EW Physics

3 x 2h lectures
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* Today: Reminder
  - Group and Lie algebra
  - Spinors, Clifford Algebra and Chiral components
  - Gauge transformations
    Abelian & non Abelian case

Standard Model Particles

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{EM}$
$L_L = \begin{pmatrix} \nu_{eL} \\ l_L \end{pmatrix}$	1	2		$\begin{matrix} 0 \\ -1 \end{matrix}$
l_R	1	1		-1
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2		$\begin{matrix} 2/3 \\ -1/3 \end{matrix}$
u_R	3	1		$2/3$
d_R	3	1		$-1/3$
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	1	2		$\begin{matrix} +1 \\ 0 \end{matrix}$

<u>SM gauge group</u> $SU(3)_c \times SU(2)_L \times U(1)_Y$ $\hookrightarrow \neq U(1)_{EM}$

<u>gauge fields</u>: 8 gluons, Z $W^\pm$ γ $(\leftrightarrow B_\mu, W_\mu^{1,2,3})$

1. Dirac Field

$$(i\not{\partial} - m)\Psi = 0$$

$$\not{\partial} = \partial_\mu \gamma^\mu$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \mathbb{1}$$

$\mu, \nu = 0, 3$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

$\gamma^{5\dagger} = \gamma^5$, $(\gamma^5)^2 = \mathbb{1}$

$$\{\gamma^\mu, \gamma^5\} = 0$$

Weyl or Chiral representation

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\sigma^\mu = \{\mathbb{1}, \sigma^i\} \qquad \mu = 0, \dots, 3$$
$$\bar{\sigma}^\mu = \{\mathbb{1}, -\sigma^i\} \qquad i = 1, \dots, 3$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} \quad \begin{cases} L = \dfrac{\mathbb{1} - \gamma_5}{2} \\ R = \dfrac{\mathbb{1} + \gamma_5}{2} \end{cases}$$

$$\psi_L = L\,\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$$

$$L = \frac{1-\gamma_5}{2} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\psi \longleftrightarrow \gamma^\alpha$$

$$\psi_L \longleftrightarrow \sigma^\mu$$

$$S^{\mu\nu} = \frac{i}{4}\left[\gamma^\mu, \gamma^\nu\right]$$

4x4 representation of the Lorentz Algebra

$$\Lambda = e^{-\frac{i}{2}\omega_{\mu\nu} S^{\mu\nu}}$$

Lorentz trf acting on spinors

$$\begin{cases} \omega_{0i} = \beta_i \rightarrow \text{Boost} \\ \omega_{kj}\,\varepsilon^{kji} = \theta_i \rightarrow \text{Rotations} \end{cases}$$

$$\psi_{\substack{L\\R}} \rightarrow e^{i\sigma^j/2(\pm i\beta_j - \theta_j)}\,\psi_{\substack{L\\R}} = \Lambda_{\substack{L\\R}}\psi_{\substack{L\\R}}$$

Λ_L^* is equivalent Λ_R

$$i\sigma_2 \psi_L^* \equiv \chi_R$$

<u>Check:</u> $\chi_L^\dagger i\sigma_2 \psi_L$

Lorentz invariant

$$(i\sigma_2 \psi_R^*)^\dagger i\sigma_2 \psi_L$$

$$\equiv \psi_R^\dagger \psi_L$$

$$\bar{\Psi}\Psi = \Psi^\dagger \gamma_0 \Psi = \psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R$$

2. GAUGE SYMMETRIES

GLOBAL SYMMETRIES

$$\Psi \to e^{i\alpha(x)}\Psi$$

$$\mathcal{L} \supset \bar{\Psi}\, i \not{\partial}\, \Psi$$

$$\to \bar{\Psi}\, e^{-i\alpha}\, i\, \gamma^{\mu} \partial_{\mu} \left(e^{i\alpha} \Psi \right)$$

$$\bar{\Psi}\, i\, \gamma^{\mu} \left(i\, \partial_{\mu}\alpha + \partial_{\mu} \right) \Psi$$

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - iqA_\mu$$

A_μ transform as $A_\mu + \frac{1}{q}\partial_\mu \alpha$

$$\bar{\Psi} i \not{D} \Psi$$

$$\longrightarrow \bar{\Psi} e^{-i\alpha} \gamma^\mu i \left(\partial_\mu - iqA'_\mu\right) e^{i\alpha} \Psi$$

$$\bar{\Psi} \gamma^\mu i \left(\partial_\mu + i \cancel{\partial_\mu \alpha} - iq\left(A_\mu + \cancel{\tfrac{1}{q}\partial_\mu \alpha}\right)\right) \Psi$$

$$= \bar{\Psi} \gamma^\mu i \left(\partial_\mu - i\underline{\underline{qA_\mu}}\right) \Psi = \bar{\Psi} i \not{D} \Psi$$

$$\mathcal{L} \supset +q \bar{\Psi} \gamma^\mu \Psi A_\mu \equiv q J^\mu A_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

