Topical Lectures on DM@Nikhef
14 December 2017

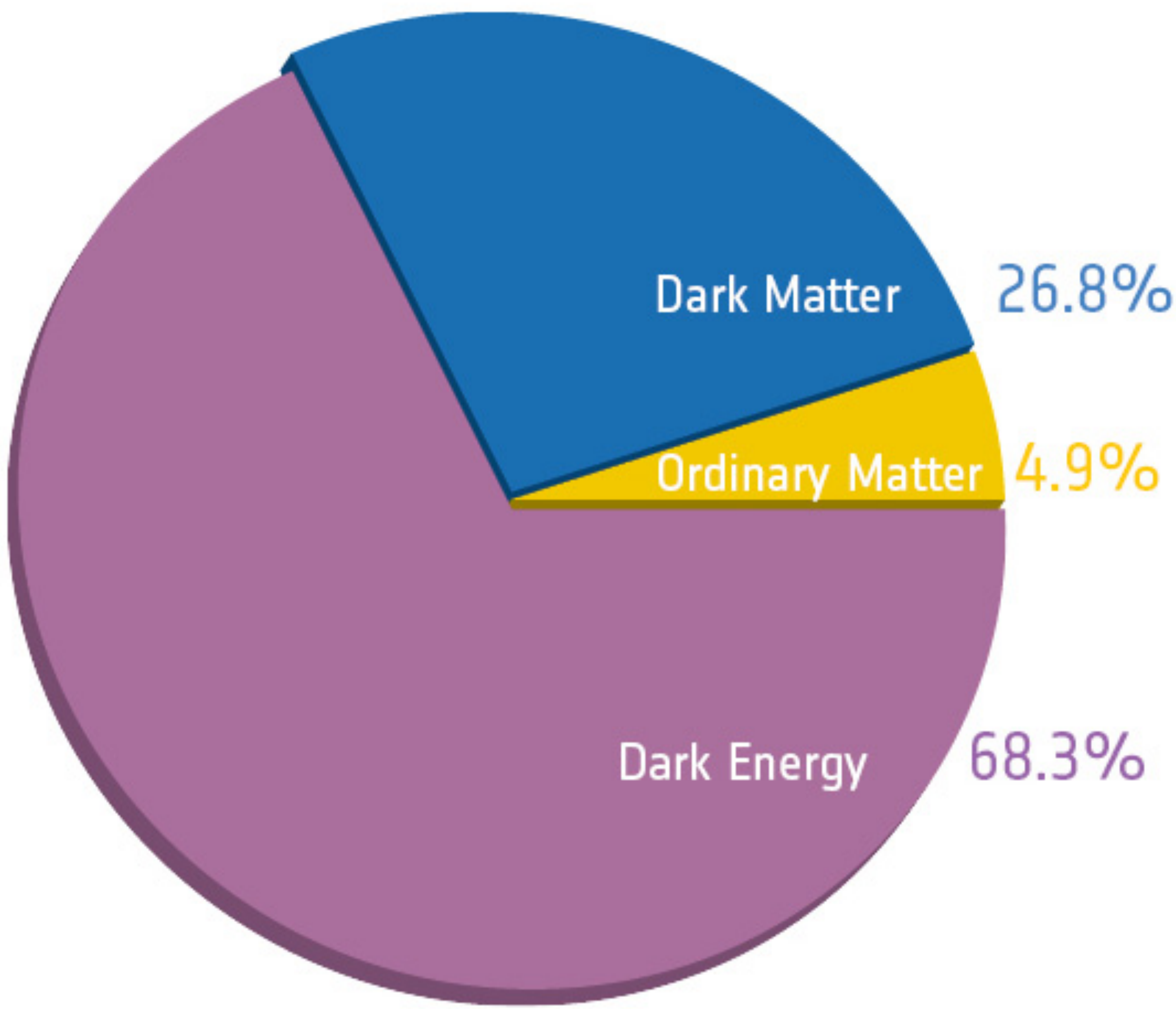
Dark matter distribution
Large and small scale structure

Shin'ichiro Ando
GRAPPA, University of Amsterdam

GRAPPA
GRavitation AstroParticle Physics Amsterdam

Result from all the cosmology data

Planck 2015



- CMB, galaxy power spectrum, weak lensing, supernova Ia, etc.
- 27% of the total energy / 85% of the total matter is made of dark matter
- Properties of dark matter
  - **Collisionless** ?
  - **Non-baryonic**
  - **Doesn't interact with photons** ?
  - **Cold** (or warm; hot dark matter erases too many structures) ?

Key questions to reveal nature of dark matter

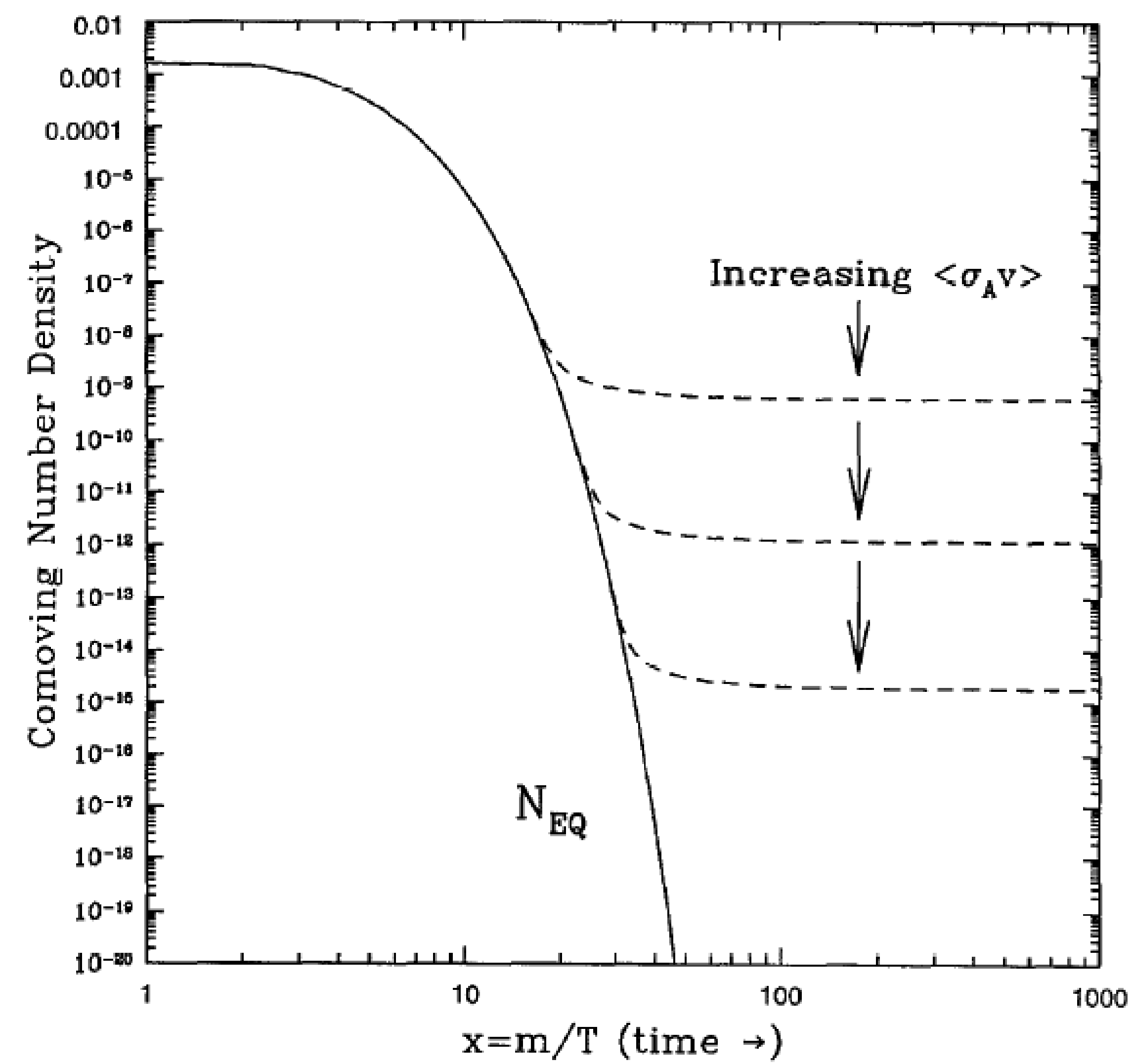
- Does dark matter ***interact with standard model*** particles such as photons? Is “dark” matter really dark?

- Is dark matter ***collisionless*** with each other?

- How ***cold*** is dark matter?

Dark matter candidate: WIMP

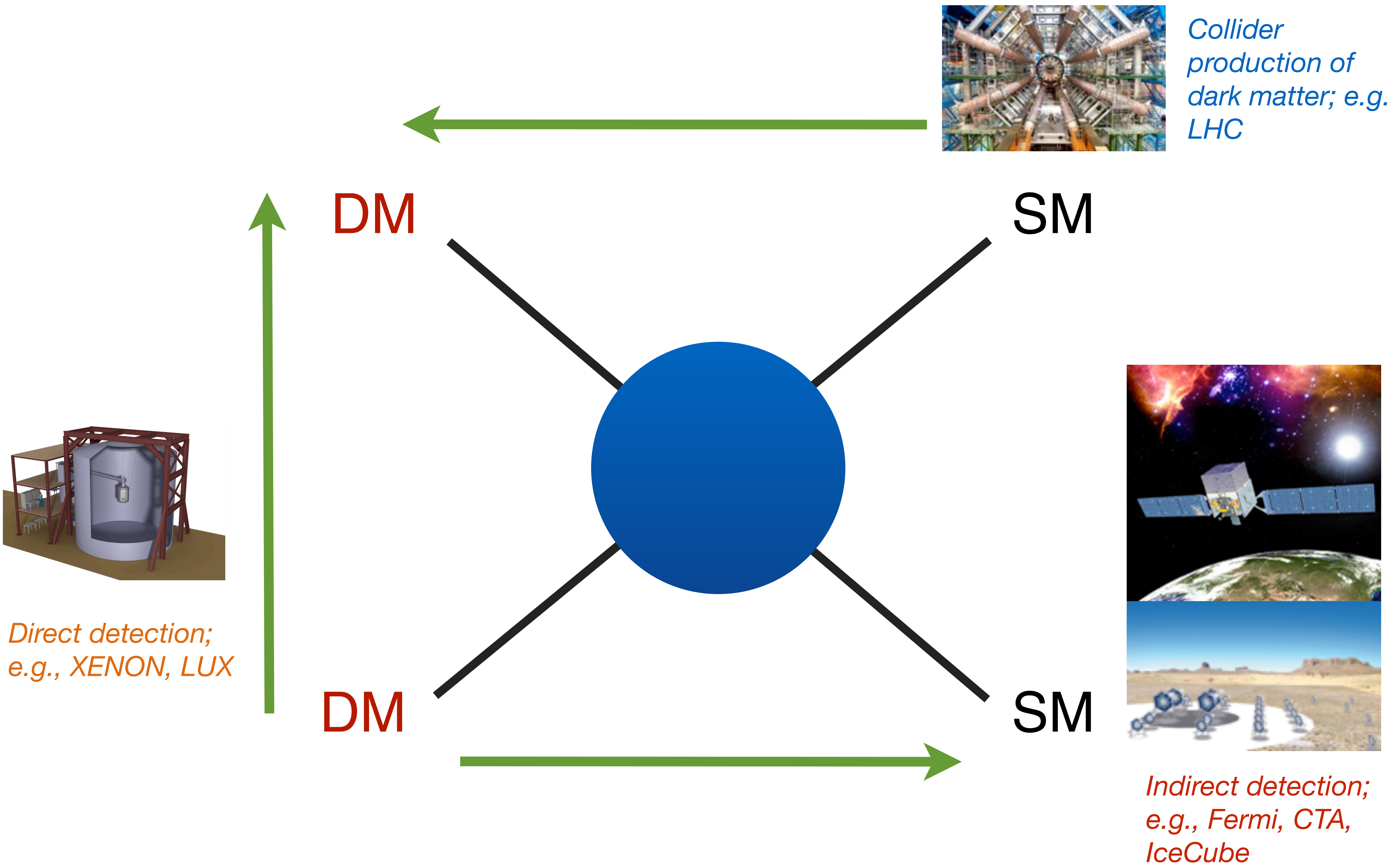
- Weakly Interacting Massive Particle (**WIMP**)
- Current dark matter density: determined by competition between Hubble expansion and annihilation
  - Later, expansion becomes too fast for WIMPs to annihilate (thermal freeze-out)
- WIMP models can naturally explain the relic abundance
- E.g., neutralino predicted by supersymmetry



$$\Omega_\chi h^2 \simeq \frac{3 \times 10^{-27}\ \mathrm{cm}^3\ \mathrm{s}^{-1}}{\langle \sigma_{\mathrm{ann}} v \rangle}$$

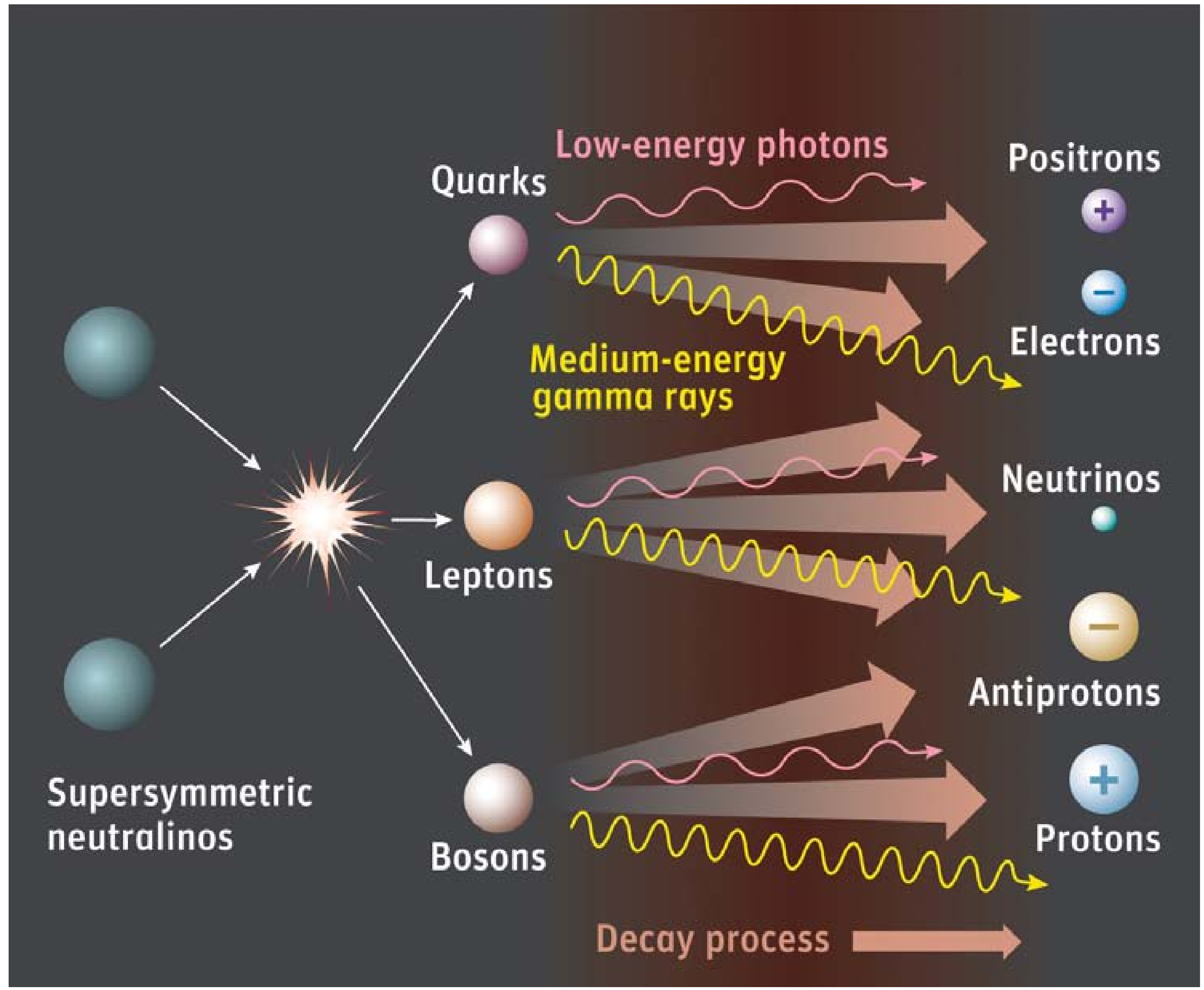
$$\langle \sigma_{\mathrm{ann}} v \rangle \sim \alpha^2 (100\ \mathrm{GeV})^{-2} \sim 3 \times 10^{-26}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$$

Three routes to dark matter



Dark matter annihilation

- WIMPs annihilate into standard model particles (photons, positrons, neutrinos, etc.)
- Each of these particles carry a fraction of WIMP mass energy (E ~ GeV–TeV)
- Annihilation rate is proportional to density squared and to annihilation cross section and relative velocity: $\sigma_{ann}v$



Rate of annihilation: Simple consideration

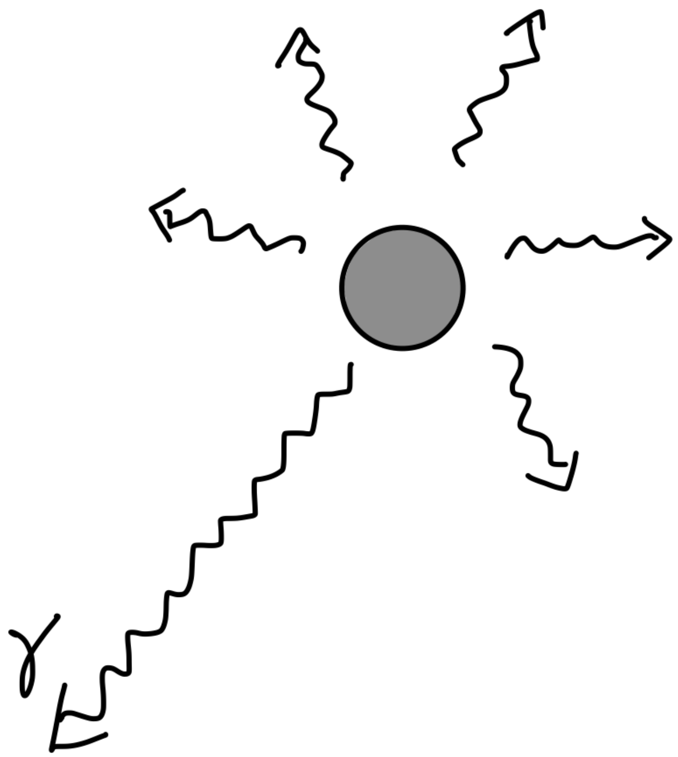
- Suppose you are a WIMP particle in a region of mass density ρ_χ
- Other WIMP particles are around you with velocity *v*, and if one of them hit you, you are both eliminated
- Incoming flux of the other WIMPs is $n_\chi v = \dfrac{\rho_\chi v}{m_\chi}$
- You encounter the others with the rate of $n_\chi \sigma v$
- If we look at this region of unit volume, such encounters happen at the rate of

$$\frac{n_\chi^2 \sigma v}{2} = \frac{\rho_\chi^2 \sigma v}{2 m_\chi}$$

: rate of annihilation per volume

Gamma-ray flux from dark matter annihilation

Case of single halos



Annihilation rate per volume

$$\frac{\langle \sigma v \rangle \rho_\chi^2}{2 m_\chi^2}$$

Differential gamma-ray luminosity

$$\mathcal{L}(E) = \frac{\langle \sigma v \rangle}{2 m_\chi^2} \frac{dN_{\gamma,\mathrm{ann}}}{dE} \int dV \rho_\chi^2$$

Differential flux

$$\mathcal{F}(E, z) = \frac{\mathcal{L}((1+z)E)}{4\pi r^2}$$

***r*: comoving distance to the halo**

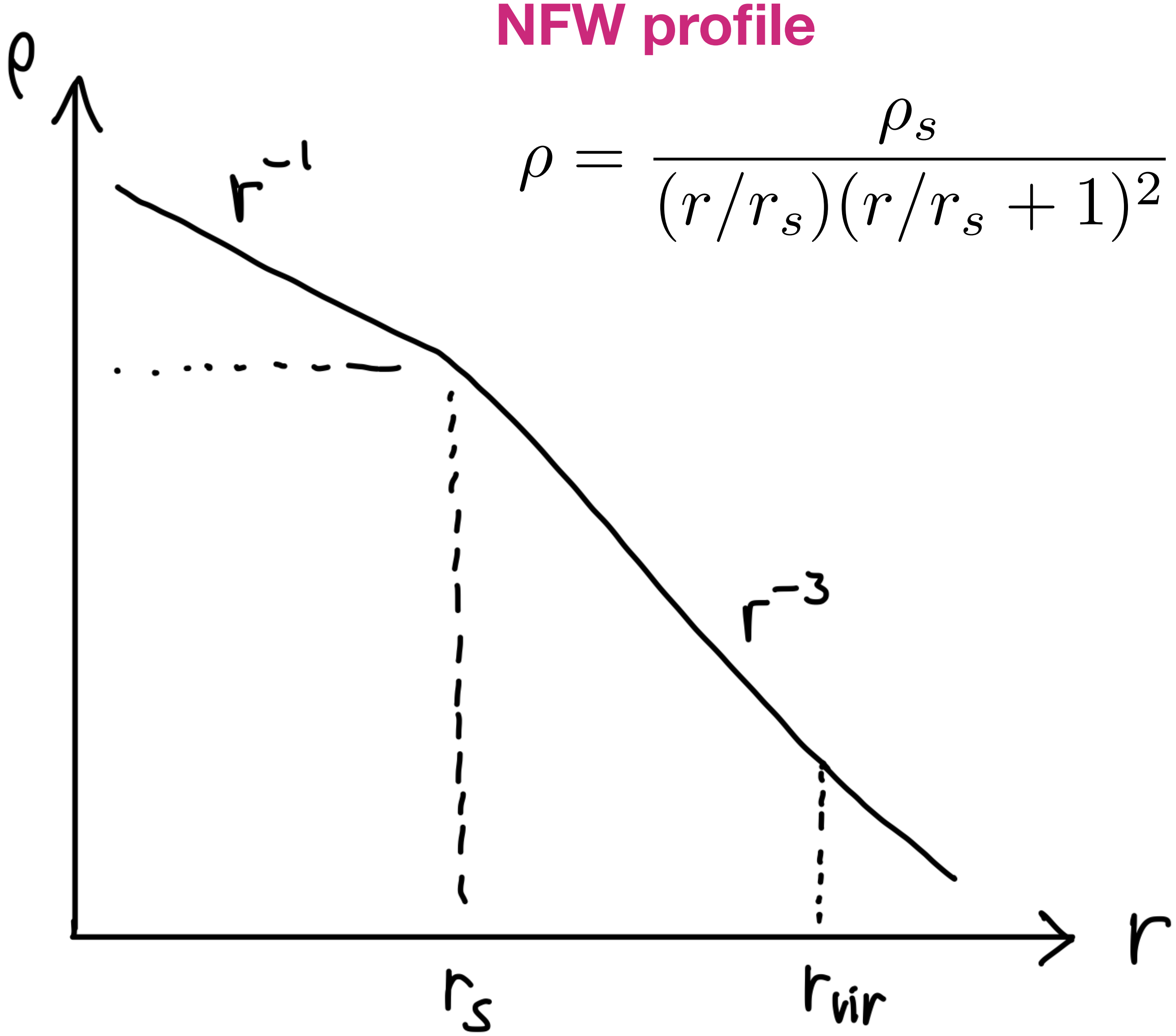
Gamma-ray flux from dark matter annihilation

Case of single halos

Halo mass *M* at redshift *z*

NFW profile

$$\rho = \frac{\rho_s}{(r/r_s)(r/r_s+1)^2}$$



Virial radius

$$r_{\rm vir} = \left(\frac{3M}{4\pi\Delta_{\rm vir}(z)\rho_c(z)}\right)^{1/3}$$

Scale radius

$$r_s = \frac{r_{\rm vir}}{c_{\rm vir}}$$

Characteristic density

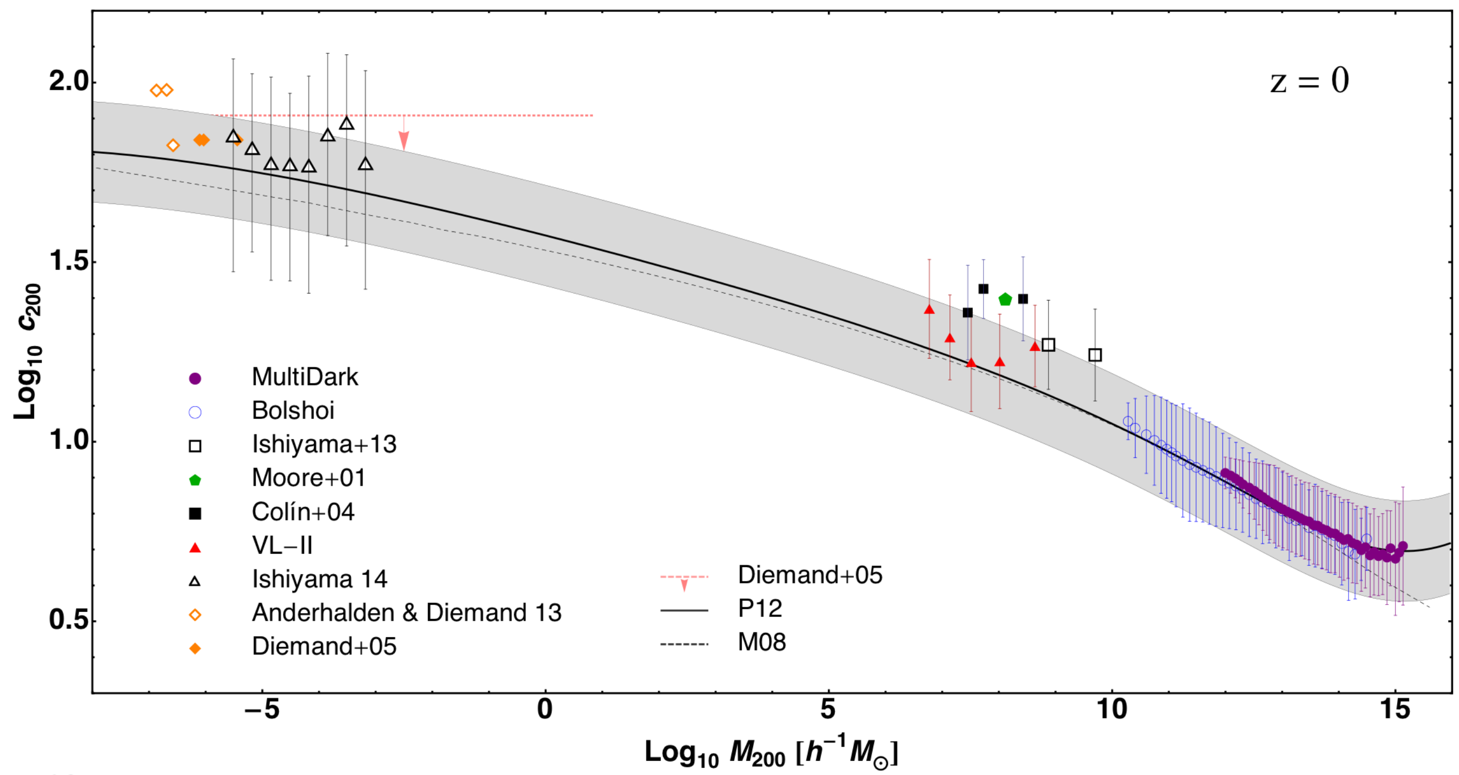
$$\rho_s = \frac{M}{4\pi r_s^3[\ln(1+c_{\rm vir}) - c_{\rm vir}/(1+c_{\rm vir})]}$$

