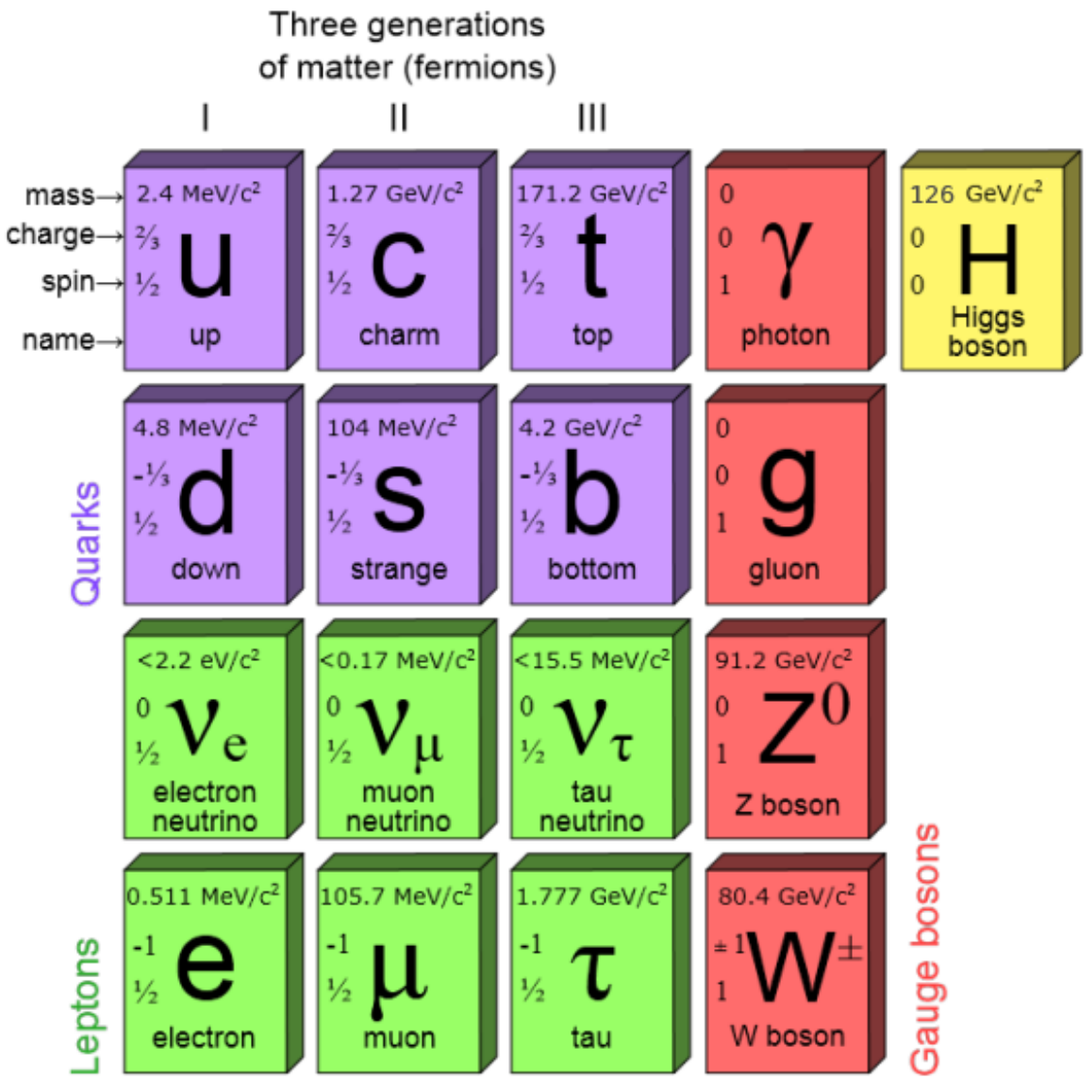
- Typical mass scales:

light quarks ~ few MeV

pion ~ 140 MeV

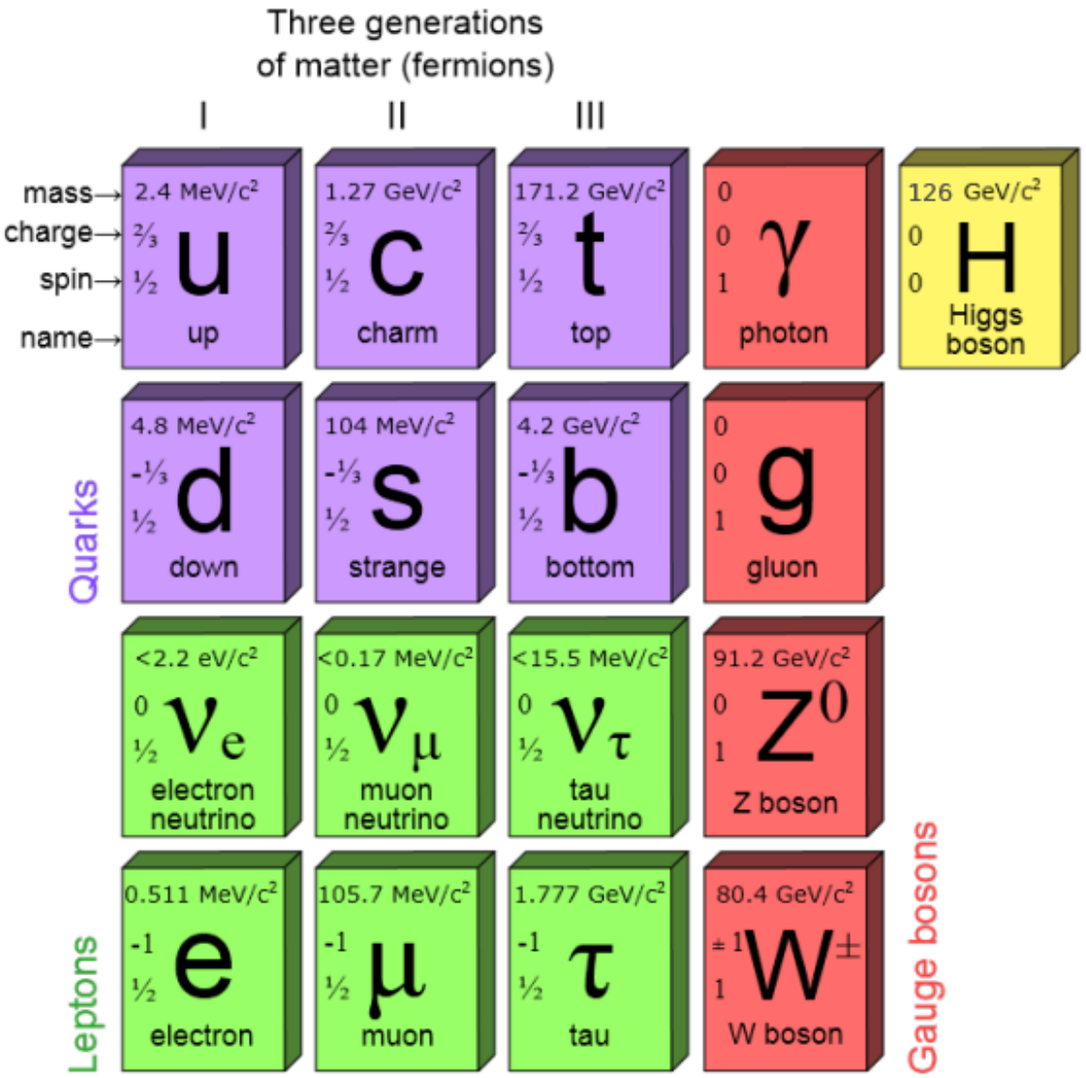
proton ~ 1 GeV

Standard Model



- Quantum Field Theory:
- Quantum Chromodynamics
- Electro-Weak = QED + weak interaction

Standard Model



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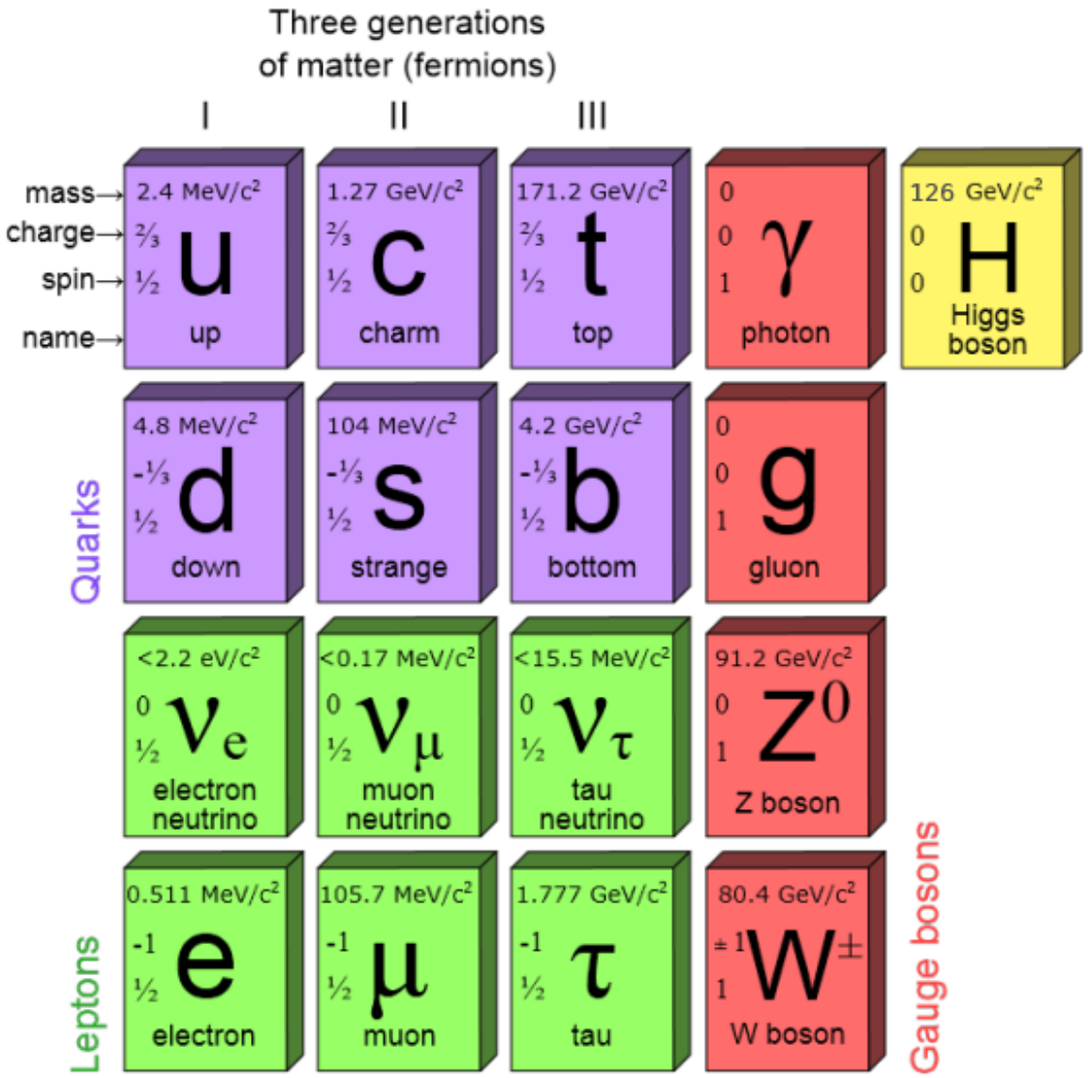
- Electro-Weak = QED + weak interaction
- EW+QCD=Standard Model
- Very successful, but many puzzles remain

Standard Model



- Quantum Field Theory:
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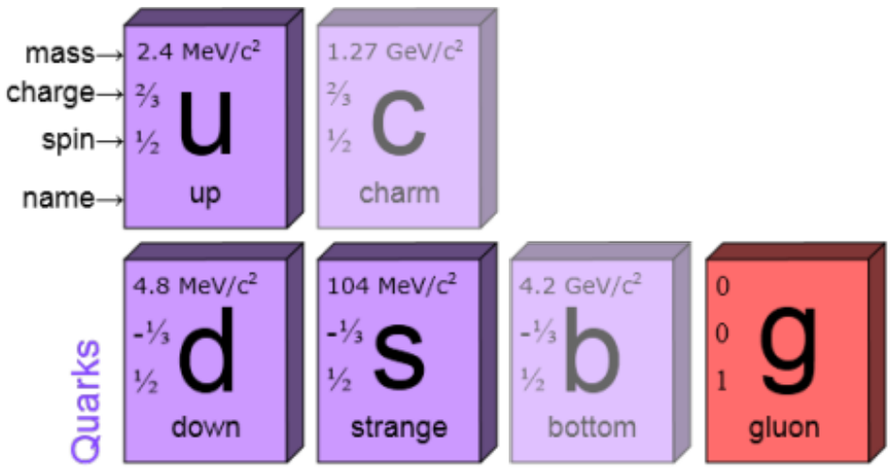
- Electro-Weak = QED + weak interaction
- EW+QCD=Standard Model
- Very successful, but many puzzles remain

- QED – most developed, i.e. electron magnetic moment:

$$a_e(exp) = 1159652180.73(0.28) \times 10^{-12}$$
$$a_e(the) = 1159652181.13(0.11)(0.37)(0.02)(0.77) \times 10^{-12}$$

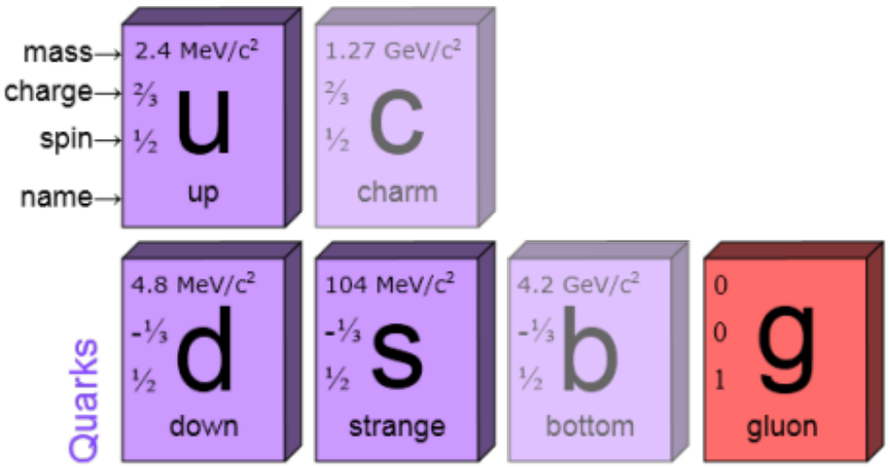
Aoyama et al. PRD85 (2012)

Quantum Chromodynamics



- Quantum Chromodynamics

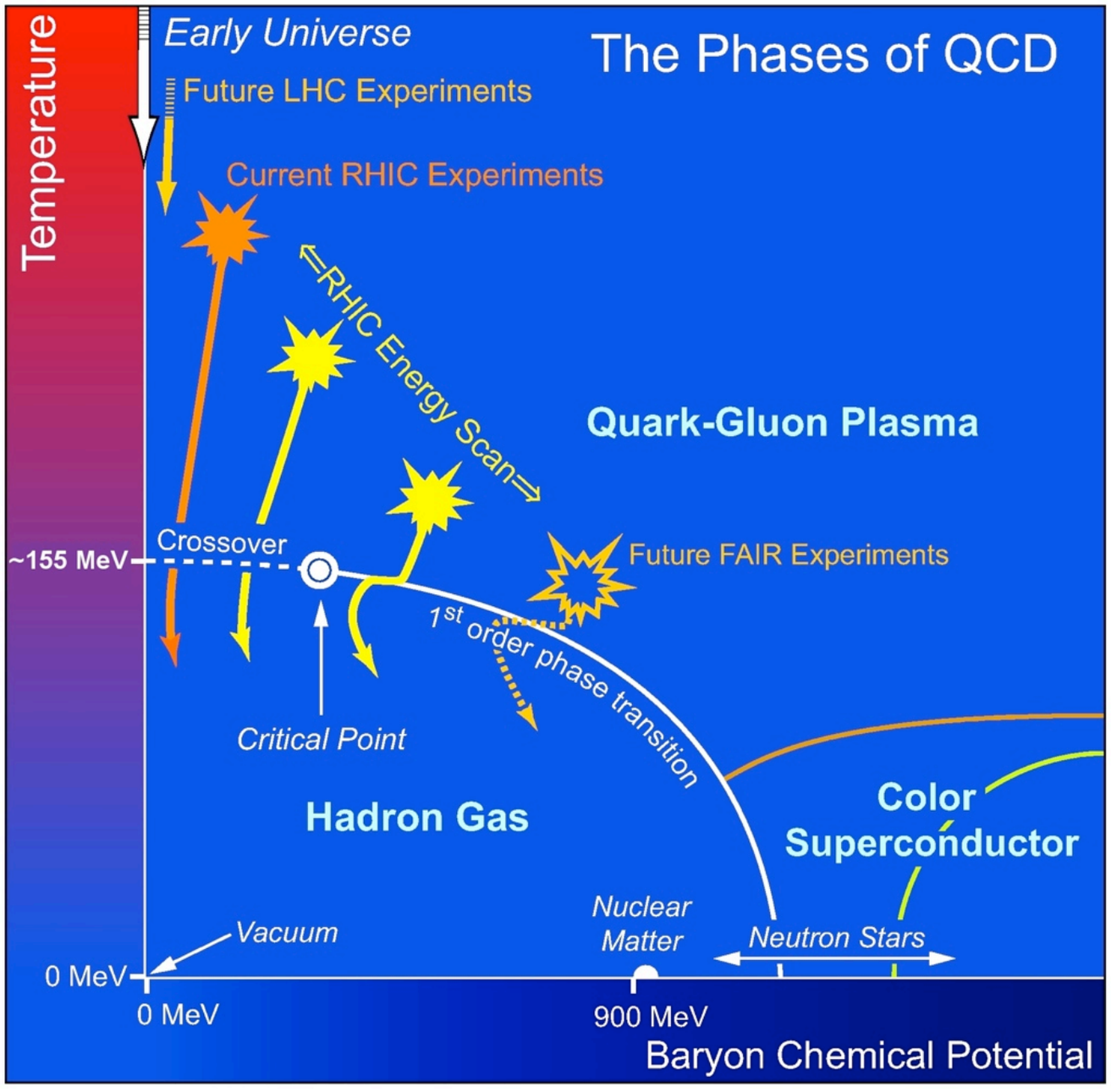
Quantum Chromodynamics



- Quantum Chromodynamics
- Zero temperature: properties of individual hadrons, masses, decays, etc.
- Finite temperature: collective behavior, response to change in external parameters, thermodynamics

QCD Thermodynamics

QCD phases



- Asymptotic freedom suggests weakly interacting phase at high temperature or density

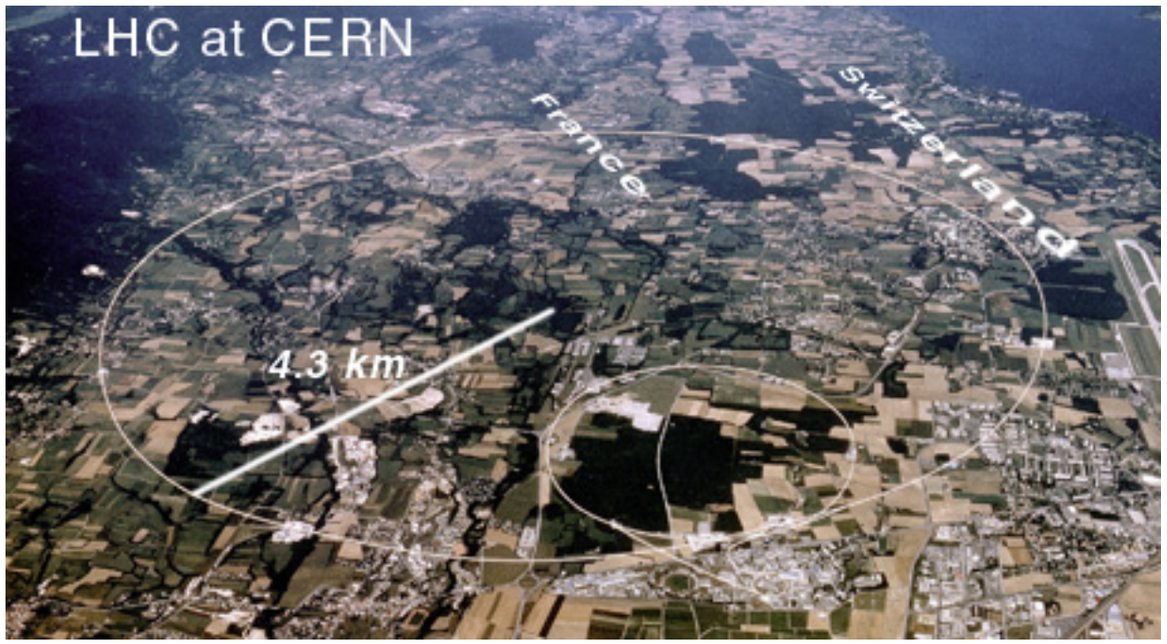
Collins and Perry (1975), Cabbibo and Parisi (1975)

- Need very high T, about 10^{12} K!

