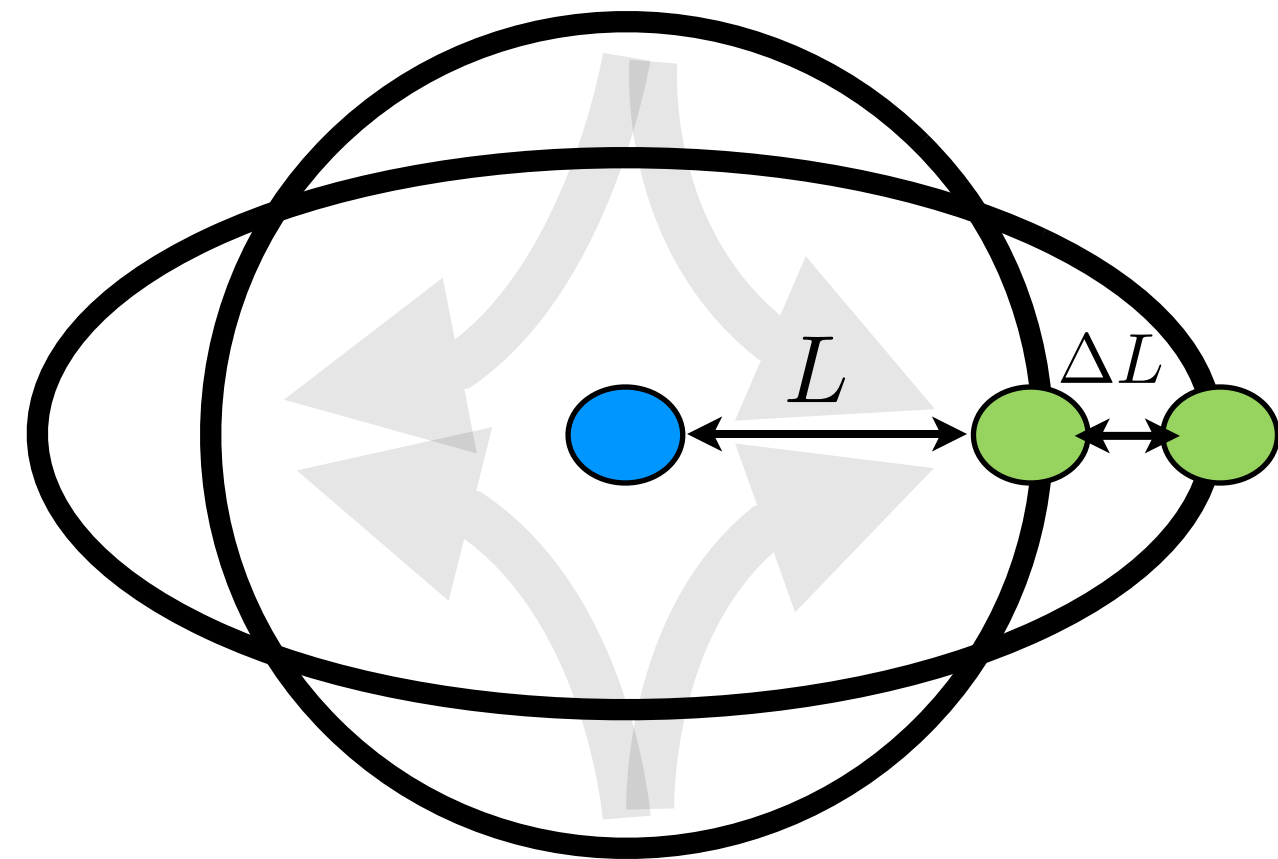


Gravitational wave Detectors

Lecture 2

Problem 1: Consider a wave traveling into the screen (z-direction) and two test masses. One is at the origin and the other is at a distance L along the x-axis in unperturbed spacetime. What is the distance L^* in the gravitational-wave spacetime?

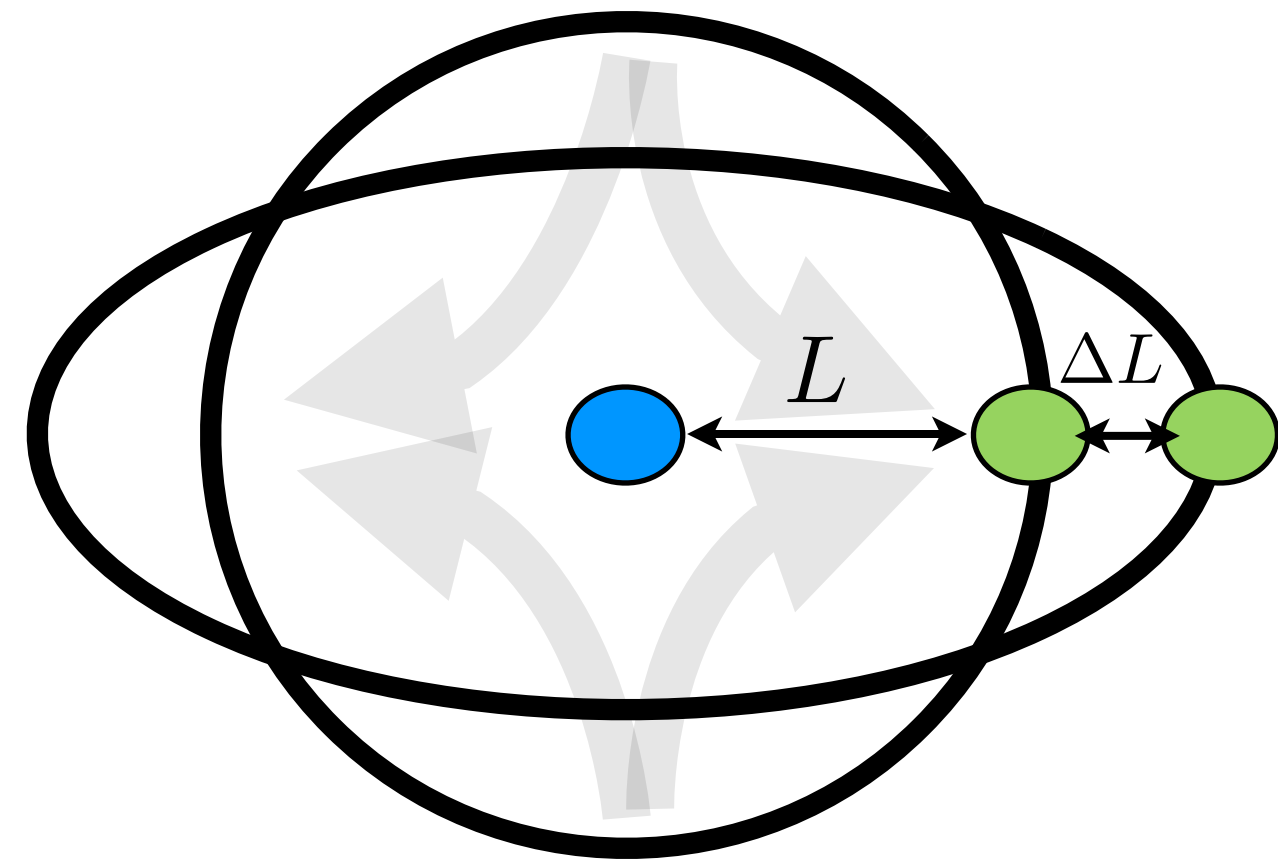


$$h_{\alpha\beta}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cos [\omega (t - z/c)]$$

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Problem 1: Consider a wave traveling into the screen (z-direction) and two test masses. One is at the origin and the other is at a distance L along the x-axis in unperturbed spacetime. What is the distance L^* in the gravitational-wave spacetime?



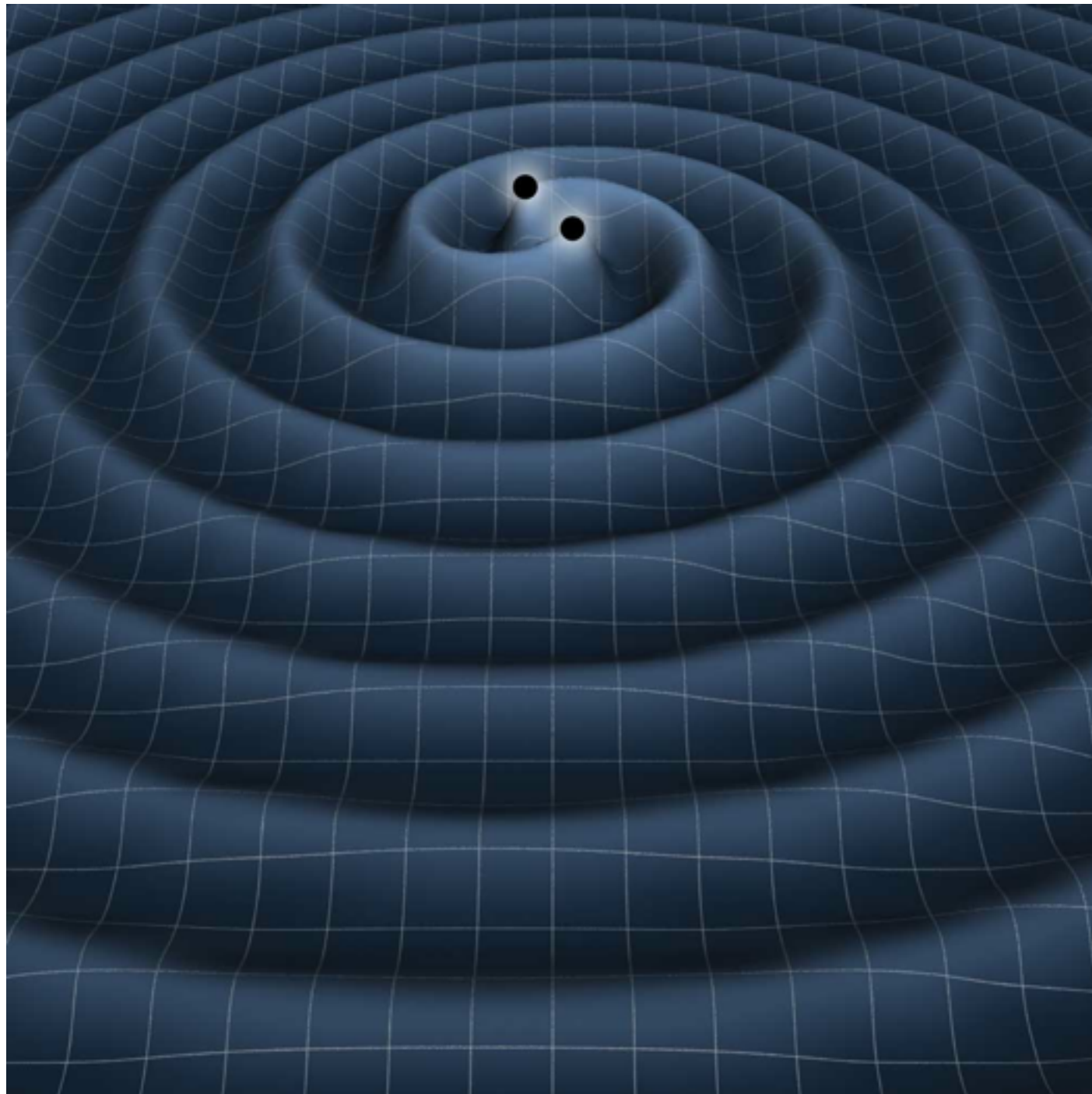
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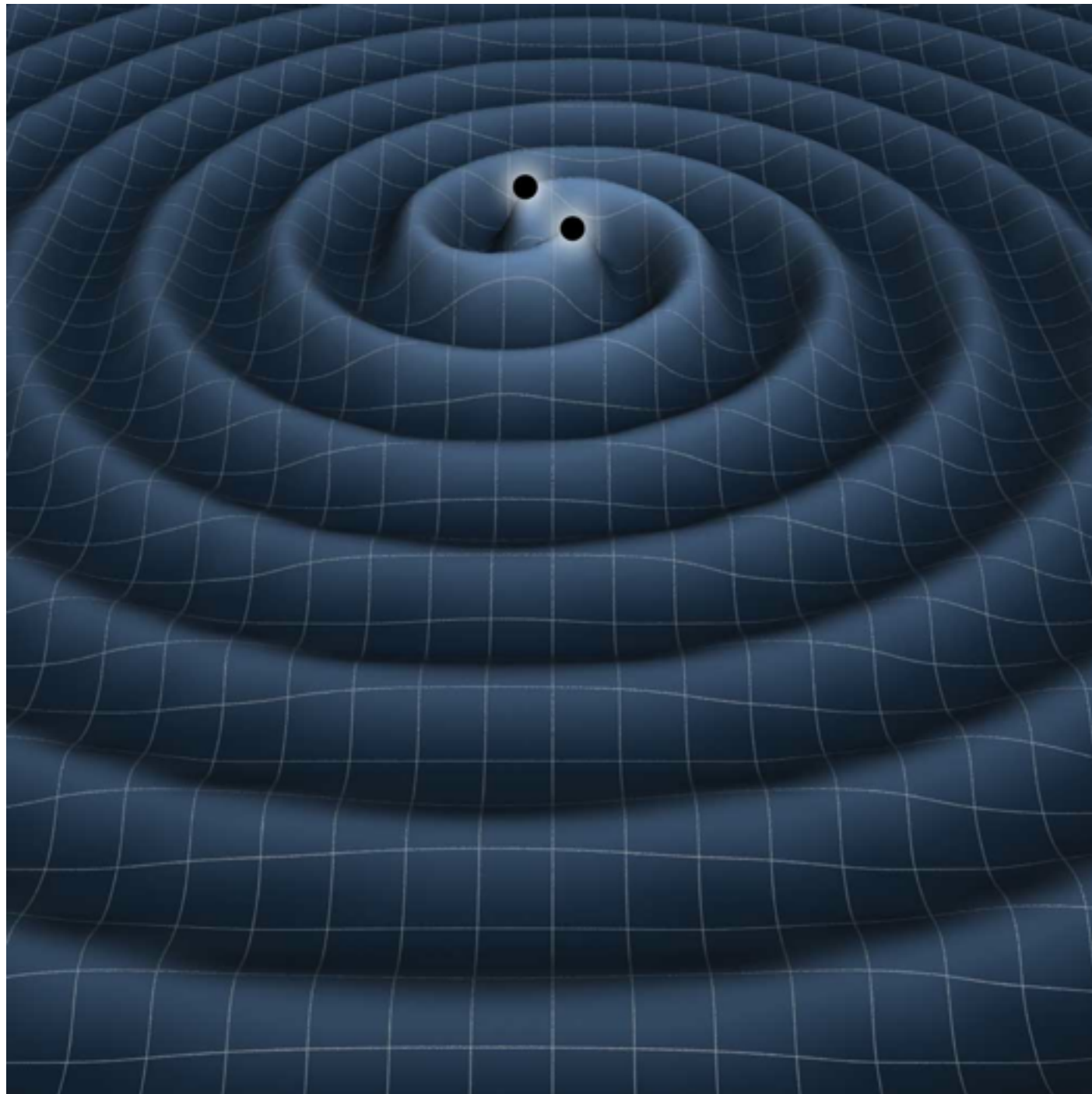
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$L^*(t) = L + \frac{Lh_+}{2} \cos(\omega t) \longrightarrow h = \frac{\Delta L}{L} \quad \text{Strain!}$$

Problem 2: Electromagnetic waves are produced by moving **charged** particles. Gravitational waves are produced by moving **massive** particles. How does the strength of gravitational waves compare to electromagnetic waves?



Problem 2: Electromagnetic waves are produced by moving **charged** particles. Gravitational waves are produced by moving **massive** particles. How does the strength of gravitational waves compare to electromagnetic waves?



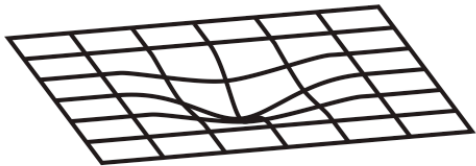
Very very weak as a natural consequence of the weakness of gravitational forces compared to electromagnetic forces.

Between two protons, EM force is 10^{36} times stronger than gravitational attraction.

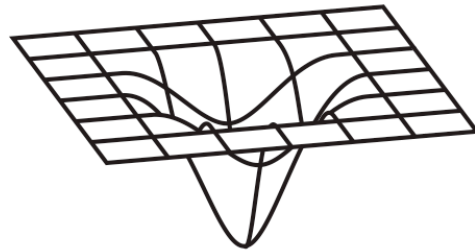
Only objects with strong gravitational fields moving very fast will produce non-negligible GWs.

