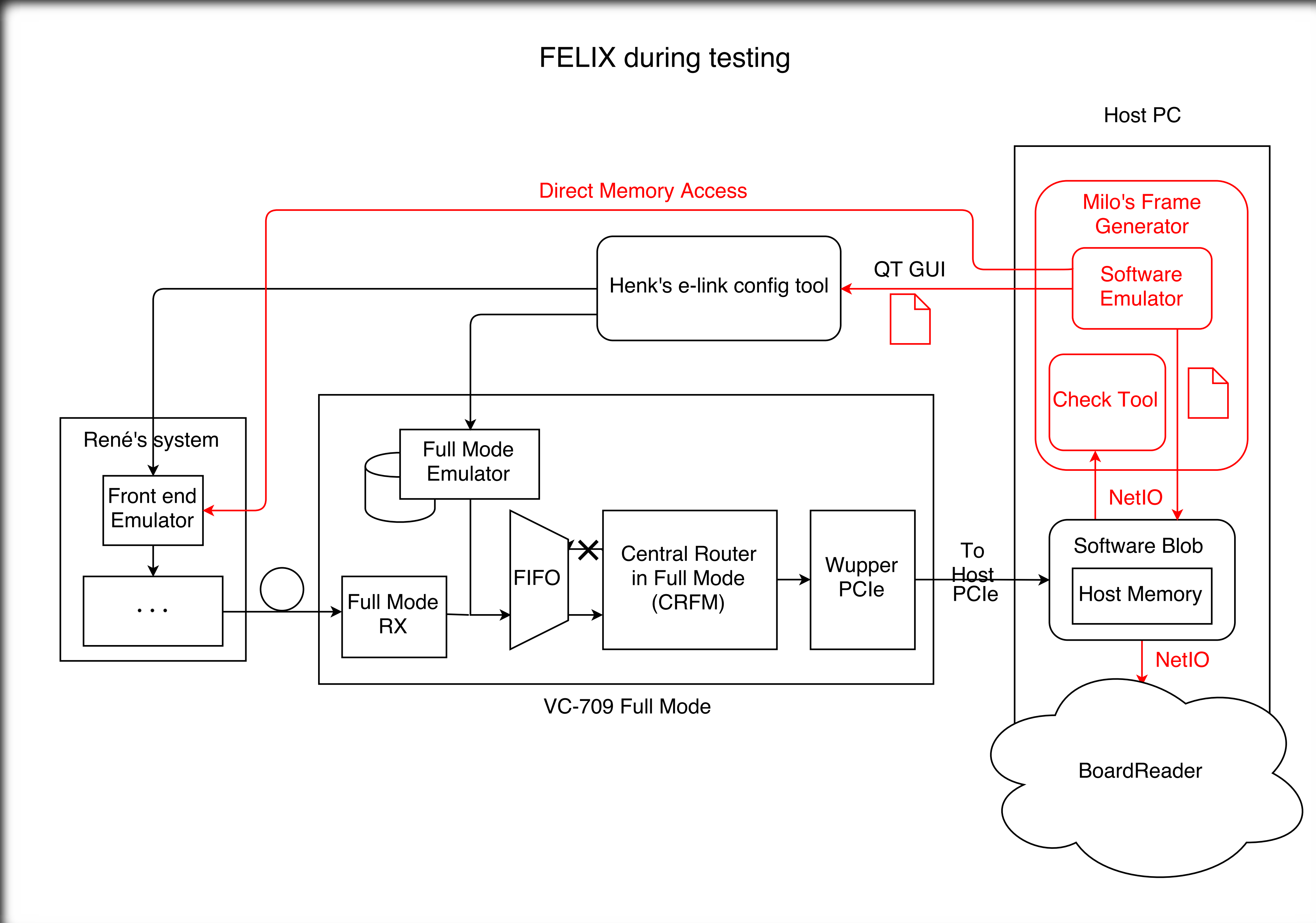
MILO VERMEULEN 18-7-2017

NEUTRINO OSCILLATION SIMULATIONS

PROTODUNE DAQ

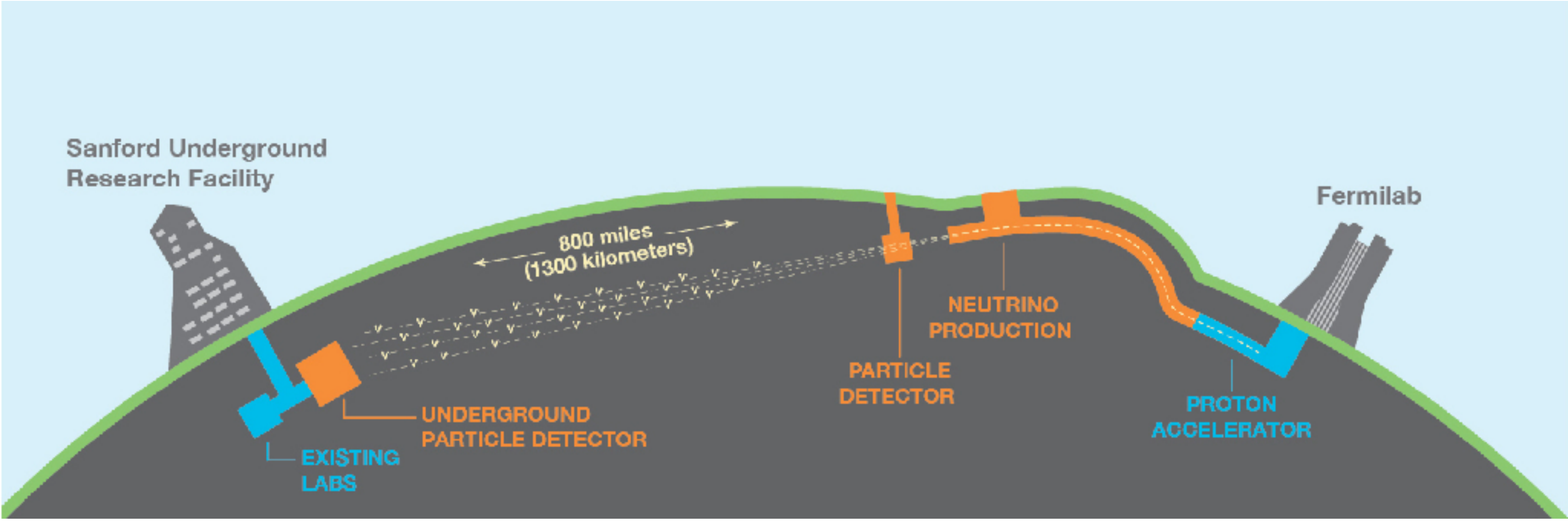


OBJECTIVES

- To simulate neutrino oscillations through vacuum
  - Easily done for arbitrary energy and distance
- To simulate neutrino oscillations through matter
  - Analytically by approximation
    - Does not work so far
  - Numerically

VACUUM OSCILLATIONS

- Use the well-known PMNS matrix and best values for oscillation parameters
- $$\begin{aligned} P(\nu_i \to \nu_j) = & |U_{i0}U_{j0}|^2 + |U_{i1}U_{j1}|^2 + |U_{i2}U_{j2}|^2 + \\ & 2\Re\left(U^\dagger_{i0}U_{j0}U_{i1}U^\dagger_{j1}e^{-i2.54\cdot\Delta m^2_{12}L/E}\right) + \\ & 2\Re\left(U^\dagger_{i0}U_{j0}U_{i2}U^\dagger_{j2}e^{-i2.54\cdot\Delta m^2_{13}L/E}\right) + \\ & 2\Re\left(U^\dagger_{i1}U_{j1}U_{i2}U^\dagger_{j2}e^{-i2.54\cdot\Delta m^2_{23}L/E}\right) \end{aligned}$$
- From Mark Thompson's *Modern Particle Physics*



Sanford Underground Research Facility
Fermilab
800 miles (1300 kilometers)
NEUTRINO PRODUCTION
PARTICLE DETECTOR
PROTON ACCELERATOR
UNDERGROUND PARTICLE DETECTOR
EXISTING LABS