QCD Thermodynamics

- What is the order of the transition to QGP at μ_B=0?
- What is the transition temperature?
- What are the signatures of deconfinement and chiral symmetry restoration?
- What is the structure of the phase diagram at μ_B>0?
- What is the equation of state of QGP?
- What happens to the QCD spectrum close to the transition?
- How do the interactions get screened in the plasma?
- At what temperatures does the asymptotic regime (weakly interacting gas) become valid?

Heavy-ion collisions



RHIC, BNL:

PHENIX, STAR

gold-gold

LHC, CERN:

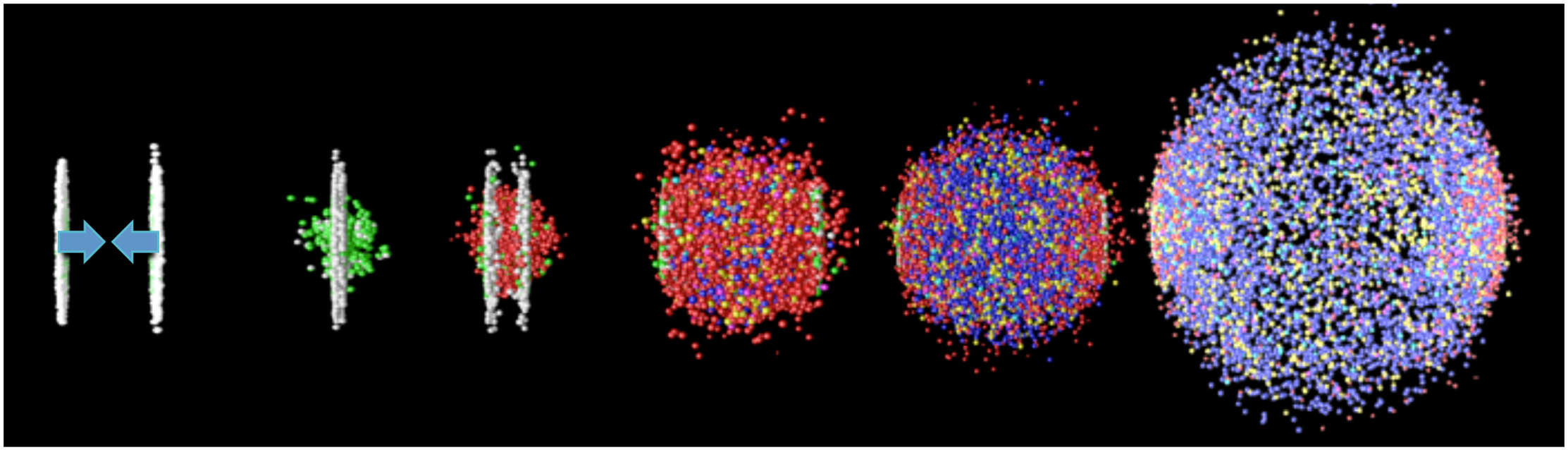
ALICE, ATLAS, CMS

lead-lead

Discovery: QGP – perfect fluid $\eta/s \leqslant 0.2$

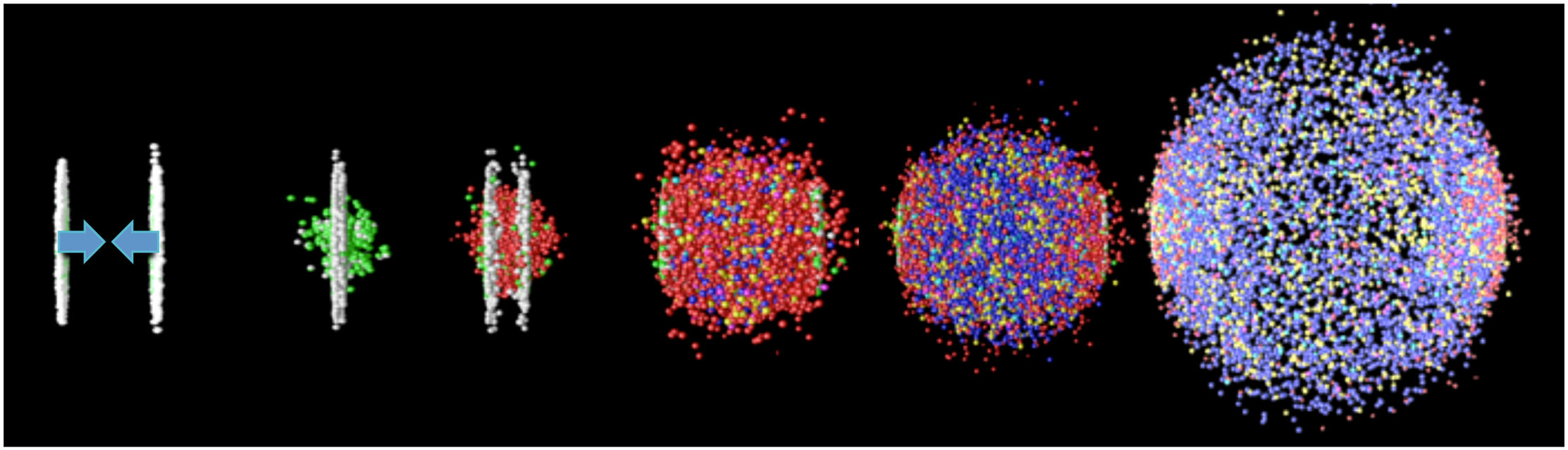
BRAHMS, PHOBOS, STAR, PHENIX, NPA 757 (2005)

Heavy-ion collisions



- Collide heavy nuclei at almost the speed of light

Heavy-ion collisions



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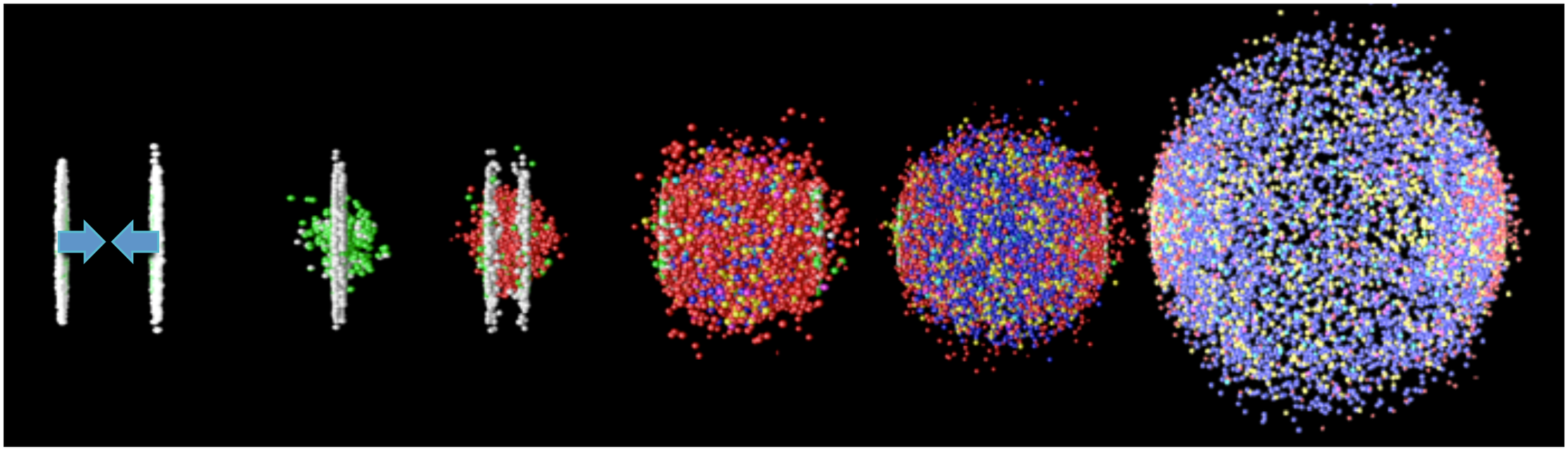
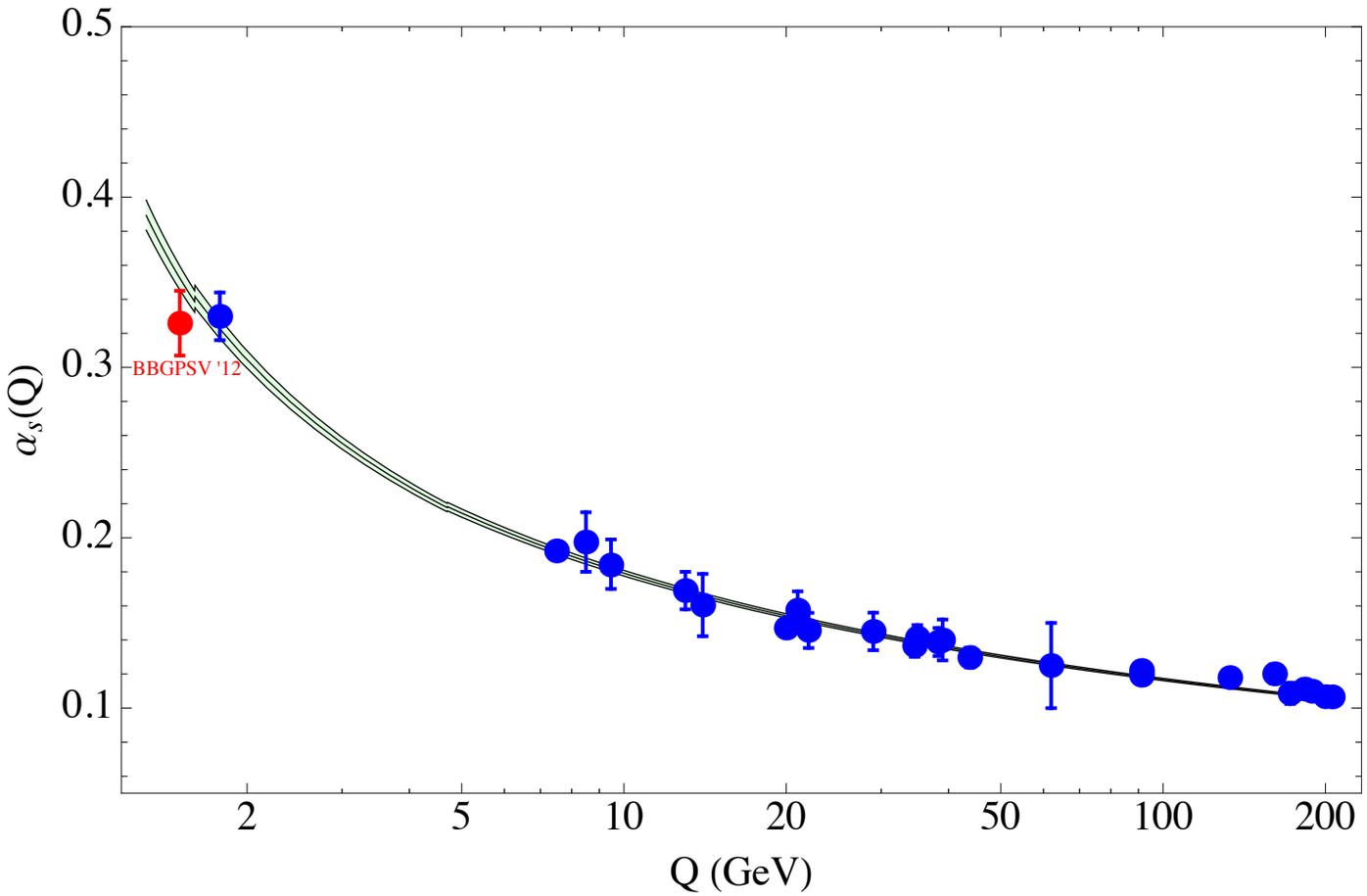


- Collide heavy nuclei at almost the speed of light
- If the energy is high enough, hadrons melt and the system is in the quark-gluon plasma phase
- The system expands, cools down and breaks into the hadrons of the final state, which are detected

Theoretical methodology

QCD: running coupling constant



Bazavov et al. PRD86 (2012)

- Small at large energy scale – asymptotic freedom

Gross and Wilczek; Politzer (1973)

- Large at low energies (where we live)

Quantum field theory

$$\begin{aligned}
\langle \mathcal{O} \rangle &= \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi]\mathcal{D}[\bar{\psi}]\mathcal{D}[A]\ \mathcal{O} \exp(-\mathcal{S}_E(T,V,\vec{\mu})) \\
\mathcal{Z}(T,V,\vec{\mu}) &= \int \mathcal{D}[\psi]\mathcal{D}[\bar{\psi}]\mathcal{D}[A] \exp(-\mathcal{S}_E(T,V,\vec{\mu}))
\end{aligned}$$

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\mathcal{S}_E(T,V,\vec{\mu}) &= -\int_0^{1/T} dx_0 \int_V d^3\mathbf{x} \mathcal{L}^E(\vec{\mu}) \\
\mathcal{L}^E(\vec{\mu}) &= \mathcal{L}^E_{QCD} + \sum_{f=u,d,s} \mu_f \bar{\psi}_f \gamma_0 \psi_f
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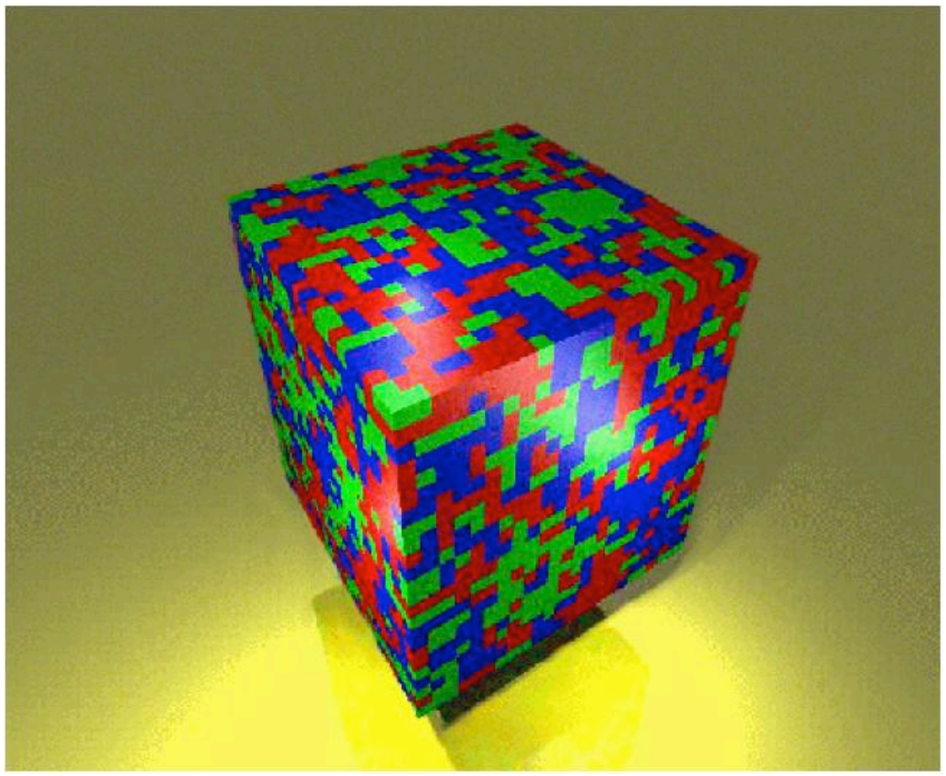
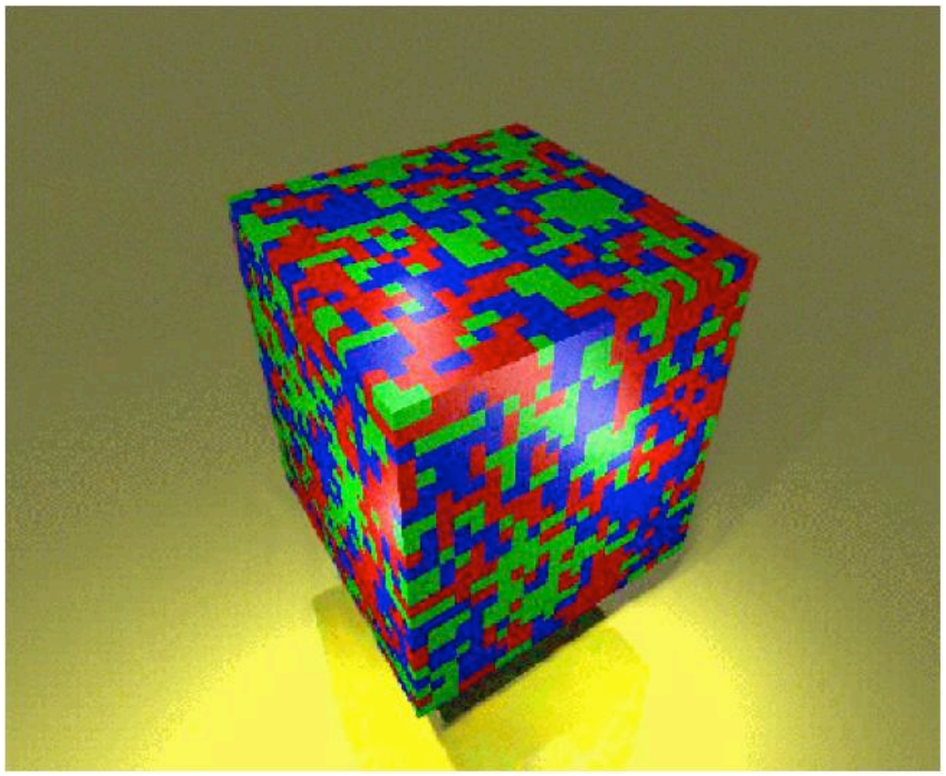
\mathcal{L}^E_{QCD} &= -\frac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu} \\
&\quad - \sum_{f=u,d,s} \bar{\psi}^\alpha_f(x) \left((\gamma^E_\mu)_{\alpha\beta} D^E_\mu + m_f \delta_{\alpha\beta} \right) \psi^\beta_f(x)
\end{aligned}
$$

