Topical Lectures on DM@Nikhef
13 December 2017

Dark matter distribution
Large and small scale structure

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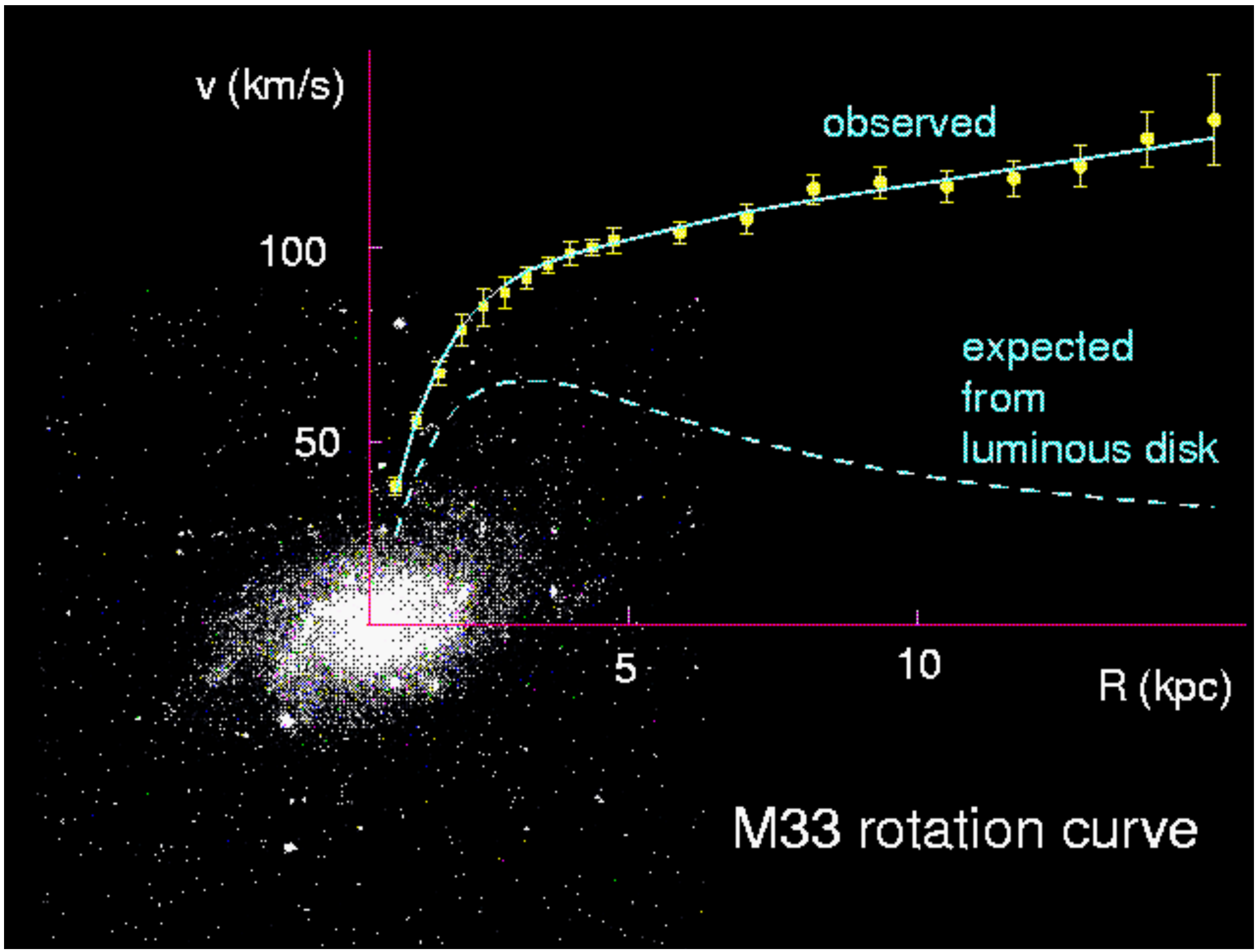
Evidence for dark matter: Rotation curves

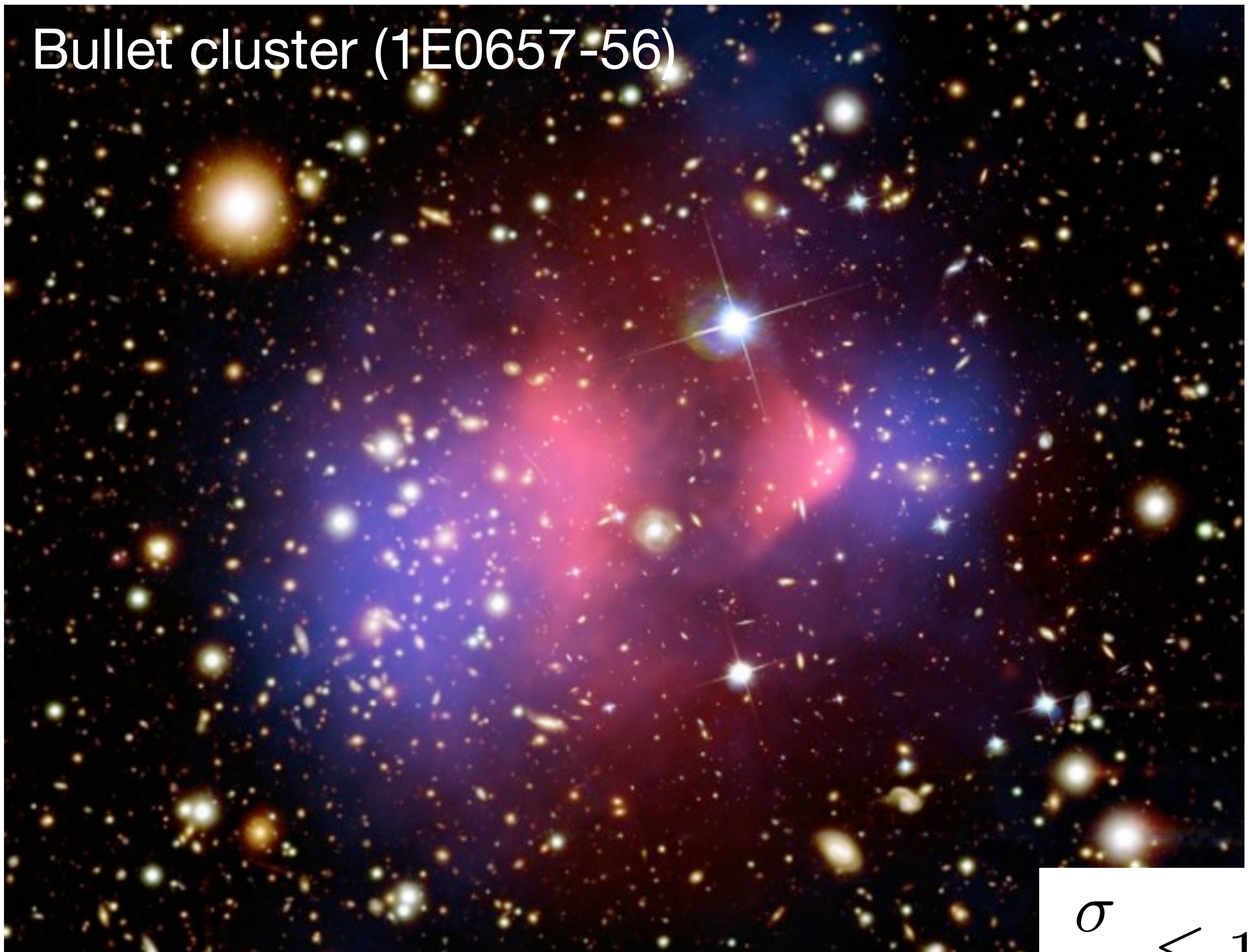


Bergström, *Rep. Prog. Phys.* **63**, 793 (2000)

$$v^2 = \frac{GM(<r)}{r}$$

- Inferring enclosed mass for luminous matter (stars assuming reasonable mass-to-light ratio) significantly under-predicts rotation-curve data
- *Implication*: "Dark" matter exists (but it doesn't exclude not-so-bright stars or black holes)

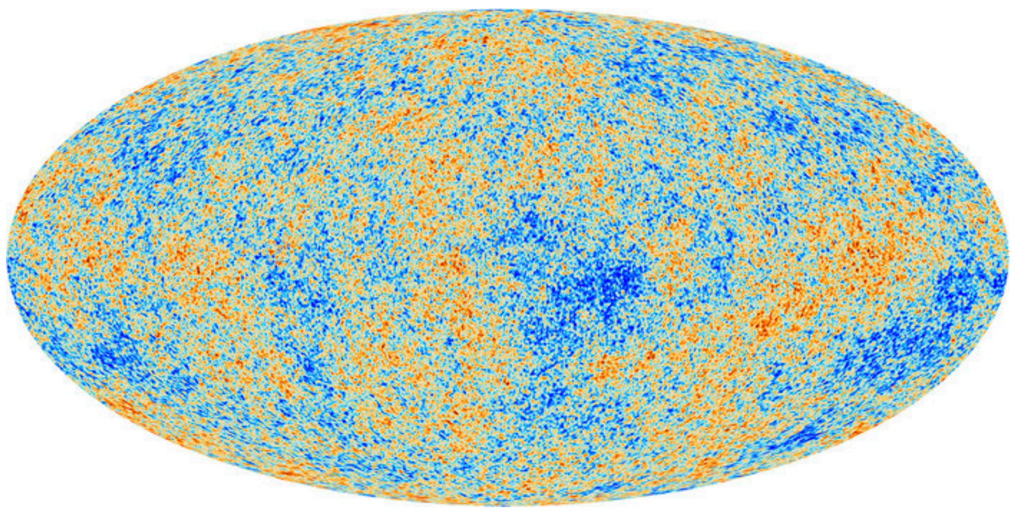
Evidence for dark matter: Bullet clusters