Strong Gravity Candidates

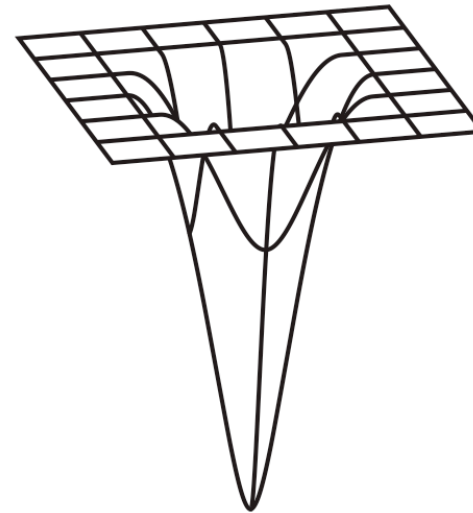
Sun



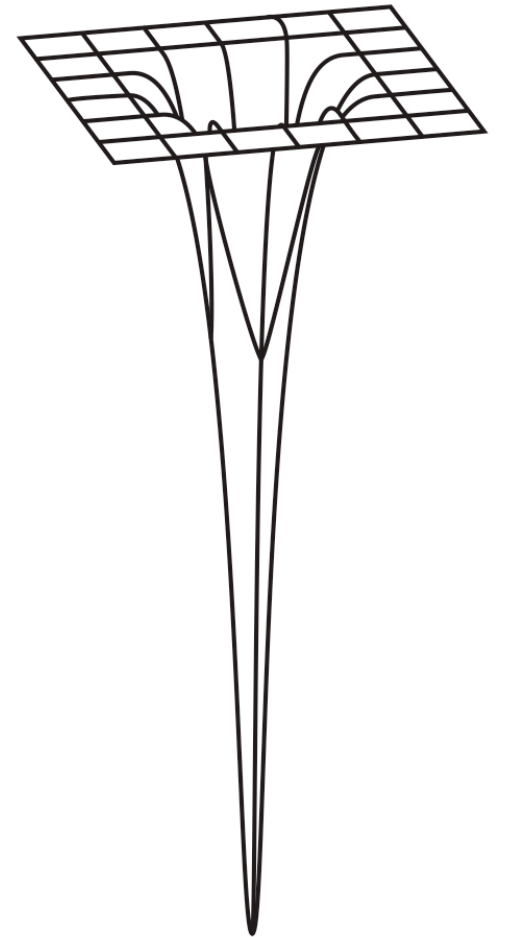
White dwarf



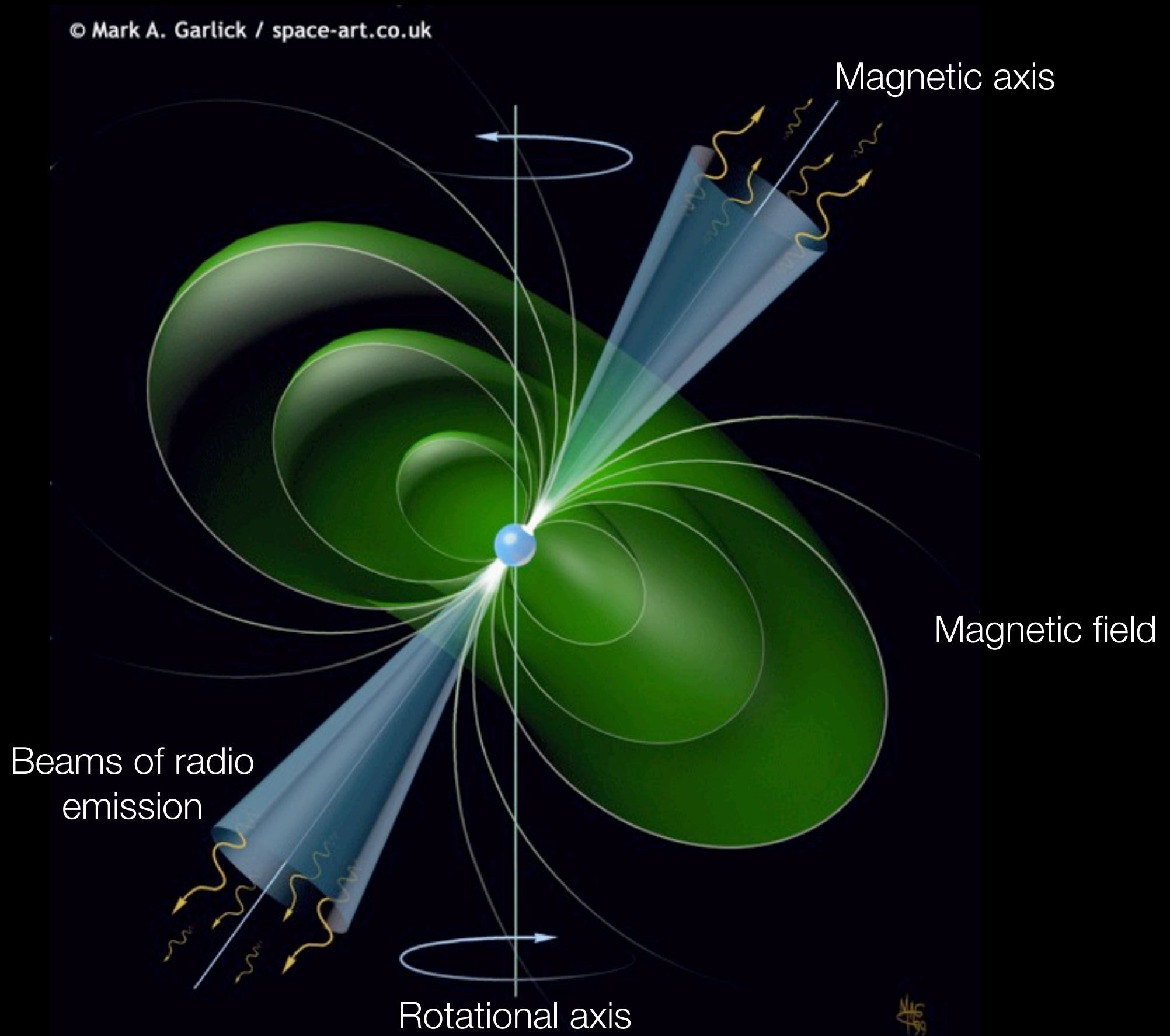
Neutron star



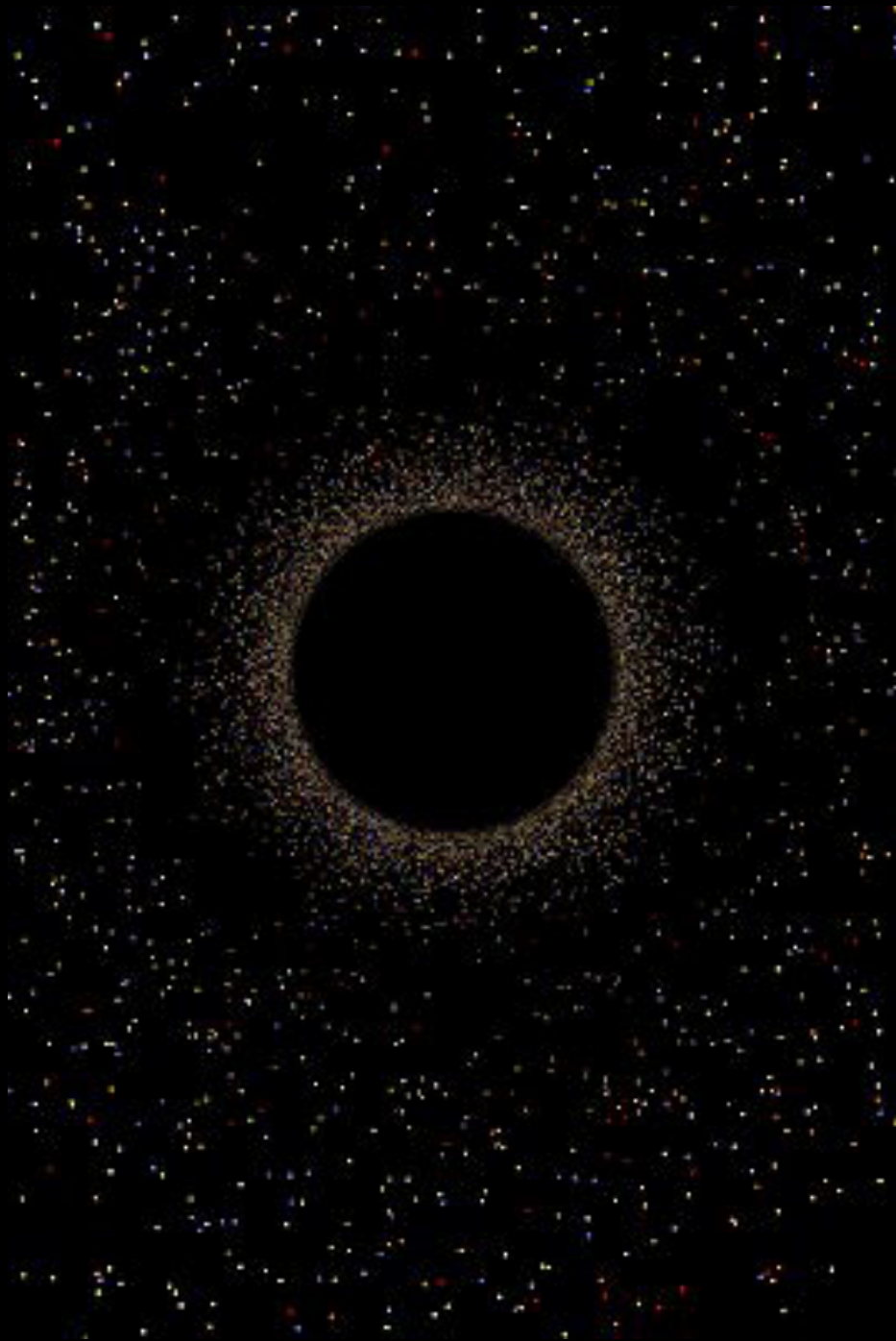
Black hole



Neutron Stars



Black Holes



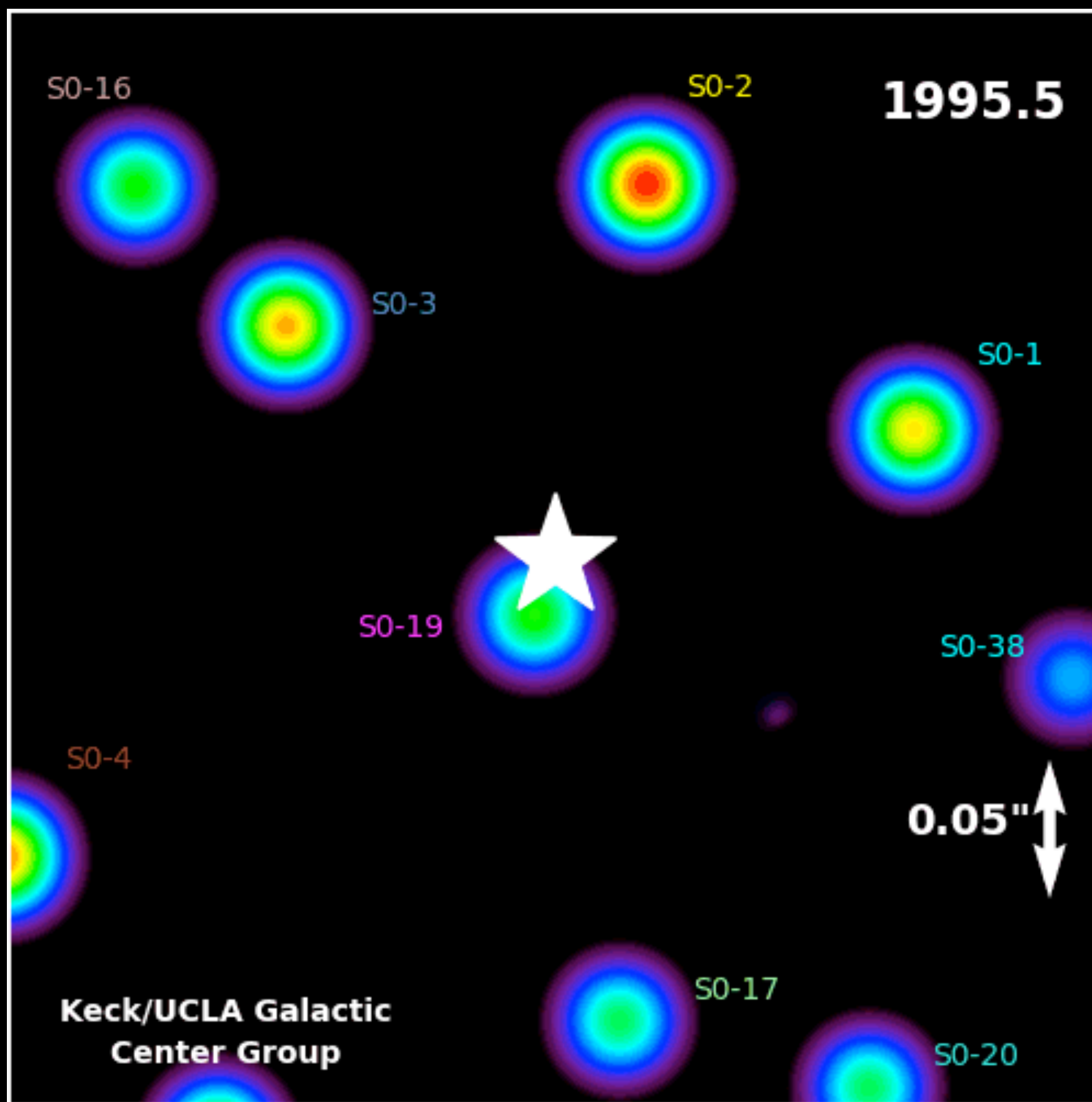
Maximum neutron star mass about 3 solar masses.

More massive than that: nothing will stop collapse.

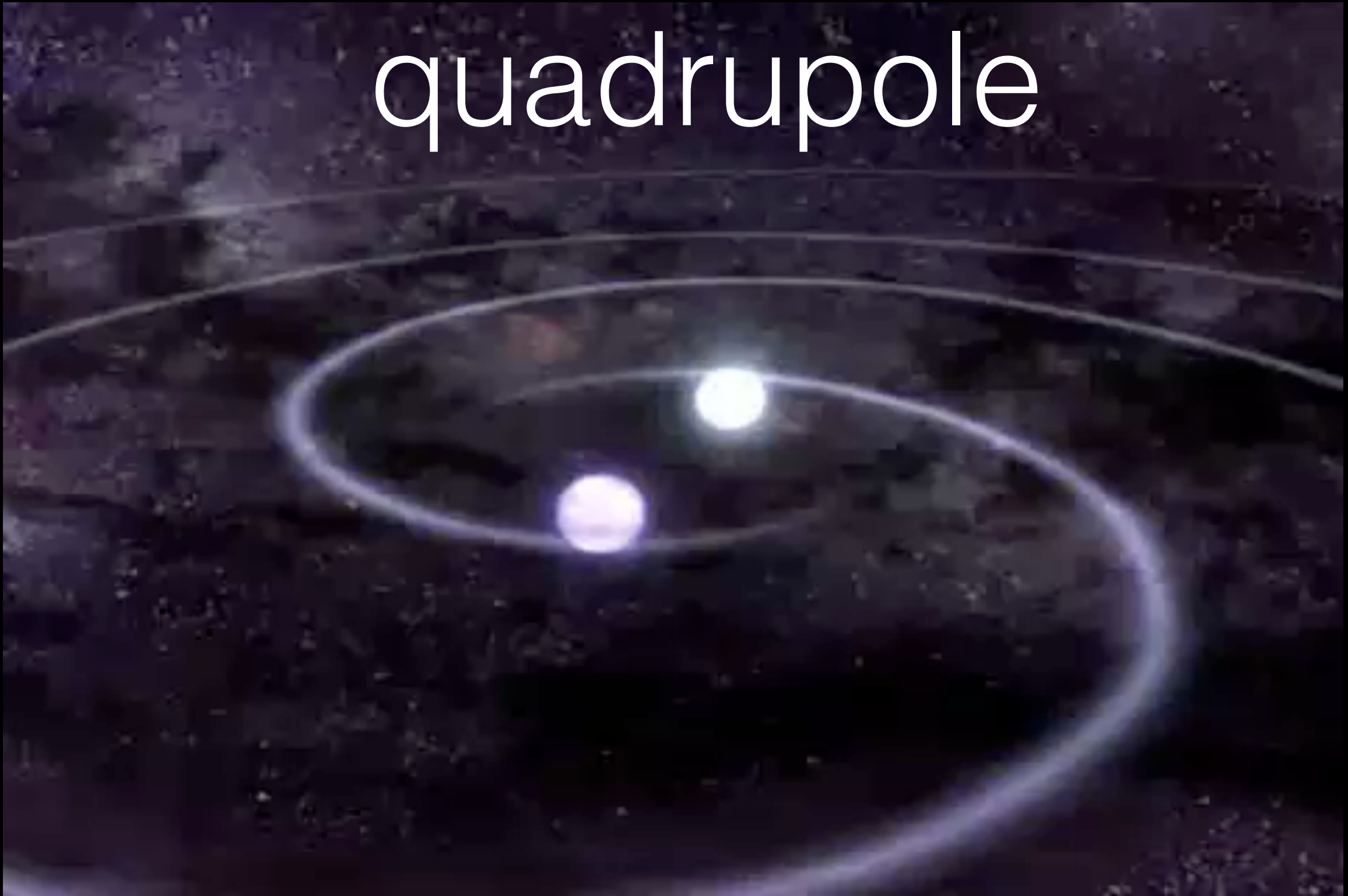
Gravitational force becomes so intense that even light can't escape: **black hole**.

The radius at which the escape speed from the black hole equals the speed of light is called the **event horizon**.

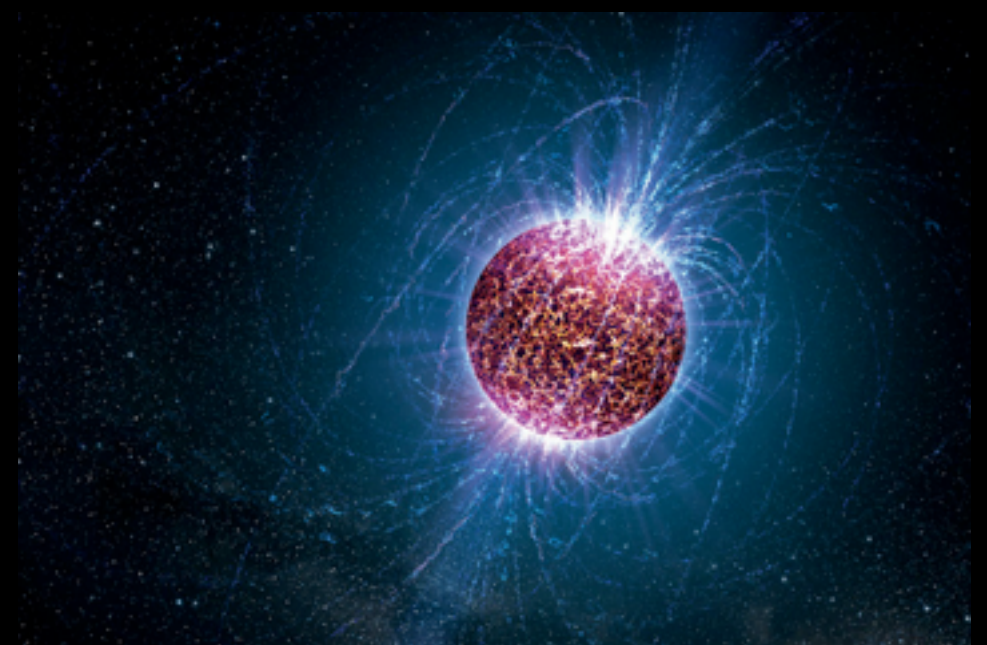
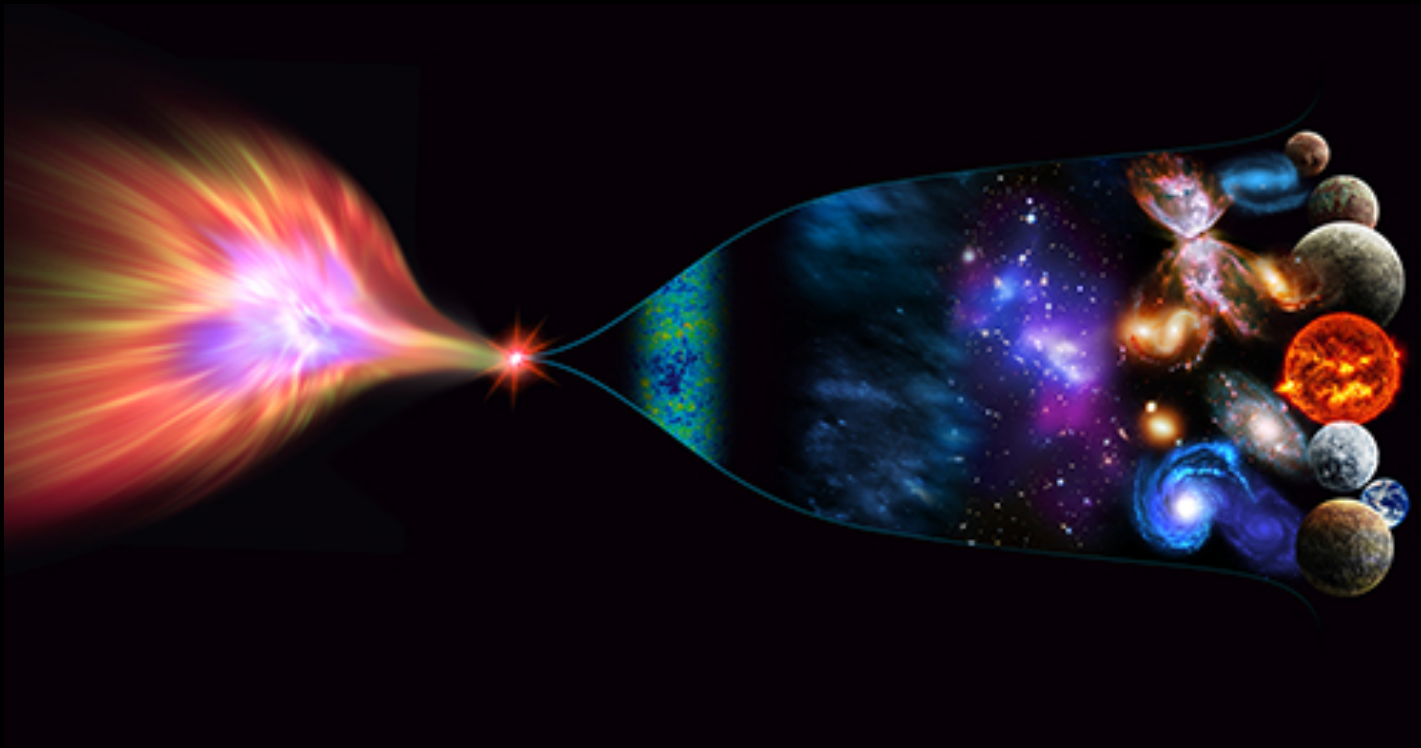
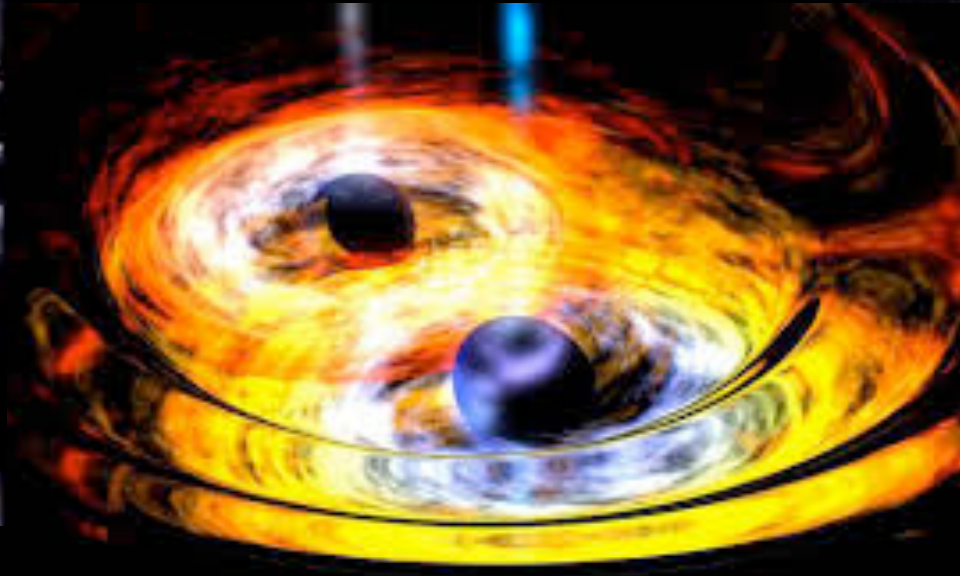
For the Sun, event horizon is about 3km.