Lattice gauge theory



- Euclidean space-time
- Hypercubic lattice
- This is gauge-invariant regularization, cutoff π/a
- Fermions integrated out

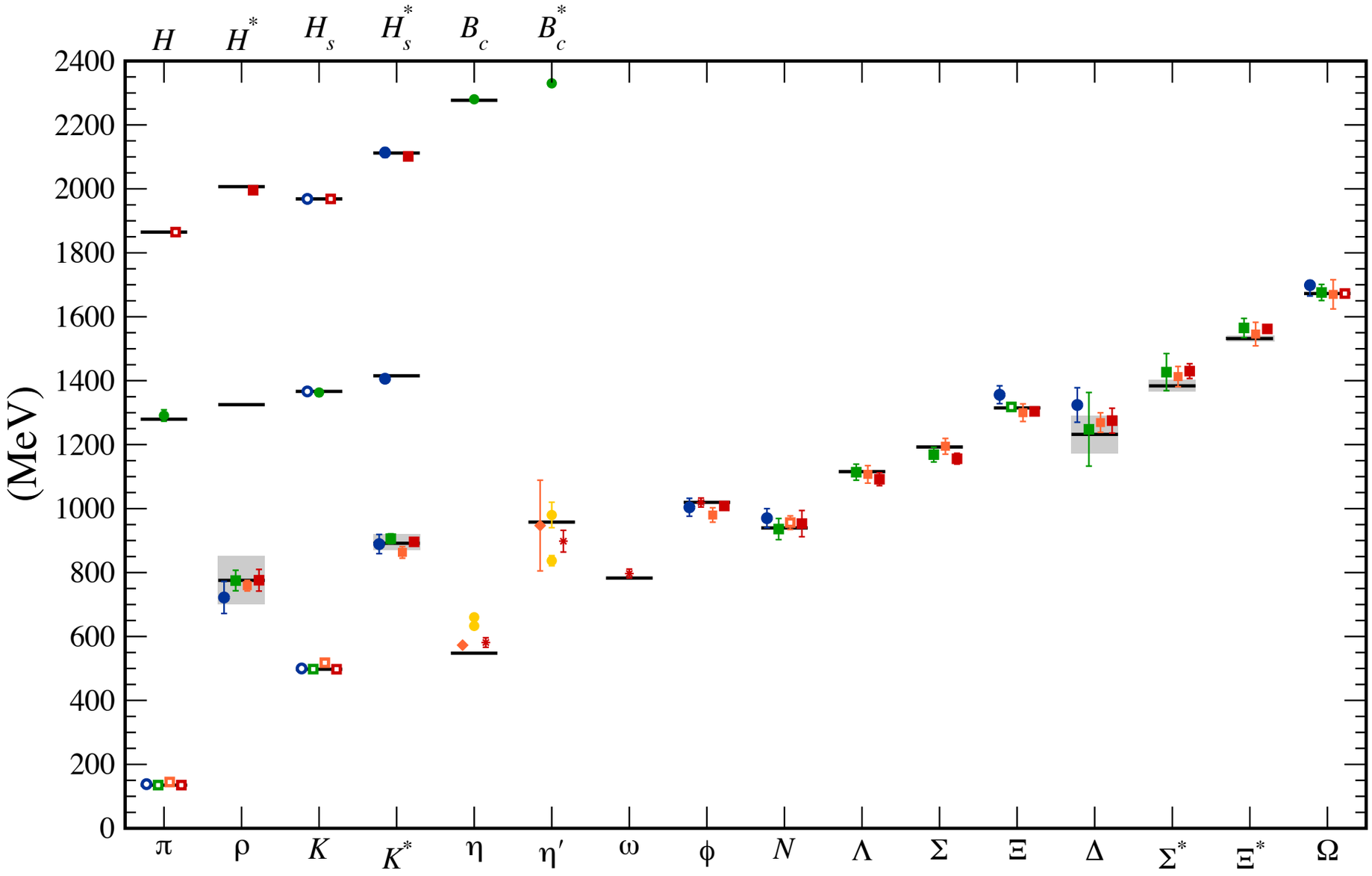
Lattice gauge theory



- Euclidean space-time
- Hypercubic lattice
- This is gauge-invariant regularization, cutoff π/a
- Fermions integrated out

- Evaluate path integrals by Monte Carlo – random walk in the phase space
- Physics recovered in the continuum limit

QCD spectrum



Review by Kronfeld (2012)

- Hadrons: lattice vs experiment

Finite-temperature QCD

- 1975 – QGP phase suggested

Collins and Perry (1975), Cabbibo and Parisi (1975)

- 1979 – perturbative equation of state

Kapusta (1979)

- 1981 – SU(2) pure gauge theory on the lattice

McLerran, Svetitsky (1981), Kuti et al. (1981), Engels et al. (1981)

- 1996 – SU(3) pure gauge theory on lattice, first-order phase transition, equation of state, continuum

Boyd et al. (1996)

Finite-temperature QCD

- 2006 – nature of the transition in 2+1 flavor QCD, crossover

Bernard et al. [MILC] (2005), Cheng et al. [RBC-Bielefeld] (2006), Aoki et al. [BW] (2006)

- 2012 – QCD chiral crossover temperature, physical masses, continuum

Aoki et al. [BW] (2010), Bazavov et al. [HotQCD] (2012)

- 2014 – 2+1 flavor QCD equation of state on the lattice, physical masses, continuum

Bazavov et al. [HotQCD] (2014), Borsanyi et al. [BW] (2014)

Finite-temperature QCD

- 2017 – QCD equation of state at finite density up to the sixth order in baryon chemical potential

Bazavov et al. [HotQCD] (2017)

- 2019 – QCD chiral crossover temperature, physical masses, continuum, high precision

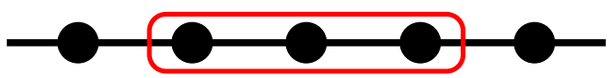
Bazavov et al. [HotQCD] (2019)

Improved actions and continuum limit

Improved actions

- Need to discretize a first order differential operator:

$$\frac{df}{dx} \quad \rightarrow \quad \frac{1}{2a}\left[f(x+a) - f(x-a)\right] + O(a^2)$$



- To reduce discretization error either use a finer grid, but more computationally expensive...

Improved actions

- Need to discretize a first order differential operator:

$$\frac{df}{dx} \quad \rightarrow \quad \frac{1}{2a}\left[f(x+a) - f(x-a)\right] + O(a^2)$$

- To reduce discretization error either use a finer grid, but more computationally expensive...
- ...or use a smarter finite difference:

$$\frac{df}{dx} \quad \rightarrow \quad \frac{2}{3a}\left[f(x+a) - f(x-a)\right] - \frac{1}{12a}\left[f(x+2a) - f(x-2a)\right] + O(a^4)$$

- Note: same continuum limit

HISQ action

- Highly – more improvement than previously
- Improved – adding higher-order (irrelevant) operators suppresses discretization effects

Symanzik (1980)

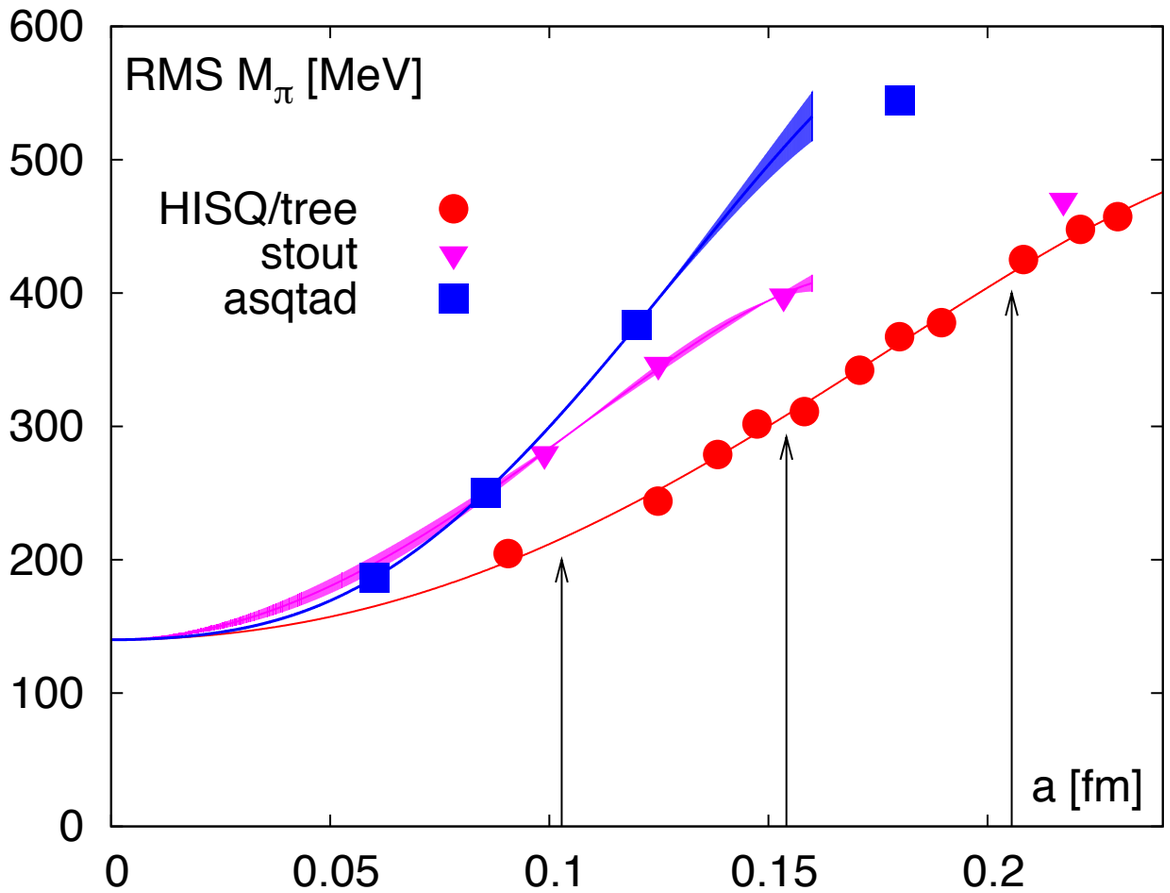
- Staggered – particular fermion discretization scheme which partially deals with the fermion doubling problem

Kogut and Susskind (1975)

- Quarks

Follana et al. [HPQCD] PRD75 (2007), Bazavov et al. [MILC] PRD82 (2010)

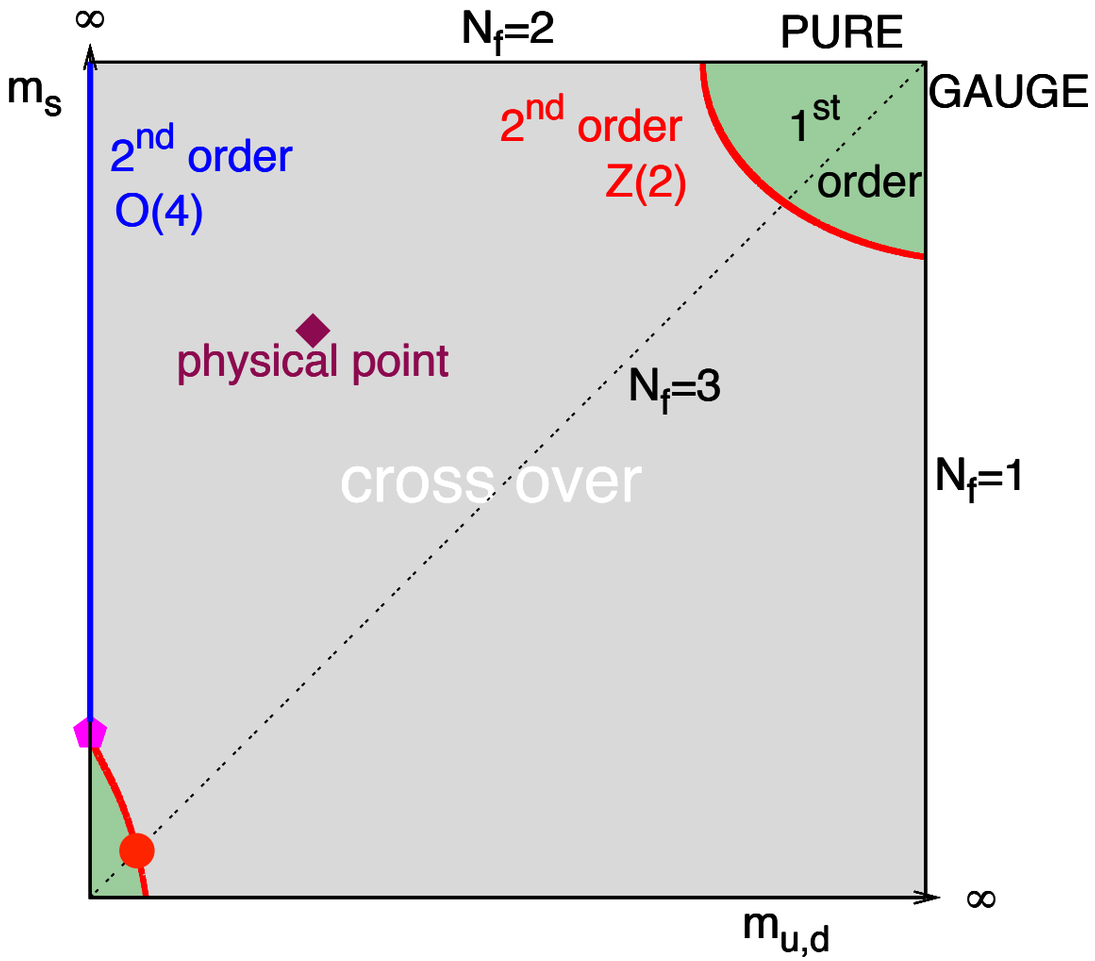
Why continuum limit is a big deal?



- Lattice artifacts make the spectrum heavier, the lightest states are the most affected
- Simulation cost scales as $\left(\frac{L}{a}\right)^4 \frac{1}{a} \frac{1}{m_\pi^2 a} \sim \frac{1}{a^6}$

Criticality in QCD and chiral crossover

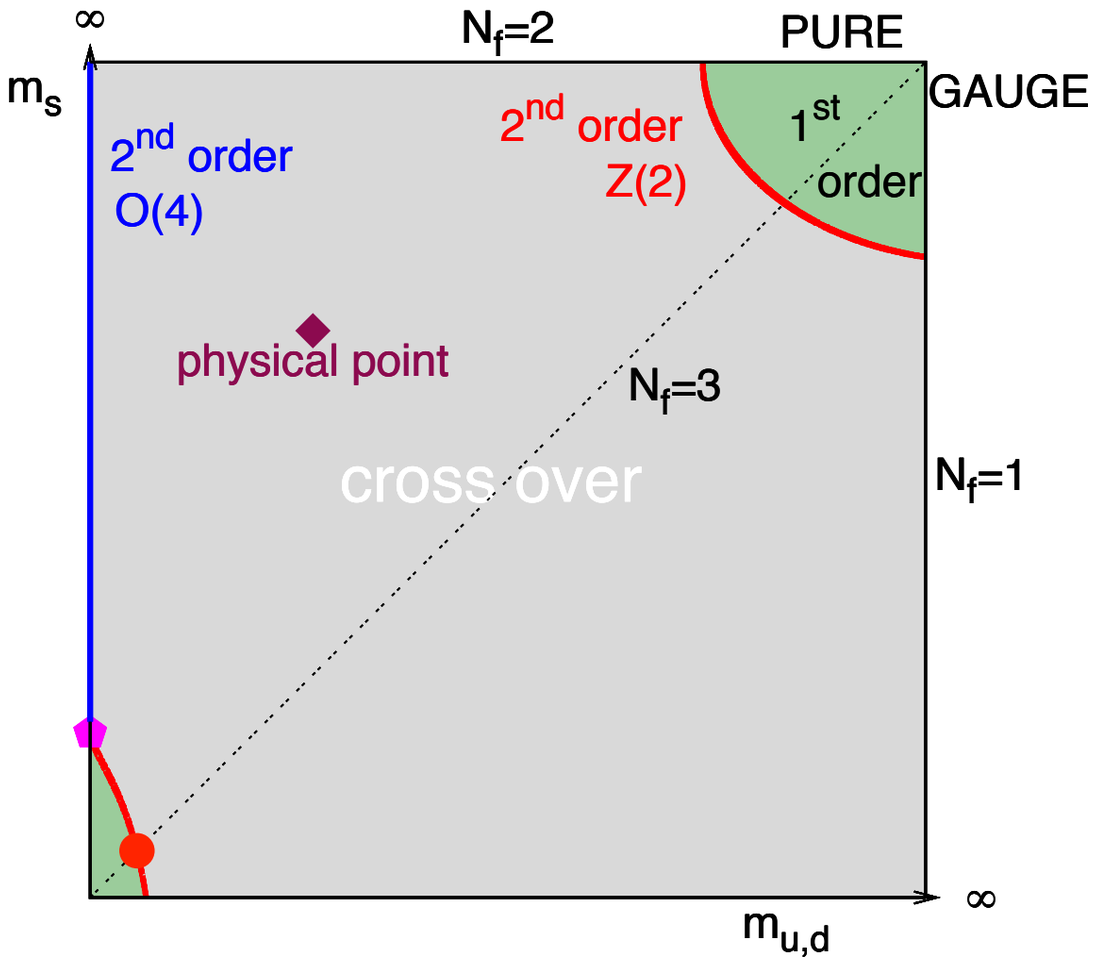
Criticality in QCD



Pisarski and Wilczek (1984)

- Pure gauge – infinitely heavy quarks
- QCD – almost massless quarks
- Criticality in the chiral limit