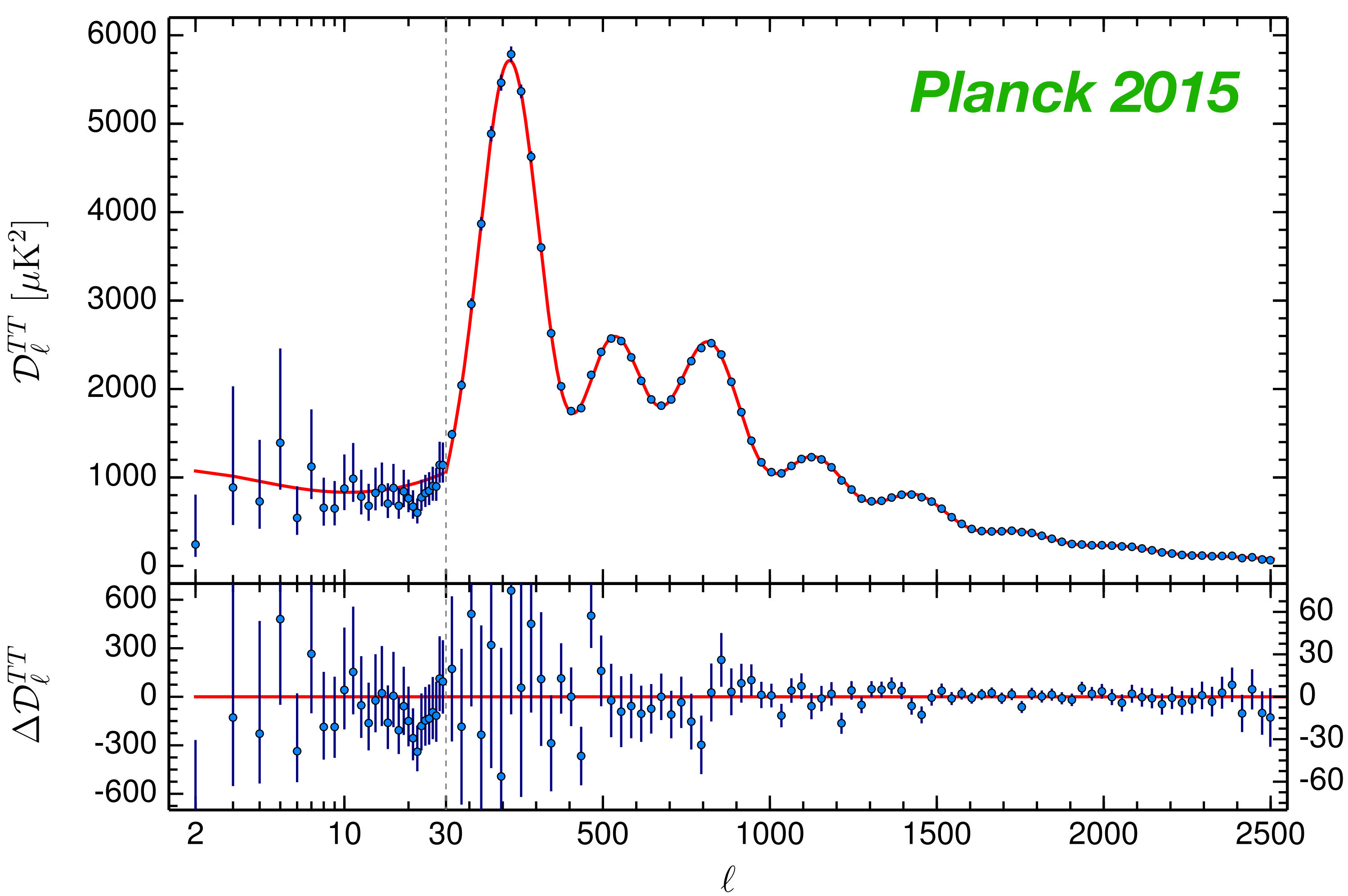


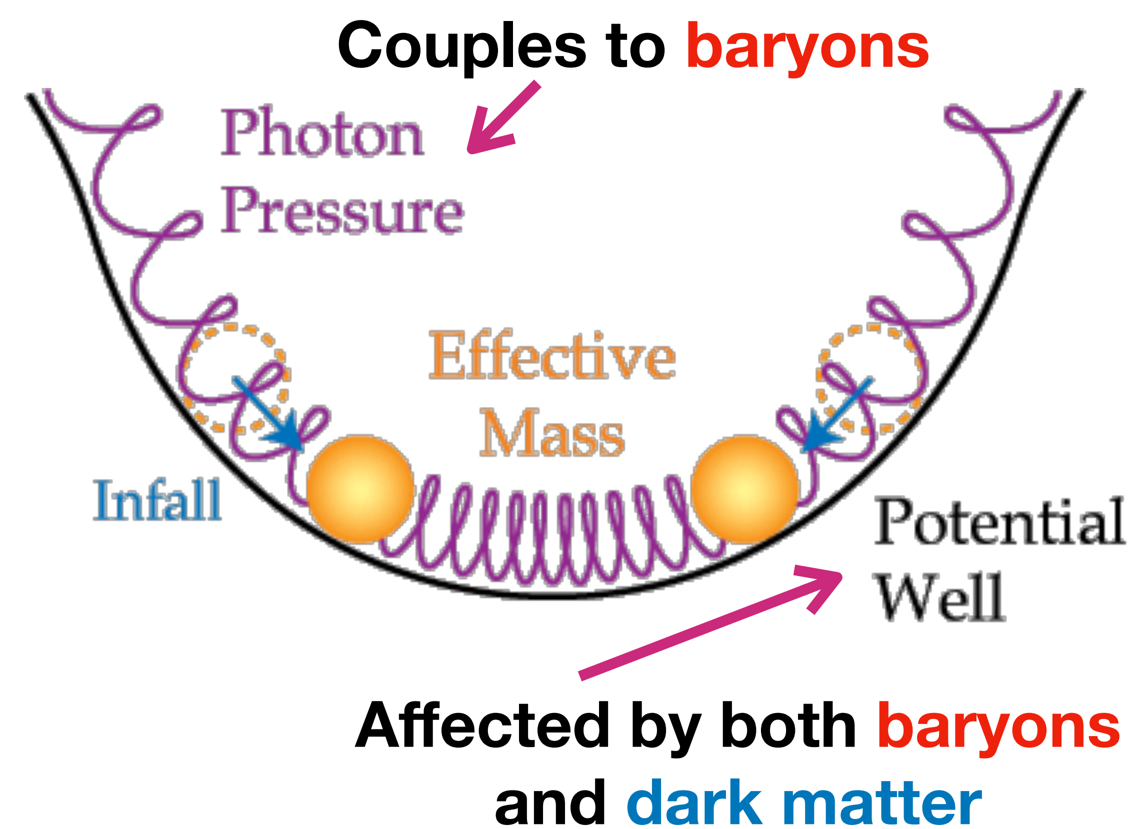
- Red: X-ray image (baryonic gas)
- Blue: Weak gravitational lensing (dark matter)
- Gas is collisional (Coulomb force) so feels drag from each other; dark matter goes through
- *Implication*: Dark matter is collionless

$$\frac{\sigma}{m} \lesssim 1\ \mathrm{cm}^2/\mathrm{g} = 2\ \mathrm{barn}/\mathrm{GeV}$$

Evidence for dark matter: Baryon acoustic oscillation (BAO)

