Semi-Inclusive Jet Functions and small-R resummation in SCET

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Outline

- Inclusive Jet Production
- The Jet Fragmentation Function
- Conclusions

Kang, FR, Vitev - in preparation

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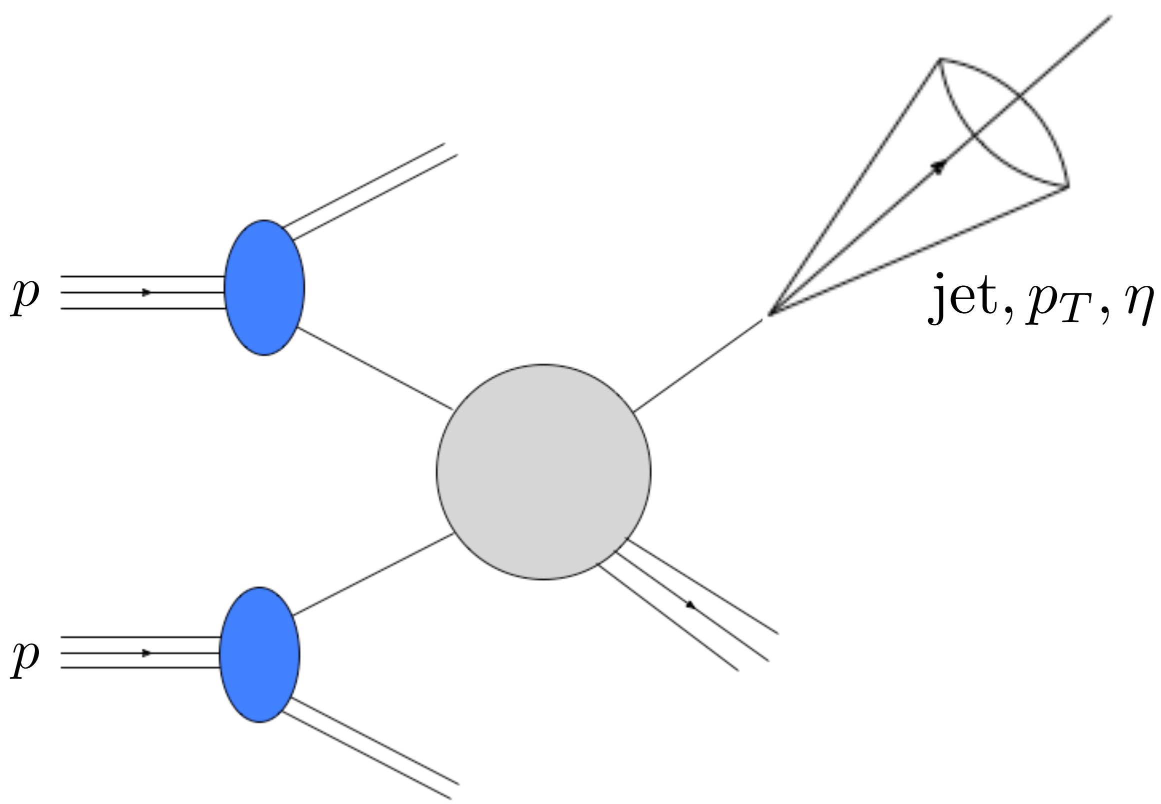
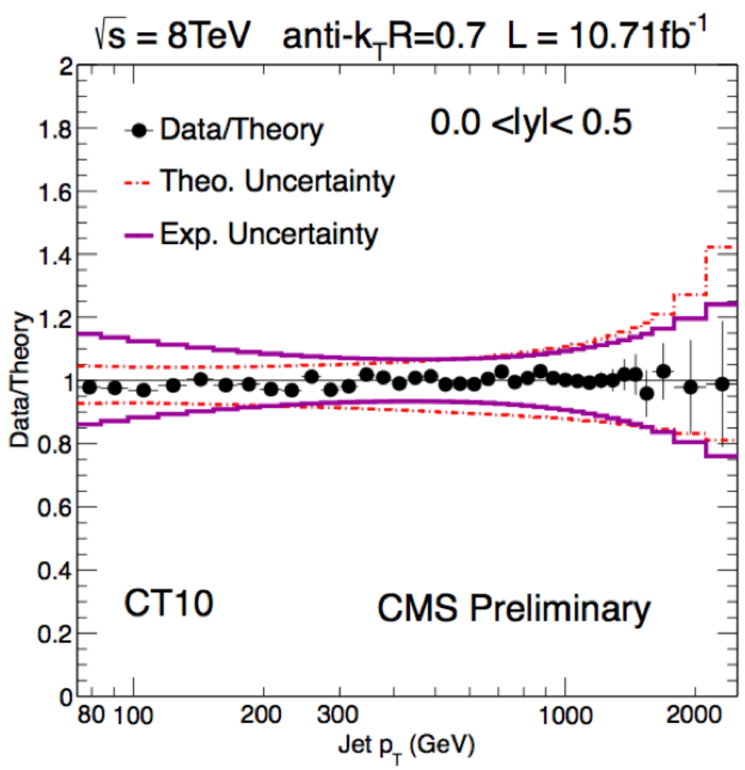
- Conclusions

Kang, FR, Vitev - in preparation

Inclusive Jet Production $pp \to \text{jet}X$

- Large theoretical uncertainties especially at high p_T
- PDFs are constrained by collider jet data, especially $g(x), \Delta g(x)$
- Determination of α_s
- High p_T jets are a promising observable for the search of BSM physics at the LHC
- Jet quenching studies in heavy-ion collisions
- …

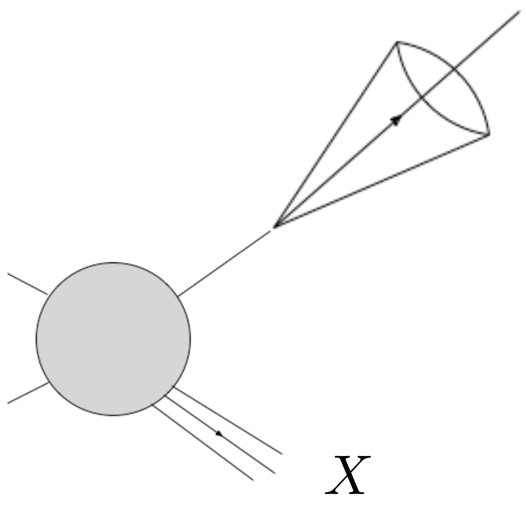




Inclusive Jet Production $pp \to \mathrm{jet}X$

Earlier work in standard pQCD (MC or NJA)

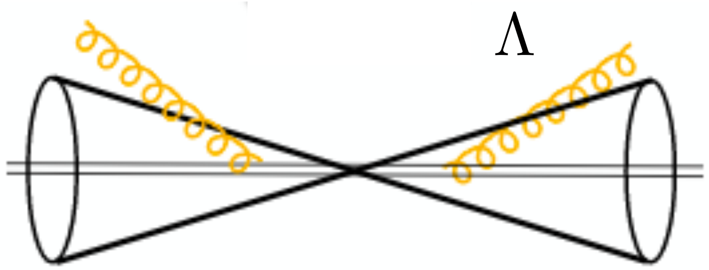
Ellis, Kunszt, Soper `90, Aversa, Chiappetta, Greco, Guillet `89,
Jäger, Stratmann, Vogelsang `04,
Mukherjee, Vogelsang `12,
Currie, Gehrmann-De Ridder, Glover, Pires `14,
de Florian, Hinderer, Mukherjee, FR, Vogelsang `14
Dasgupta, Dreyer, Salam, Soyez `15, `16
...



$\sim (\alpha_s \ln R)^n$

Earlier work within SCET for exclusive jet production

Ellis, Vermilion, Walsh, Hornig, Lee `10,
Chien, Hornig, Lee `15
Becher, Neubert, Rothen, Shao `16
...



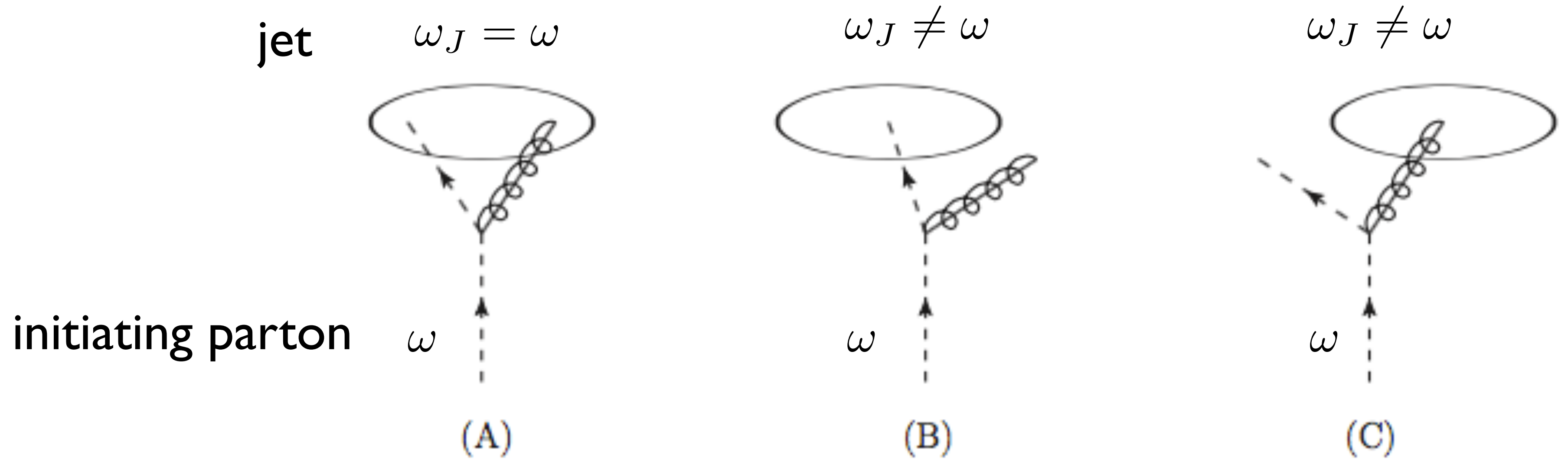
$\sim (\alpha_s \ln^2 R)^n$

Semi-inclusive jet function

Operator definition - quark jet function:

$$\langle 0|\chi_{n,\omega}^{a\alpha}(x)|X\rangle\langle X|\bar{\chi}_{n,\omega}^{b\beta}(0)|0\rangle = \delta^{ab}\left(\frac{\not{n}}{2}\right)^{\alpha\beta}\int\frac{dz}{z}\delta\left(z-\frac{\bar{n}\cdot p_J}{\omega}\right)J_q(z,\omega_J)$$

where $z = \omega_J/\omega$



Semi-inclusive jet function

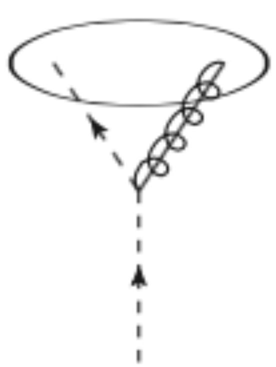
Leading order $$J_q^{(0)}(z, \omega_J) = \delta(1-z)$$

Semi-inclusive jet function

Leading order $J_q^{(0)}(z, \omega_J) = \delta(1-z)$

Next-to-leading order

(A) (B) (C) (D)



(A)

$$J_q(z, \omega_J) = \delta(1-z)\frac{\alpha_s}{\pi}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon}\int_0^1 dx \hat{P}_{qq}(x,\epsilon)\int \frac{dq_\perp}{q_\perp^{1+2\epsilon}}\Theta_{\rm alg}$$

where: anti-k$_{\rm T}$: $\Theta_{\text{anti-k}_{\rm T}} = \theta\left(x(1-x)\omega_J \tan\frac{R}{2} - q_\perp\right)$

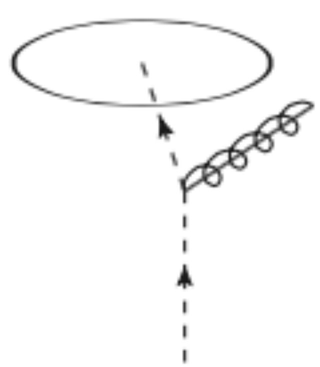
$$\hat{P}_{qq}(x,\epsilon) = C_F\left[\frac{1+x^2}{1-x} - \epsilon(1-x)\right]$$

$$x = \frac{\ell^- - q^-}{\ell^-}$$

Semi-inclusive jet function

Leading order $$J_q^{(0)}(z,\omega_J) = \delta(1-z)$$

Next-to-leading order

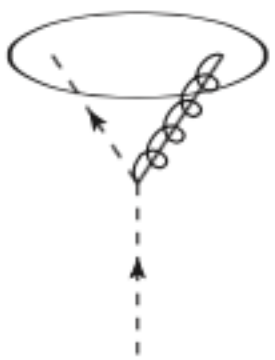


$$J_{q\to q(g)}(z,\omega_J) = \frac{\alpha_s}{\pi}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon}\hat{P}_{qq}(z,\epsilon)\int\frac{dq_\perp}{q_\perp^{1+2\epsilon}}\Theta_{\text{alg}}$$

where: $$\Theta_{\text{anti-k}_\text{T}} = \theta\left(q_\perp - (1-z)\omega_J \tan\frac{R}{2}\right)$$