Time-varying mass quadrupole



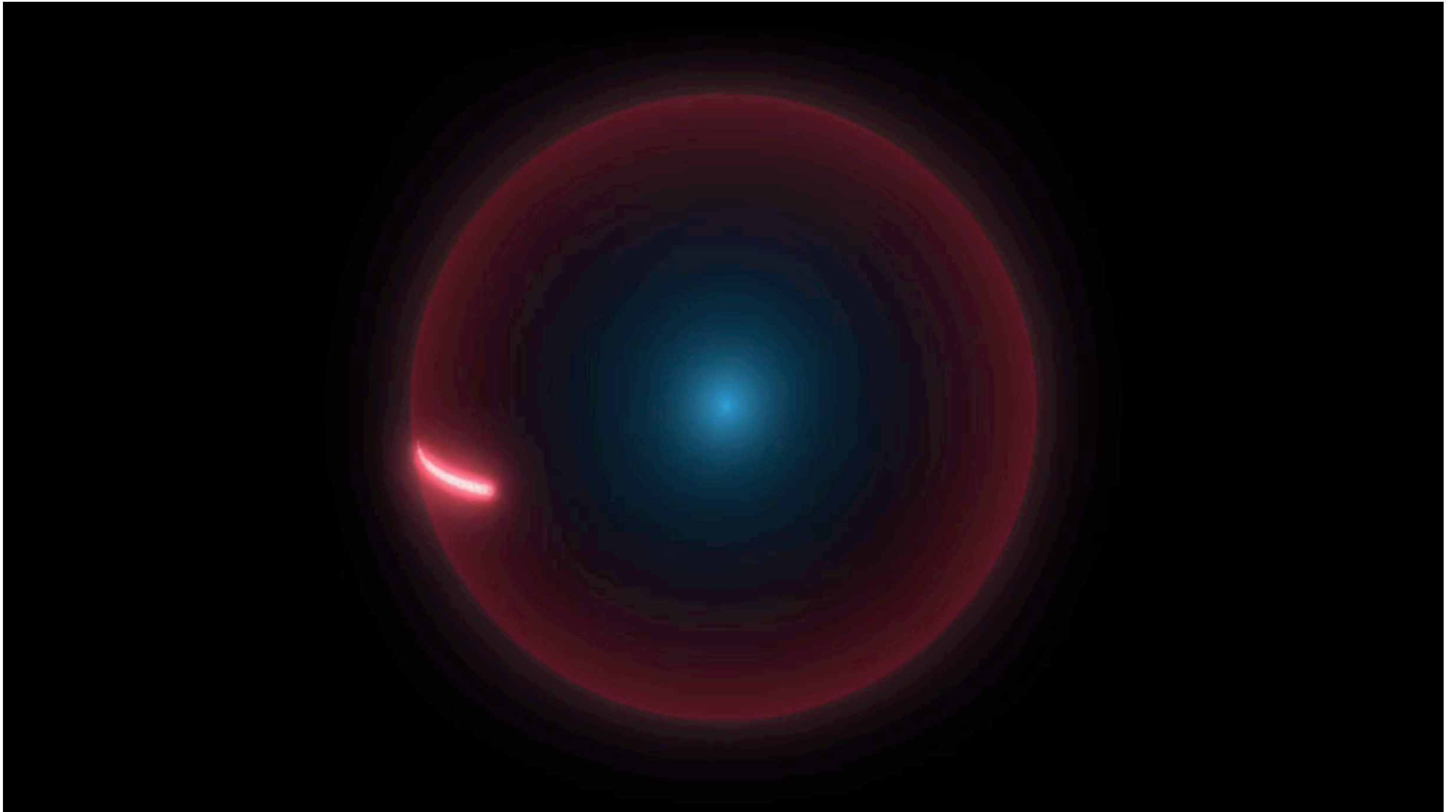
Astrophysical Sources



Problem 3: What is a rough estimate of the amplitude of the perturbations we might expect from a dynamical astrophysical source in the Virgo cluster? What frequency of oscillation will these perturbations have?

$$h \simeq \frac{2G}{c^4 r} \ddot{I} \sim \frac{G E_{\text{kinetic}}^{\text{non-symm}}}{c^4 r}$$

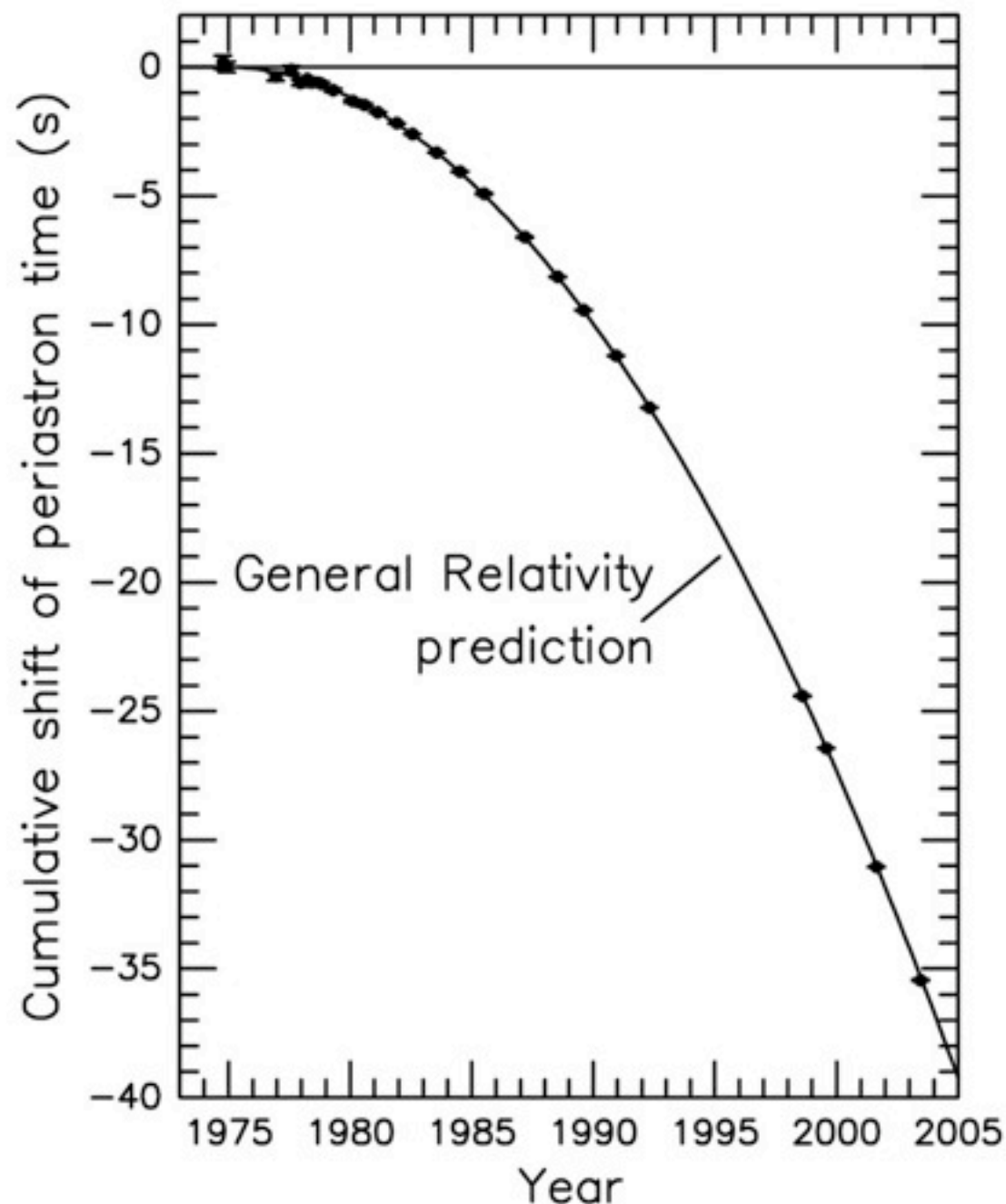
A Very Small Strain Measurement



First Observational Evidence

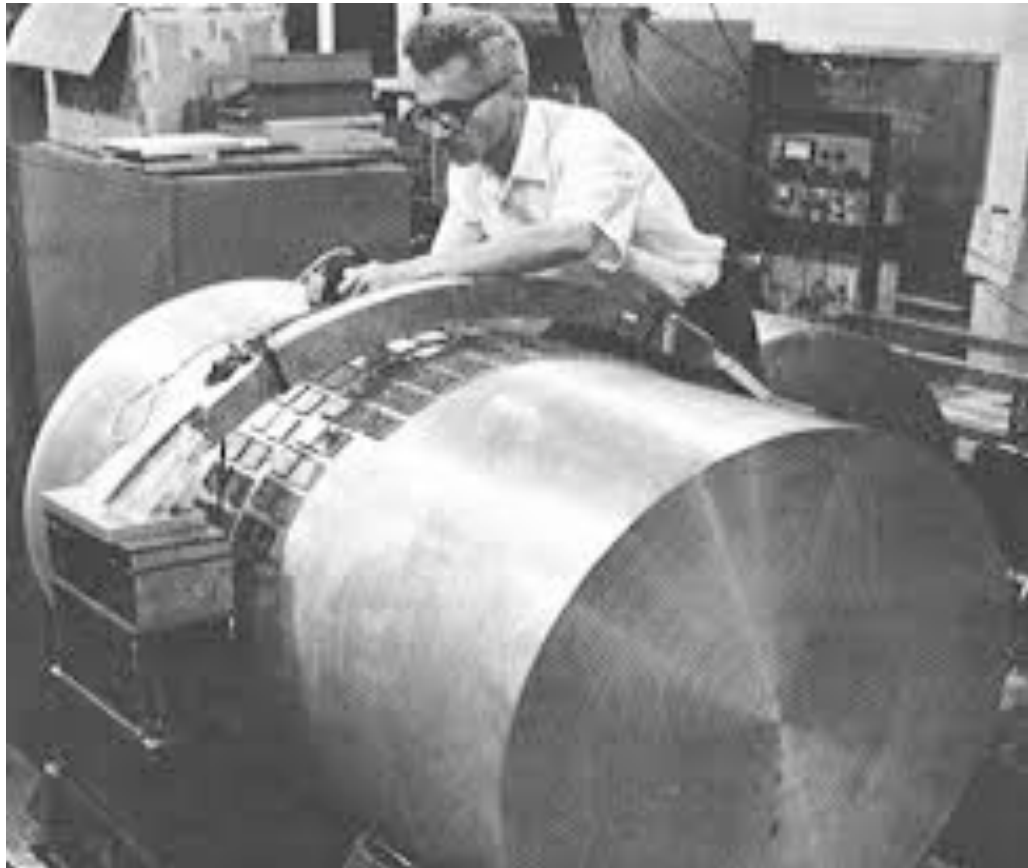
Hulse-Taylor Binary Pulsar

Pulsar-Neutron Star System



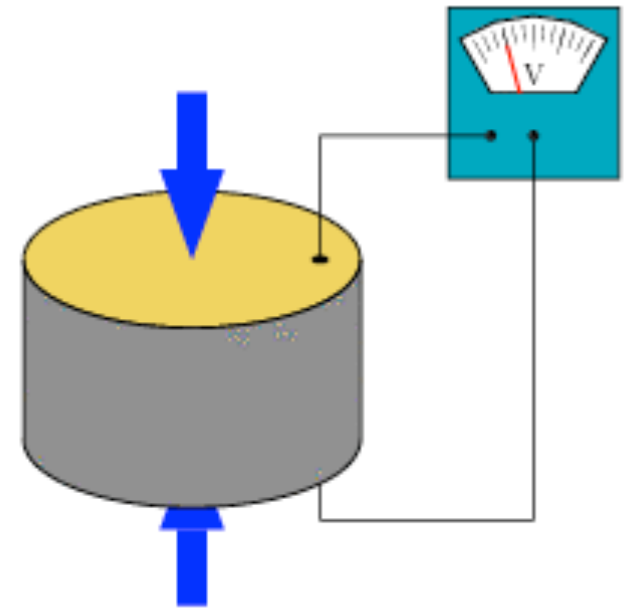
- Period: 7.75 hours
- Discovered by R. Hulse and J. Taylor
- Awarded 1993 Nobel prize

Bar Detectors



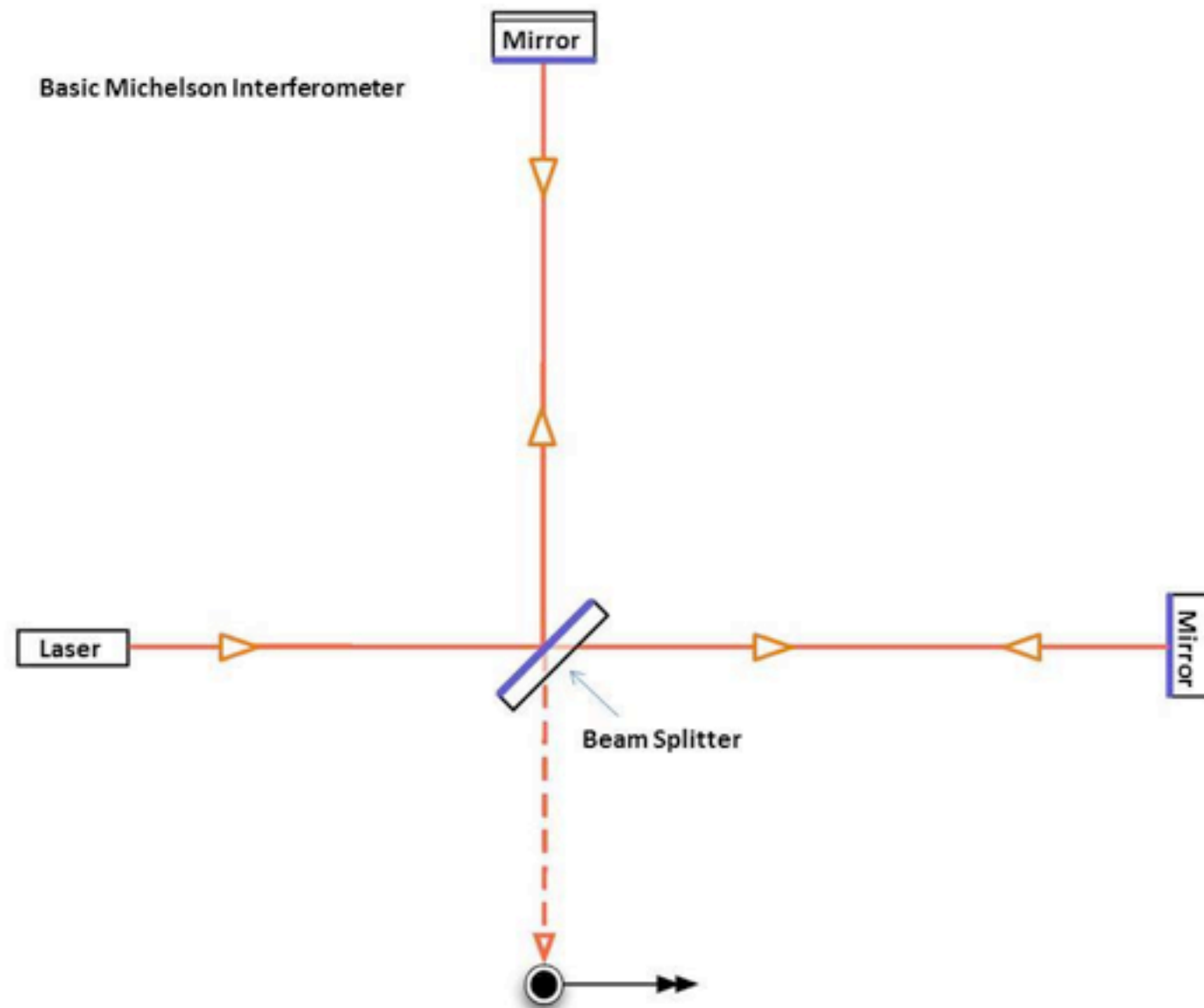
Joseph Webber -
University of Maryland
1961: proposed to use
resonant bar detectors to
detect GWs

Piezoelectric sensors



Only sensitive over
very narrow range of
frequencies

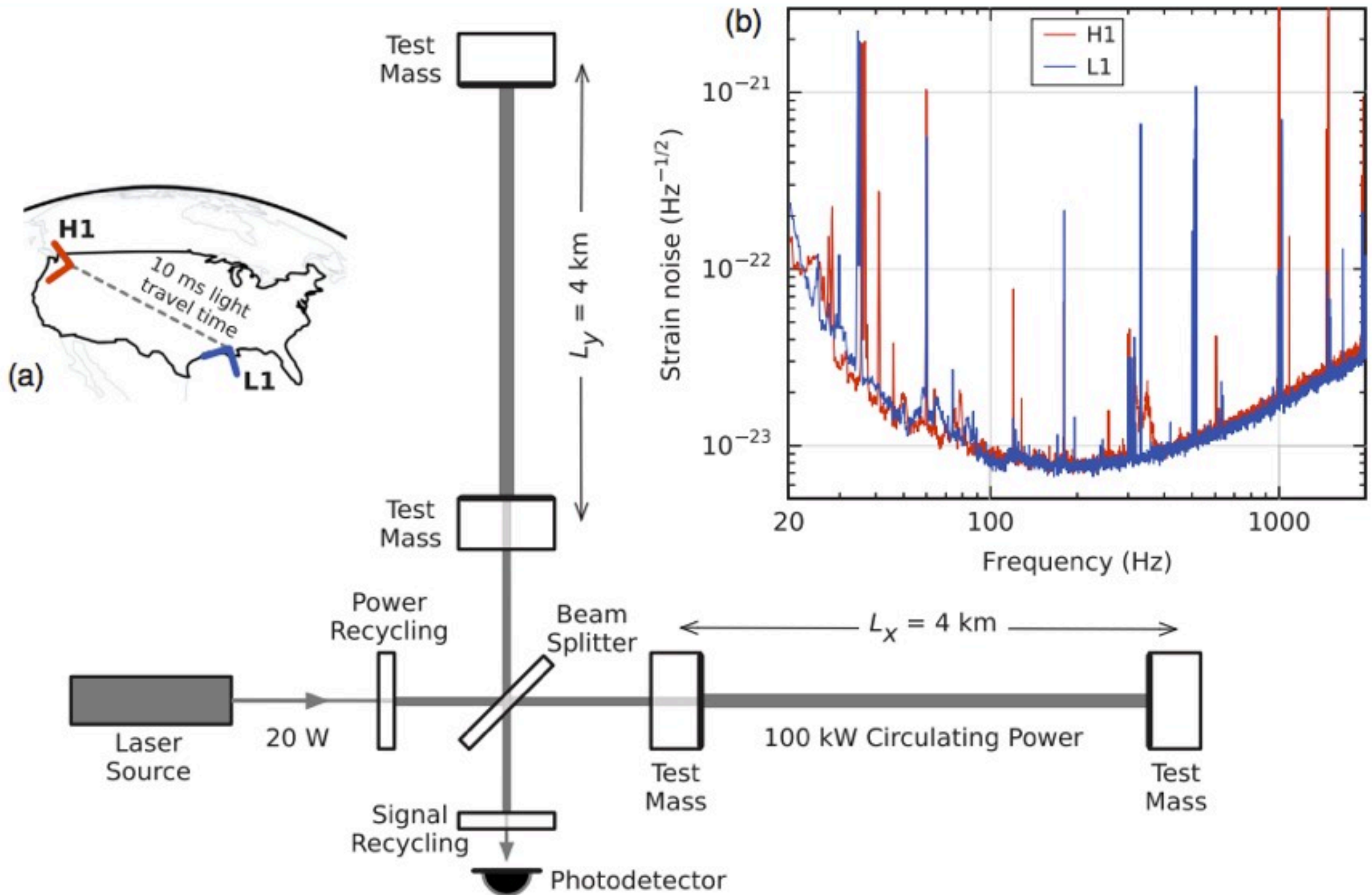
Interferometric Detectors



- 1970s: Development of precision measurement laser technology
- Early 1980s: Prototypes built in Glasgow, Garching, MIT
- 1992: National Science Foundation approves the Laser Interferometer Gravitational-wave Observatory (LIGO)
- 1994: Construction begins...



