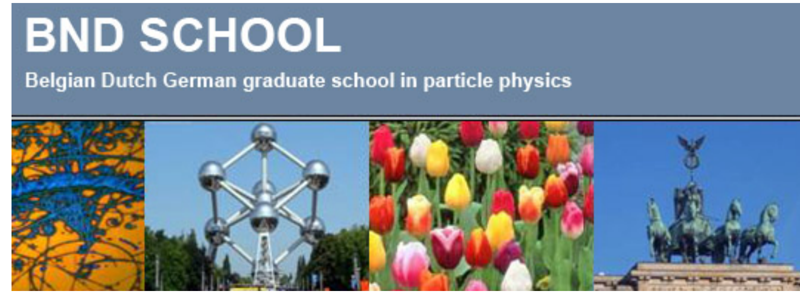
CERN



Accelerators

Part 1 of 3 : Introduction & Transverse Motion

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Three Lectures

1. Introduction and Transverse Optics

2. Longitudinal Motion, Diagnostics, Possible Limitations

3. Injection/Extraction, Collider Specifics and CERN Upgrade Projects,

All you ever wanted to ask about accelerators

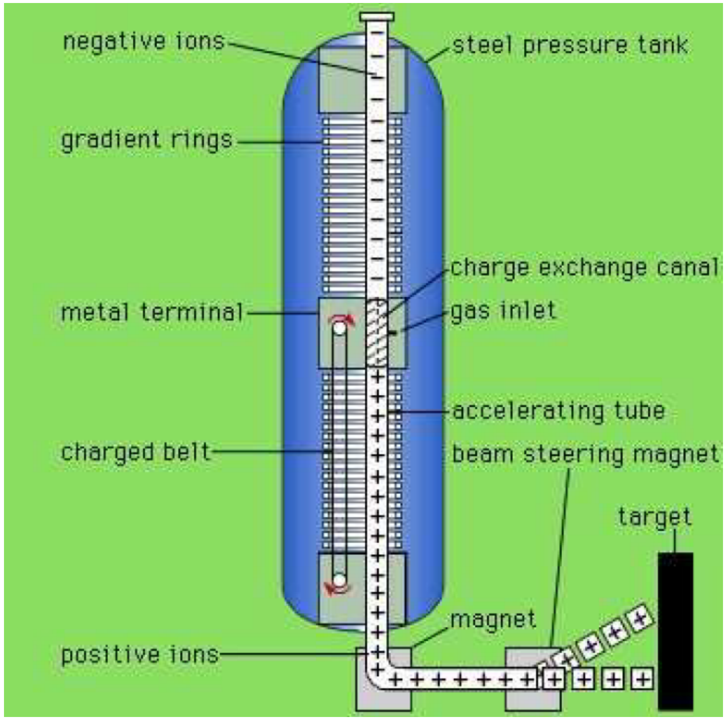
Topics

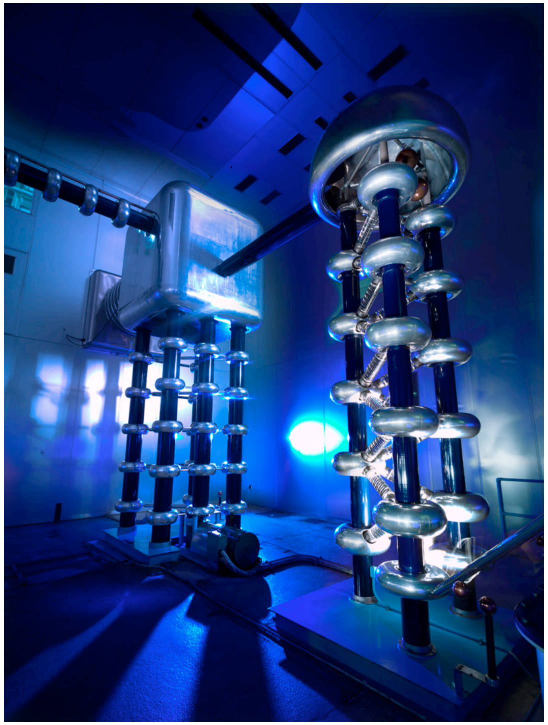
- A brief Word on Accelerator History
- The CERN Accelerator Complex
- A Brief Word on Relativity & Units
- Transverse Motion

A brief Word on Accelerator History

Cockroft & Walton / van de Graaff

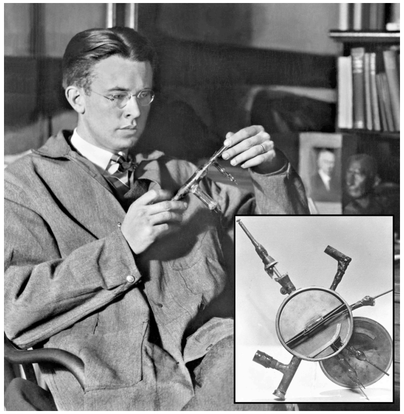
- 1932: First accelerator – single passage 160 - 700 keV
- **Static voltage** accelerator
- Limited by the high voltage needed



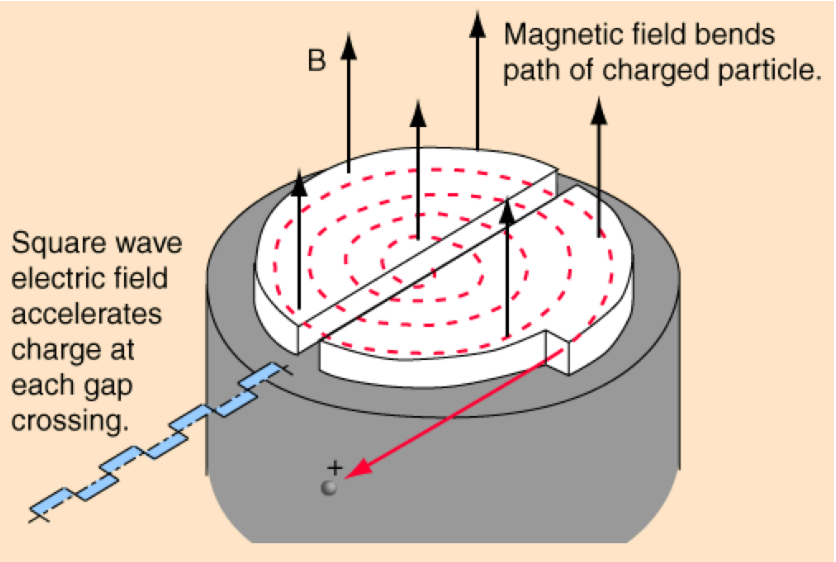


Cyclotron

- 1932: 1.2 MeV – 1940: 20 MeV (E.O. Lawrence, M.S. Livingston)
- **Constant magnetic field** resulting in E = 80 keV for 41 turns
- Alternating voltage between the two D's
- **Increasing particle orbit radius**
- Development lead to the synchro-cyclotron to cope with the relativistic effects (Energy ~ 500 MeV)

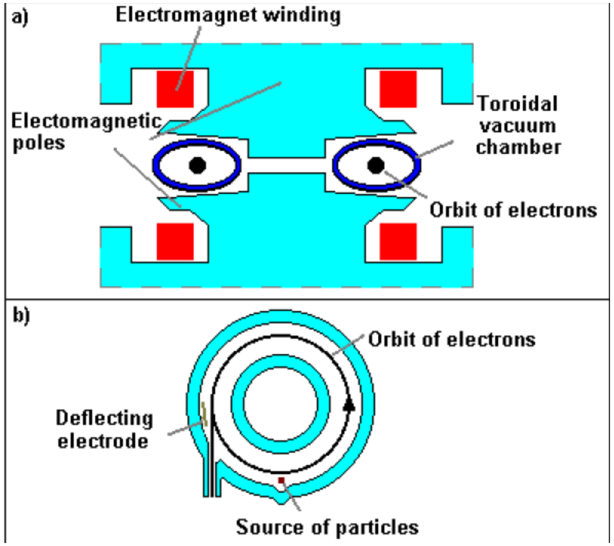


In 1939 Lawrence received the Noble prize for his work.



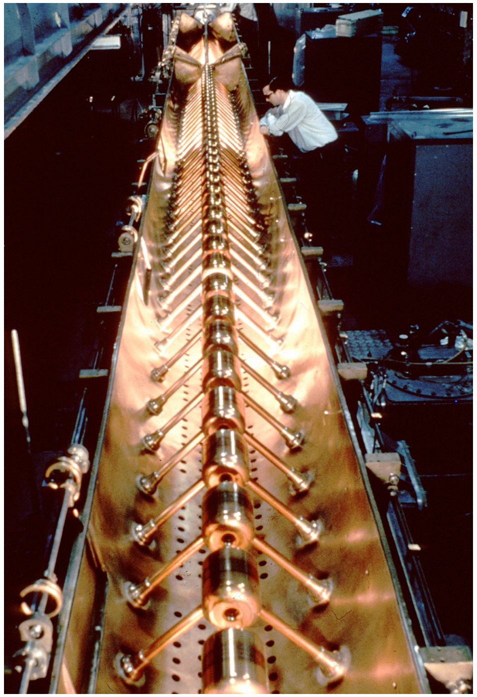
Betatron

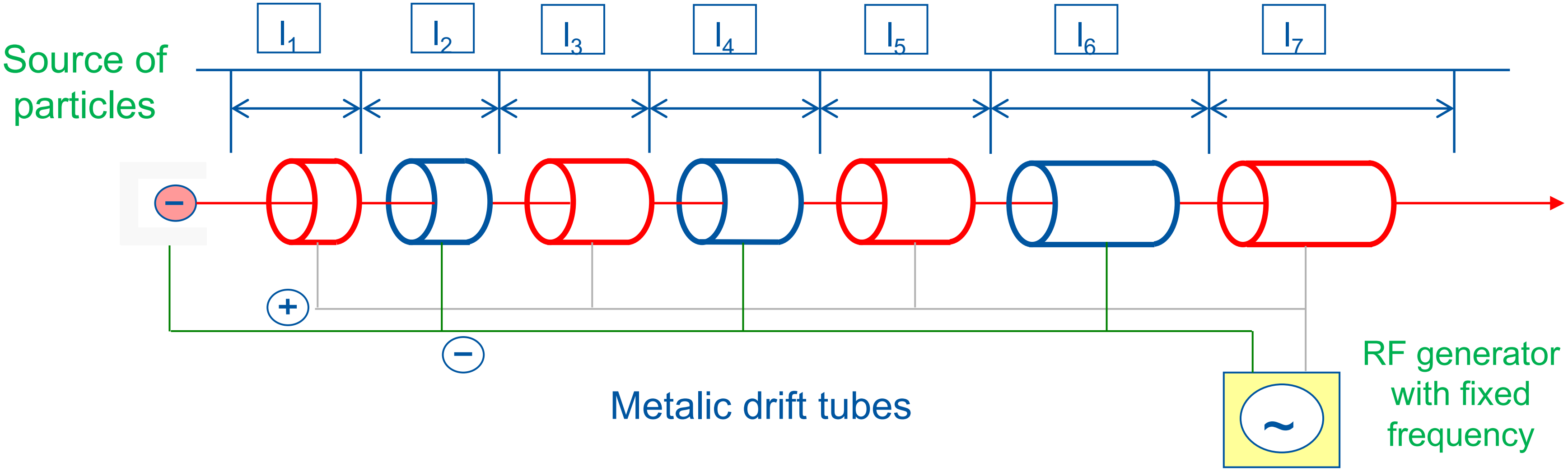
- 1940: Kerst 2.3 MeV and very quickly 300 MeV
- First machine to accelerate electrons to energies higher than from electron guns
- It is actually a transformer with a beam of electrons as secondary winding
- The magnetic field is used to bend the electrons in a circle, but also to accelerate them
- A deflecting electrode is use to deflect the particles for extraction.



Linear Accelerator

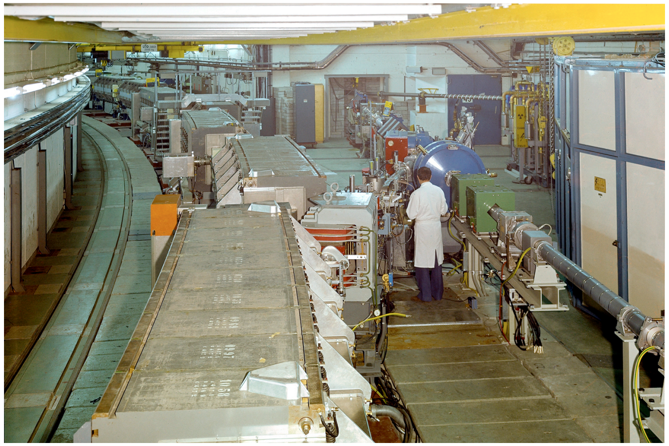
- Many people involved: Wideroe, Sloan, Lawrence, Alvarez,….
- Main development took place between 1931 and 1946.
- Development was also helped by the progress made on high power high frequency power supplies for radar technology.
- Today still the first stage in many accelerator complexes.
- Limited by energy due to length and single pass.