Semi-inclusive jet function

$\overline{\text{MS}}$ scheme, anti-k_T


(A)

$$J_q(z,\omega_J) = \delta(1-z)\frac{\alpha_s}{2\pi}C_F\left[\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}+\frac{1}{\epsilon}L+\frac{1}{2}L^2+\frac{3}{2}L+\frac{13}{2}-\frac{3\pi^2}{4}\right]$$

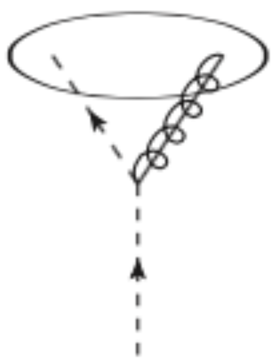
Essentially the same result as in the exclusive case

Ellis, Vermilion, Walsh, Hornig, Lee `10

where $L = \ln\left(\frac{\mu^2}{\omega_J^2\tan^2(R/2)}\right)$

Semi-inclusive jet function

$\overline{\mathrm{MS}}$ scheme, anti-$\mathrm{k_T}$



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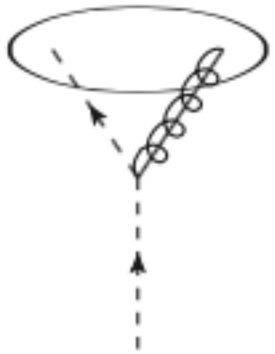
Exclusive cross section: double logarithmic dependence $\ln^2 R$

multiplicative renormalization

Semi-inclusive jet function

$\overline{\text{MS}}$ scheme, anti-k_T



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$$J_q(z,\omega_J) = \delta(1-z)\frac{\alpha_s}{2\pi}C_F\left[\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}+\frac{1}{\epsilon}L+\frac{1}{2}L^2+\frac{3}{2}L+\frac{13}{2}-\frac{3\pi^2}{4}\right]$$

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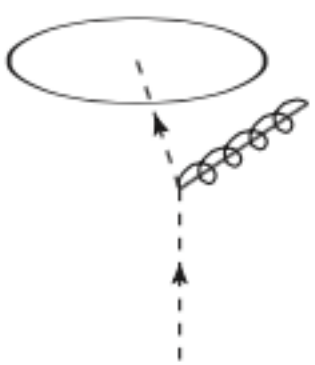


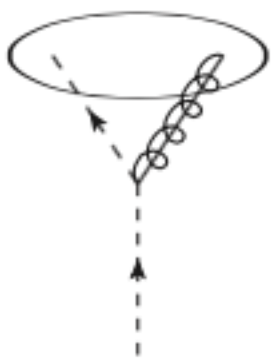
(B)

$$J_q(z,\omega_J) = \frac{\alpha_s}{2\pi}\delta(1-z)\left[-\frac{1}{\epsilon^2}-\frac{1}{\epsilon}L-\frac{1}{2}L^2+\frac{\pi^2}{12}\right] + \frac{\alpha_s}{2\pi}\left[\left(\frac{1}{\epsilon}+L\right)\frac{1+z^2}{(1-z)_+}-2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+-(1-z)\right]$$

(C) …

Semi-inclusive jet function

$\overline{\text{MS}}$ scheme, anti-k_T



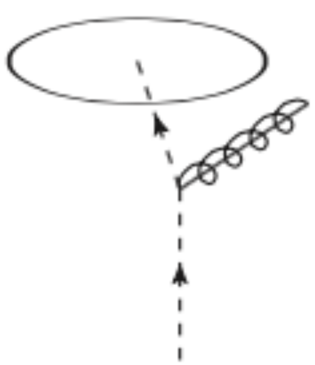
(A)

$$J_q(z,\omega_J) = \delta(1-z)\frac{\alpha_s}{2\pi}C_F\left[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon}L + \frac{1}{2}L^2 + \frac{3}{2}L + \frac{13}{2} - \frac{3\pi^2}{4}\right]$$

Essentially the same result as in the exclusive case

Ellis, Vermilion, Walsh, Hornig, Lee `10

where $L = \ln\left(\frac{\mu^2}{\omega_J^2 \tan^2(R/2)}\right)$



(B)

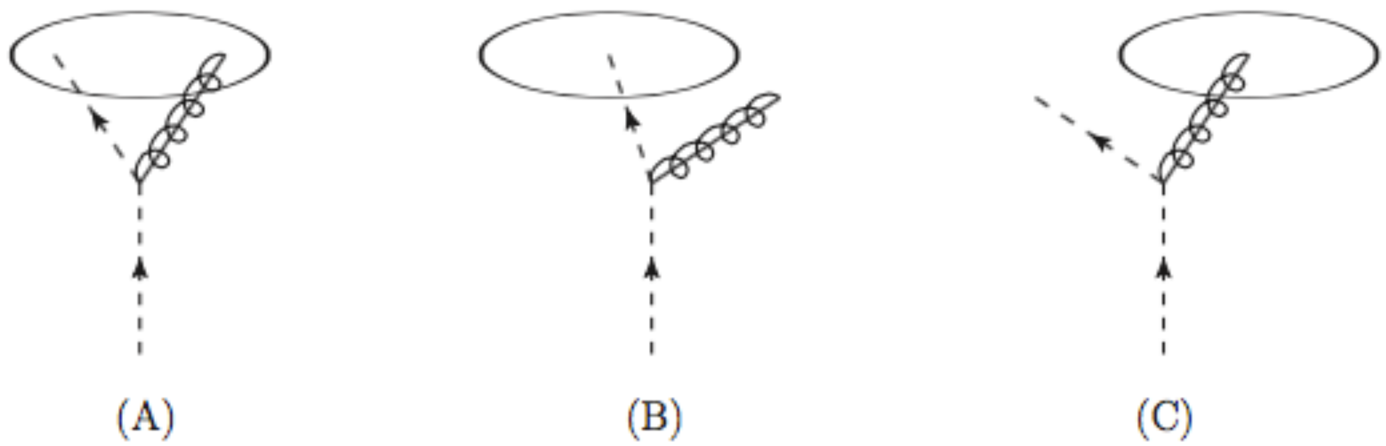
$$J_q(z,\omega_J) = \frac{\alpha_s}{2\pi}\delta(1-z)\left[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon}L - \frac{1}{2}L^2 + \frac{\pi^2}{12}\right] + \frac{\alpha_s}{2\pi}\left[\left(\frac{1}{\epsilon}+L\right)\frac{1+z^2}{(1-z)_+} - 2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - (1-z)\right]$$

(C) … ⟶ only a single logarithmic $\ln R$ remains

Semi-inclusive jet function

$\overline{\text{MS}}$ scheme, anti-$\mathrm{k_T}$

$$J_q^{(1)}(z,\omega_J) = J_{q\to qg}(z,\omega_J) + J_{q\to q(g)}(z,\omega_J) + J_{q\to (q)g}(z,\omega_J)$$



Semi-inclusive jet function

$\overline{\mathrm{MS}}$ scheme, anti-$\mathrm{k_T}$

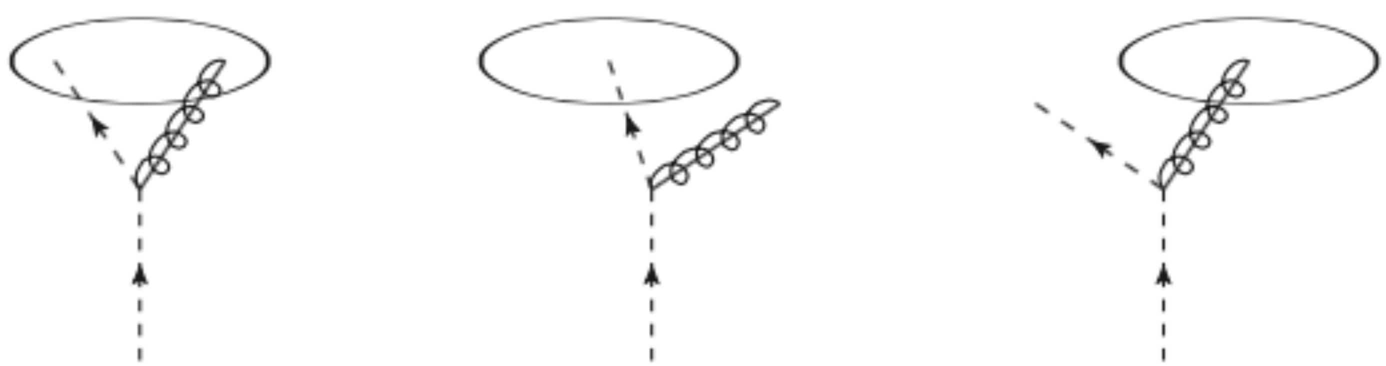
$$J_q^{(1)}(z,\omega_J) = J_{q\to qg}(z,\omega_J) + J_{q\to q(g)}(z,\omega_J) + J_{q\to (q)g}(z,\omega_J)$$

$$= \frac{\alpha_s}{2\pi}\left(\frac{1}{\epsilon}+L\right)\left[P_{qq}(z)+P_{gq}(z)\right]$$

$$- \frac{\alpha_s}{2\pi}\left\{C_F\left[2\left(1+z^2\right)\left(\frac{\ln(1-z)}{1-z}\right)_+ + (1-z)\right] - \delta(1-z)d_J^{q,\mathrm{alg}}\right.$$

$$\left. + P_{gq}(z)2\ln(1-z) + C_F z\right\}$$

where $d_J^{q,\mathrm{anti\text{-}k_T}} = C_F\left(\frac{13}{2}-\frac{2\pi^2}{3}\right)$



(A) (B) (C)

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Renormalization and RG evolution

Bare - renormalized semi-inclusive jet function

$$J_{i,\text{bare}}(z,\omega_J) = \sum_j \int_z^1 \frac{dz'}{z'} Z_{ij}\left(\frac{z}{z'},\mu\right) J_j(z',\omega_J,\mu)$$

RG equation

$$\mu\frac{d}{d\mu}J_i(z,\omega_J,\mu) = \sum_j \int_z^1 \frac{dz'}{z'} \gamma_{ij}^J\left(\frac{z}{z'},\mu\right) J_j(z',\omega_J,\mu)$$

Anomalous dimension

$$\gamma_{ij}^J(z,\mu) = -\sum_k \int_z^1 \frac{dz'}{z'} (Z)^{-1}_{ik}\left(\frac{z}{z'},\mu\right) \mu\frac{d}{d\mu} Z_{kj}(z',\mu)$$

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Renormalization and RG evolution

We find

$$\gamma_{ij}^{J}(z,\mu) = \frac{\alpha_s(\mu)}{\pi} P_{ji}(z)$$

$\longrightarrow$

$$\mu \frac{d}{d\mu} J_i(z,\omega_J,\mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'},\mu\right) J_j(z',\omega_J,\mu)$$

DGLAP evolution equation like for FFs. Resums single $\ln R$: LL_R, NLL_R

see also
Dasgupta, Dreyer, Salam, Soyez `15, `16

Renormalization and RG evolution

$\overline{\mathrm{MS}}$ scheme, $\mathrm{anti\text{-}k_T}$

$$J_q^{(1)}(z,\omega_J) = J_{q\to qg}(z,\omega_J) + J_{q\to q(g)}(z,\omega_J) + J_{q\to (q)g}(z,\omega_J)$$

$$= \frac{\alpha_s}{2\pi} L \left[P_{qq}(z) + P_{gq}(z)\right]$$

$$- \frac{\alpha_s}{2\pi}\left\{C_F\left[2\left(1+z^2\right)\left(\frac{\ln(1-z)}{1-z}\right)_+ + (1-z)\right] - \delta(1-z)d_J^{q,\mathrm{alg}}\right.$$

$$\left. + P_{gq}(z) 2\ln(1-z) + C_F z\right\}$$

where $d_J^{q,\mathrm{anti\text{-}k_T}} = C_F\left(\frac{13}{2} - \frac{2\pi^2}{3}\right)$ $L = \ln\left(\frac{\mu^2}{\omega_J^2 \tan^2(R/2)}\right)$ $\mu = \mu_J \sim \omega_J \tan(R/2)$