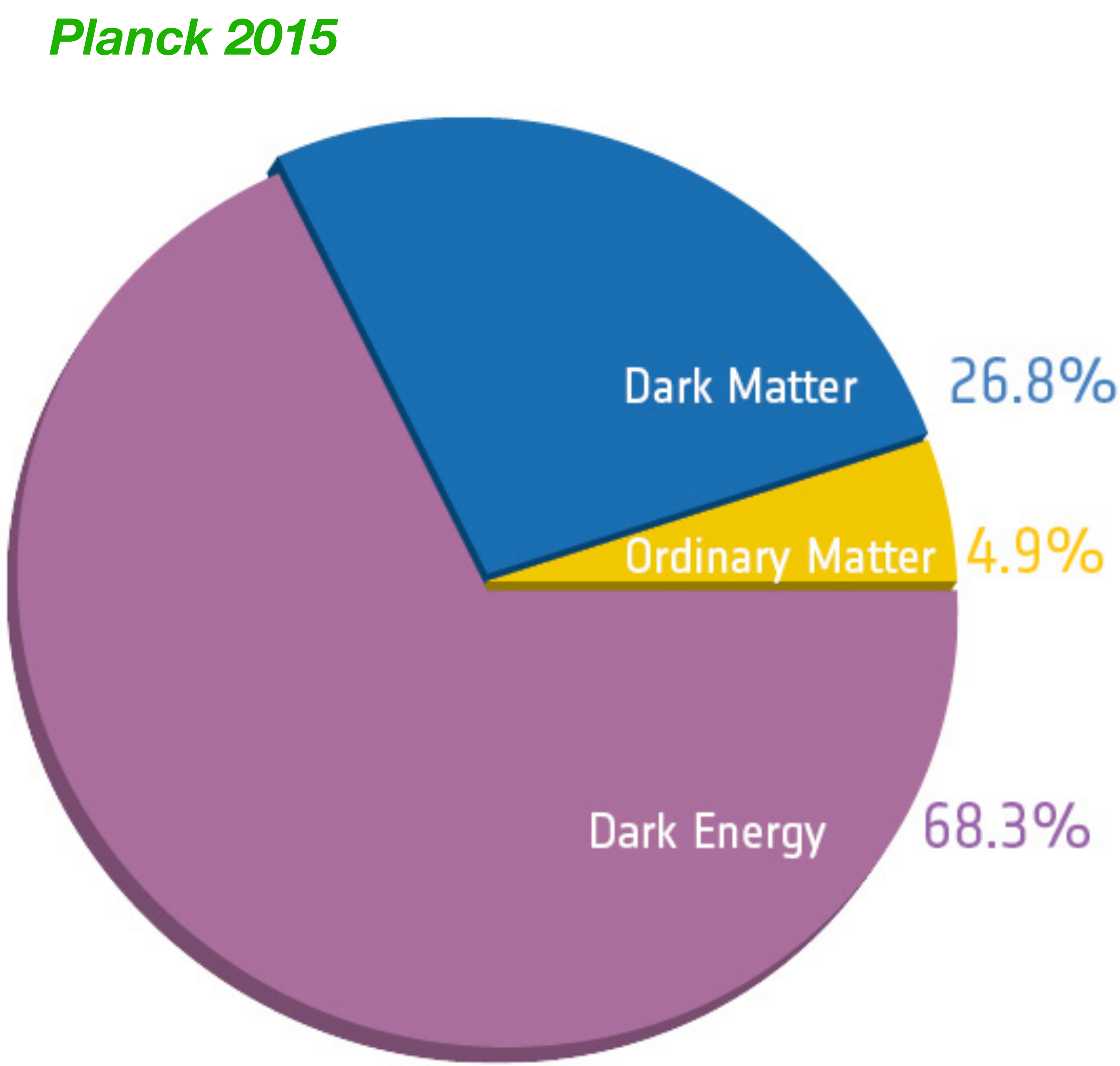






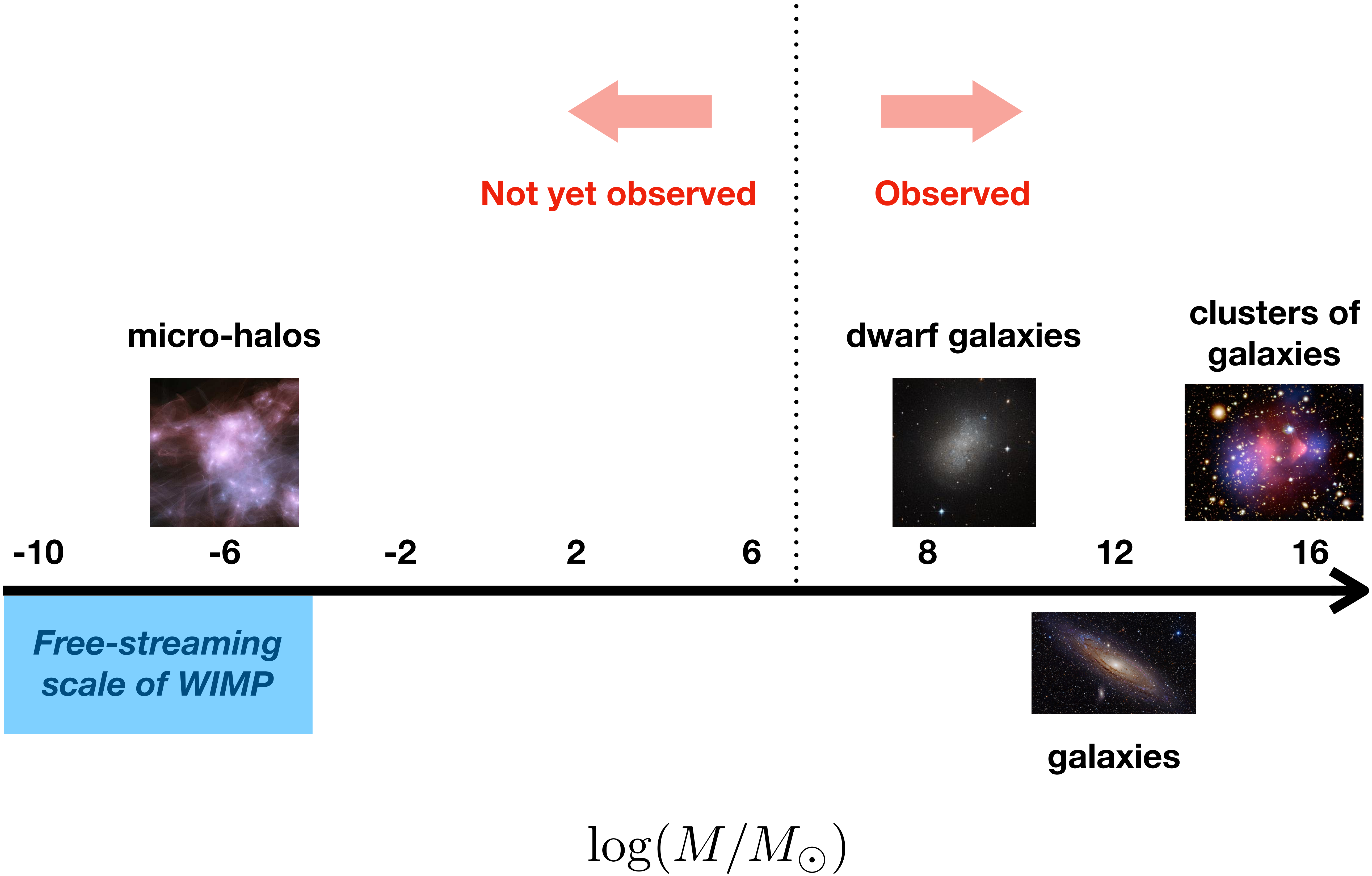
- Measured both in CMB and galaxy power spectrum
- *Implication*: Dark matter can ***not*** be made of baryons

Result from all the cosmology data



- CMB, galaxy power spectrum, weak lensing, supernova Ia, etc.
- 27% of the total energy / 85% of the total matter is made of dark matter
- Properties of dark matter
  - **Collisionless**
  - **Non-baryonic**
  - **Doesn't interact with photons**
  - **Cold** (or warm; hot dark matter erases too many structures)

Dark matter: Origin of all the structures



Dark matter: Origin of all the structures

- How do dark matter structures form? — *Spherical collapse model*

Today

- What is abundance, mass distribution, etc.? — *Halo mass function*

Today

- Impact on dark matter annihilation in cosmological halos — *Indirect dark matter searches*

Tomorrow

- Implications for properties of dark matter particles — *Cold, warm, self-interacting?*

Tomorrow

Spherical collapse model

- Deriving two **magic numbers** *analytically*

  - Over-density of virialized halos compared with background

$$\Delta_{\rm vir} = 18\pi^2$$

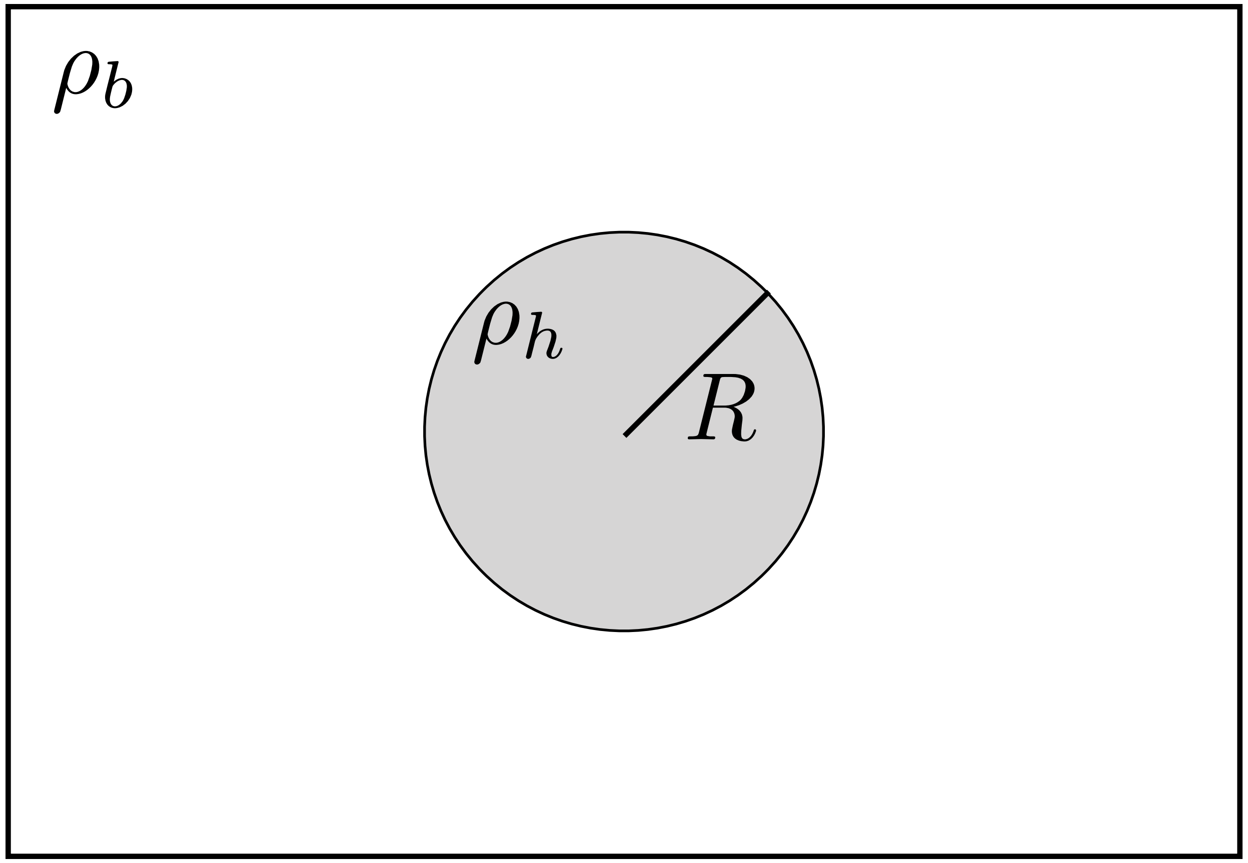
Useful for simulations to find halos

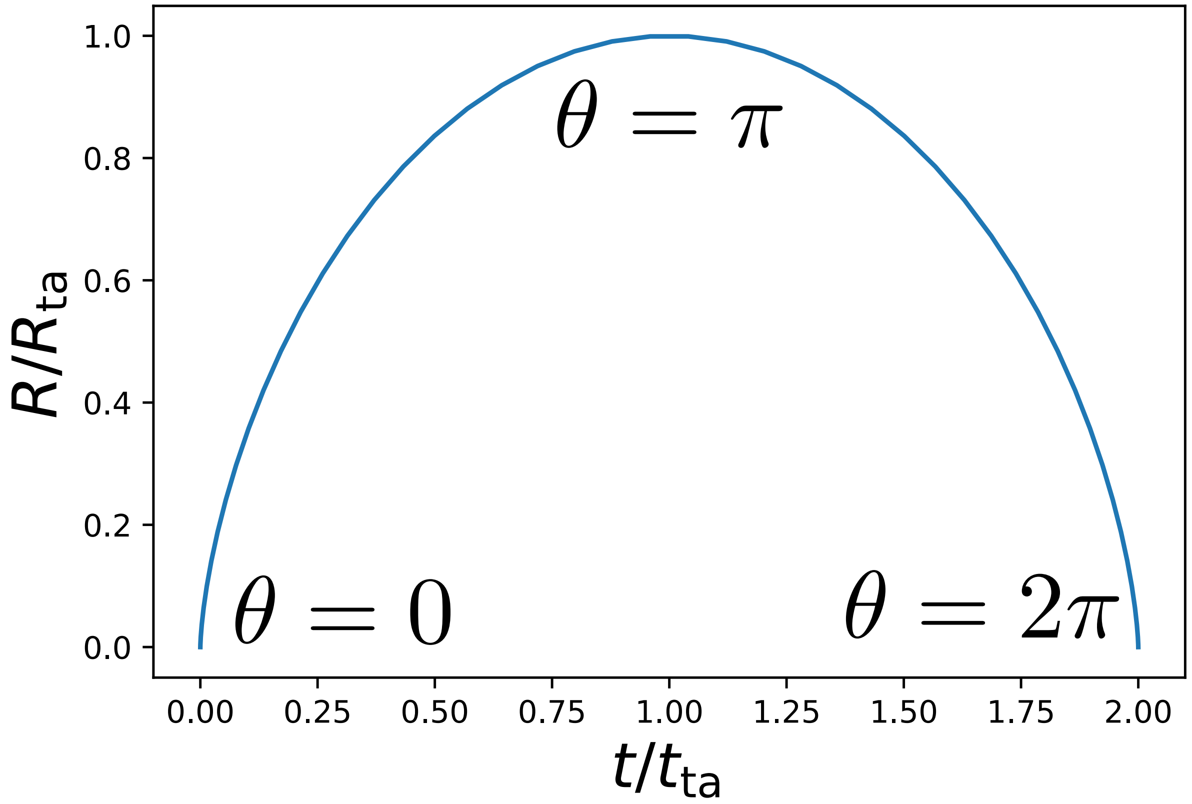
  - Linear extrapolation of over-density for halos that *just* collapsed

$$\delta_c = 1.686$$

Useful for analytic calculations to estimate number of halos

Spherical collapse model





Parameterized solution
(cf., expanding *closed* Universe)

