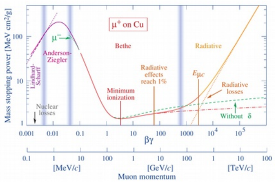

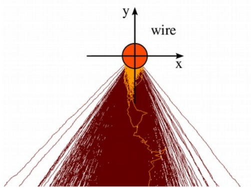

Tracking – 2
Basic Principles of Detectors

Jochen Kaminski
University of Bonn

BND – summer school
Callantsoog, Netherlands
4.-6.9.2017

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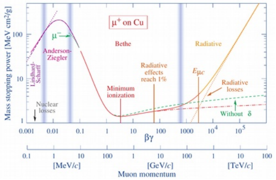
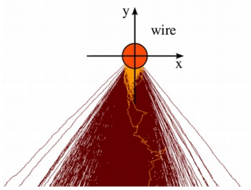


Overview Course



Lecture 1: Tracking - Basics and Reconstruction

Lecture 2: Basic Principle of Detectors
- Energy Loss
- Drift and Diffusion
- Gas Amplification
- Signal Generation

Lecture 3: Gaseous Detectors

Lecture 4: Semiconductor Detectors

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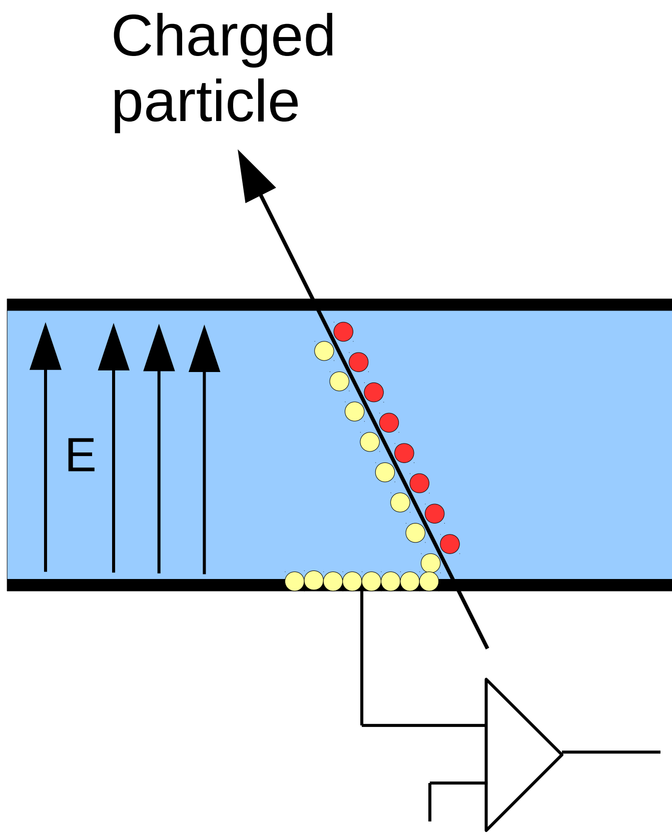
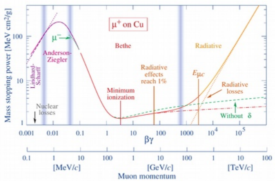
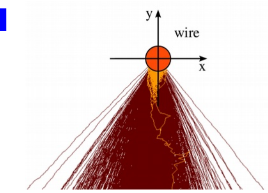
Detection Principles of Charged Particles



- Particle passing the detector volume
- Ionizing the gas atoms along its path
- If an electric field is applied the charges are separated
- They drift towards the electrodes
- The signal is amplified and readout by the electronics



Detection Principle of Neutral Particles



Convert particles into charged particles by some specific processes:

- Photons: depending on energy there are different processes
  Low energy: photoelectric effect - e^- are hit from central shells of atoms.
  Medium energy: Compton effect – photon scatters with quasi free e^-
  High energy: Pair production – photon 'decays' into e^+e^-

- Neutrons:
  Slow neutrons have high cross section for nuclear reactions inducing α/p-emission, e.g. $^3He+n \rightarrow {}^3H+{}^1H$ or $^{10}B + n \rightarrow \alpha + Li$
  Fast neutrons can kick out low Z nuclei (e.g. H) from material