Parameter mapping

Considering the correct ordering of the eigenvectors (eq. (21)) and following the above described steps, one can determine the complete parameter mapping for all regions of the $\hat{A}$ parameter space. Comprising, one obtains the following expressions for the mixing parameters in matter:

$$\sin\theta'_{13} = \frac{\sin 2\theta_{13}}{\sqrt{2\hat{C}(\mp\hat{A}+\hat{C}\pm\cos 2\theta_{13})}} \pm \frac{\alpha\,\hat{A}\sin^2\theta_{12}\sin^2 2\theta_{13}}{2\hat{C}^2\sqrt{2\hat{C}(\pm\hat{A}+\hat{C}\mp\cos 2\theta_{13})}} \tag{27a}$$

$$\sin\theta'_{12} = \alpha\frac{\hat{C}\sin 2\theta_{12}}{|\hat{A}|\cos\theta_{13}\sqrt{2\hat{C}(\mp\hat{A}+\hat{C}\pm\cos 2\theta_{13})}} \tag{27b}$$

$$\sin\theta'_{23} = \sin\theta_{23} + \alpha\cos\delta\frac{\hat{A}\sin 2\theta_{12}\sin\theta_{13}\cos\theta_{23}}{\pm 1+\hat{C}\mp\hat{A}\cos 2\theta_{13}} \tag{27c}$$

$$\sin\delta' = \sin\delta\left(1-\alpha\frac{\cos\delta}{\tan 2\theta_{23}}\frac{2\hat{A}\sin 2\theta_{12}\sin\theta_{13}}{\pm 1+\hat{C}\mp\hat{A}\cos 2\theta_{13}}\right) \tag{27d}$$

Here, in the expressions with choices for the sign, the upper sign holds for $\hat{A} < \cos 2\theta_{13}$ and the lower sign holds for $\hat{A} > \cos 2\theta_{13}$. Higher orders than $\mathcal{O}(\alpha)$ are omitted. To take into account also θ_{23} and δ, which were factored out at the beginning, the equations (15a-d) were applied. The expansion of $\sin\delta'$ given here does not hold for $\theta_{23} \to 0$.

From this parameter mapping it is possible to derive the following quantities:

$$\sin^2 2\theta'_{13} = \frac{\sin^2 2\theta_{13}}{\hat{C}^2} + \alpha\frac{2\hat{A}(-\hat{A}+\cos 2\theta_{13})\sin^2\theta_{12}\sin^2 2\theta_{13}}{\hat{C}^4} \tag{28a}$$

$$\sin 2\theta'_{12} = \alpha\frac{2\hat{C}\sin 2\theta_{12}}{|\hat{A}|\cos\theta_{13}\sqrt{2\hat{C}(\mp\hat{A}+\hat{C}\pm\cos 2\theta_{13})}} \tag{28b}$$

$$\sin 2\theta'_{23} = \sin 2\theta_{23} + \alpha\cos\delta\frac{2\hat{A}\sin 2\theta_{12}\sin\theta_{13}\cos 2\theta_{23}}{\pm 1+\hat{C}\mp\hat{A}\cos 2\theta_{13}} \tag{28c}$$