→ Full agreement with standard pQCD result

Mukherjee, Vogelsang `12
Kaufmann, Mukherjee, Vogelsang `15

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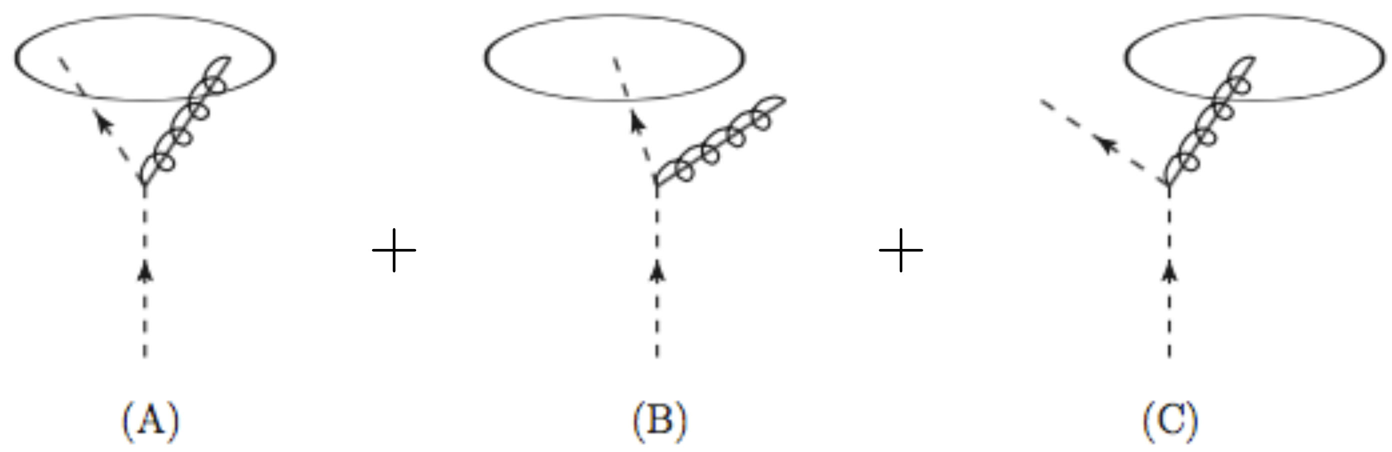
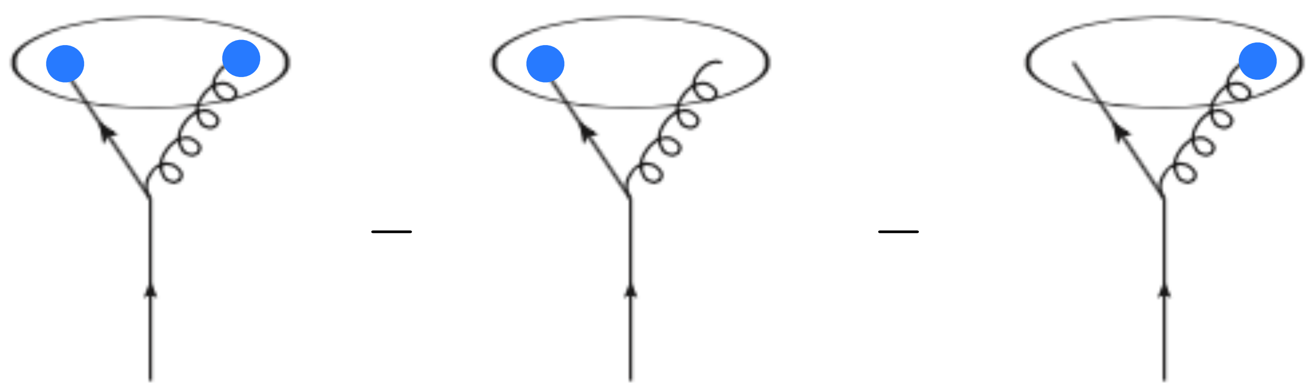
Renormalization and RG evolution

SCET:



pQCD:



NJA: using a collinear splitting

Matching for $pp \to \text{jet}X$

Cross section

$$\frac{d\sigma^{pp\to\text{jet}X}}{dp_T d\eta} = \frac{2p_T}{s}\sum_{a,b,c}\int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a,\mu)\int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b,\mu)\int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} H_{ab}^c(\hat{s},\hat{p}_T,\hat{\eta},\mu)\, J_c(z_c,\omega_J,\mu)$$

PDFs — hard functions — evolved jet functions

- Hard functions are the same for both $pp \to \text{jet}X$ and $pp \to hX$
- Need separate evolution for $J_c^{(0)},\ J_c^{(1)}$:

$$\left(H_{ab}^{c,(0)} + H_{ab}^{c,(1)}\right)\left(J_c^{(0)} + J_c^{(1)}\right) = \left(H_{ab}^{c,(0)} + H_{ab}^{c,(1)}\right) J_c^{(0)} + H_{ab}^{c,(0)} J_c^{(1)} + \mathcal{O}(\alpha_s^2)$$

- Depending on the accuracy of the evolution, we can do $\text{NLO} + \text{LL}_R$

  $\text{NLO} + \text{NLL}_R$

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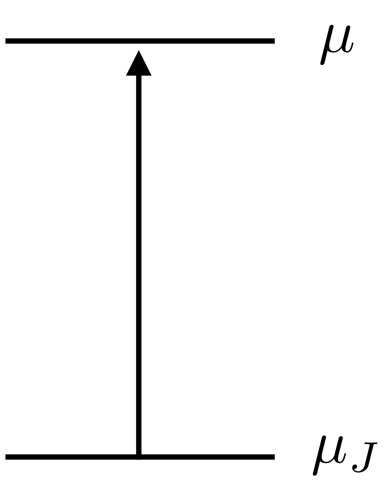
Jet function evolution

$$\frac{d}{d\log\mu^2}\begin{pmatrix} J_S(z,\omega_J,\mu) \\ J_g(z,\omega_J,\mu)\end{pmatrix} = \frac{\alpha_s(\mu)}{2\pi}\begin{pmatrix} P_{qq}(z) & 2N_f P_{gq}(z) \\ P_{qg}(z) & P_{gg}(z)\end{pmatrix} \otimes \begin{pmatrix} J_S(z,\omega_J,\mu) \\ J_g(z,\omega_J,\mu)\end{pmatrix}$$



initial condition contains distributions in $1-z$

where $J_S(z,\omega_J,\mu) = \sum_{q,\bar{q}} J_q(z,\omega_J,\mu) = 2N_f J_q(z,\omega_J,\mu)$ (singlet jet function)

Jet function evolution

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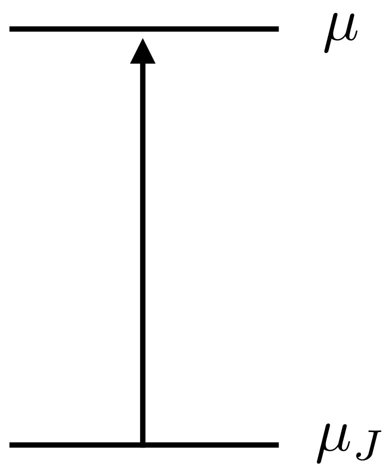

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where $$J_S(z,\omega_J,\mu) = \sum_{q,\bar{q}} J_q(z,\omega_J,\mu) = 2N_f J_q(z,\omega_J,\mu)$$ (singlet jet function)

solve in Mellin space:

$$f(N) = \int_0^1 dz\, z^{N-1} f(z)$$

$$(f\otimes g)(N) = f(N)\,g(N)$$

Jet function evolution

solve in Mellin space to LL_R

$$\begin{pmatrix} J_S(N,\omega_J,\mu) \\ J_g(N,\omega_J,\mu) \end{pmatrix} = \left[e_+(N) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_J)}\right)^{-r_-(N)} + e_-(N) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_J)}\right)^{-r_+(N)} \right] \begin{pmatrix} J_S(N,\omega_J,\mu_J) \\ J_g(N,\omega_J,\mu_J) \end{pmatrix}$$

where $$e_\pm(N) = \frac{1}{r_\pm(N) - r_\mp(N)} \begin{pmatrix} P_{qq}(N) - r_\mp(N) & 2N_f P_{gq}(N) \\ P_{qg}(N) & P_{gg}(N) - r_\mp(N) \end{pmatrix}$$

$$r_\pm(N) = \frac{1}{2\beta_0} \left[P_{qq}(N) + P_{gg}(N) \pm \sqrt{(P_{qq}(N) - P_{gg}(N))^2 + 4P_{qg}(N)P_{gq}(N)} \right]$$

see
Vogt \`04 (Pegasus),
Anderle, FR, Stratmann \`15

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Jet function evolution

solve in Mellin space to LL_R

$$\begin{pmatrix} J_S(N,\omega_J,\mu) \\ J_g(N,\omega_J,\mu) \end{pmatrix} = \left[e_+(N) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_J)} \right)^{-r_-(N)} + e_-(N) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_J)} \right)^{-r_+(N)} \right] \begin{pmatrix} J_S(N,\omega_J,\mu_J) \\ J_g(N,\omega_J,\mu_J) \end{pmatrix}$$

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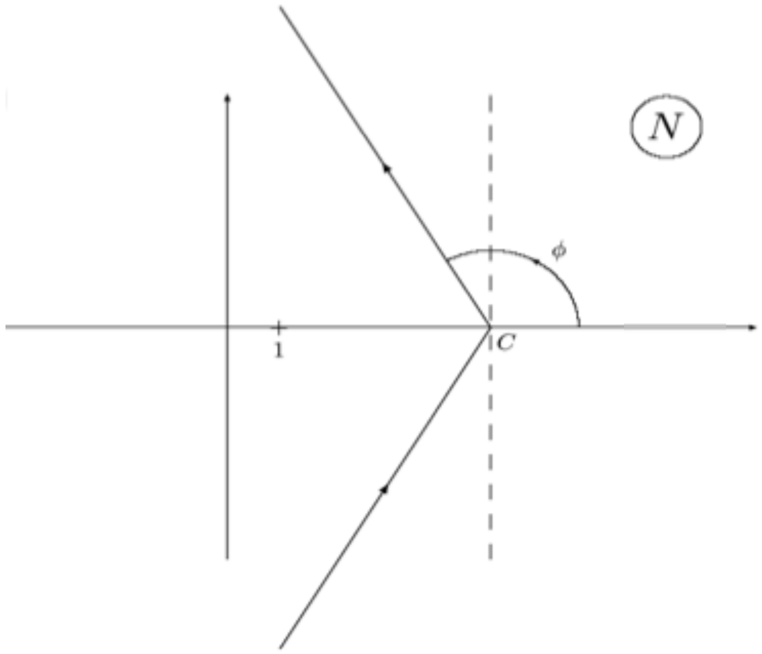
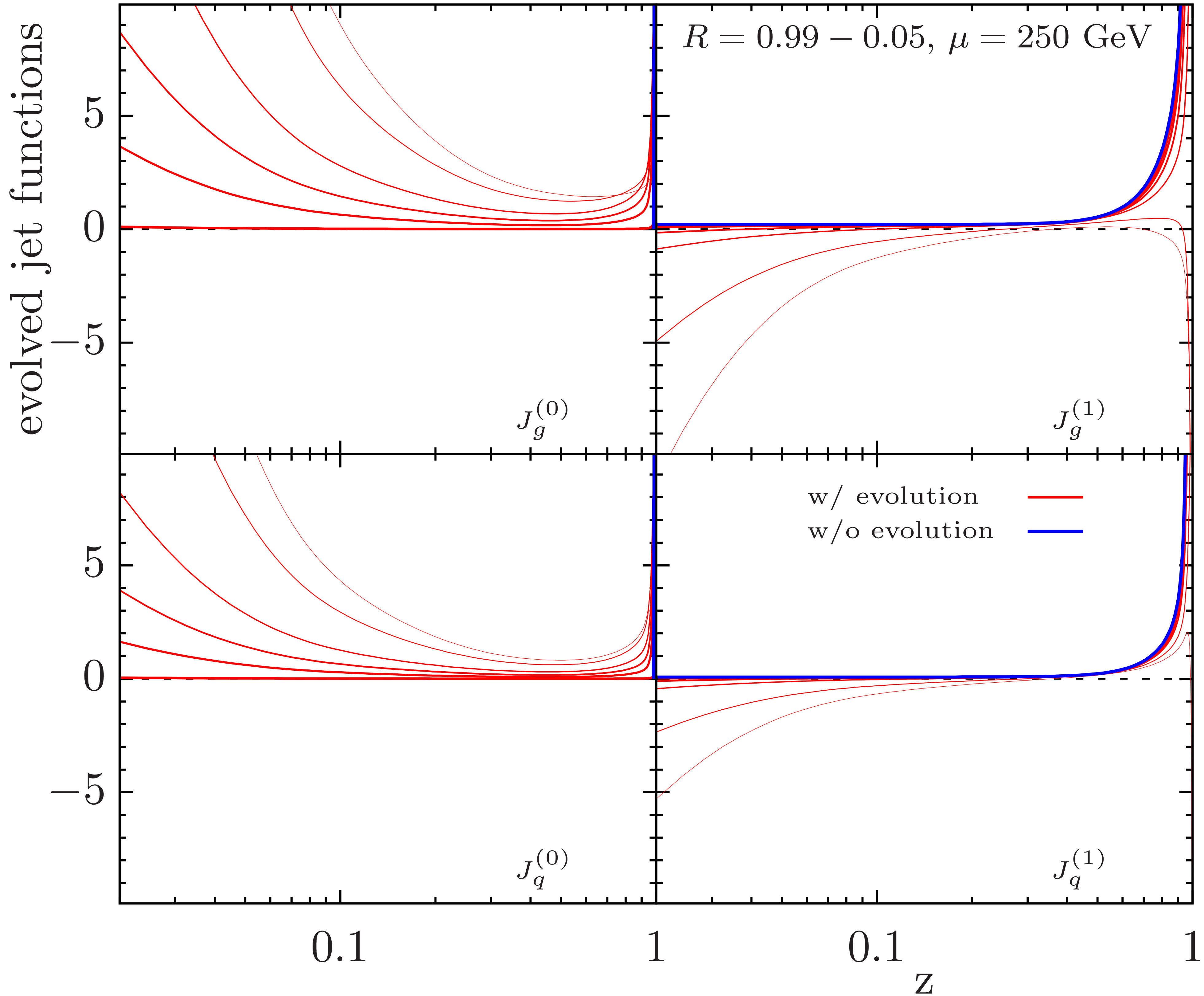
see
Vogt `04 (Pegasus),
Anderle, FR, Stratmann `15

$$r_\pm(N) = \frac{1}{2\beta_0} \left[P_{qq}(N) + P_{gg}(N) \pm \sqrt{(P_{qq}(N) - P_{gg}(N))^2 + 4P_{qg}(N)P_{gq}(N)} \right]$$