All relevant parameters derived as a function of *M* and *z*

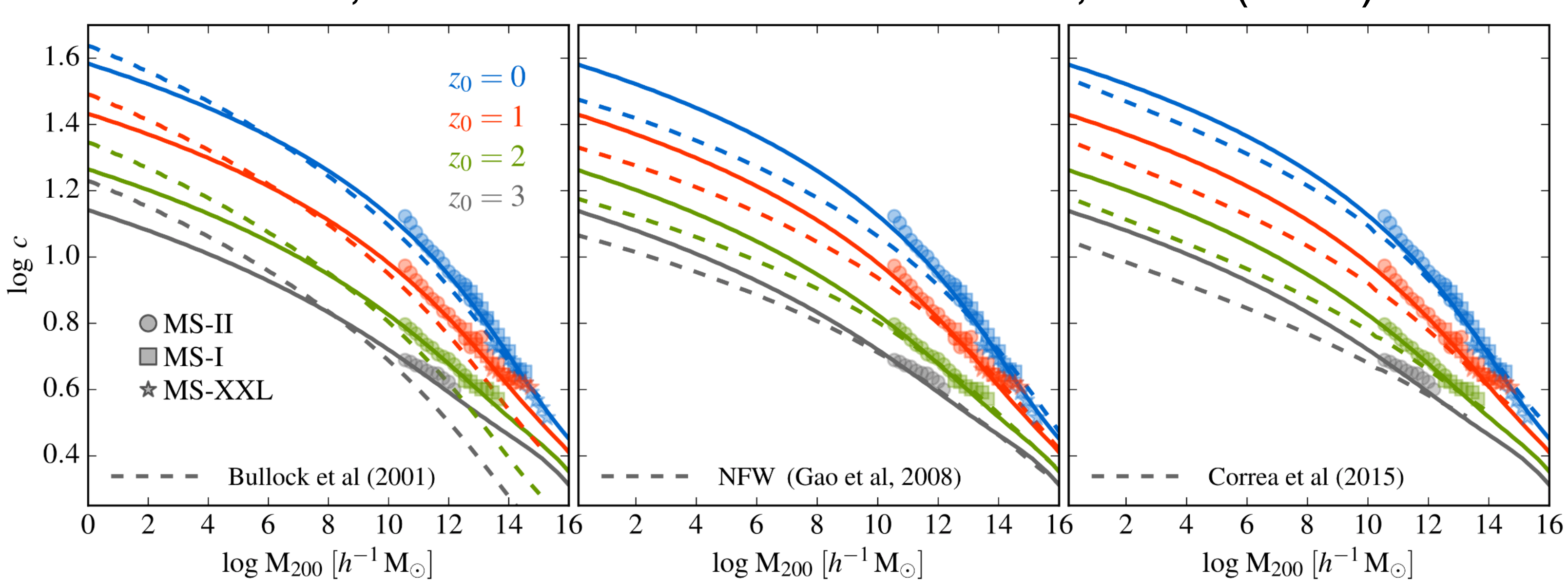
Gamma-ray flux from dark matter annihilation

Case of single halos

Sanchez-Conde, Prada, *Mon. Not. R. Astron. Soc.* **442**, 2271 (2014)



Ludlow et al., *Mon. Not. R. Astron. Soc.* **460**, 1214 (2016)



Halo concentration

$$c_{\rm vir} = \frac{r_{\rm vir}}{r_s}$$

- Halo concentration-mass relation is well calibrated through simulations
  - From largest to smallest halos
  - From low to high redshifts ($0 < z < 5$)
- About 20-30% scatter from one halo to another

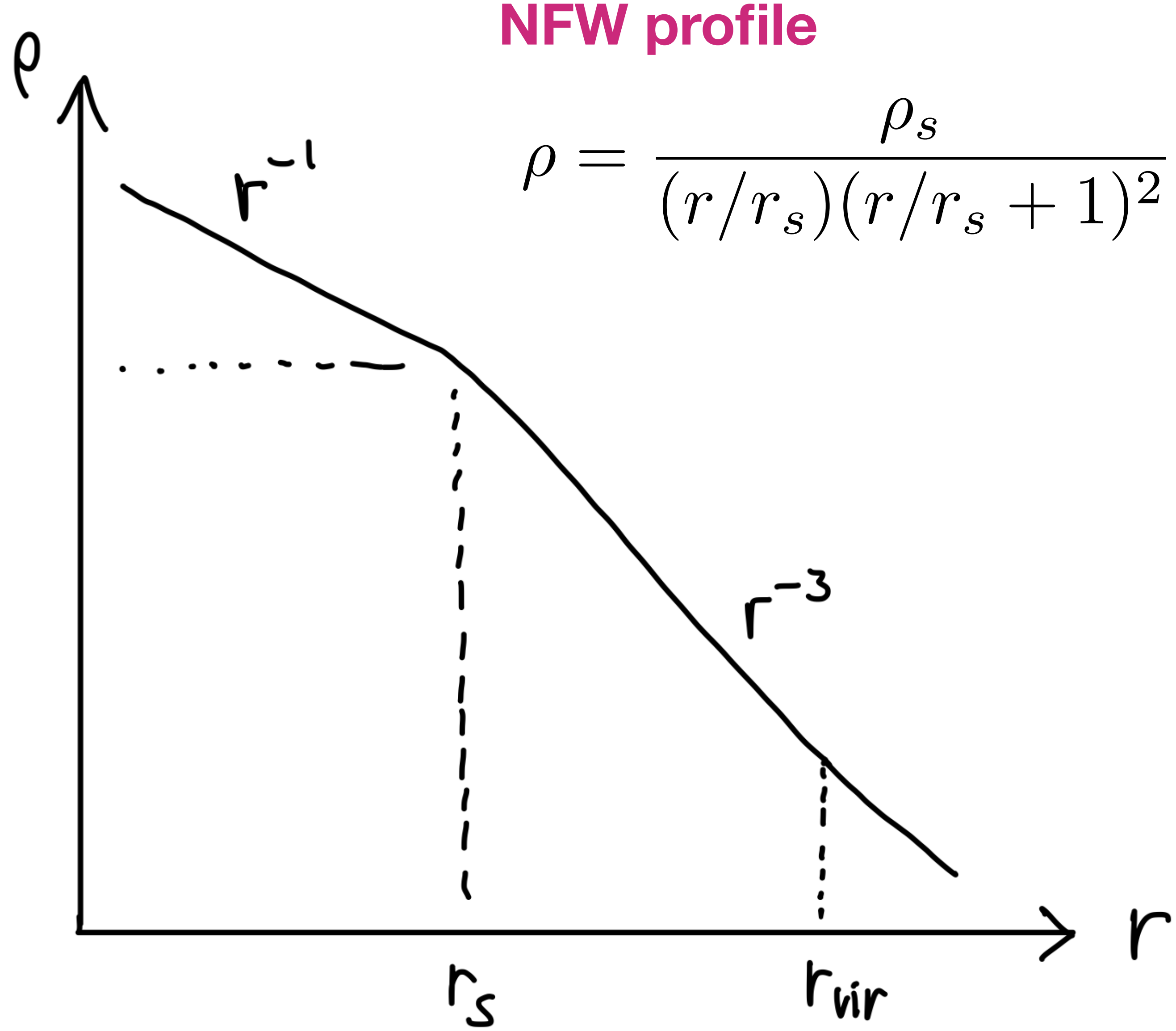
Gamma-ray flux from dark matter annihilation

Case of single halos

Halo mass *M* at redshift *z*

NFW profile

$$\rho = \frac{\rho_s}{(r/r_s)(r/r_s+1)^2}$$



Virial radius

$$r_{\rm vir} = \left(\frac{3M}{4\pi\Delta_{\rm vir}(z)\rho_c(z)}\right)^{1/3}$$

Scale radius

$$r_s = \frac{r_{\rm vir}}{c_{\rm vir}}$$

Characteristic density

$$\rho_s = \frac{M}{4\pi r_s^3[\ln(1+c_{\rm vir}) - c_{\rm vir}/(1+c_{\rm vir})]}$$

$$\int dV \rho^2 = \frac{4\pi\rho_s^2 r_s^3}{3}\left[1 - \frac{1}{(1+c_{\rm vir})^3}\right]$$

Gamma-ray flux from dark matter annihilation

Case of single halos

- A quick exercise: ***How many dark matter annihilations are happening per second in the entire Milky Way?***

Milky Way halo: $M = 10^{12} M_\odot$

$$r_{\rm vir} = \left(\frac{3M}{4\pi\Delta_{\rm vir}\rho_c}\right)^{1/3} \sim 200\ {\rm kpc}$$

$$r_s = \frac{r_{\rm vir}}{c_{\rm vir}} \sim 20\ {\rm kpc} \qquad (c_{\rm vir} = 10)$$

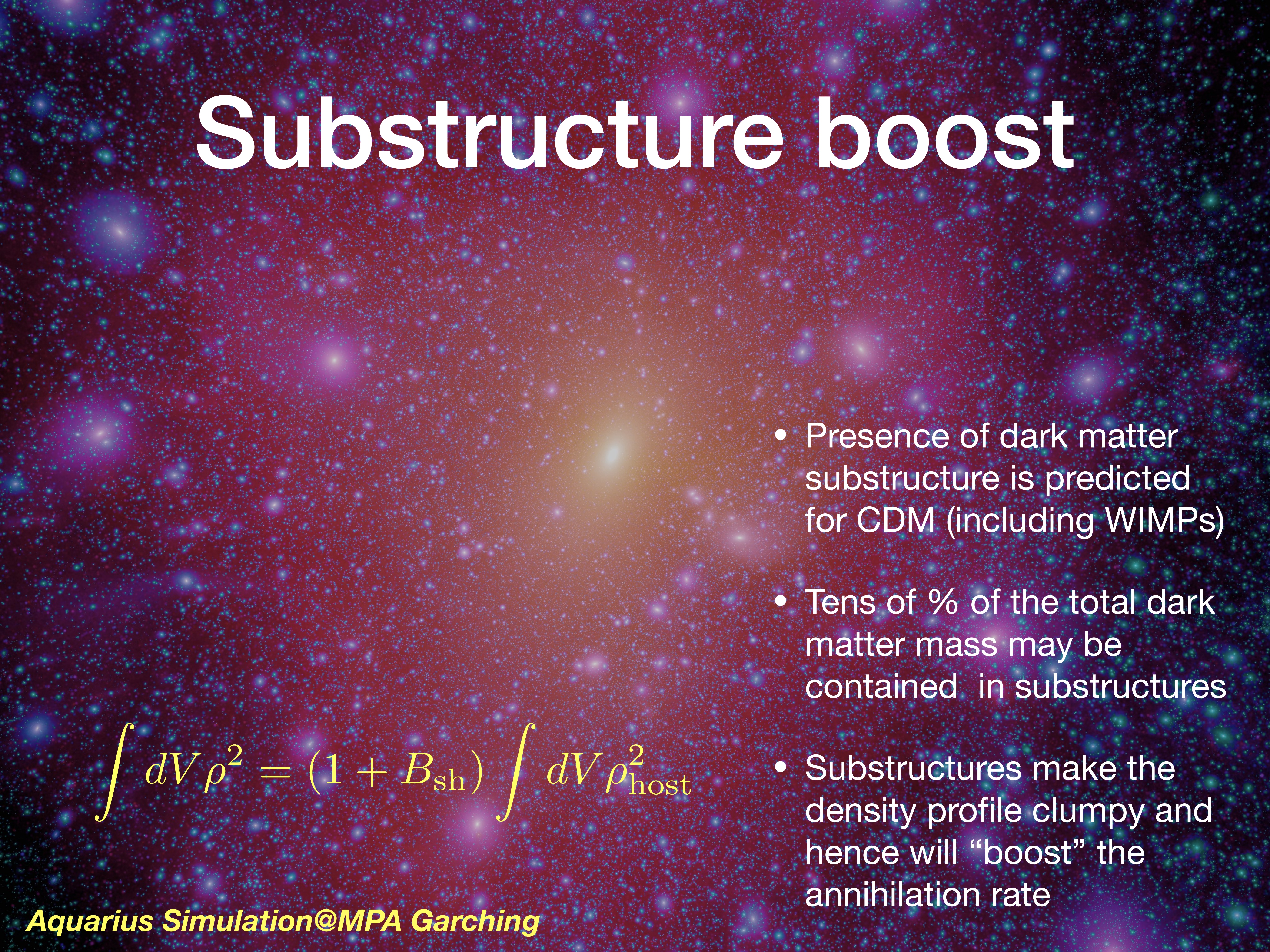
$$\rho_s = \frac{M}{4\pi r_s^3[\ln(1+c_{\rm vir}) - c_{\rm vir}/(1+c_{\rm vir})]}$$

$$\sim 0.3\ {\rm GeV\ cm^{-3}}$$

Annihilation rate:

$$\frac{\langle\sigma v\rangle}{2m_\chi^2}\int dV \rho_\chi^2 = 6\times 10^{37}\ {\rm s^{-1}}\left(\frac{\langle\sigma v\rangle}{3\times 10^{-26}\ {\rm cm^3\ s^{-1}}}\right)\left(\frac{m_\chi}{100\ {\rm GeV}}\right)^{-2}$$

Substructure boost



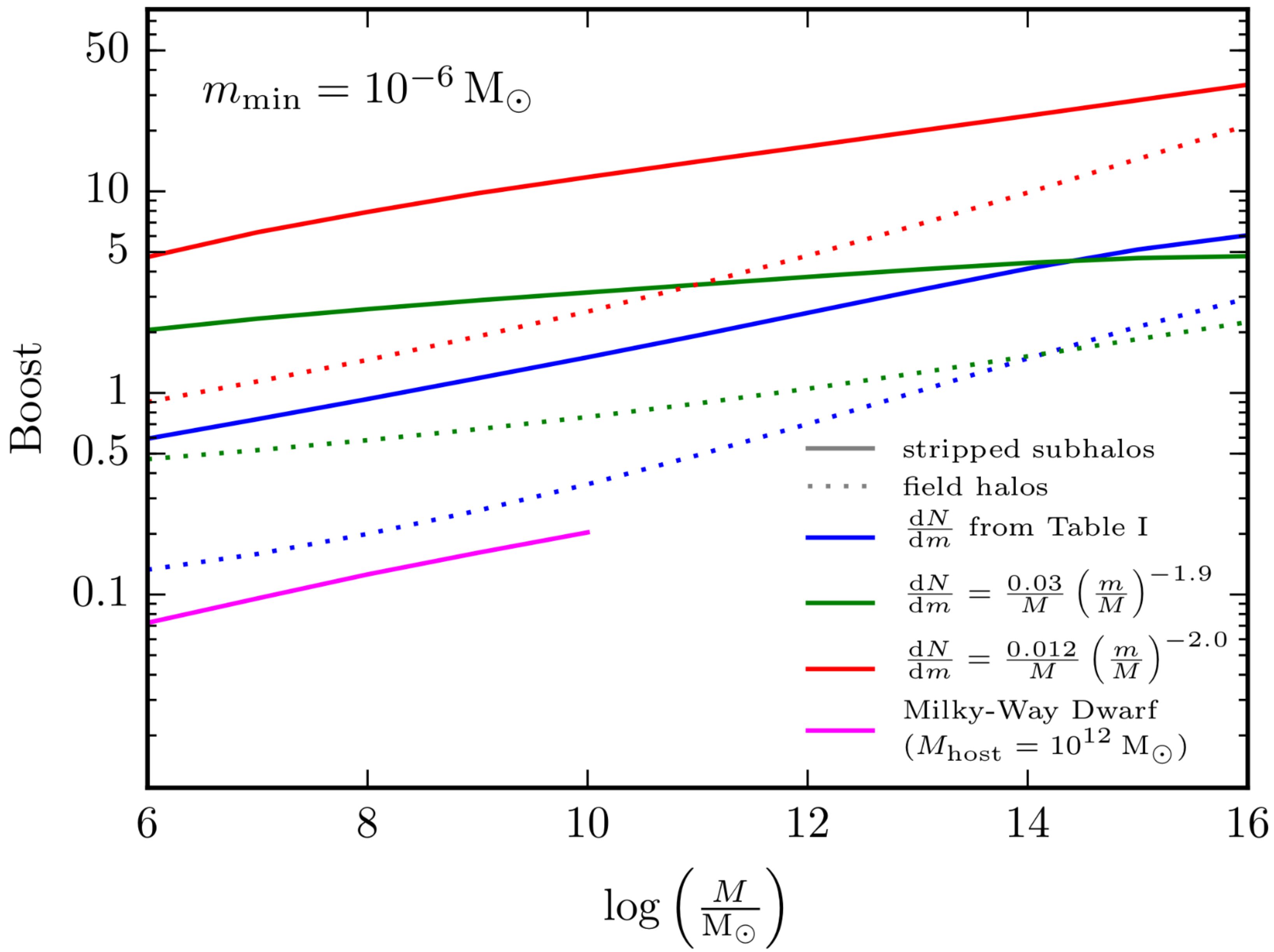
Aquarius Simulation@MPA Garching

$$\int dV \rho^2 = (1 + B_{\rm sh}) \int dV \rho_{\rm host}^2$$

- Presence of dark matter substructure is predicted for CDM (including WIMPs)
- Tens of % of the total dark matter mass may be contained in substructures
- Substructures make the density profile clumpy and hence will "boost" the annihilation rate

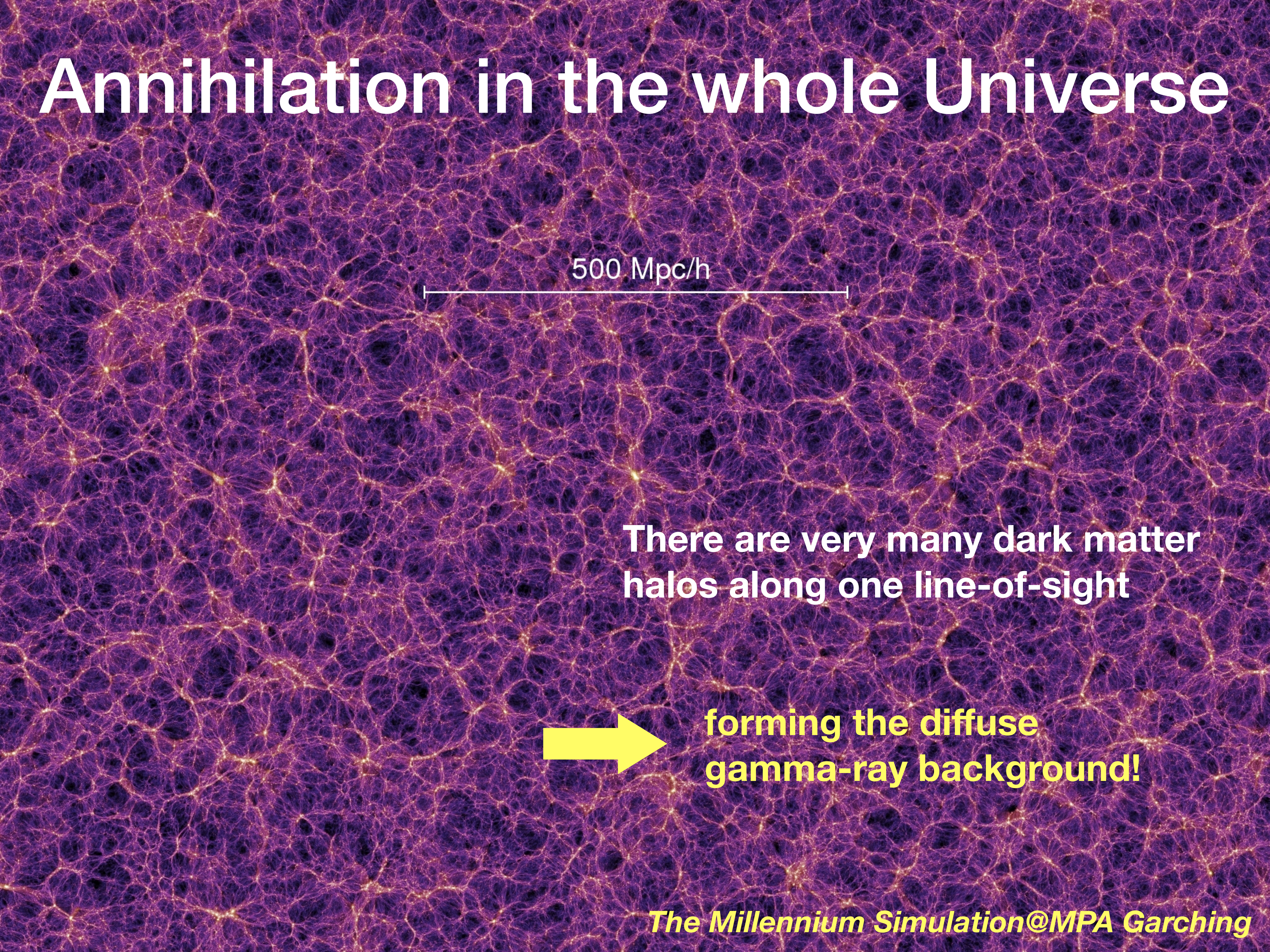
Substructure boost

Bartels, Ando, *Phys. Rev. D* **92**, 123508 (2015)



- Typical substructure boost: a few to ~10 for Milky Way size halos
- Depends on properties of subhalos such as mass function, tidal stripping, etc.
- It is the only probe of micro-halos that formed the earliest

Annihilation in the whole Universe

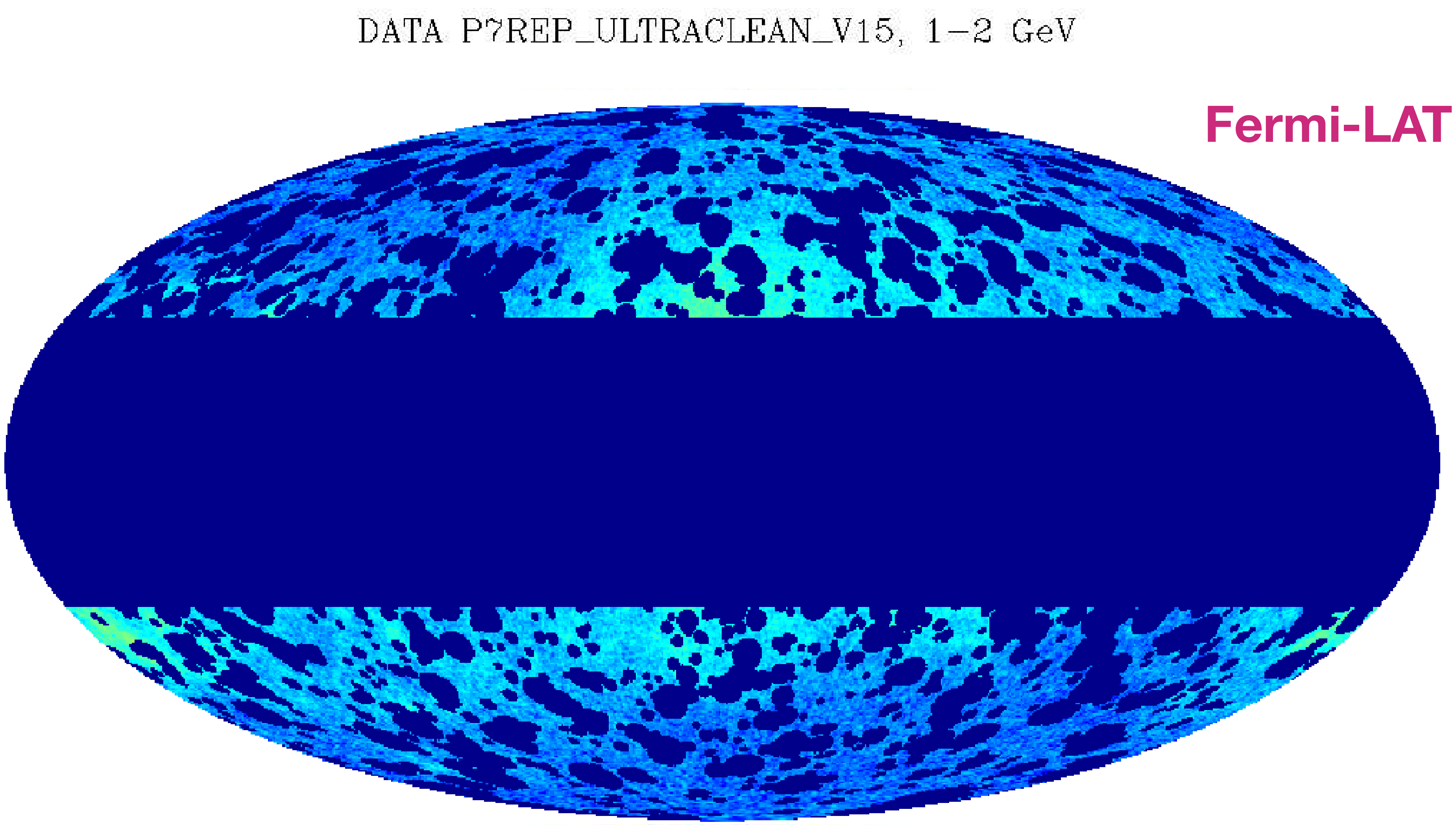


There are very many dark matter halos along one line-of-sight

forming the diffuse gamma-ray background!