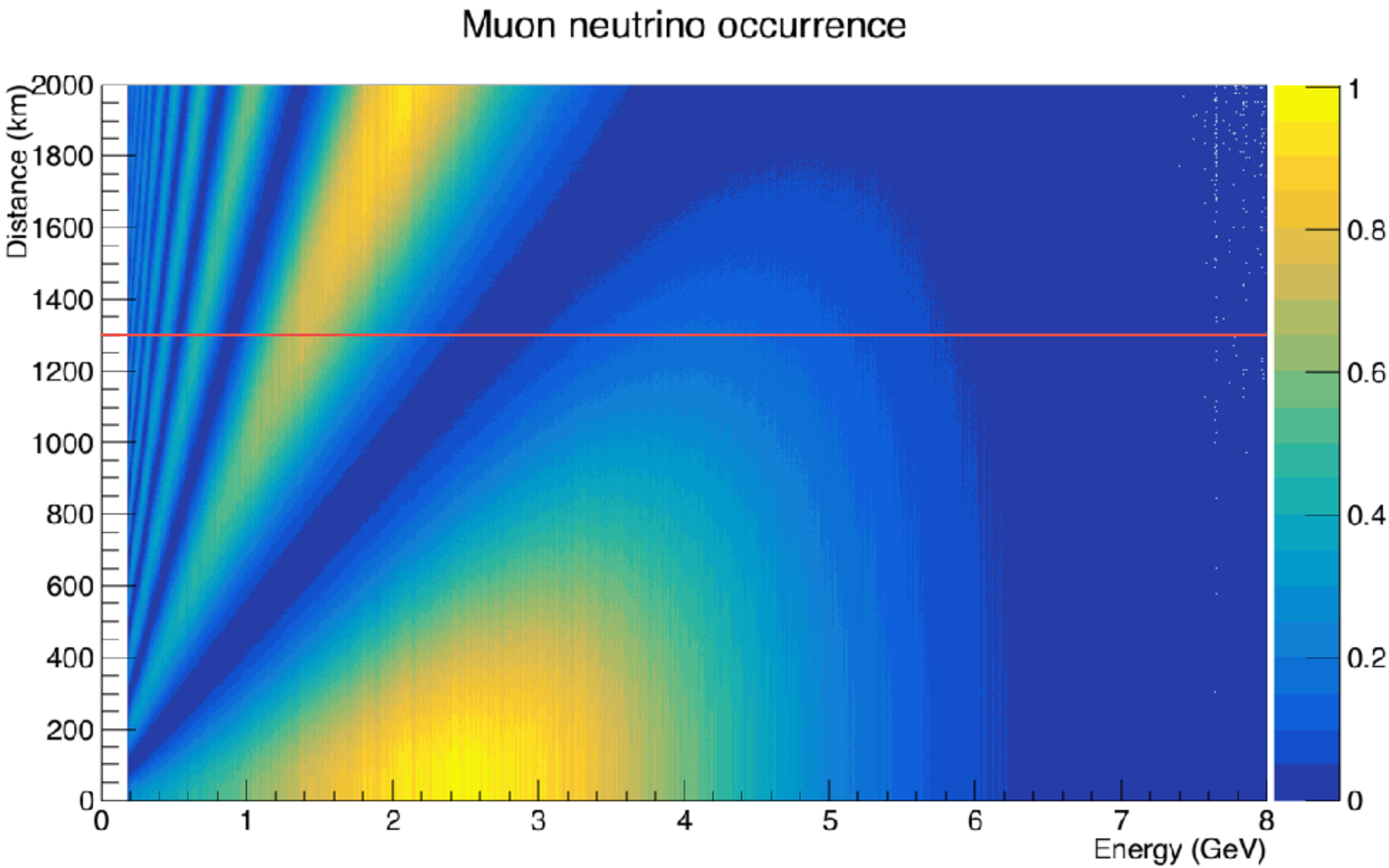
For the mass squared differences one obtains:

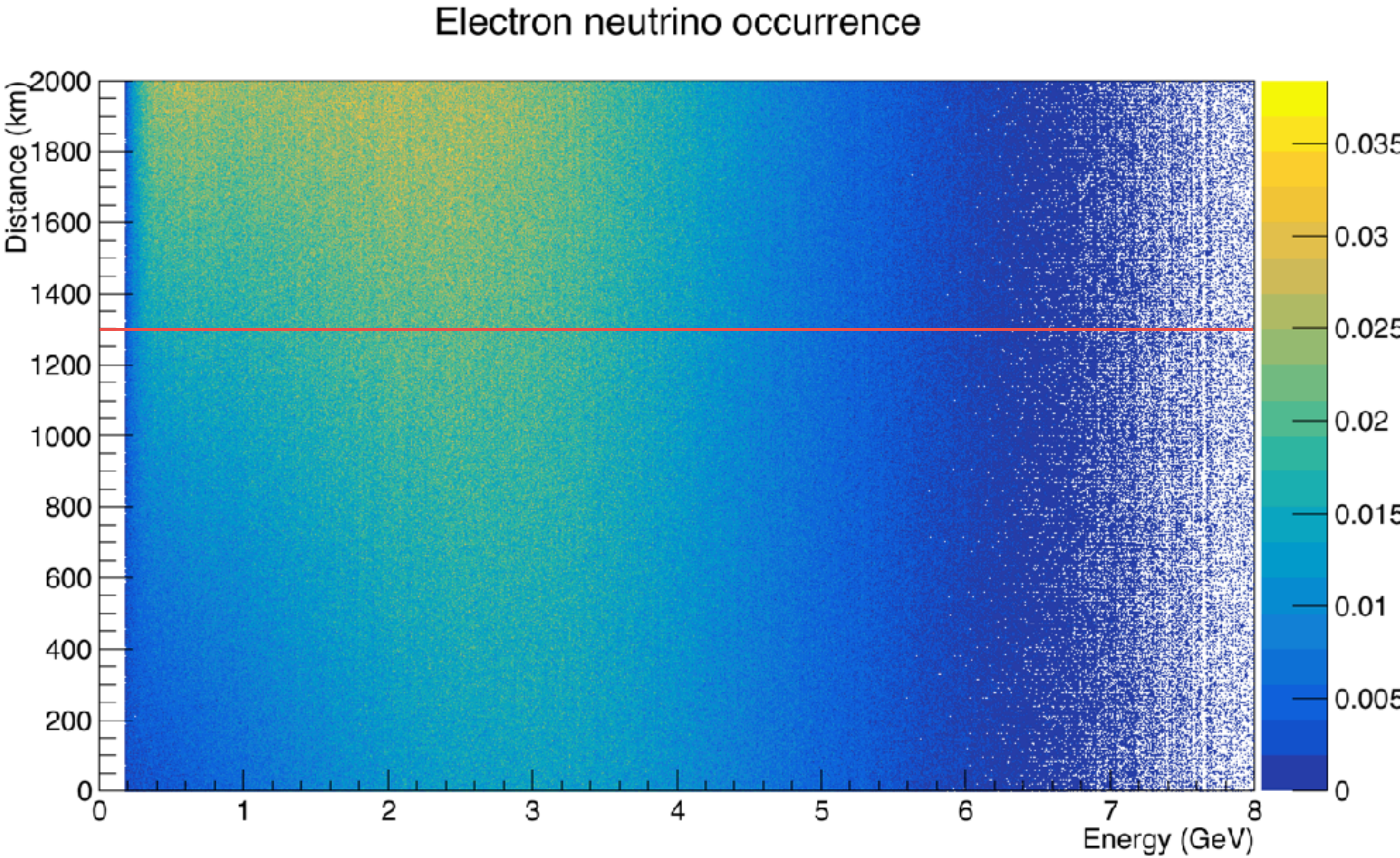
$$(\Delta m^2_{21}{}', \Delta m^2_{31}{}', \Delta m^2_{32}{}') = \begin{cases} (\ \ \Delta m^2_3, \ \ \Delta m^2_2, \ \ \Delta m^2_1) & \text{for} \quad \hat{A} < \cos 2\theta_{13} \\ (-\Delta m^2_1, -\Delta m^2_2, -\Delta m^2_3) & \text{for} \quad \hat{A} > \cos 2\theta_{13} \end{cases} \tag{29}$$