Mellin inverse

$$J_{S,g}(z,\omega_J,\mu) = \frac{1}{2\pi i} \int_{\mathcal{C}_N} dN\, z^{-N} J_{S,g}(N,\omega_J,\mu)$$





LL_R DGLAP evolution

see Vogt `04 (Pegasus), Anderle, FR, Stratmann `15

Jet function evolution

- PDFs, FFs are $\sim (1-z)^{\alpha} \to 0$ for $z \to 1$
- Evolved jet functions are divergent for $z \to 1$

$\longrightarrow$ Adopt a prescription used for quarkonium fragmentation functions

Bodwin, Chao, Chung, Kim, Lee, Ma `16

Jet function evolution

Bodwin, Chao, Chung, Kim, Lee, Ma `16

Introduce a cut off ε :

$$\int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_c(z_c)}{dvdz} J_c(z_c) = \int_{z_c^{\min}}^{1-\varepsilon} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_c(z_c)}{dvdz} J_c(z_c) + \int_{1-\varepsilon}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_c(z_c)}{dvdz} J_c(z_c)$$

Jet function evolution

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rewrite the 2nd term:

$$\int_{1-\varepsilon}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_c(z_c)}{dvdz} J_c(z_c) = \int_{1-\varepsilon}^{1} \frac{dz_c}{z_c^2} \left[\frac{d\hat{\sigma}_c(z_c)}{dvdz} z_c^{-N} \right] \left[z_c^N J_c(z_c) \right]$$

$$= \left[\frac{d\hat{\sigma}_c(z_c)}{dvdz} \right]_{z_c=1} \times \left[\int_0^1 dz_c\, z_c^{N-2} J_c(z_c) - \int_0^{1-\varepsilon} dz_c\, z_c^{N-2} J_c(z_c) \right]$$

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Jet function evolution

Bodwin, Chao, Chung, Kim, Lee, Ma `16

Introduce a cut off ε :

$$\int_{z_c^{\min}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_c(z_c)}{dvdz} J_c(z_c) = \int_{z_c^{\min}}^{1-\varepsilon} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_c(z_c)}{dvdz} J_c(z_c) + \int_{1-\varepsilon}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_c(z_c)}{dvdz} J_c(z_c)$$

rewrite the 2nd term:

$$\int_{1-\varepsilon}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_c(z_c)}{dvdz} J_c(z_c) = \int_{1-\varepsilon}^{1} \frac{dz_c}{z_c^2} \left[\frac{d\hat{\sigma}_c(z_c)}{dvdz} z_c^{-N}\right] \left[z_c^N J_c(z_c)\right]$$

$$= \left[\frac{d\hat{\sigma}_c(z_c)}{dvdz}\right]_{z_c=1} \times \left[\int_0^1 dz_c\, z_c^{N-2} J_c(z_c) - \int_0^{1-\varepsilon} dz_c\, z_c^{N-2} J_c(z_c)\right]$$

1. Check that calculation is independent of ε and N
2. Check that the calculation agrees with NLO for $R \to 1$

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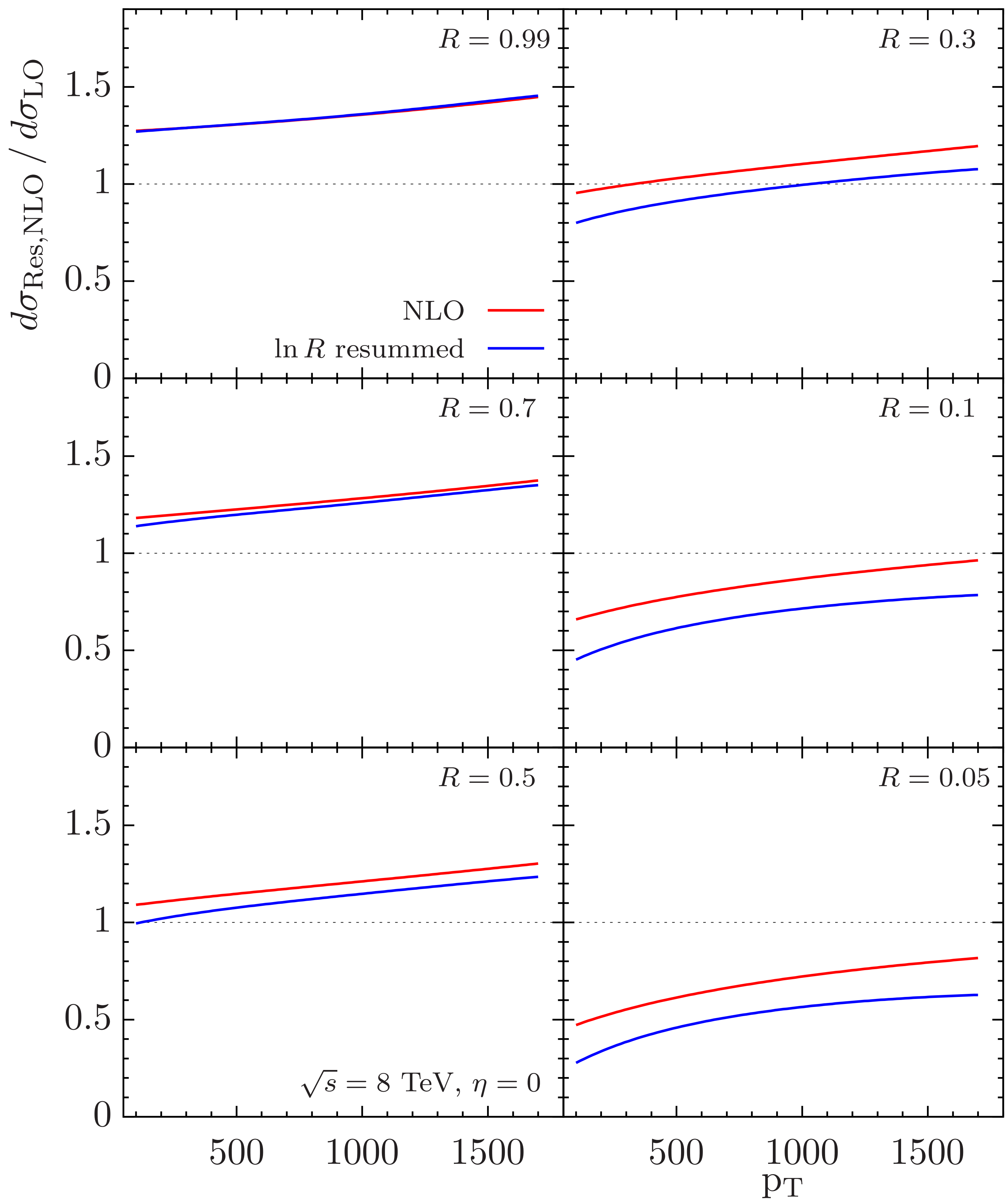
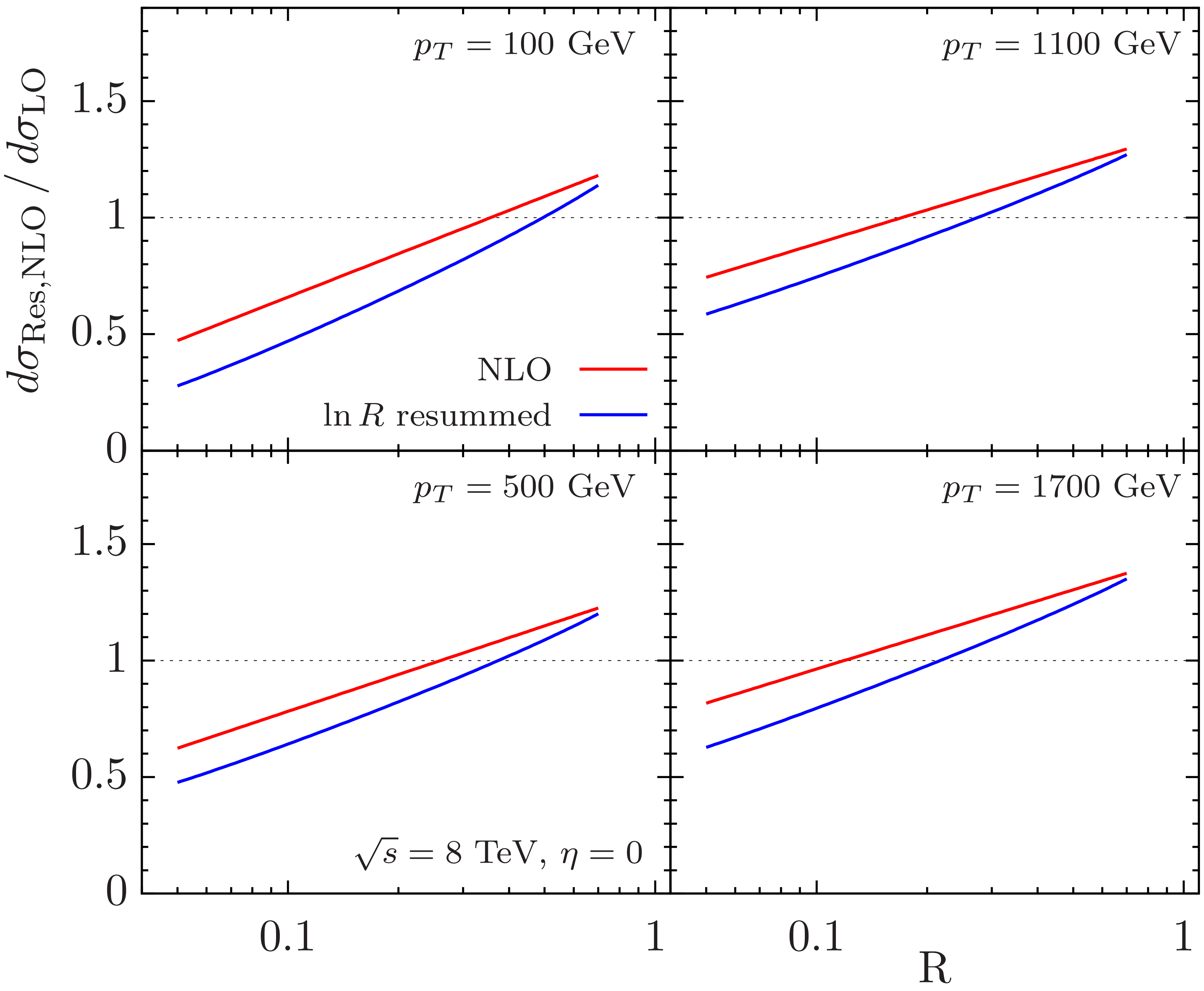