Synchrotrons

- 1943: M. Oliphant described his synchrotron invention in a memo to the UK Atomic Energy directorate
- 1959: CERN-PS and BNL-AGS
- **Varying magnetic field** and radio frequency give a **fixed particle radius**
- Phase stability
- Important focusing of particle beams (Courant – Snyder)
- Providing beam for fixed target physics
- **Paved the way to colliders**



Accelerators and Their Use

Today: ~ **30’000 accelerators** operational world-wide*

The **large majority** is used in **industry** and **medicine**

- Industrial applications: ~ 20’000*
- Medical applications: ~ 10’000*

Les than a fraction of a percent is used for **research** and discovery science

- Cyclotrons
- Synchrotron light sources (e^-)
- Lin. & Circ. accelerators/Colliders

These lectures will mainly concentrate on **Synchrotron** machines
That form the source of particle for the majority of accelerator based experiments

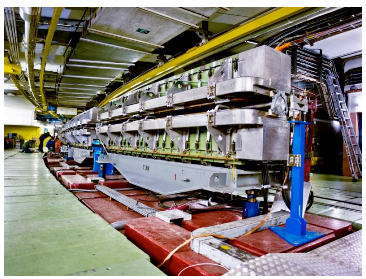
**Source: World Scientific Reviews of Accelerator Science and Technology*
A.W. Chao

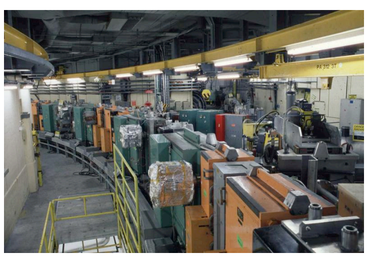
Fixed Target vs. Colliders

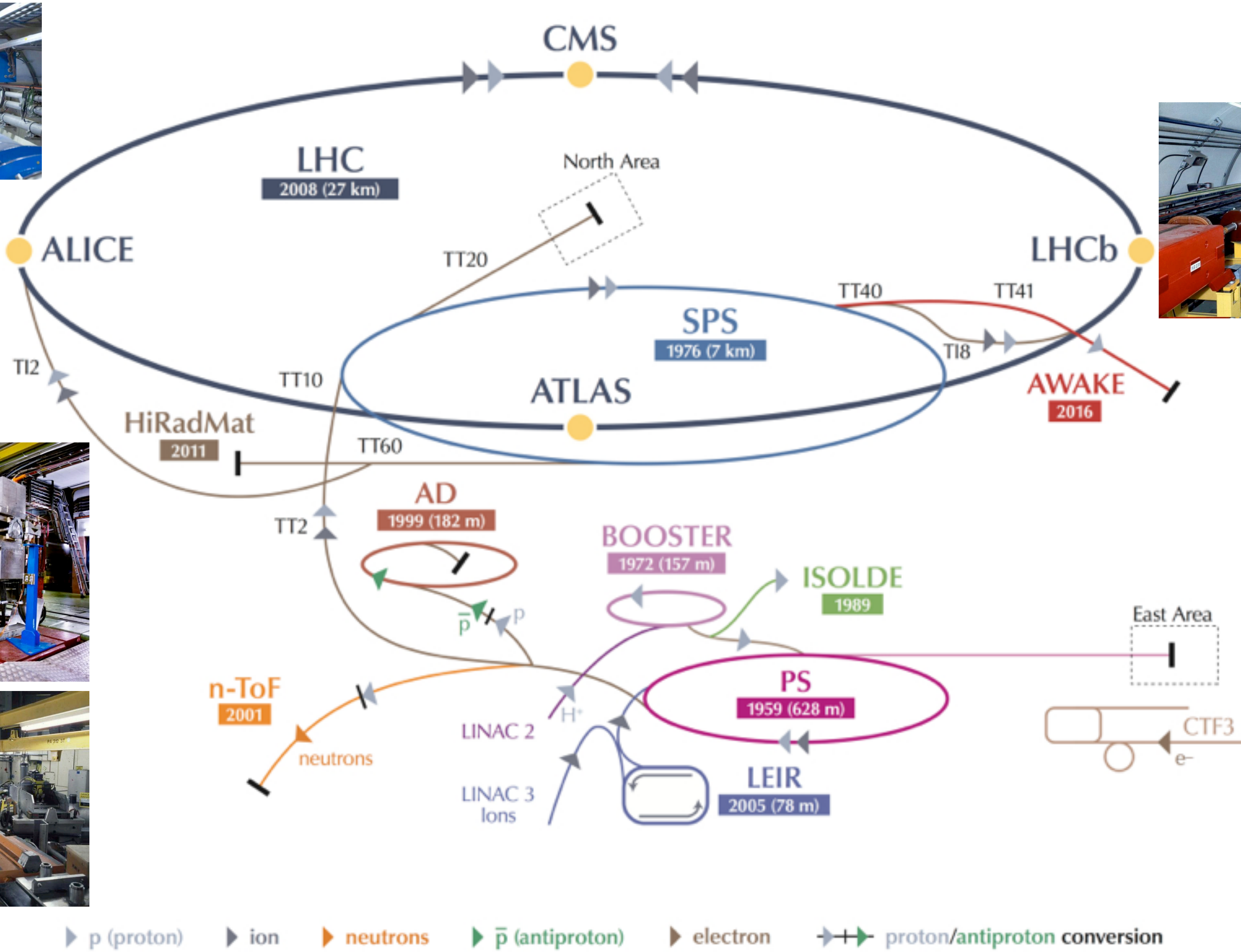
Fixed Target	Collider
	
$E_{sec} \propto \sqrt{E_{primary}}$	$E = \sqrt{E_{beam1}^2 + E_{beam2}^2}$
Much of the energy is lost in the target and only part is used to produce secondary particles	All energy will be available for particle production

The CERN Accelerator Complex

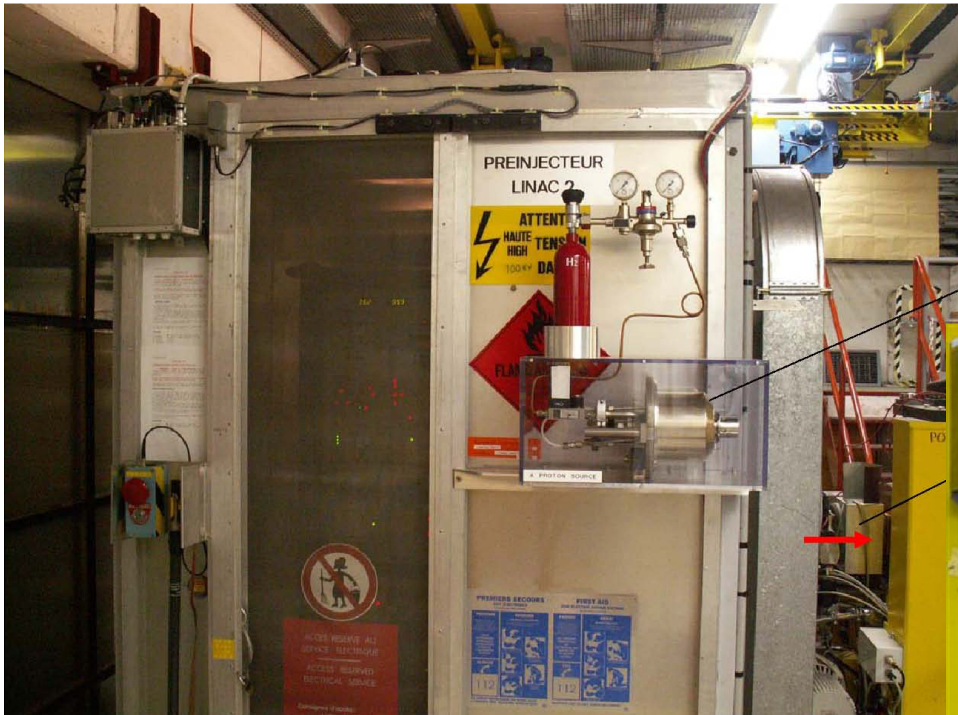
The CERN Accelerator Complex







LINAC 2



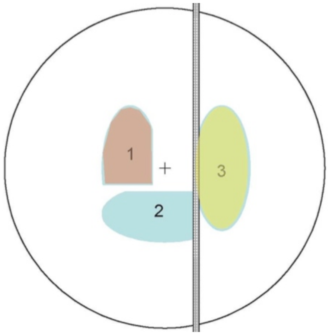
- Duoplasmatron proton source
- Extract protons at 90 keV from H_2

- Accelerates beam up to 50 MeV over a length of 33m, using Alvarez structures
- Provides a beam pulse every 1.2s

PS Booster

- 1st Synchrotron in the chain with 4 superposed rings
- Circumference of 157m
- Increases proton energy from 50 MeV to 1.4 GeV on a 1.2s cycle

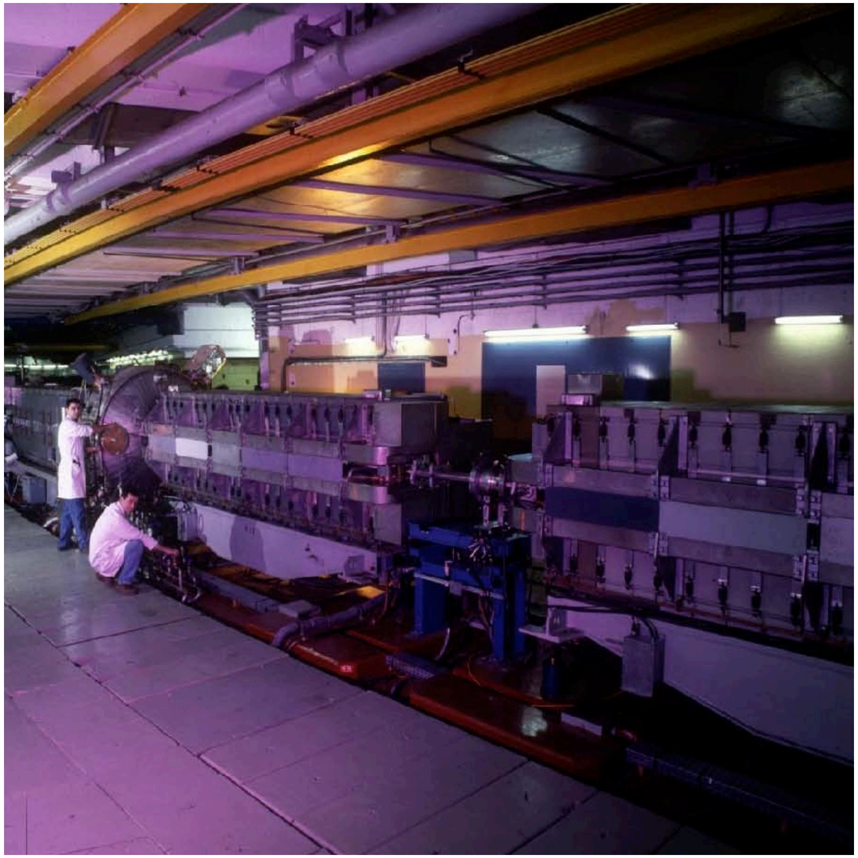
- The LINAC2 pulse is distributed over the four rings, using kicker magnets
- Each ring will inject over multiple turns, accumulating beam in the horizontal phase space
- This means that the beam size (transverse emittance) increases when the intensity increases → ~ constant density



The PS Booster determines the transverse Brightness of the LHC beam

PS

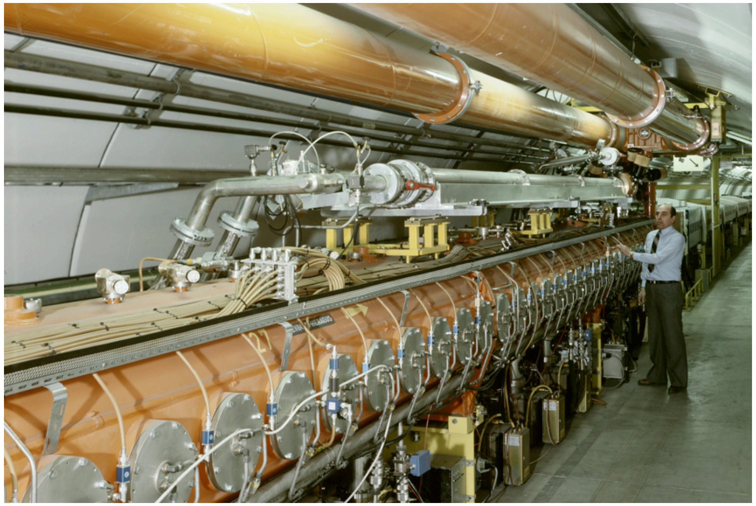
- The oldest operating synchrotron at CERN
- Circumference of 628m
  - 4 x PSB circumference
- Increases proton energy from 1.4 GeV to a range of energies up to 26 GeV
- Cycle length varies depending on the final energy, but ranges from 1.2s to 3.6s



- The many different RF systems allow for complex RF gymnastics:
  - 10 MHz, 13/20 MHz, 40 MHz, 80 MHz, 200 MHz
- Various types of extractions:
  - Fast extraction
  - Multi-turn extraction (MTE)
  - Slow extraction

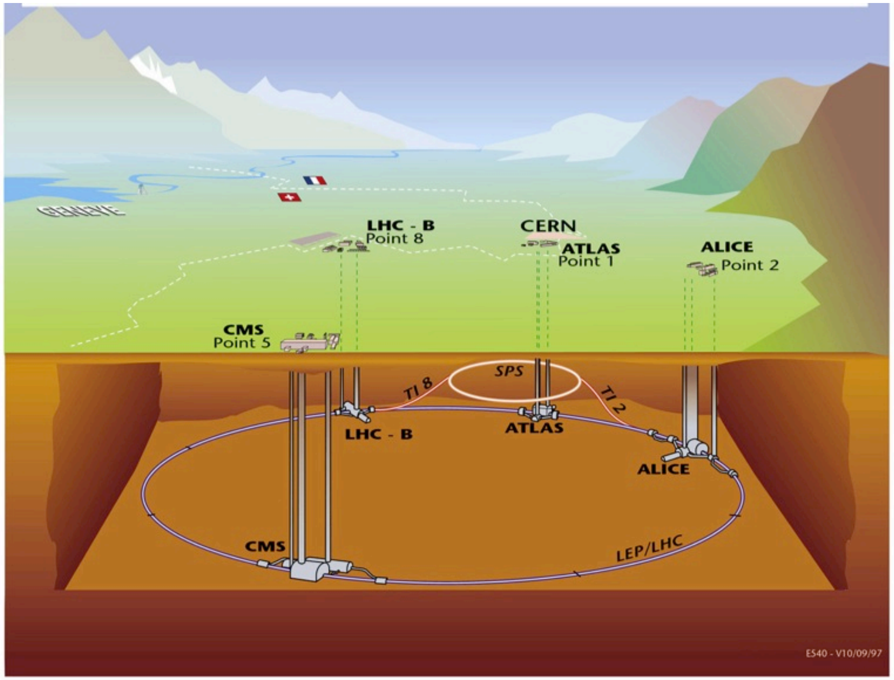
SPS

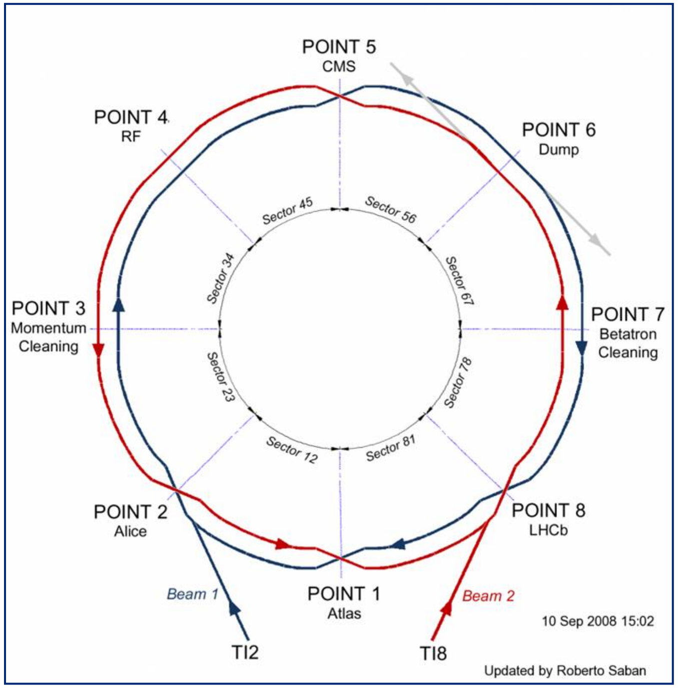
- The first synchrotron in the chain at about 30m under ground
- Circumference of 6.9 km
  - 11 x PS circumference
- Increases proton beam energy up to 450 GeV with up to ~$5x10^{13}$ protons per cycle