Broadband Detectors



Toward an Interferometric GW Detector

Proper distance between test masses in curved
spacetime of passing GW

$$ds^2 = -c^2 dt^2 + (1 + h) dx^2 + (1 - h) dy^2$$

Changing path lengths of interferometer's x and y arms

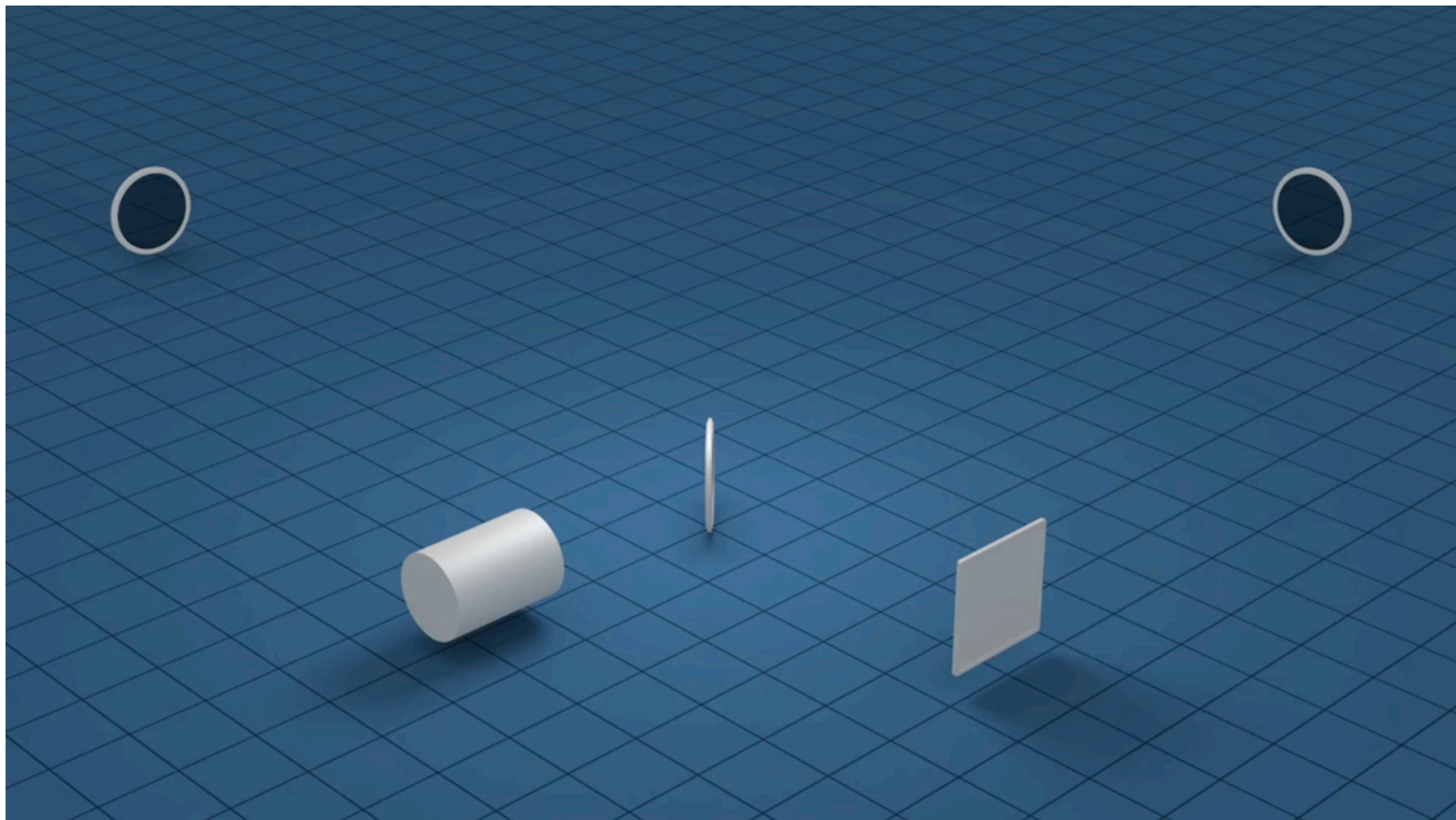
$$dx = \frac{cdt}{\sqrt{1+h}} \approx c \left(1 - \frac{h}{2}\right) dt = L_x dt$$

$$dy = \frac{cdt}{\sqrt{1-h}} \approx c \left(1 + \frac{h}{2}\right) dt = L_y dt$$

$$L_x - L_y = c \times h \times dt = L \times h$$

$$L = c \times dt$$

$$h = \frac{\Delta L}{L}$$



Toward an Interferometric GW Detector

Strain Sensitivity Goal: $h \sim 10^{-21}$

Measurable change
in wavelength:

$$h := \frac{\Delta l}{l} \sim \frac{\lambda_{\text{laser}}}{l} \sim \frac{10^{-6}}{10^3} = 10^{-9}$$

Increase optical path
length (Fabry-Perot):

$$h \sim \frac{\Delta l}{l_{\text{eff}}} \sim \frac{\lambda_{\text{laser}}}{\lambda_{\text{GW}}} \sim \frac{10^{-6}}{10^6} = 10^{-12}$$

Not there yet!

Toward an Interferometric GW Detector

Improve optical path length measurements:

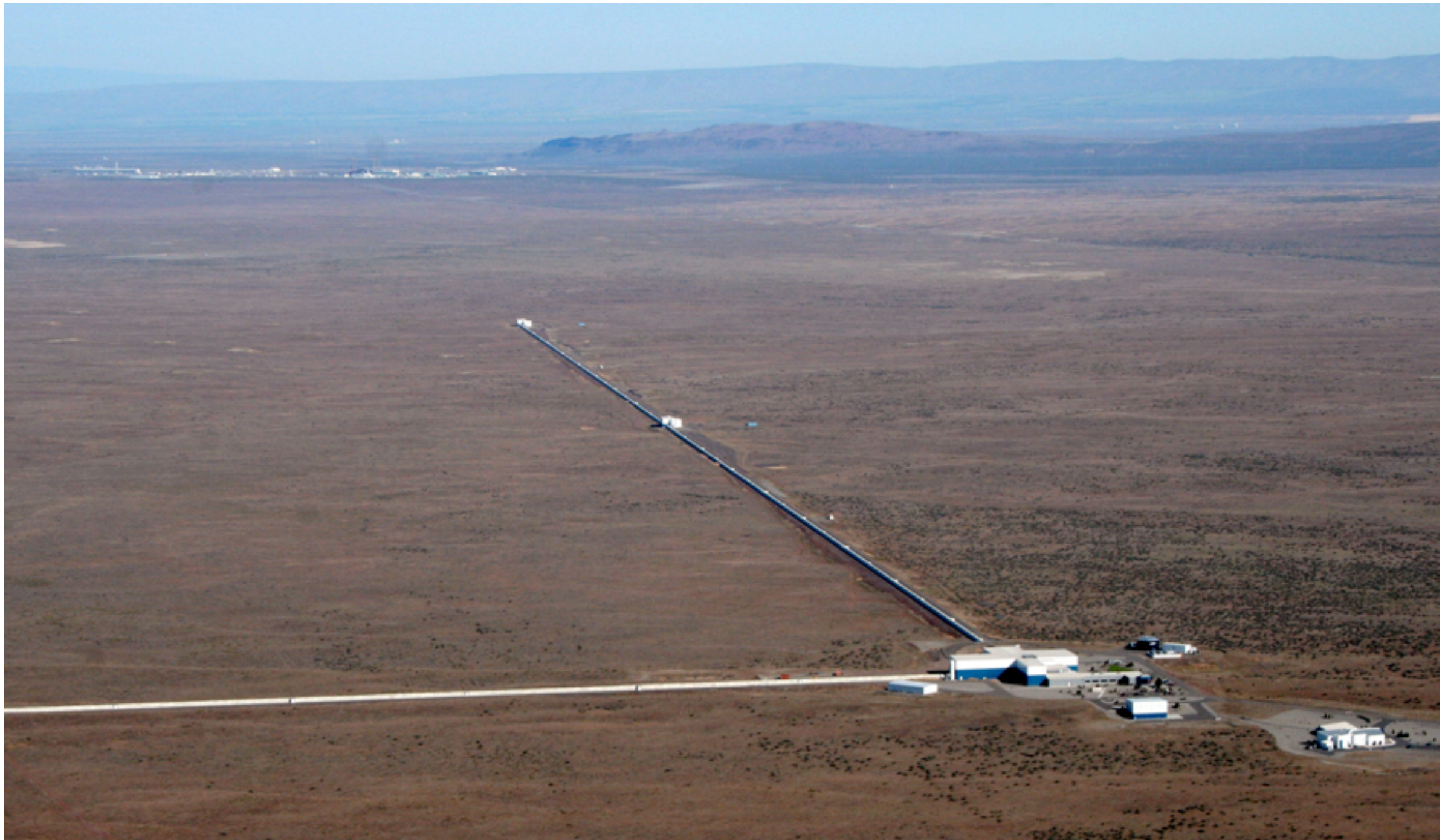
$$h \sim \frac{\Delta l}{l_{\text{eff}}} \sim \frac{\sqrt{N_{\text{photons}}}}{N_{\text{photons}}} \frac{\lambda_{\text{laser}}}{\lambda_{\text{GW}}} \sim \frac{\sqrt{10^{16}}}{10^{16}} \frac{10^{-6}}{10^6} = 10^{-20}$$

Increase laser power (power recycling):

$$h \sim \frac{\sqrt{N_{\text{photons}}}}{N_{\text{photons}}} \frac{\lambda_{\text{laser}}}{\lambda_{\text{GW}}} \sim \frac{\sqrt{10^{17}}}{10^{17}} \frac{10^{-6}}{10^6} = 10^{-21}$$

It can be done!

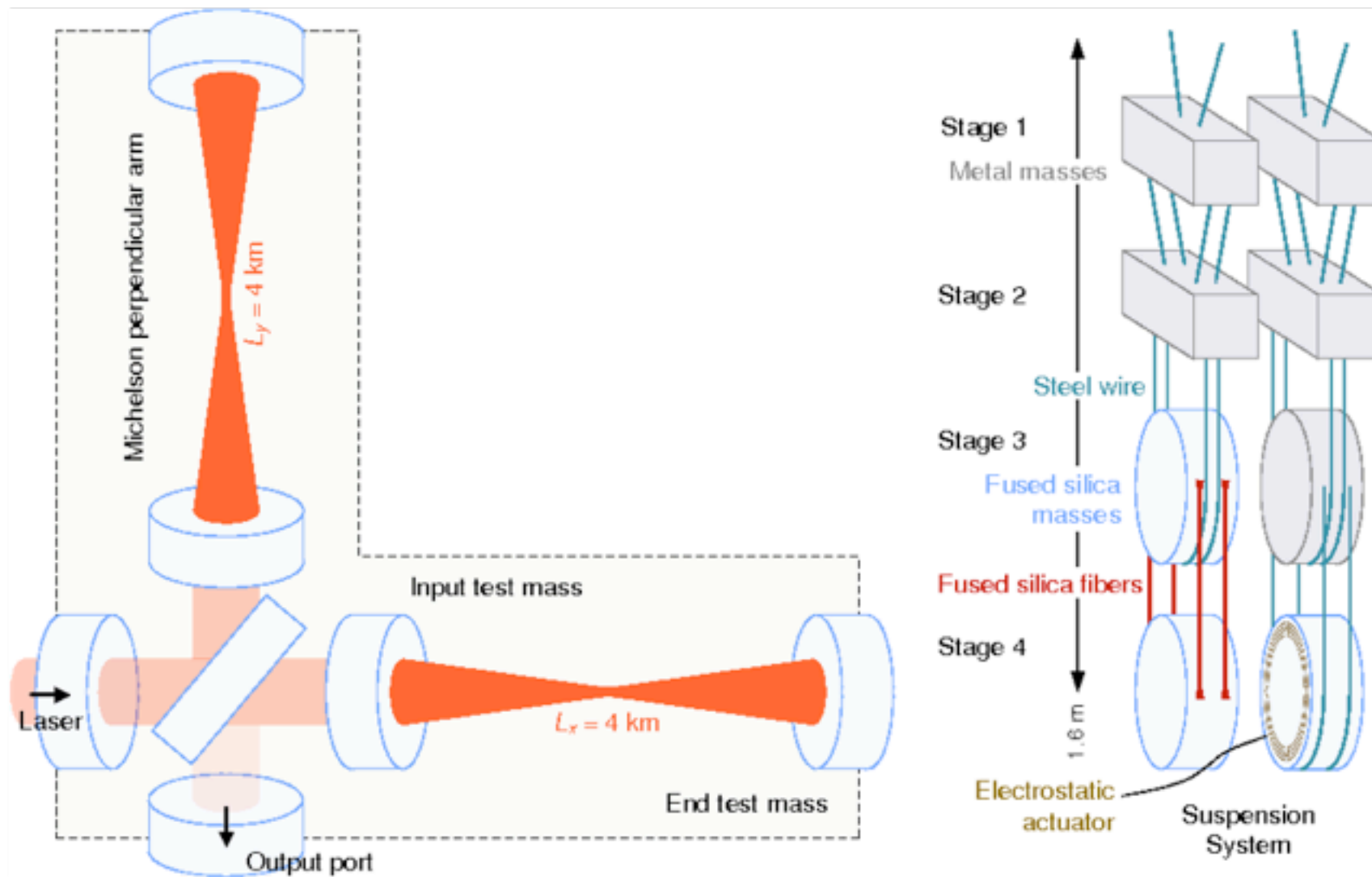
The Laser Interferometer Gravitational-wave Observatory: LIGO-Hanford