Criticality in QCD



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- Pure gauge – infinitely heavy quarks
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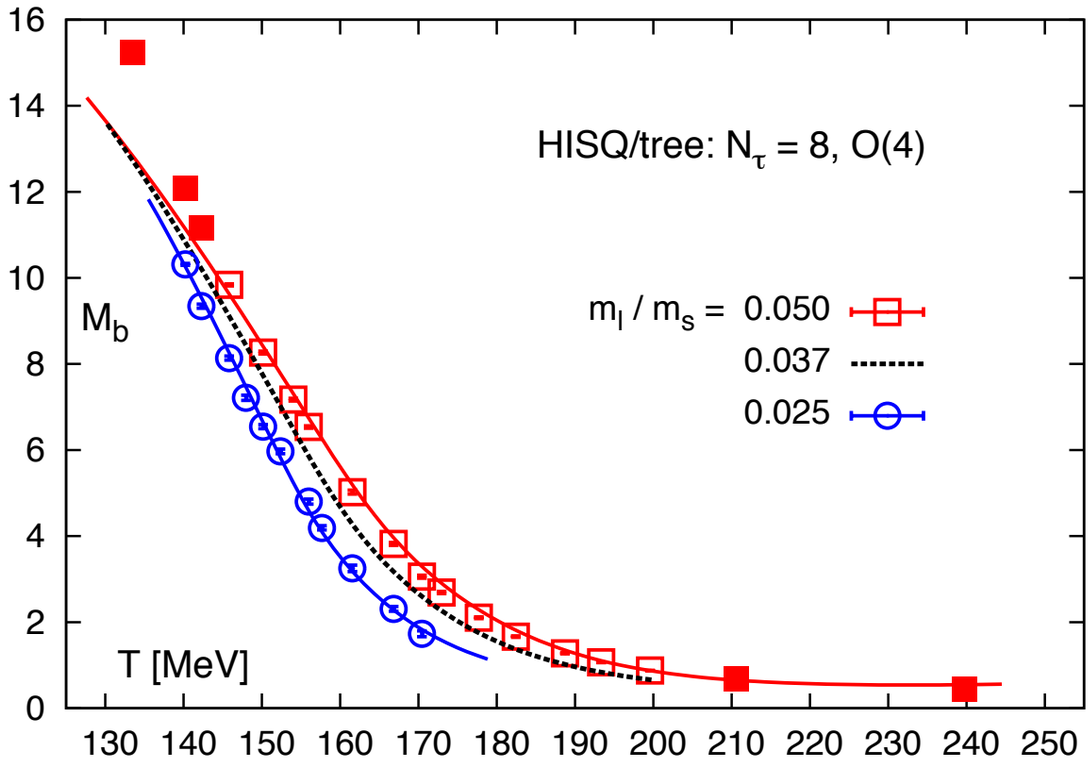
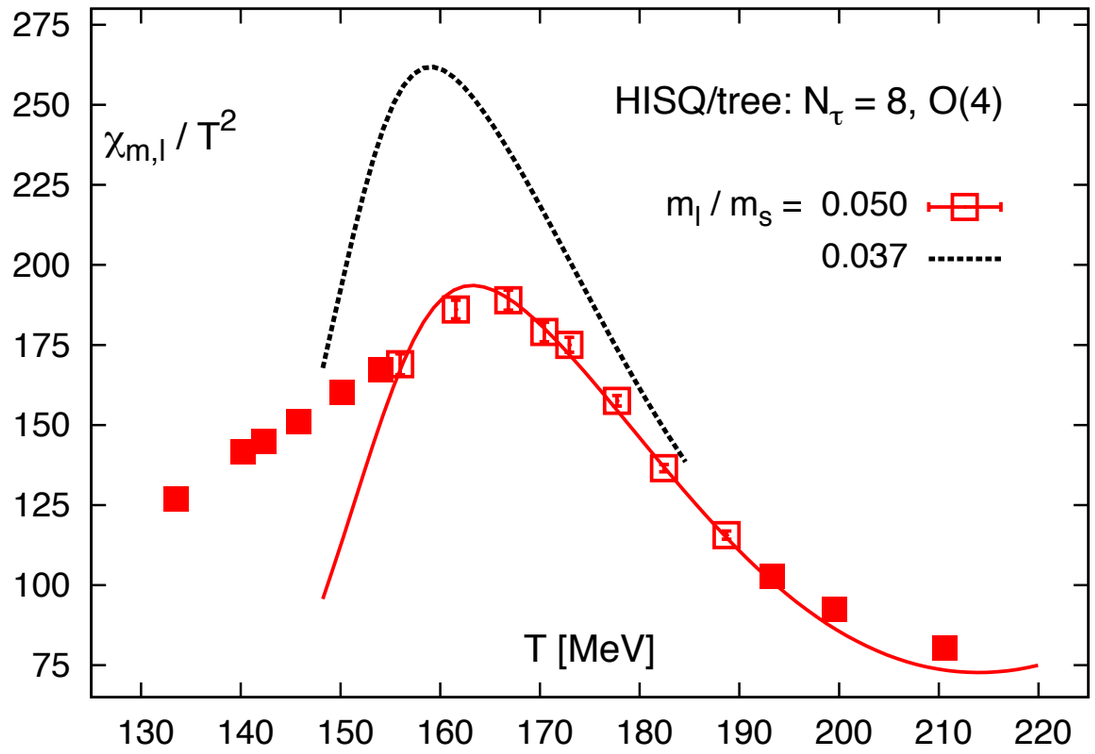
- Criticality in the chiral limit

- Order parameter – chiral condensate $\langle\bar{\psi}\psi\rangle \sim \dfrac{d\ln Z}{dm}$
- Chiral susceptibility $\chi \sim \langle(\bar{\psi}\psi)^2\rangle - \langle\bar{\psi}\psi\rangle^2$

Chiral symmetry

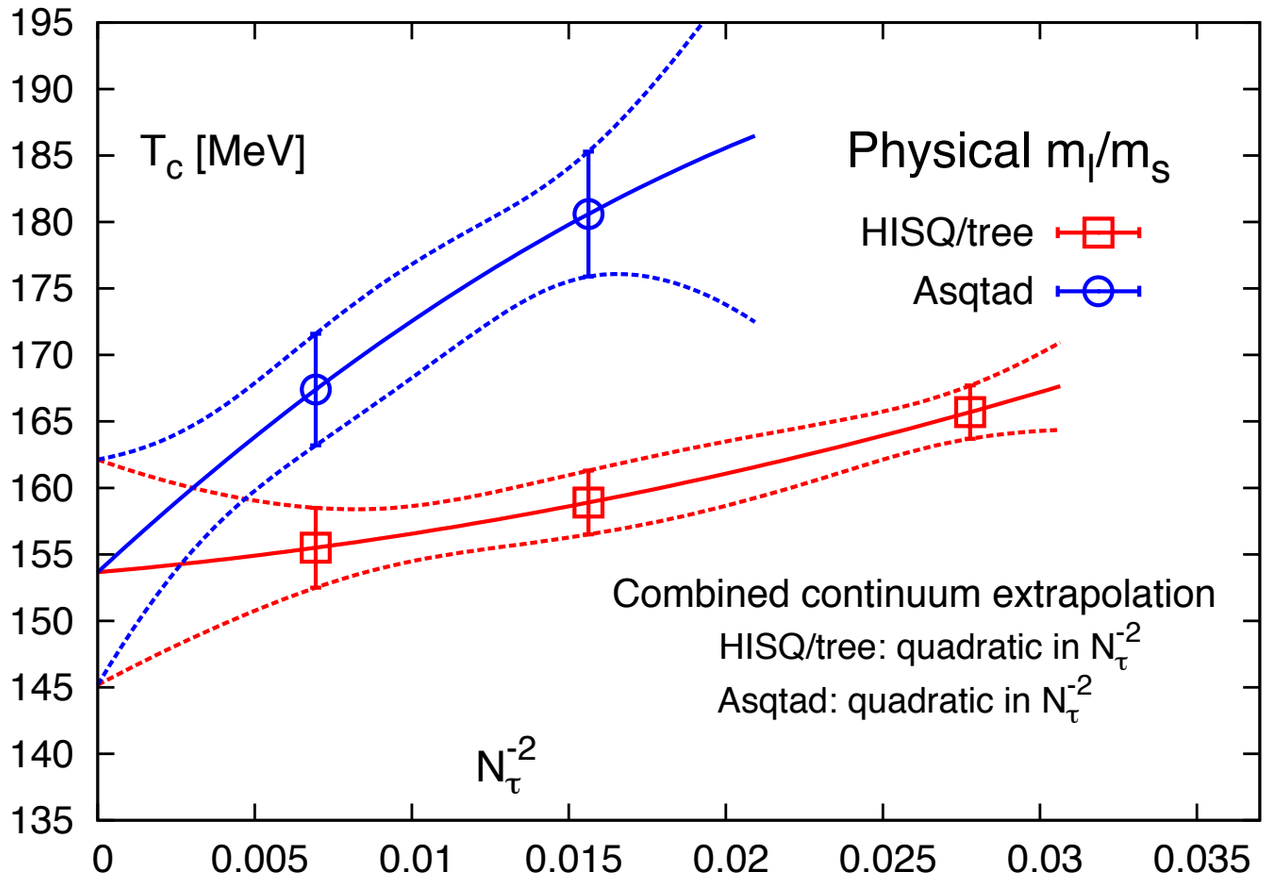




Bazavov et al. [HotQCD] PRD85 (2012)

- Susceptibility – diverges in the chiral limit
- Define crossover temperature from the peak

Crossover temperature (2012)

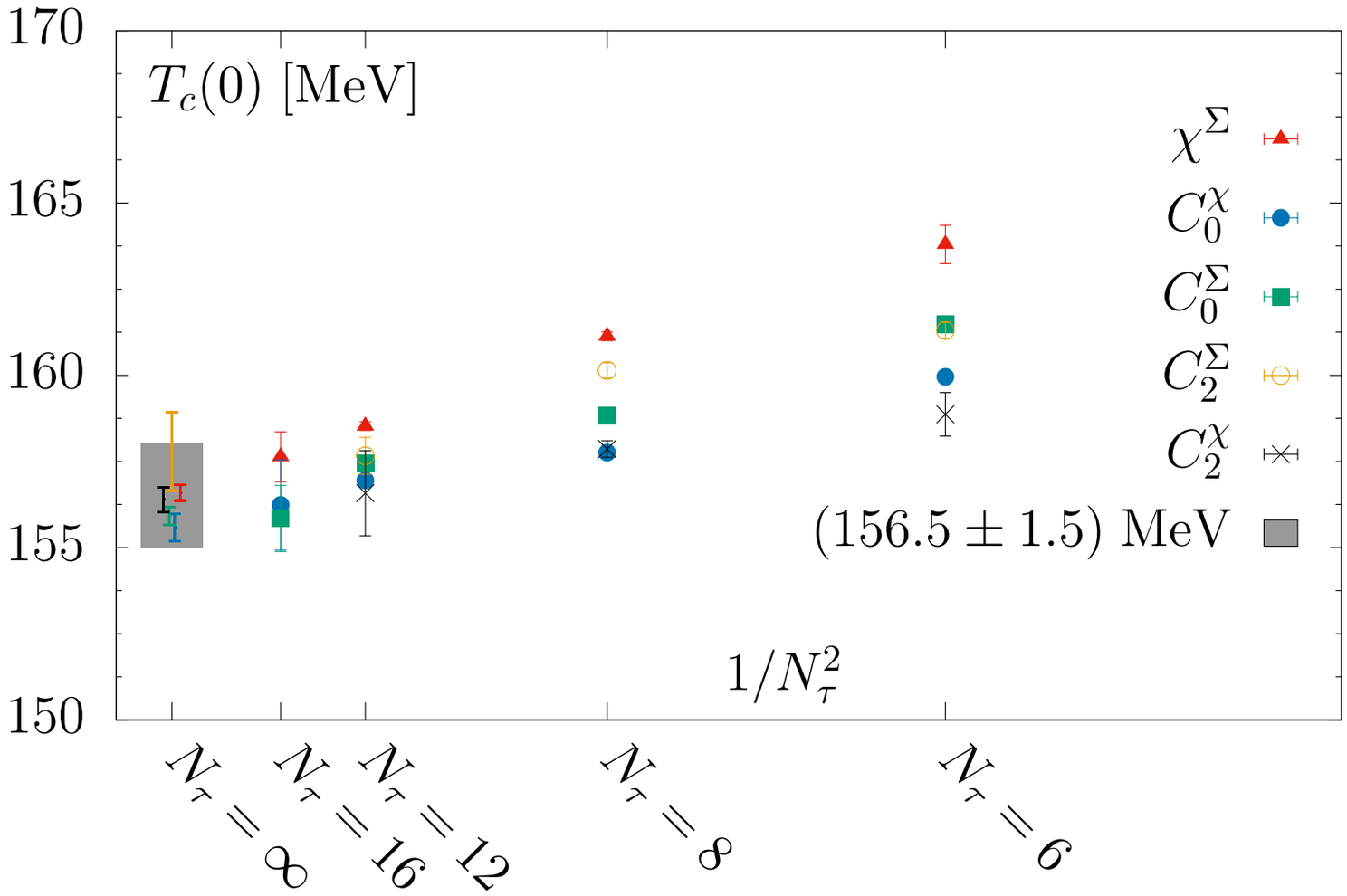


Bazavov et al. [HotQCD] PRD85 (2012)

- Continuum extrapolation at the physical mass:

$$T_c = 154(9)\ \text{MeV}$$

Crossover temperature (2019)



Steinbrecher, PhD thesis (2018), Bazavov et al. [HotQCD] accepted PLB (2019)

- Continuum extrapolation, five quantities:

$$T_c = 156.5(1.5) \text{ MeV}$$

Deconfinement

Fluctuations of conserved charges

$$\chi_2^X \sim \left\langle (\delta N_X)^2 \right\rangle, \qquad \delta N_X = N_X - \langle N_X \rangle$$

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- Generalized susceptibilities

$$\chi_{mn}^{XY} = \left. \frac{\partial^{(m+n)} [p(\hat{\mu}_X, \hat{\mu}_Y)/T^4]}{\partial \hat{\mu}_X^m \partial \hat{\mu}_Y^n} \right|_{\vec{\mu}=0}$$

$$X, Y = B, S, Q$$

$$\hat{\mu} = \mu/T$$

$$\vec{\mu} = (\mu_B, \mu_S, \mu_Q)$$

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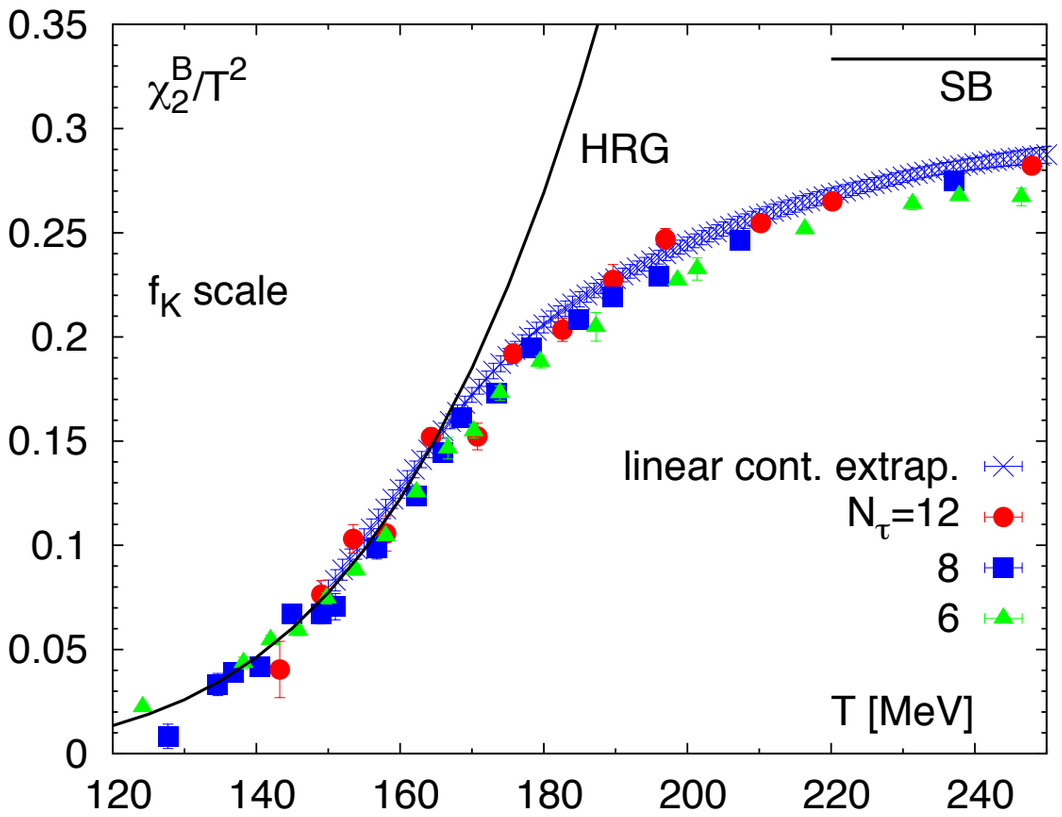
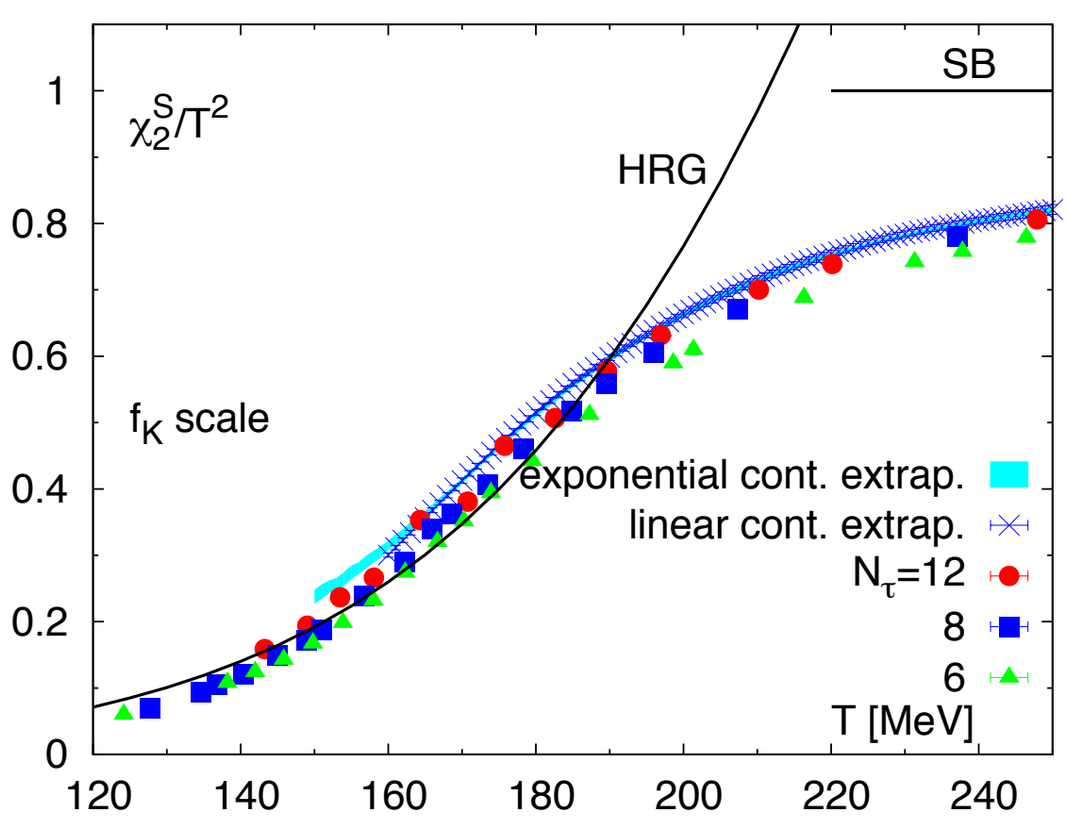
- At low temperatures the partition function of QCD can be well approximated by a non-interacting gas of resonances – Hadron Resonance Gas (HRG) model