- Provides slow extracted beam to the North Area
- Provides fast extracted beam to LHC, AWAKE and HiRadMat

LHC

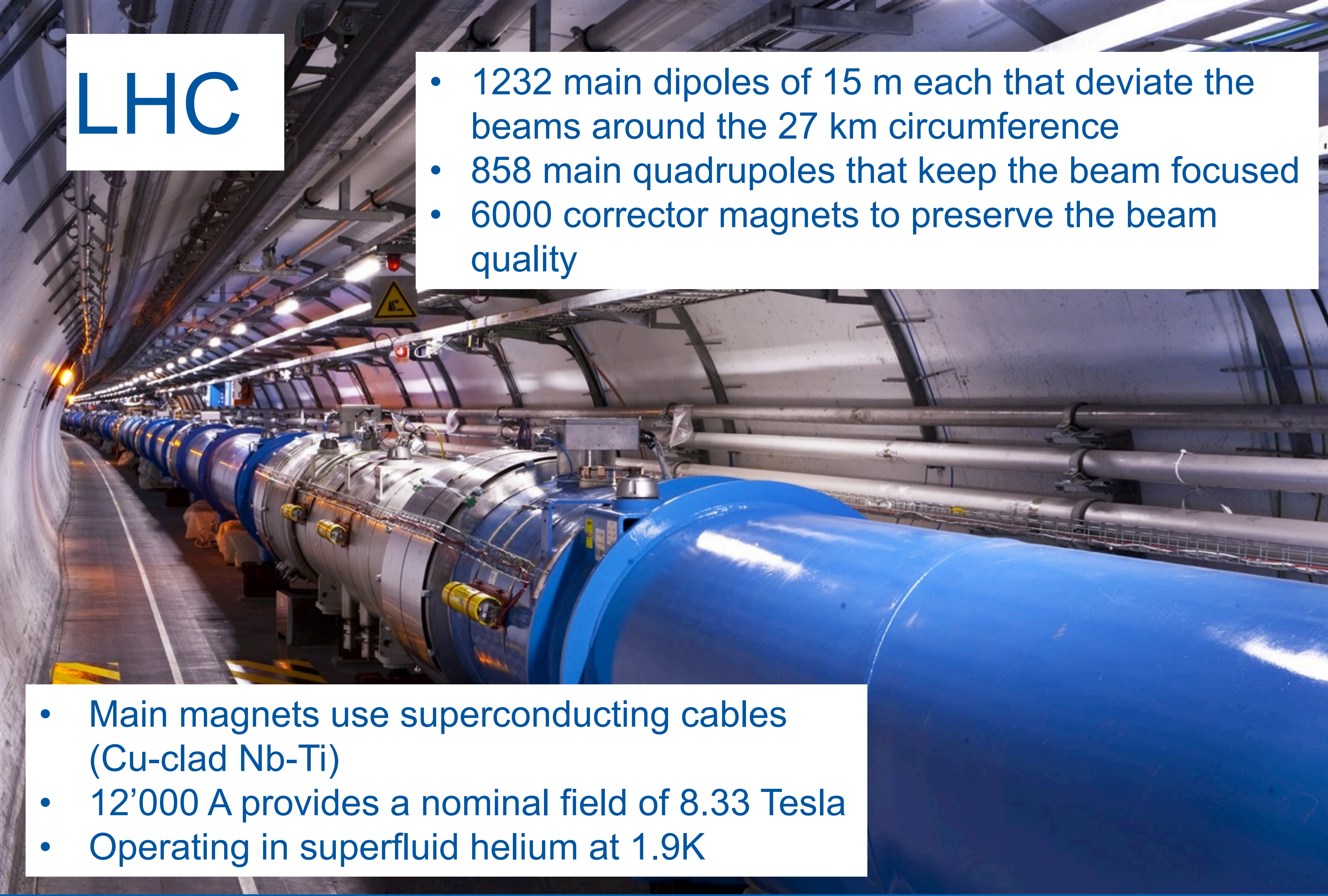




- Situated on average ~100 m under ground
- Four major experiments (ATLAS, CMS, ALICE, LHCb)
- Circumference 26.7 km
- Two separate beam pipes going through the same cold mass 19.4 cm apart
- 150 tonnes of liquid helium to keep the magnets cold and superconducting

LHC

- 1232 main dipoles of 15 m each that deviate the beams around the 27 km circumference
- 858 main quadrupoles that keep the beam focused
- 6000 corrector magnets to preserve the beam quality



- Main magnets use superconducting cables (Cu-clad Nb-Ti)
- 12’000 A provides a nominal field of 8.33 Tesla
- Operating in superfluid helium at 1.9K

LHC: Luminosity

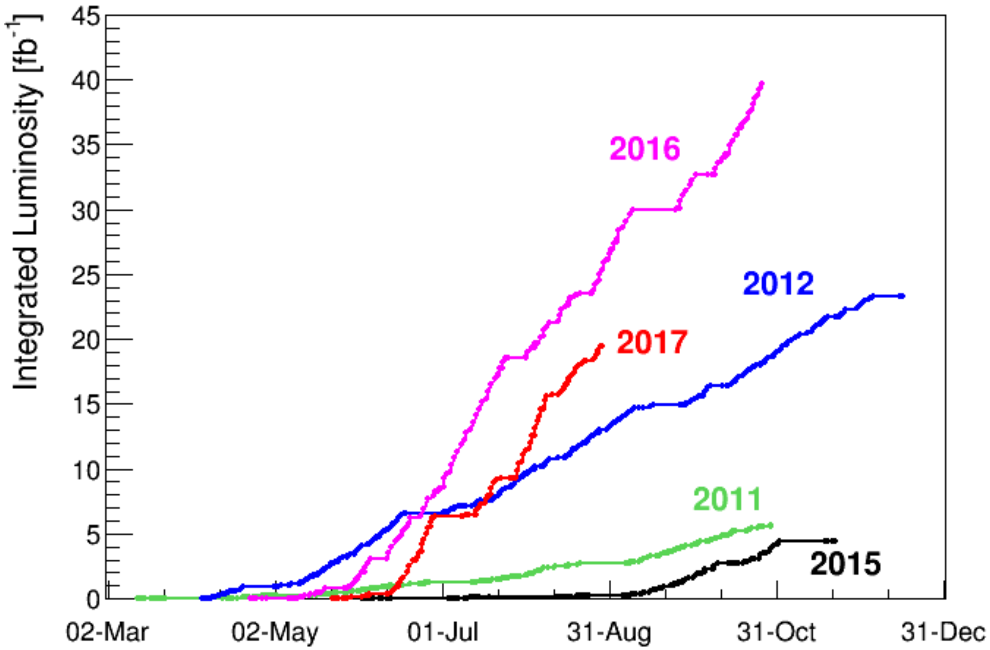
$$LUMINOSITY = \frac{N_{event/sec}}{\sigma_r} = \frac{N_1 N_2 f_{rev} n_b}{4\pi\sigma_x\sigma_y} F$$

Intensity per bunch

Number of bunches

Geometrical Correction factors

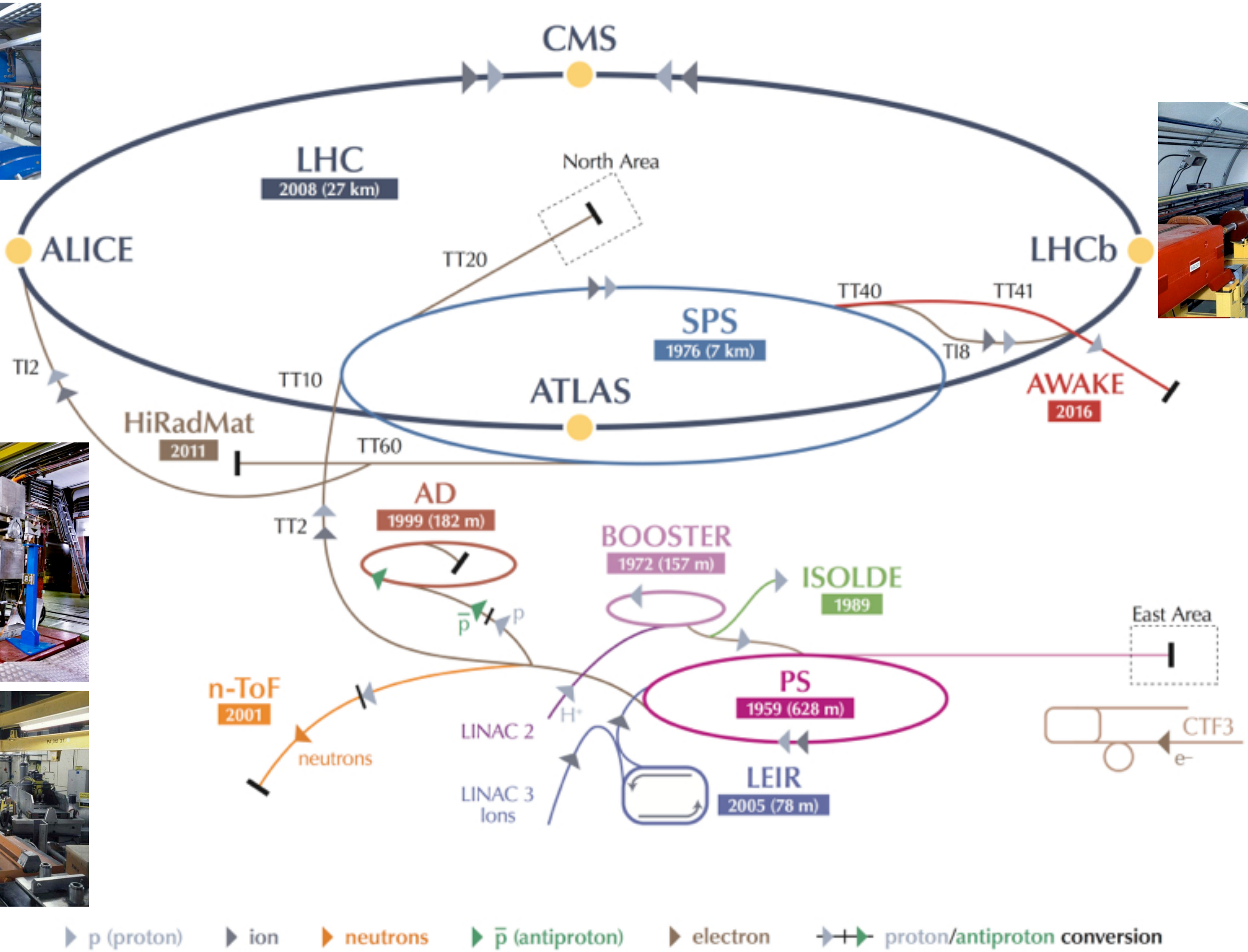
Beam dimensions

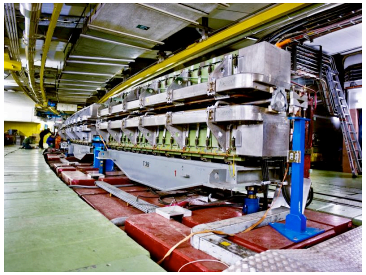


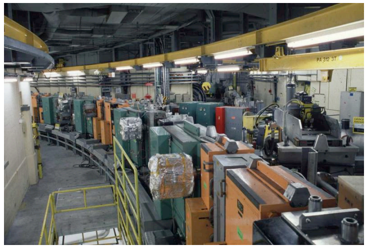
Maximise Luminosity:

- Bunch intensity
- Transverse beam size
- Beam size at collision points (optics functions)
- Crossing angle
- Machine availability