LL_R DGLAP evolution

see also
Dasgupta, Dreyer, Salam, Soyez `15, `16



LL_R DGLAP evolution

Outline

- Inclusive Jet Production
- The Jet Fragmentation Function
- Conclusions

Kang, FR, Vitev - in preparation

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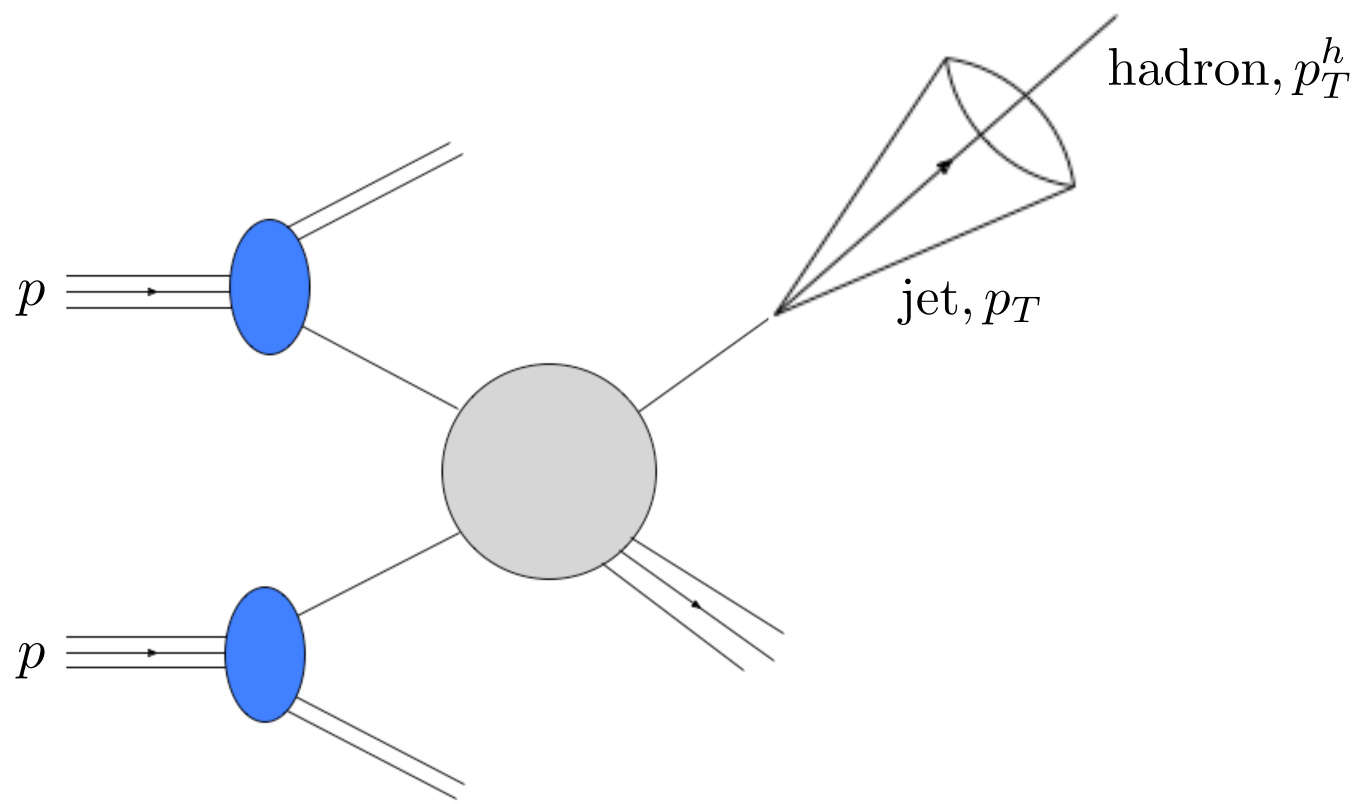
Jet fragmentation function $pp \to (\text{jet}h)X$

- Jet substructure observable studying the distribution of hadrons inside a jet

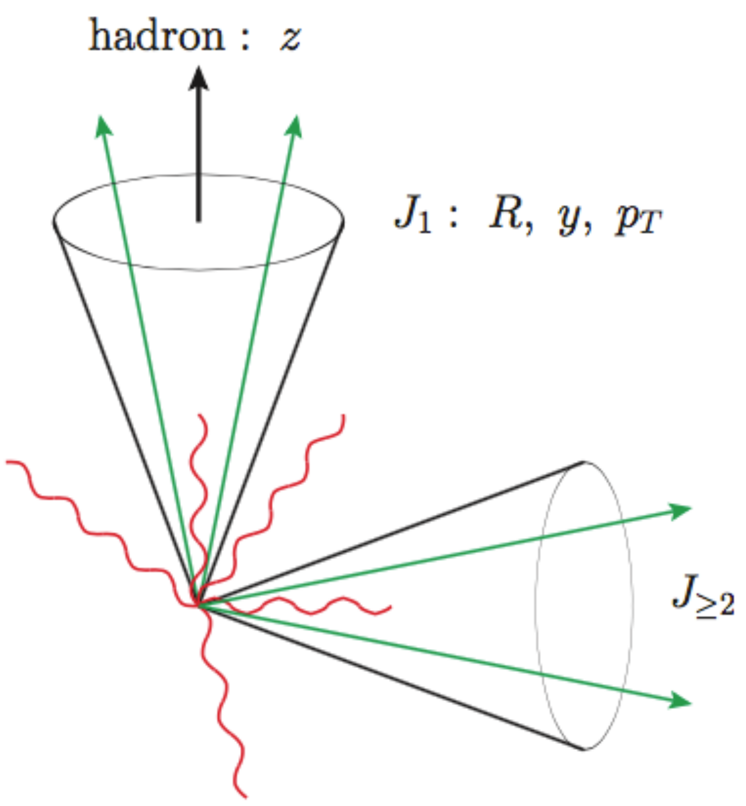
- Provides further constraints for fits of fragmentation functions
- Possible studies include spin correlations and
- the modification in heavy ion collisions



Jet fragmentation function

Definition: $$F(z, p_T) = \frac{d\sigma^h}{dy dp_T dz} \Big/ \frac{d\sigma}{dy dp_T}$$

where $$z \equiv p_T^h / p_T$$

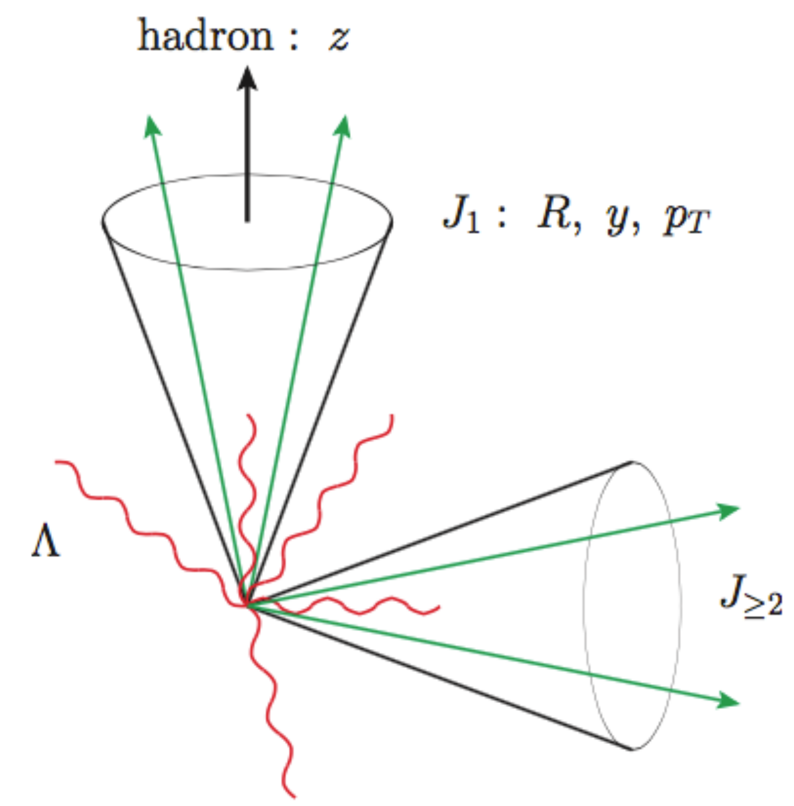


It describes the longitudinal momentum distribution of hadrons inside a reconstructed jet

Jet fragmentation function in pp

- Fragmenting jet function studies within SCET
  Procura, Stewart `10; Liu `11; Jain, Procura, Waalewijn `11 and '12; Procura, Waalewijn `12; Bauer, Mereghetti `14; Baumgart, Leibovich, Mehen, Rothstein `14, Bain, Dai, Hornig, Leibovich, Makris, Mehen `16, Chien, Kang, FR, Vitev, Xing `15, ...

- Jet fragmentation function studies at NLO for pp
  Arleo, Fontannaz, Guillet, Nguyen `14, Kaufmann, Mukherjee, Vogelsang `15



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Jet fragmentation function in pp

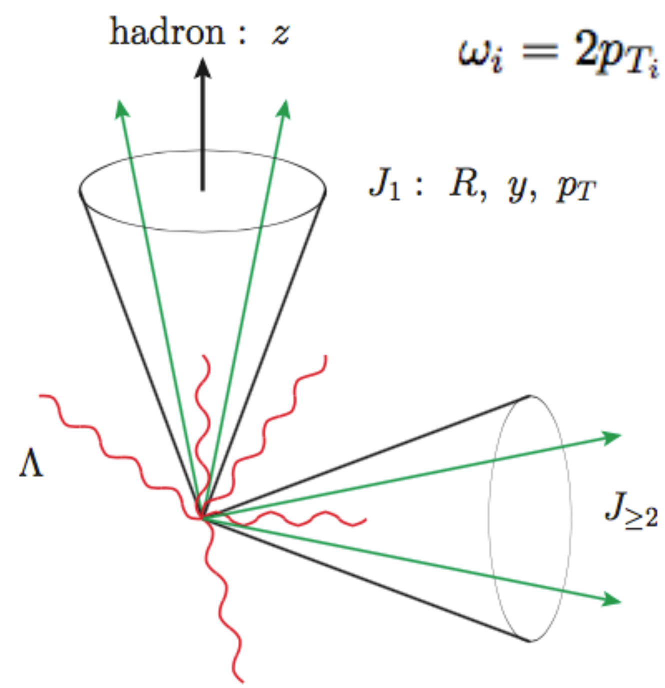
Chien, Kang, FR, Vitev, Xing `15

Exclusive case:

$$\frac{\dfrac{d\sigma^h}{dy_i dp_{T_i} dz}}{\dfrac{d\sigma}{dy_i dp_{T_i}}} = \frac{H(y_i, p_{T_i}, \mu)\, \mathcal{G}^h_{\omega_1}(z,\mu) J_{\omega_2}(\mu) \cdots J_{\omega_N}(\mu) S_{n_1 n_2 \cdots n_N}(\Lambda,\mu)}{H(y_i, p_{T_i}, \mu) J_{\omega_1}(\mu) \cdots J_{\omega_N}(\mu) S_{n_1 n_2 \cdots n_N}(\Lambda,\mu)} = \frac{\mathcal{G}^h_{\omega_1}(z,\mu)}{J_{\omega_1}(\mu)}$$

Li, Li, Yuan `13
Chien, Vitev `15

$$\longrightarrow \quad F(z, p_T) = \frac{1}{\sigma_{\rm tot}} \sum_{i=q,g} \int_{\rm PS} dy\, dp_T' \frac{d\sigma^i}{dy\, dp_T'} \frac{\mathcal{G}_i^h(\omega, R, z, \mu)}{J^i(\omega, R, \mu)} + \mathcal{O}\left(\frac{\Lambda}{Q}\right) + \mathcal{O}(R)$$



Numerically similar but conceptually different to *Kaufmann, Mukherjee, Vogelsang `15*

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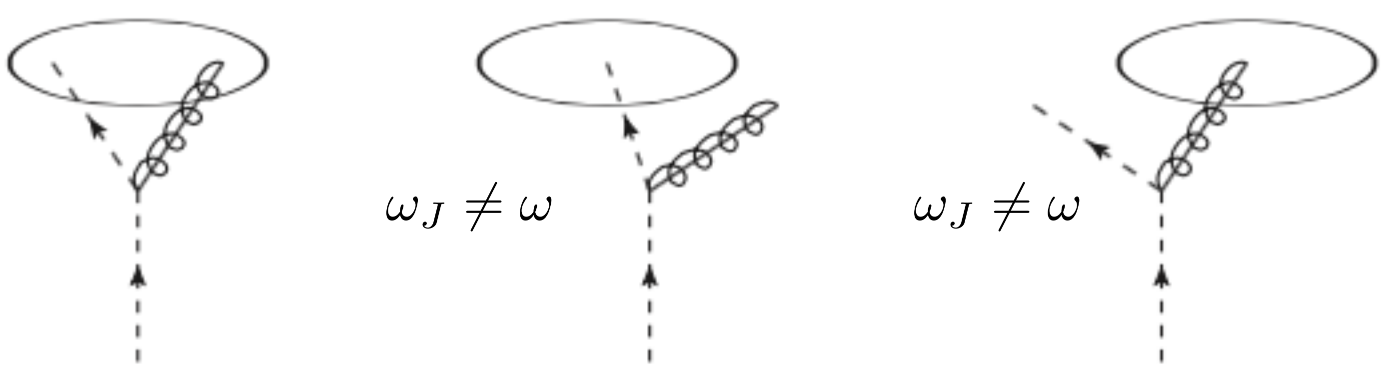
Semi-inclusive fragmenting jet function

$\mathcal{G}_q(z, z_h, \omega_J)$ where $z = \frac{\omega_J}{\omega}\,, \quad z_h = \frac{\omega_h}{\omega_J}$

Leading-order, e.g. $\mathcal{G}_q^{q,(0)}(z, z_h, \omega_J) = \delta(1-z)\delta(1-z_h)$

NLO

- fragmenting parton $\quad \omega_h \neq \omega_J \quad \omega_h = \omega_J \quad \omega_h = \omega_J$



- jet $\quad \omega_J = \omega \quad \omega_J \neq \omega \quad \omega_J \neq \omega$
- initiating parton $\quad \omega \quad \omega \quad \omega$