Hagedorn (1965), Dashen, Ma and Bernstein (1969)

Fluctuations of conserved charges

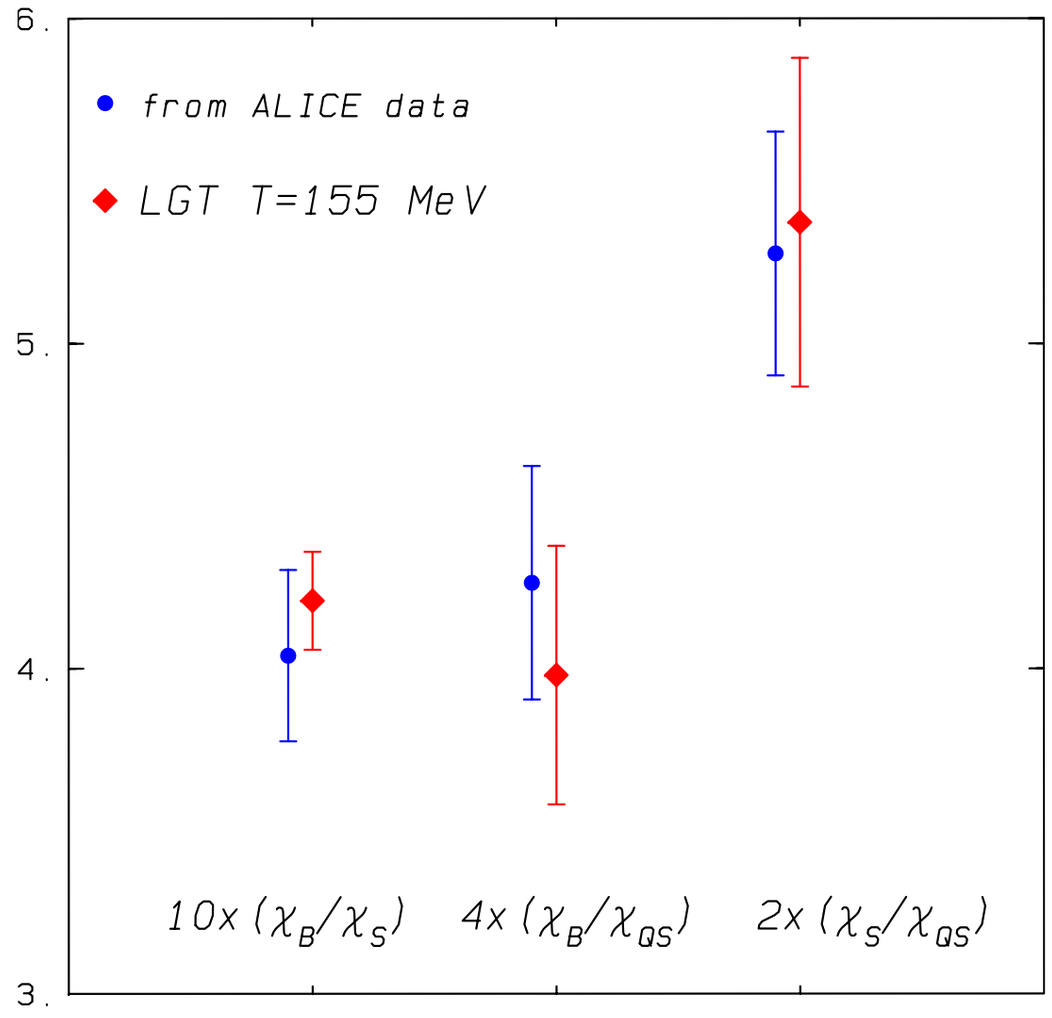




Bazavov et al. [HotQCD] PRD86 (2012)

- Baryon number fluctuations (left)
- Strangeness fluctuations (right)

Fluctuations of conserved charges



- Comparison with ALICE data

Braun-Munzinger, Kalweit, Redlich, Stachel, PLB747 (2015)

QCD equation of state

QCD equation of state

- The partition function

$$Z = \int DU\, D\bar{\psi} D\psi \exp\{-S\}$$

- The trace anomaly

$$\Theta^{\mu\mu} \equiv \varepsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a}$$

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$$\frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}$$

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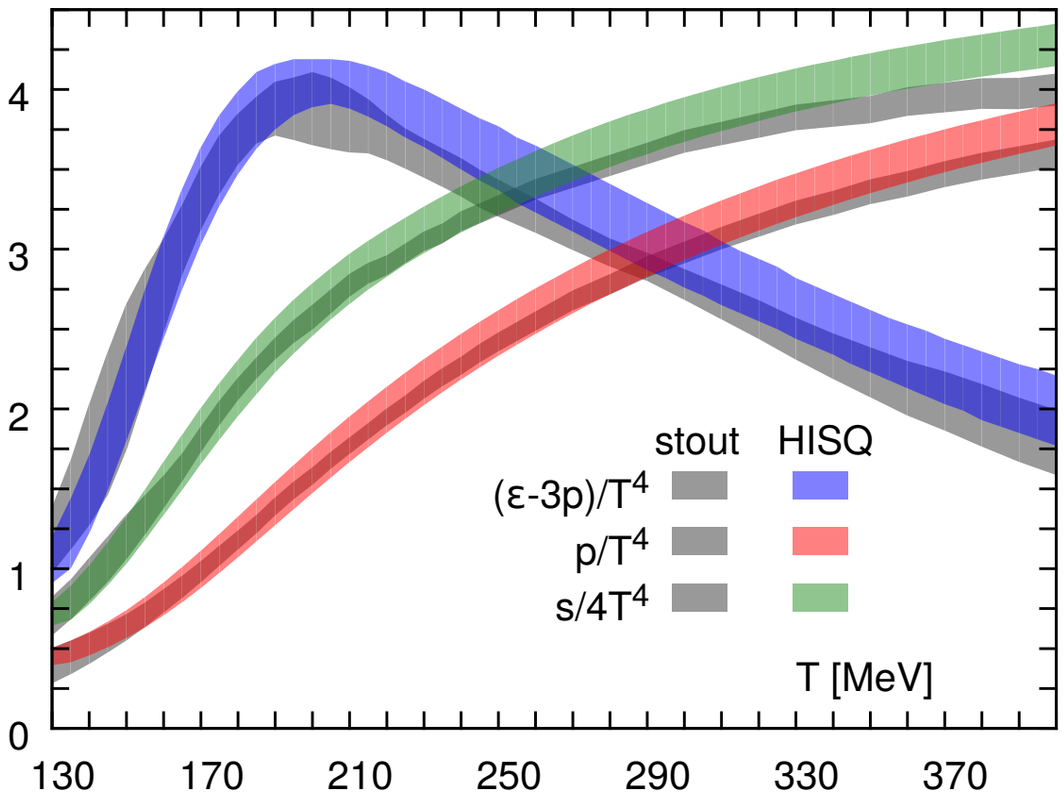
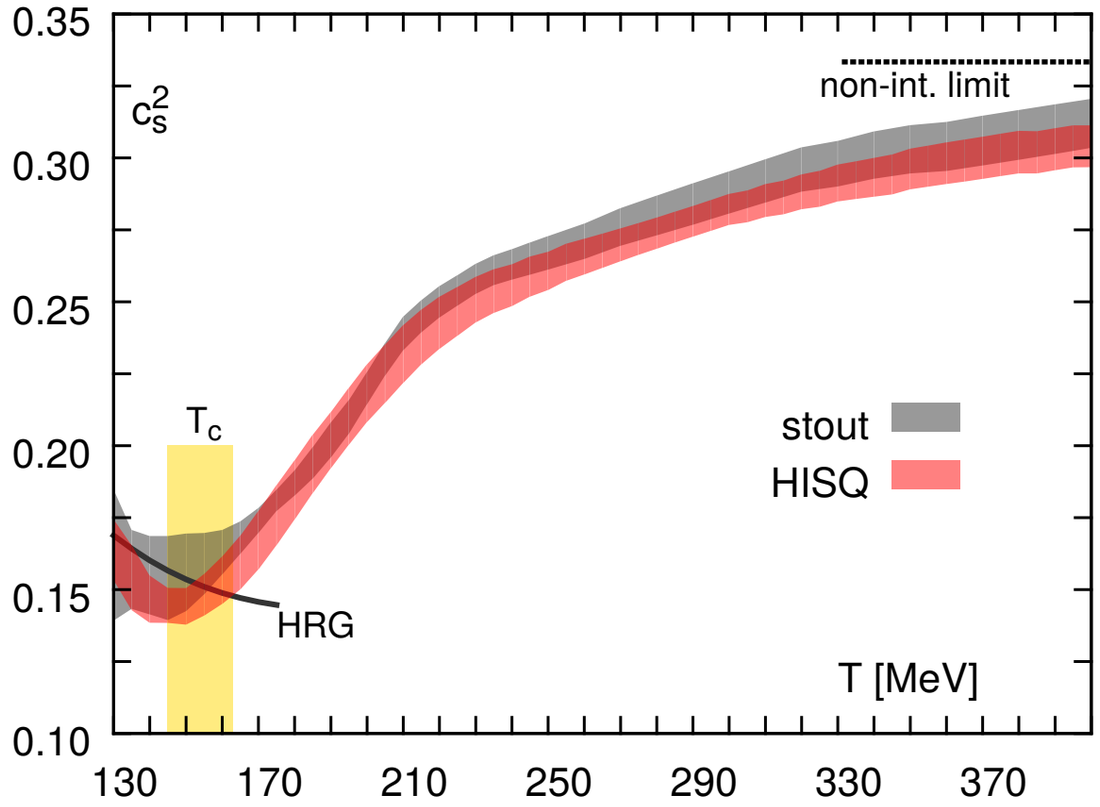
- Requires additive renormalization due to breaking of the Lorentz symmetry (high computational cost)

QCD equation of state

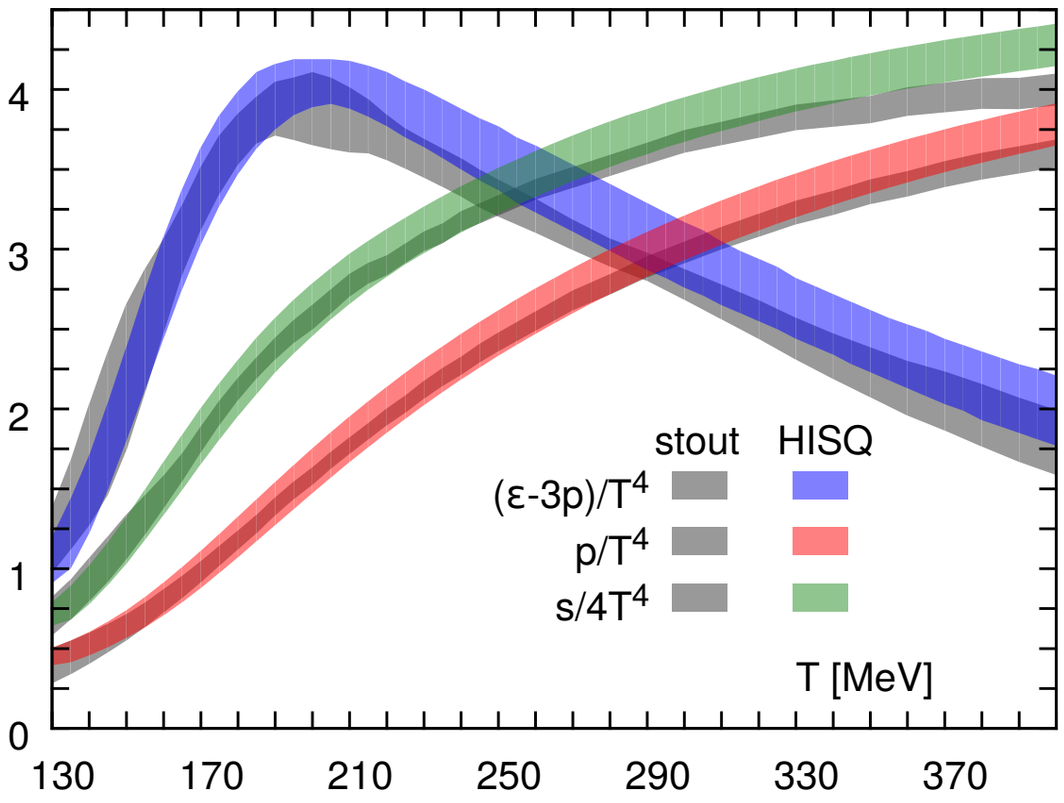


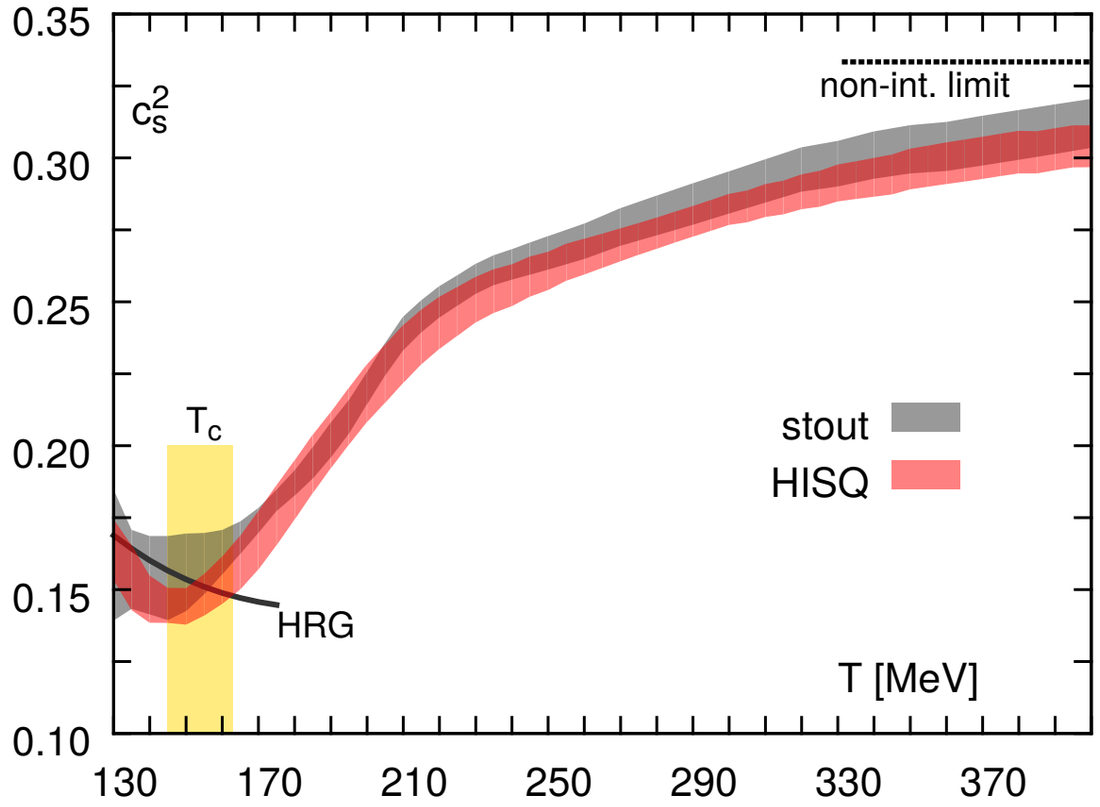


Borsanyi et al [WB] PLB730 (2014), Bazavov et al. [HotQCD] PRD90 (2014)

- Trace anomaly, *p*, *s* (left) and speed of sound (right) at zero baryon chemical potential

QCD equation of state



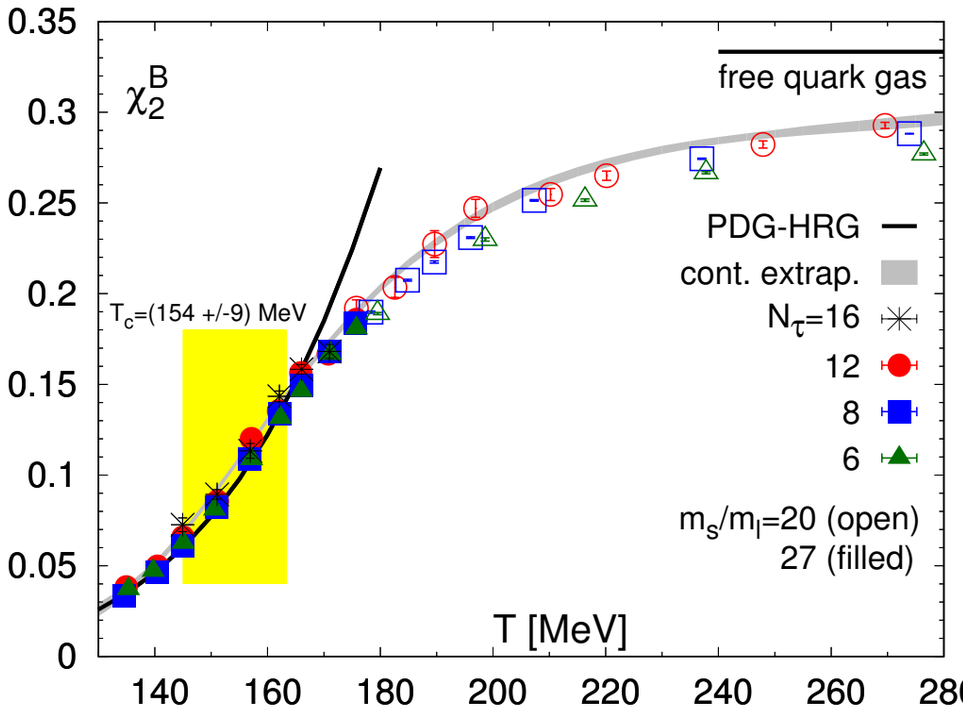


Borsanyi et al [WB] PLB730 (2014), Bazavov et al. [HotQCD] PRD90 (2014)

- Trace anomaly, *p*, *s* (left) and speed of sound (right) at zero baryon chemical potential
- Additional contributions at $\mu_B > 0, \;\; \mu_Q = \mu_S = 0$