$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$R = A^2(1 - \cos\theta)$$

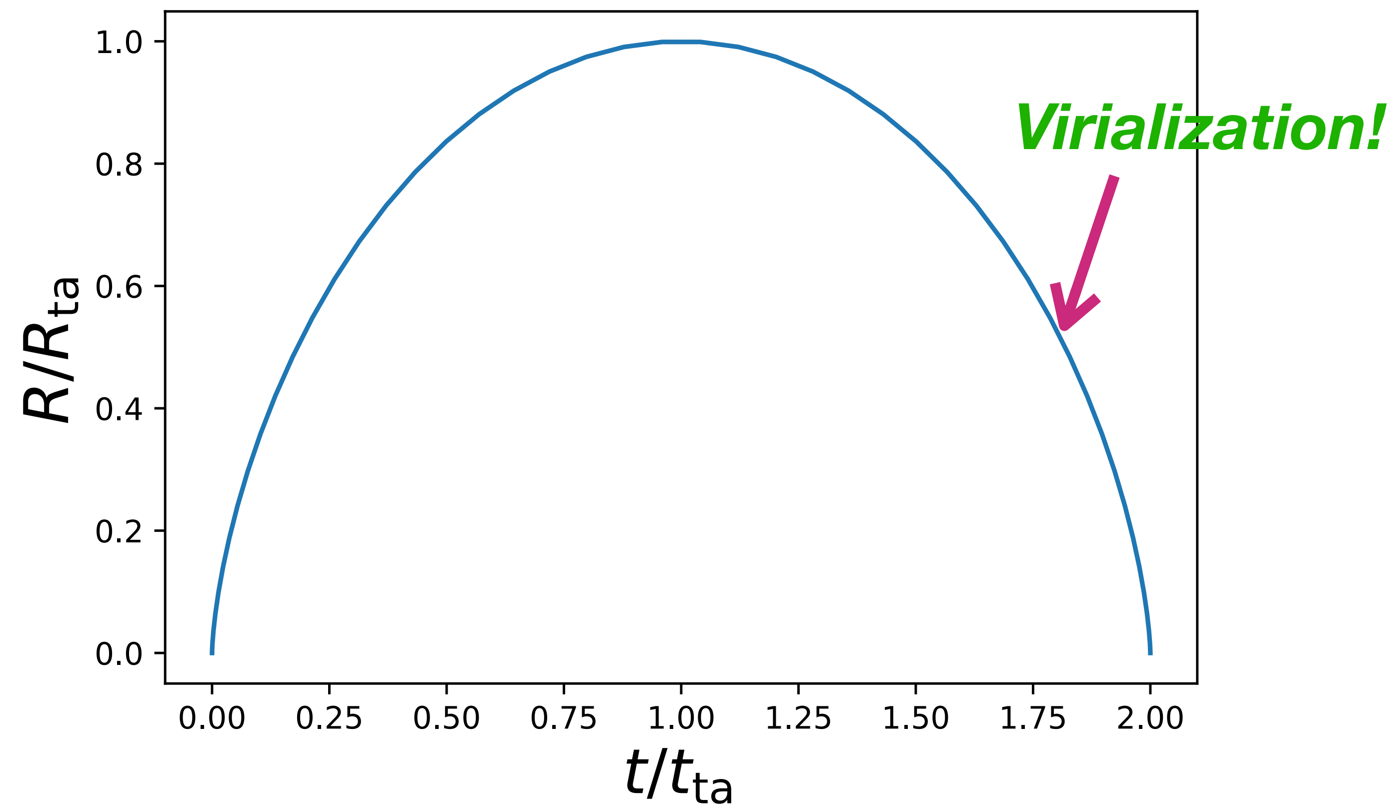
Spherical collapse model

When do halos virialize?

Virial theorem:

$$2K_{\rm vir} + U_{\rm vir} = 0$$

(for $1/R$ potential)



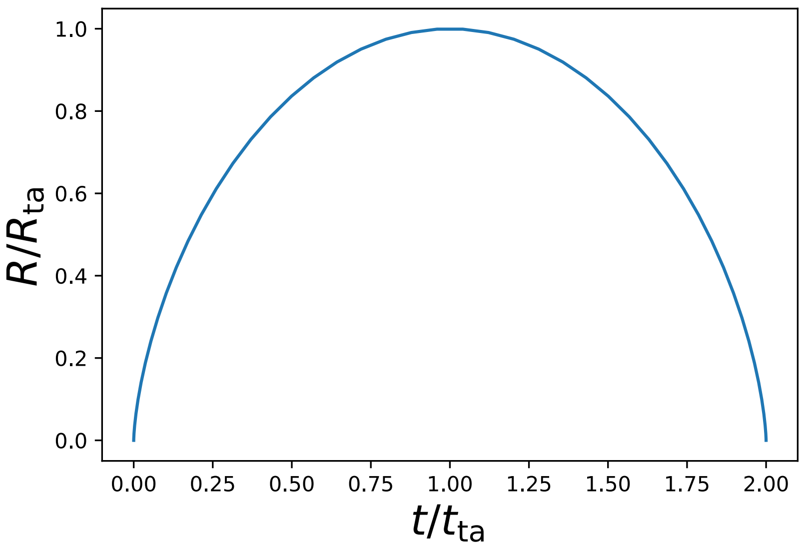
Total energy conservation:

$$K_{\rm vir} + U_{\rm vir} = U_{\rm ta}$$

$$U_{\rm vir} = 2U_{\rm ta}$$

$$R_{\rm vir} = \frac{R_{\rm ta}}{2}$$

Spherical collapse model



$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$R = A^2(1 - \cos\theta)$$

$$R_{\rm vir} = \frac{R_{\rm ta}}{2}$$

How dense is a virialized halo compared with background?

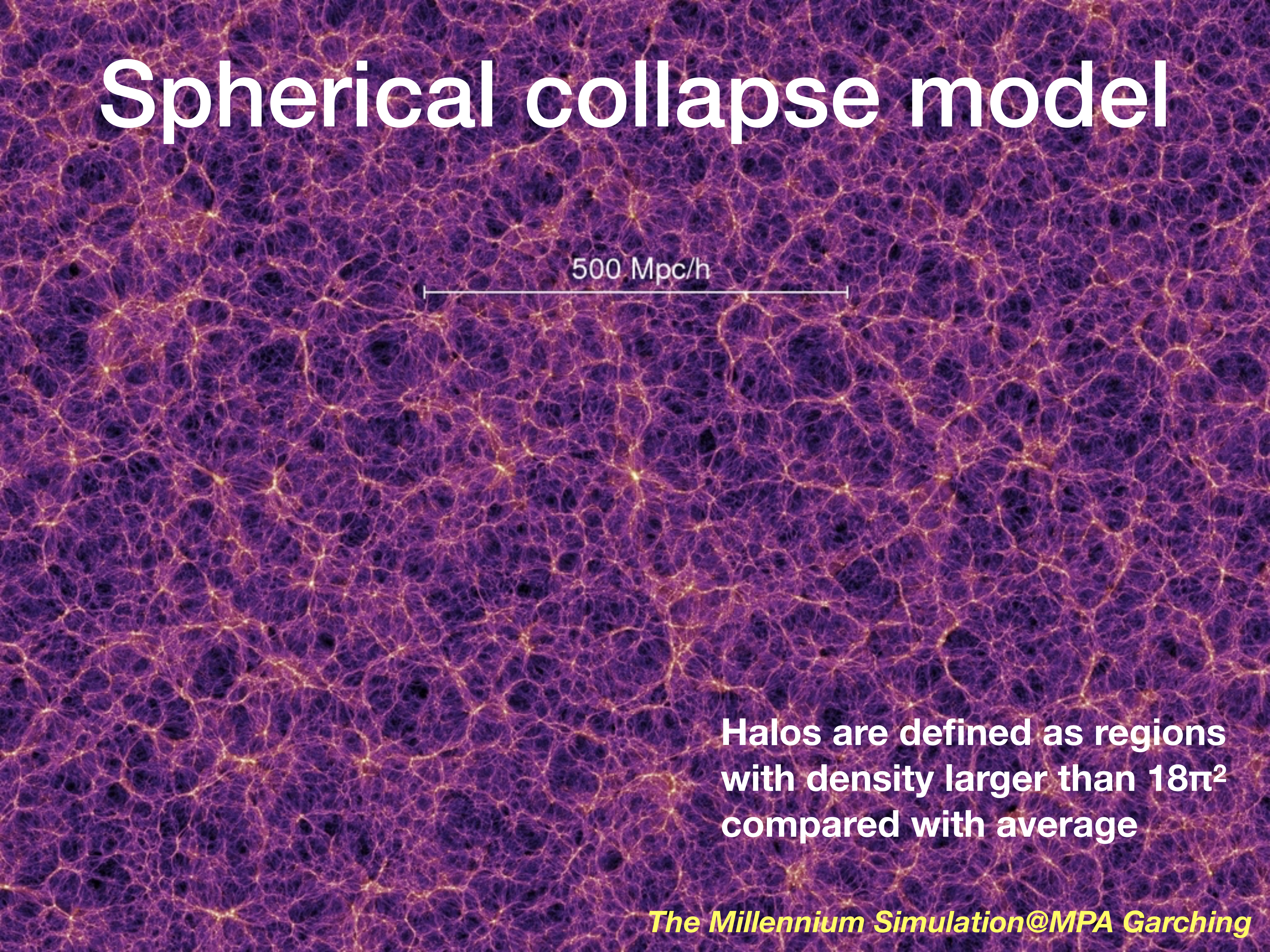
$$\rho_{\rm vir} = \frac{3M}{4\pi R_{\rm vir}^3} = \frac{6M}{\pi R_{\rm ta}^3} = \frac{3\pi}{G t_{\rm col}^2}$$

$$(t_{\rm col} = 2t_{\rm ta})$$

$$\rho_b(t_{\rm col}) = \frac{1}{6\pi G t_{\rm col}^2}$$

$$\frac{\rho_{\rm vir}}{\rho_b(t_{\rm col})} = 18\pi^2$$

Spherical collapse model



Halos are defined as regions with density larger than $18\pi^2$ compared with average

The Millennium Simulation@MPA Garching

Spherical collapse model

- Deriving two **magic numbers** *analytically*

✔ Over-density of virialized halos compared with background

$$\Delta_{\rm vir} = 18\pi^2$$

Useful for simulations to find halos

- Linear extrapolation of over-density for halos that *just* collapsed

$$\delta_c = 1.686$$

Useful for analytic calculations to estimate number of halos

Spherical collapse model

- Deriving two **magic numbers** *analytically*

✔ Over-density of virialized halos compared with background

$$\Delta_{\rm vir} = 18\pi^2 + 82[\Omega_m(z) - 1] - 39[\Omega_m(z) - 1]^2$$

ΛCDM: Bryan & Norman (1998)

- Linear extrapolation of over-density for halos that *just* collapsed

$$\delta_c = 1.686$$

Useful for analytic calculations to estimate number of halos

Analytic model of halo mass function

- It is not possible to describe non-linear evolution of density fully analytically

- However, one can extrapolate behavior in linear regime (that can be solved analytically), as if it continues

  - *What does this $18\pi^2$ collapsed region correspond to, in terms of linear over-density, δ_L?*

- One can estimate the number of halos of given mass (i.e., halo mass function), by using this threshold δ_c and by assuming density distribution is Gaussian (excellent approximation for CMB, hence must be true with linear extrapolation)

Over-density: Linear extrapolation

$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$R = A^2(1 - \cos\theta)$$

Exact solution:

$$\rho_h = \frac{3M}{4\pi R^3} = \frac{3M}{4\pi A^6}\frac{1}{(1-\cos\theta)^3}$$

$$\rho_b = \frac{1}{6\pi G t^2} = \frac{M}{6\pi A^6}\frac{1}{(\theta - \sin\theta)^2}$$

$$\Rightarrow \quad \delta = \frac{\rho_h}{\rho_b} - 1 = \frac{9(\theta-\sin\theta)^2}{2(1-\cos\theta)^3} - 1$$

Over-density: Linear extrapolation

$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$R = A^2(1 - \cos\theta)$$

$$\xrightarrow[\theta \ll 1]{} \quad t \approx \frac{A^3}{6\sqrt{GM}}\theta^3$$

Linear extrapolation:

$$\delta = \frac{9(\theta - \sin\theta)^2}{2(1-\cos\theta)^3} - 1 \approx \frac{3}{20}\theta^2 \quad \Rightarrow \quad \delta_L \approx \frac{3}{20}\left(\frac{6\sqrt{GM}}{A^3}t\right)^{2/3}$$