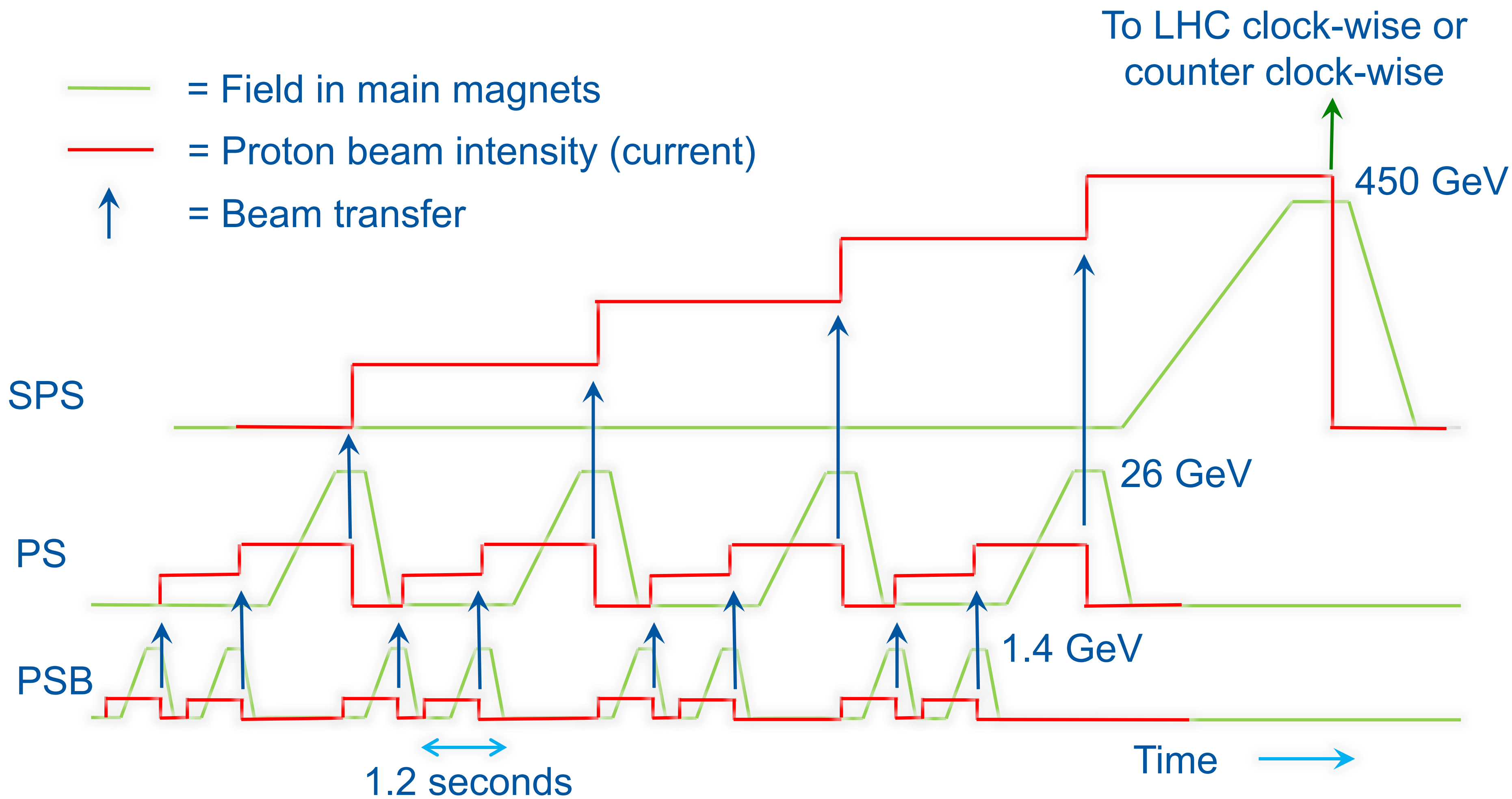
The CERN Accelerator Complex



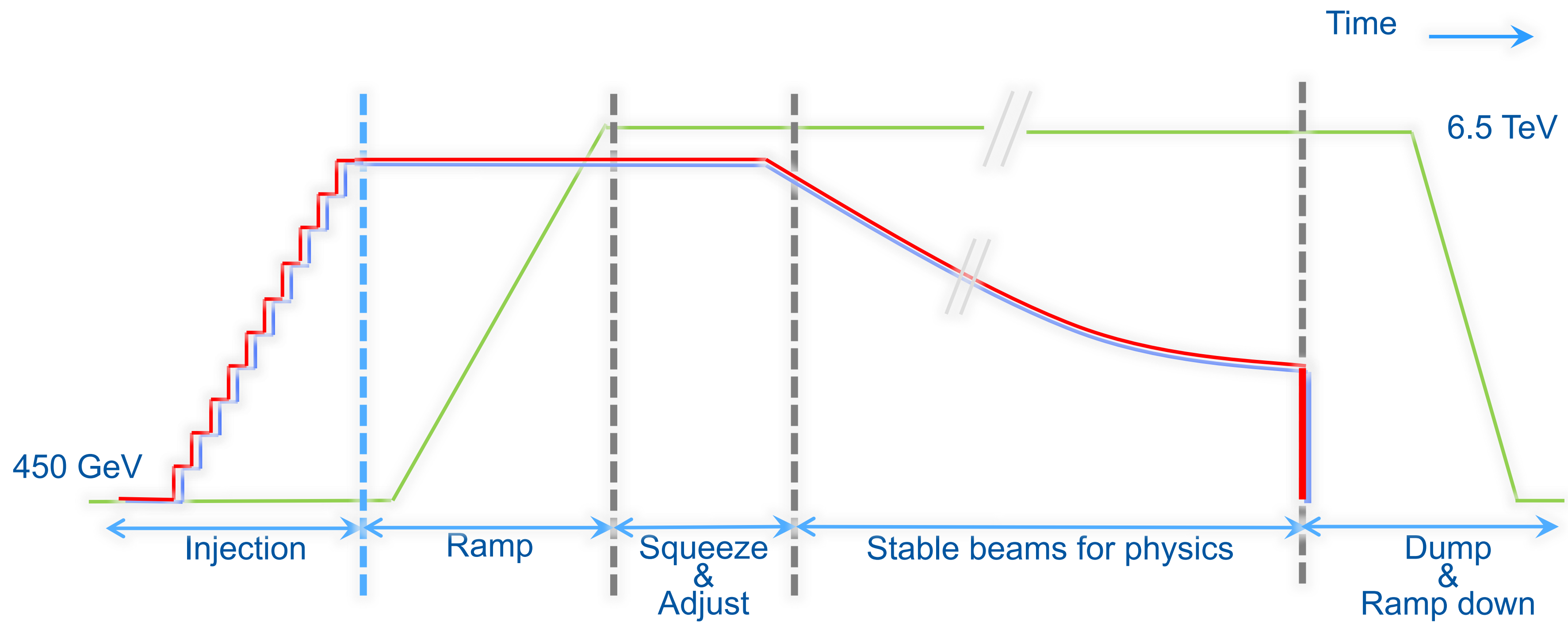




Filling the LHC and Satisfying Fixed Target users



How does the LHC fit in this ?

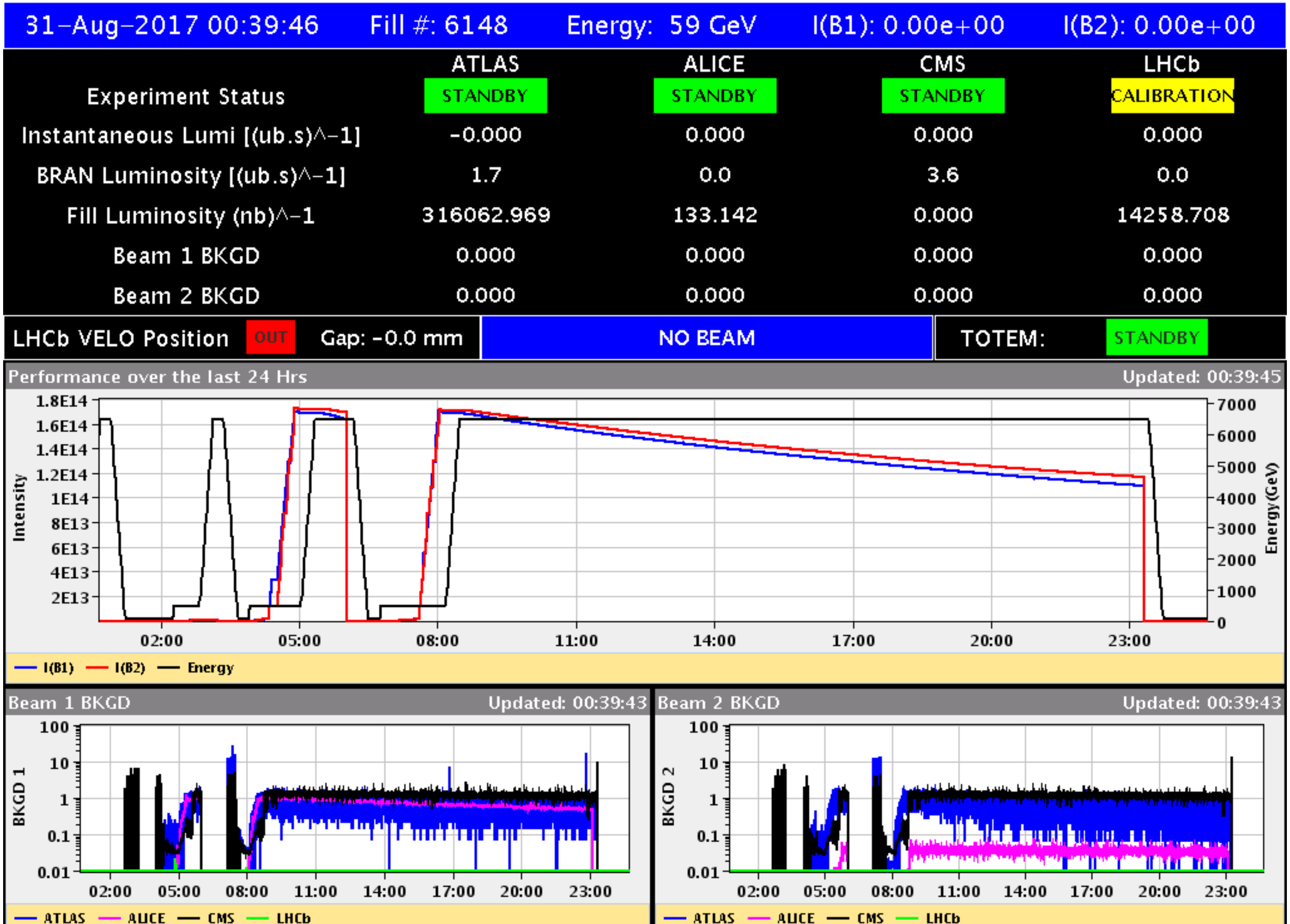


= Field in main magnets
= Beam 1 intensity (current)
= Beam 2 intensity (current)

The LHC is built to collide protons at 7 TeV per beam, which is **14 TeV centre of Mass**

In 2012 it ran at 4 TeV per beam, 8 TeV c.o.m.

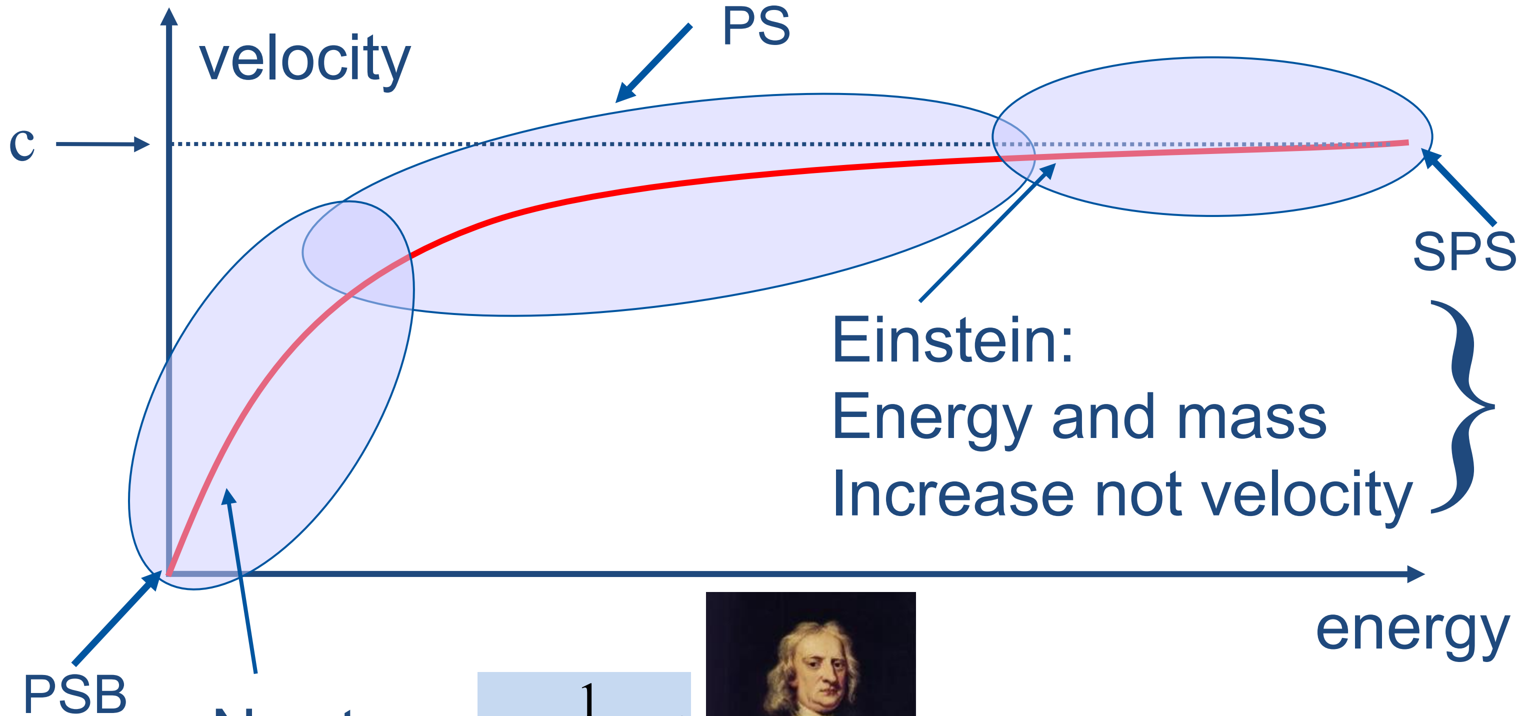
Since 2015 it runs at 6.5 TeV per beam, 13 TeV c.o.m

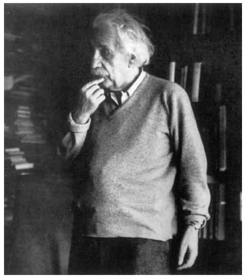


URL: https://op-webtools.web.cern.ch/vistar/vistars.php?usr=LHC1

A Brief Word on Relativity & Units

Towards Relativity





Basic Relativity

Einstein's formula:

$E = mc^2$ which for a mass at rest is: $E_0 = m_0 c^2$

The ratio between the real velocity and the velocity of light is the **relative velocity**

$$\beta = \frac{v}{c}$$

The ratio between the total energy and the rest energy is the **Lorentz factor**

$$\gamma = \frac{E}{E_0} = \frac{1}{\sqrt{1-\beta^2}}$$

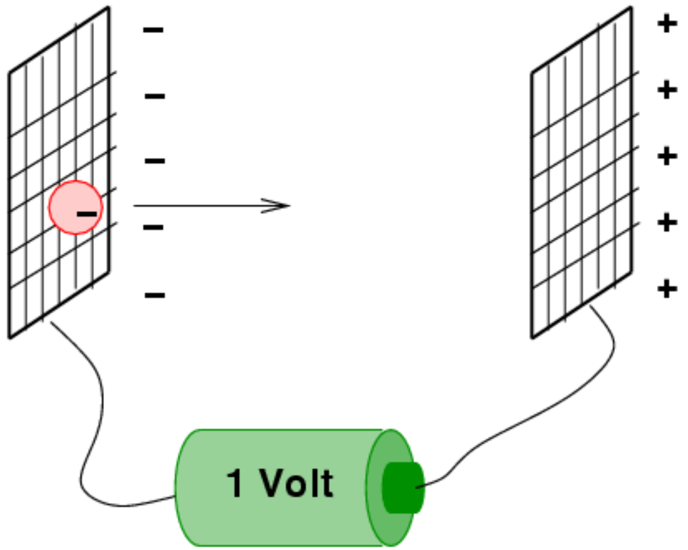
We can write: $\beta = \frac{mvc}{mc^2}$

Momentum is: $p = mv$

$$\beta = \frac{pc}{E} \quad \Leftrightarrow \quad p = \frac{E\beta}{c}$$

The Units

- The energy acquired by an electron in a potential of 1 Volts is defined as being 1 eV
- Hence 1 eV = 1.6 x 10^{-19} Joules
- Units:
  - Energy: eV
  - Momentum: eV/c
  - Mass: eV/c^2
- The unit eV is too small to be used today, we use:
  - 1 KeV = 10^3, MeV = 10^6, GeV = 10^9, TeV = 10^{12}



$$p = \frac{E\beta}{c}$$

when $\beta = 1$: value for energy [eV] and momentum [eV/c] are equal
when $\beta < 1$: value for energy [eV] and momentum [eV/c] are not equal

Q1: The LHC Beam Stored Energy

- The LHC runs with 2556 bunches per beam
- Each bunch is populated with 1.15×10^{11} protons
- The center of mass energy during collisions is 14 TeV
- Both beams have the same $B\rho$

- What is the total stored energy per beam at the start of the squeeze ?