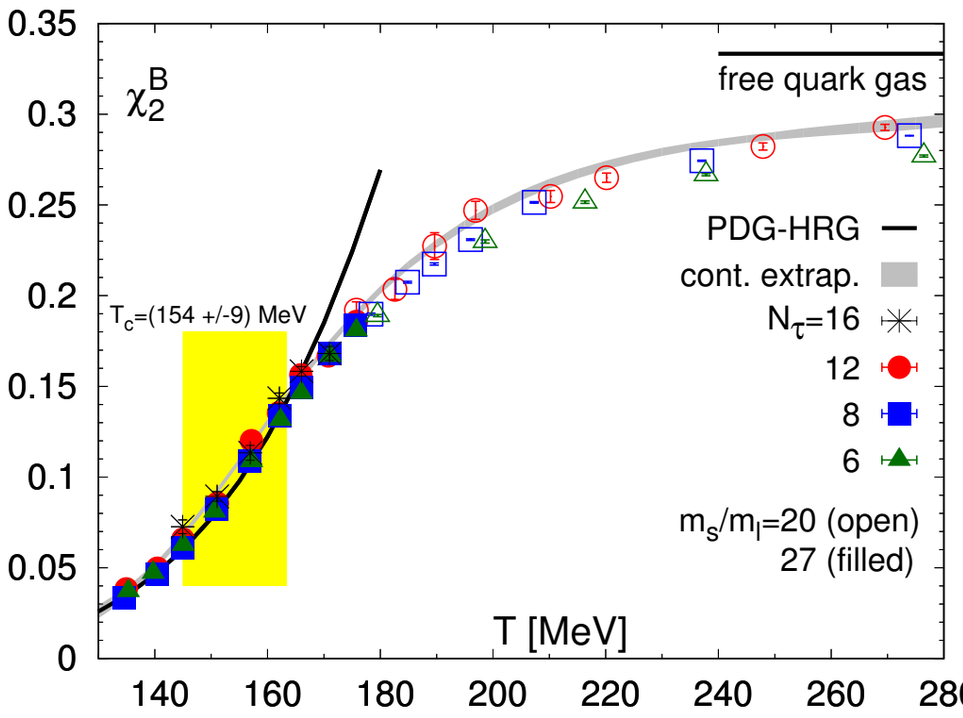
$$\frac{\Delta P}{T^4} = \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2 \left(1 + \frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2 + \frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4 + \; ...\right)$$

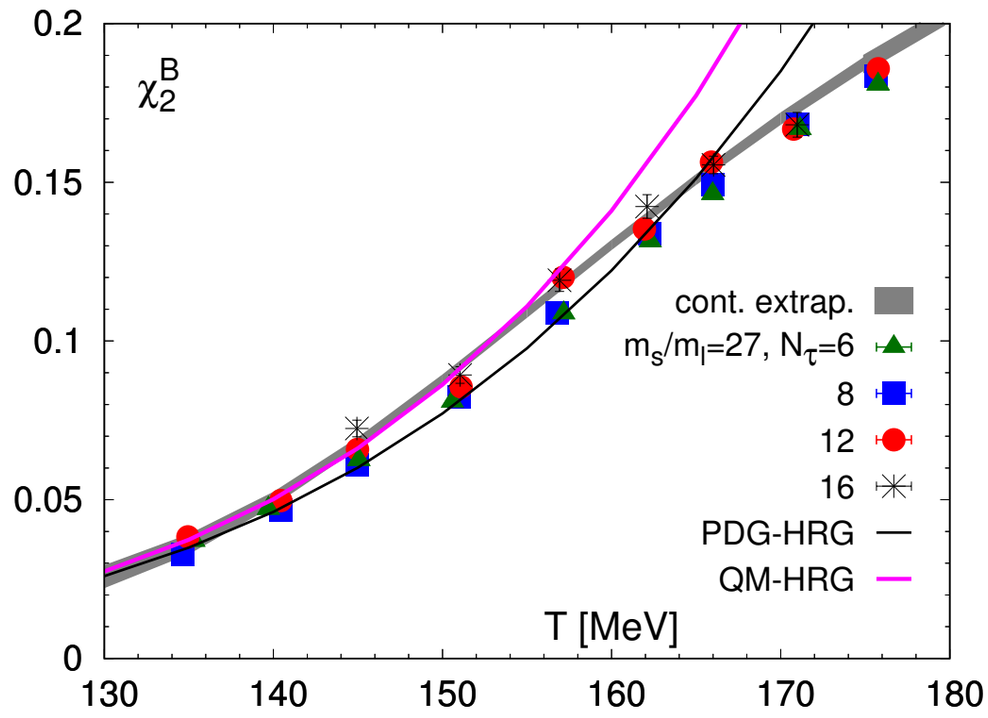
QCD equation of state at $O(\mu_B^6)$



Bazavov et al. [HotQCD] PRD95 (2017), Borsanyi et al [WB] JHEP10 (2018)

QCD equation of state at $O(\mu_B^6)$





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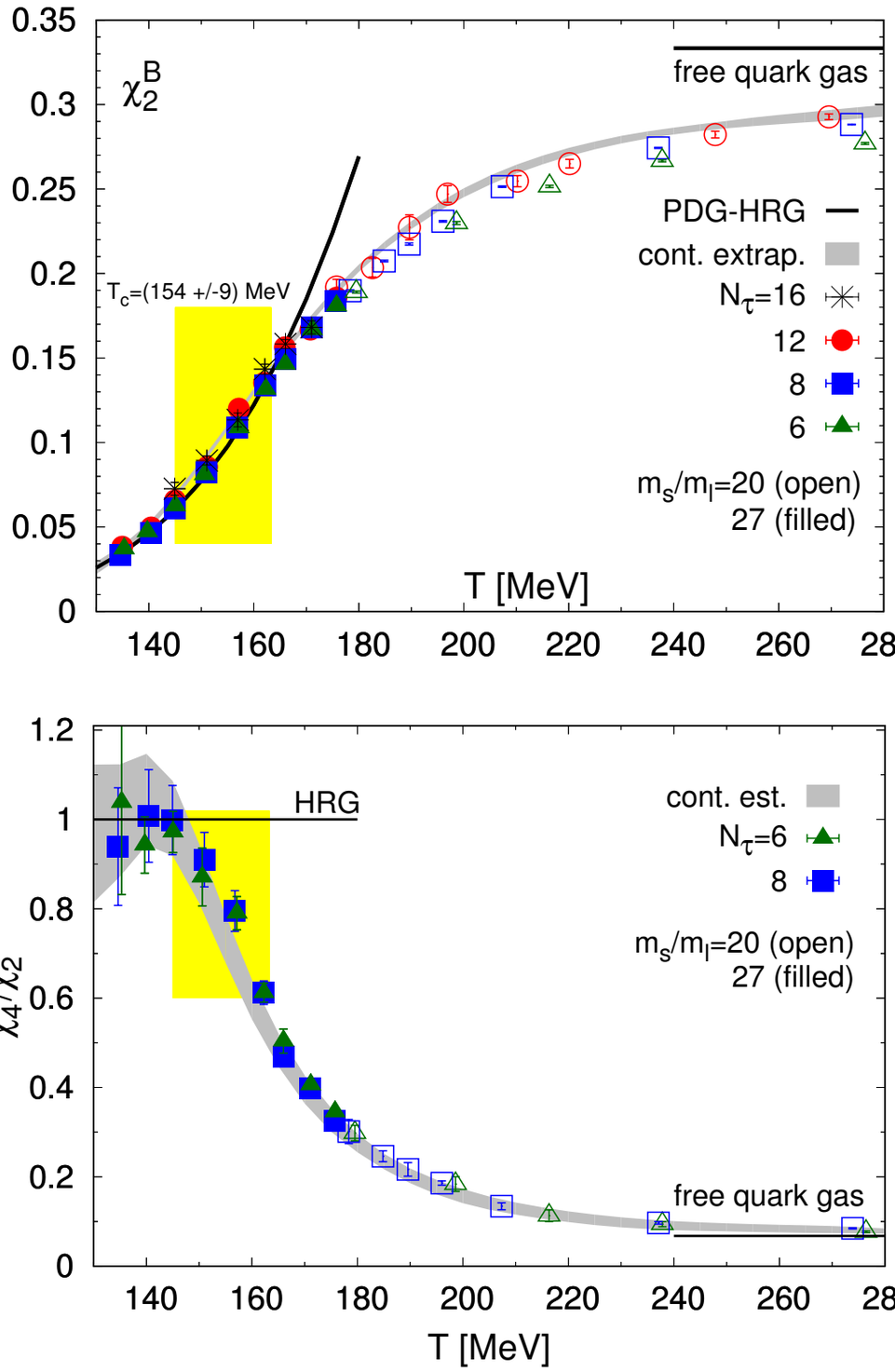

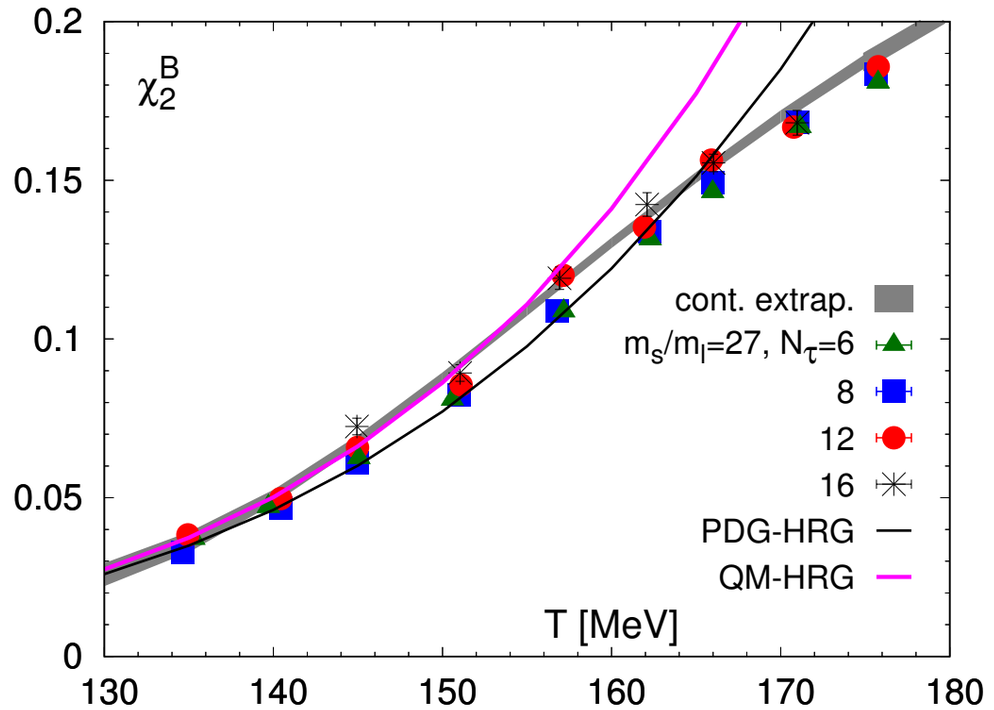

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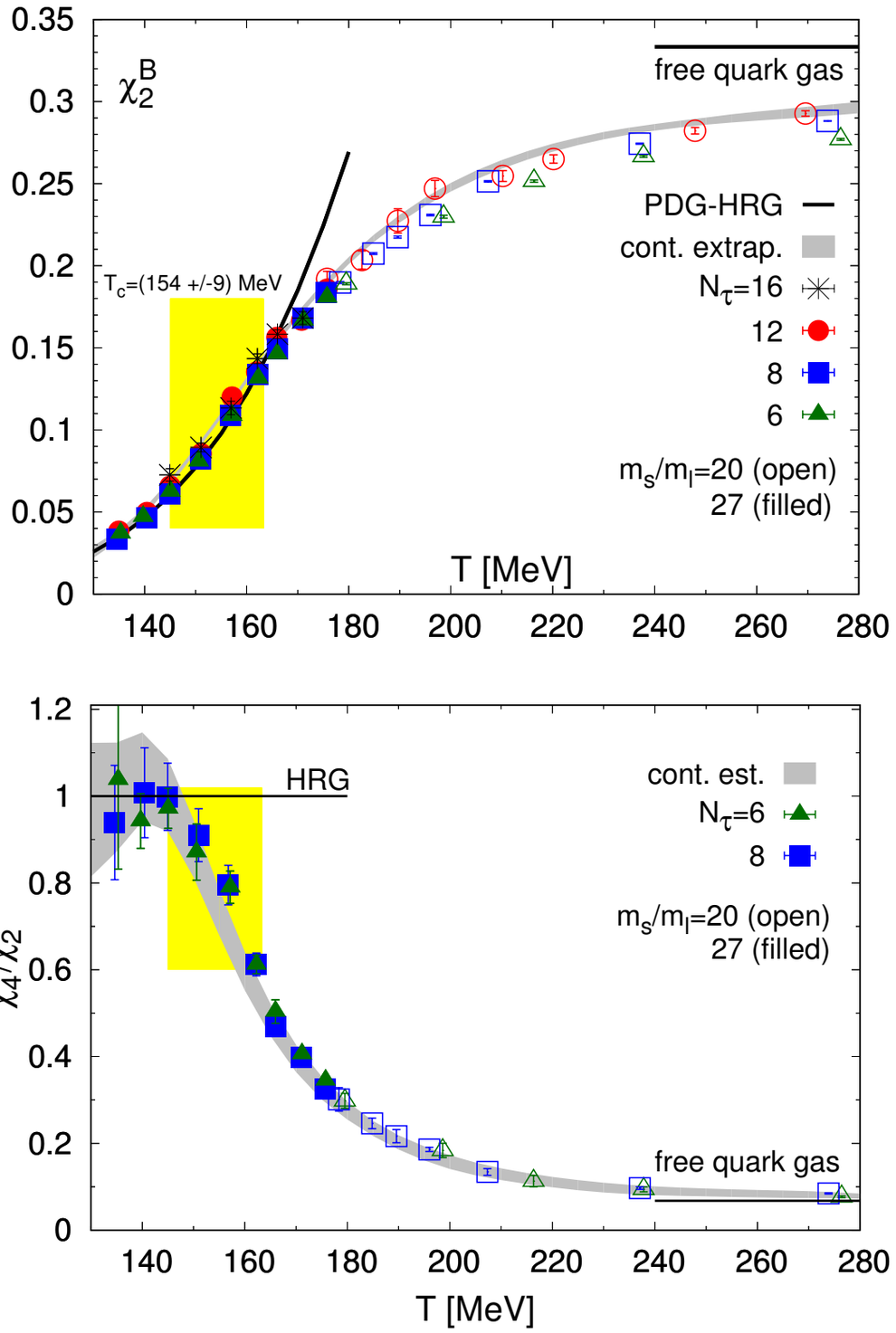