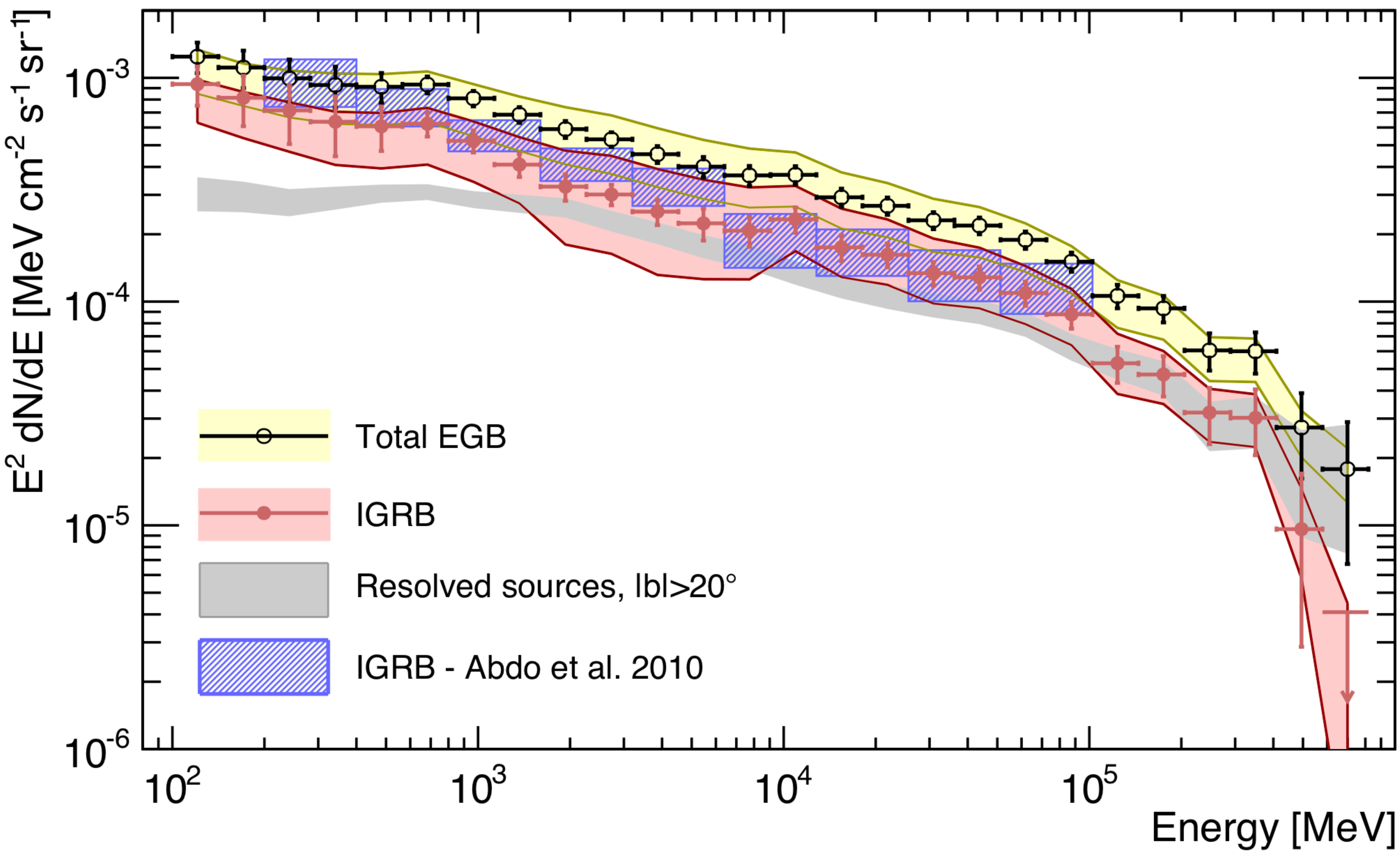
The Millennium Simulation@MPA Garching

Diffuse gamma-ray background



- No dark matter signals have been found around any sources (Galactic center, dwarf galaxies, etc.)
- Hints of dark matter might be hidden in this unresolved map

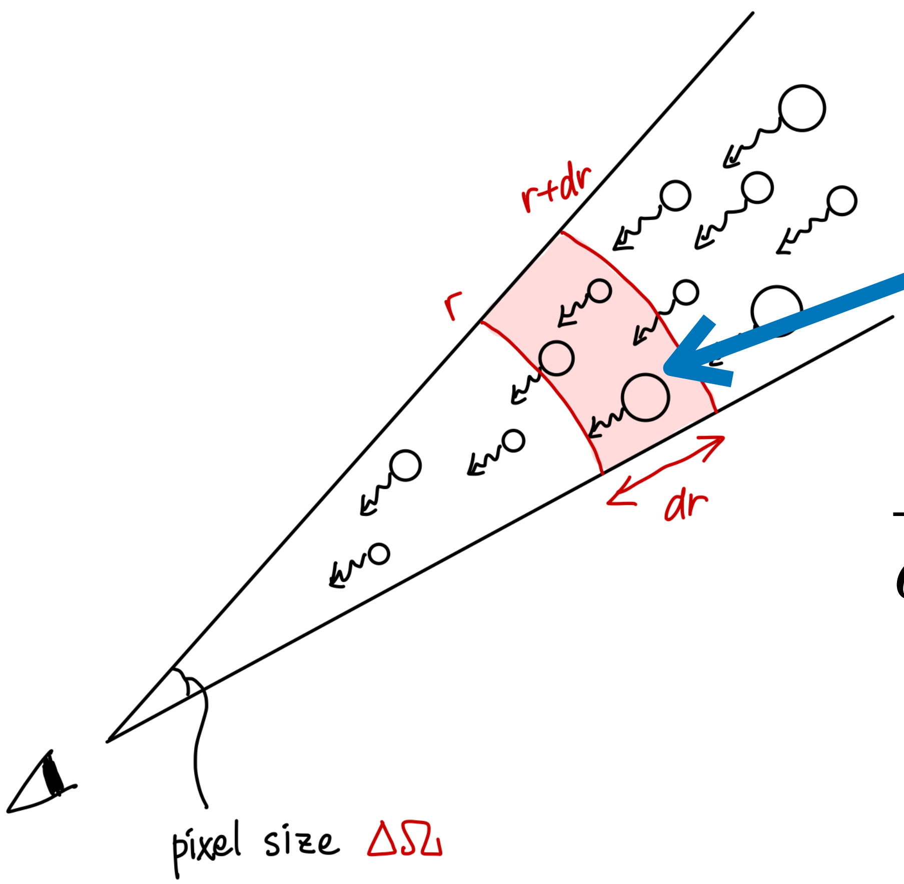
Energy spectrum of the gamma-ray background



Ackermann et al., *Astrophys. J.* **799**, 86 (2015)

Gamma-ray flux from dark matter annihilation

Contribution from all the halos



Number of halos between *M* and *M+dM* in this volume

$$\frac{d^3N}{dr\,dM\,d\Omega}dr\,dM\,\Delta\Omega = \Delta\Omega\; r^2 dr \frac{dn(M,z)}{dM}dM$$

Intensity (flux per unit solid angle) from all the halos

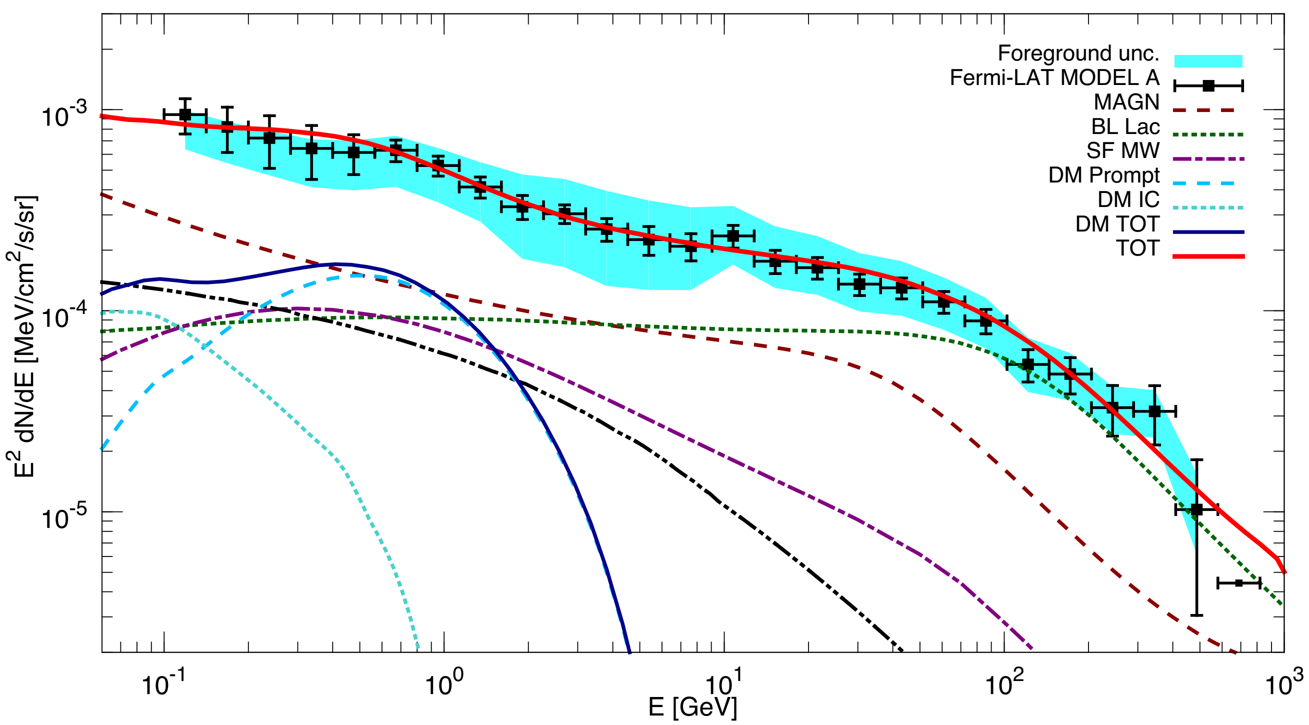
$$I(E) = \int dr \int dM \frac{d^3N}{dr\,dM\,d\Omega}\frac{1}{4\pi r^2}\mathcal{L}((1+z)E|M,z)$$

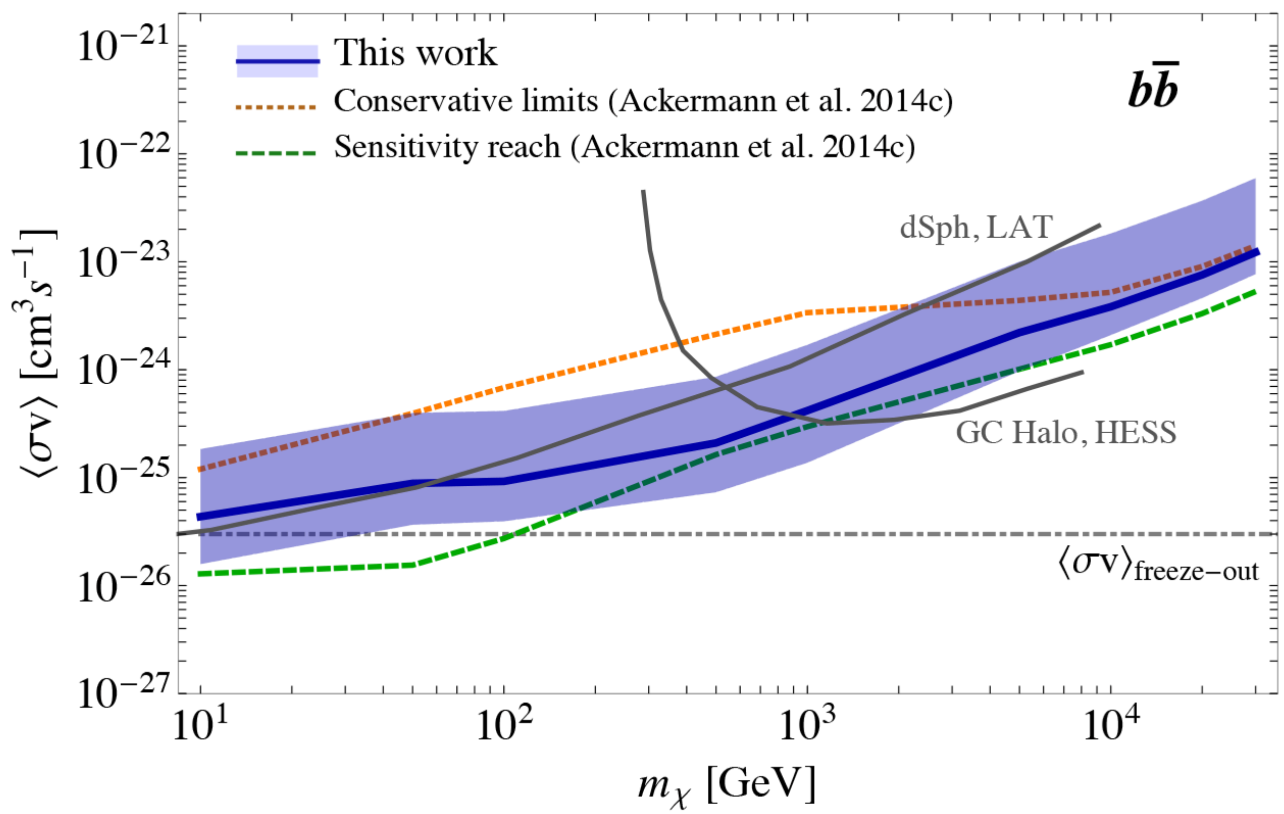
$$= \frac{1}{4\pi}\int dr \int dM \frac{dn(M,z)}{dM}\mathcal{L}((1+z)E|M,z)$$

Gamma-ray background from WIMP annihilation

Di Mauro, Donato, *Phys. Rev. D* **91**, 123001 (2015)

Ajello et al., *Astrophys. J.* **800**, L27 (2015)



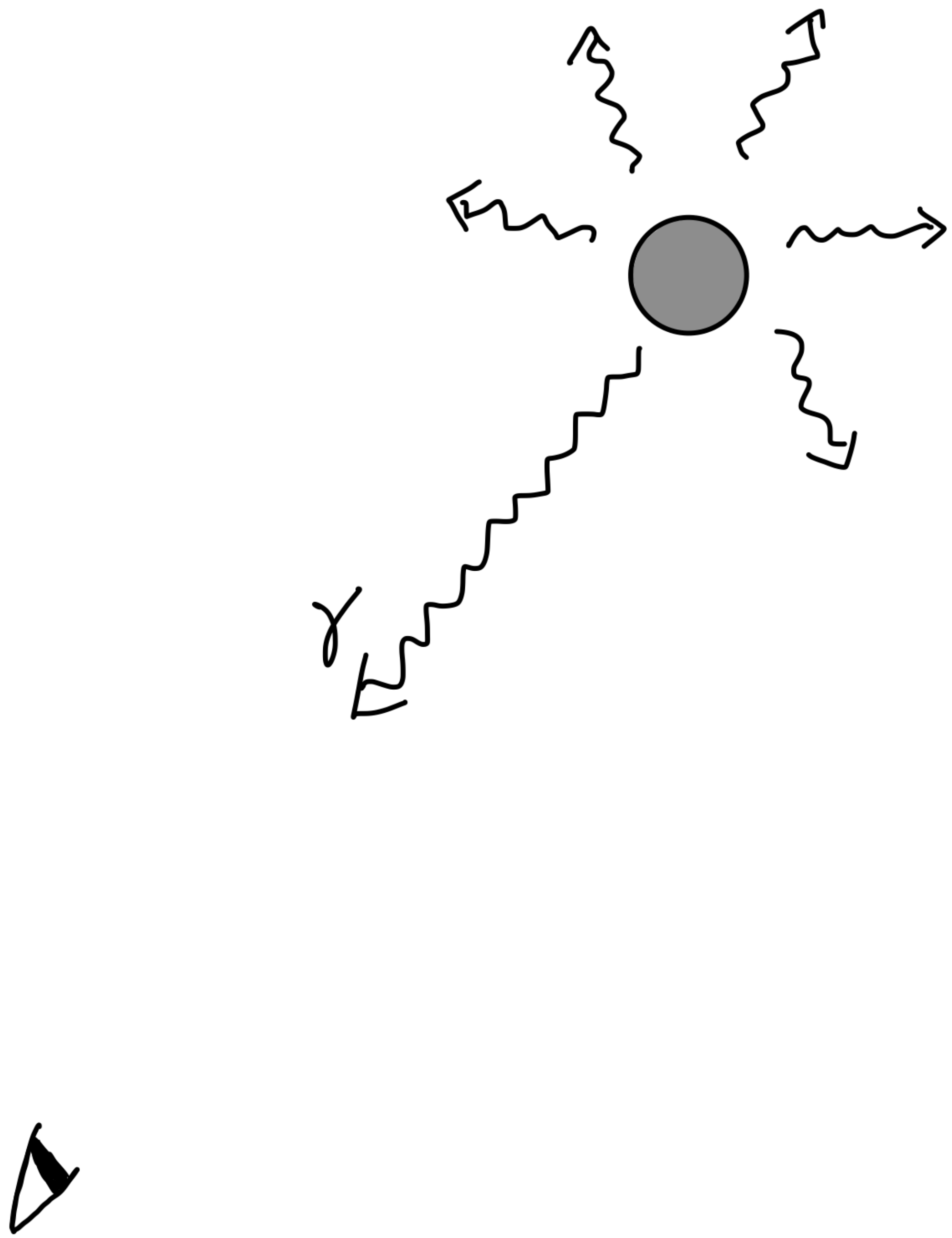


Most, if not all, data can be explained by ordinary astrophysical sources

Stringent constraint on annihilation rate

Gamma-ray flux from dark matter ~~annihilation~~ *decay*

Case of single halos



~~Annihilation~~ rate per volume

Decay

$$\cancel{\frac{\langle \sigma v \rangle \rho_\chi^2}{2 m_\chi^2}} \qquad \frac{\rho_\chi}{m_\chi \tau_\chi}$$

Differential gamma-ray luminosity

$$\mathcal{L}(E) = \cancel{\frac{\langle \sigma v \rangle}{2 m_\chi^2} \frac{dN_{\gamma,\mathrm{ann}}}{dE} \int dV \rho_\chi^2}$$

$$\frac{M}{m_\chi \tau_\chi} \frac{dN_{\gamma,\mathrm{decay}}}{dE}$$

Differential flux

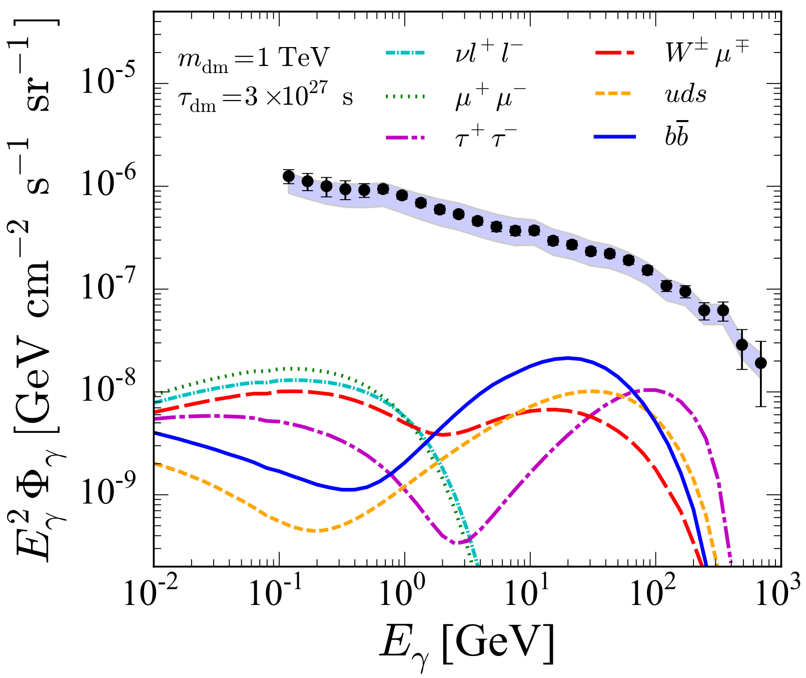
$$\mathcal{F}(E, z) = \frac{\mathcal{L}((1+z)E)}{4\pi r^2}$$

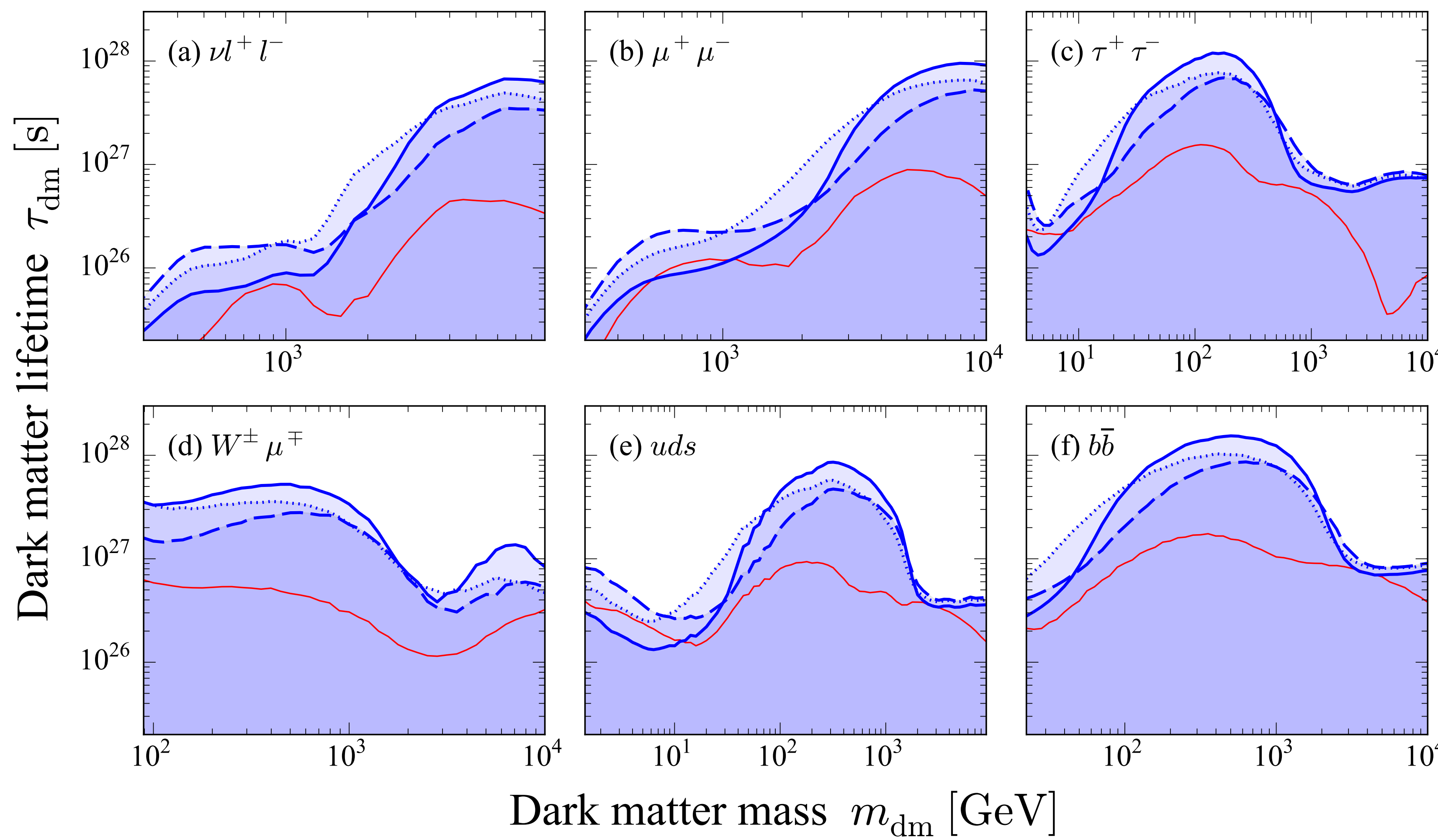
Gamma-ray flux from dark matter decay

Contribution from all the halos

$$I(E) = \frac{\Omega_\chi \rho_c}{4\pi m_\chi \tau_\chi} \int dr \, \frac{dN_{\gamma,\mathrm{decay}}}{dE'}\bigg|_{E'=(1+z)E}$$

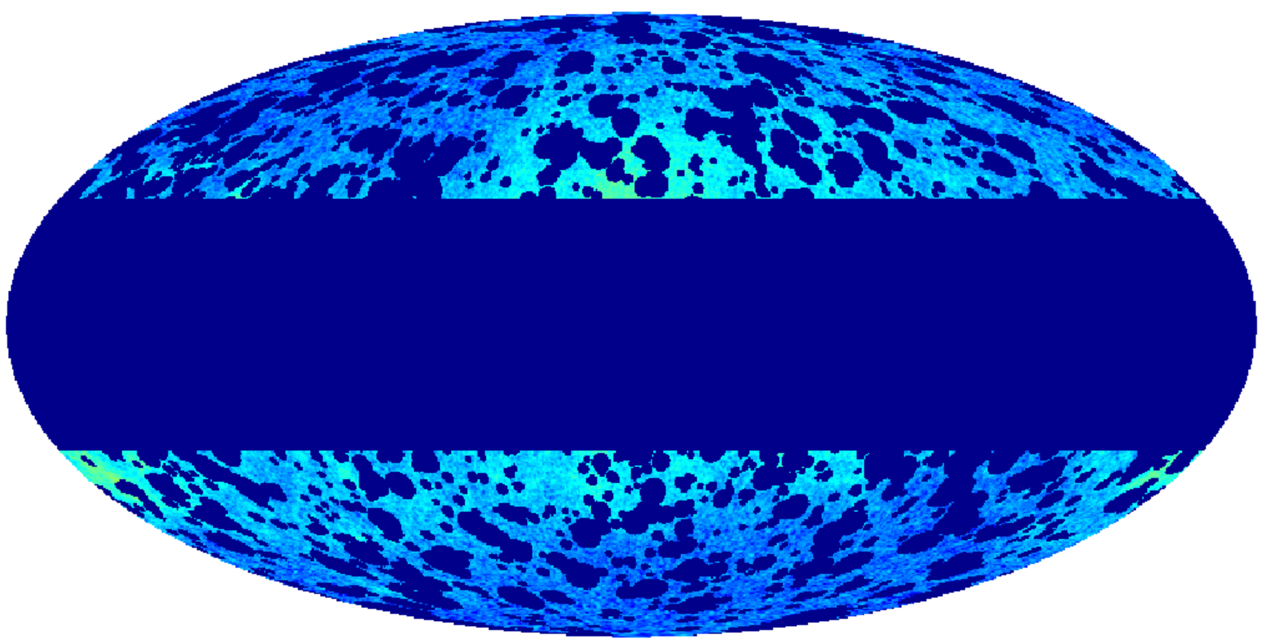
Ando, Ishiwata, *JCAP* **1506**, 024 (2015)





- Robust results independent of subhalo abundance

Multipole expansion



Spherical harmonic expansion of intensity map:

$$I(\hat{\mathbf{n}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$$

Expansion coefficients:

$$a_{\ell m} = \int d\hat{\mathbf{n}} \; I(\hat{\mathbf{n}}) Y^*_{\ell m}(\hat{\mathbf{n}})$$

Angular power spectrum (estimator):

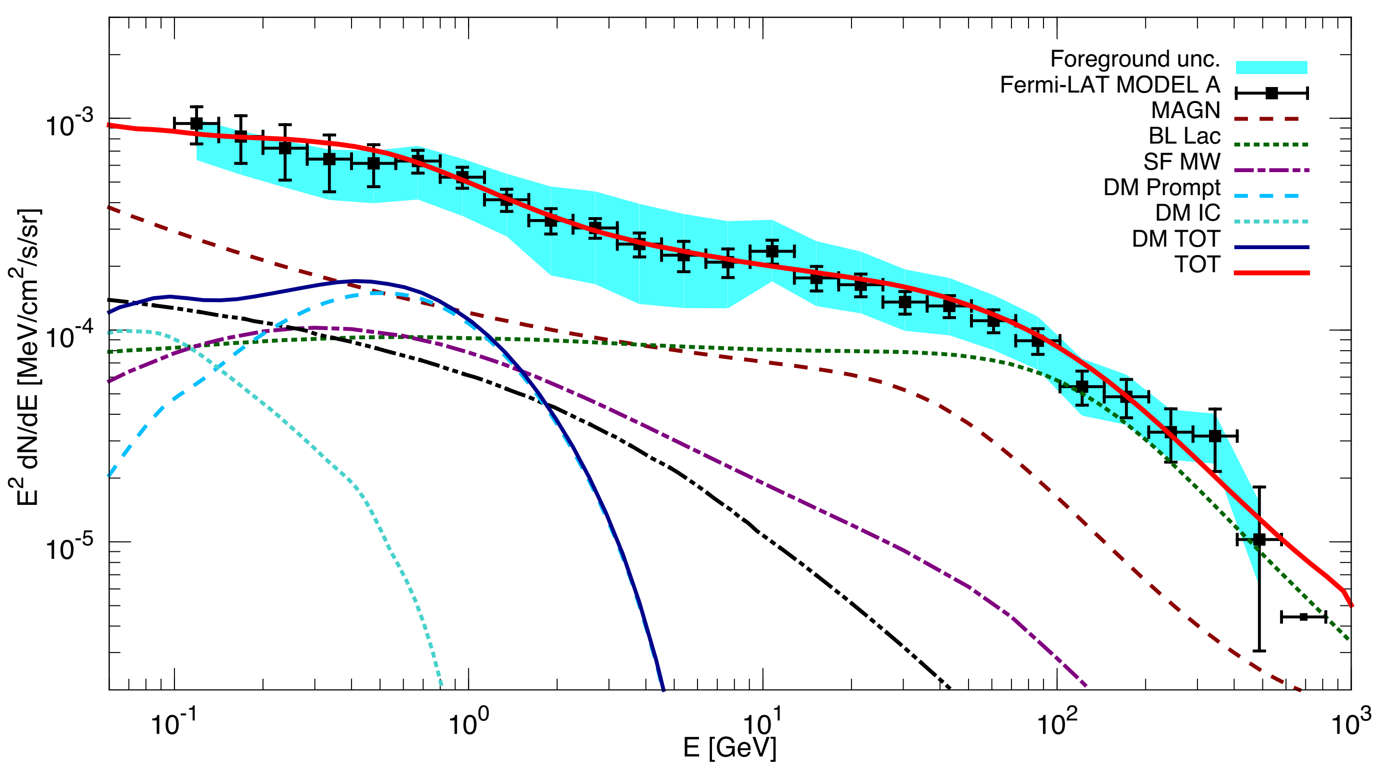
$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

Monopole: Mean (energy spectrum)