- Could you come up with a more known example that represents the same stored energy ?

Q1a: The LHC Beam Stored Energy

- The LHC runs with 2556 bunches per beam
- Each bunch is populated with 1.15×10^{11} protons
- The center of mass energy during collisions is 14 TeV
- Both beams have the same $B\rho$

- What is the total stored energy per beam at the start of the squeeze ?

  - The Squeeze is on the high energy flat top where the beams are at 14 TeV center of mass, which is 7 TeV per beam if $B\rho$ for the two beams is identical

  - $1\text{eV} = 1.6 \times 10^{-19}$ Joules

  - 2556 bunches of 1.15×10^{11} protons each is 2.94×10^{14} protons per beam

  - $E_{stored} = 2.94 \times 10^{14} \times 1.6 \times 10^{-19} \times 7 \times 10^{12} = 330$ MJoules

Q1b: The LHC Beam Stored Energy

- LHC stored beam energy = 330 MJoules
- Could you come up with a more known example that represents the same stored energy ?
  - TGV train weight = 380000 kg
  - Velocity of 150 km/h corresponds to 41.67 m/s
  - E_{stored} = 380000 x 41.67^2 = 330 Mjoules

…but then concentrated in the size of a needle

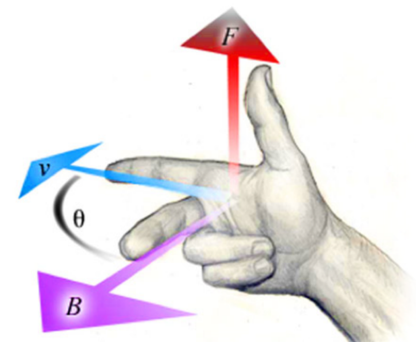
Transverse Motion

Lorentz Force

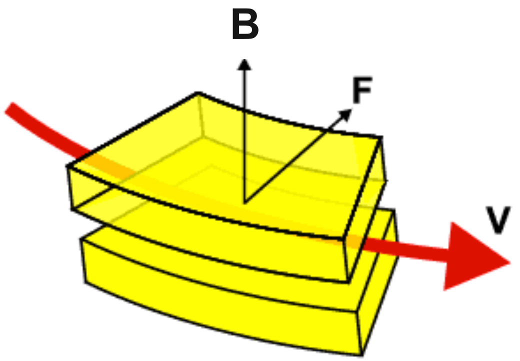
- Lorentz Formula: $\vec{F} = e\vec{E} + e\vec{v}\times\vec{B}$

- **Transverse motion** in accelerators is dominated by **magnetic forces**:

$$\vec{F} = e\vec{v}\times\vec{B}$$



Right hand rule for + charge



Moving electron in a dipole field

- **Radius of curvature in the magnet**
- Linear motion before and after

Magnetic Rigidity

- The **Lorentz Force** can be seen as a **Centripetal Force**

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{m\vec{v}^2}{\rho}$$

- ρ is the particle's **radius of curvature** in the magnetic field

$$B\rho = \frac{mv}{q} = \frac{p}{q}$$

- ***Bρ*** is the **magnetic rigidity**

$$B\rho[\mathrm{Tm}] = \frac{mv}{q} = \frac{p[GeV/c]}{q} \Rightarrow B\rho = 3.3356\,p$$

If in an accelerator all magnetic fields are scaled with the momentum during acceleration of the particles, the trajectories remain the same

Q2: Building the Next Particle Collider

- Two options:
  - Use the existing LHC tunnel and replace existing magnets with high field superconducting magnets (20 T)
    - What centre of mass energy could we reach ?
  - Build a new tunnel and use the 20 T magnets to reach a centre of mass energy of 100 TeV
    - What would the circumference of the tunnel be if only 66% of the space can be allocated to dipoles ?
- Some input:
  - LHC: C= 26658.883 m, r = 4242.9 m, ρ = 2804 m
  - General: Max. dipole length is 15 m for transport reasons

Q2a: High Energy LHC

- Use the **existing LHC tunnel** and replace existing magnets with **high field superconducting magnets**
- Beam rigidity:

$$B\rho = 3.3356\, p$$

- **ρ = 2804 m** (fixed by tunnel geometry and filling factor)
- Vigorous R&D for **20 T dipole magnets is** on-going (Nb_3SN and HTS)

$$p = \frac{2804 \times 20}{3.3356} \Rightarrow \text{~16.5 TeV per beam} \Rightarrow \mathbf{33\ TeV_{cm}}$$

Q2b: Future Circular Collider

- 100 TeV center of mass is 50 TeV per beam

$$B\rho = 3.3356\,p$$

- The radius of curvature: $\rho = \dfrac{3.3356 \times 50{\times}10^3}{20} = 8339\,m$

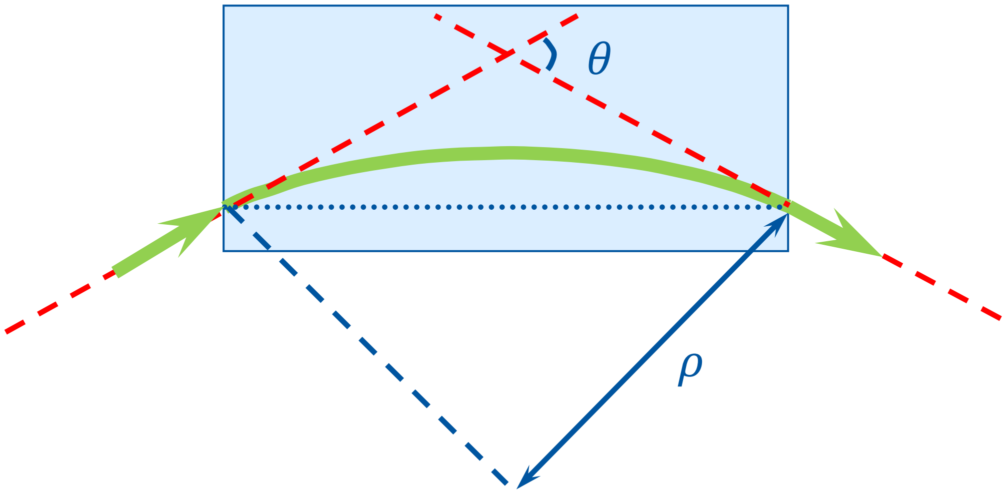
- 66% filling factor: $r = \dfrac{\rho}{filling\ factor} = \dfrac{8339}{0.66} = 12635\,m$

- Circumference: $C = 2\pi r = 2\pi \times 12635 = 79388\,m$

- The 100 TeV collider with 20 T dipole magnets would result in a circumference of 80 km

This Lens Approximation

- If the path length through a transverse magnetic field is short compared to the bend radius of the particle, then we can think of the particle receiving a transverse "kick", which is proportional to the integrated field

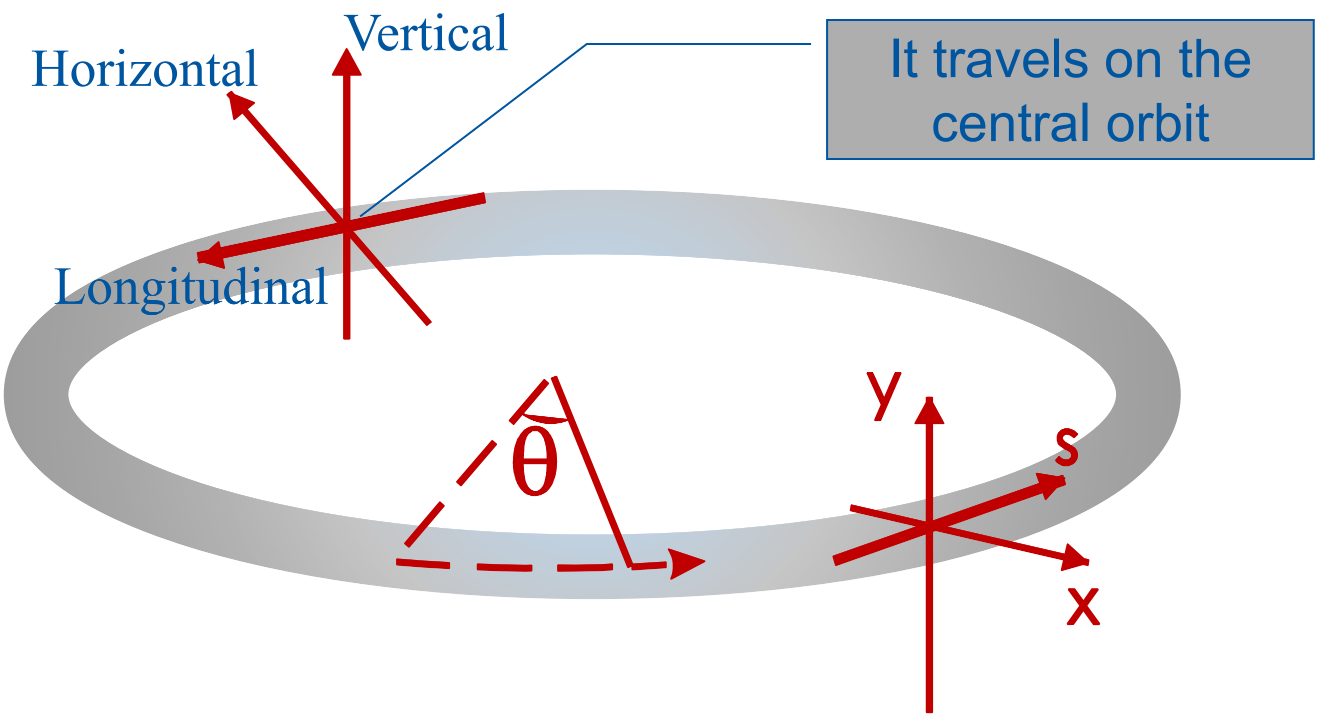


- Since the angles in magnets are small, thin lens approximation is widely used for transverse optics calculations, including in simulations codes such as MADX, SixTrack, etc.

Radii & Small Angles

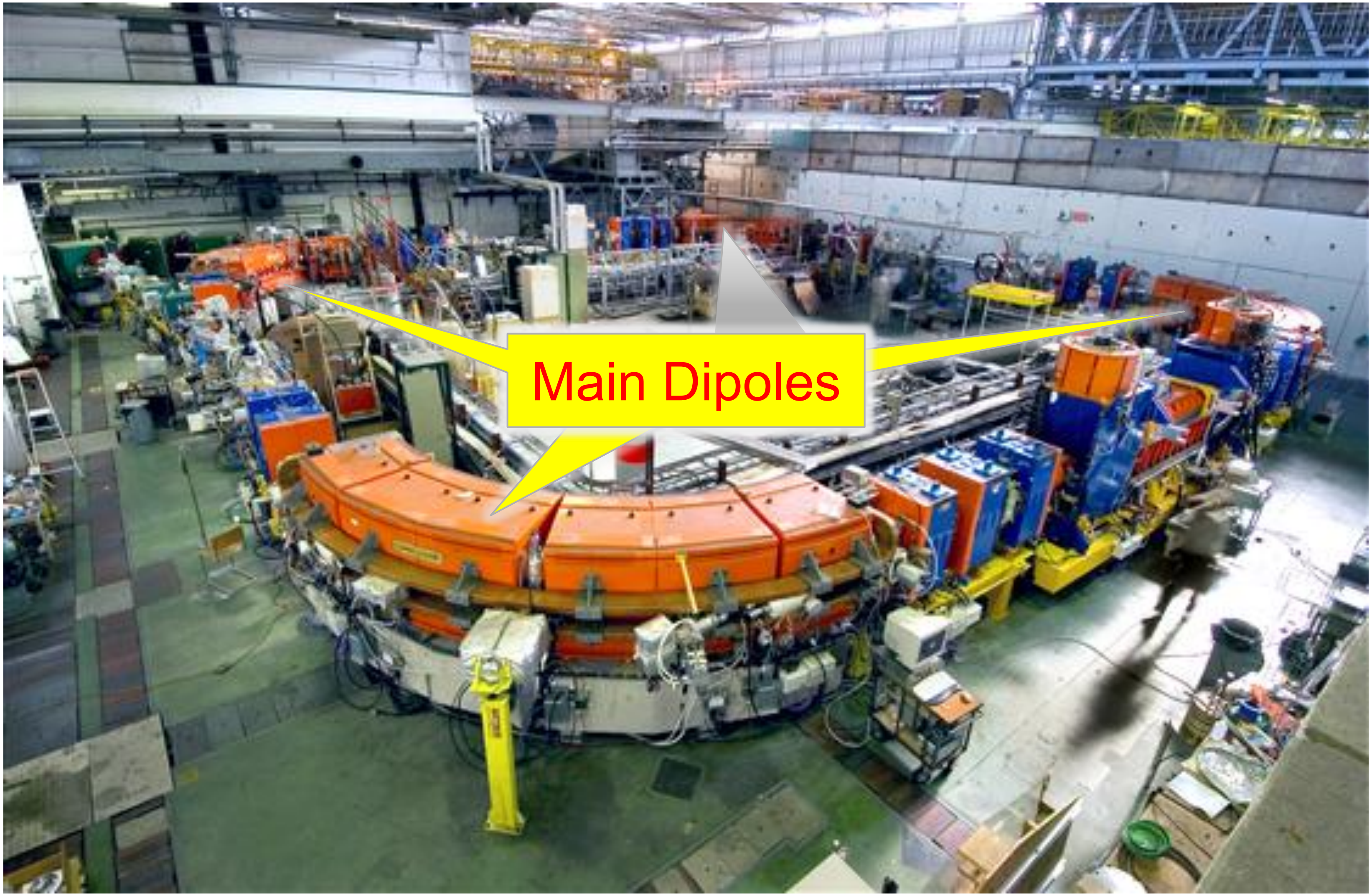
- LHC circumference = 26658.883 m
  - Therefore the radius r = 4242.9 m
- There are 1232 main dipoles to make 360˚
  - This means that each dipole deviates the beam by only 0.29˚
- The dipole length = 14.3 m
  - The total dipole length is thus 17617.6 m, which occupies 66.09 % of the total circumference
- The bending radius ρ is therefore
  - $\rho = 0.6609 \times 4242.9$ m → $\rho =$ 2804 m
- Apart from dipole magnets there are also straight sections in our collider
  - These are used to house RF cavities, diagnostics equipment, special magnets for injection, extraction etc.