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Semi-inclusive jet function

quark-quark:

$$\mathcal{G}_{q,\mathrm{bare}}^{q}(z, z_h, \omega_J, \mu) = \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi}\left(-\frac{1}{\epsilon} - L\right) P_{qq}(z_h)\delta(1-z)$$
$$+ \frac{\alpha_s}{2\pi}\left(\frac{1}{\epsilon} + L\right) P_{qq}(z)\delta(1-z_h)$$
$$+ \delta(1-z)\frac{\alpha_s}{2\pi}\left[2C_F(1+z_h^2)\left(\frac{\ln(1-z_h)}{1-z_h}\right)_+ + C_F(1-z_h) + 2P_{qq}(z_h)\ln z_h\right]$$
$$- \delta(1-z_h)\frac{\alpha_s}{2\pi}\left[2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C_F(1-z)\right],$$

$\overline{\mathrm{MS}}$ scheme, $\mathrm{anti}{-}k_T$

Semi-inclusive jet function

quark-quark:

IR

UV

$$\mathcal{G}_{q,\text{bare}}^{q}(z, z_h, \omega_J, \mu) = \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi}\left(-\frac{1}{\epsilon} - L\right)P_{qq}(z_h)\delta(1-z)$$
$$+ \frac{\alpha_s}{2\pi}\left(\frac{1}{\epsilon} + L\right)P_{qq}(z)\delta(1-z_h)$$
$$+ \delta(1-z)\frac{\alpha_s}{2\pi}\left[2C_F(1+z_h^2)\left(\frac{\ln(1-z_h)}{1-z_h}\right)_+ + C_F(1-z_h) + 2P_{qq}(z_h)\ln z_h\right]$$
$$- \delta(1-z_h)\frac{\alpha_s}{2\pi}\left[2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C_F(1-z)\right],$$

$\overline{\text{MS}}$ scheme, $\text{anti}-k_T$

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Renormalization and RG evolution

Bare - renormalized:

$$\mu \frac{d}{d\mu} \mathcal{G}_i^j(z, z_h, \omega_J, \mu) = \sum_j \int_z^1 \frac{dz'}{z'} \gamma_{ik}^{\mathcal{G}}\left(\frac{z}{z'}, \mu\right) \mathcal{G}_k^j(z', z_h, \omega_J, \mu)$$

where

$$\gamma_{ij}^{\mathcal{G}}(z, \mu) = \frac{\alpha_s(\mu)}{\pi} P_{ji}(z)$$

$$\mu \frac{d}{d\mu} \mathcal{G}_i(z, z_h, \omega_J, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}\right) \mathcal{G}_j(z', z_h, \omega_J, \mu)$$

… same DGLAP RG equations as before, resums $\ln R$

$$\longrightarrow \quad \frac{d}{d\log\mu^2} \begin{pmatrix} \mathcal{G}_S^h(z, z_h, \omega_J, \mu) \\ \mathcal{G}_g^h(z, z_h, \omega_J, \mu) \end{pmatrix} = \frac{\alpha_s(\mu)}{2\pi} \begin{pmatrix} P_{qq}(z) & 2N_f P_{gq}(z) \\ P_{qg}(z) & P_{gg}(z) \end{pmatrix} \otimes \begin{pmatrix} \mathcal{G}_S^h(z, z_h, \omega_J, \mu) \\ \mathcal{G}_g^h(z, z_h, \omega_J, \mu) \end{pmatrix}$$

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Matching

- onto standard collinear fragmentation functions

$$\mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \sum_j \int_{z_h}^1 \frac{dz_h'}{z_h'} \mathcal{J}_{ij}(z, z_h', \omega_J, \mu)\, D_j^h\left(\frac{z_h}{z_h'}, \mu\right)$$

FFs:

$$D_i^j(z, \mu) = \delta_{ij}\delta(1-z) + \frac{\alpha_s}{2\pi} P_{ji}(z)\left(-\frac{1}{\epsilon}\right)$$

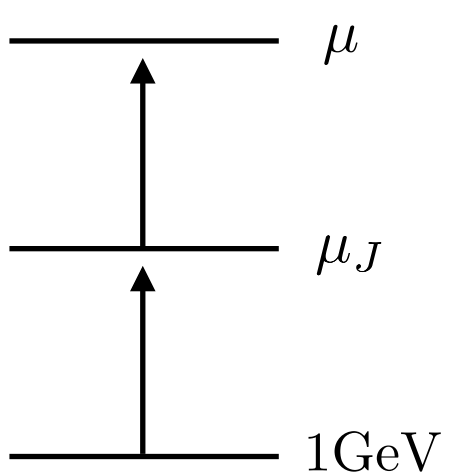
$$\begin{aligned}\mathcal{J}_{qq}(z, z_h, \omega_J, \mu) = \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi}\Bigg\{ & L\,\left[P_{qq}(z)\delta(1-z_h) - P_{qq}(z_h)\delta(1-z)\right] \\ & + \delta(1-z)\left[2C_F(1+z_h^2)\left(\frac{\ln(1-z_h)}{1-z_h}\right)_+ + C_F(1-z_h) + \mathcal{I}_{qq}^{\rm alg}(z_h)\right] \\ & - \delta(1-z_h)\left[2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C_F(1-z)\right]\Bigg\},\end{aligned}$$

$\overline{\rm MS}$ scheme, $\text{anti}-k_T$

→ Full agreement with standard pQCD result *Kaufmann, Mukherjee, Vogelsang `15*

Matching

- onto standard collinear fragmentation functions



$$\mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \sum_j \int_{z_h}^1 \frac{dz_h'}{z_h'} \mathcal{J}_{ij}\left(z, z_h', \omega_J, \mu\right) D_j^h\left(\frac{z_h}{z_h'}, \mu\right)$$

FFs:

$$D_i^j(z, \mu) = \delta_{ij}\delta(1-z) + \frac{\alpha_s}{2\pi} P_{ji}(z)\left(-\frac{1}{\epsilon}\right)$$

… 2 DGLAPs now

$$\begin{aligned}\mathcal{J}_{qq}(z, z_h, \omega_J, \mu) = \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi}\Bigg\{ & L\left[P_{qq}(z)\delta(1-z_h) - P_{qq}(z_h)\delta(1-z)\right] \\ & + \delta(1-z)\left[2C_F(1+z_h^2)\left(\frac{\ln(1-z_h)}{1-z_h}\right)_+ + C_F(1-z_h) + \mathcal{I}_{qq}^{\mathrm{alg}}(z_h)\right] \\ & - \delta(1-z_h)\left[2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C_F(1-z)\right]\Bigg\},\end{aligned}$$

$\overline{\mathrm{MS}}$ scheme, $\mathrm{anti}{-}k_T$

→ Full agreement with standard pQCD result *Kaufmann, Mukherjee, Vogelsang `15*

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Matching

- onto standard collinear fragmentation functions

$$\mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \sum_j \int_{z_h}^1 \frac{dz_h'}{z_h'} \mathcal{J}_{ij}\left(z, z_h', \omega_J, \mu\right) D_j^h\left(\frac{z_h}{z_h'}, \mu\right)$$

- at the hard scale

$$\frac{d\sigma^{pp\to(\text{jet}h)X}}{dp_T d\eta dz_h} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a, \mu) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b, \mu) \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} H_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu)\, \mathcal{G}_c^h(z_c, z_h, \omega_J, \mu)$$

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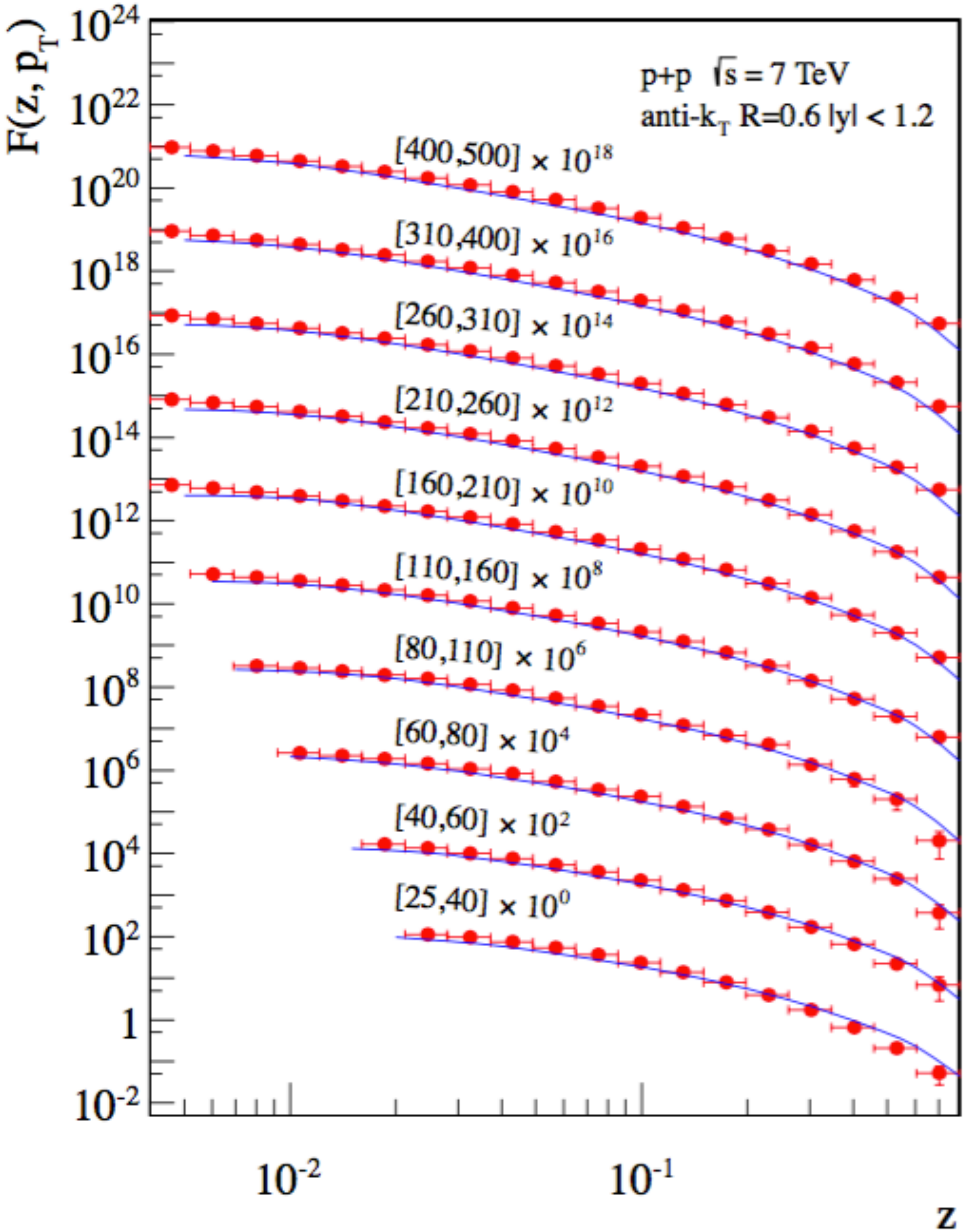