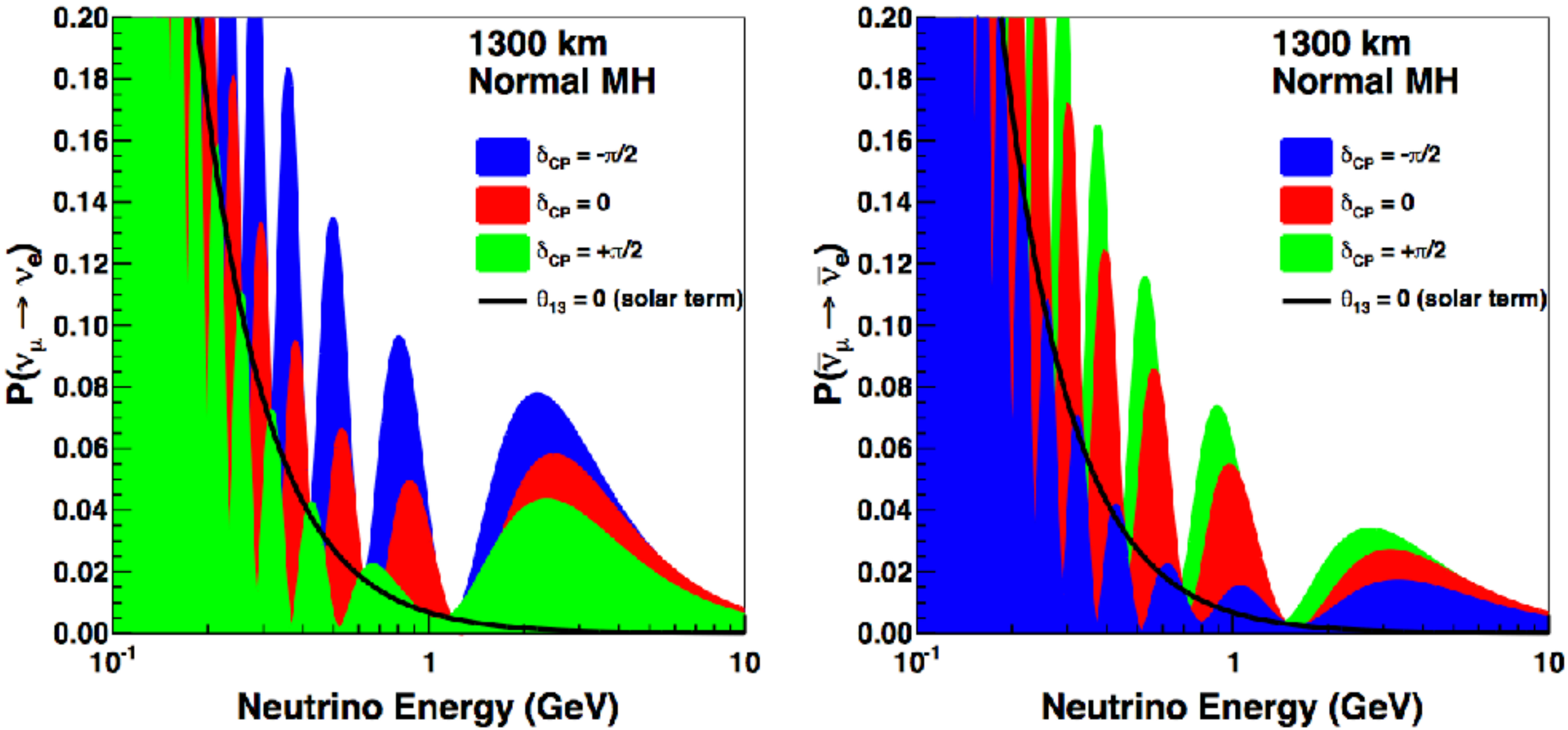
with

$$\begin{aligned} \Delta m^{2\prime}_1 &:= \Delta(\lambda_3 - \lambda_2) \\ &= \frac{1}{2}(1 + \hat{A} + \hat{C})\Delta - \alpha\Delta\left(\cos^2\theta_{12} - \frac{(-1 + \hat{C} + \hat{A}\cos 2\theta_{13})\sin^2\theta_{12}}{2\hat{C}}\right), \quad (30a) \\ \Delta m^{2\prime}_3 &:= \Delta(\lambda_2 - \lambda_1) \\ &= \frac{1}{2}(-1 - \hat{A} + \hat{C})\Delta + \alpha\Delta\left(\cos^2\theta_{12} - \frac{(1 + \hat{C} - \hat{A}\cos 2\theta_{13})\sin^2\theta_{12}}{2\hat{C}}\right), \quad (30b) \\ \Delta m^{2\prime}_2 &:= \Delta(\lambda_3 - \lambda_1) \\ &= \hat{C}\Delta + \alpha\frac{\Delta(-1 + \hat{A}\cos 2\theta_{13})\sin^2\theta_{12}}{\hat{C}}. \quad (30c) \end{aligned}$$

M. Freund, 2001


Muon neutrino occurrence
Distance (km)
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
1
2
3
4
5
6
7
8
Energy (GeV)
1
0.8
0.6
0.4
0.2
0


Electron neutrino occurrence
Distance (km)
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
1
2
3
4
5
6
7
8
Energy (GeV)
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0



DUNE Conceptual Design Report, 2016

PLANS

- Redo oscillation implementation
- Compare analytically to CDR
- Perform numerical simulations