Coordinate System

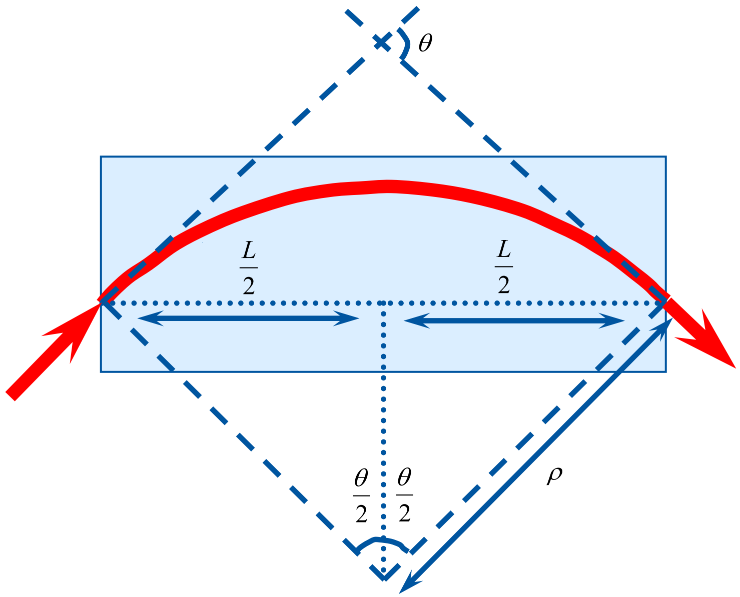


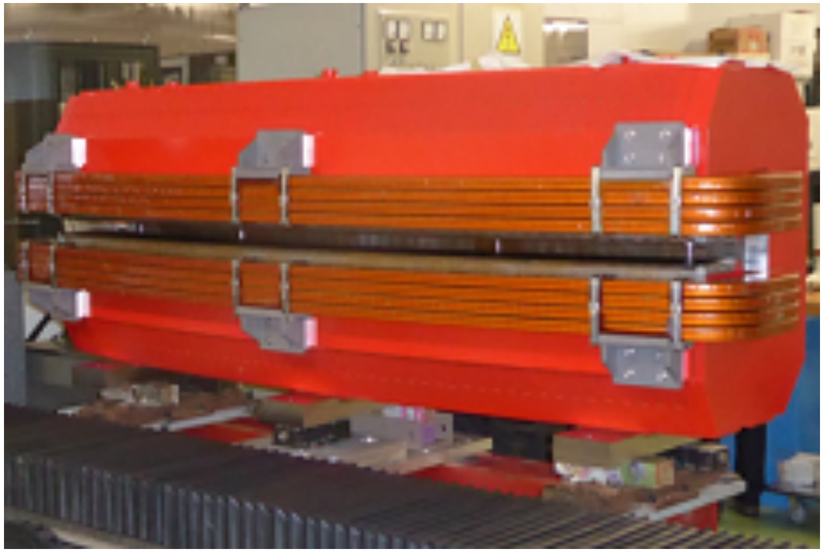
- We can speak of a: **Rotating Cartesian Co-ordinate System**

Make Particles Circulate



Dipole Magnet



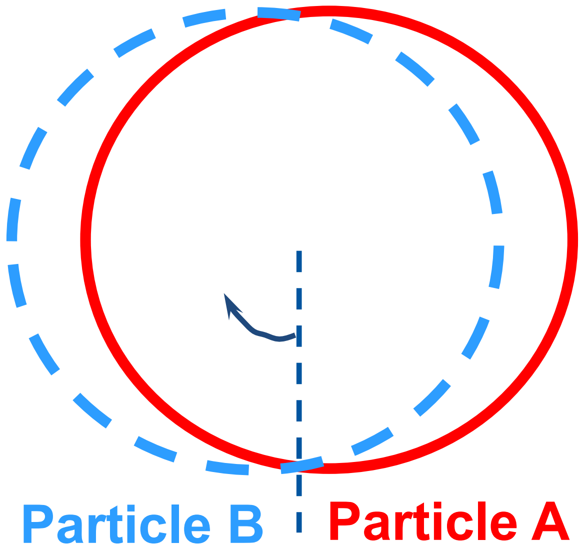


- A magnet with a uniform dipolar field deviates a particle by an angle θ in one plane
- The angle θ depends on the length L and the magnetic field B.

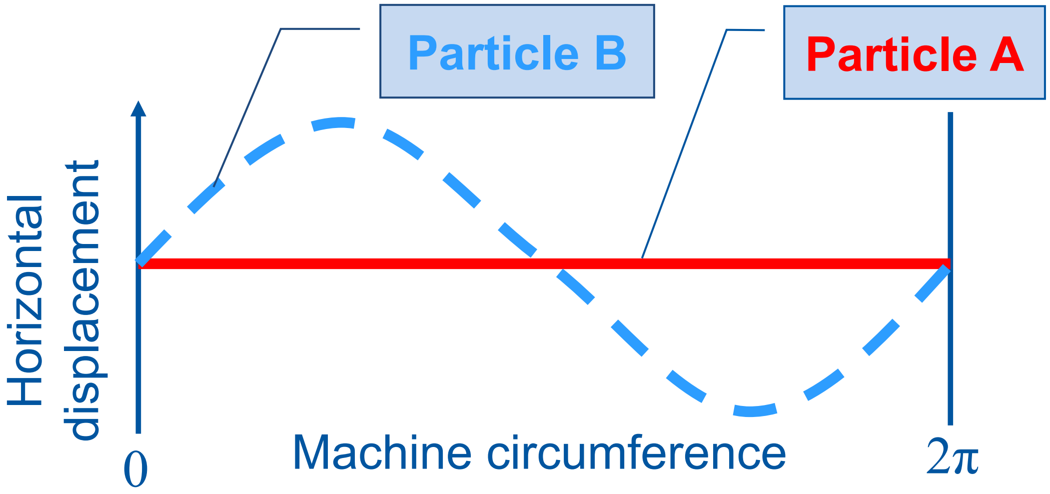
$$\sin\left(\frac{\theta}{2}\right) = \frac{L}{2\rho} = \frac{1}{2}\frac{LB}{(B\rho)} \quad \Rightarrow \quad \sin\left(\frac{\theta}{2}\right) = \frac{\theta}{2} \quad \Rightarrow \quad \theta = \frac{LB}{(B\rho)}$$

Oscillatory Motion of Particles

Two charged Particles in a homogeneous magnetic field



Horizontal motion

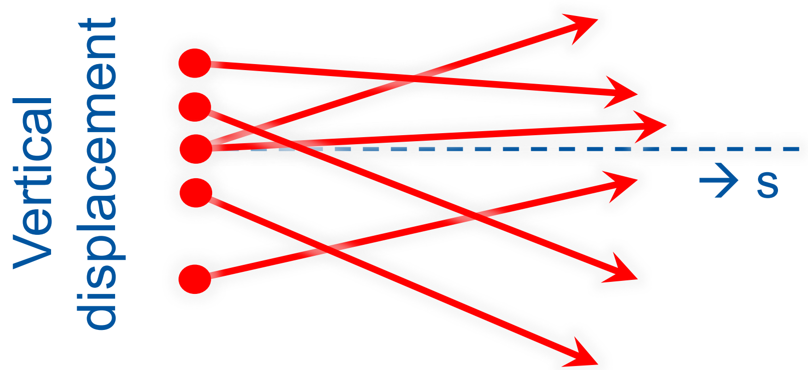


Different particles with different initial conditions in a homogeneous magnetic field will cause oscillatory motion in the horizontal plane → **<u>Betatron Oscillations</u>**

Oscillatory Motion of Particles

The horizontal motion seems to be “stable”…. What about the vertical plane ?

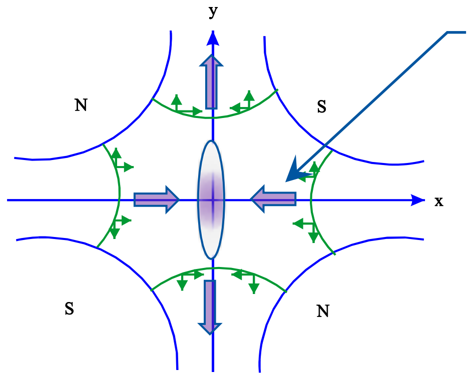
Many particles many initial conditions



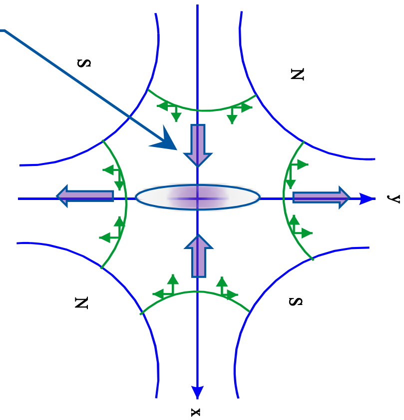
Many different angles

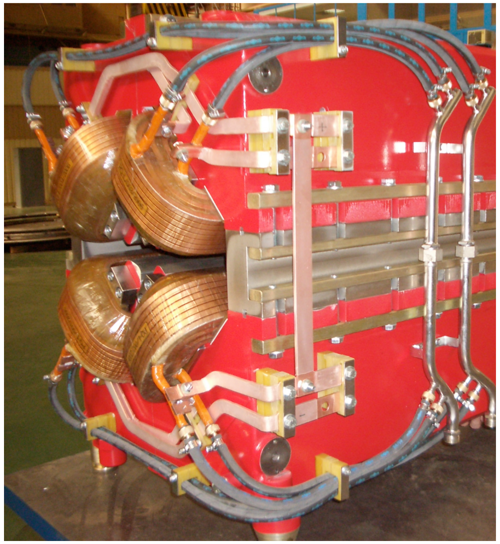
Focusing Particle Beams

Focusing Quadrupole	De-focusing Quadrupole

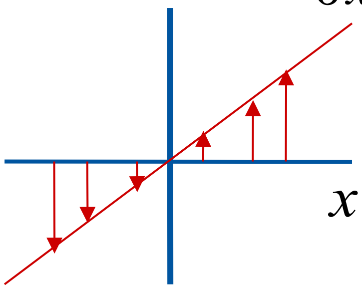


Force on particles

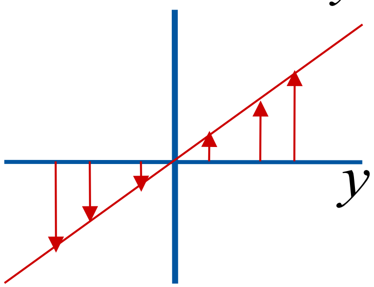




$$B_y = \frac{\partial B_y}{\partial x} x$$



$$B_x = \frac{\partial B_x}{\partial y} y$$



Field gradient : $K = \frac{\partial B_y}{\partial x} \, [\mathrm{Tm^{-1}}]$

Normalised gradient : $k = \frac{K}{B\rho} \, [\mathrm{m^{-2}}]$

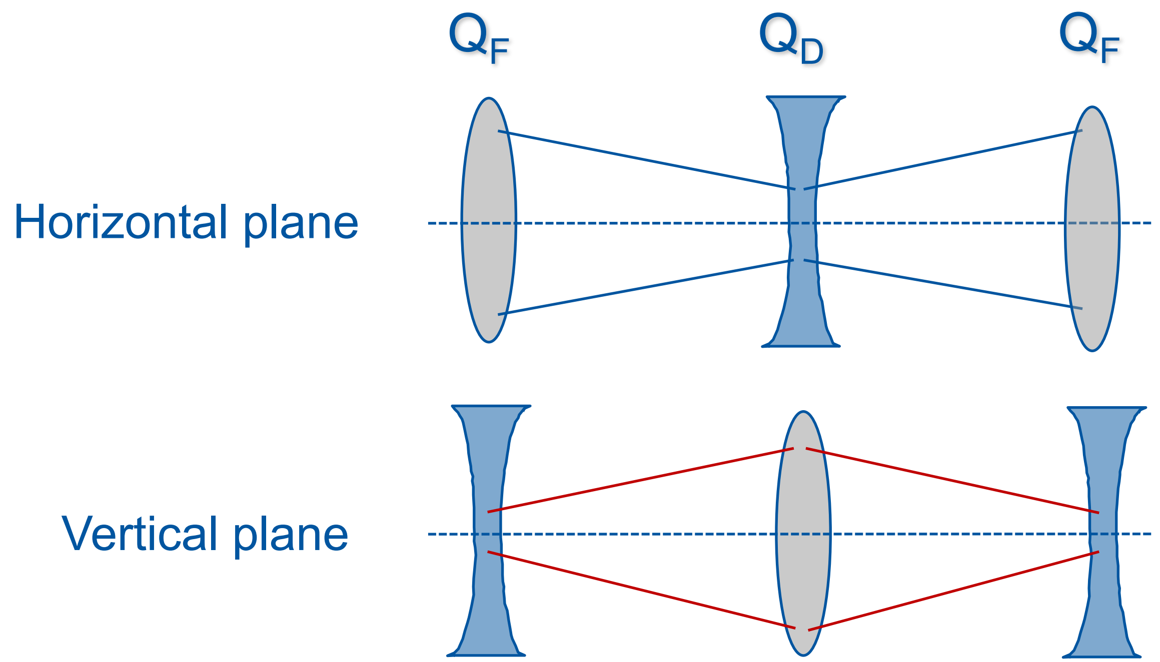
Restoring 'kick' : $\Delta\theta = -\frac{K}{B\rho} l \, x \, [\mathrm{rad}]$

FODO Cell

- Using a combination of **focusing (Q_F)** and **defocusing (Q_D) quadrupoles** solves our problem
  - It will keep the beams focused in **both planes** when the position in the accelerator, type, strength and length of the quadrupoles are well chosen.
- By now our "virtual" accelerator is composed of:
  - **Dipoles**, constrain the beam to some closed path (orbit)
  - **Focusing and Defocusing Quadrupoles**, provide horizontal and vertical focusing in order to constrain the beam in transverse directions
- A combination of focusing and defocusing sections that is very often used is the so called: **FODO lattice**
  - A configuration of magnets where focusing and defocusing magnets are alternated and are separated by non-focusing drift spaces