At collapse: $(\theta = 2\pi)$

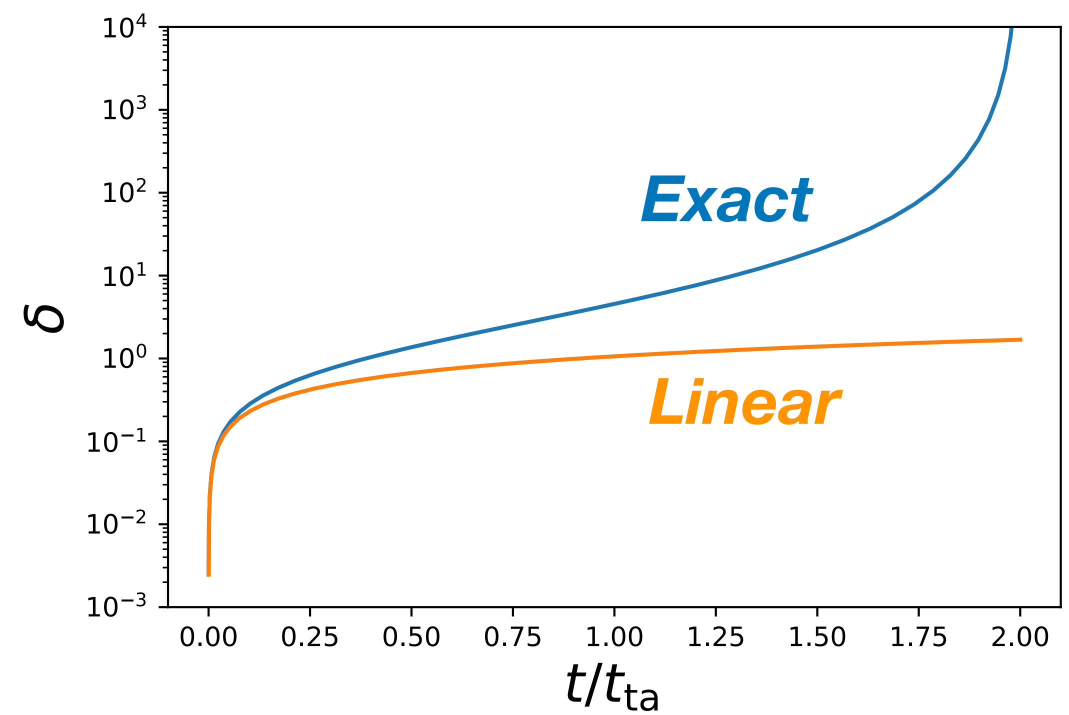
$$t_{\rm col} = \frac{2\pi A^3}{\sqrt{GM}} \quad \Rightarrow \quad \delta_L = \frac{3}{20}(12\pi)^{2/3} \approx 1.686$$

Over-density: Linear extrapolation

Exact

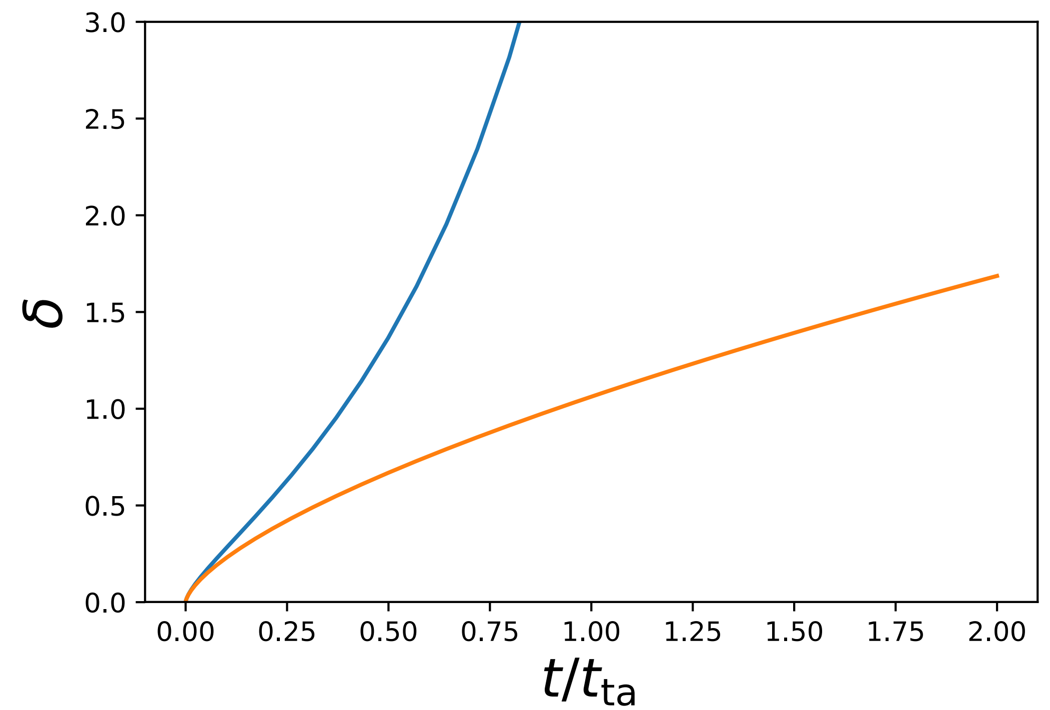
$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$\delta = \frac{9(\theta - \sin\theta)^2}{2(1-\cos\theta)^3} - 1$$

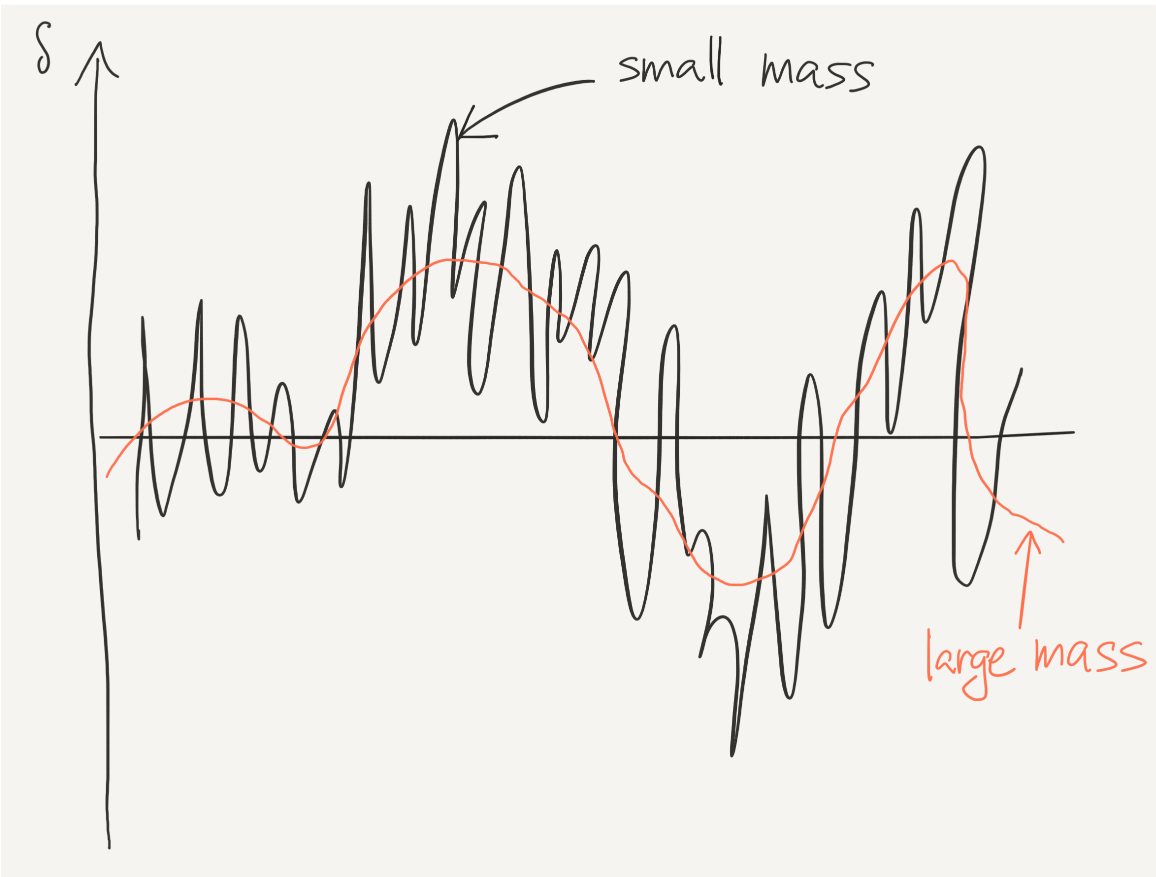


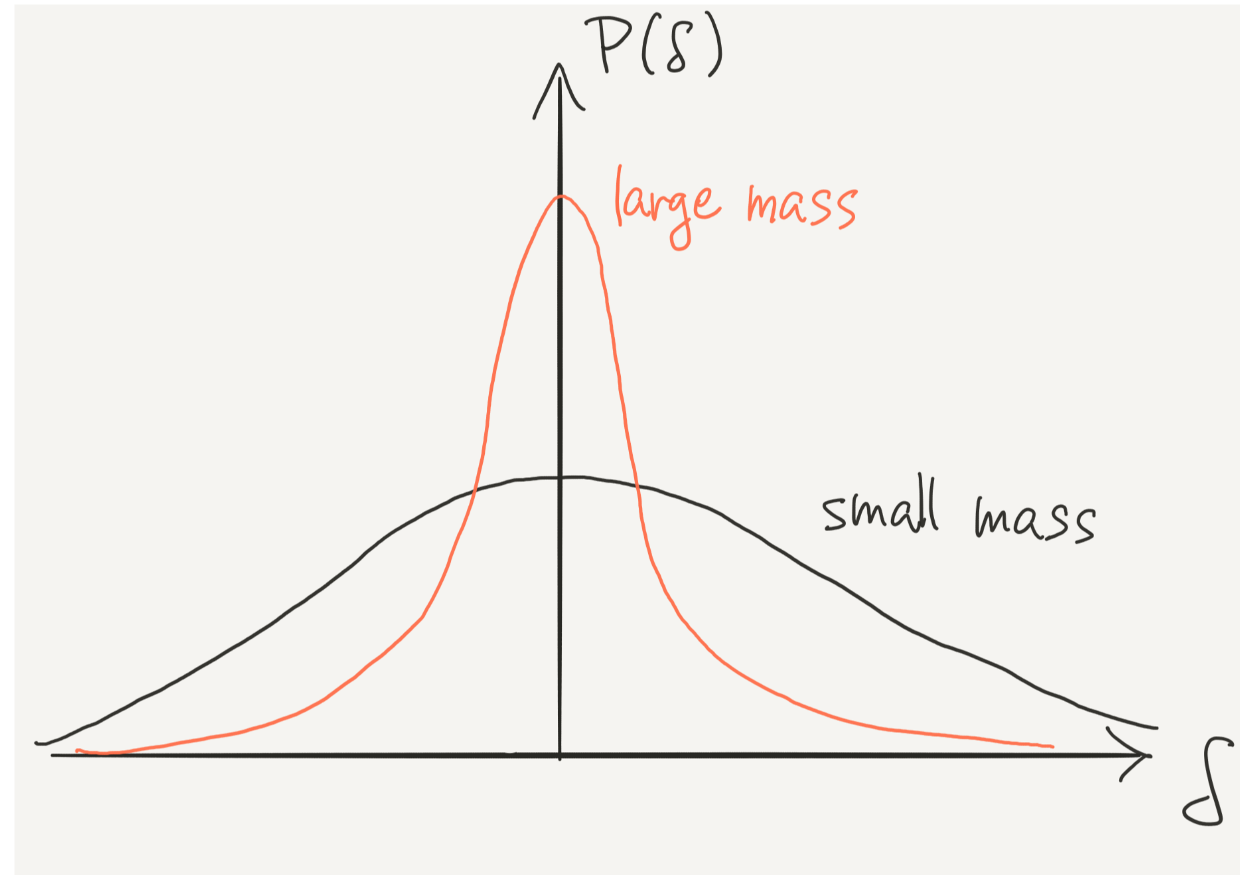
Linear

$$\delta_L \approx \frac{3}{20}\left(\frac{6\sqrt{GM}}{A^3}t\right)^{2/3}$$



Gaussian random field





Density field smeared over *R*, given by

$$M = \frac{4\pi}{3}\bar{\rho}R^3$$

rms over-density:

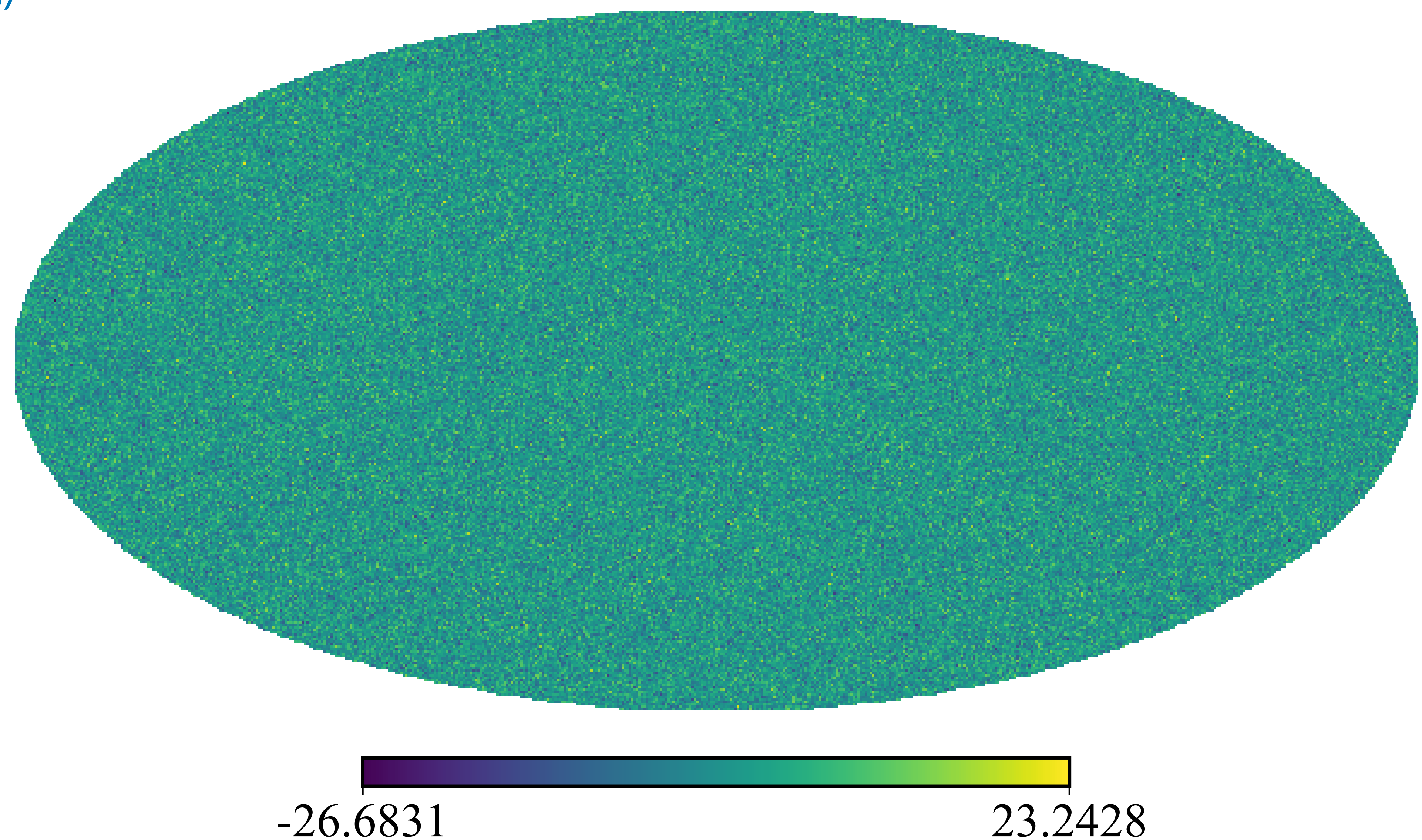
$$\sigma^2(M) = \langle \delta_R^2 \rangle = \int \frac{d^3k}{(2\pi)^3} P_{\rm lin}(k) W_R^2(k)$$

Redshift evolution:

$$\sigma(M, z) = \sigma(M) D(z)$$

Gaussian random field: simple example

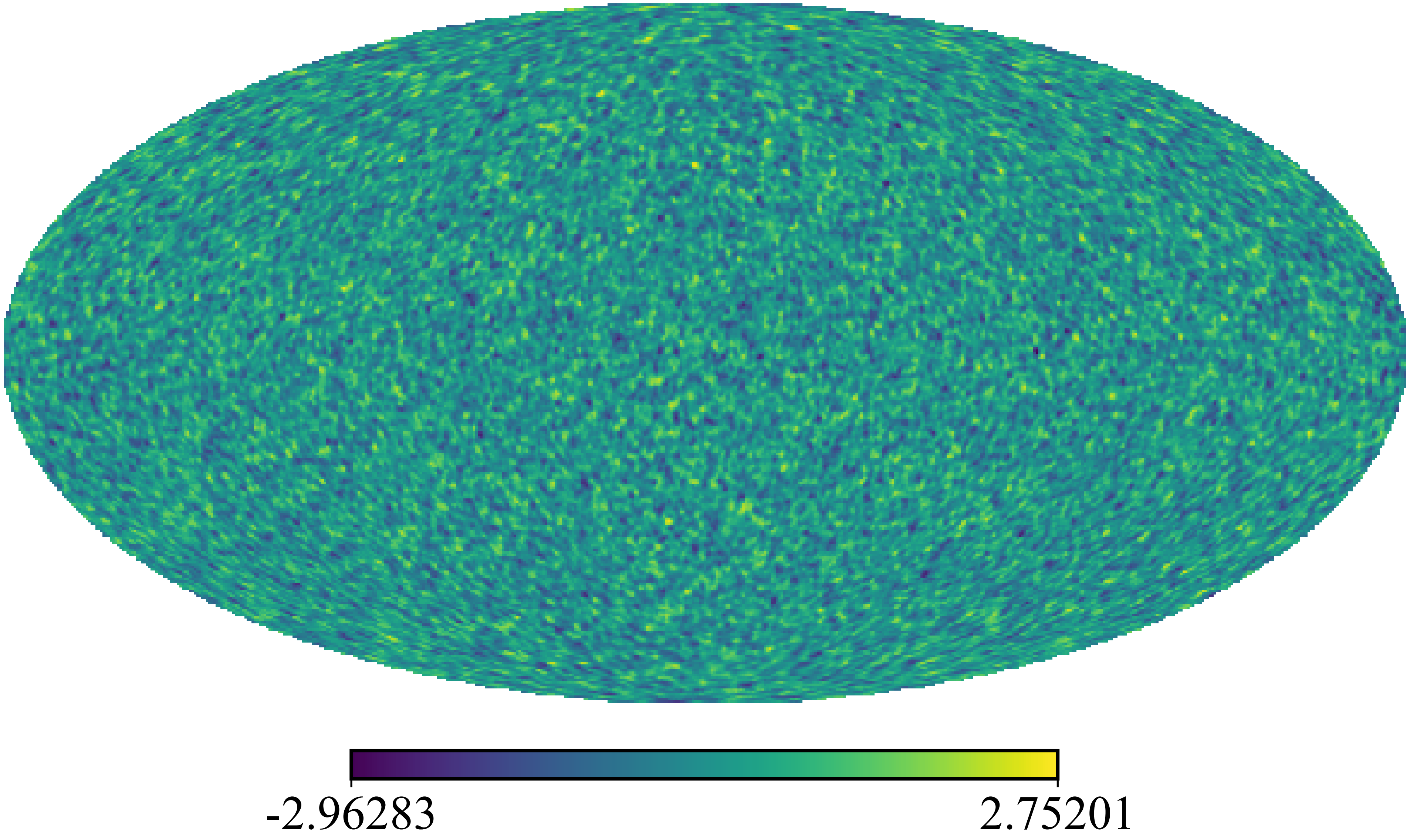
Mean = 0
SD = 5
Pixel size = $(0.23\ \mathrm{deg})^2$