The Laser Interferometer Gravitational-wave Observatory: LIGO-Livingston



Advanced LIGO

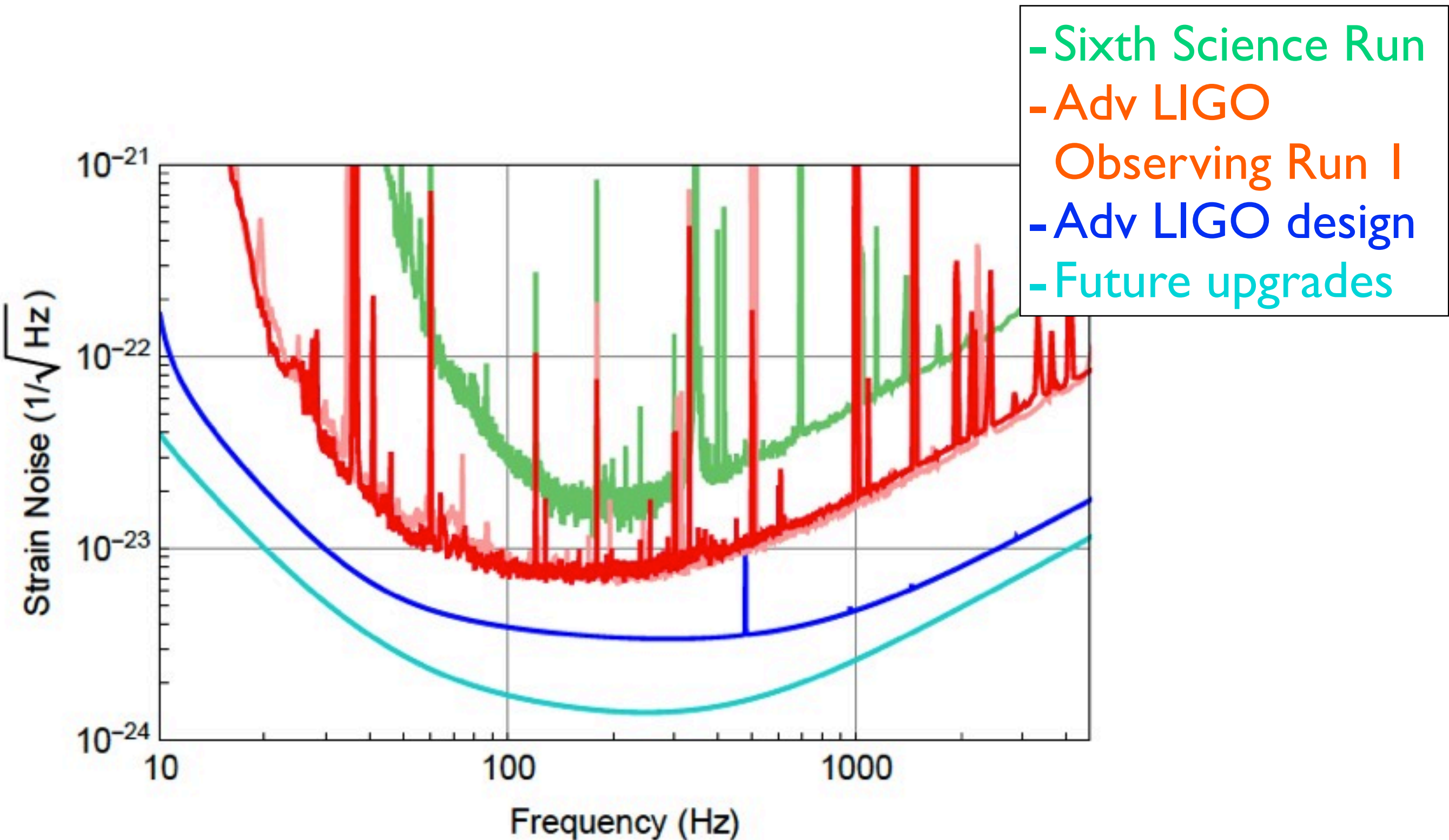


Began Sept 2015

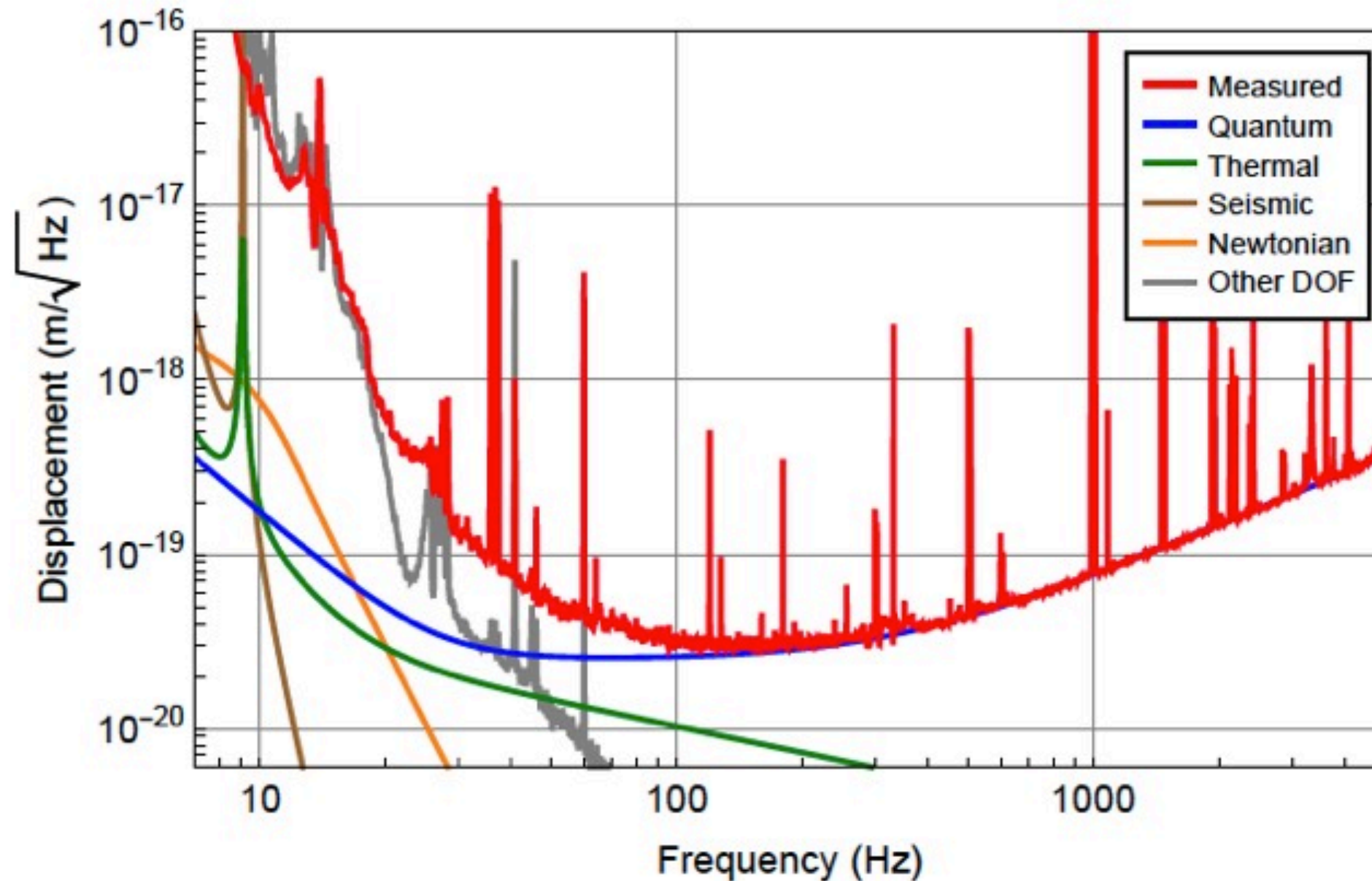
Target: factor of 10
improvement in
sensitivity

- Signal recycling to improve frequency response
- Better seismic isolation and test mass suspension for gains at low frequency
- Higher laser power, larger test masses, improved mirror coating at mid/high frequencies

LIGO Sensitivity

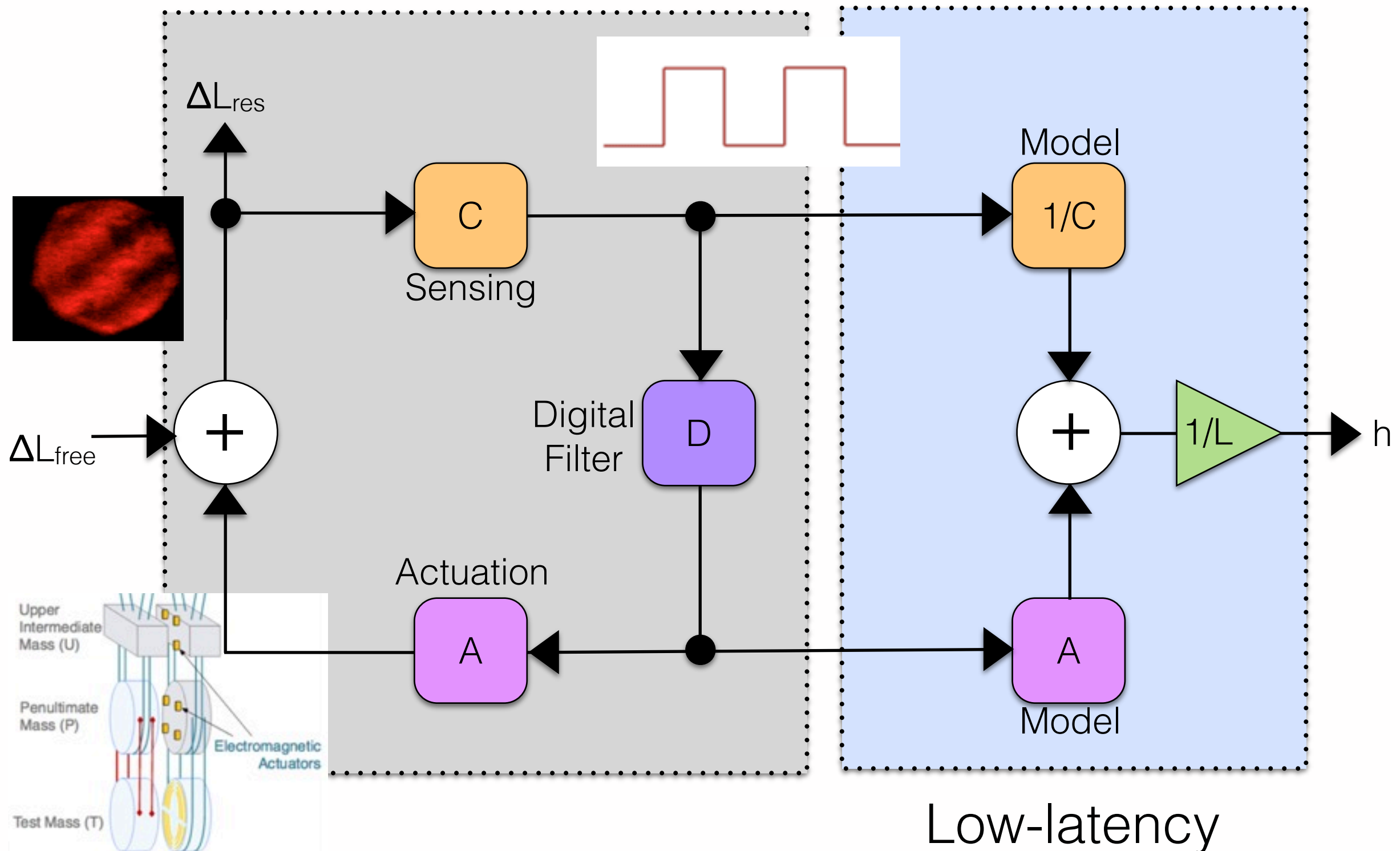


LIGO Noise Sources



- Quantum - shot noise and radiation pressure
- Thermal - suspensions, test masses, coatings
- Seismic - ground displacement attenuation through seismic isolation system and suspensions
- DOF - cross couplings from auto alignment system and auxiliary lengths
- Newtonian - gravitational noise from density perturbations due to surface ground motion
- Strong lines - violin modes of suspension wires, suspensions, AC power line and its harmonics, calibration lines

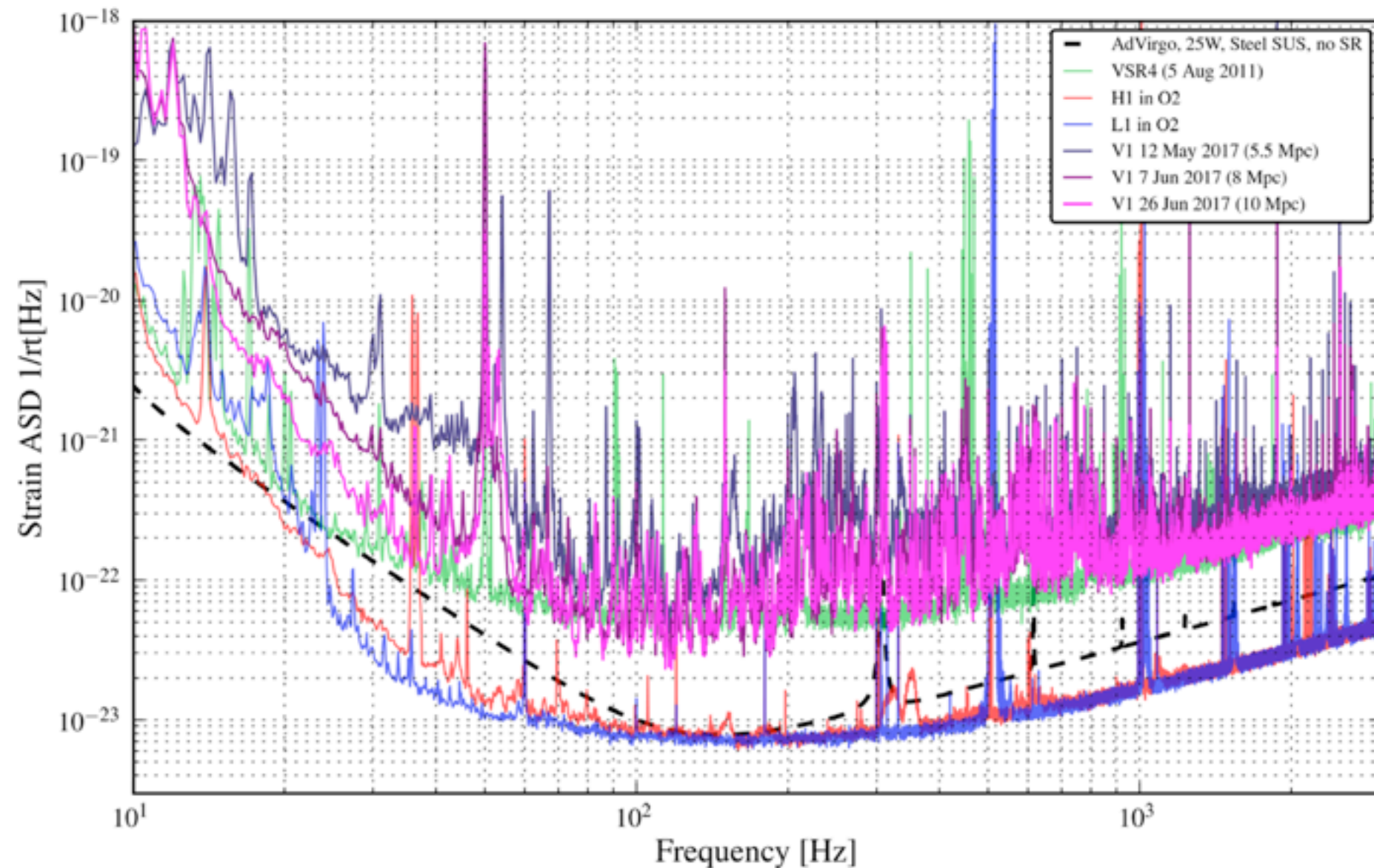
Calibration



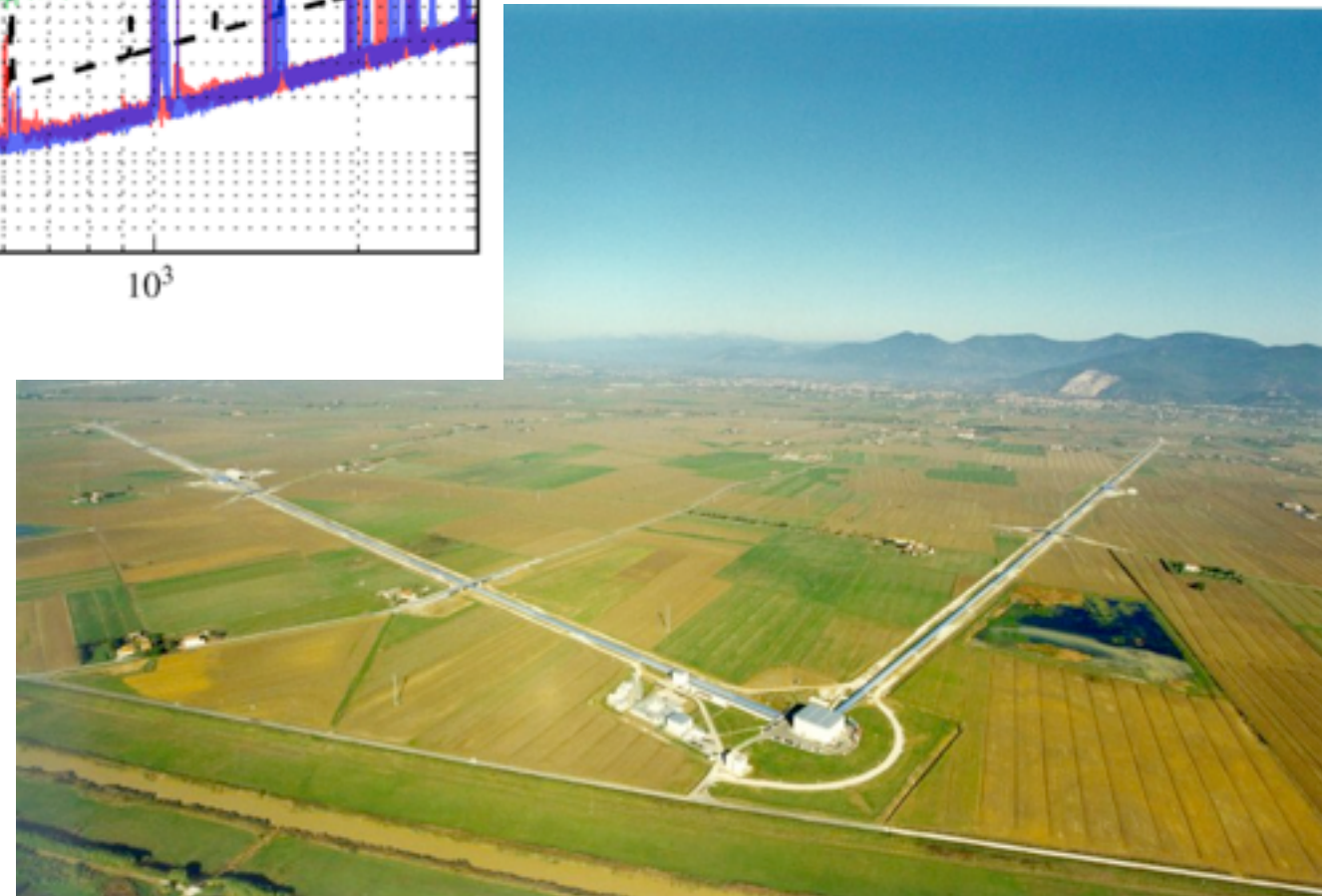
Realtime interferometer
control

Low-latency
Calibration:
~8seconds

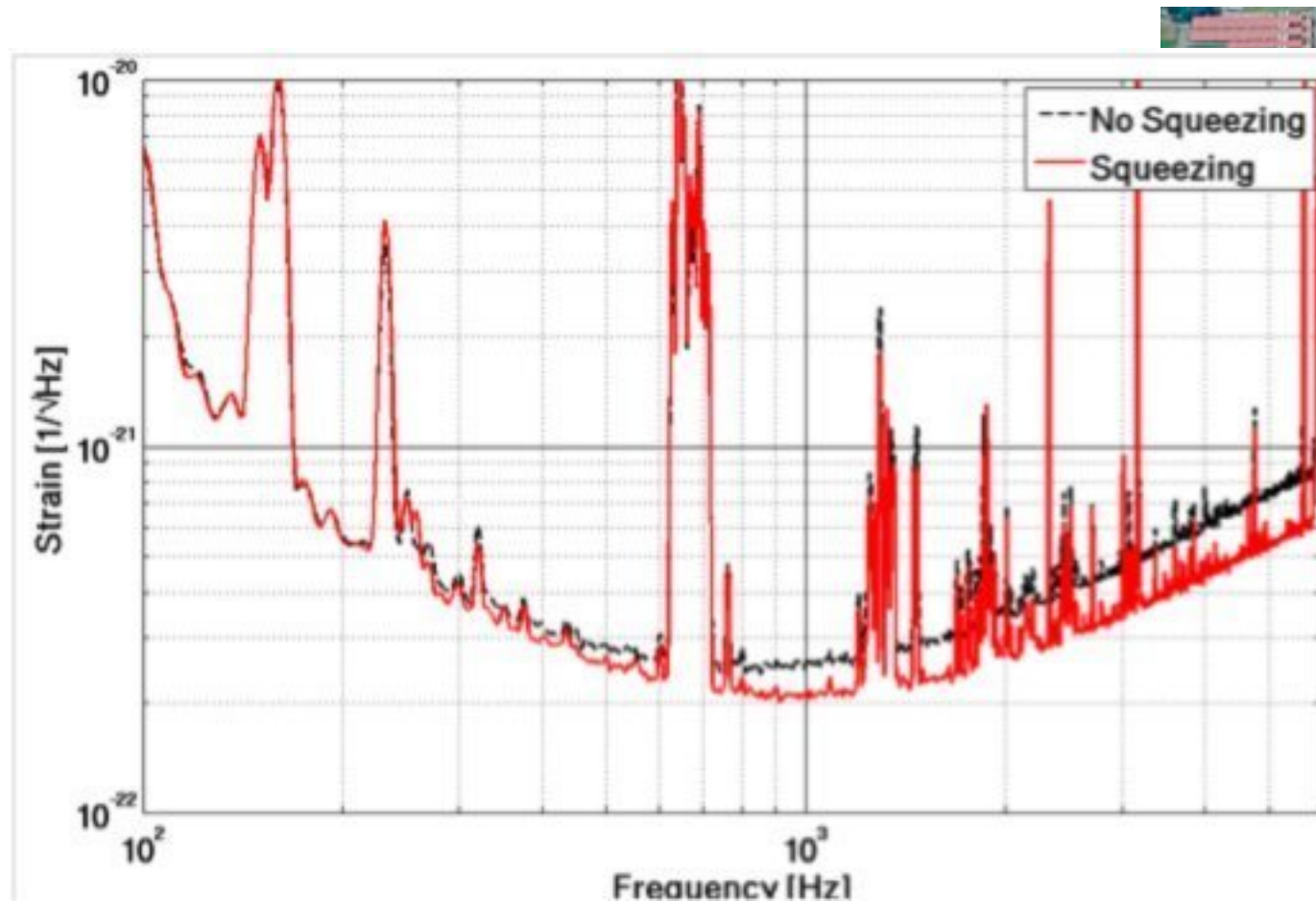
Advanced Virgo



- Operated in Cascina, Italy
- 3km-long arms
- France, Italy, Netherlands, Poland, Hungary