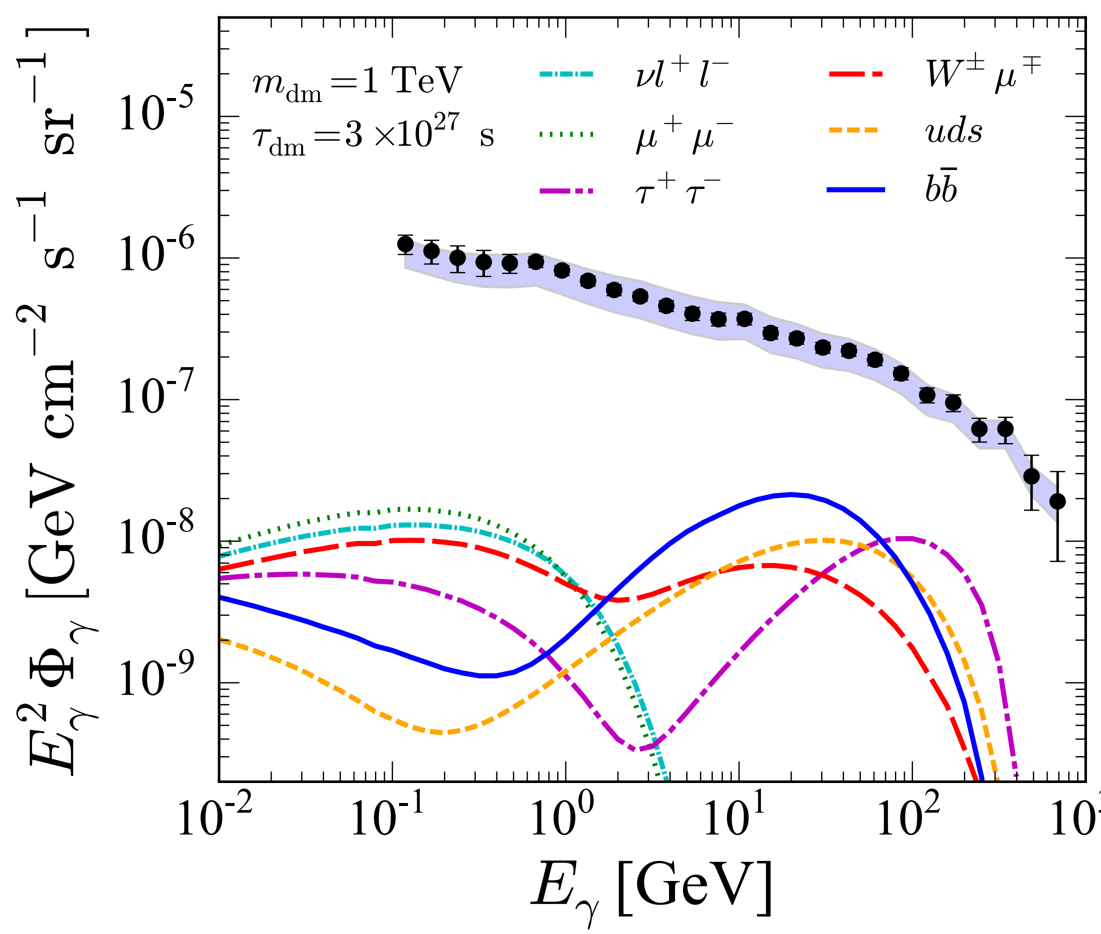
$$a_{00} = \sqrt{4\pi}\langle I(\hat{\mathbf{n}})\rangle, \quad C_0 = 4\pi\langle I(\hat{\mathbf{n}})\rangle^2$$

This is equivalent to *mean* intensity

Di Mauro, Donato, *Phys.Rev. D* **91**, 123001 (2015)



Ando, Ishiwata, *JCAP* **1506**, 024 (2015)



Angular power: Beyond monopole

Measured C_l

$$C_\ell^{\rm total} = C_N + C_P + C_\ell^{\rm correlation}$$

***NOISE* (1p)**

Shot noise of photons

$$\propto \langle I(\hat{\mathbf{n}})\rangle^2 / N_{\rm event}$$

Intrinsic clustering of sources

***SIGNAL* (2p)**

Shot noise due to discreteness of sources

$$\propto 1/N_{\rm source}$$

***SIGNAL* (1p)**

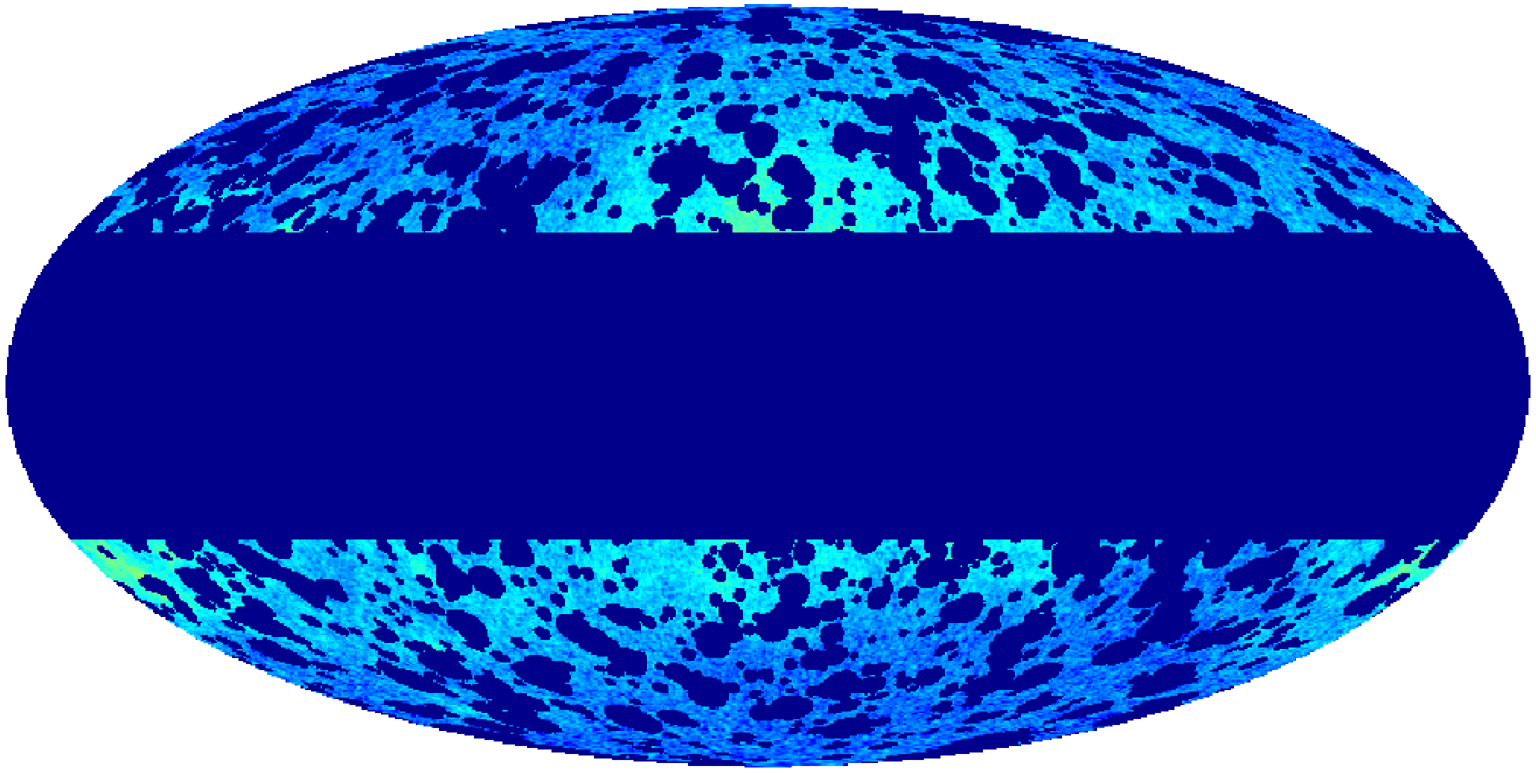
One can use information of all the multipoles up to $\ell_{\rm max} = \dfrac{180^\circ}{\theta_{\rm PSF}}$

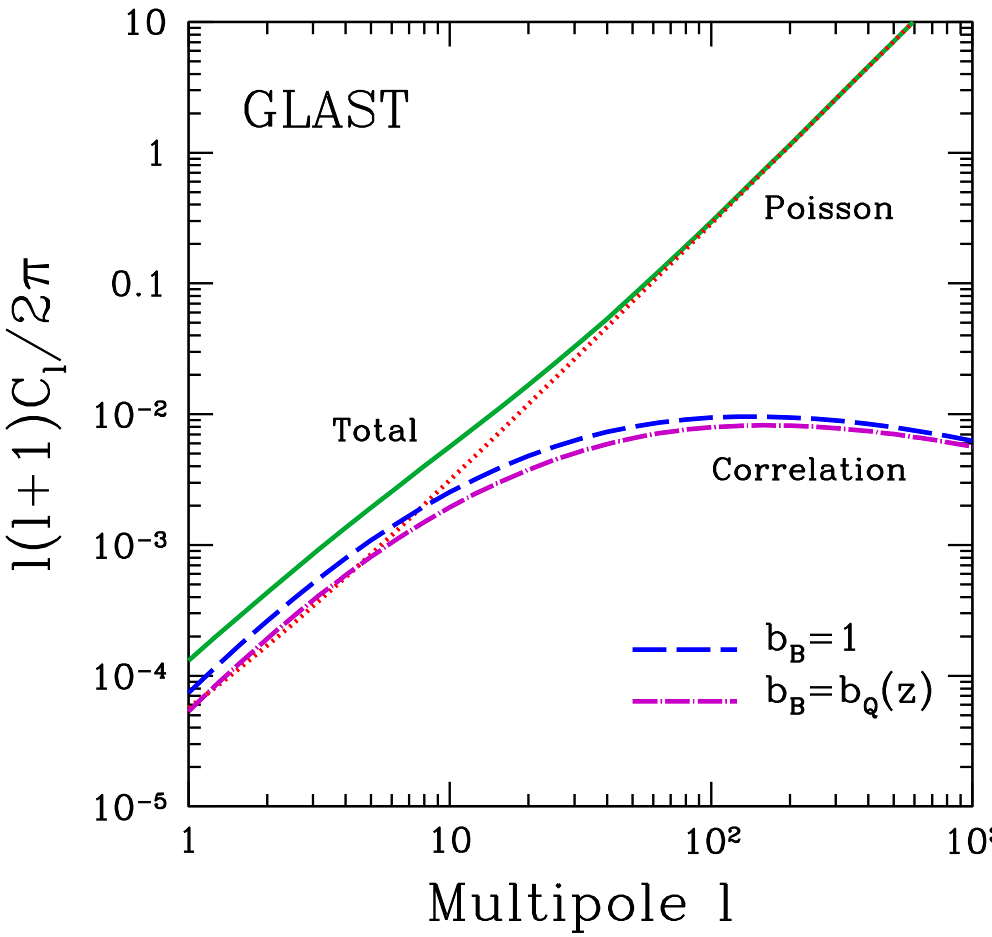
Angular power: Beyond monopole

Measured C_l

$$C_\ell^{\mathrm{total}} = C_N + C_P + C_\ell^{\mathrm{correlation}}$$

Example 1: **Blazars (point sources) for Fermi γ background**





Ando et al., *Phys. Rev. D* **75**, 063519 (2007)

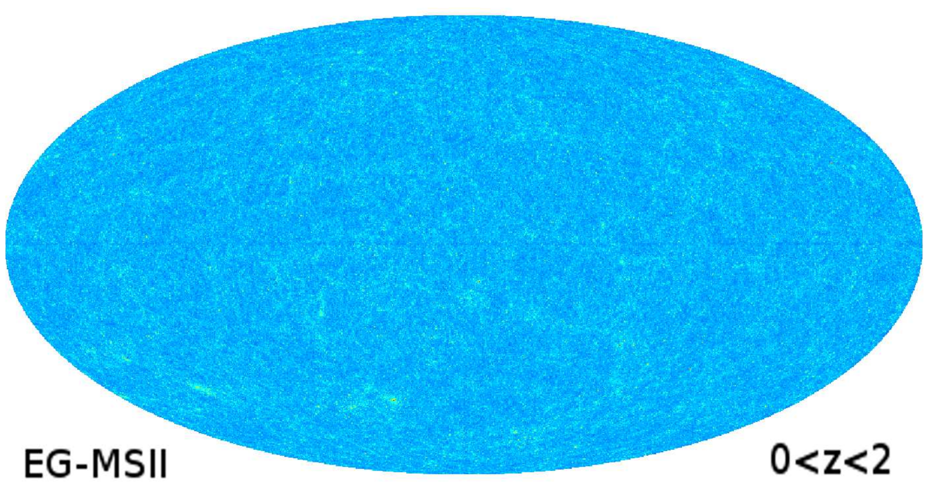
Angular power: Beyond monopole

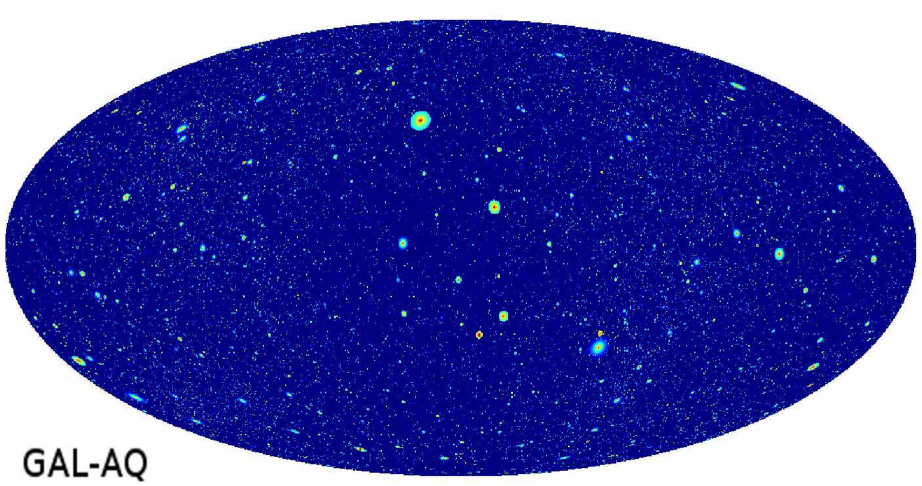
Measured C_l

$$C_\ell^{\rm total} = C_N + C_P + C_\ell^{\rm correlation}$$

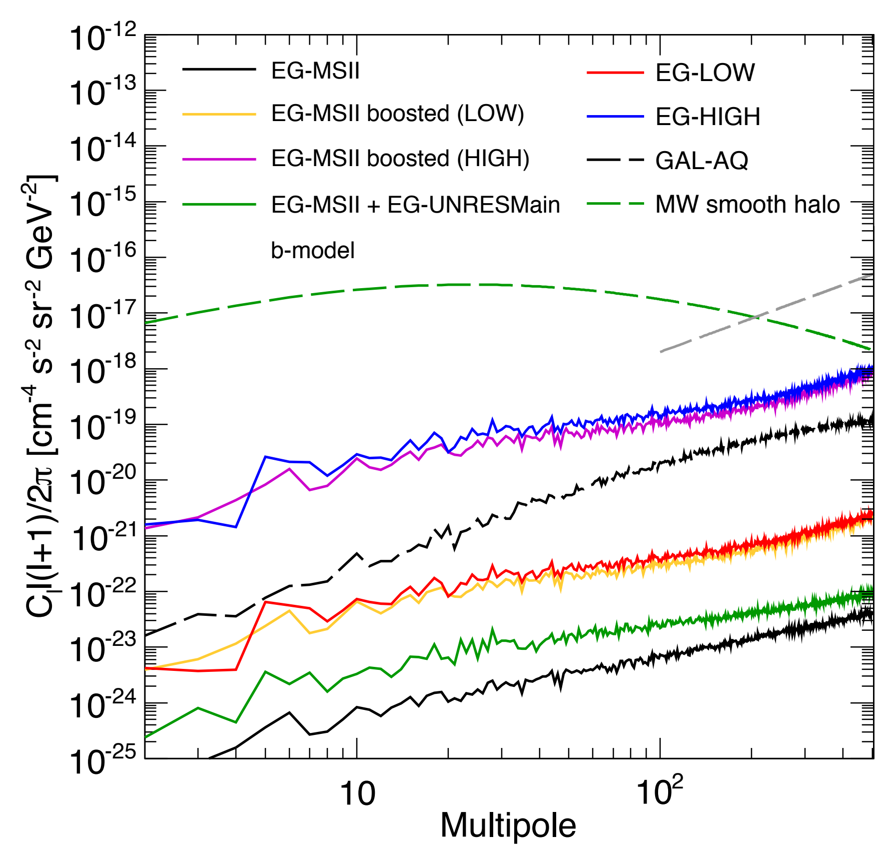
Example 2: **Dark matter annihilation**

Fornasa et al., *Mon. Not. R. Astron. Soc.* **429**, 1529 (2013)

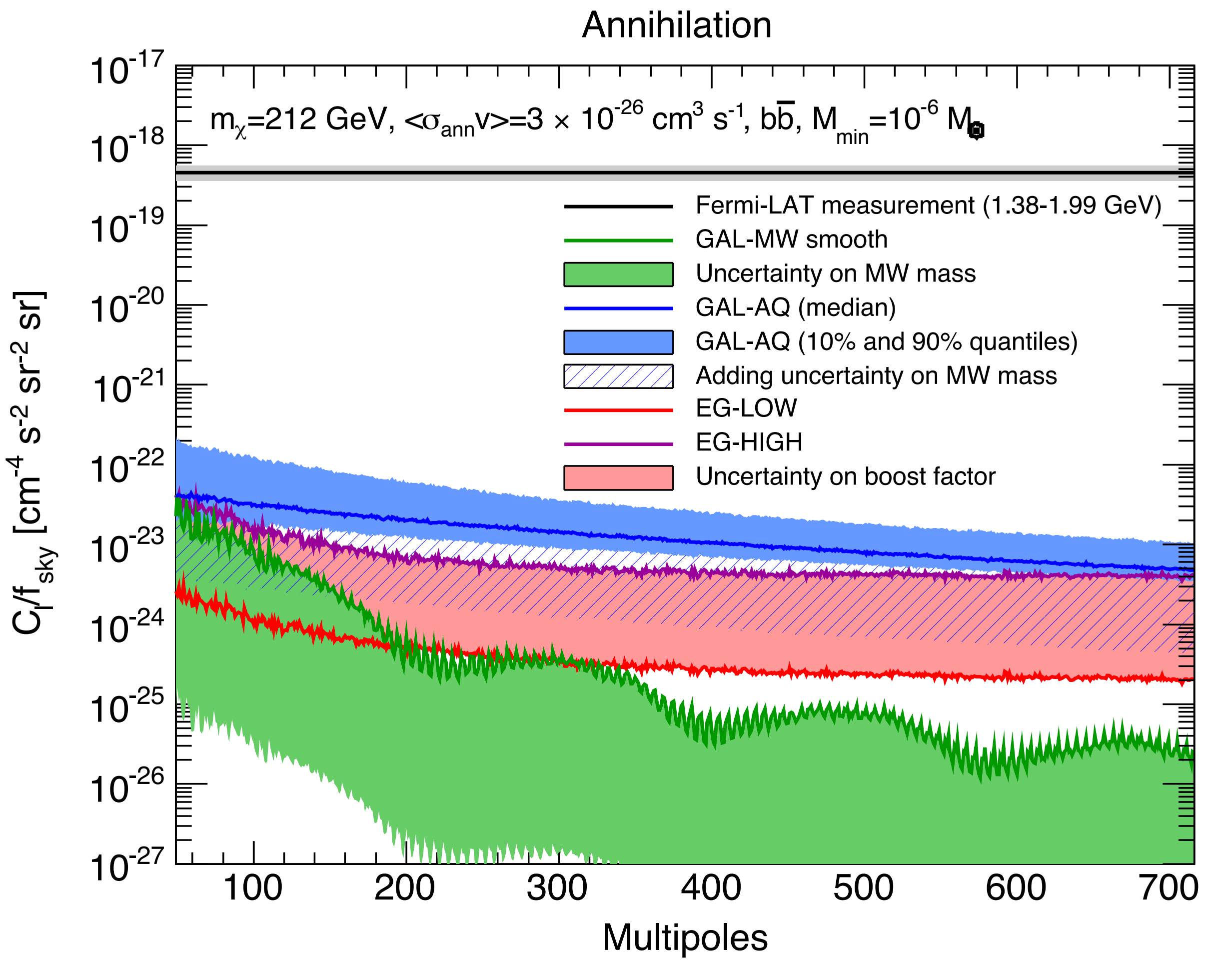




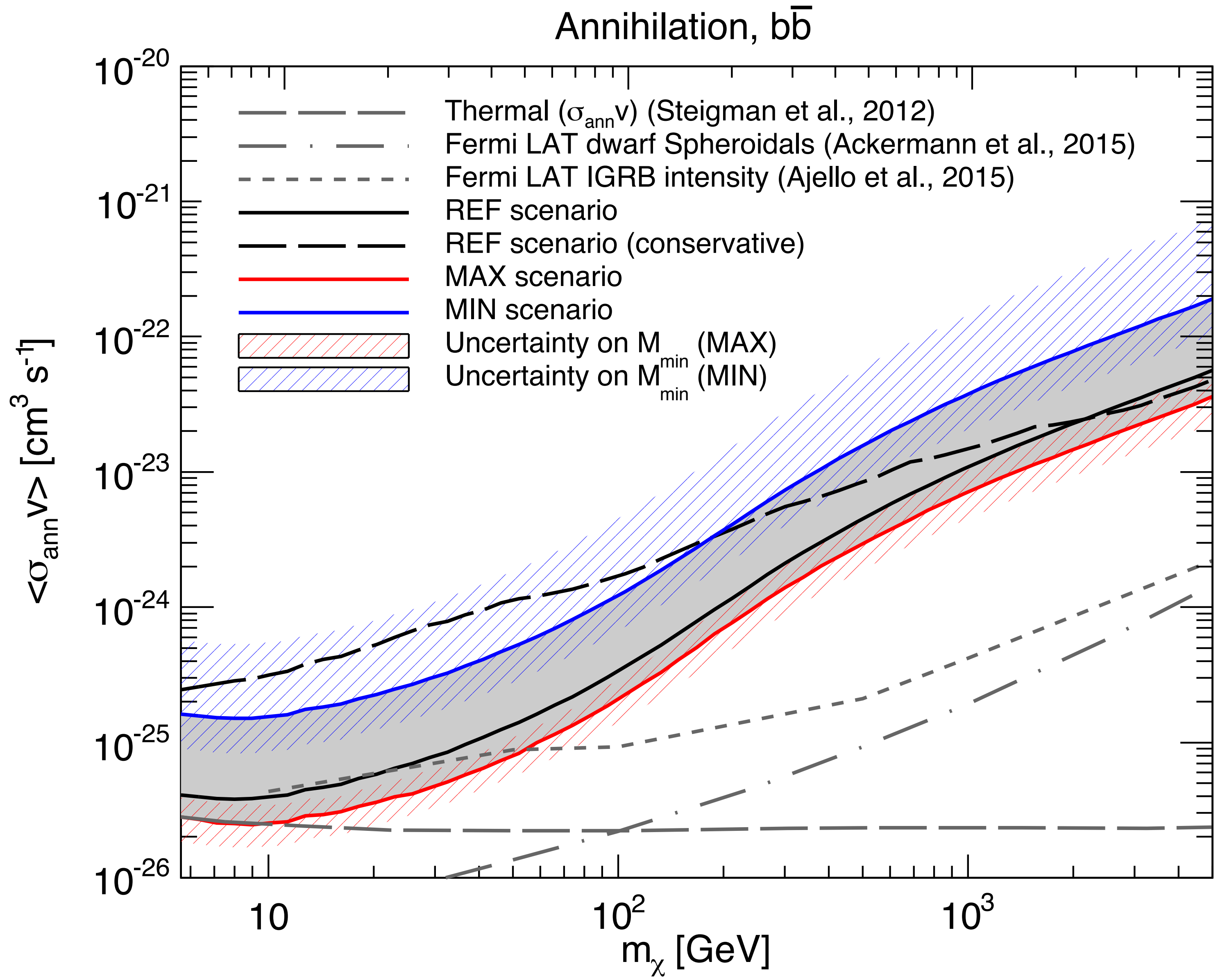
- Characteristic angular pattern due to ρ^2 dependence



Constraints on dark matter annihilation



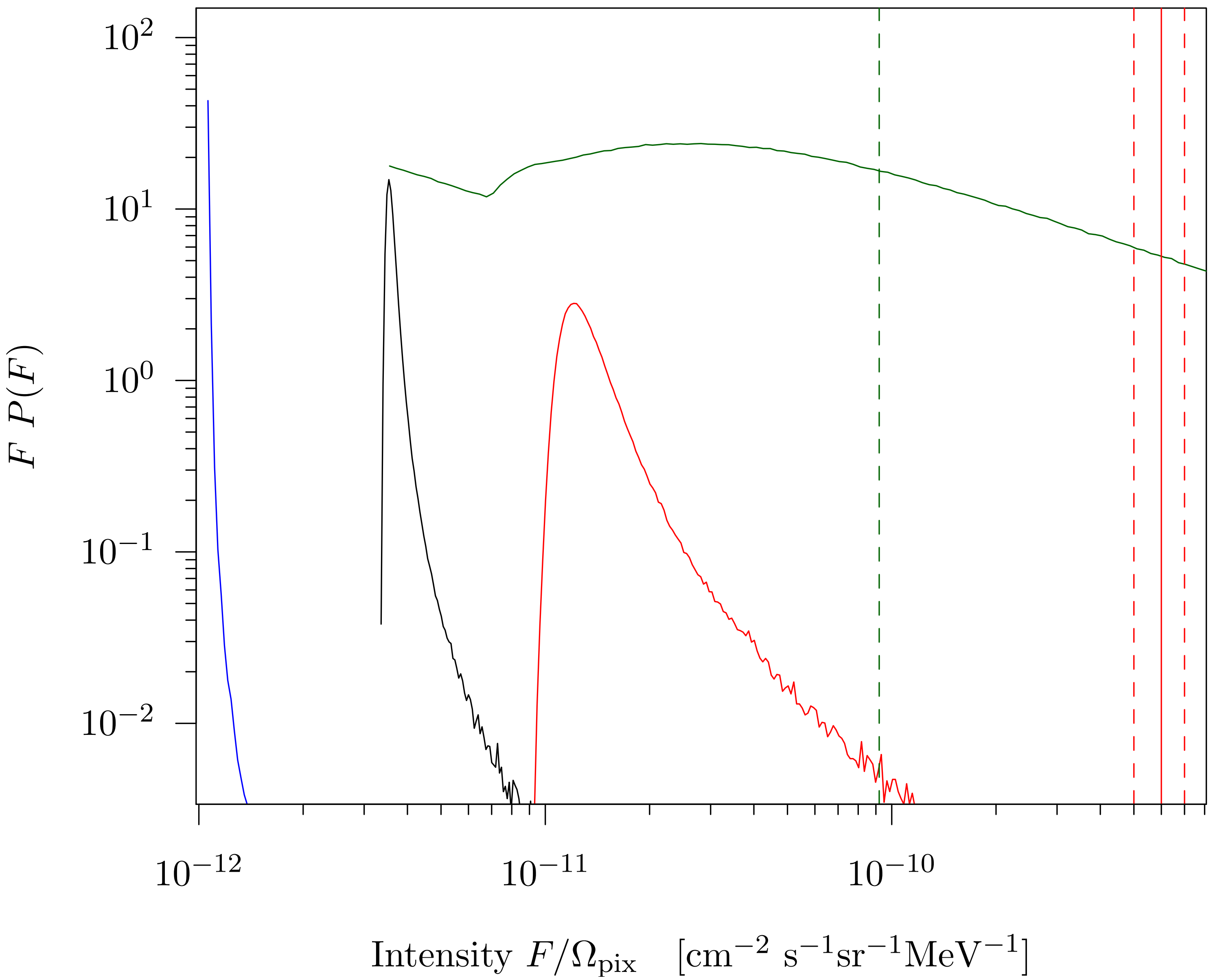
Fornasa et al. *Phys. Rev. D* **94**, 123005 (2016)