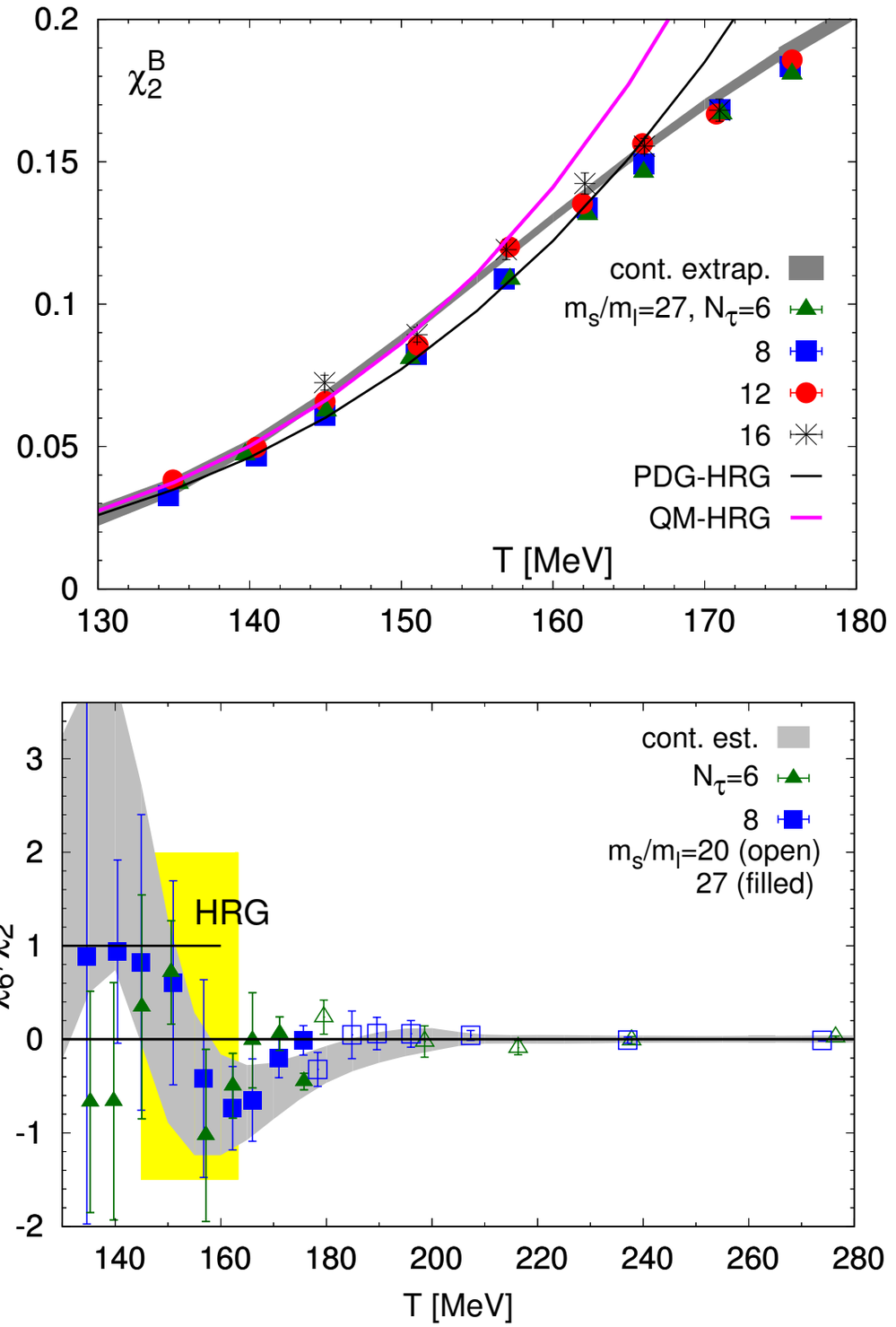

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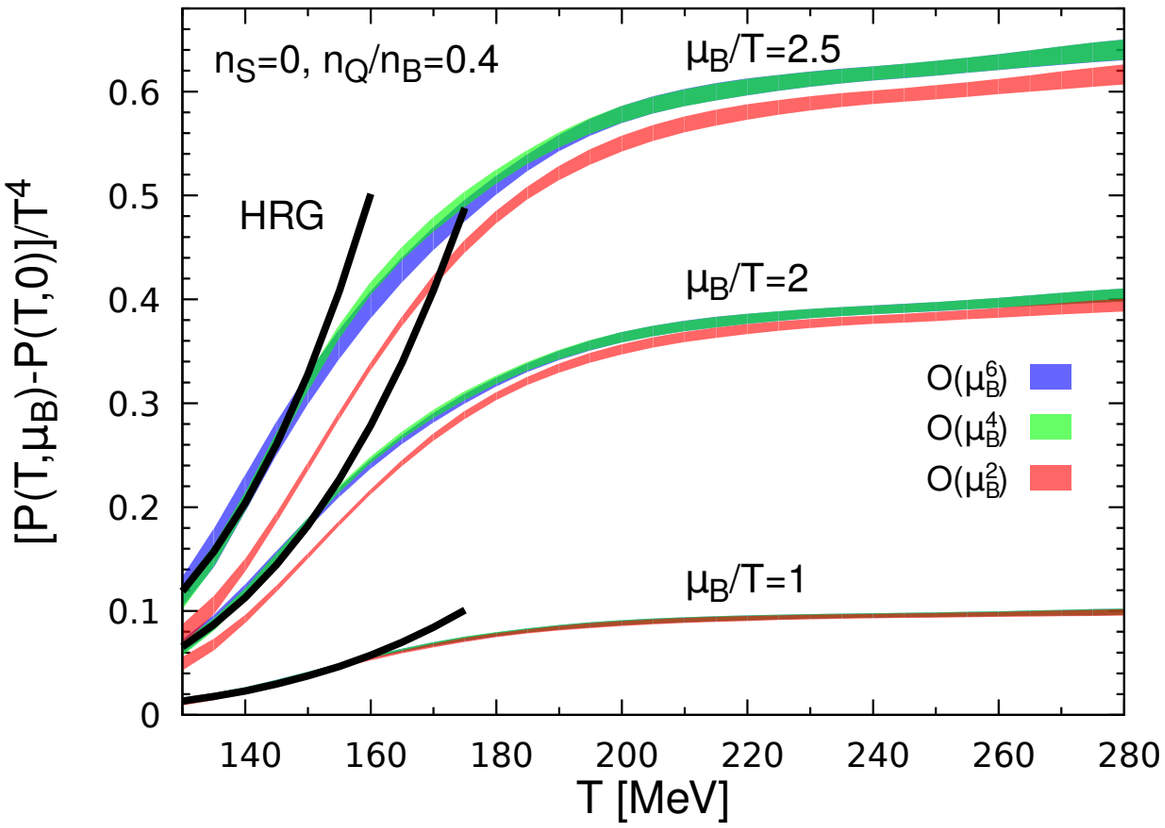
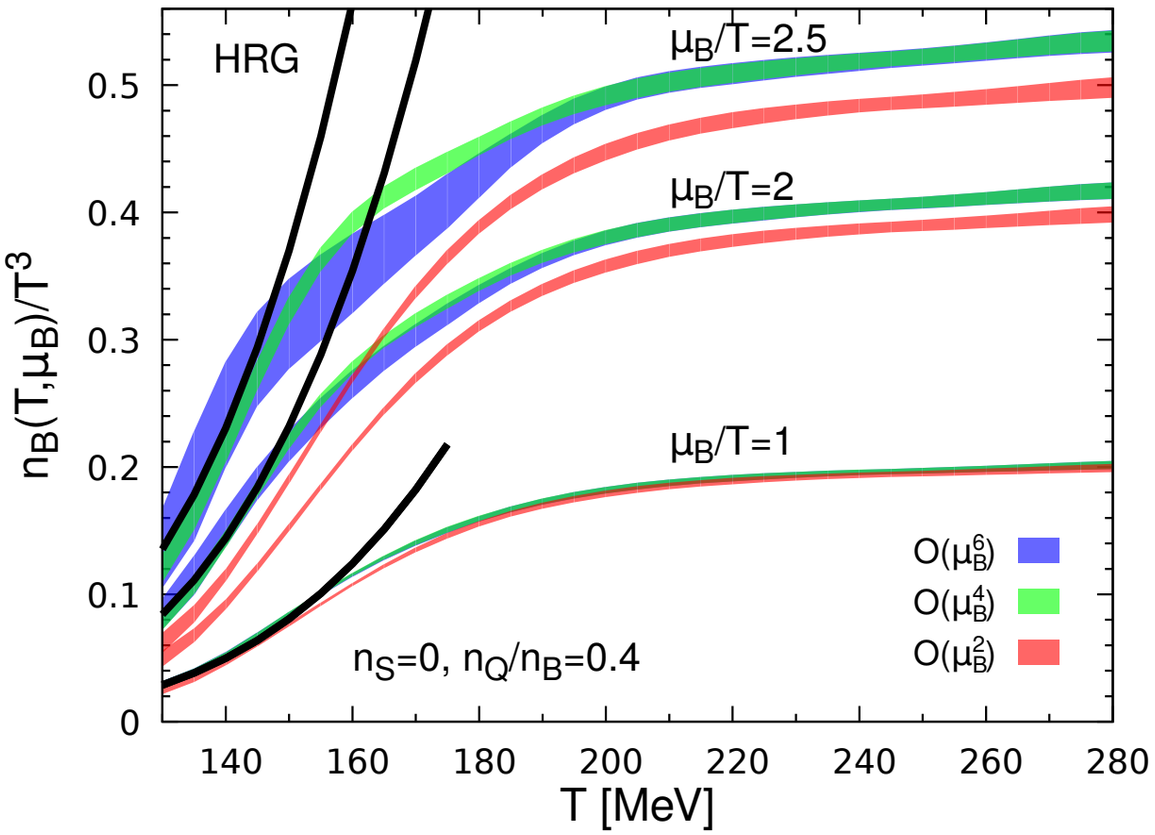
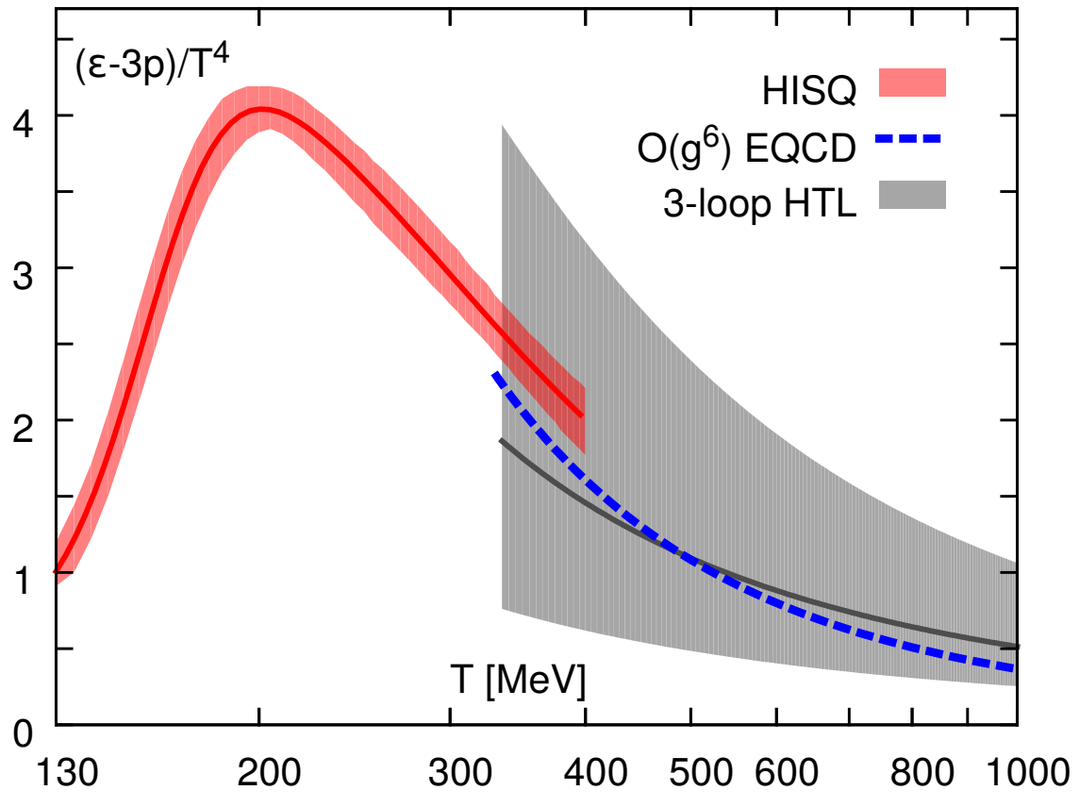


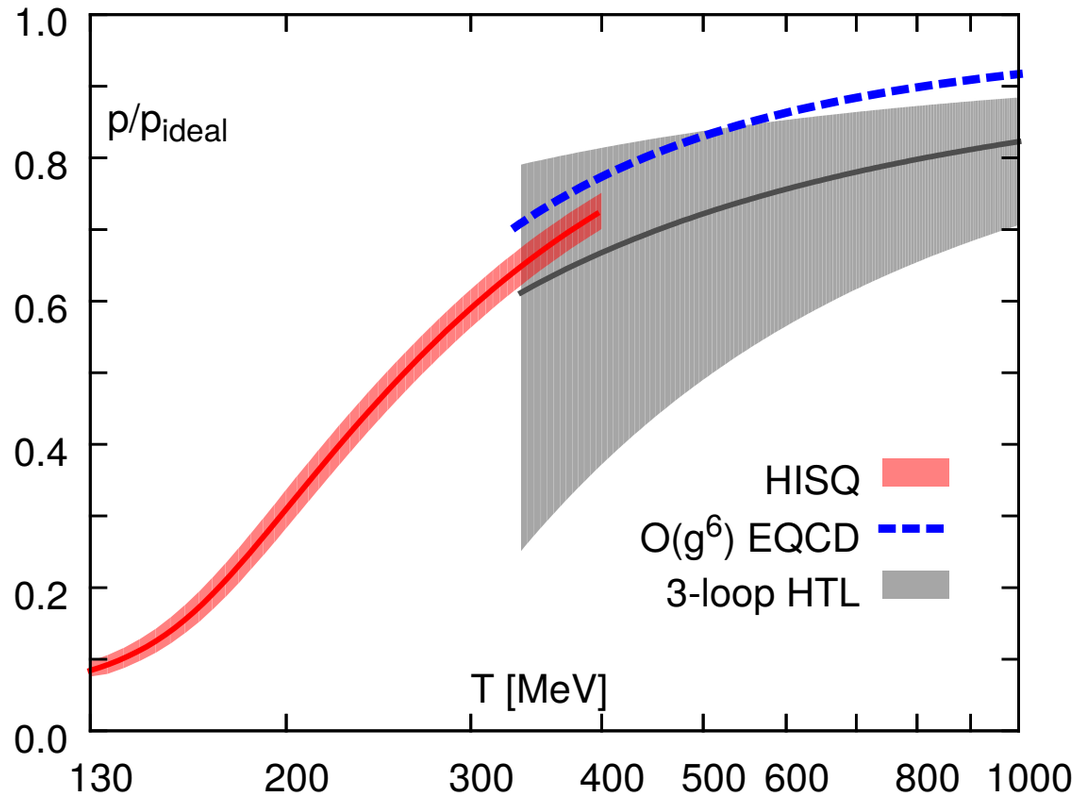


Bazavov et al. [HotQCD] PRD95 (2017)

- The contribution to the pressure due to finite chemical potential (left) and the baryon number density (right) for strangeness neutral systems

QCD equation of state at high temperature



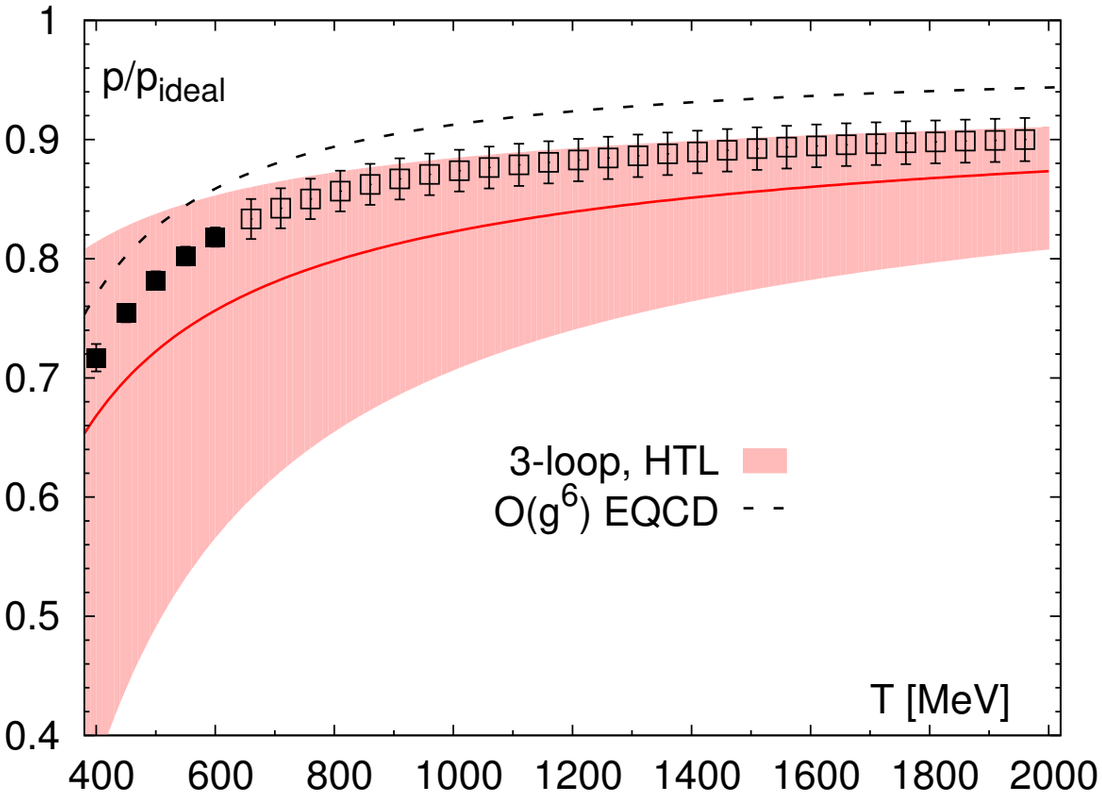


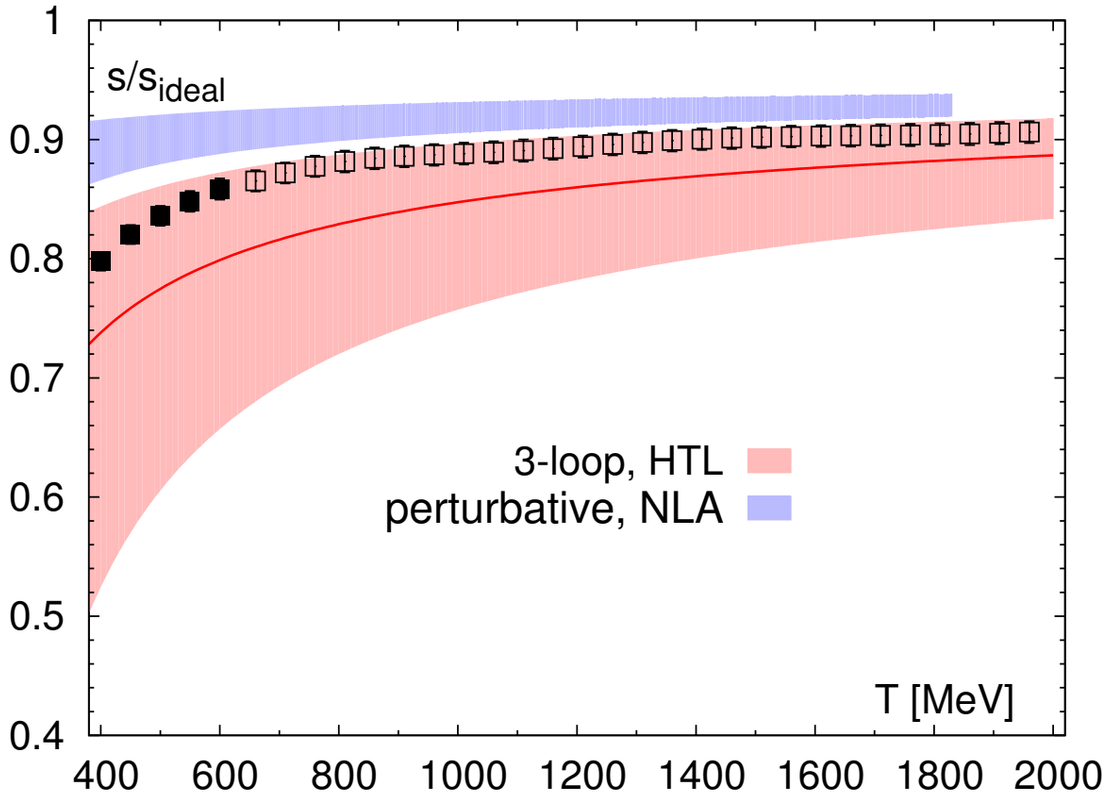
Bazavov et al. [HotQCD] PRD90 (2014)

- Comparison with perturbative calculations

Laine and Schroeder (2006), Haque et al. (2014)

QCD equation of state at high temperature



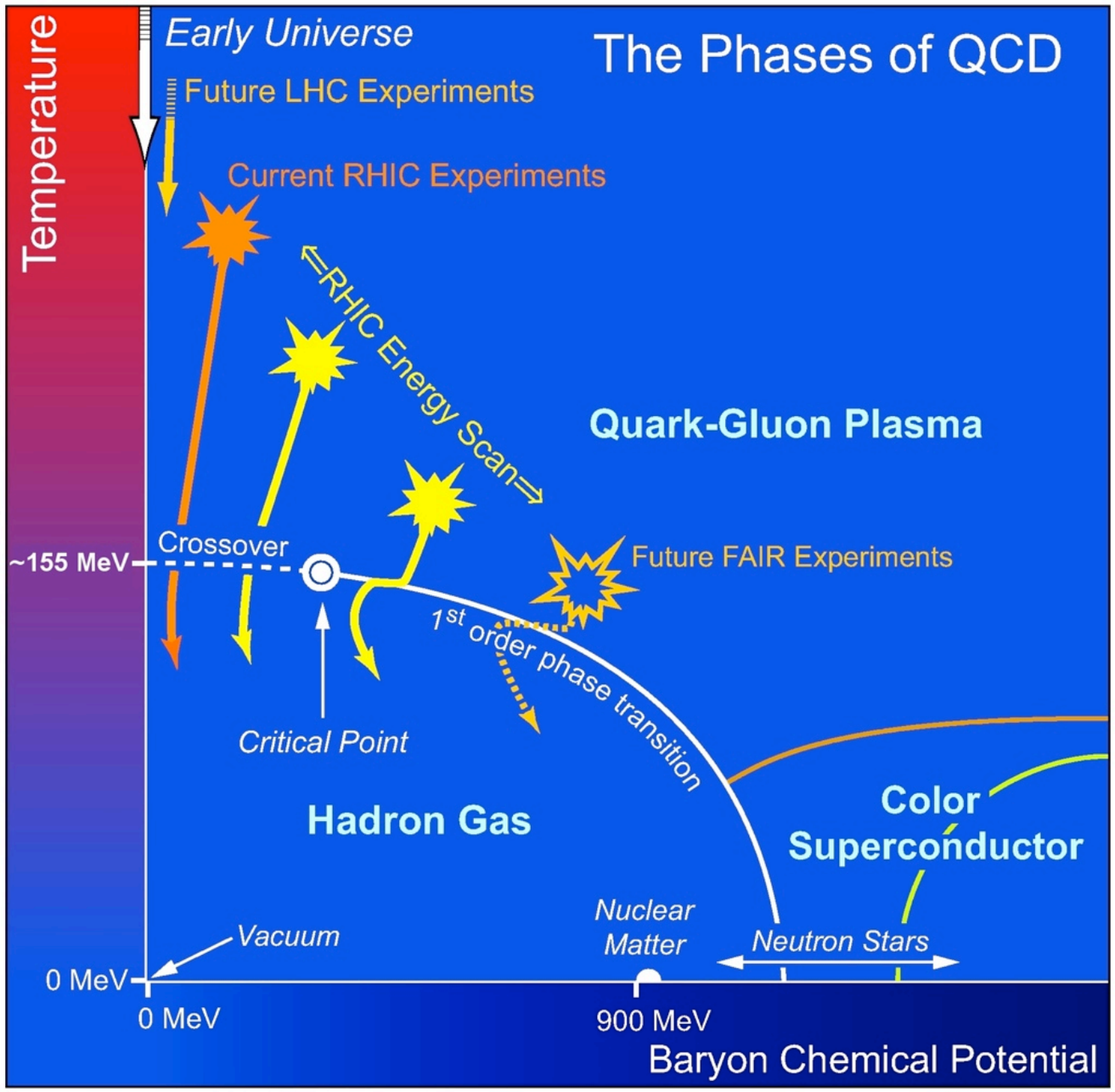


Bazavov, Petreczky, Weber PRD97 (2018)