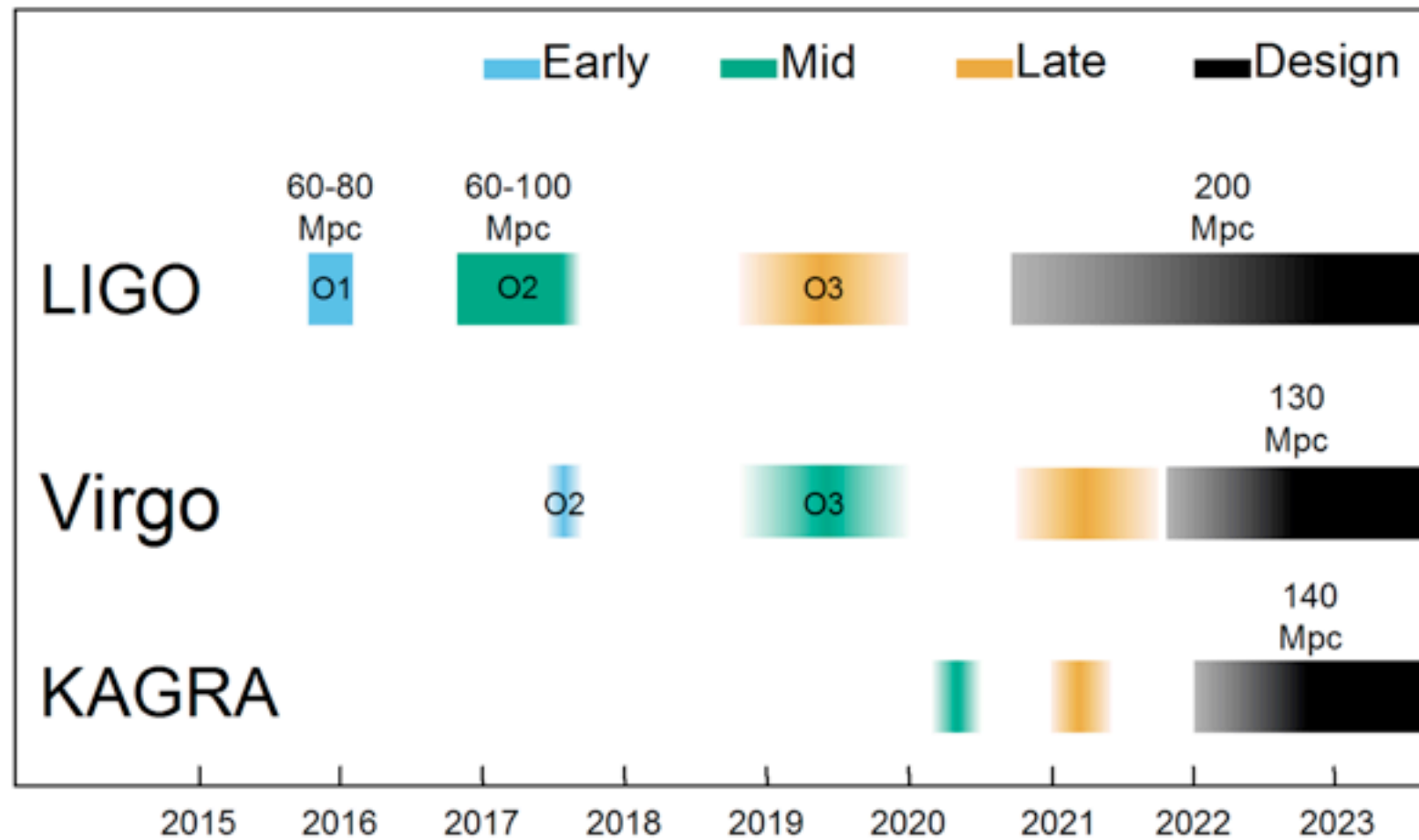


GEO 600



- GEO600, test-bed for new technologies: laser stabilization, absorption-free optics, control engineering, vibration damping, etc.
- “Squeezed light” technology - mitigating shot-noise

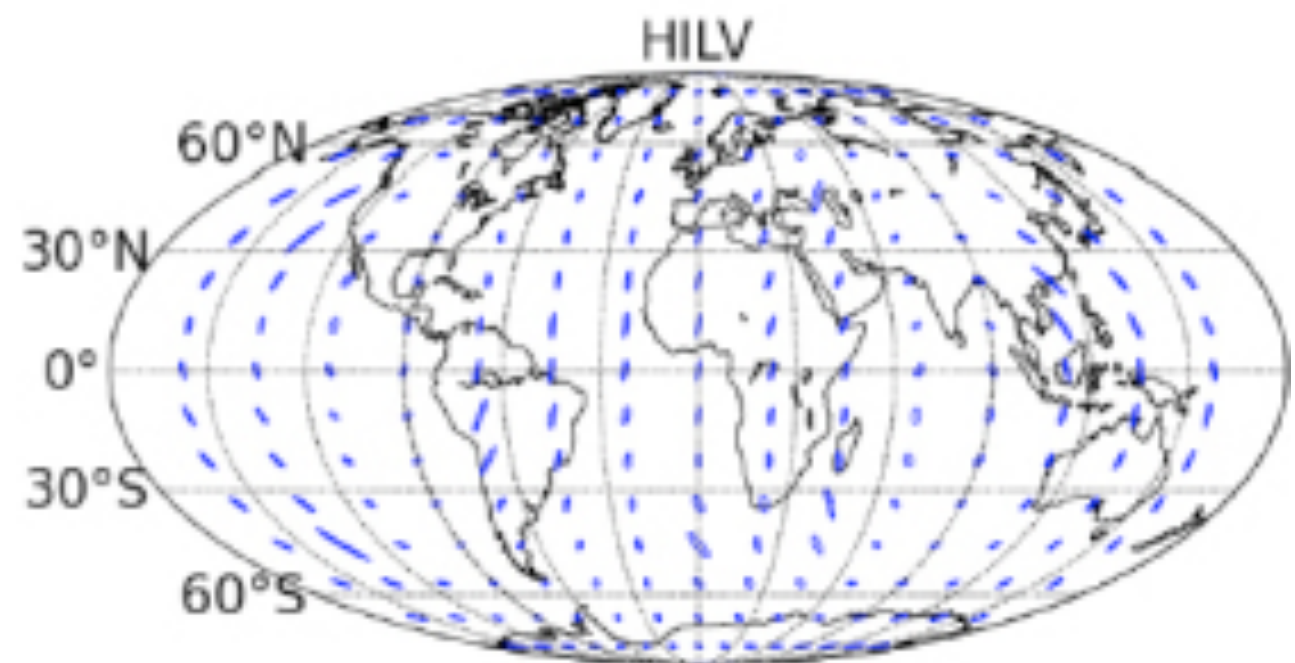
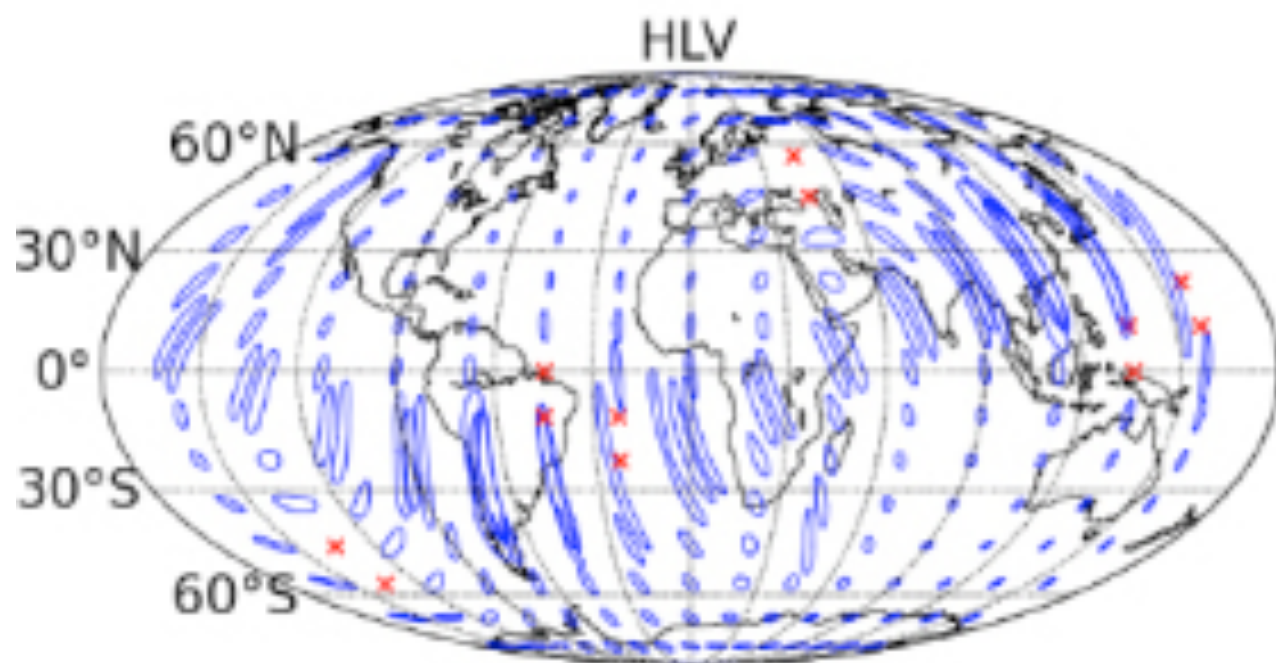
Kamioka Gravitational Wave Detector - KAGRA



- Operated in Kamioka Observatory in Japan
- 3km-long arms
- Cryogenic mirrors

LIGO-India

Expected to come online 2022+
Will use hardware from LIGO Laboratory

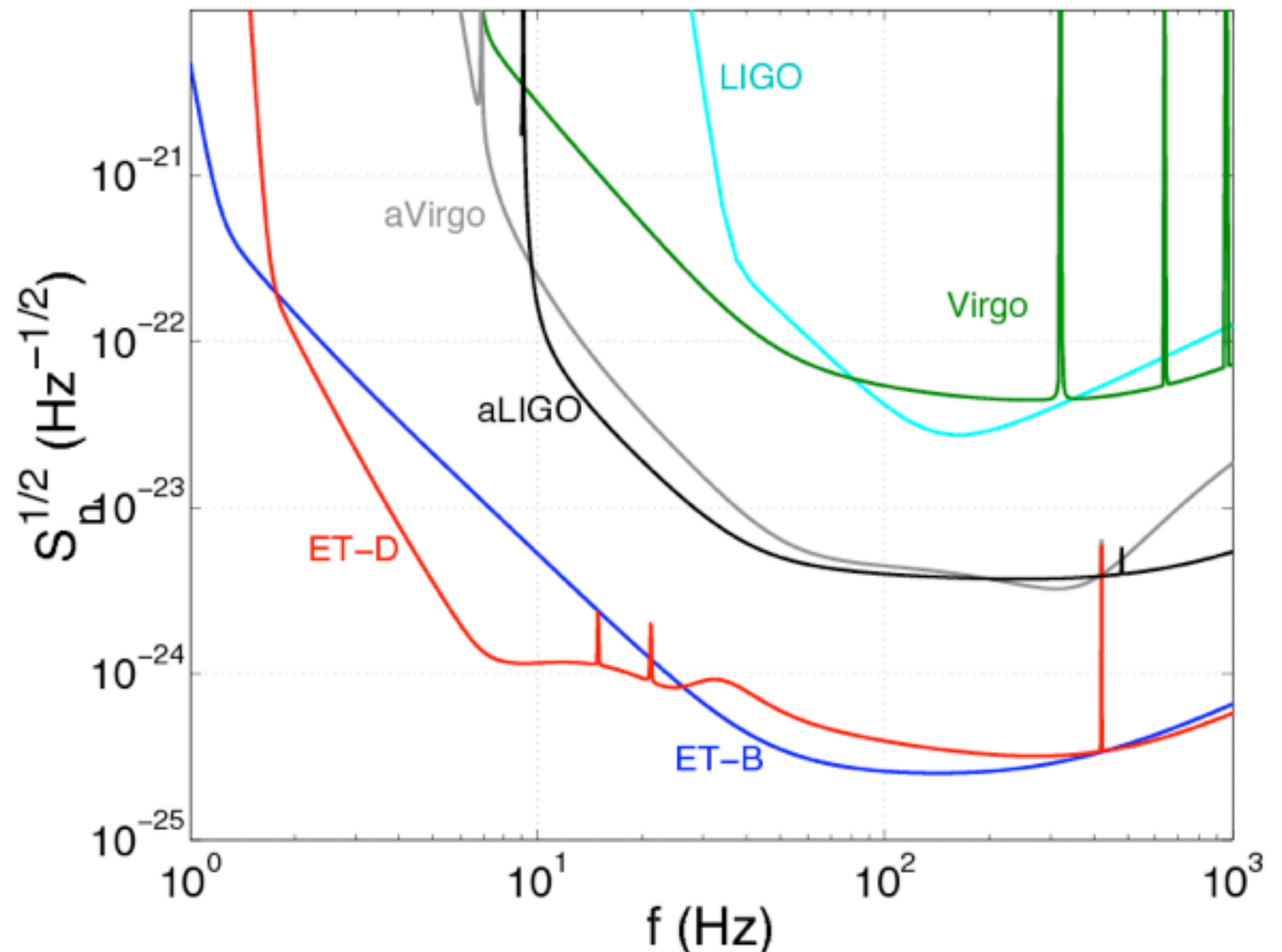


Localization of GW source for detector network without
and with LIGO-India, improved by ~order of magnitude.

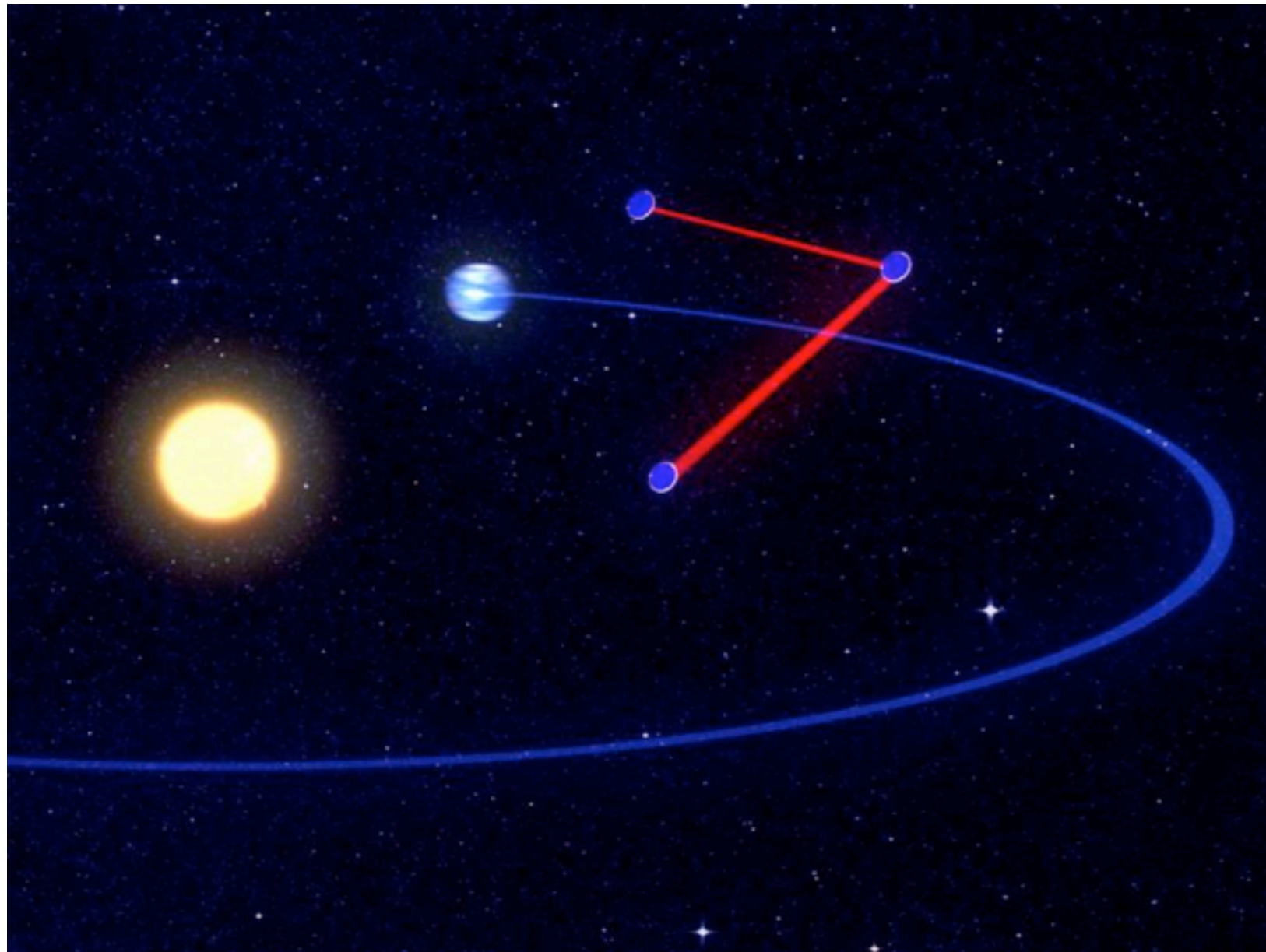
Third-Generation Ground Detectors

For example, the Einstein Telescope under study by institutes in Europe

- 10km arms
- Underground to reduce seismic noise
- Cryogenic facilities to reduce thermal noise