- Analysis including C_P components and dark matter
- Limits on annihilation cross section are comparable to spectral analysis at small masses

Complementary approach using one-point statistics

- Flux PDF is highly non-Gaussian, featuring long power-law tail
- Power spectrum does *not* capture all the statistical information



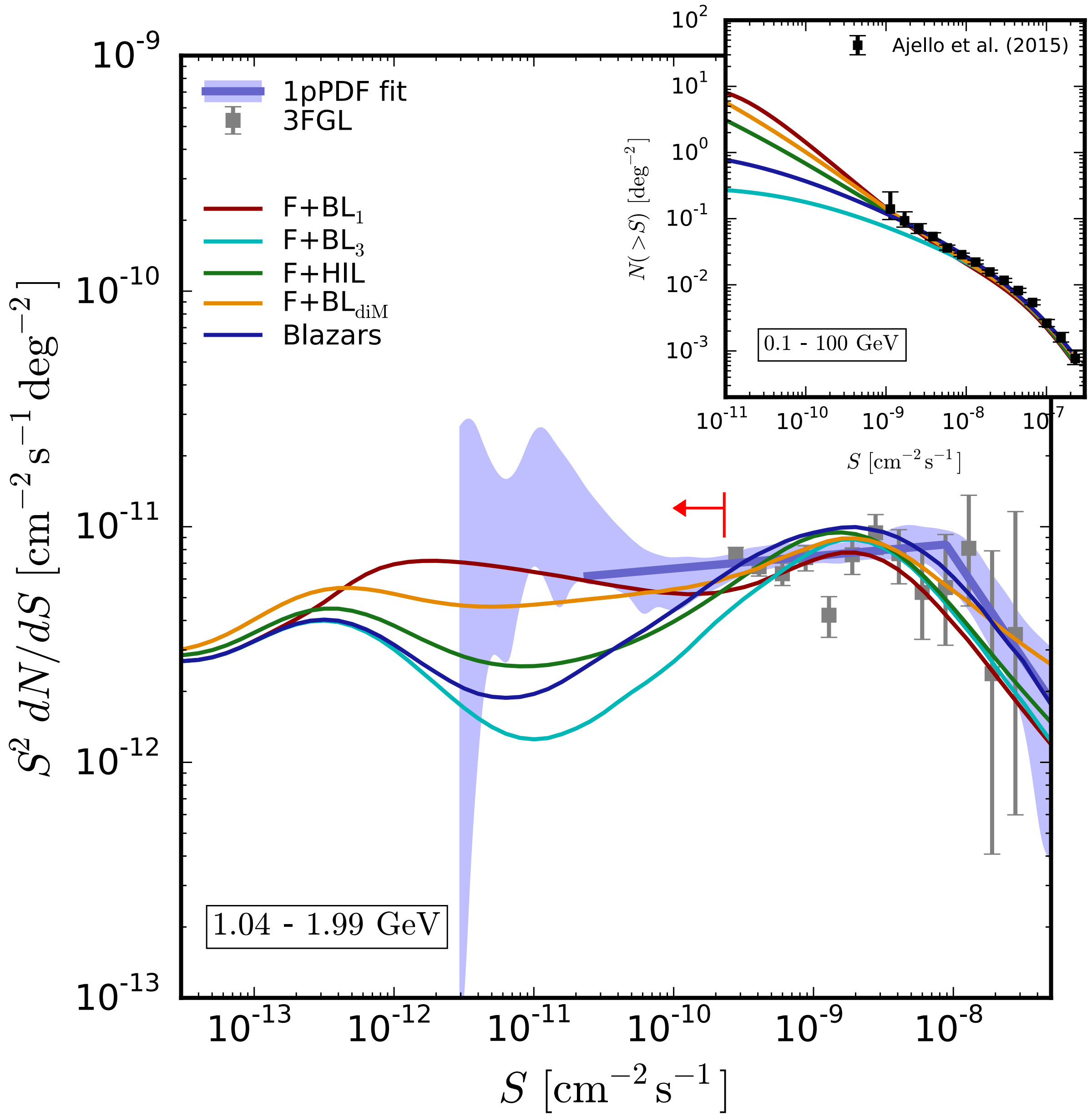
Advantage:
It includes **all the statistical information** in single pixels

Disadvantage:
Analysis much more complicated

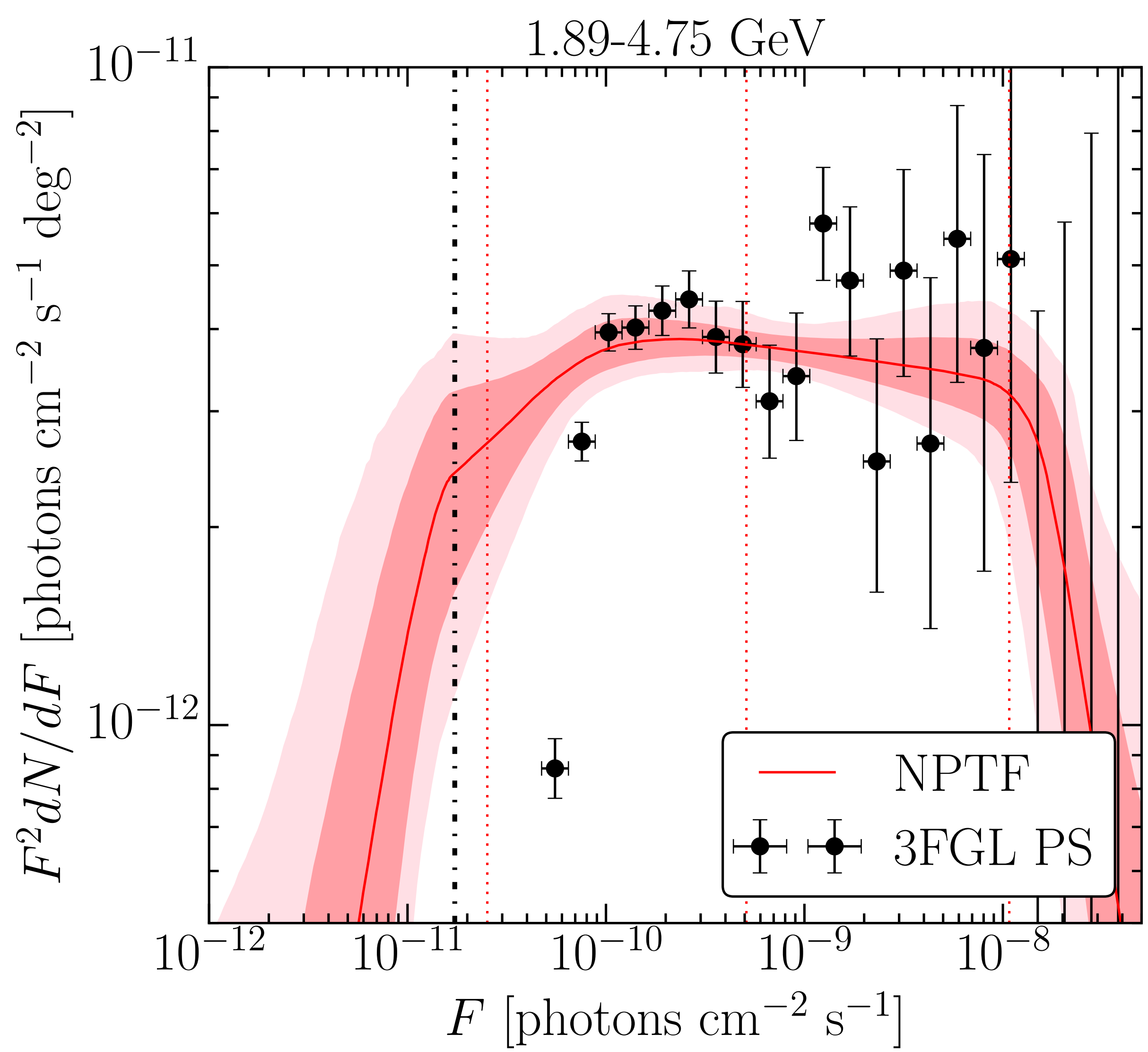
Feyereisen, Ando, Lee, *JCAP* **1509**, 027 (2015)

Complementary approach using one-point statistics

- Flux PDF is highly non-Gaussian, featuring long power-law tail
- Power spectrum does *not* capture all the statistical information



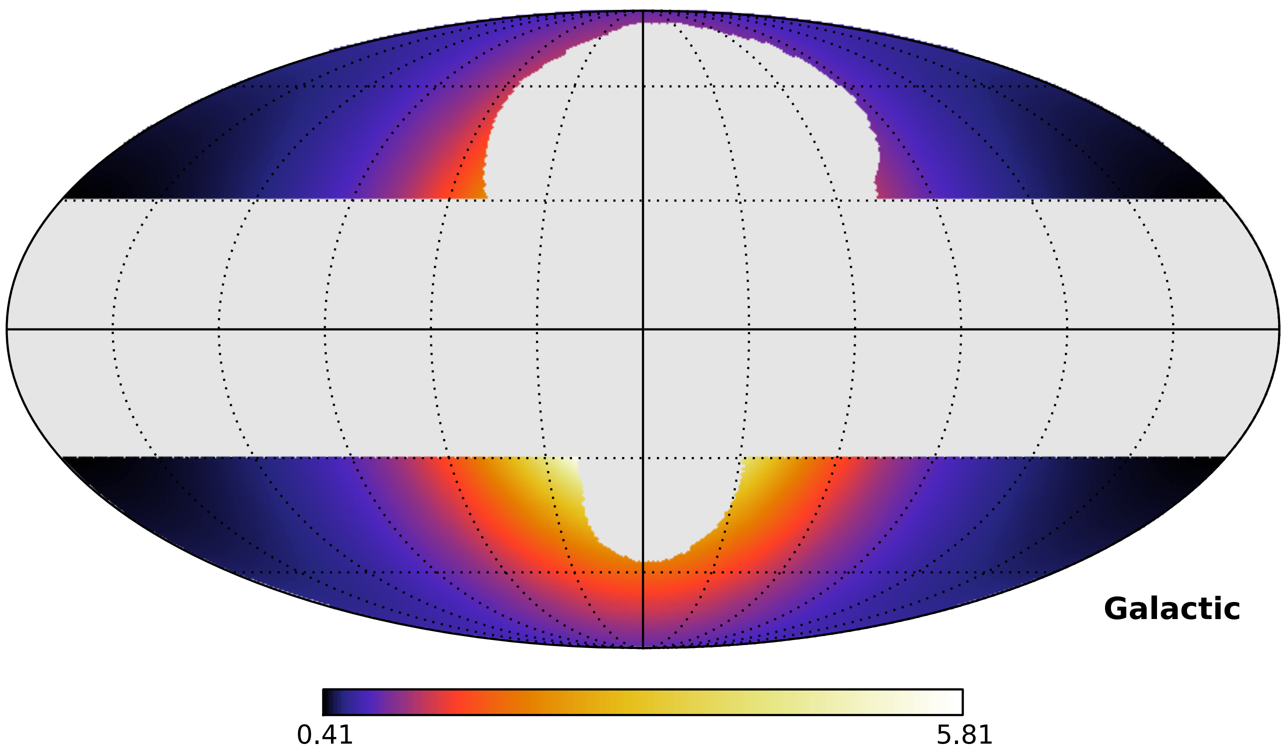
Zechlin et al., *Astrophys. J.* **826**, L31 (2016)

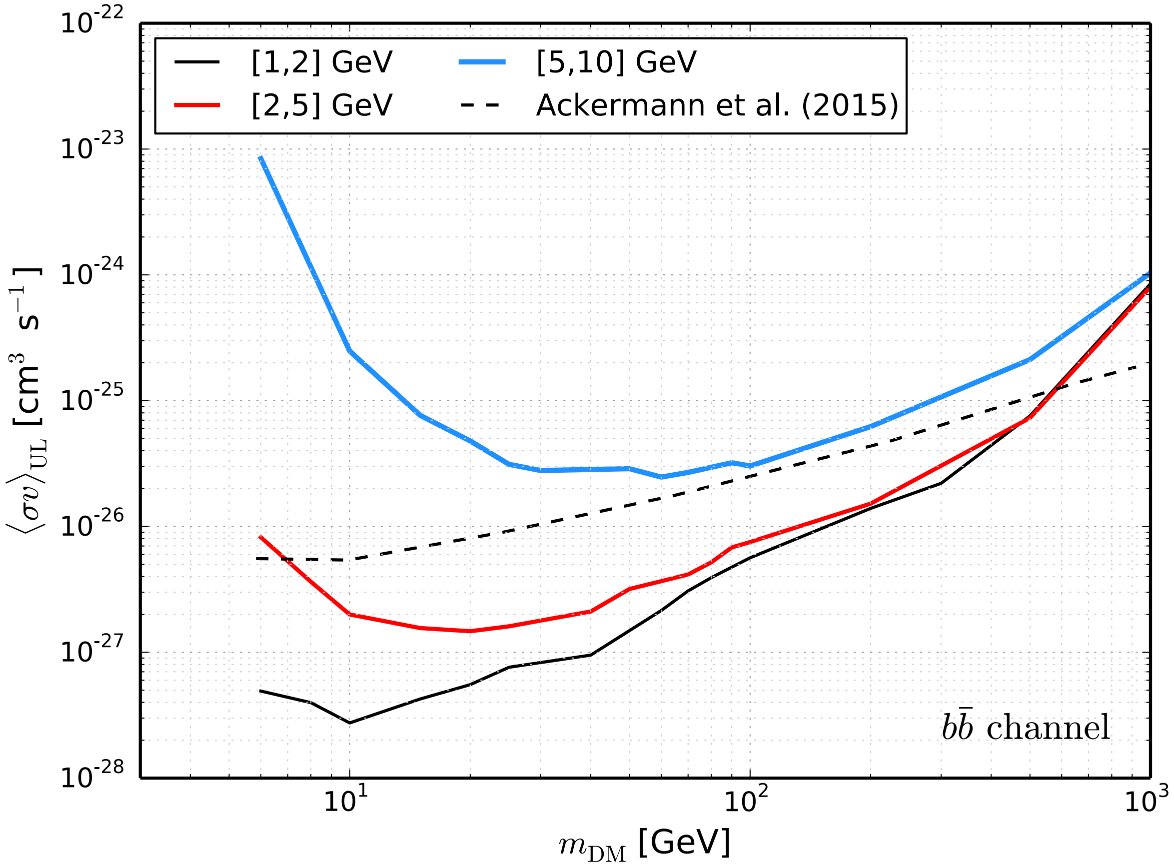


Lisanti et al., *Astrophys. J.* **832**, 117 (2016)

Constraining Galactic dark matter with one-point PDF

Zechlin, Manconi, Donato, arXiv:1710.01506 [astro-ph.HE]

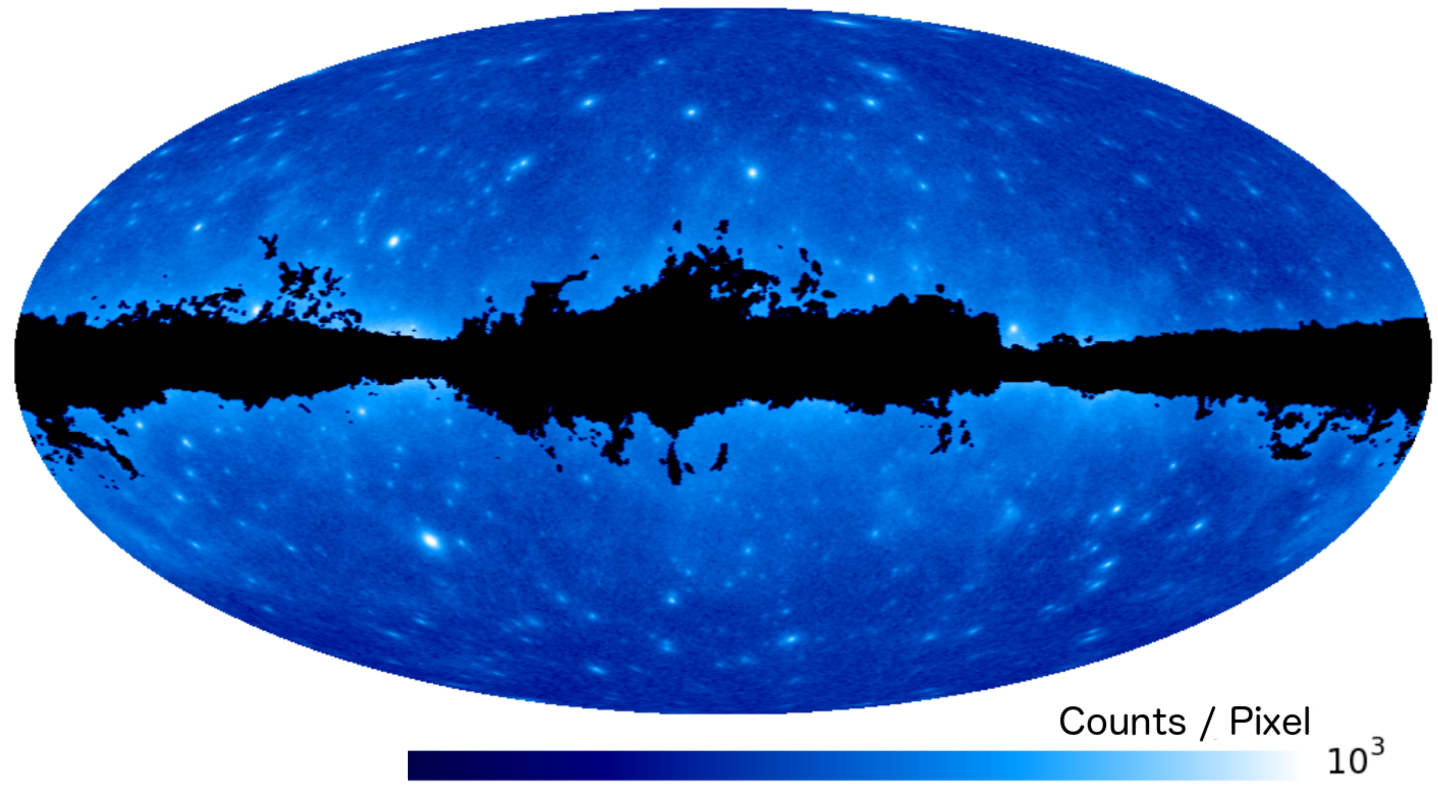




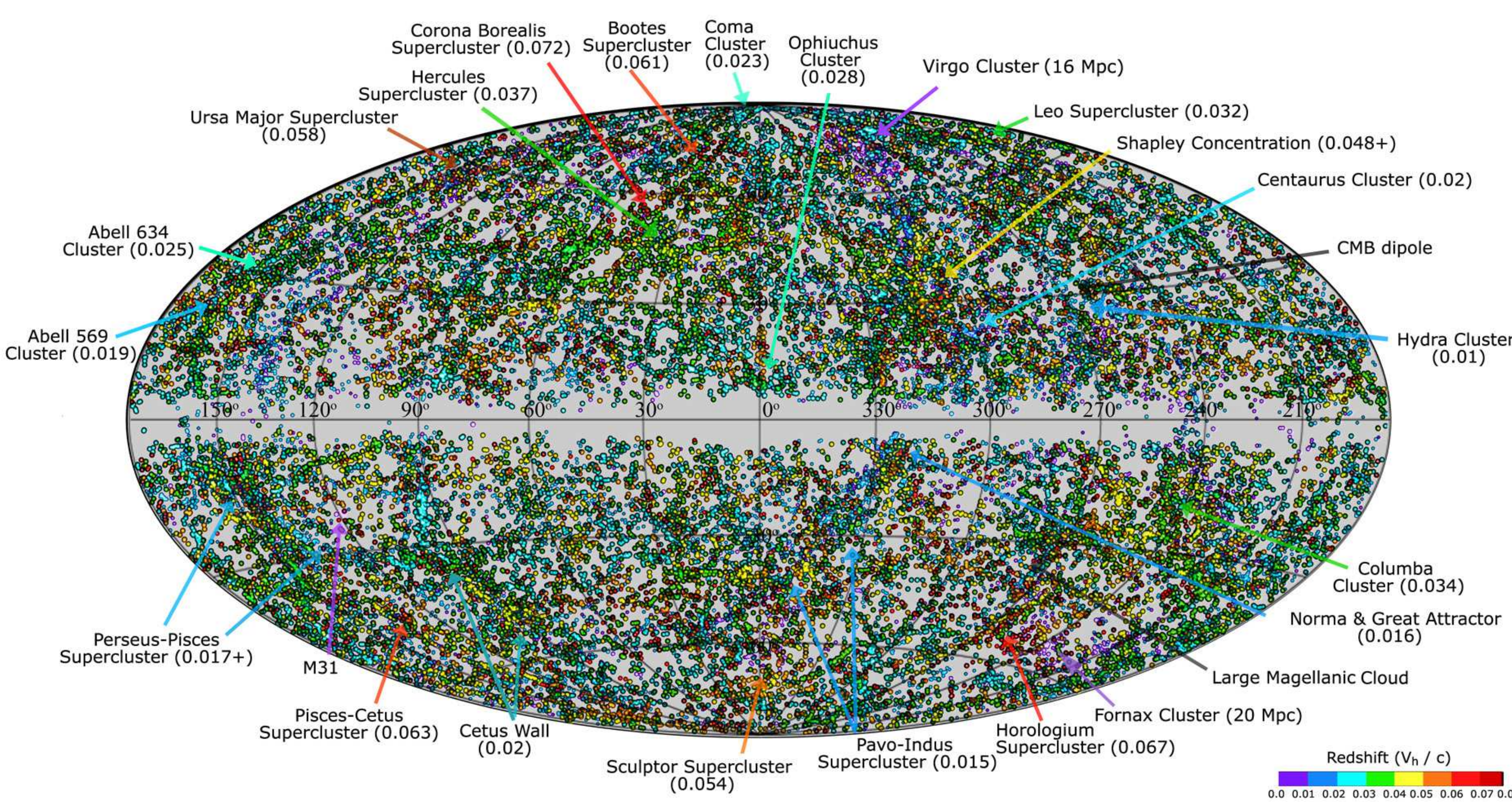
- Constraining Galactic dark matter annihilation using Einasto density profile and high-latitude data
- Surprisingly stringent constraints on the annihilation cross section, partly because of better calibration of unresolved point sources
- Order of magnitude uncertainty according to foreground models

Cross correlation with galaxy distribution

Are these two maps similar to each other?



Fermi-LAT, *Astrophys. J.* **799**, 86 (2015)

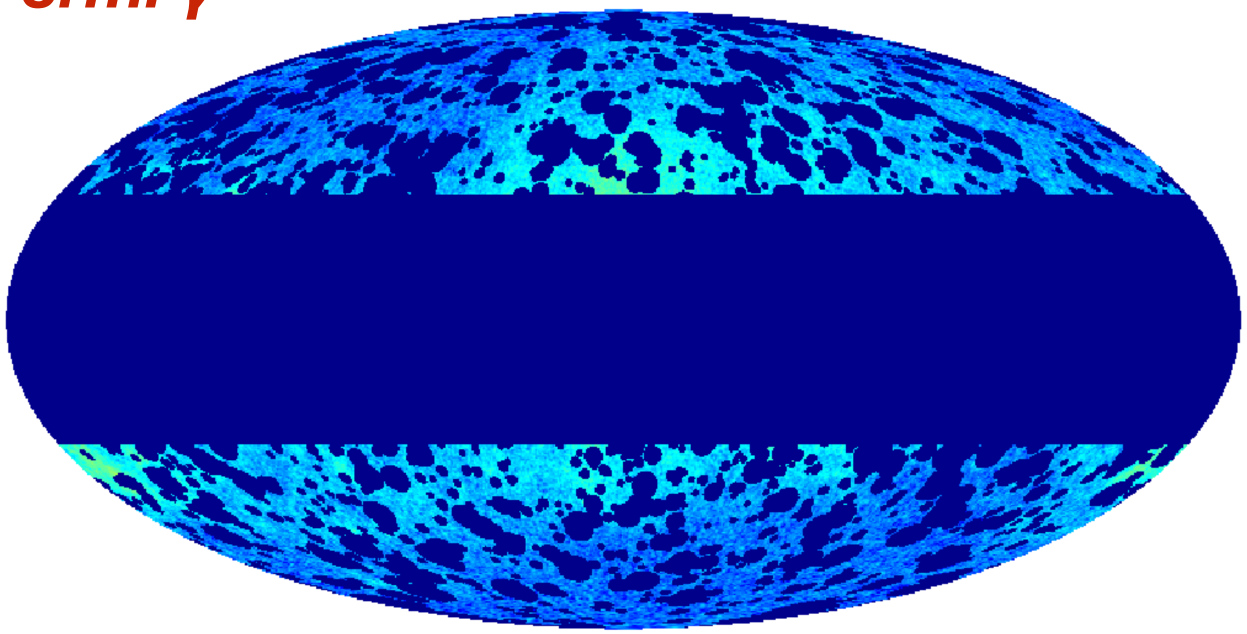


Huchra et al., *Astrophys. J. Suppl. Ser.* **199**, 26 (2011)

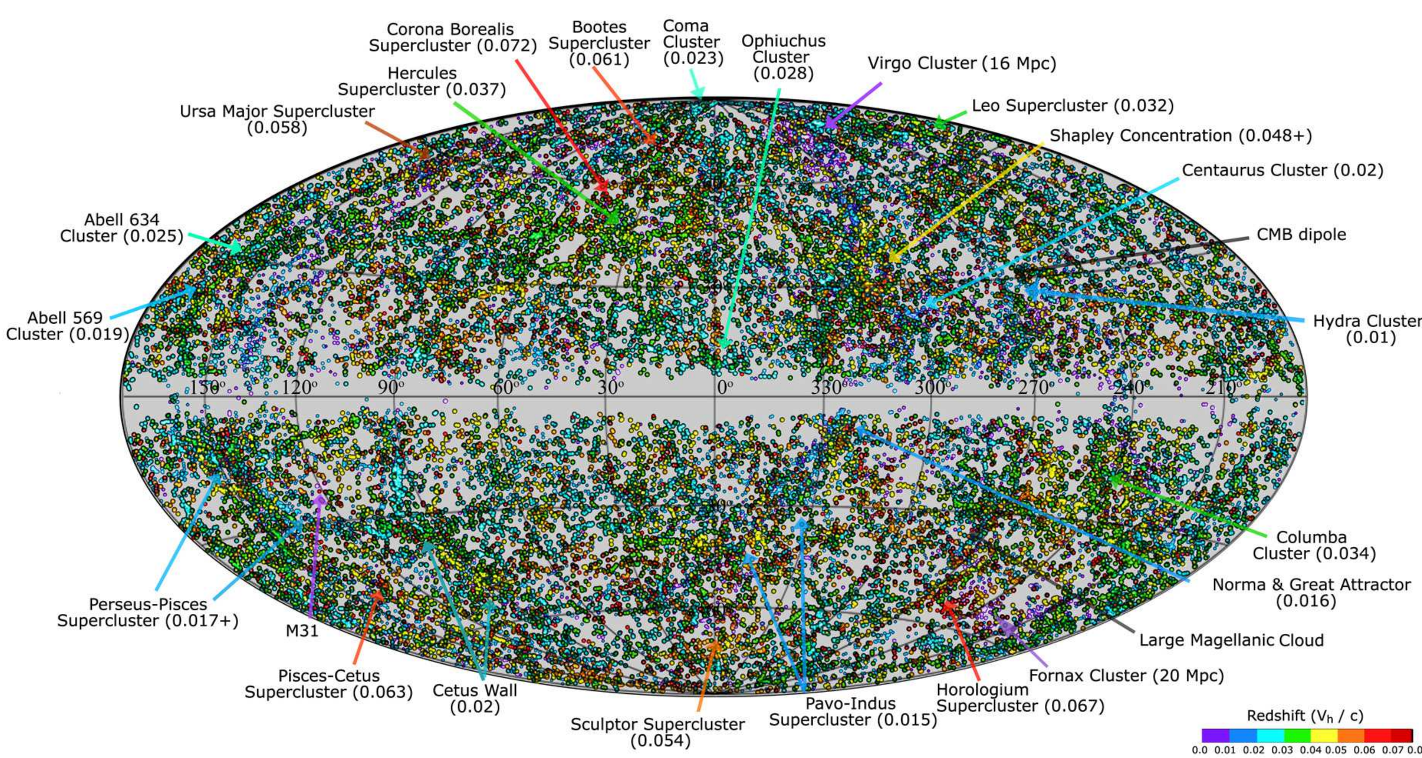
They must be, since both gamma-ray sources and galaxies trace dark matter distribution!