EM Field strength

$$A_\mu \to A_\mu + \frac{1}{q} \partial_\mu \alpha .$$

$F_{\mu\nu}$ is invariant under U(1) trsf.

$$\mathcal{L}_{Kin} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i \bar{\Psi} \not{D} \Psi - m \bar{\Psi} \Psi$$

EOM A_μ: $\partial_\mu F^{\mu\nu} = -q \bar{\Psi} \gamma^\nu \Psi$

$\partial_\nu \partial_\mu F^{\mu\nu} = -q \partial_\nu \bar{\Psi} \gamma^\nu \Psi \Rightarrow$

$\Downarrow$

$\partial_\nu J^\nu = 0$

NB $\star$ $[D_\mu, D_\nu] = -iq F_{\mu\nu}$

$\star$ $\frac{1}{2} m_A^2 A_\mu A^\mu$ does not respect the gauge sym

3. Abelian and non Abelian trsf

- $U(N)$ or $SU(N)$ $\equiv$ unitary groups

$$U \in U(N) \rightarrow U^{\dagger}U = \mathbb{1}$$
$$SU(N) \rightarrow \det(U) = 1$$

- $O(N)$ or $SO(N)$ $\equiv$ orthog. groups

$$O \in O(N) \rightarrow O^{T}O = \mathbb{1}$$
$$\in SO(N) \rightarrow \det(O) = \mathbb{1}$$

$$G(\alpha) = \mathbb{1} + i\alpha^a T^a + O(\alpha^2)$$

groups with this structure

$\equiv$ Lie Groups

T^a (generators) satisfy commutation rel

$$[T^a, T^b] = i f^{abc} T^c$$

Lie algebra

structure constant

$\underline{SU(2)} \longrightarrow T^a = \frac{\sigma^a}{2}$

$$U = \mathbb{1} + i\alpha^a T^a + \dots$$

$$U^\dagger U = \mathbb{1} = \left(\mathbb{1} - i\alpha^a T^{a\dagger}\right)\left(\mathbb{1} + i\alpha^a T^a\right)$$

$$\cancel{\mathbb{1}} = \cancel{\mathbb{1}} + i\alpha^a\left(-T^{a\dagger} + T^a\right) + \dots$$

$$\Rightarrow T = T^\dagger$$

$$\begin{pmatrix} \times & \times \\ & \times \end{pmatrix} \quad \begin{matrix} \to 2 \\ \to 2 \end{matrix} \quad \Rightarrow 4$$

* $\det U = 1$

$\Rightarrow \mathrm{tr}(T) = 0$ $\qquad \Rightarrow$ 3 generators

4. Invariance under Non Abelian gauge symmetries

$$U = e^{i\alpha^a T^a} = e^{i\alpha^a \sigma^a/2}$$

$a = 1, \dots, 3$

$$U_1 U_2 \neq U_2 U_1$$

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$A = 1, 2$

$i, j = 1, \dots, 4$

$$\mathcal{L} = \bar{\Psi} \not{\partial} \Psi = \Psi^{+}_{Ai} \gamma^0_{ik} \gamma^\mu_{kj} \partial_\mu \Psi_{Aj}$$

- $\Psi \to U\Psi = e^{i\alpha^a \sigma_a/2}\,\Psi$

$$\Psi'_A = U_{AB}\Psi_B \;;\; A,B = 1,2$$

- $\bar{\Psi}\not{\partial}\Psi \to \bar{\Psi}\,U^+\gamma^\mu\partial_\mu(U\Psi)$

$$= \bar{\Psi}\,\gamma^\mu\,(\partial_\mu + U^+\partial_\mu U)\,\Psi$$

- $D_\mu = \partial_\mu - igW_\mu = \partial_\mu - ig\,W_\mu^a\,T^a$

$$\bar{\Psi}\not{D}\Psi \to \bar{\Psi}\,\gamma^\mu\,(\partial_\mu + U^+\partial_\mu U - ig\,U^+W'_\mu U)\Psi$$

For gauge invariance, I need

$$W'_\mu = U W_\mu U^+ - \frac{i}{g}(\partial_\mu U)\,U^+$$

$$\longrightarrow W'^{a}_{\mu} = W^{a}_{\mu} + \frac{1}{g}\,\partial_\mu \alpha^a - \epsilon^{abc}\alpha^b W^c_\mu$$

$$[T^a, T^b] = i\underbrace{f^{abc}}_{\epsilon^{abc}} T^c$$

$$[D_\mu, D_\nu] = -ig\,F_{\mu\nu} = -ig\,F^a_{\mu\nu}\,\frac{\sigma^a}{2}$$

$$\longrightarrow F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon^{abc} W_{\mu b} W_{\nu c}$$

$$\mathcal{L}_{kin} = -\frac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu} = -\frac{1}{2}\,\mathrm{Tr}\left(F^{\mu\nu} F_{\mu\nu}\right)$$

check! (use $\sigma^a \sigma^b = \delta^{ab}\,\mathbb{1} + i\epsilon^{abc}\sigma^c$)