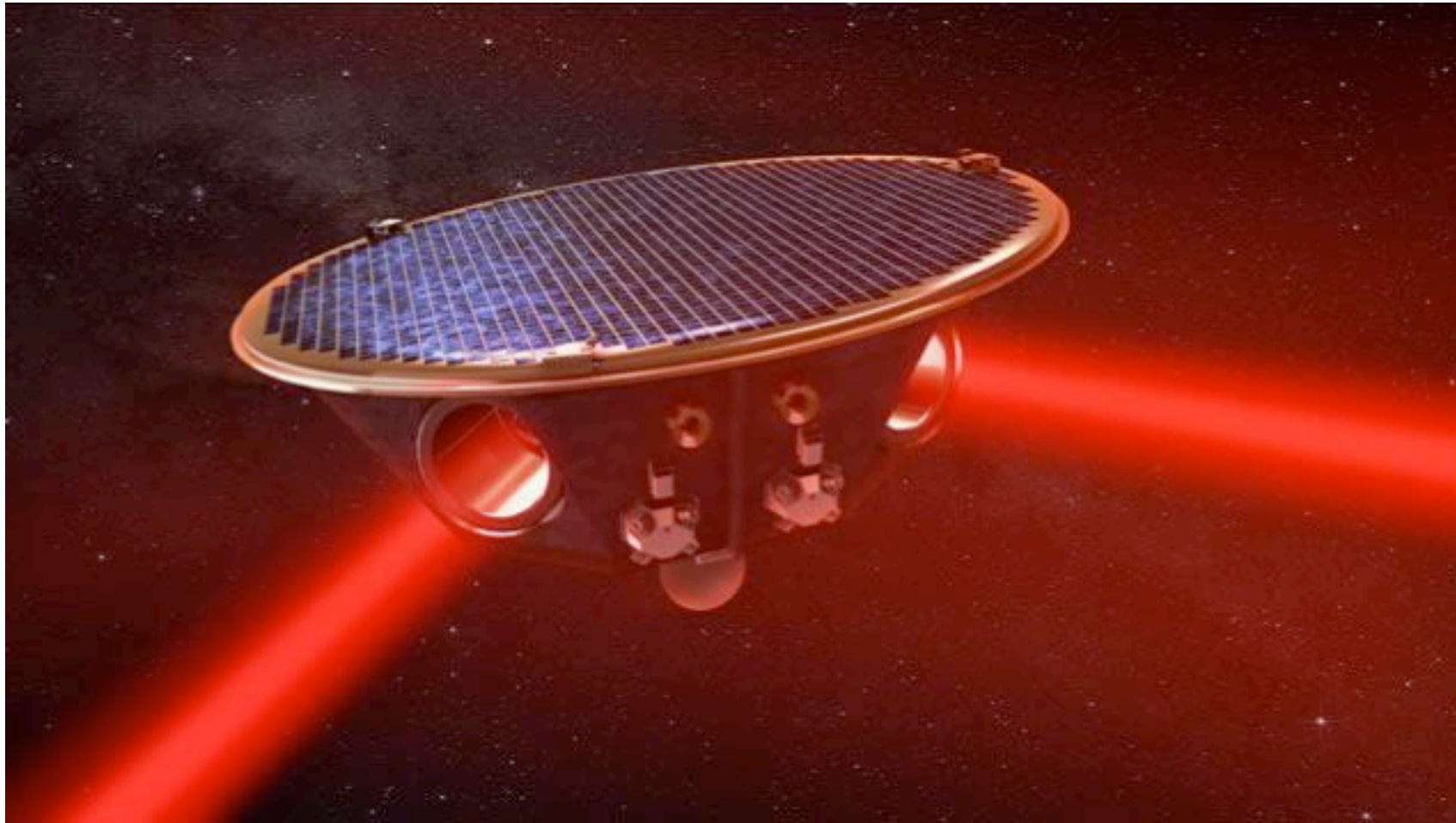


Space-based Detectors: eLISA



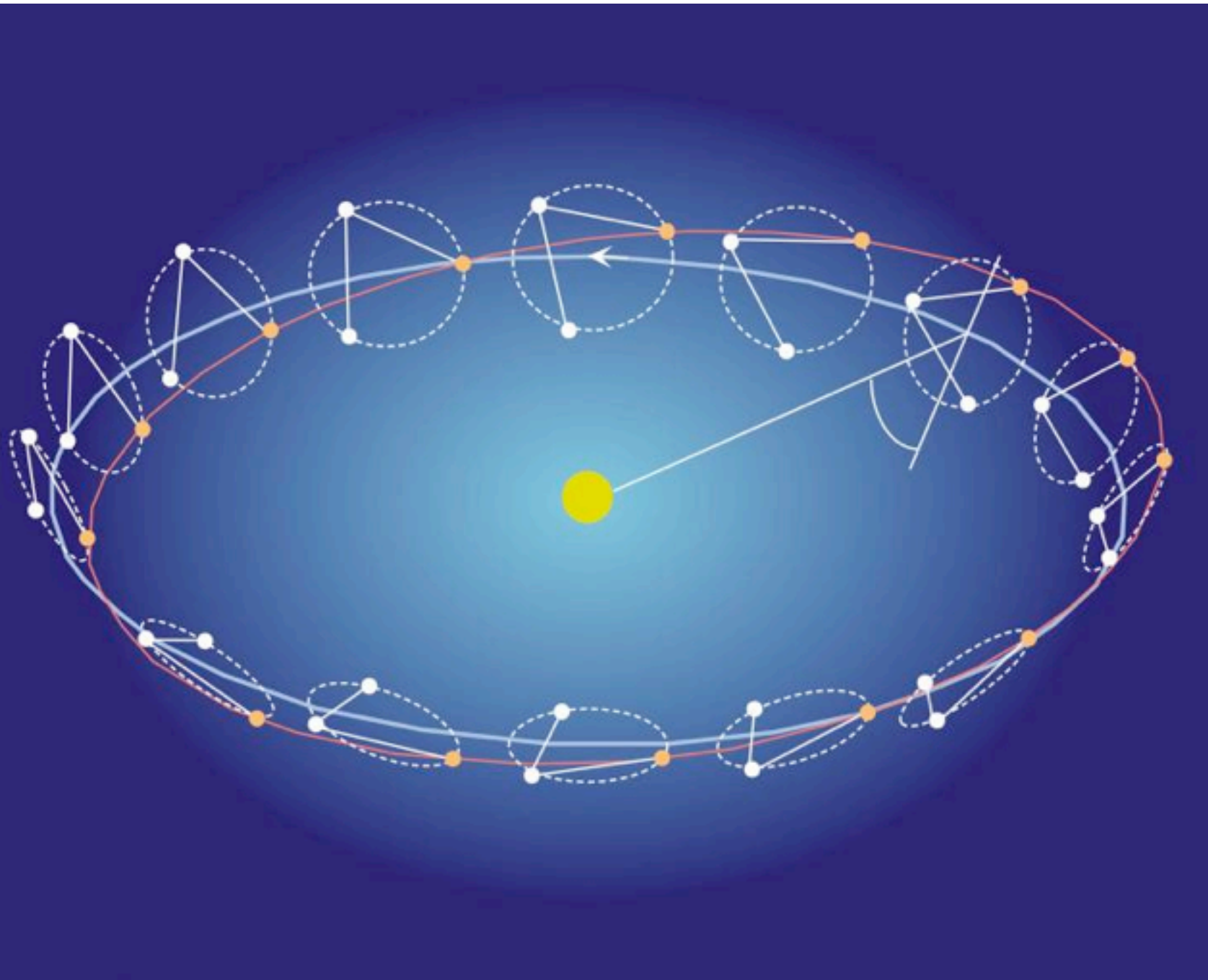
Three spacecraft in Earth-like orbits around Sun.
Arm length of 1 million km, equilateral triangle formation.

Space-based Detectors: eLISA



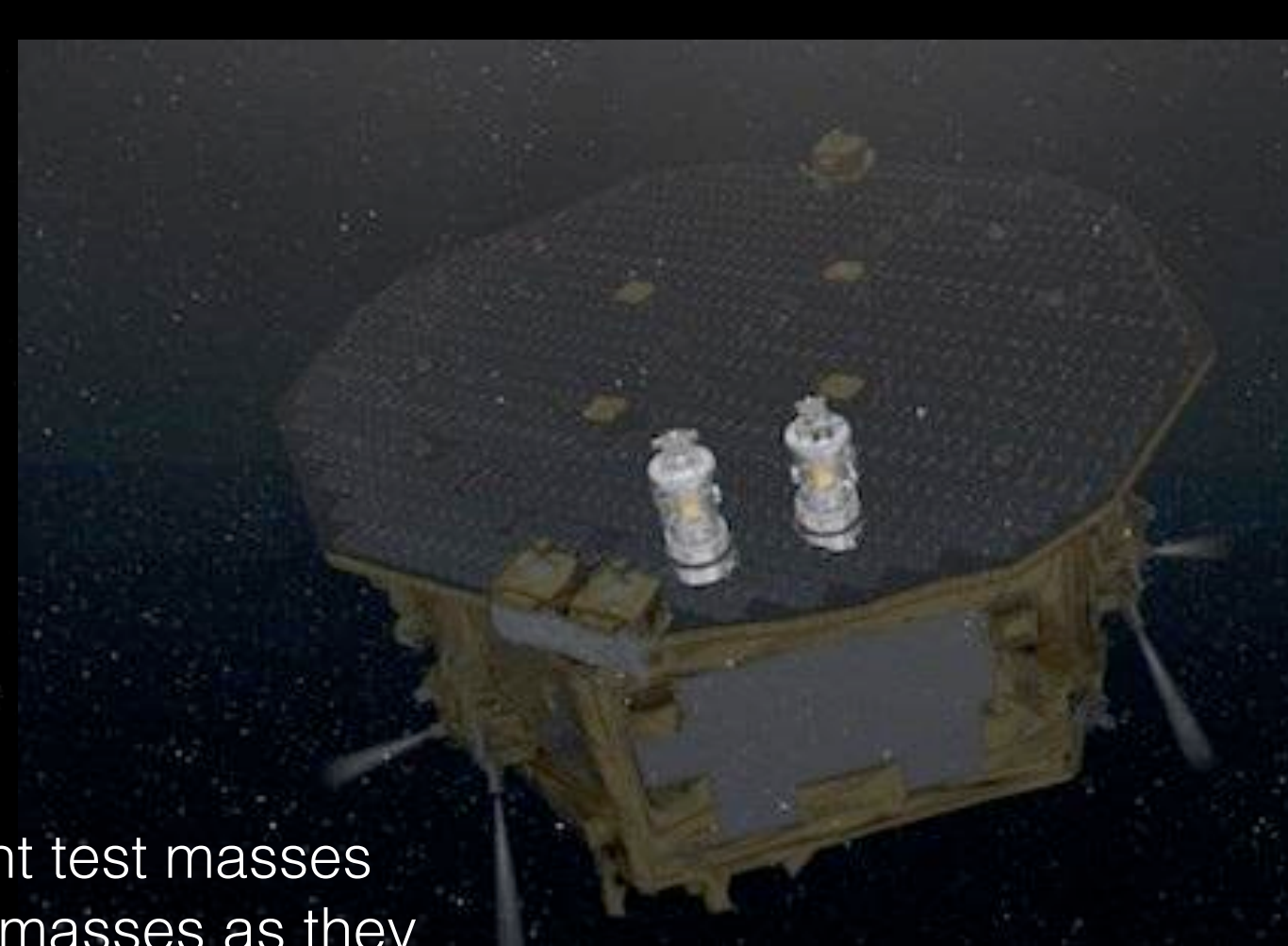
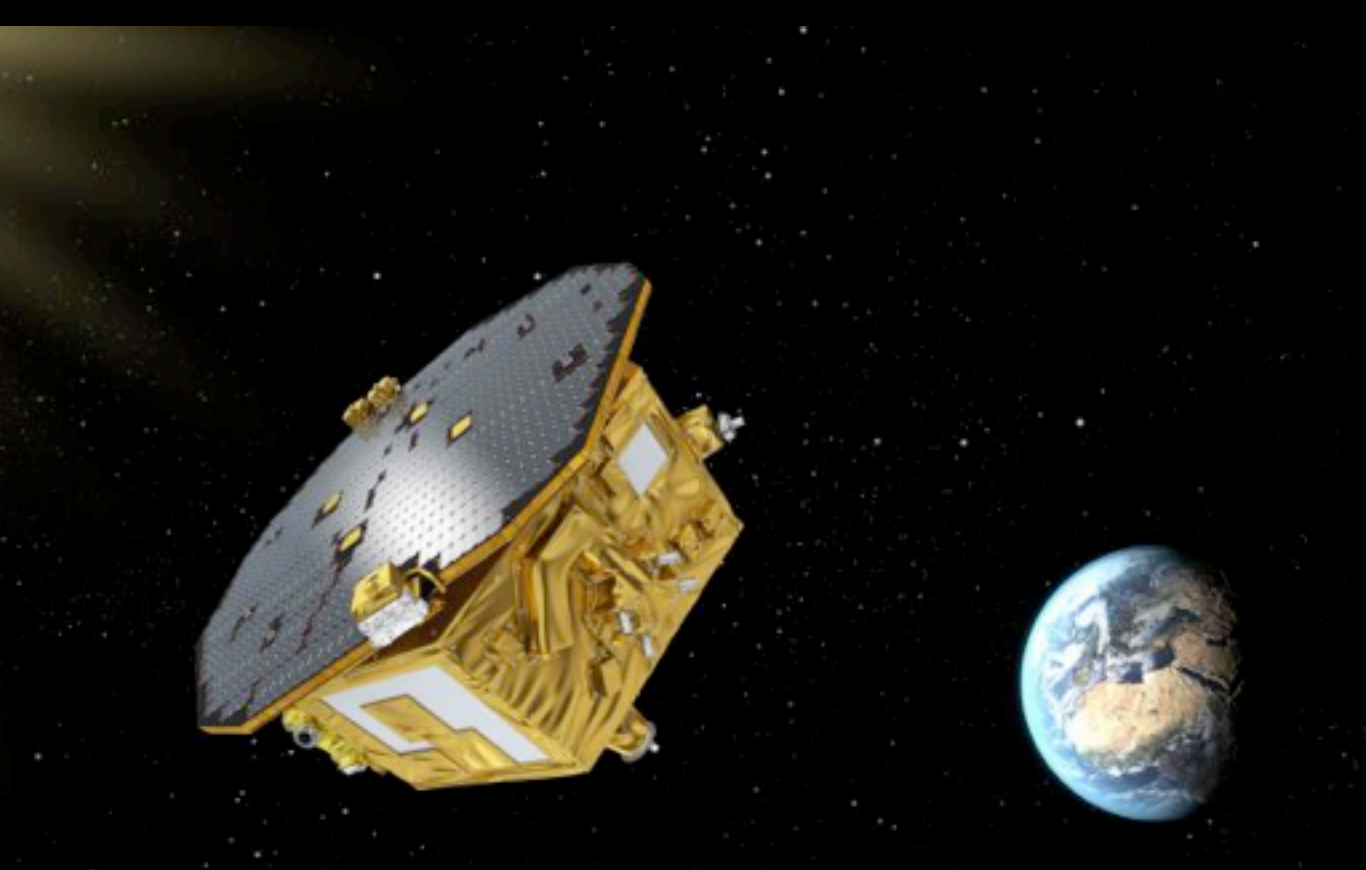
Each spacecraft carries lasers. They're too far apart for the laser light to reflect back and forth. Instead, when light from one arrives at the other, the onboard laser of receiving craft amplifies and returns it (like reflection): **active mirrors**.

Space-based Detectors: eLISA

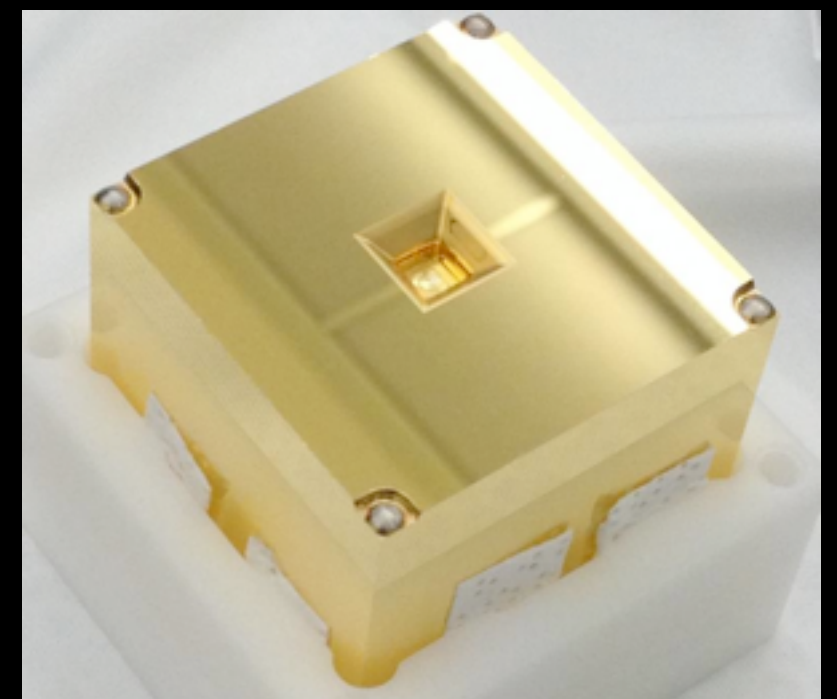
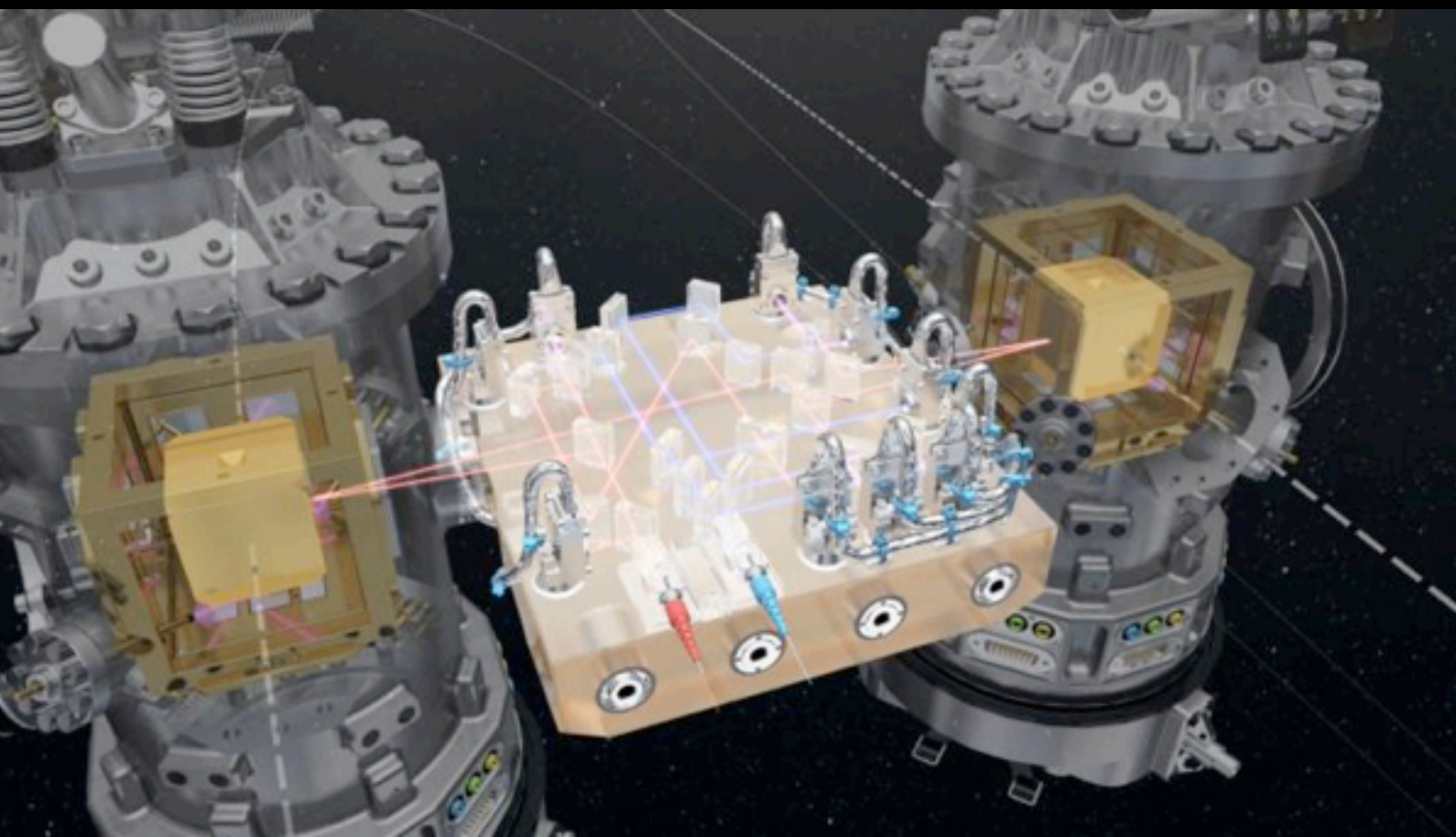


Changing orientation of triangle through one orbit will allow observers to determine direction of sources.