Gaussian random field: simple example

Mean = 0
SD = 5
Pixel size = $(0.23 \text{ deg})^2$

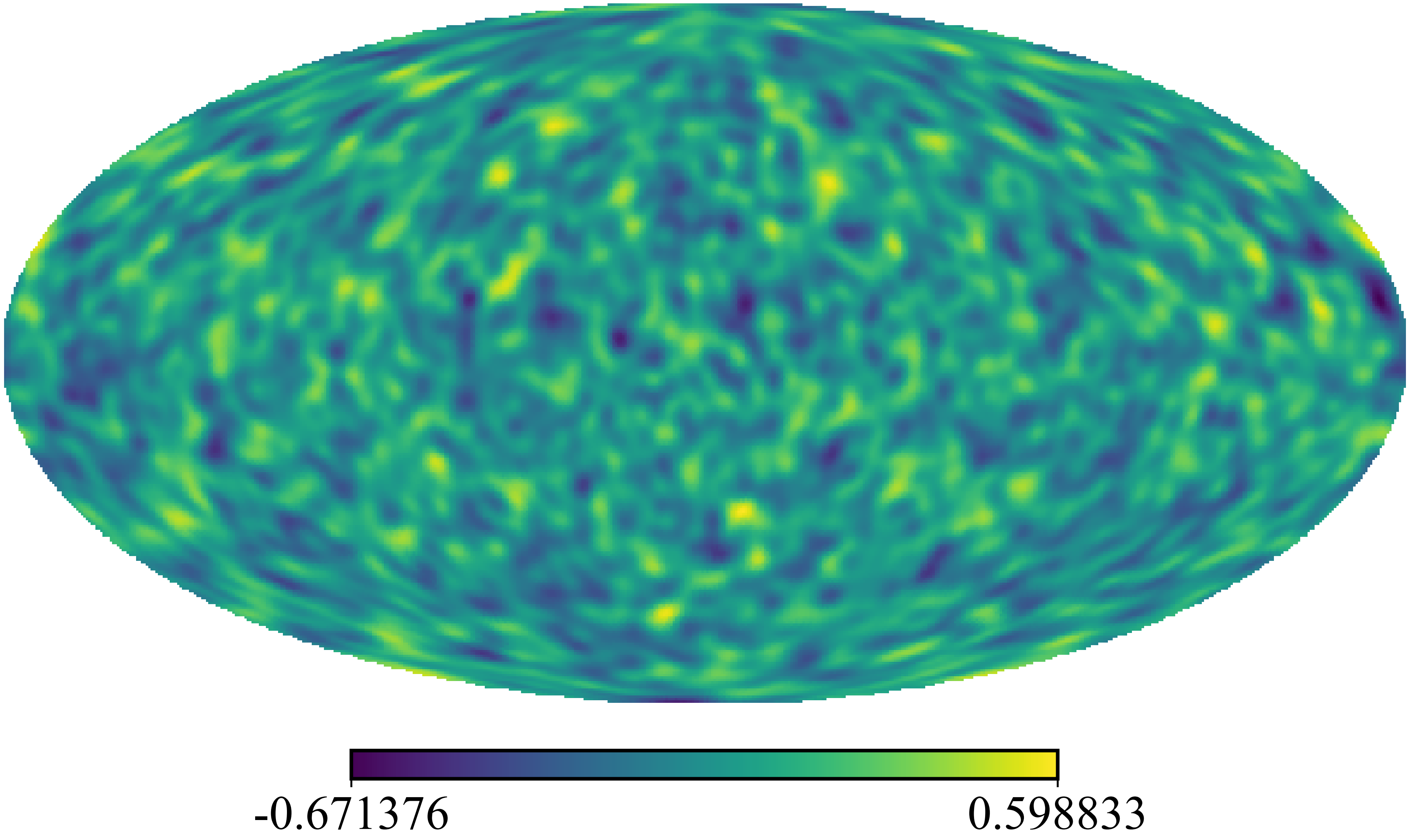
Gaussian smoothing: σ = 0.5 deg



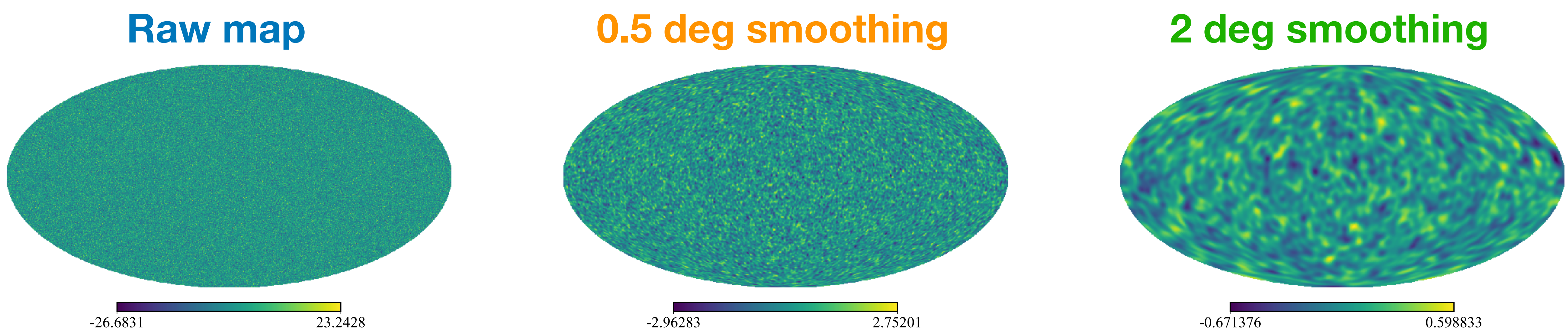
Gaussian random field: simple example

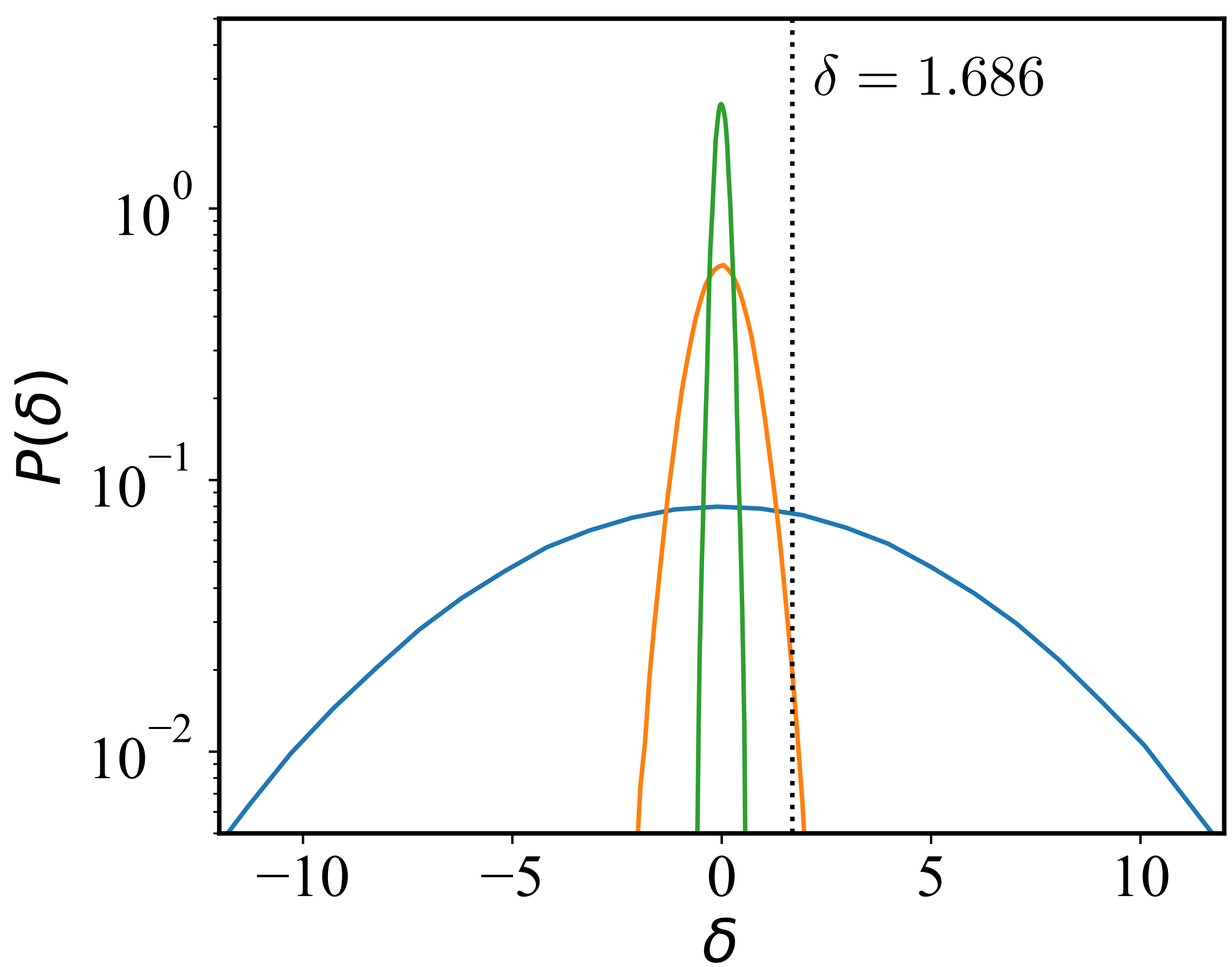
Mean = 0
SD = 5
Pixel size = (0.23 deg)^2

Gaussian smoothing: σ = 2 deg



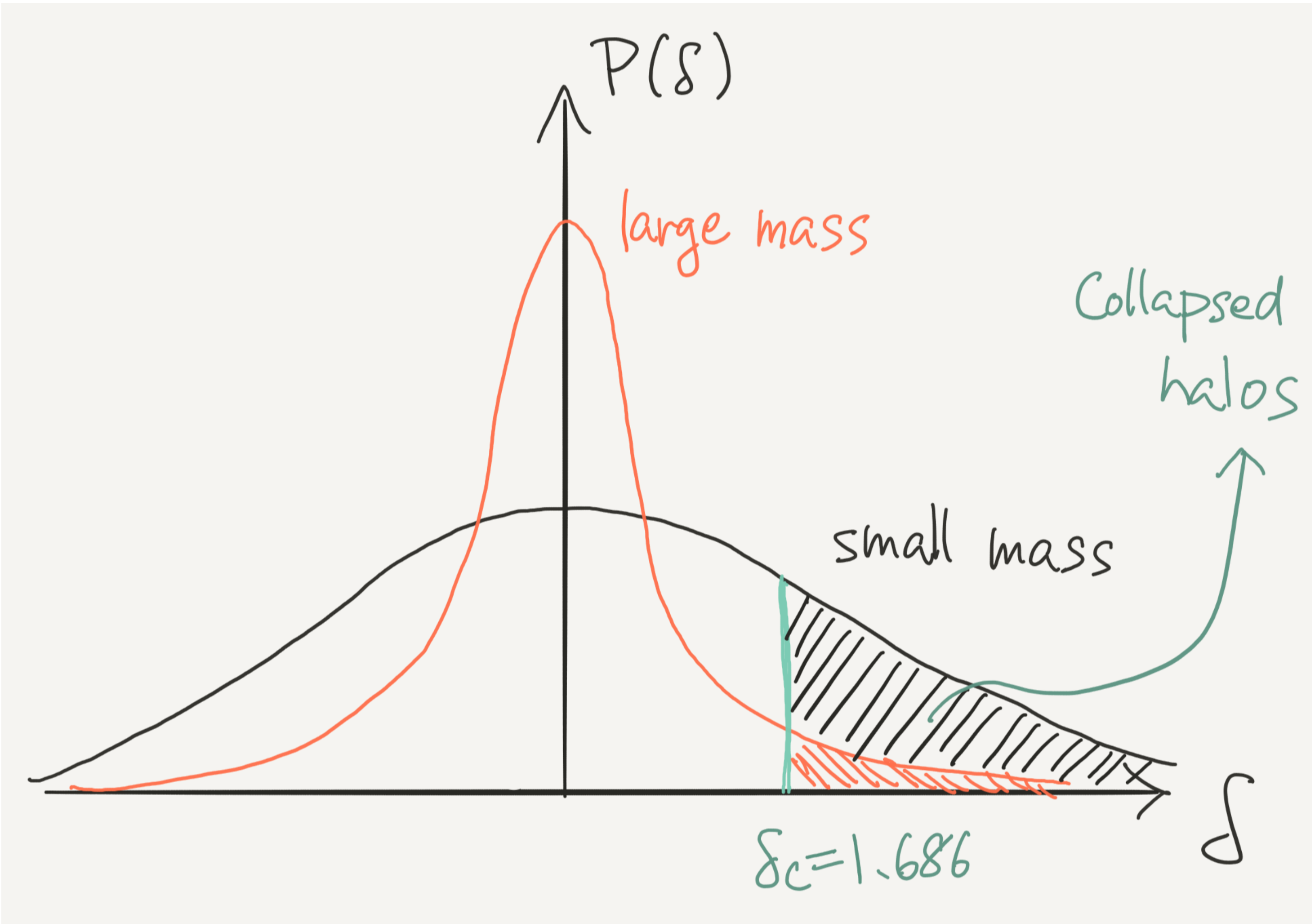
Gaussian random field: simple example





Smaller structures form first and then merge and accrete to form larger structures