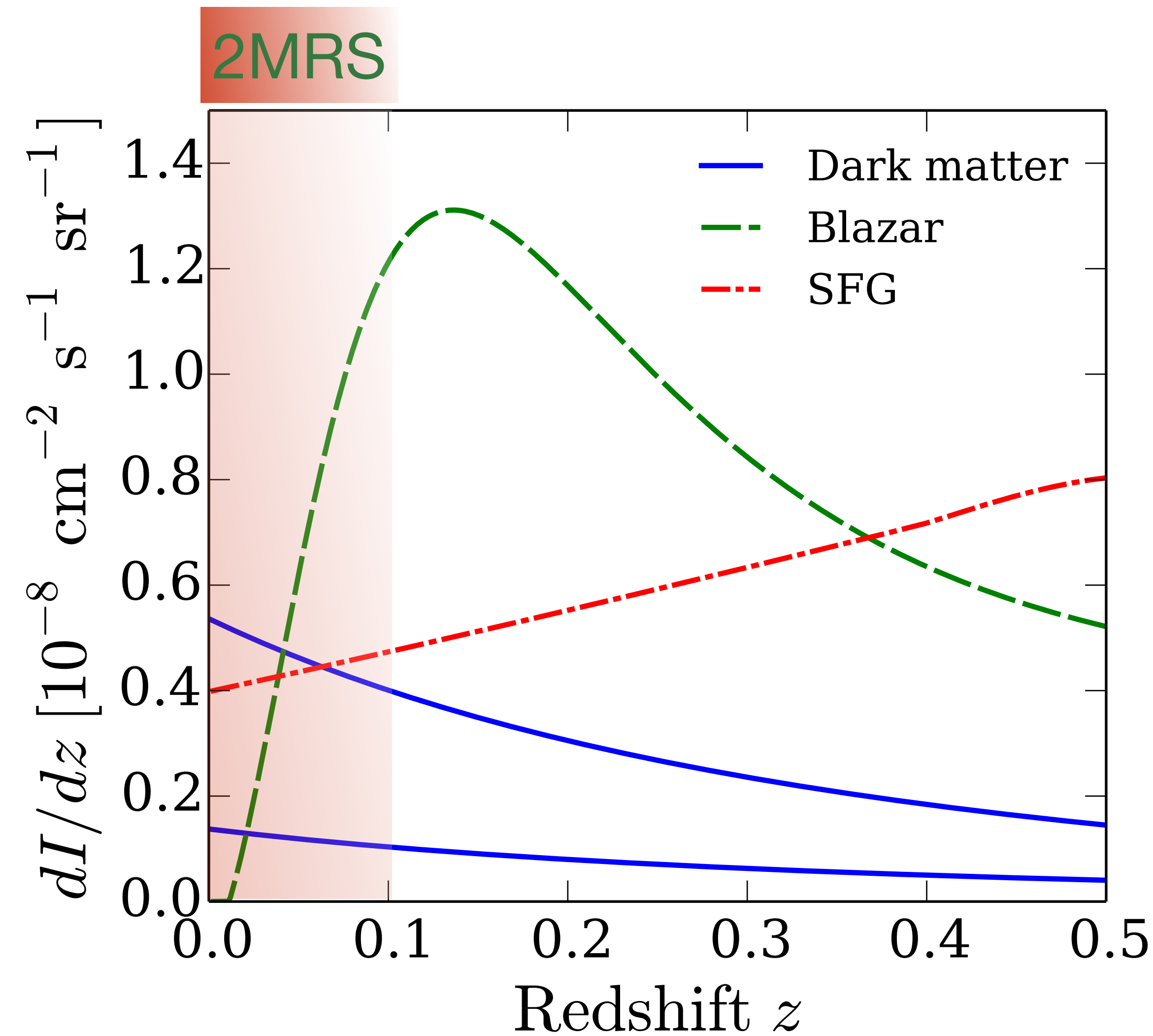
Gamma-ray background tomography

Fermi γ



2MRS galaxies

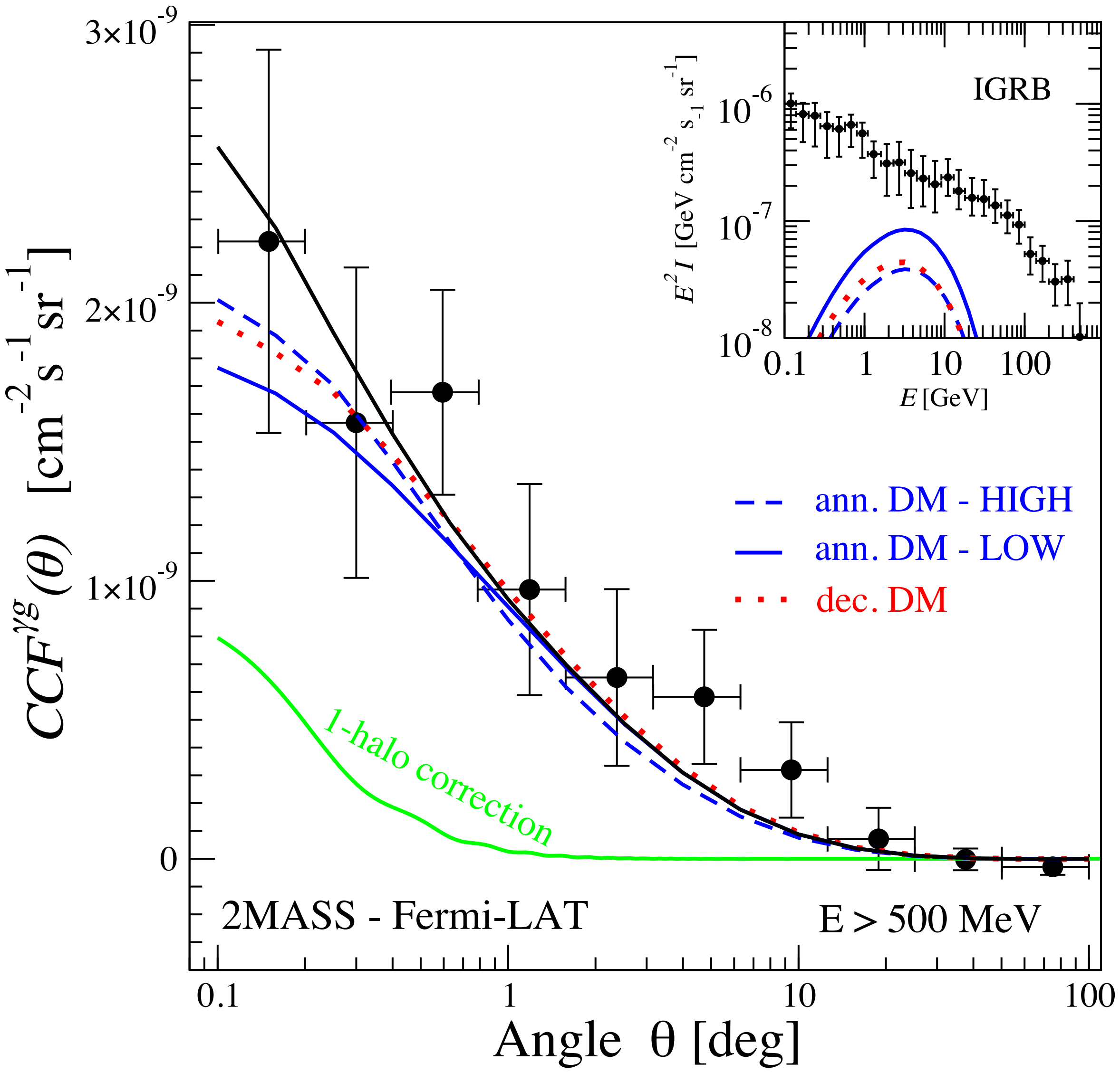




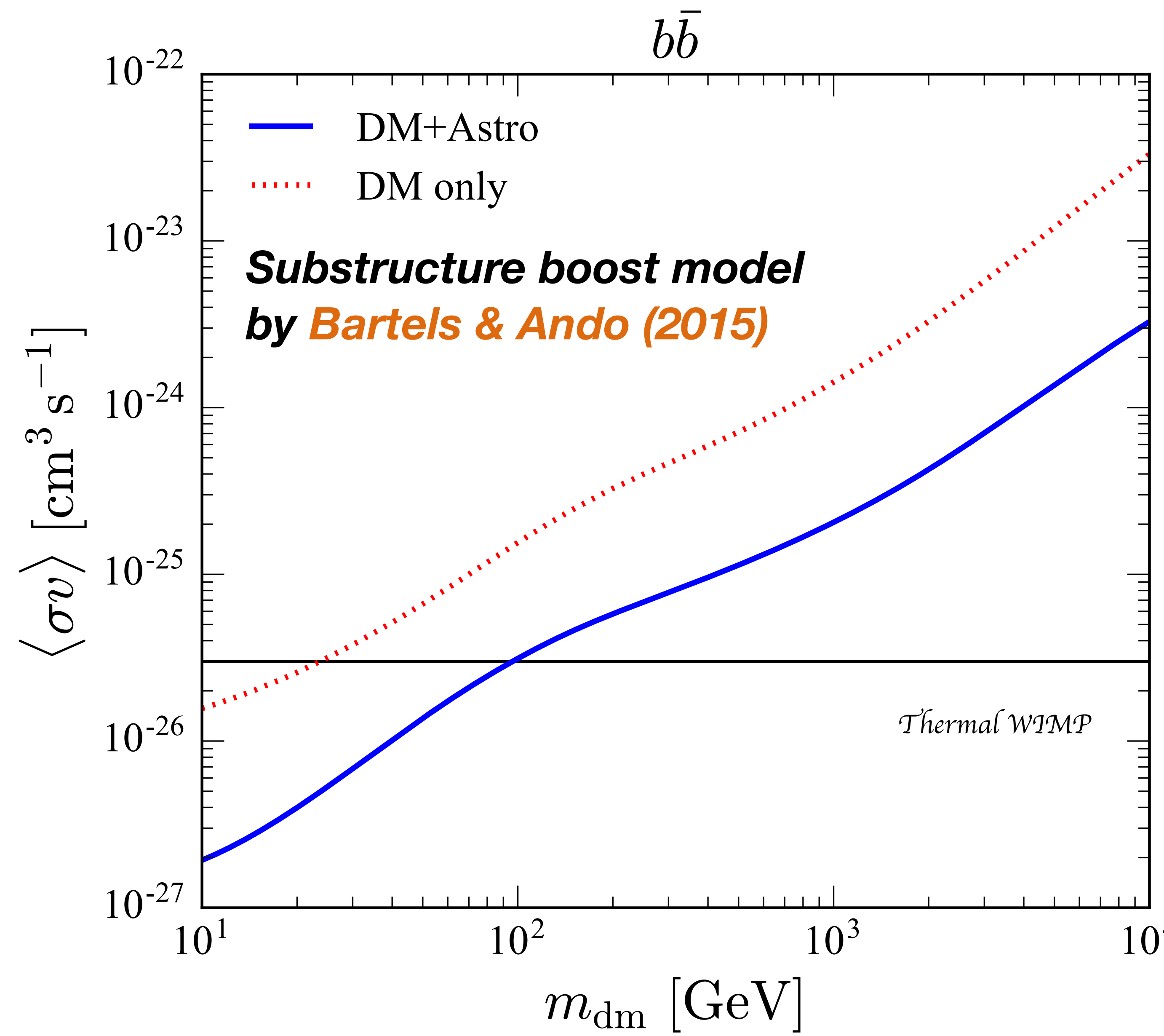
- **DM contribution is relatively more important at z < 0.1**
- Cross-correlating with 2MRS galaxies will single out this redshift region!

Cross correlation with 2MASS: DM constraints

Regis et al., *Phys. Rev. Lett.* **113**, 241301 (2015)



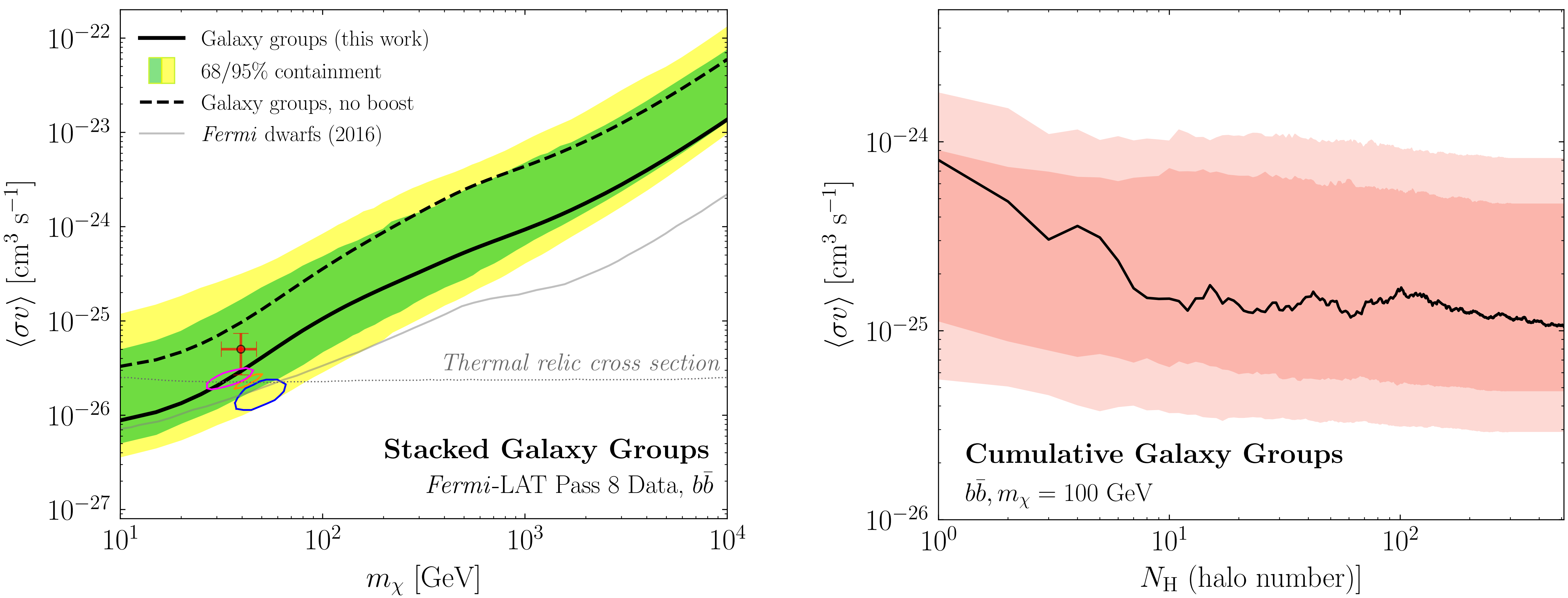
Ando, Ishiwata, *JCAP* **1606**, 045 (2016)



- Very competitive bounds even in the dark matter only case

Complementary method: Stacking groups

Lisanti et al., arXiv:1708.09385 [astro-ph.CO]



- Stacking analysis of hundreds of 2MRS galaxy groups (Tully 2015+2017)

Summary: WIMP annihilation

- WIMPs keep annihilating ever since, forming the diffuse gamma-ray background
- The data from Fermi-LAT yields **competitive upper limits on WIMP annihilation cross section**
  - This is the **only probe of the smallest halos** ever formed (micro-halos)
- Other statistical probes started to be explored, especially in recent years
  - **Angular power spectrum** (Ando & Komatsu 2006, Fornasa et al. 2016, etc.)
  - **One-point fluctuation statistics** (Feyereisen, Ando, Lee 2015, etc.)
  - **Cross correlation with gravitational tracers** (Ando et al. 2015, Cuoco et al. 2017, etc.)
  - **Stacking of nearby galaxy groups** (Lisanti et al. 2017)

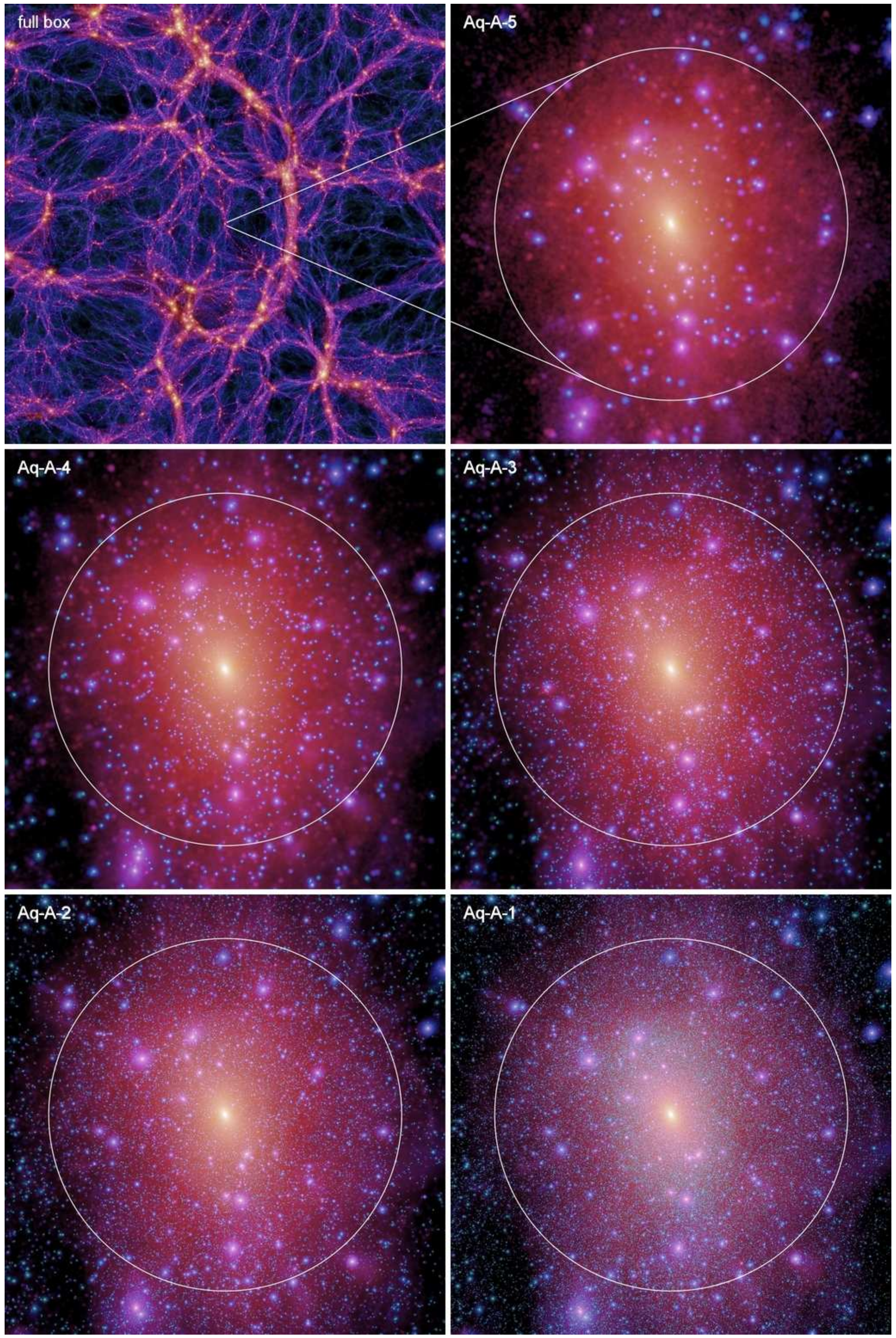
Key questions to reveal nature of dark matter

- Does dark matter ***interact with standard model*** particles such as photons? Is “dark” matter really dark?
- Is dark matter ***collisionless*** with each other?
- How ***cold*** is dark matter?

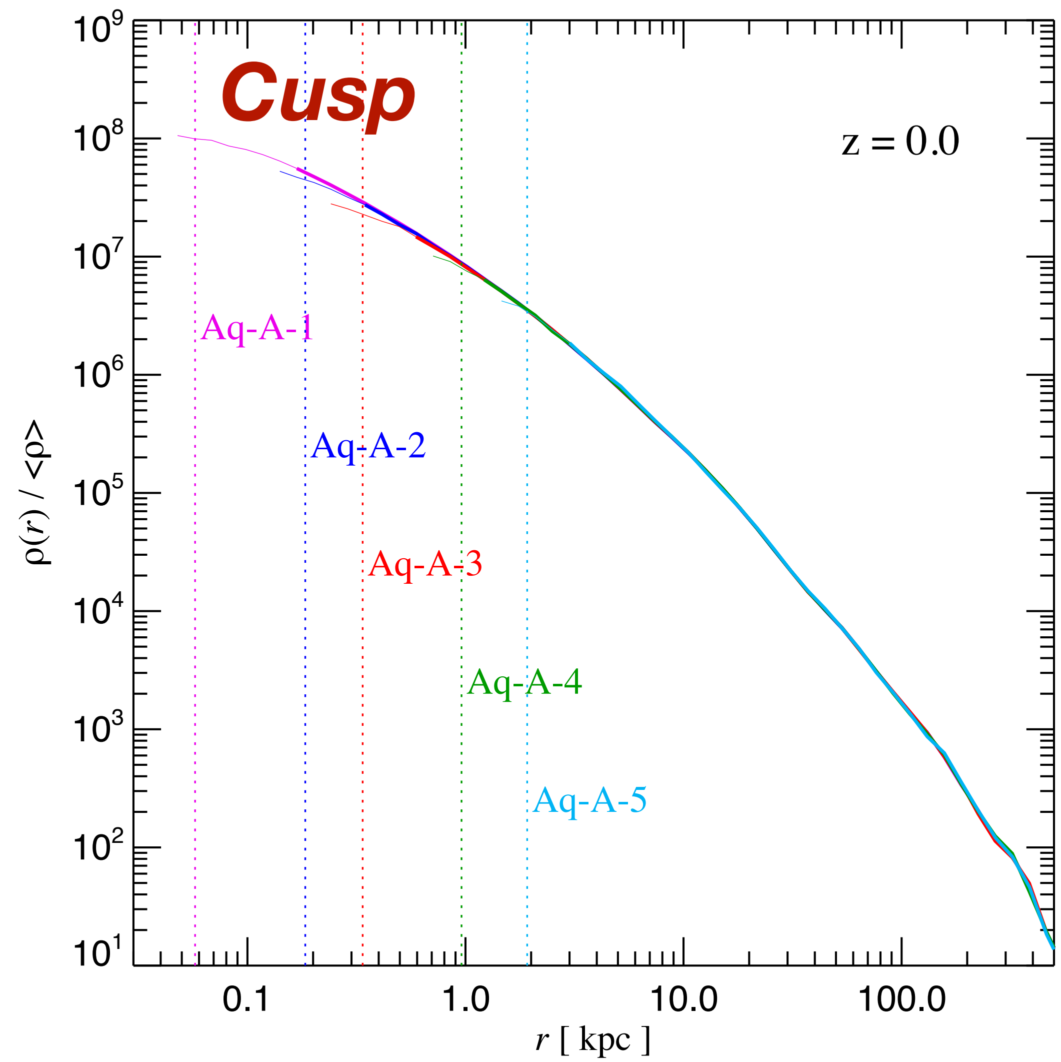
Issues with CDM at small scales

- CDM hypothesis is largely successful
- Distributions and clustering of halos are in very good agreement with cosmological/astrophysical data at large scales (~ larger than galaxies)
- However, there are a few **small-scale (<10—100 kpc) issues**
    - **Cusp vs core**
    - **Missing satellites**
    - **Too-big-too-fail**
    - **Diversity**

Small scale structure in CDM: recap



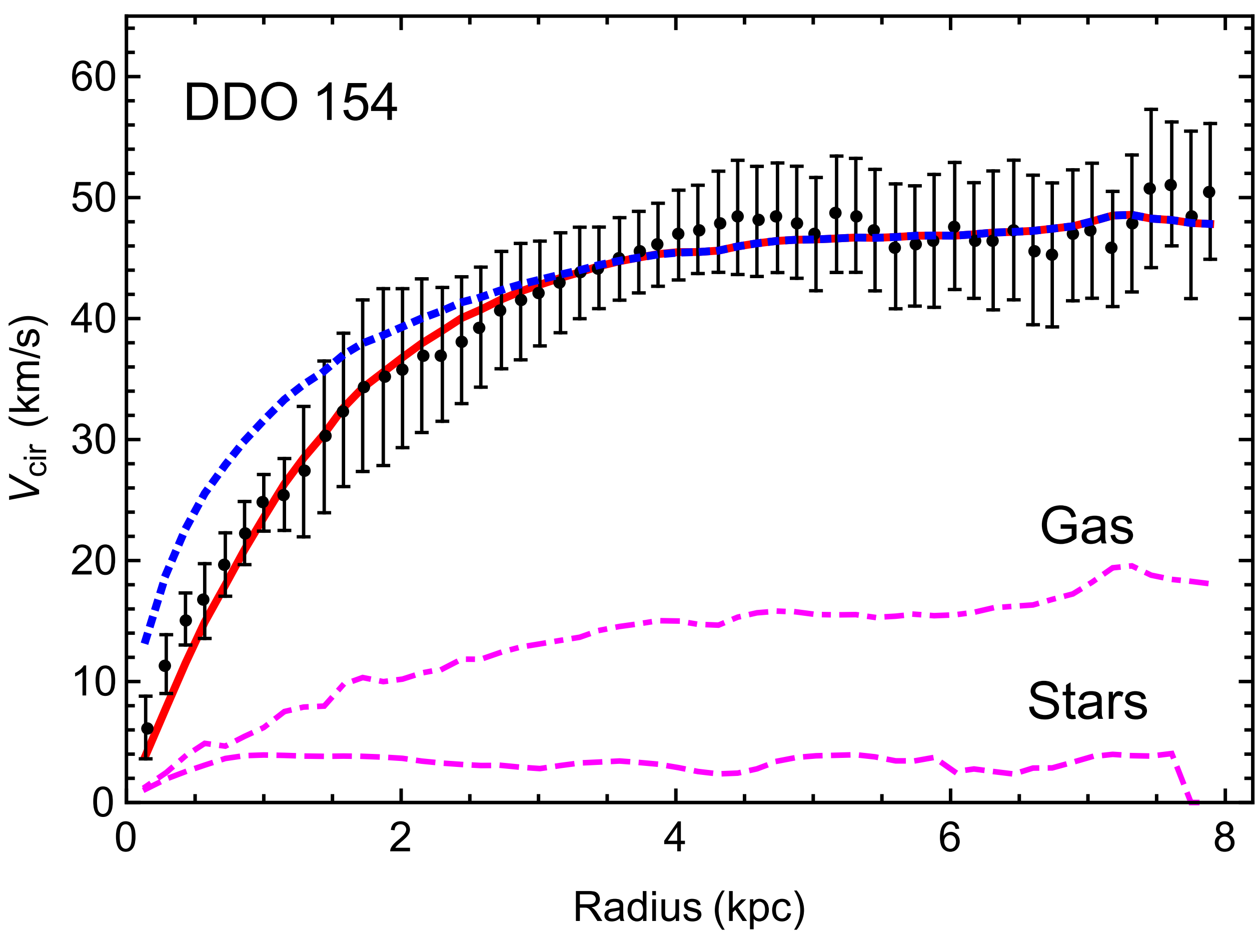
Springel et al., *Mon Not. R. Astron Soc.* **391**, 1685 (2008)