LISA pathfinder operates 1.5 mill km from Earth toward the Sun, orbiting the Sun-Earth langrangian point L1.



Two inertial sensors surrounding independent test masses and an optical bench in-between monitor test masses as they free-fall through space



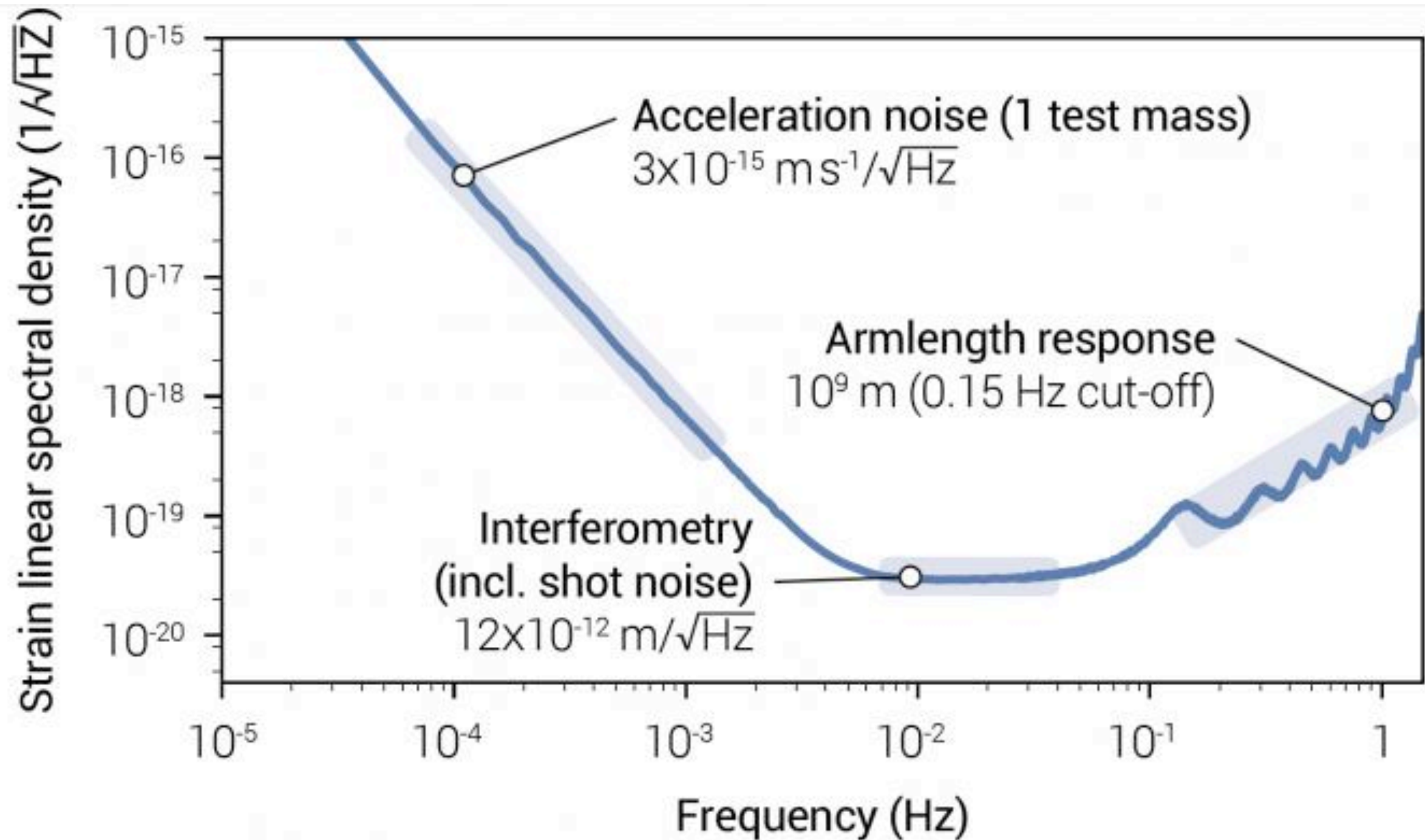
One of LISA Pathfinder's two test masses, cube of solid gold-platinum alloy, 1.96kg.

Space-based Detectors: eLISA

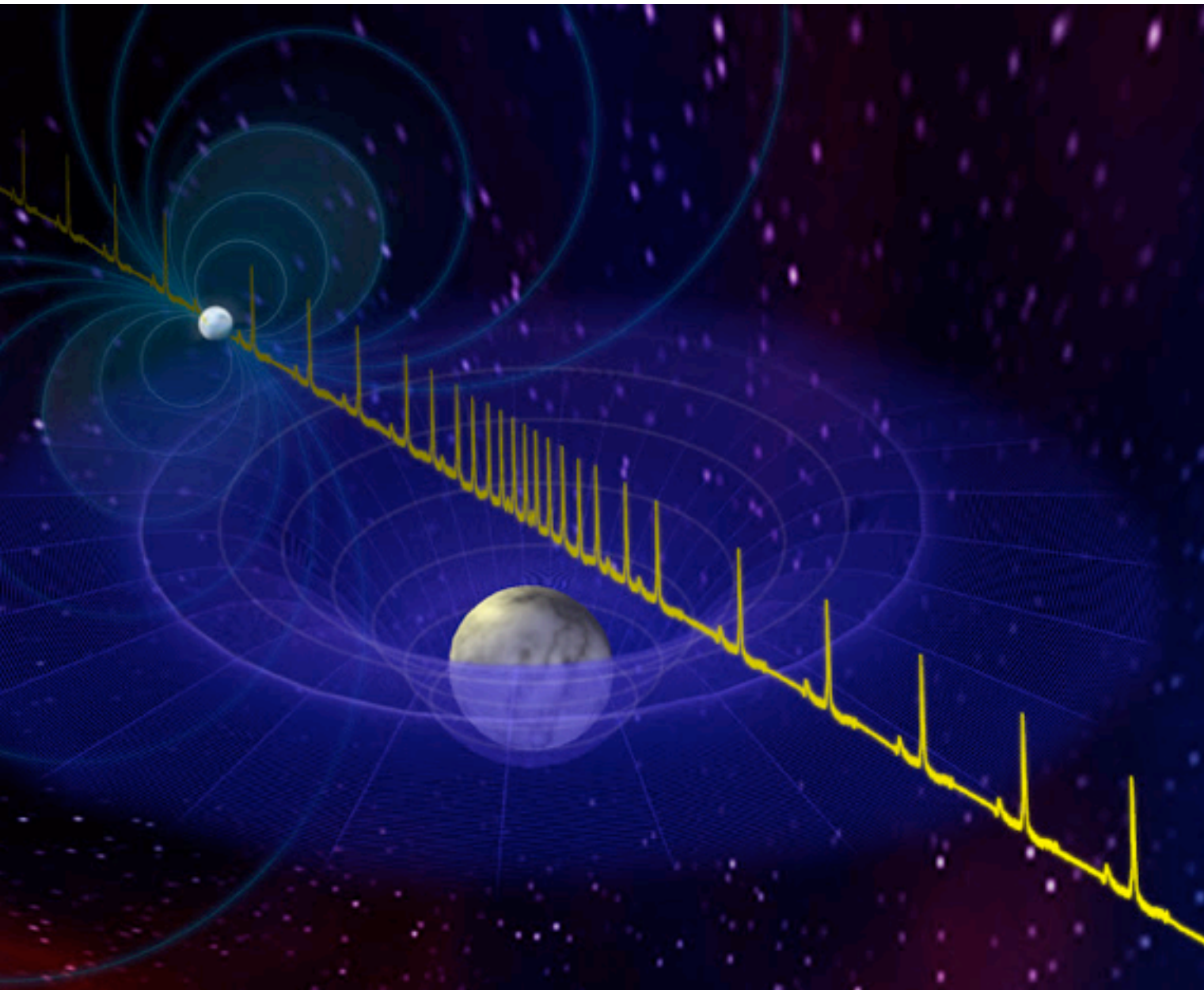


Microthrusters will make minuscule shifts in order to keep the spacecraft centered on one of the masses. This will isolate the two cubes from all external and internal forces except gravity, placing them in the most precise freefall ever obtained.

LISA Sensitivity



Pulsar Timing Arrays



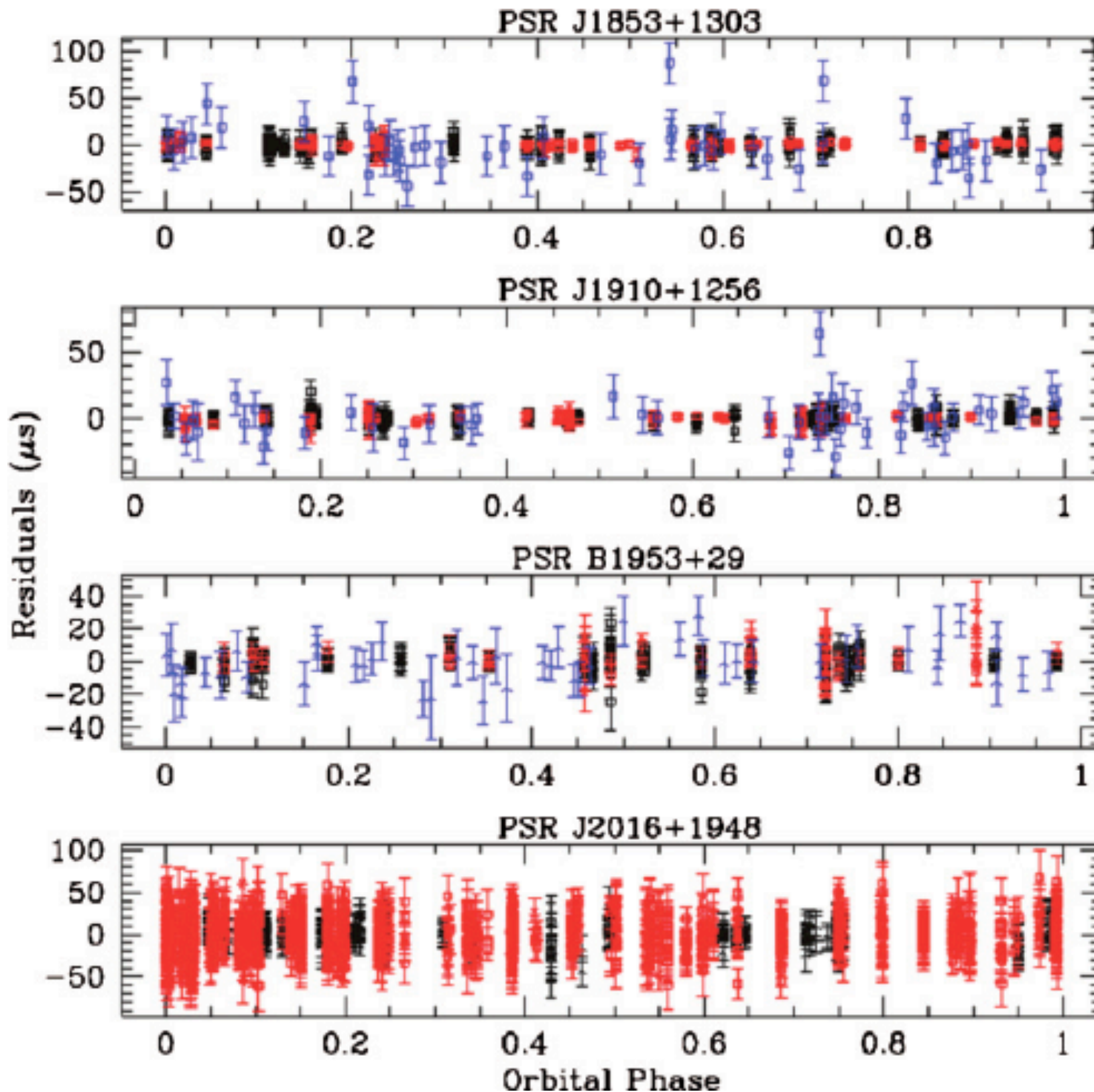
Millisecond pulsars - rotate with incredible stability, can be used as precise clocks.

The time of arrival (TOA) of pulse can be usually measured to:

$$\frac{\delta}{\text{SNR}}$$

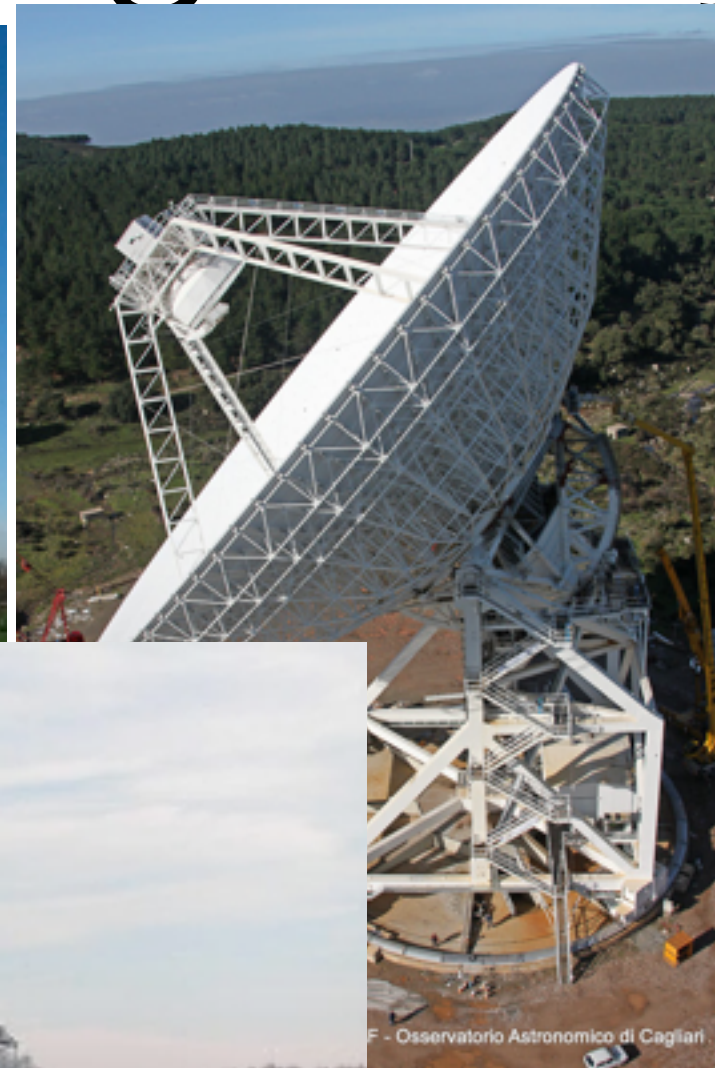
δ - pulse width; few hundred microseconds

Pulsar Timing Arrays



Difference between predicted and measured TOAs can have RMS scatter as little as 100 nanoseconds over timescales of many years.

Pulsar Timing Arrays



Germany
UK
France
Italy
Netherlands
US
Puerto Rico
Australia



Pulsar Timing Arrays



Pulsar timing brings together astronomers who use the world's most sensitive radio telescopes to observe and discover millisecond pulsars.

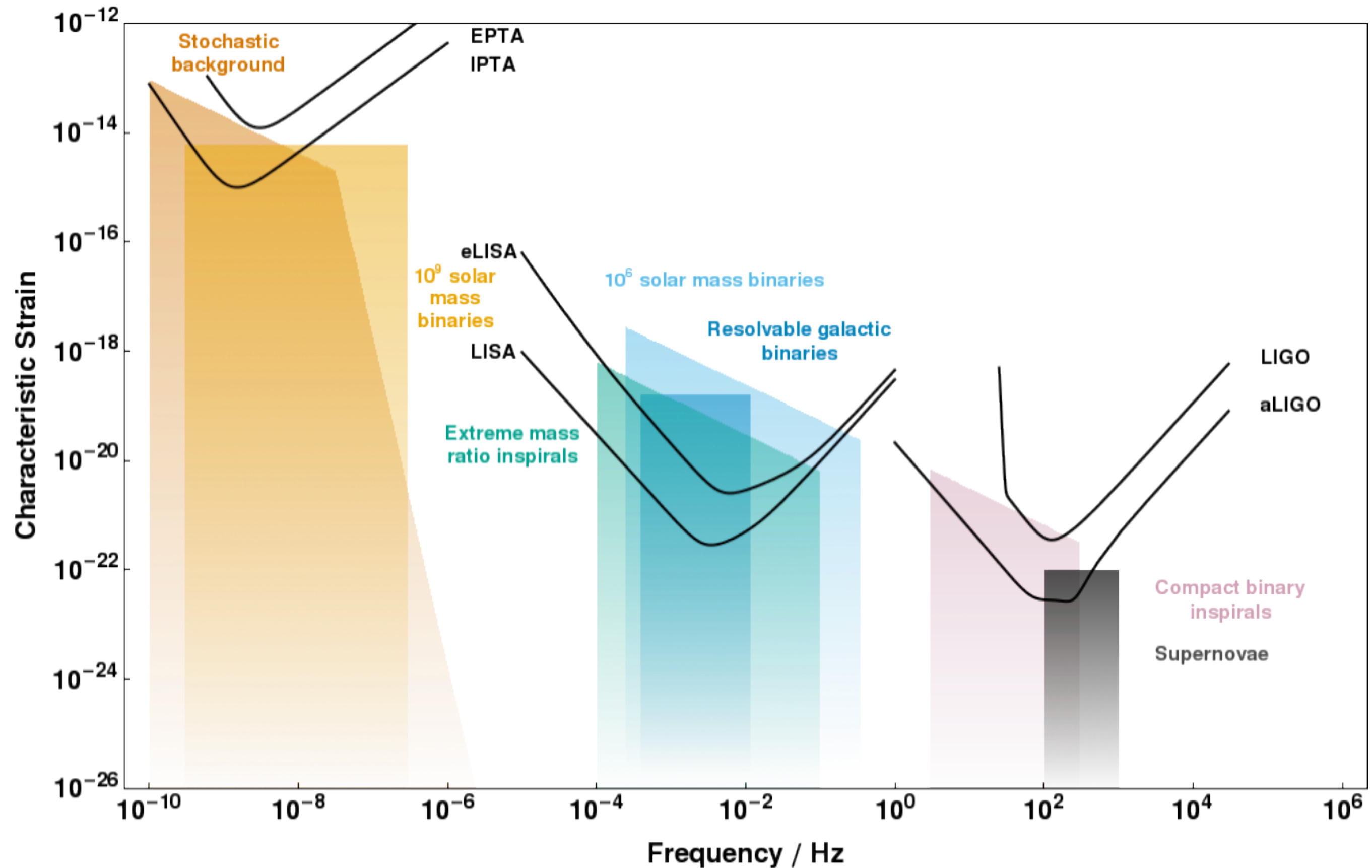
They look for the influence of GWs on timing residuals from ultra-precise millisecond pulsars, in particular for correlations in timing residuals between pairs of pulsars

Pulsar Timing Arrays



Sensitive to GWs with periods between the cadence of pulsar timing observations (weeks) and the span of the dataset (years), ie nanohertz frequencies.

Gravitational-wave Spectrum



Extra Slides

$$\Delta L \equiv L_x - L_y = n\lambda \quad \text{constructive interference}$$

$$\Delta L \equiv L_x - L_y = \left(n + \frac{1}{2}\right) \lambda \quad \text{destructive interference}$$

$$n = 0, 1, 2, \dots$$