Press-Schechter mass function



Fraction of collapsed halos

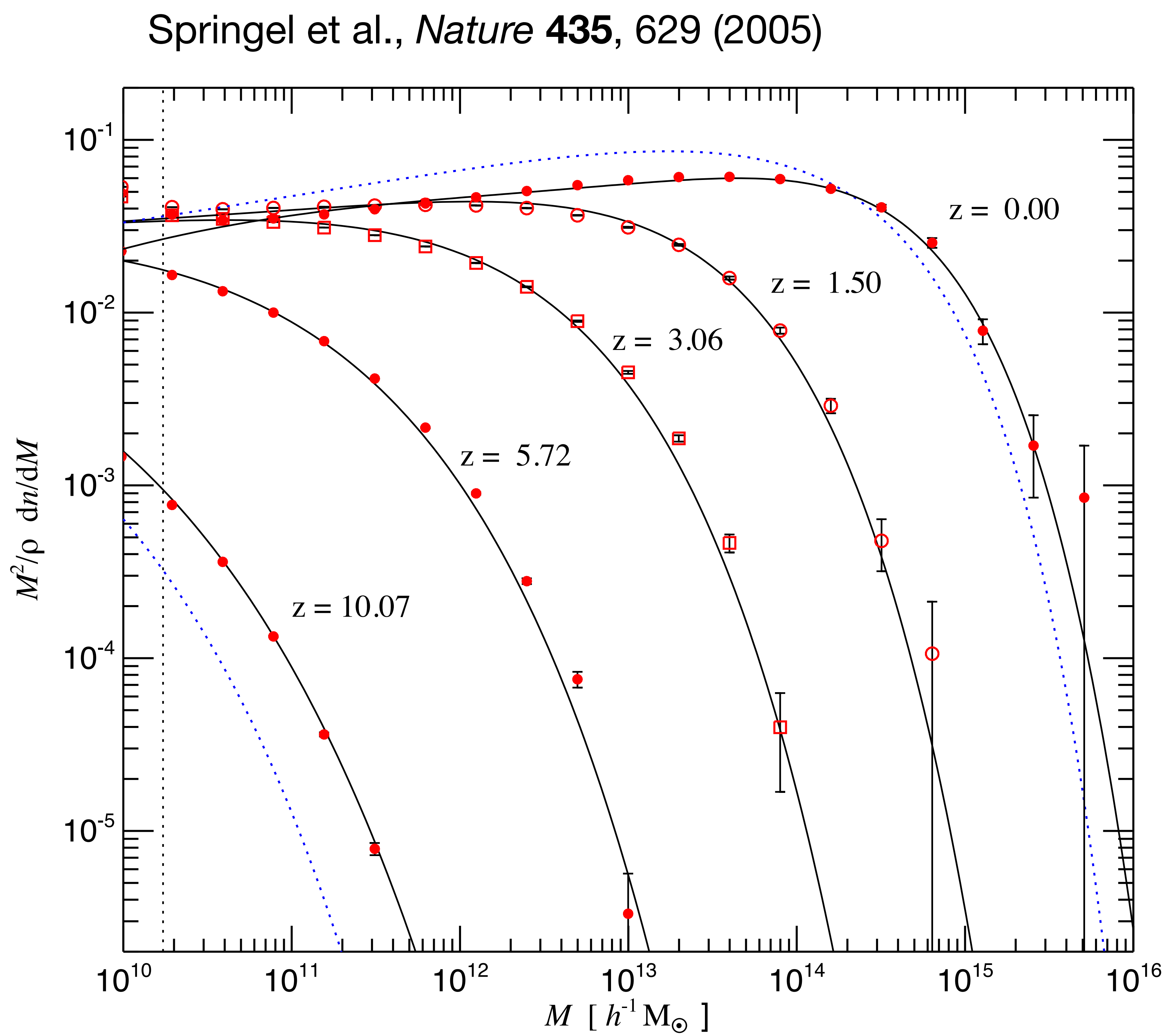
$$\int_{\delta_c}^{\infty} d\delta \; P(\delta|M,z)$$

Press-Schechter mass function $[\nu \equiv \delta_c/\sigma(M,z)]$

$$\frac{dn}{d\ln M} = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M}\nu\exp\left(-\frac{\nu^2}{2}\right)\frac{d\ln\sigma^{-1}}{d\ln M}$$

Comparison with numerical simulations

Springel et al., *Nature* **435**, 629 (2005)



- Reasonable agreement with the Millennium simulations (**Red** points)
  - **Blue**: Press-Schechter mass function
  - **Black**: Jenkins et al. (2001) mass function
- Other representative models include Sheth & Tormen (2001), Tinker et al. (2008), many of which are based on ***ellipsoidal collapse*** model

Spherical collapse model

- Deriving two **magic numbers** *analytically*

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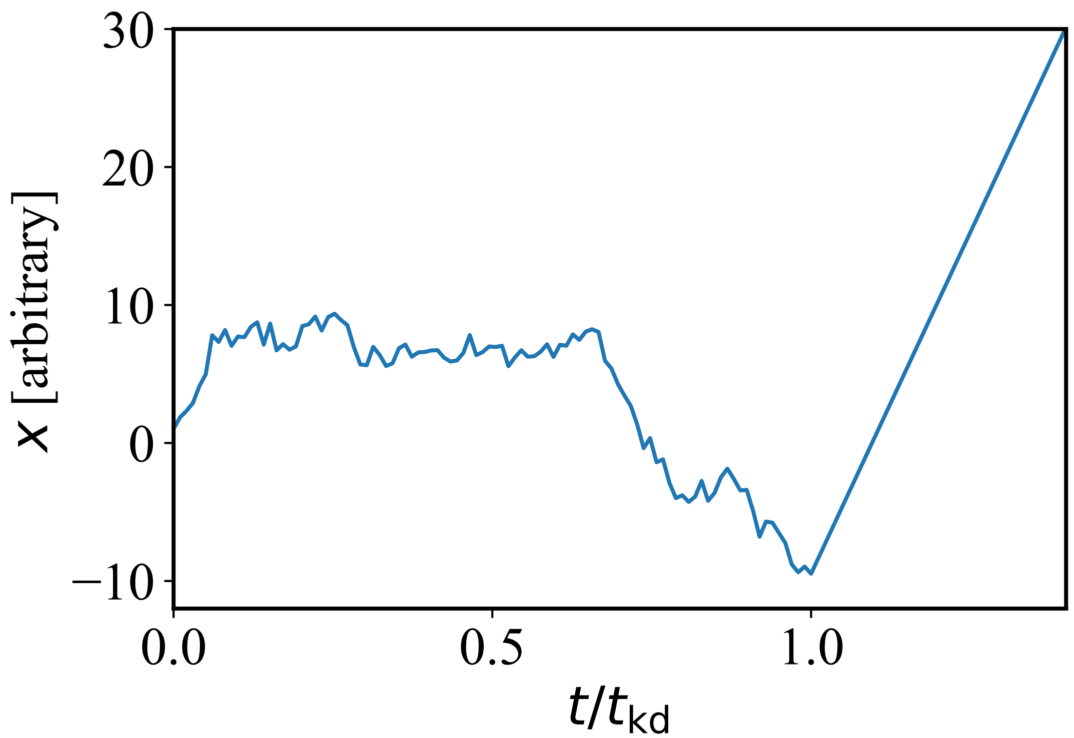
Useful for analytic calculations to estimate number of halos

What is the smallest structure?

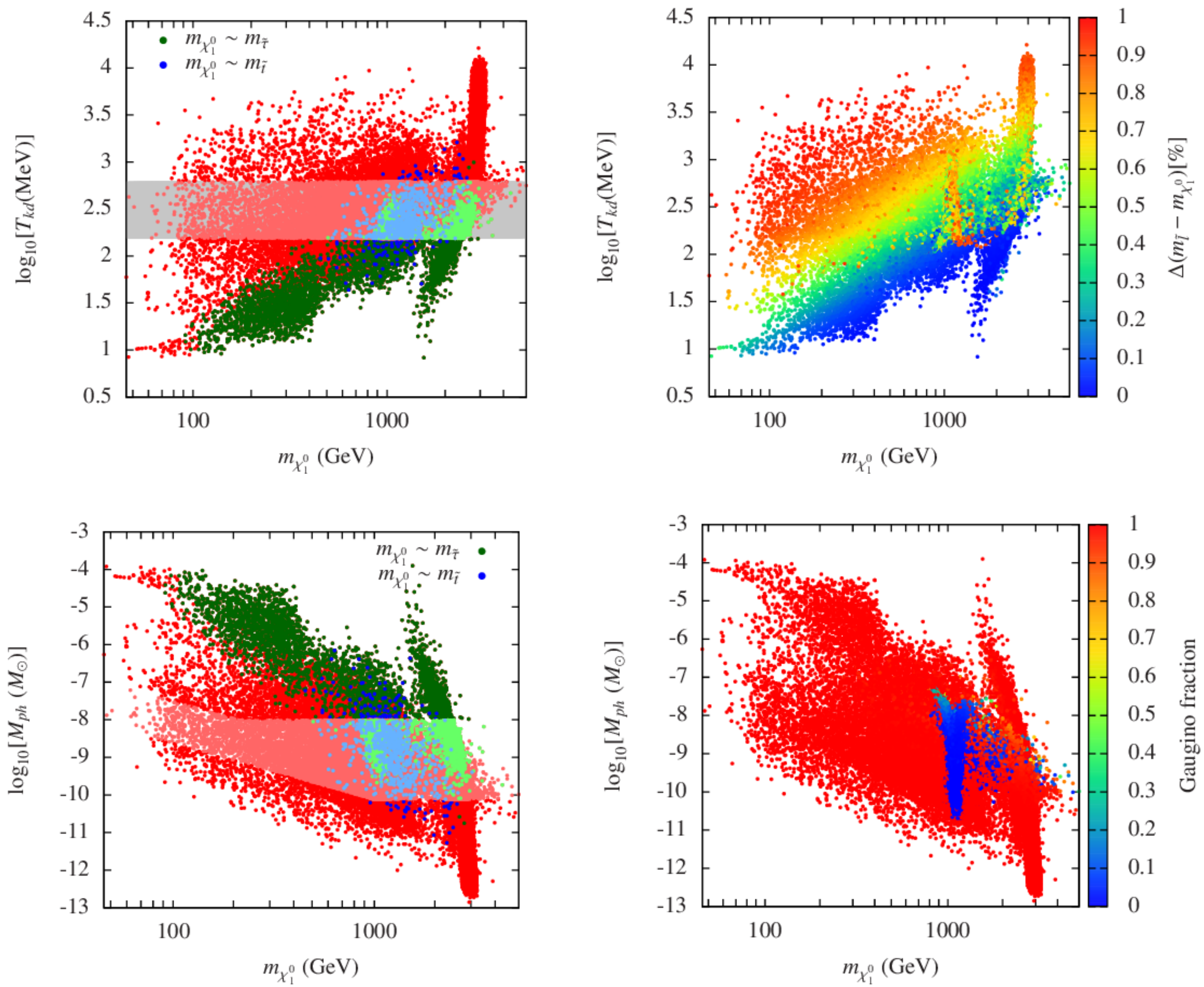
E.g., 1d random walk followed by free-streaming



- In the WIMP scenario:
- After **chemical decoupling**, WIMPs can still interact with baryons and leptons through scattering
- When this gets slower than Hubble expansion (**kinetic decoupling**), WIMPs start free-streaming
- **All the structures below this *kd+free-streaming* scale will be washed away**
- Finding small halos is key to distinguish different dark matter models

What is the smallest structure?

Diamanti, Cabrera-Catalan, Ando, *Phys. Rev. D* **92**, 065029 (2015)



- MCMC parameter scan for 9-parameter MSSM
- Typical kinetic decoupling temperature: a few MeV
- Typical smallest halo mass: $10^{-12} - 10^{-4} M_\odot$