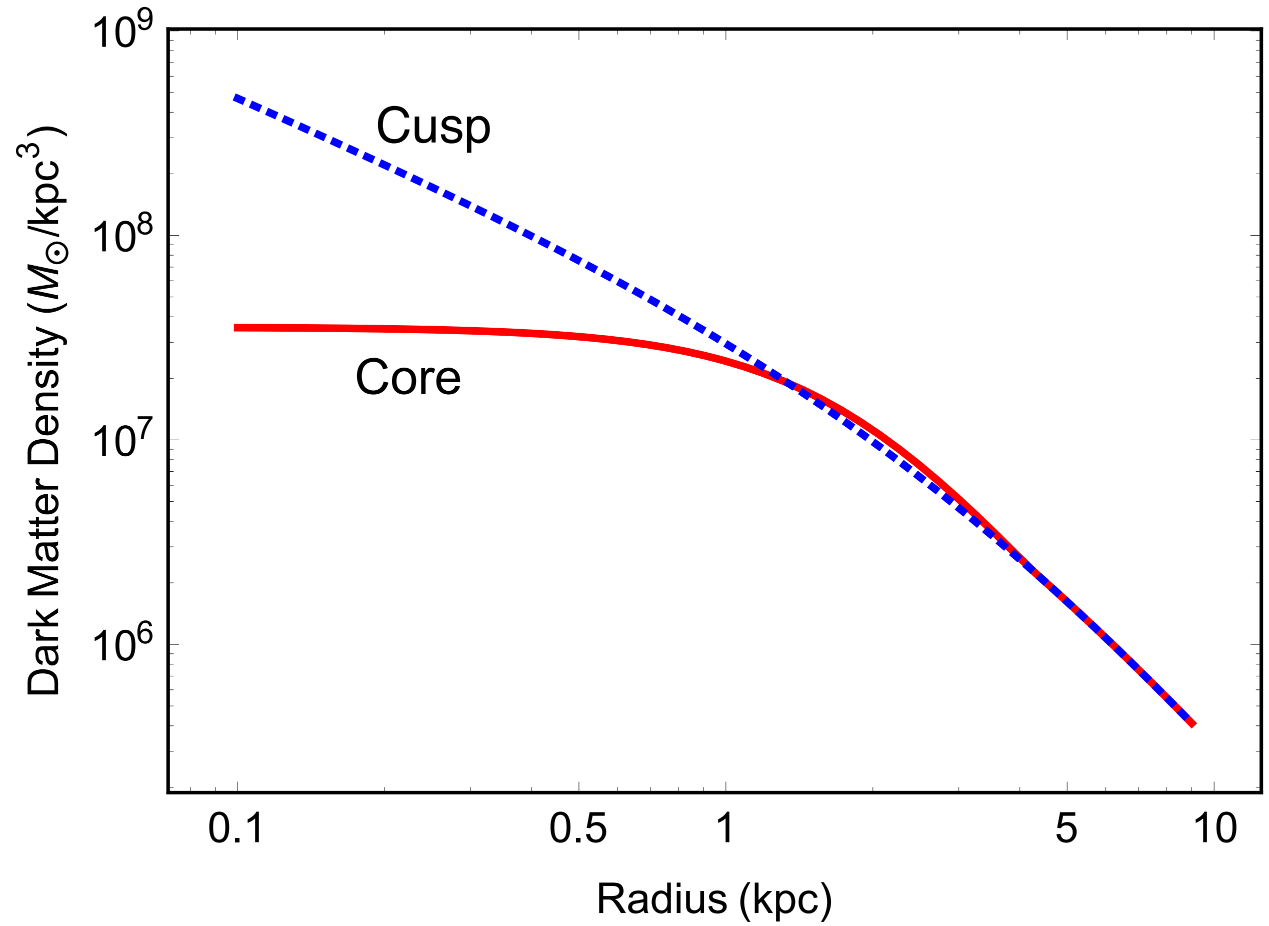


- Universal density profile; ρ_s and r_s correlated
- Lots of substructure

Cups vs core problem

Tulin, Yu, arXiv:1705.02358 [hep-ph]

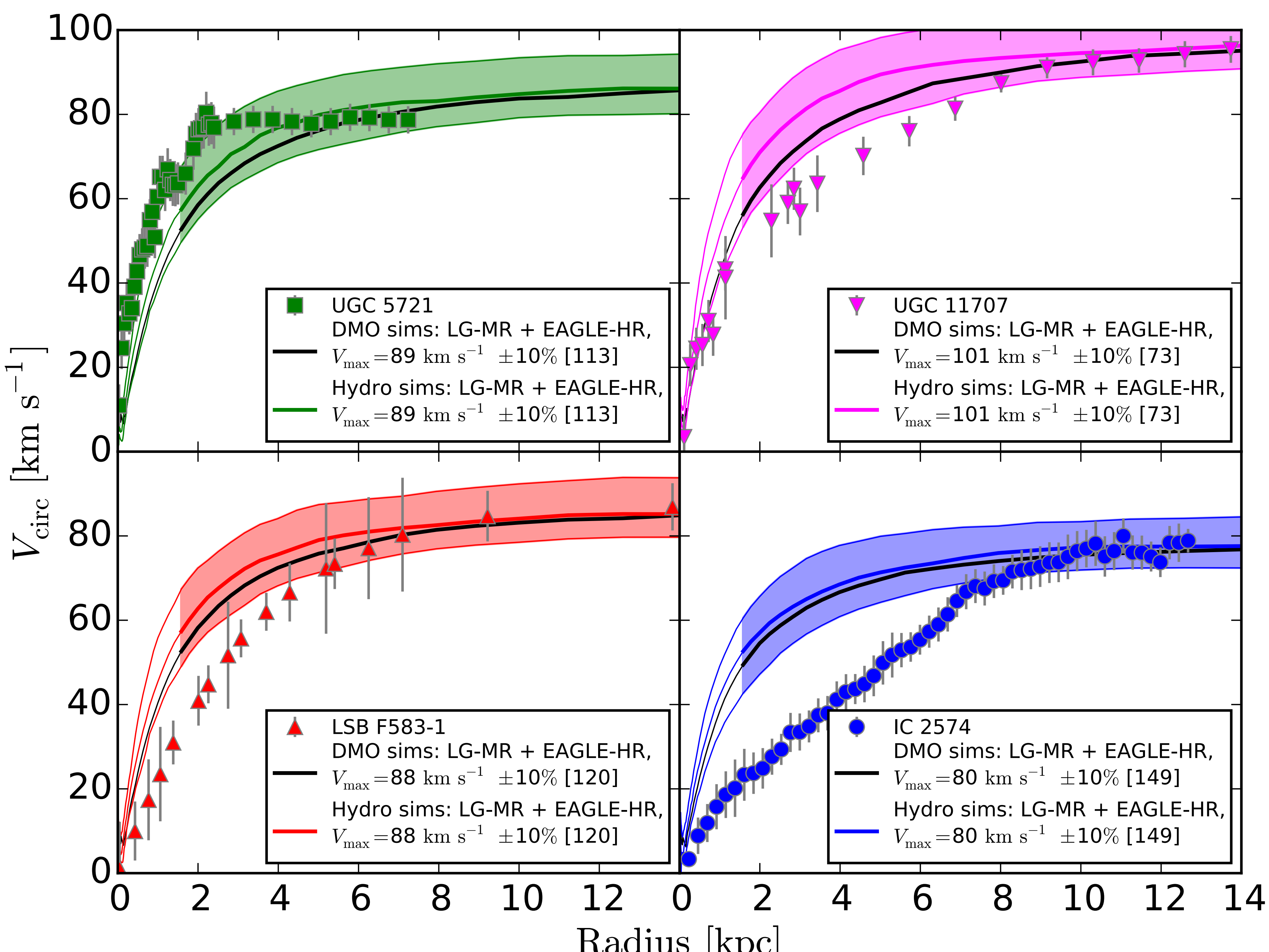




- Dark matter dominated systems (e.g., dwarf spheroidal galaxies) appear to have a shallow density core, rather than steep cusp

Diversity problem

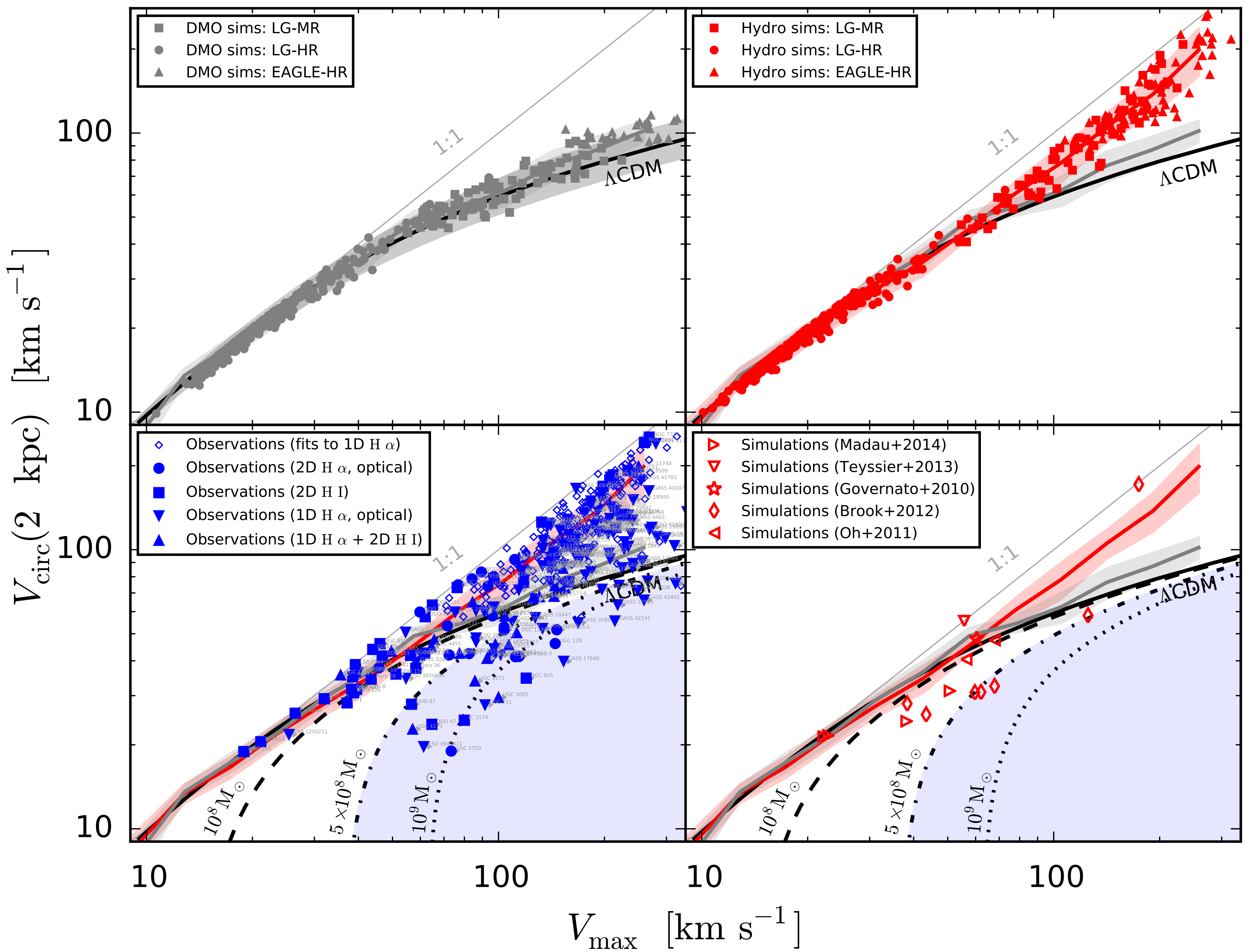
Oman et al., *Mon. Not. R. Astron. Soc.* **452**, 3650 (2015)



- Samples of galaxy rotation curves of similar maximum circular velocity
- Surprising **diversity in galaxy rotation curves**
- **In contrast with robust prediction of CDM simulations** (colored bands)

Diversity problem

Oman et al., *Mon. Not. R. Astron. Soc.* **452**, 3650 (2015)



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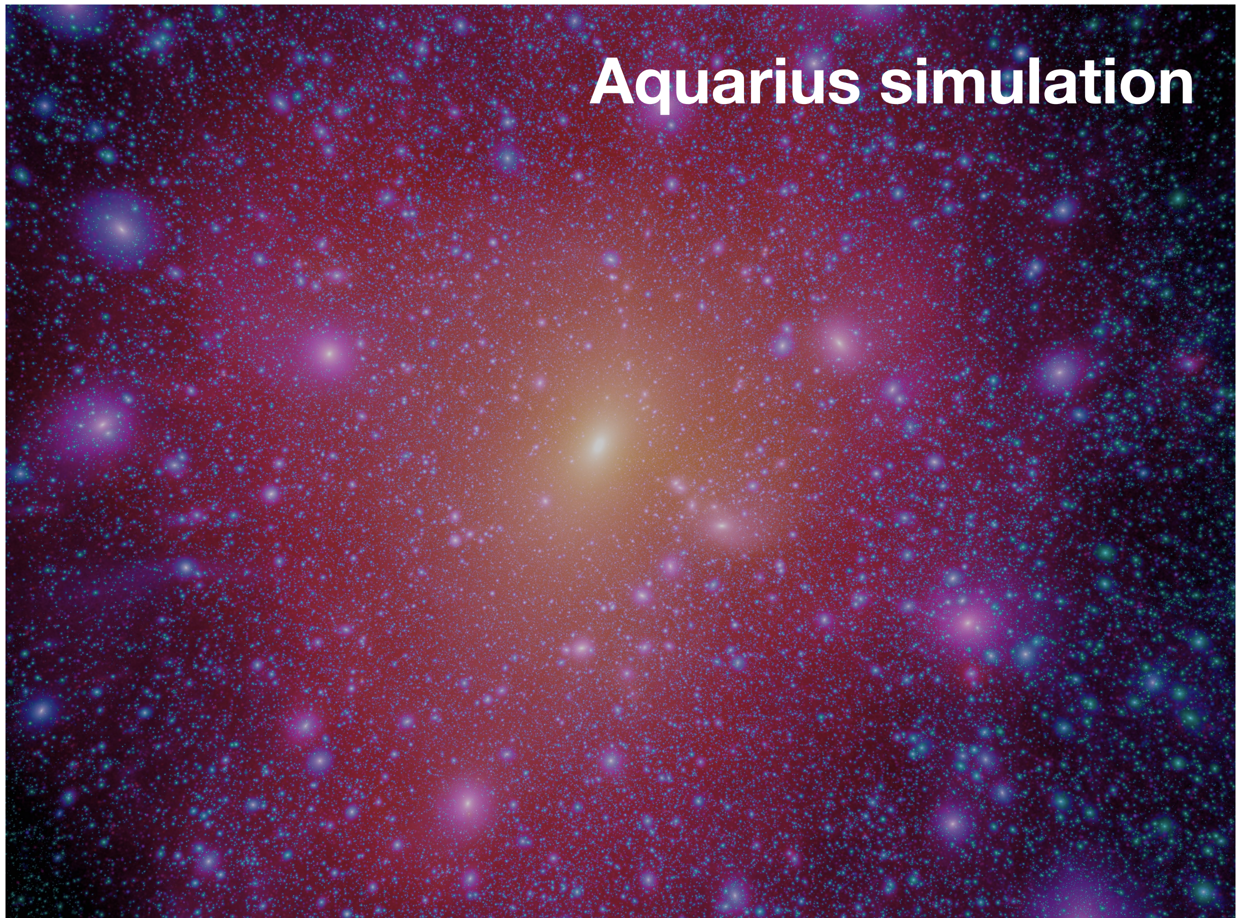
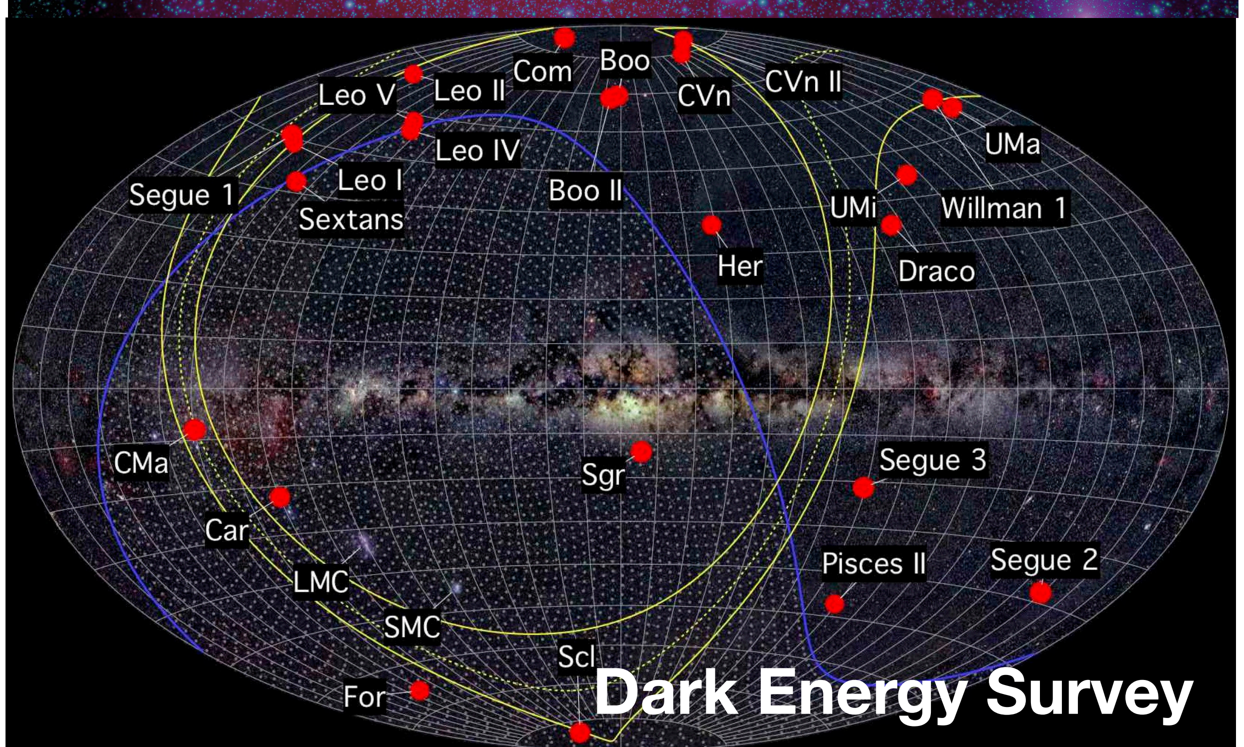
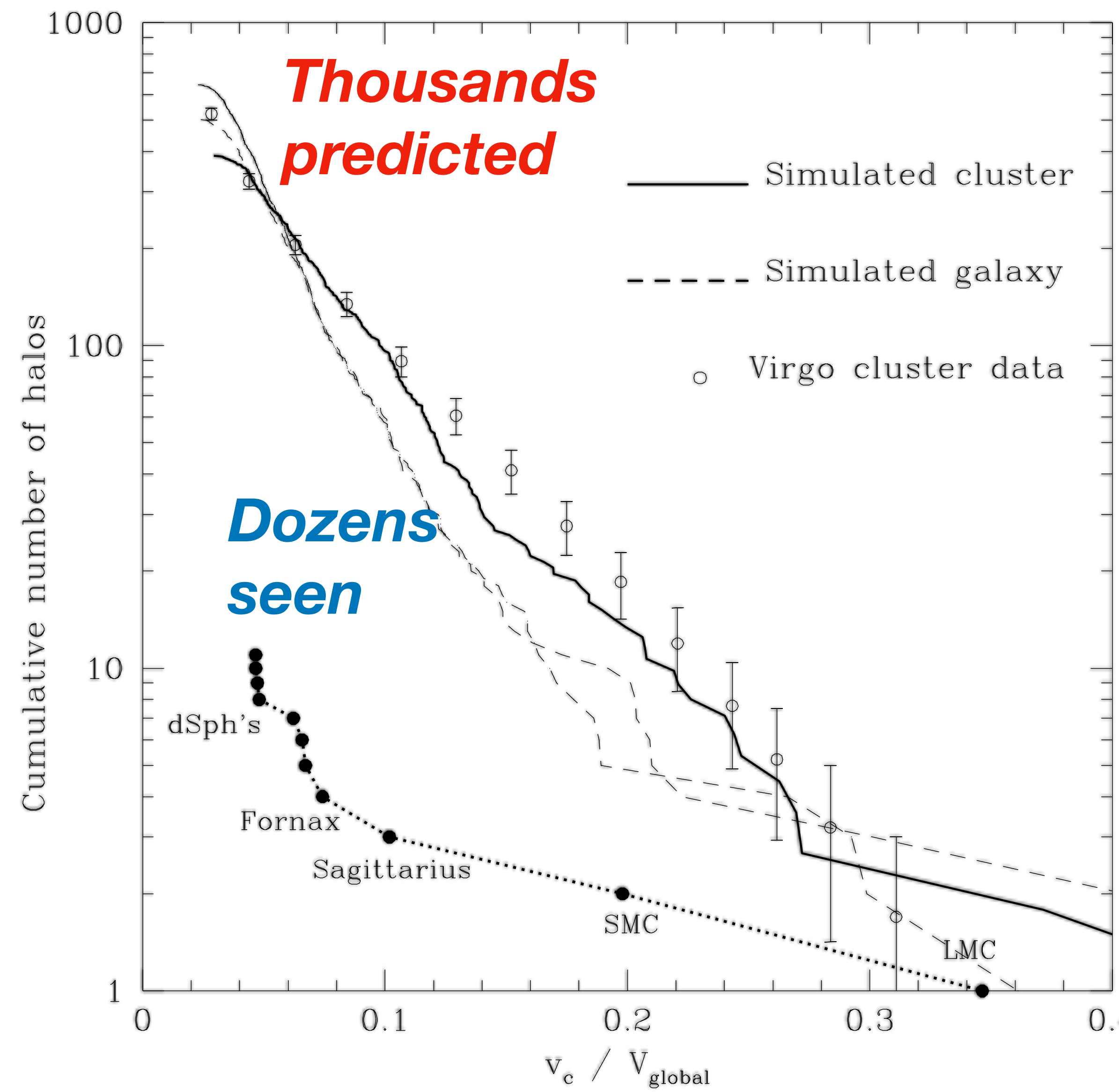
Missing satellite problem



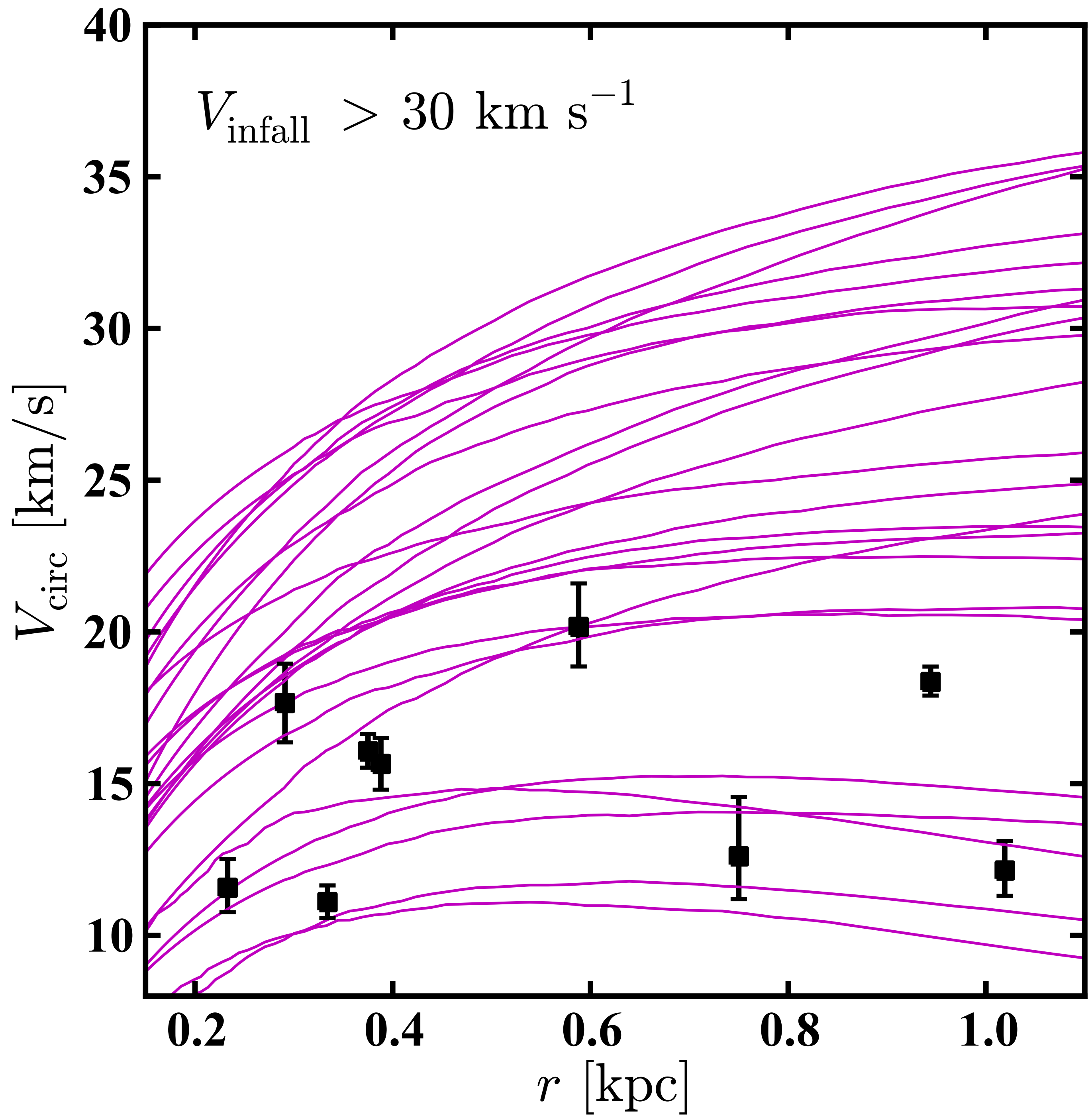




Moore et al. *Astrophys. J.* **524**, L19 (1999)

Where are these substructures? Are they hidden, or nonexistent?

Too-big-to-fail problem



- CDM simulations predict existence of many halos with large circular velocity than observed
- These would be dense halos, so it is hard to imagine that they fail to form stars

Boylan-Kolchin et al., *Mon. Not. R. Astron. Soc.* **422**, 1203 (2012)

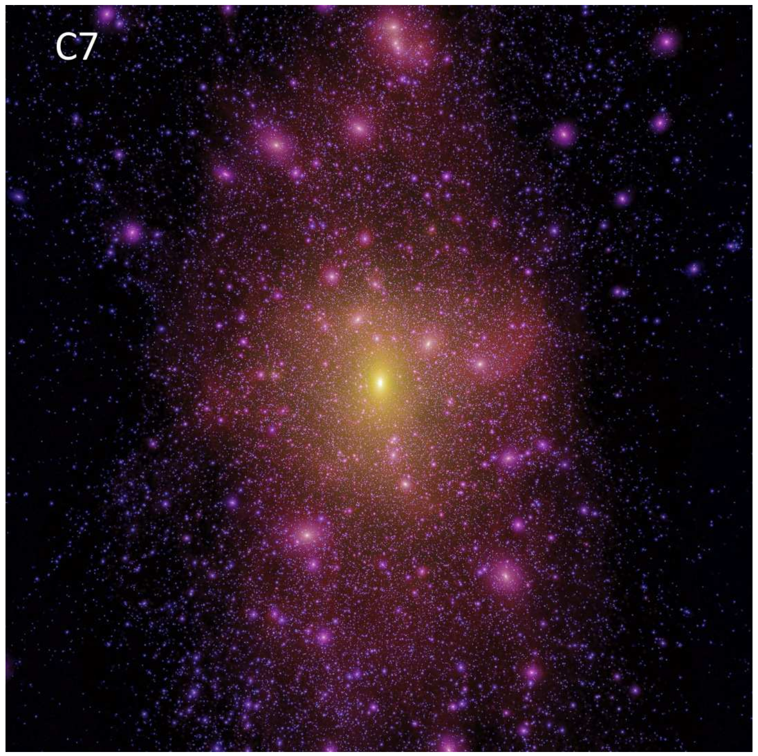
Solutions?