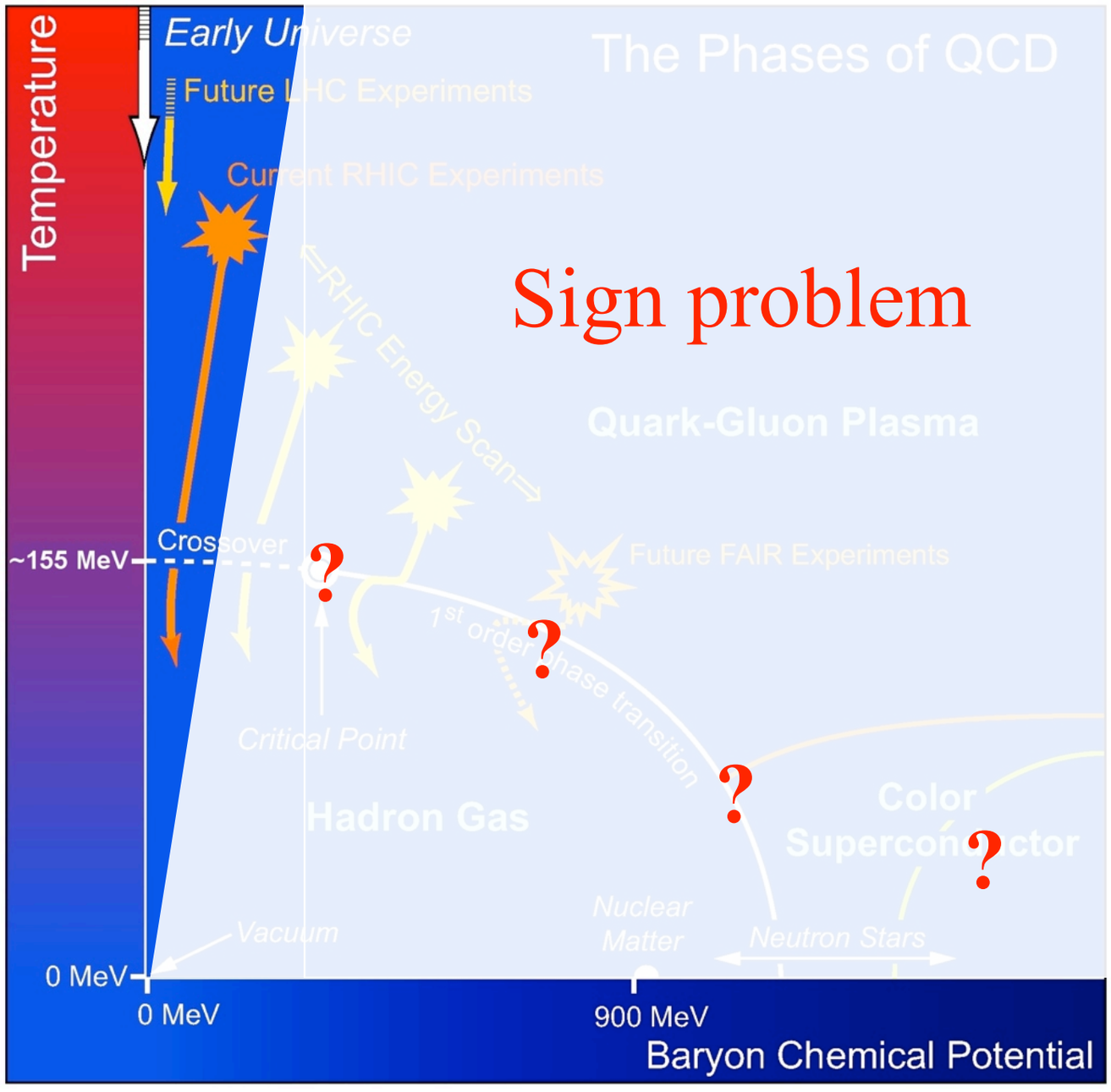
- Update of the equation of state at high temperature, reaching up to 2 GeV

Future challenges

1.Finite baryon chemical potential



1.Finite baryon chemical potential



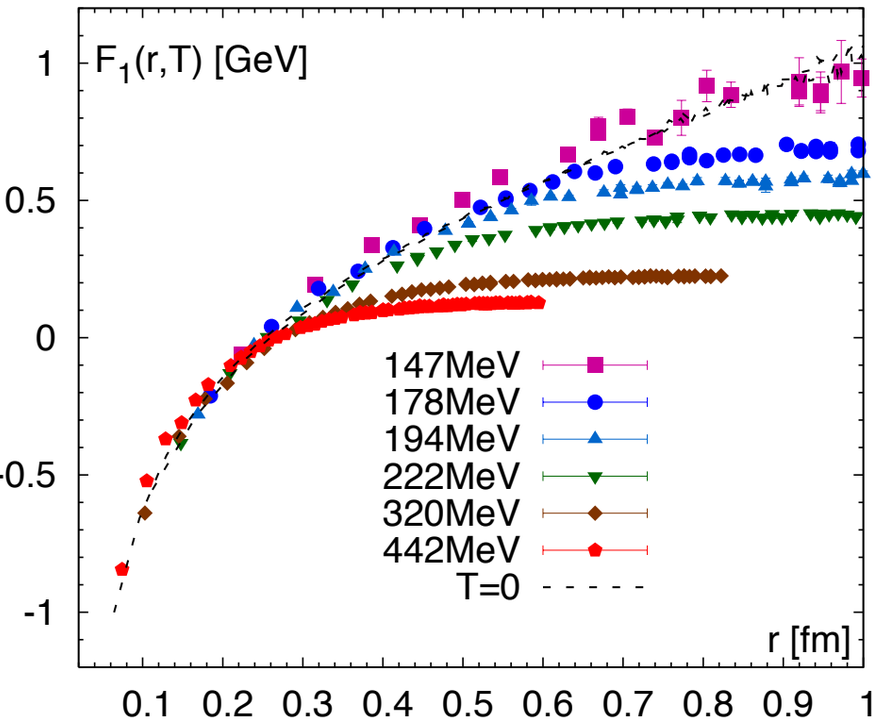
1.Finite baryon chemical potential

- RHIC beam energy scan phase 2
- New experiments at lower energies at FAIR, GSI and NICA, Dubna

- No agreement on the location of the critical point and phases at high density on the lattice
- Progress in the Taylor expansion method, but higher orders are very noisy – high computational cost
- Need more work!

2.Heavy flavor and spectral functions

- Free energy of static quark anti-quark pair – screening in the plasma

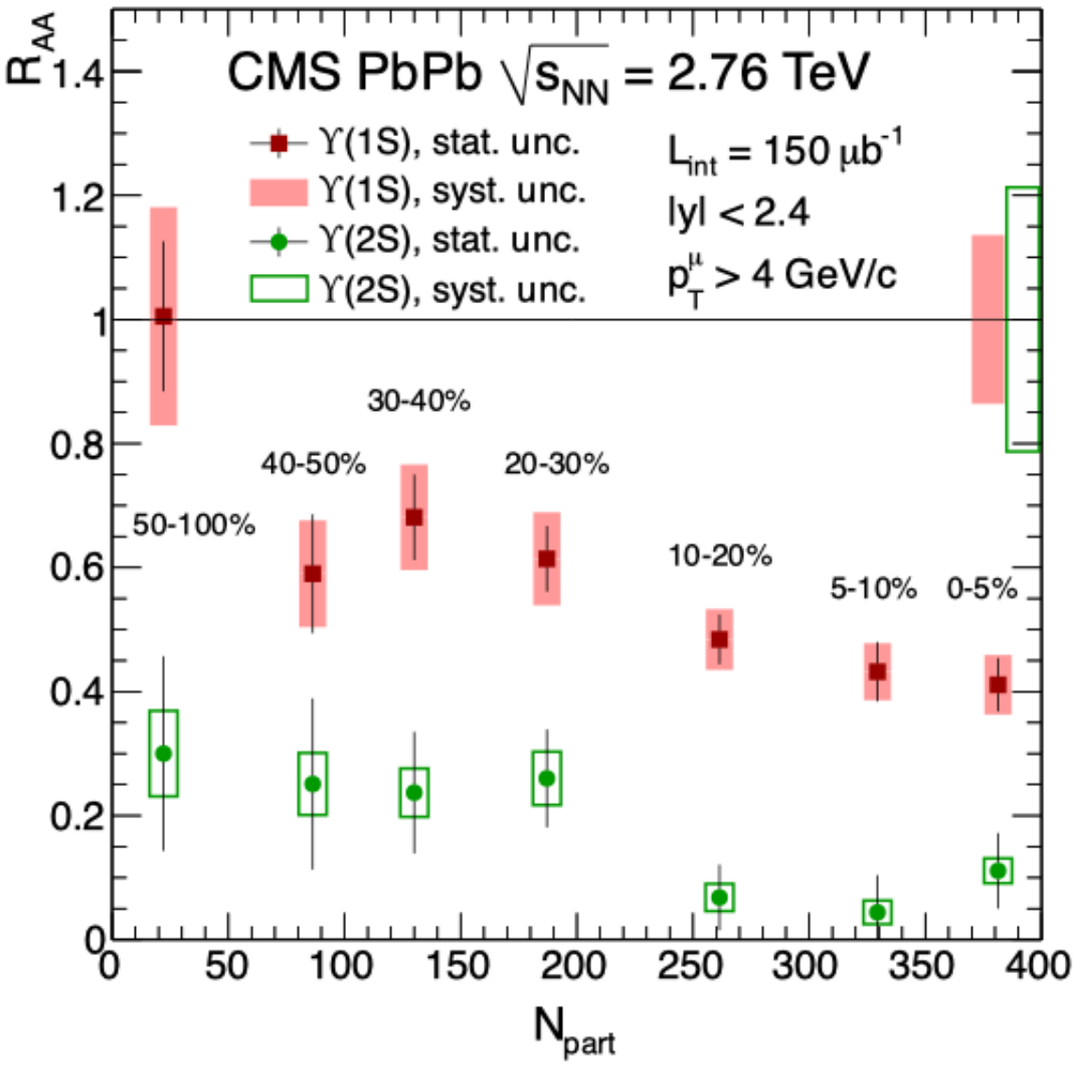


Bazavov and Petreczky (2012)

- Suppression of heavy quarkonia states due to screening was suggested as a signature of QGP

Matsui and Satz (1986)

2.Heavy flavor and spectral functions



$$R_{AA} = \frac{\mathcal{L}_{pp}}{T_{AA} N_{\mathrm{MB}}} \frac{\Upsilon(nS)|_{\mathrm{PbPb}}}{\Upsilon(nS)|_{pp}} \frac{\varepsilon_{pp}}{\varepsilon_{\mathrm{PbPb}}}$$

$$R_{AA}(\Upsilon(1S)) = 0.56 \pm 0.08(\mathrm{stat}) \pm 0.07(\mathrm{syst}),$$

$$R_{AA}(\Upsilon(2S)) = 0.12 \pm 0.04(\mathrm{stat}) \pm 0.02(\mathrm{syst}),$$

$$R_{AA}(\Upsilon(3S)) = 0.03 \pm 0.04(\mathrm{stat}) \pm 0.01(\mathrm{syst})$$

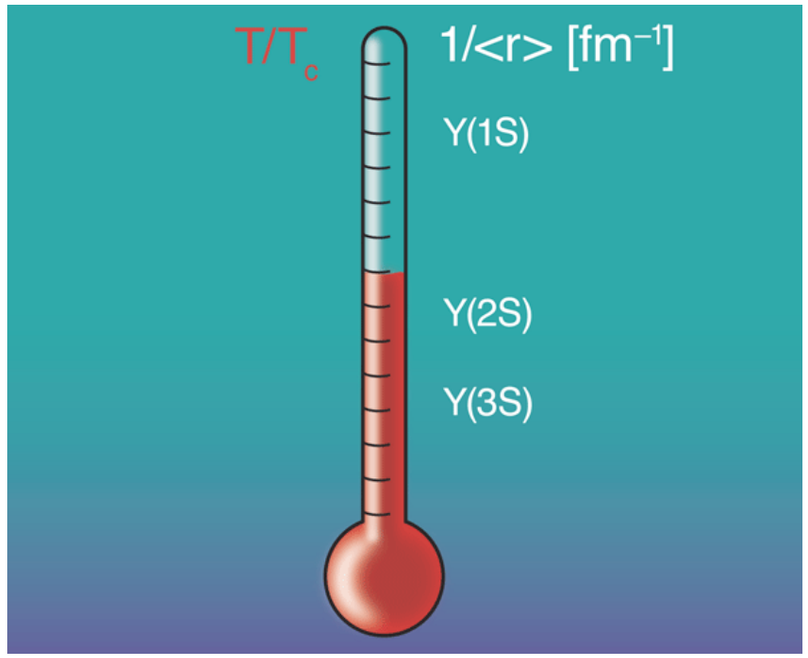
$$< 0.10(95\%\mathrm{CL}).$$

CMS, PRL109 (2012)

- Suppression of heavy states (e.g. Upsilon) is indeed observed!

2.Heavy flavor and spectral functions

- Heavy probes can serve as a thermometer for the quark-gluon plasma



Vogt (2012)

- Heavy flavor measurements are planned with the RHIC detector upgrades and at CERN
- Lattice needs to catch up!

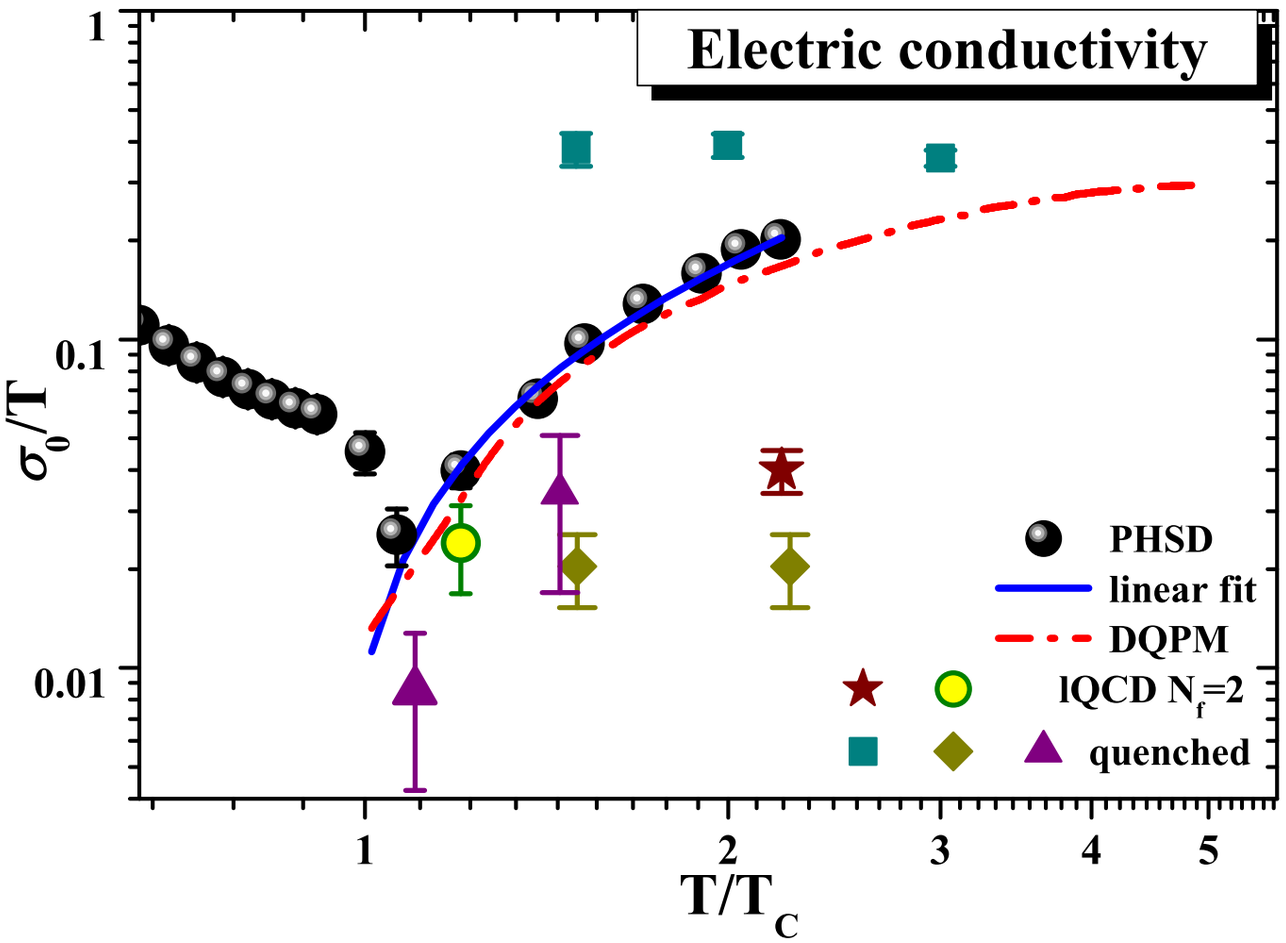
2.Heavy flavor and spectral functions

$$\sigma(\omega,p,T)=\frac{1}{2\pi}\mathrm{Im}\int_{-\infty}^{\infty}dte^{i\omega t}\int d^3xe^{ipx}\left\langle[J(x,t),J(x,0)]\right\rangle_T$$

- All information is encoded in spectral functions
- In principle, can be extracted from Euclidean correlators
- In practice, ill-defined inverse problem
- Bayesian methods are often used
- Need new methods to use all available data sets (different momenta, lattice cutoffs, etc.) to constrain the spectral function

3.Perfect fluid and transport properties

- To extract the transport coefficients one also needs to reconstruct spectral functions
- No reliable estimates of viscosity on the lattice yet



- Electric conductivity is reasonable

Cassing et al, PRL110 (2013)

Conclusion

- Finite-temperature lattice QCD can now provide quantitative answers for the heavy-ion physics
- The transition at zero baryon density is a crossover at temperature 156.5(1.5) MeV
- Fluctuations and higher-order cumulants are sensitive probes of deconfinement
- 2+1 flavor QCD equation of state is now known at the physical masses in the continuum limit
- The Taylor expansion method can reach up to the sixth order for the equation of state
- Future challenges: non-zero chemical potential, spectral functions, transport properties, ...

Thank you!