Comparison to ATLAS data at $\sqrt{s} = 7\,\mathrm{TeV}$

Light charged hadrons $h = \pi + K + p$

Using DSS FFs

de Florian, Sassot, Stratmann - '07

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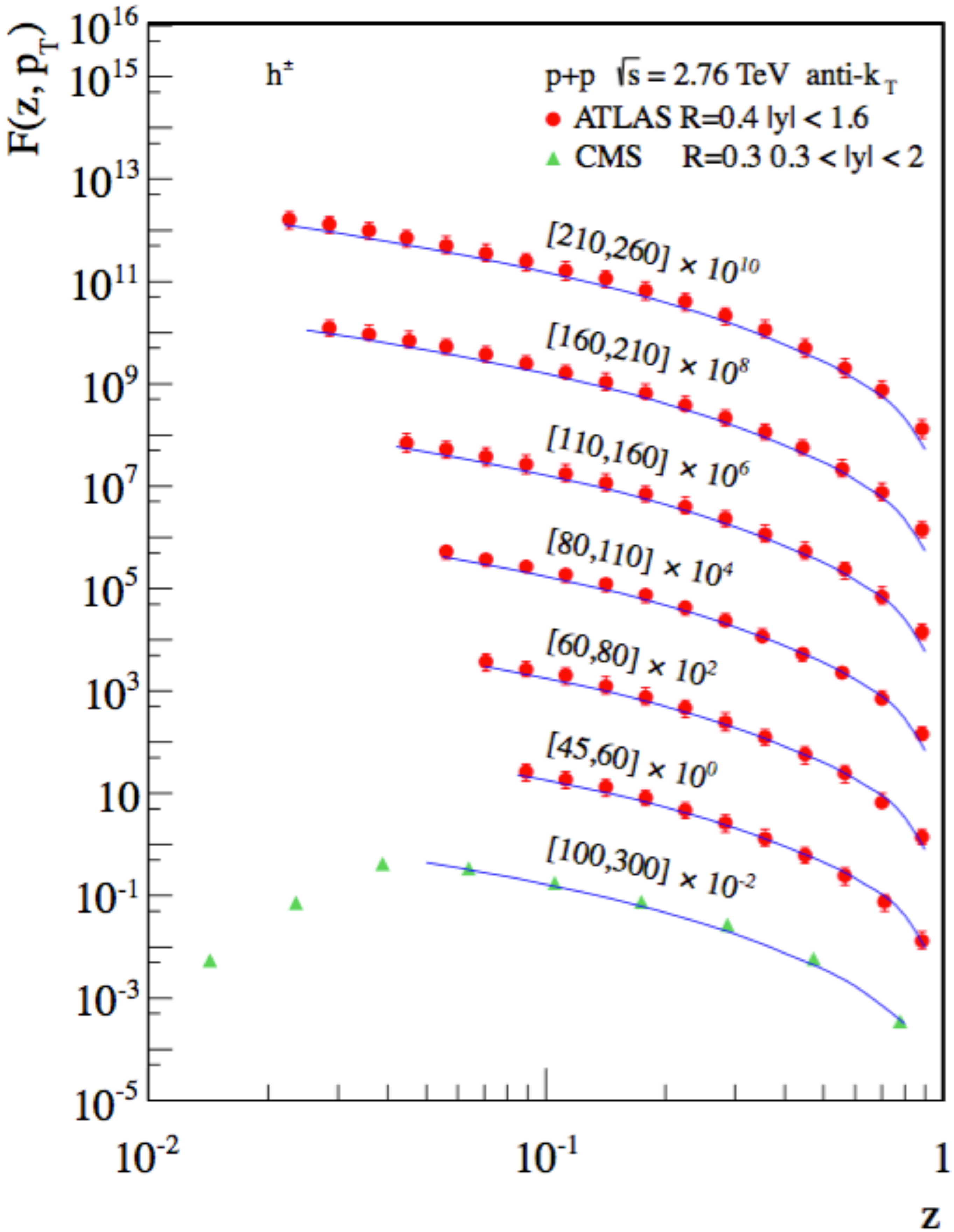
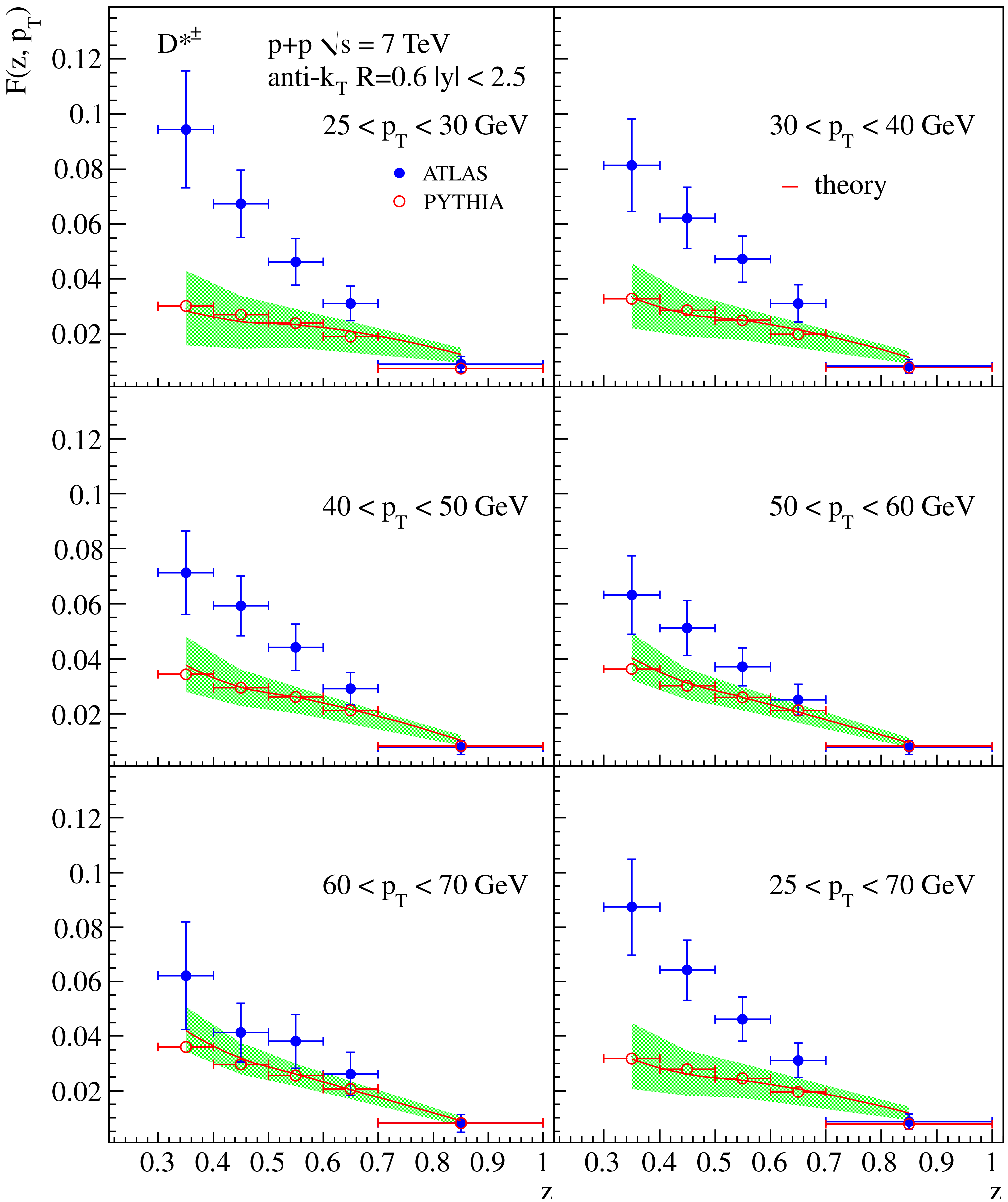


Comparison to ATLAS and CMS data at $\sqrt{s} = 2.76\,\mathrm{TeV}$

Light charged hadrons $h = \pi + K + p$

Using DSS FFs

de Florian, Sassot, Stratmann - '07



D-meson
jet fragmentation function

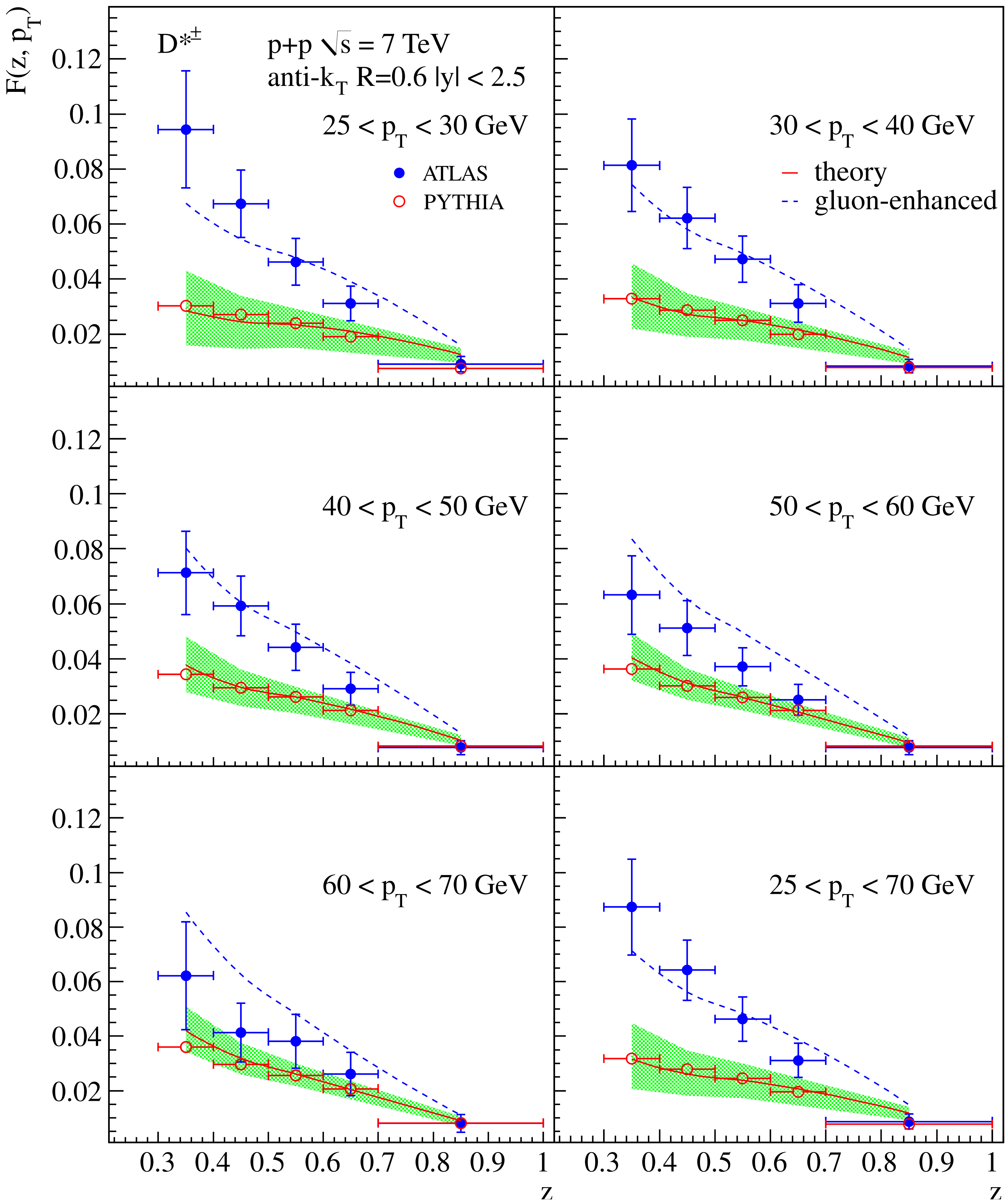
Comparison to ATLAS data
and PYTHIA simulations
at $\sqrt{s} = 7\,\mathrm{TeV}$

Using FFs from
Kneesch, Kniehl, Kramer, Schienbein - '08

ZMVNFS, $e^+e^- \to DX$

$\mu,\ \mu_J,\ \mu_G \gg m_Q$

Chien, Kang, FR, Vitev, Xing `15 (exclusive FJF)



D-meson
jet fragmentation function

$D_g^D(z,\mu) \to 2D_g^D(z,\mu)$

Comparison to ATLAS data
and PYTHIA simulations
at $\sqrt{s} = 7\,\text{TeV}$

Using FFs from
Kneesch, Kniehl, Kramer, Schienbein - '08

ZMVNFS, $e^+e^- \to DX$
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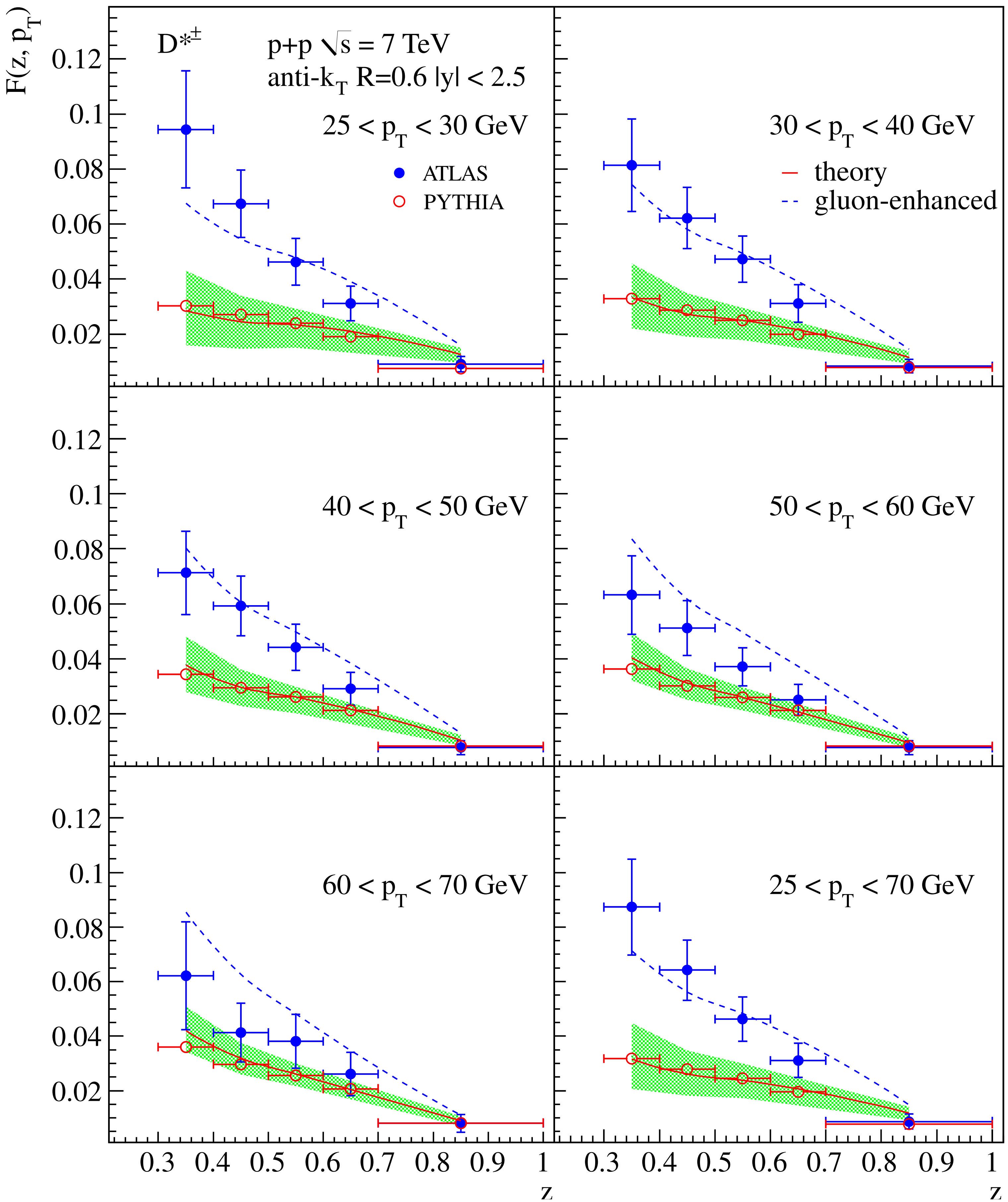