- Neutrino:
  'Inverse β-decay' converting a nucleus and ejecting an e^-

- π^0: decay in two photons

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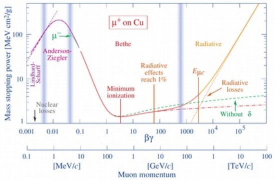
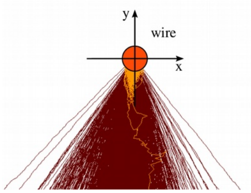


Energy Loss of Heavy Charged Particles

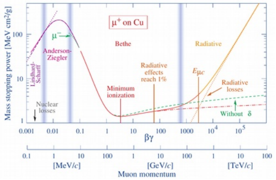


Heavy charged particles are not stopped by a single or a few interactions, but they loose energy quasi continuously by many interactions.

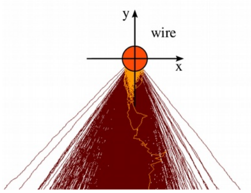
Interactions that take place are mostly based on electromagnetic interactions:

- Inelastic collisions with e^- of atomic shells: excitation and ionization
- Bremsstrahlung
- Elastic scattering off nuclei
- Cerenkov and transition radiation
- Nuclear reactions (only non-electromagnetic interaction!)

Energy loss is described by Bethe-Bloch formula.
-dE/dx is also called stopping power.



Derivation of Bethe-Bloch-Formula



1913: first calculation of *dE/dx* by Bohr using a classical picture
1930/1933: quantum-mechanical calculation by Bethe and Bloch
Later: Improved and refined calculation in particular of low/high energies

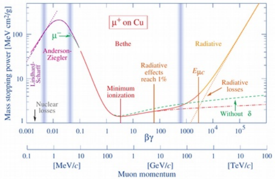
Energy loss occurs by single stochastic interactions of the incident particle with atoms.
It depends on the material type *A*, mass *M* and velocity β of the particle.

$$-\left\langle \frac{dE}{dx} \right\rangle = n_e \cdot \int_{T_{min}}^{T_{max}} T \frac{d\sigma_A(M,\beta,T)}{dT} dT$$

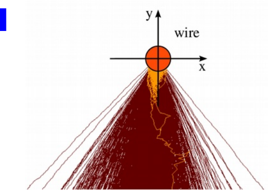
Density of electrons

Integral over range of possible energy transfers

Differential cross section for loss of kinetic energy *T* weighted by *T*



Calculation of Cross Section (I)



Considering only elastic em. interactions with a particle quasi free and at rest (i.e. $v_M >> v_e$, $\Delta T >> T_{bind}$, $M >> m_e$) → Rutherford cross section

$$\frac{d\sigma}{dQ^2} = \frac{4\pi z^2 \alpha^2 \hbar^2 c^2}{\beta^2} \frac{1}{Q^4}$$

Using the collision kinematics, one can transform the momentum transfer into the energy transfer: $Q^2 = -(p_e - p_e')^2 = 2 m_e c^2 T$.

$$\frac{d\sigma}{dT} = \frac{2\pi z^2 \alpha^2 \hbar^2}{\beta^2 m_e} \frac{1}{T^2}$$

Rutherford cross section holds only for two particles with spin 0. But the electron has spin ½: → Mott cross section

$$\frac{d\sigma}{dT} = \frac{2\pi z^2 \alpha^2 \hbar^2}{\beta^2 m_e T^2} \left(1 - \beta^2 \frac{T}{T_{max}}\right)$$

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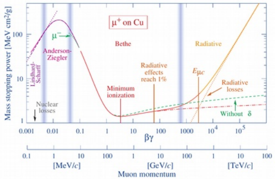
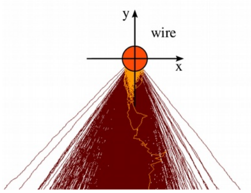


Calculation of Cross Section (II)