FODO Lattice

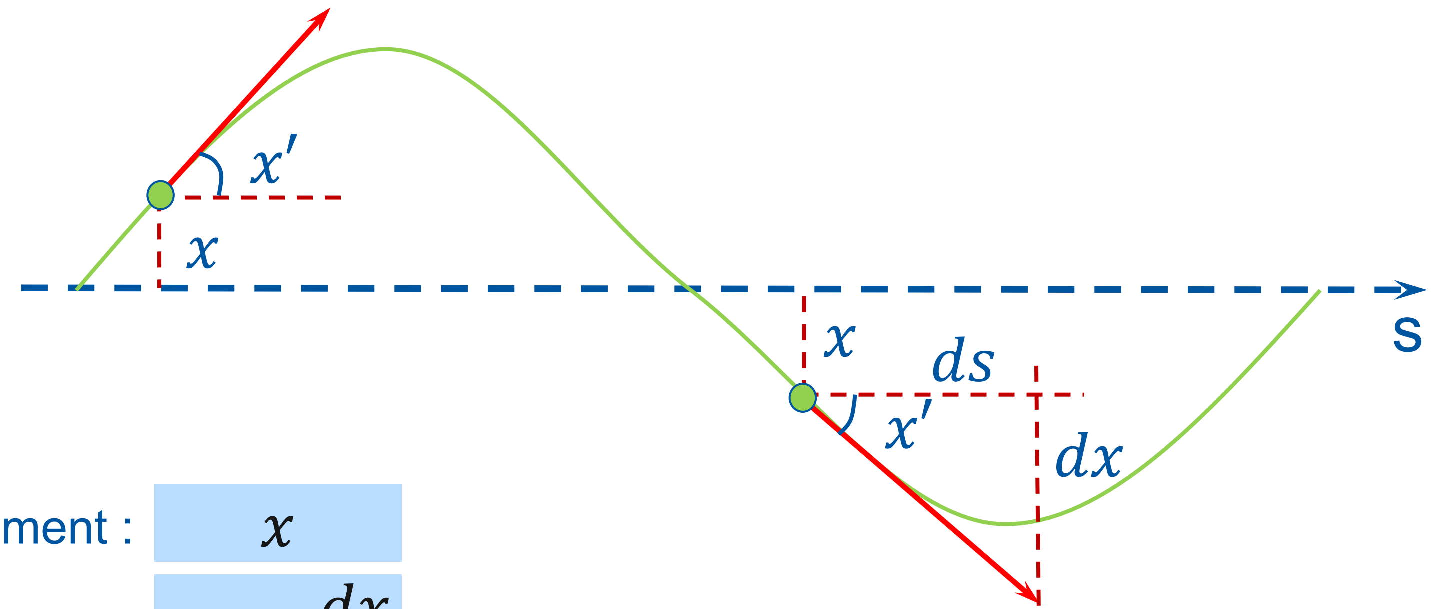
A quadrupole is defined focusing if it is oriented to focus in the horizontal plane and defocusing if it defocusses in the horizontal plane



This arrangement gives rise to **Betatron oscillations** within an envelope

Betatron Oscillations in Accelerators

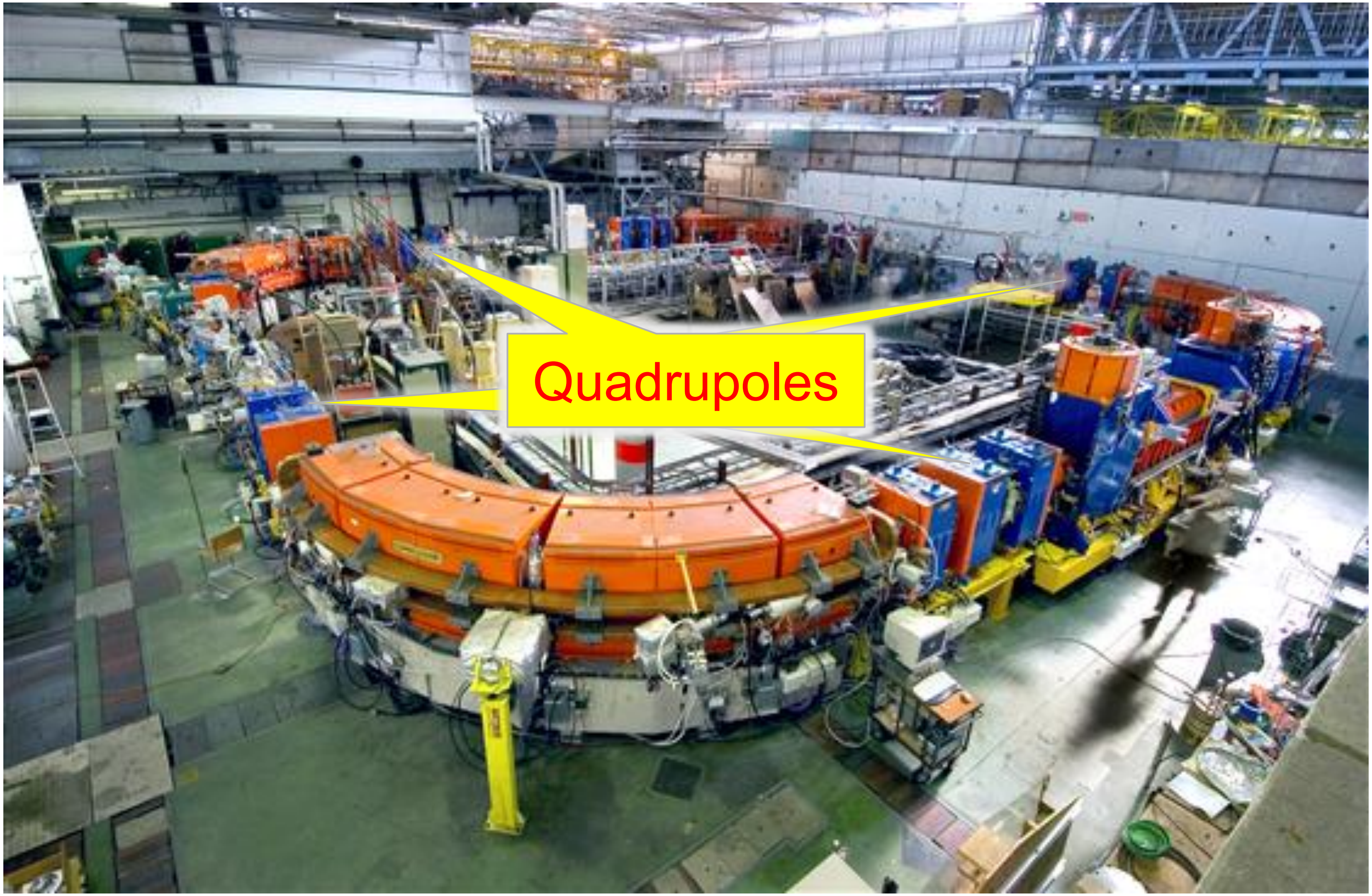
- Under the influence of the **magnetic fields** the **particle oscillate**



Displacement : x

Angle : $x' = \frac{dx}{ds}$

Focusing the Particle Beam



Matrix Formalism

- As the particle moves around the machine the values for x and x' will vary under influence of the dipoles, quadrupoles and drift spaces

- These modifications due to the different types of magnets can be expressed by a **Transport Matrix M**

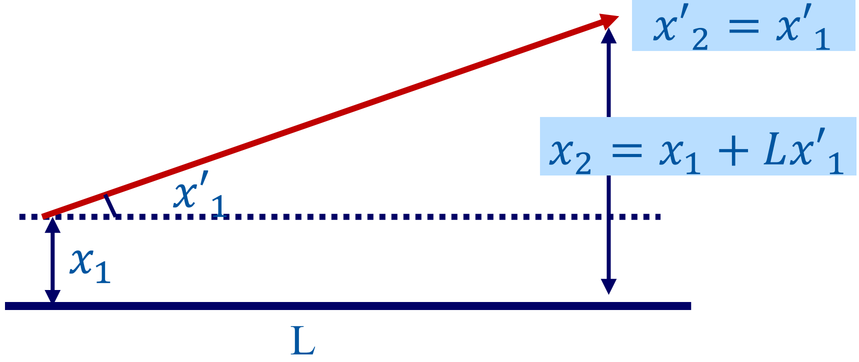
- If we know x_1 and x_1' at some point s_1 then we can calculate its position and angle after the next magnet at position s_2 using:

$$\begin{pmatrix} x(s_2) \\ x(s_2)' \end{pmatrix} = M \begin{pmatrix} x(s_1) \\ x(s_1)' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x(s_1) \\ x(s_1)' \end{pmatrix}$$

Transfer Matrices

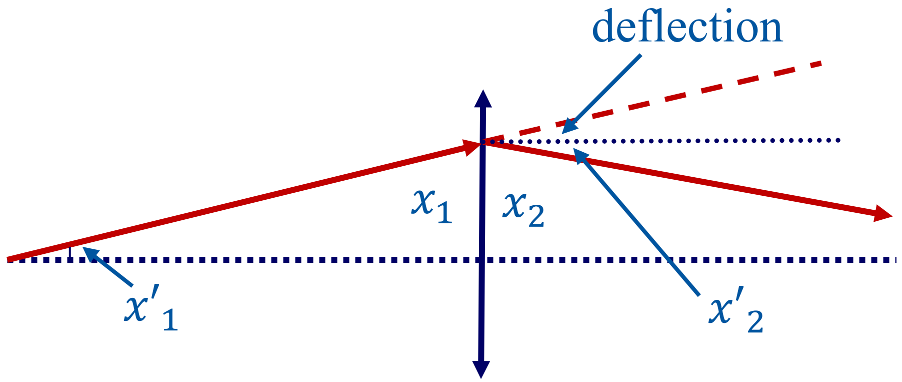
- Drift Space:

$$\begin{bmatrix} x_2 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix}$$



- Quadrupoles (thin lens):

$$\begin{bmatrix} x_2 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix}$$



Remember: $B_y \propto x$

Deflection: $\Delta\theta = -\frac{K}{B\rho} L\, x = -\frac{1}{f} x$

$x_2 = x_1$

$x'_2 = -\frac{1}{f} x_1 + x'_1$

Hill's Equation

- **Betatron oscillations** exist in both **horizontal** and **vertical** planes
- The **number of betatron oscillations per turn** is called the **betatron tune** and is defined as Q_x and Q_y
- Hill's equation describes this motion mathematically

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

- If the restoring force, K is constant in 's' then this is just a **Simple Harmonic Motion** (Like a pendulum)
- 's' is the longitudinal displacement around the accelerator

General Solutions of Hill's Equation

Position: $$x(s) = \sqrt{\varepsilon\beta_s}\cos(\varphi(s) + \varphi)$$

Angle: $$x' = -\alpha\sqrt{\varepsilon/\beta}\cos(\varphi) - \sqrt{\varepsilon/\beta}\,\sin(\varphi)\varphi$$

- ε and φ are constants determined by the initial conditions
- β(s) is the periodic envelope function given by the lattice configuration

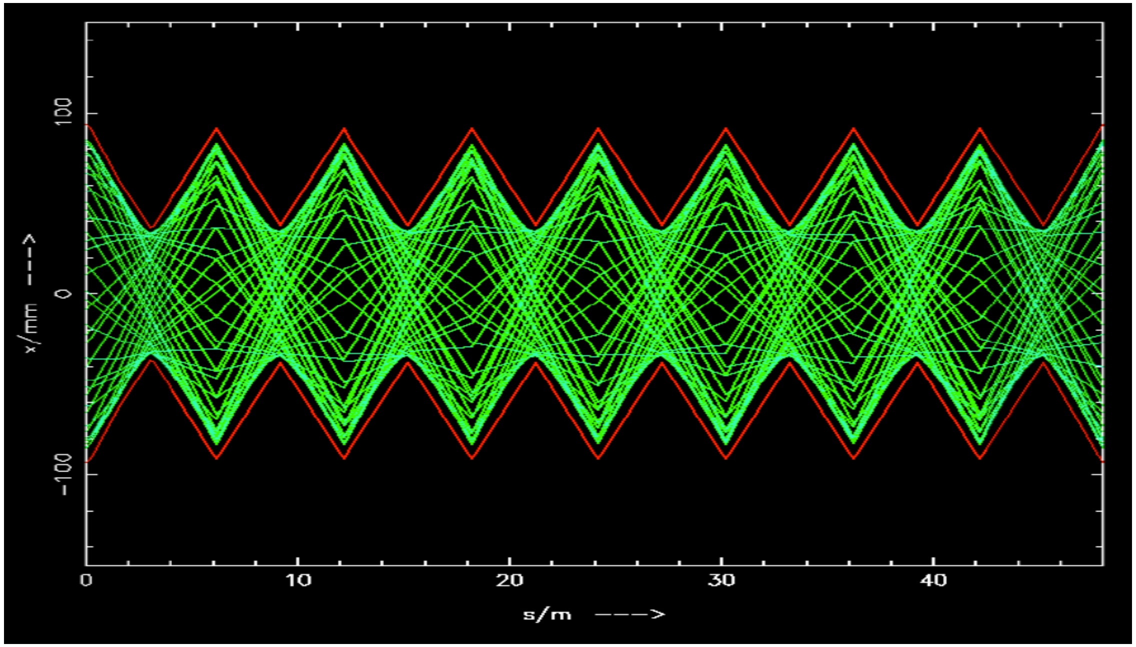
$$\varphi(s) = \int_0^s \frac{ds}{\beta(s)}$$

- φ(s) Is the phase advance over 1 turn around the machine

$$Q_{x/y} = \frac{1}{2\pi}\int_0^{2\pi} \frac{ds}{\beta_{x/y}(s)}$$

- Q_x and Q_y are the horizontal and vertical tune: the number of oscillations per turn around the machine

β function and individual particles



- The β function is the envelope function within which all particles oscillate
- The shape of the β function is determined by the lattice

Matrix Formalism and Hill's Equation

$$x = \sqrt{\varepsilon.\beta}\cos(\mu+\phi)$$

$$x = \sqrt{\varepsilon.\beta}\cos\phi$$

$$\begin{pmatrix} x(s_2) \\ x'(s_2) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x(s_1) \\ x'(s_1) \end{pmatrix}$$

$$x' = -\alpha\sqrt{\varepsilon/\beta}\cos(\mu+\phi) - \sqrt{\varepsilon/\beta}\sin(\mu+\phi)$$

$$x' = -\alpha\sqrt{\varepsilon/\beta}\cos\phi - \sqrt{\varepsilon/\beta}\sin\phi$$