D-meson
jet fragmentation function

$$D_g^D(z,\mu) \to 2D_g^D(z,\mu)$$

Comparison to ATLAS data and PYTHIA simulations at $\sqrt{s} = 7\,\mathrm{TeV}$

↓

New fit of D-FFs:

Anderle, Kang, FR, Stratmann, Vitev - work in progress

Chien, Kang, FR, Vitev, Xing `15 (exclusive FJF)

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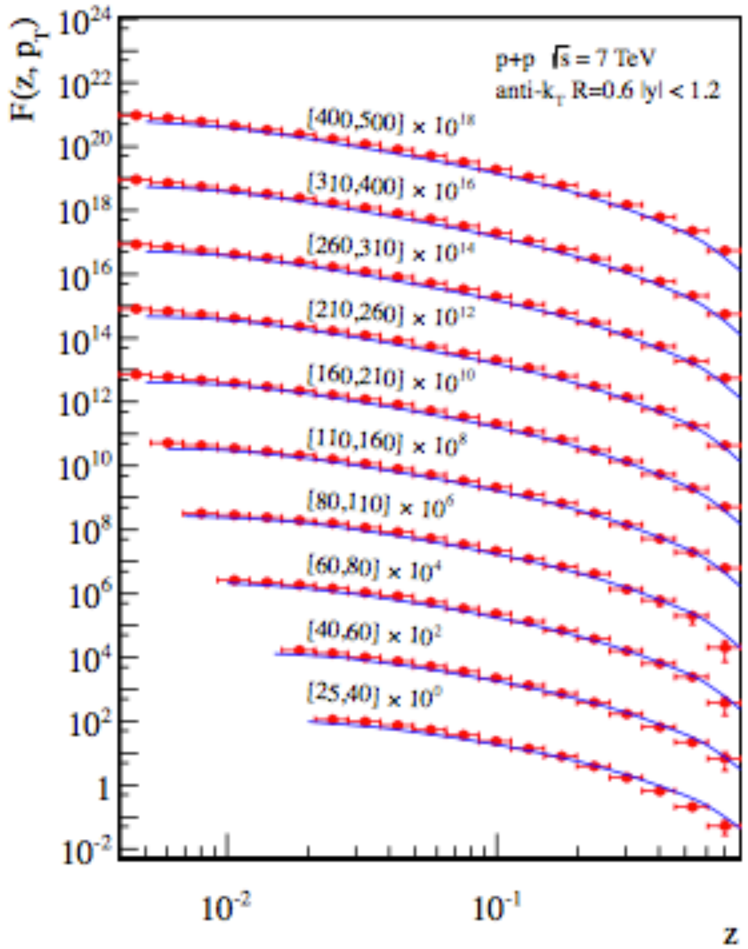
Outline

- Inclusive Jet Production
- The Jet Fragmentation Function
- Conclusions

Kang, FR, Vitev - in preparation

Conclusions

- Inclusive jet production
- The jet fragmentation function



- Threshold and small-z resummation
- Heavy quarks, J/ψ …
- Jet quenching studies beyond energy loss
- Extension to ep and eA for the EIC

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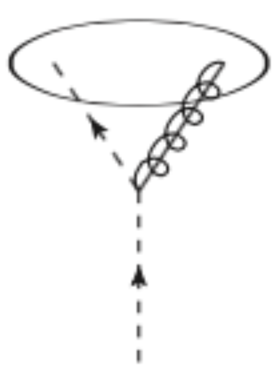
backup

Semi-inclusive jet function

Next-to-leading order



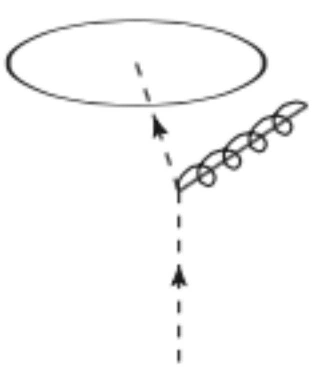
$$\mathcal{G}_{i,\mathrm{bare}}^{j,(1)}(z, z_h, \omega_J, \mu) = \delta(1-z)\frac{\alpha_s}{\pi}\frac{(e^{\gamma_E}\mu^2)^\epsilon}{\Gamma(1-\epsilon)}\hat{P}_{ji}(z_h,\epsilon)\int \frac{dq_\perp}{q_\perp^{1+2\epsilon}}\Theta_{\mathrm{alg}}$$

where: $$\Theta_{\text{anti-k}_\mathrm{T}} = \theta\left(z_h(1-z_h)\omega_J \tan\frac{R}{2} - q_\perp\right)$$

Semi-inclusive jet function

Next-to-leading order

(A) (B) (C) (D)

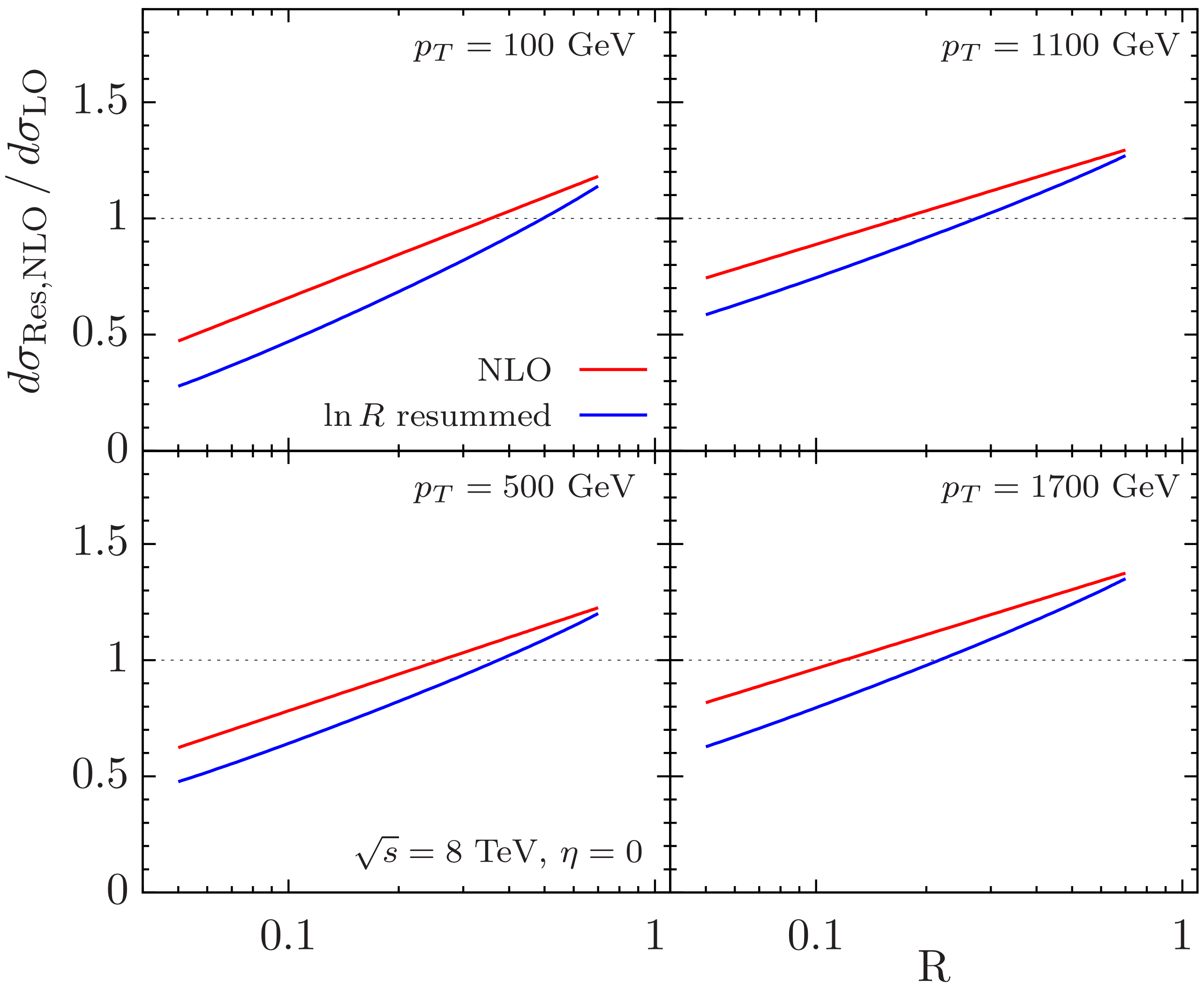


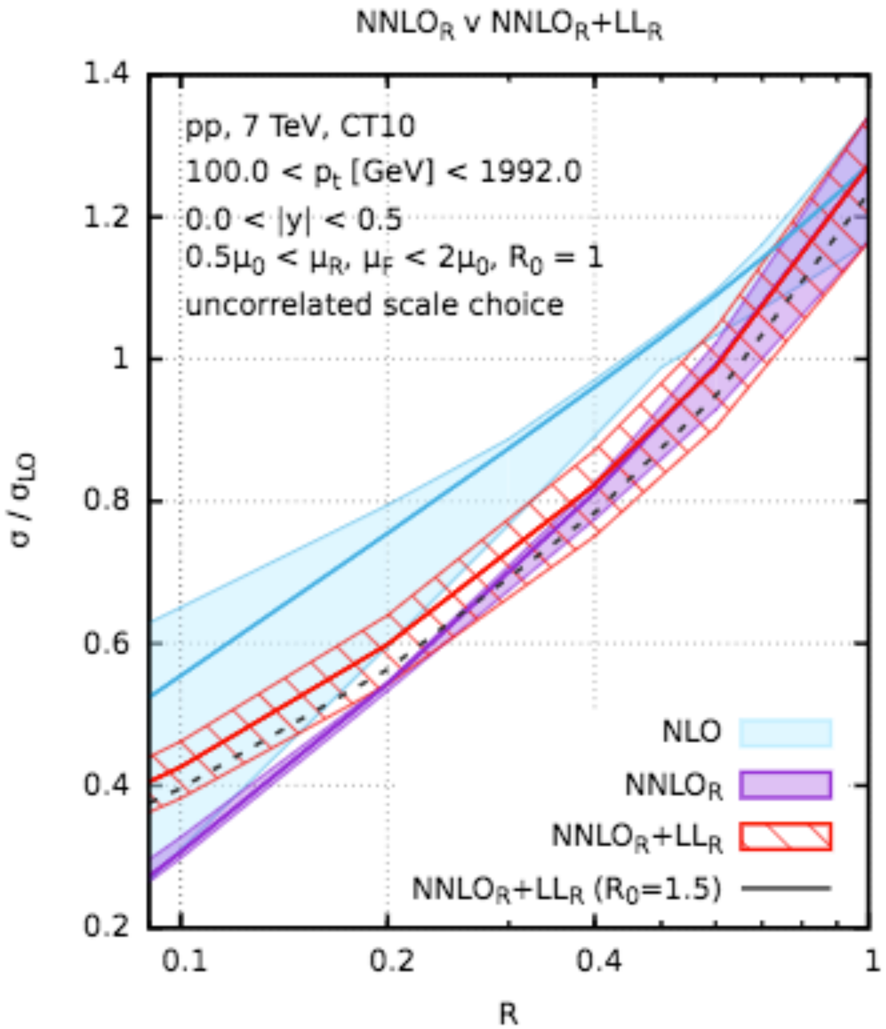
(B) + (C)

$$\mathcal{G}_{i,\mathrm{bare}}^{j,(1)}(z, z_h, \omega_J, \mu) = \delta(1-z_h)\frac{\alpha_s}{\pi}\frac{(e^{\gamma_E}\mu^2)^\epsilon}{\Gamma(1-\epsilon)}\hat{P}_{ji}(z_h,\epsilon)\int\frac{dq_\perp}{q_\perp^{1+2\epsilon}}\Theta_{\mathrm{alg}}$$

where:

$$\Theta_{\text{anti-k}_T} = \theta\left(q_\perp - (1-z)\omega_J \tan\frac{R}{2}\right)$$





Dasgupta, Dreyer, Salam, Soyez `15, `16