Doing the integration from T_{min} to T_{max}:

$$-\left\langle \frac{dE}{dx} \right\rangle = n_e \cdot \int_{T_{min}}^{T_{max}} T \frac{2\pi z^2 \alpha^2 \hbar^2}{\beta^2 m_e T^2} \left(1 - \beta^2 \frac{T}{T_{max}}\right) dT = \frac{2\pi z^2 \alpha^2 \hbar^2 n_e}{\beta^2 m_e} \cdot \int_{T_{min}}^{T_{max}} \left(\frac{1}{T} - \beta^2 \frac{1}{T_{max}}\right) dT =$$

$$= \frac{2\pi z^2 \alpha^2 \hbar^2 n_e}{\beta^2 m_e} \cdot \left(\ln T - \beta^2 \frac{T}{T_{max}}\right)\Bigg|_{T_{min}}^{T_{max}} = \frac{2\pi z^2 \alpha^2 \hbar^2 n_e}{\beta^2 m_e} \cdot \left(\ln T_{max} - \ln T_{min} - \beta^2 \frac{(T_{max} - T_{min})}{T_{max}}\right)$$

with $T_{max} - T_{min} \approx T_{max}$ and and the electron density $n_e = Z\rho N_A / A$

$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{2\pi z^2 \alpha^2 \hbar^2 Z \rho N_A}{\beta^2 m_e A} \cdot \left(\ln \frac{T_{max}}{T_{min}} - \beta^2\right)$$

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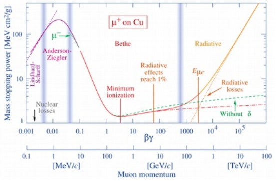
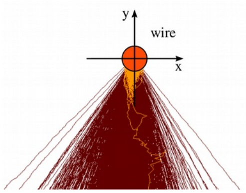


T_{min} and T_{max}

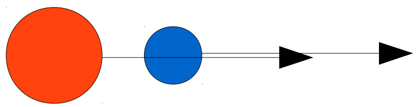


T_{max} is transferred only in a head on collision

Before collision

$p = \beta\gamma Mc,\ E = \gamma Mc^2$

After collision



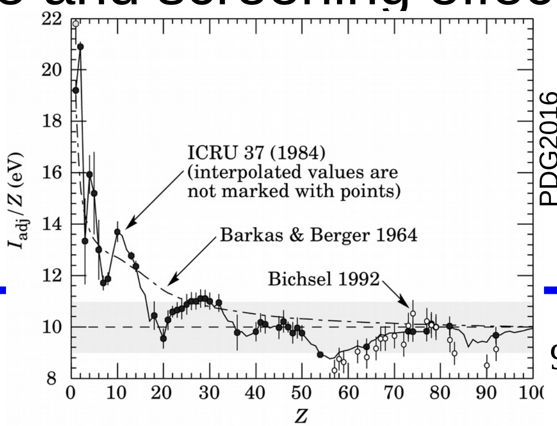
$T = E_e' - m_e c^2$

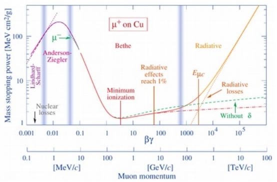
$$T_{max} = \frac{2m_e c^2 \beta^2\gamma^2}{1 + 2\gamma\frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2} \approx \begin{cases} 2m_e c^2 (\beta\gamma)^2 & \text{for} \quad \gamma m_e \ll M \\ \gamma M c^2 = E & \text{for} \quad \gamma \to \infty \\ m_e c^2 (\gamma - 1) = E - m_e c^2 & \text{for} \quad M = m_e \end{cases}$$

T_{min} is difficult to calculate, because it consists mostly of quantum mechanical transitions in the atoms, also interference and screening effects exist.

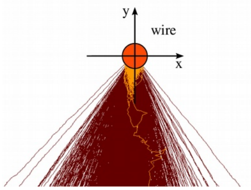
$$T_{min} = \frac{I}{2m_e c^2 \beta^2\gamma^2}$$

I = mean excitation potential





Final Form of Bethe-Bloch-Formula



$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{4\pi N_A \alpha^2 \hbar^2}{m_e} \frac{Z}{A}\rho \frac{z^2}{\beta^2} \cdot \left(\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C(\beta\gamma, I)}{Z} \right)$$

Energy loss
(in MeV/cm)