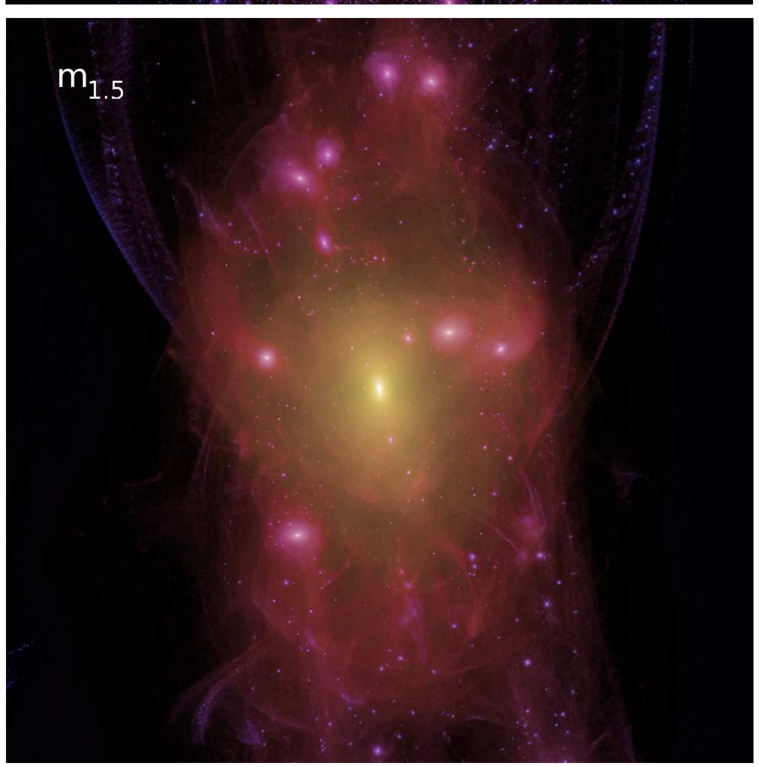
- **Baryon feedback**
  - 5% of the total matter, but exceedingly important at small scales
  - Sub-grid physics: hard to model/simulate
- **Different dark matter candidate other than CDM**
  - **Warm dark matter**: larger velocity at matter-radiation equality → wash small-scale structures away
  - **Self-interacting dark matter**: redistribution of kinetic energy in the halo central region through self-scattering

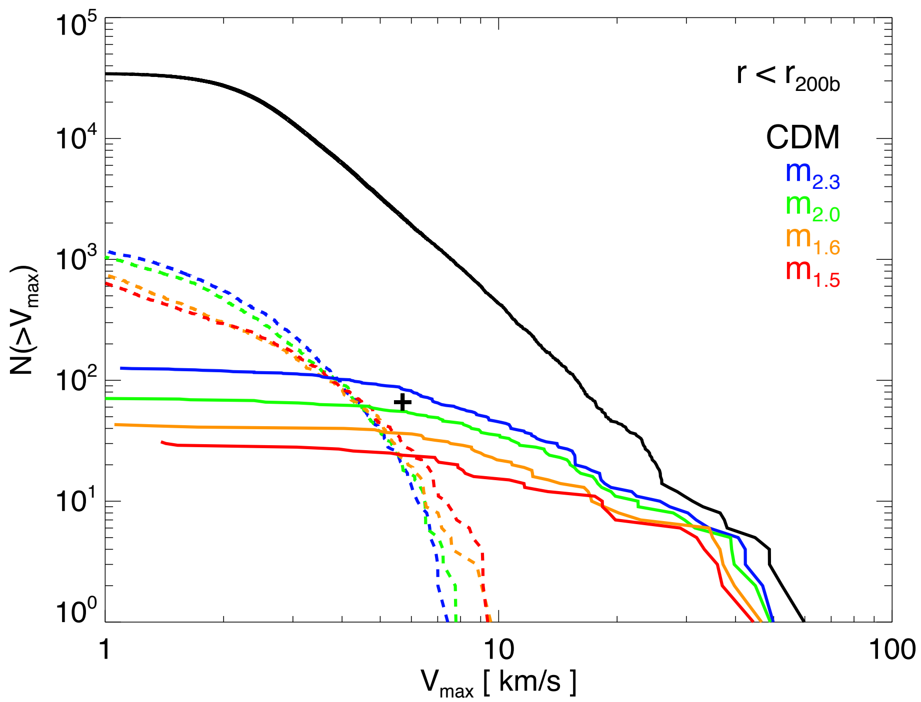
Hotness/coldness of dark matter species

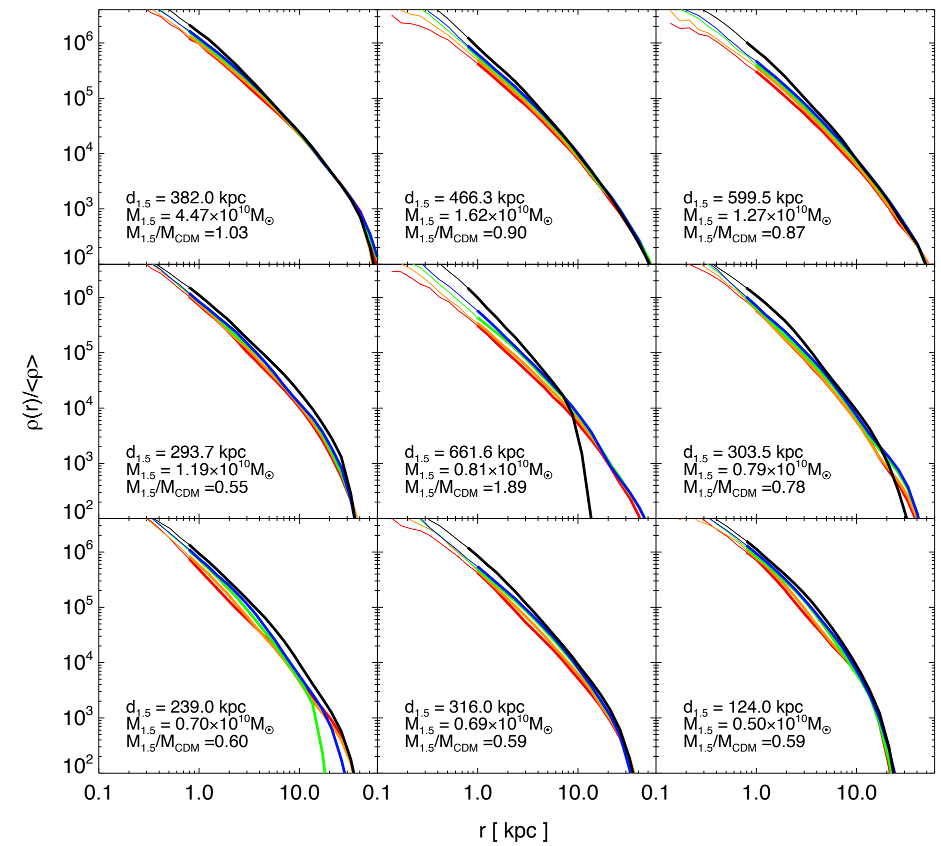
- **Cold dark matter** (e.g., GeV-TeV WIMPs): predicted smallest halos ~ Earth size or smaller
- **Hot dark matter** (e.g., sub-eV neutrinos): erase too much structures through free-streaming → already excluded
- **Warm dark matter** (e.g., keV sterile neutrinos): mass tuned such that they erase structures at sub-galactic scale

WDM simulation





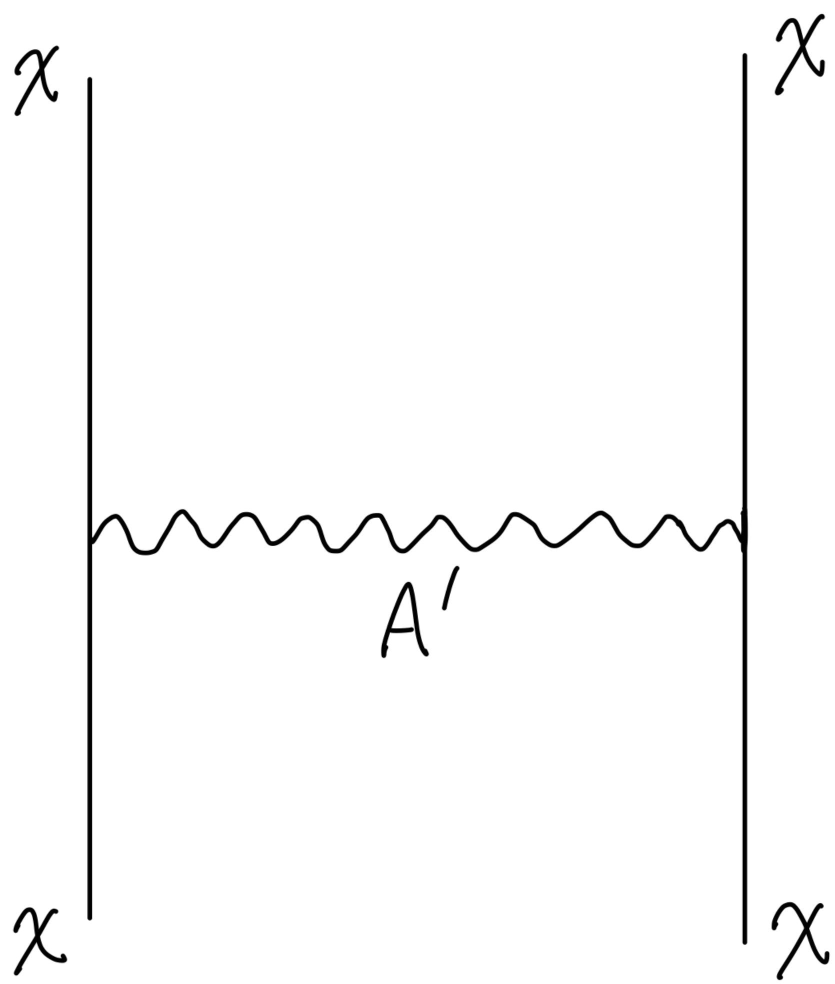




Lovell et al., *Mon. Not. R. Astron. Soc.* **439**, 300 (2014)

Small-scale substructures suppressed, but doesn’t seem to solve cusp vs core problem

Self-interacting dark matter (SIDM)

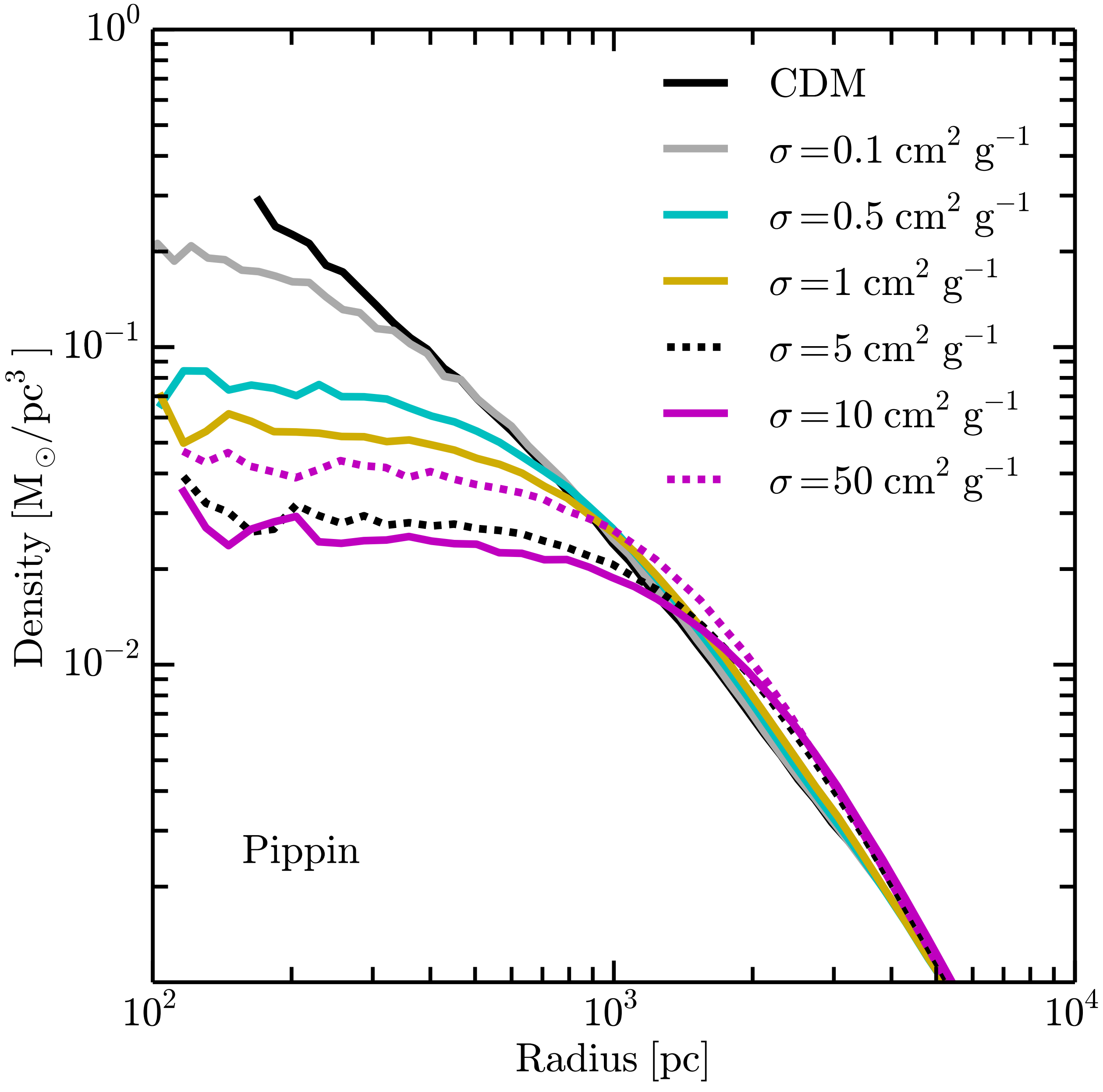


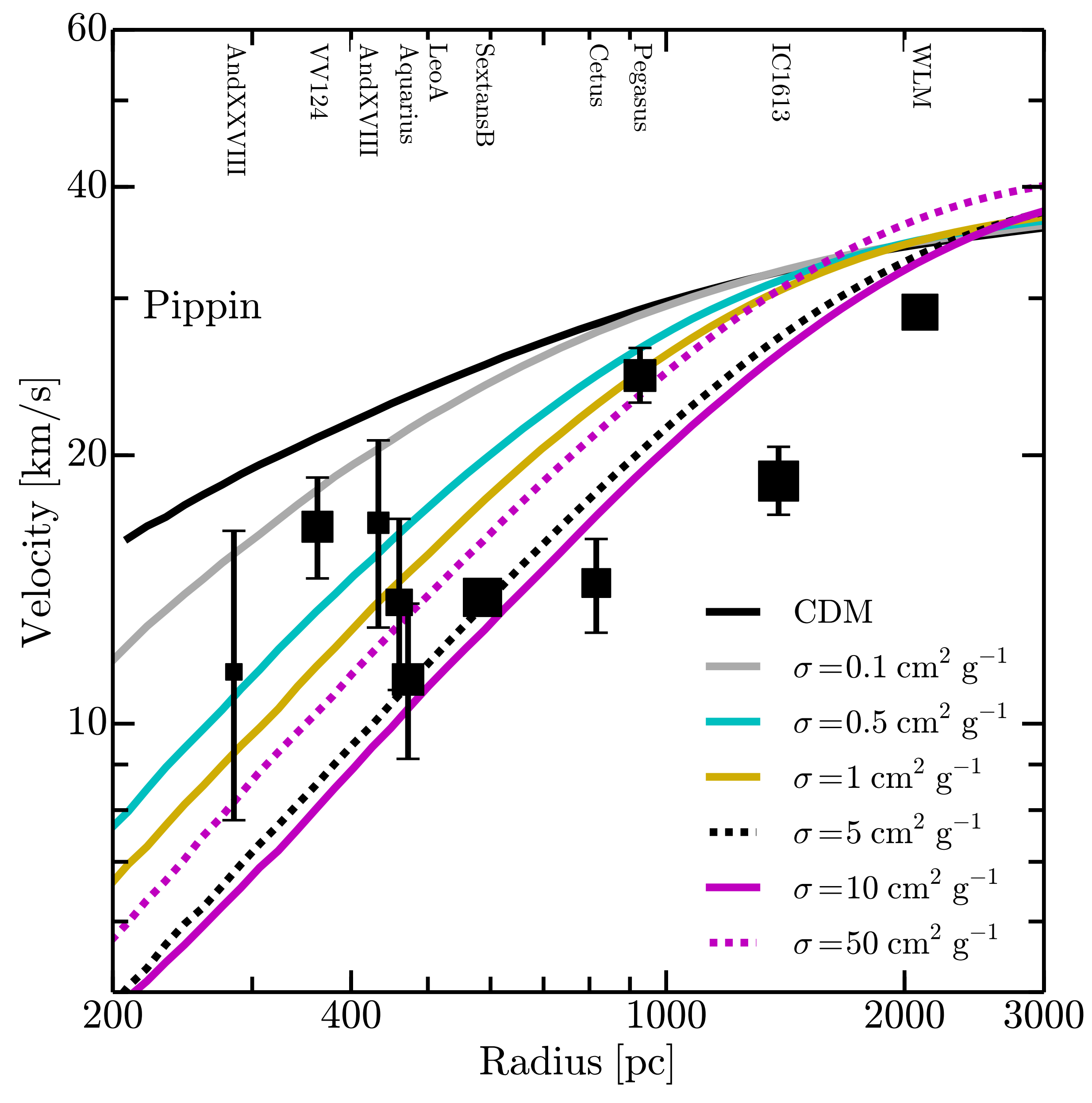
E.g., dark matter fermions mediated by massive vector boson (dark radiation)

- Proposed by Spergel & Steinhardt (2000)
- Required cross section:
$$\frac{\sigma}{m_\chi} \sim 1\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$
- This corresponds to ~1 scattering per particle in inner ~1 kpc during the Hubble time
- This is still below the upper limit from bullet clusters

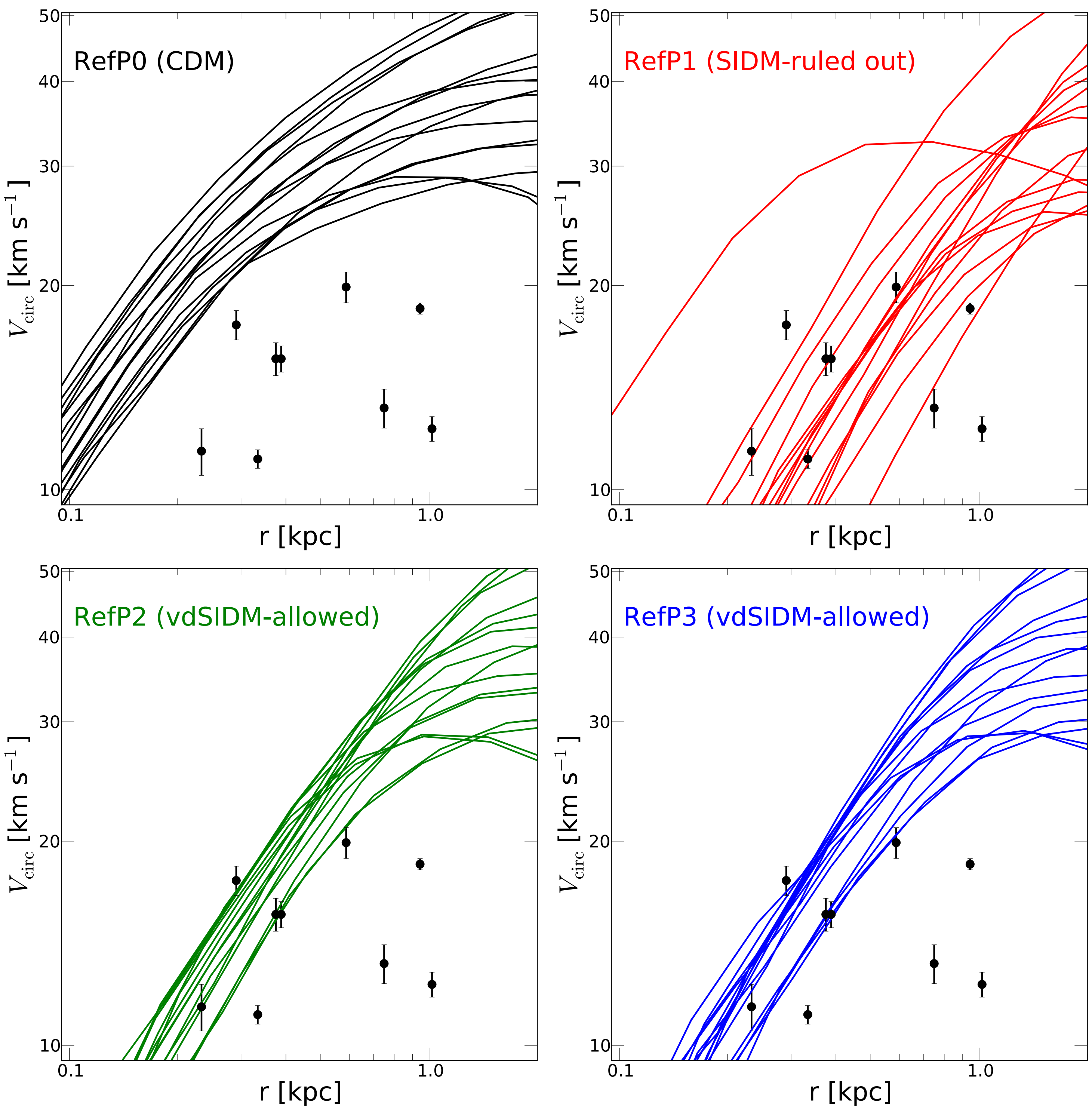
Density profile, circular velocity and SIDM

Elbert et al., *Mon. Not. R. Astron. Soc.* **453**, 29 (2015)





Too-big-to-fail problem and SIDM



$$\frac{\sigma_T}{\sigma_T^{\rm max}} \approx \begin{cases} \frac{4\pi}{22.7}\,\beta^2\,\ln\left(1+\beta^{-1}\right), & \beta < 0.1 \\ \frac{8\pi}{22.7}\,\beta^2\,\left(1+1.5\beta^{1.65}\right)^{-1}, & 0.1 < \beta < 10^3 \\ \frac{\pi}{22.7}\,\left(\ln\beta + 1 - \frac{1}{2}\ln^{-1}\beta\right)^2, & \beta > 10^3, \end{cases}$$

$$\beta = \pi v_{\rm max}^2/v^2$$

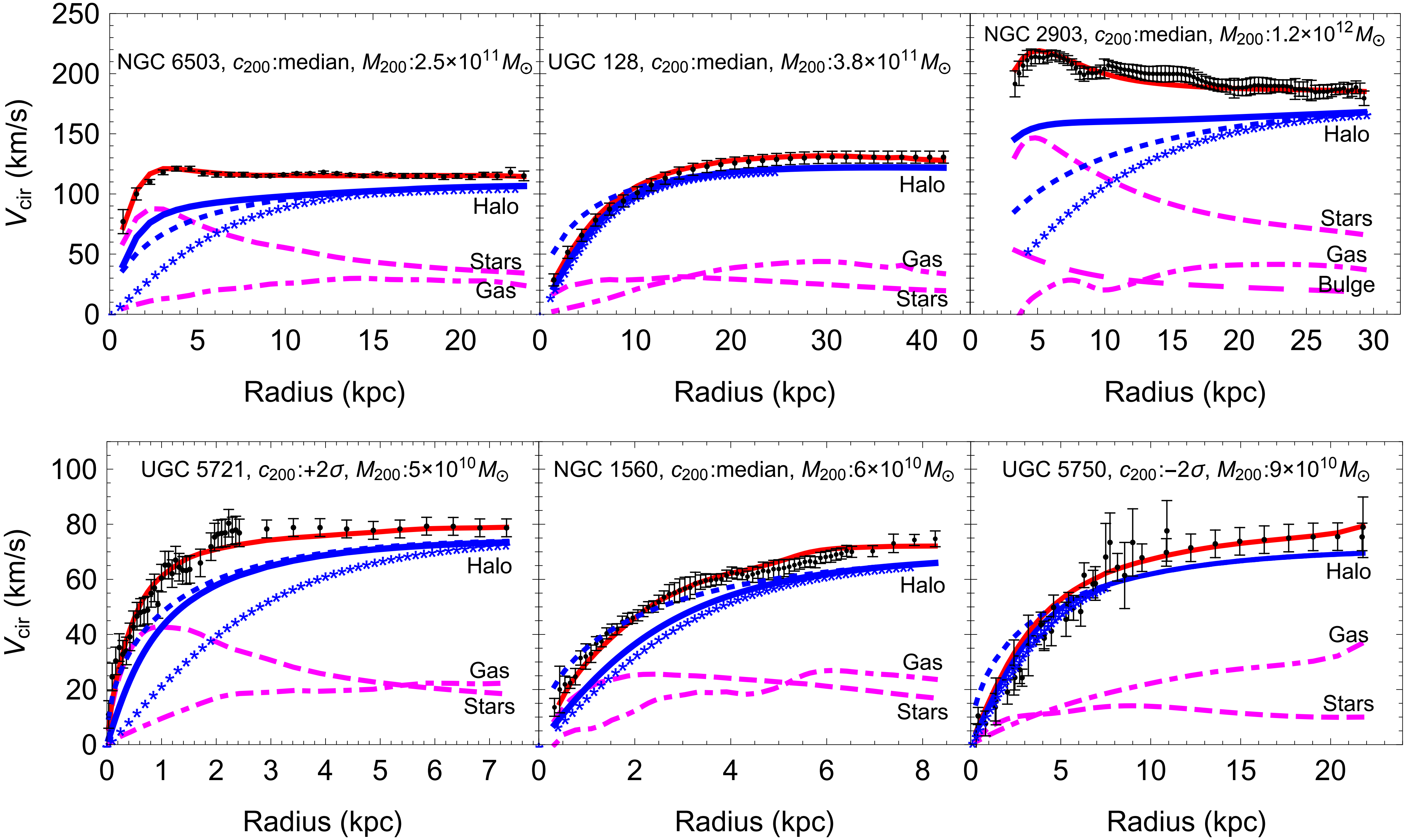
Name	Type	$\sigma_T^{\rm max}/m_\chi$ [cm^2 g^{-1}]	$v_{\rm max}$ [km s^{-1}]
RefP0	CDM	/	/
RefP1	SIDM (ruled out)	10	/
RefP2	vdSIDM (allowed)	3.5	30
RefP3	vdSIDM (allowed)	35	10

Reduction of central density also decreases the maximum circular velocity $V_{\rm max}$

Vogelsberger et al., *Mon. Not. R. Astron. Soc.* **423**, 3740 (2012)

Diversity problem and SIDM

Kamada et al., *Phys. Rev. Lett.* **119**, 111102 (2017)



Thermalization leads to baryonic influence on dark matter distribution

→ ***Diversity***

Summary: Small-scale problems of CDM

- While CDM is successful at large scales, there are several issues at small scales: ***cusp vs core, diversity, missing satellites, too-big-to-fail***

- Although baryonic physics may address these issues, they may ***hint toward particle nature of dark matter***

- Both **warm dark matter** and **self-interacting dark matter** may provide solutions to some of these issues while keeping CDM success at large scales

- **These dark matter models can be tested** through colliders, direct, and indirect detection experiments