- Assume that our transport matrix describes a complete turn around the machine.
- Therefore : $\beta(s_2) = \beta(s_1)$
- Let μ be the change in betatron phase over one complete turn.
- Then we get for $x(s_2)$:

$$x(s_2) = \sqrt{\varepsilon.\beta}\cos(\mu+\phi) = a\sqrt{\varepsilon.\beta}\cos\phi - b\alpha\sqrt{\varepsilon/\beta}\cos\phi - b\sqrt{\varepsilon/\beta}\sin\phi$$

Matrices & Twiss Parameters

- Define Twiss parameter:

$$\alpha = -\beta'/2 = -\omega\omega'$$

$$\beta = \omega^2$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

- Remember also that μ is the total betatron phase advance over one complete turn is

$$Q = \frac{\mu}{2\pi}$$

Number of betatron oscillations per turn

✓ Our transport matrix becomes now:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

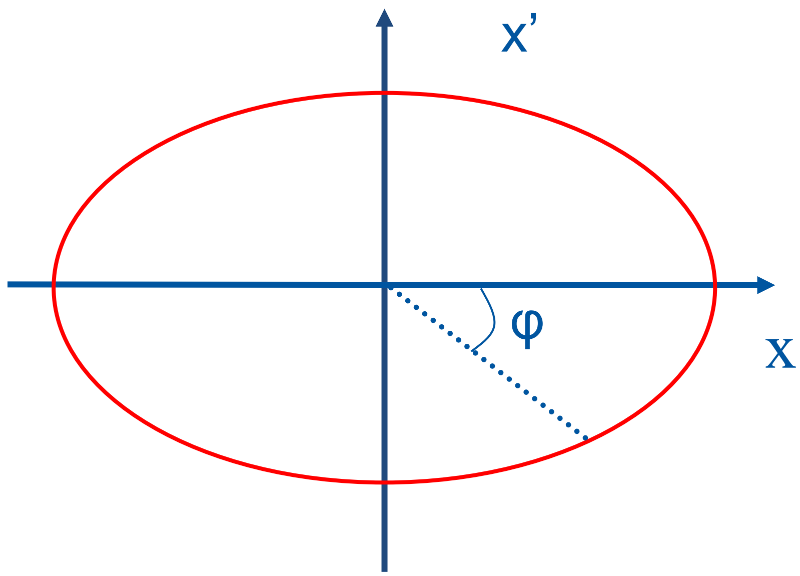
Lattice Parameters

$$\begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

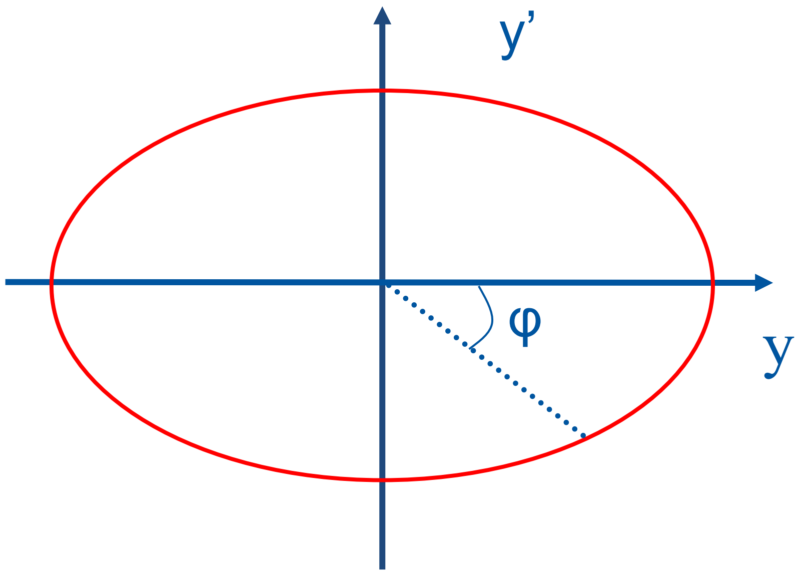
- This matrix describes one complete turn around our machine and will vary depending on the starting point (s)
- If we start at any point and multiply all of the matrices representing each element all around the machine we can calculate α, β, γ and μ for that specific point, which then will give us β(s) and Q
- If we repeat this many times for many different initial positions (s) we can calculate our Lattice Parameters for all points around the machine

Transverse Phase Space Plot

We distinguish motion in the Horizontal & Vertical Plane

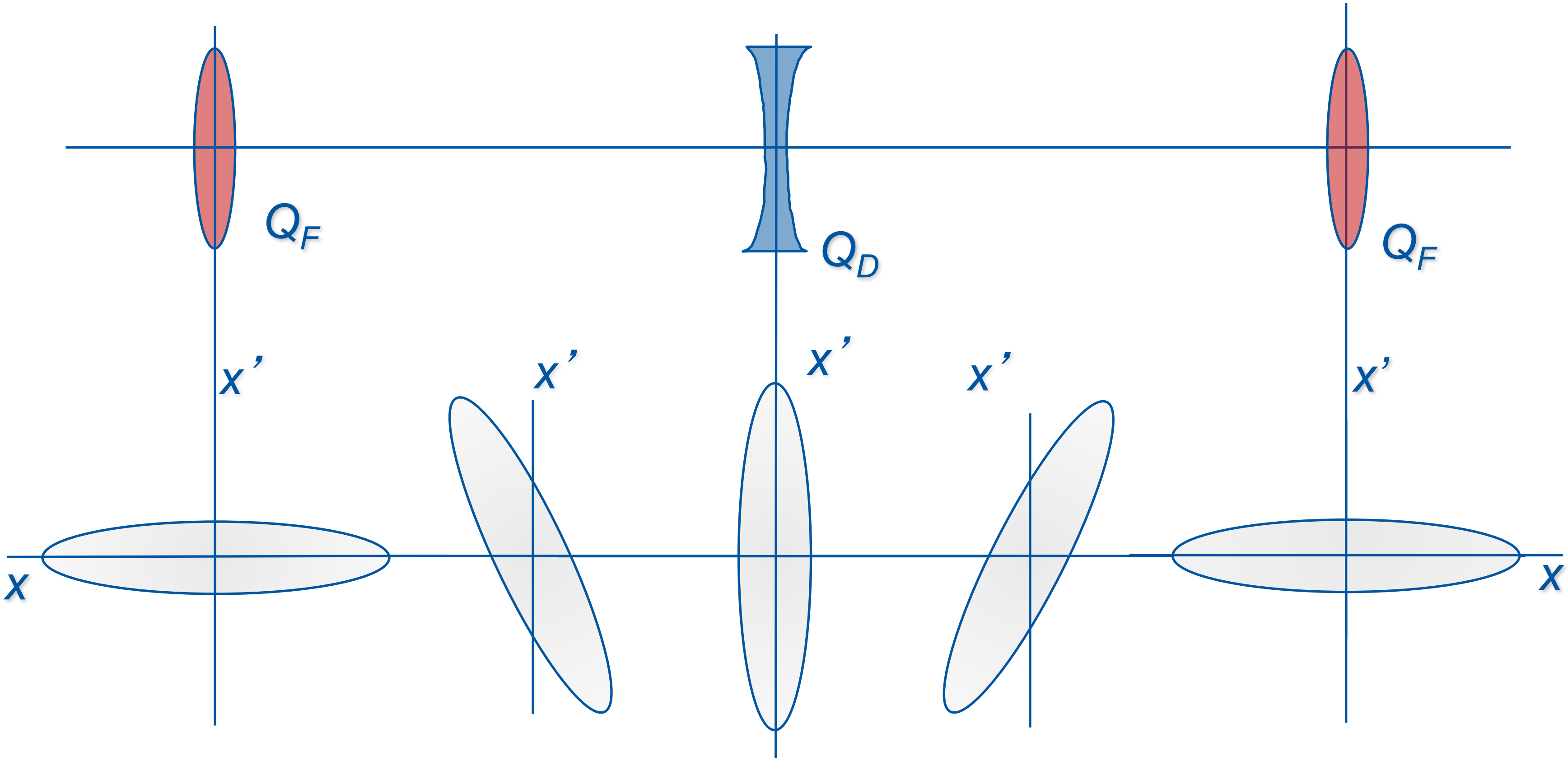


Horizontal Phase Space



Vertical Phase Space

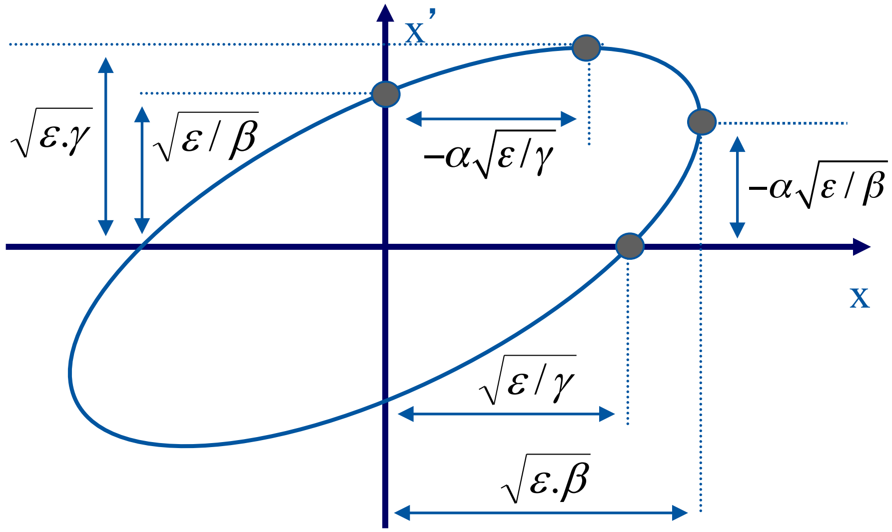
Phase Space Elipse Rotation



For each point along the machine the ellipse has a particular orientation, but the area remains the same

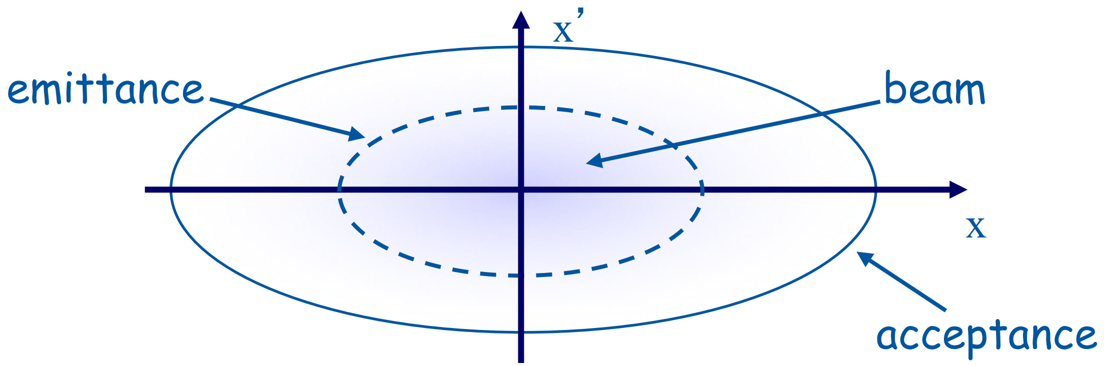
Transverse Phase Space Ellipse

- A tune Q_h = 3.5 means 3.5 horizontal betatron oscillations per turn around the machine, hence 3.5 turns on the phase space ellipse
- This means a total phase advance of 3.5 π
- Each particle, depending on it's initial conditions will turn on it's own ellipse in phase space



Transverse Emittance

- Observe all the particles at a single position on one turn and measure both their position and angle
- This will give a large number of points in our phase space plot, each point representing a particle with its co-ordinates x, x'



Symbol: ε_h or ε_v
Expressed in 1σ, 2σ,..
Units: mm mrad

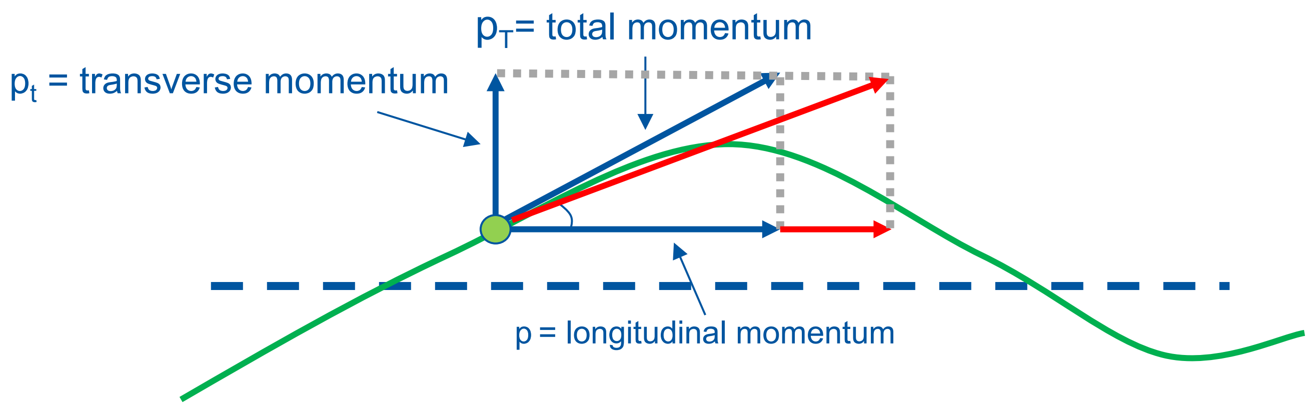
- The **emittance** is the **area** of the ellipse, which contains all, or a defined percentage, of the particles
- The **acceptance** is the maximum **area** of the ellipse, which the emittance can reach without losing particles

Adiabatic Damping of Beam Size

- For the Gaussian definition emittance the rms beam size is:

$$\sigma_x = \sqrt{\beta_x \varepsilon} \qquad \sigma_y = \sqrt{\beta_y \varepsilon}$$

- The emittance is constant at constant energy, but accelerating particles will decrease the emittance, which is called adiabatic damping



- To be able to compare emittances at different energies it is normalised to become invariant, provided the is no blow up

$$\varepsilon_x^n = \beta\gamma\varepsilon_x \qquad \varepsilon_y^n = \beta\gamma\varepsilon_y$$

Friday more …..

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