Universal constants:
K=0.307 MeV cm^2/mol

Properties of
material

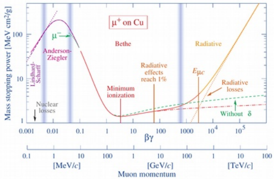
Properties of
incoming particle

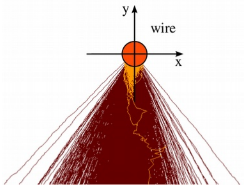
Fast drop
with $1/\beta^2$
at low γ

Slow rise
with ln(γ)
for high γ

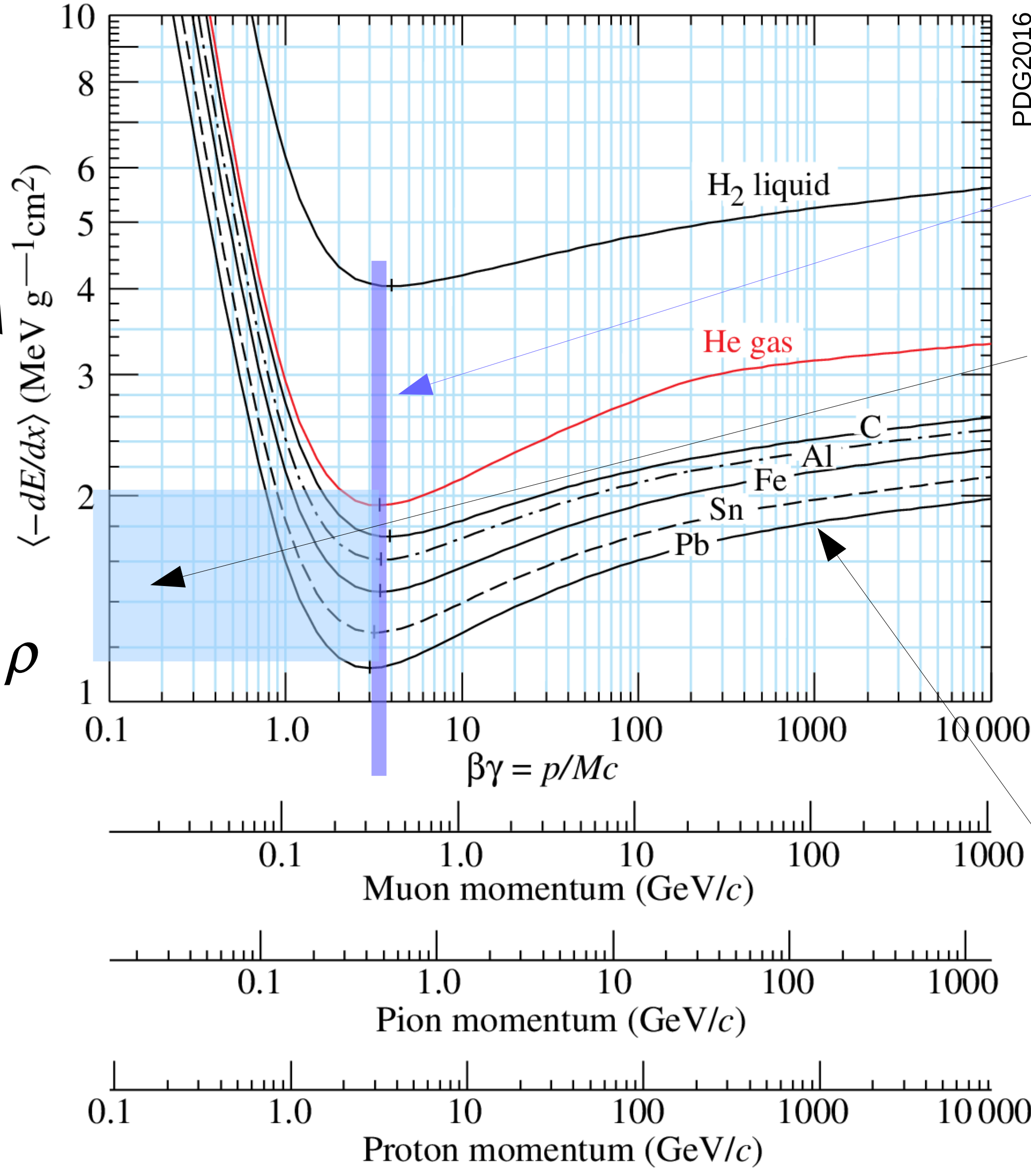
Density correction
Important at high γ

Shell correction
Important at very low γ





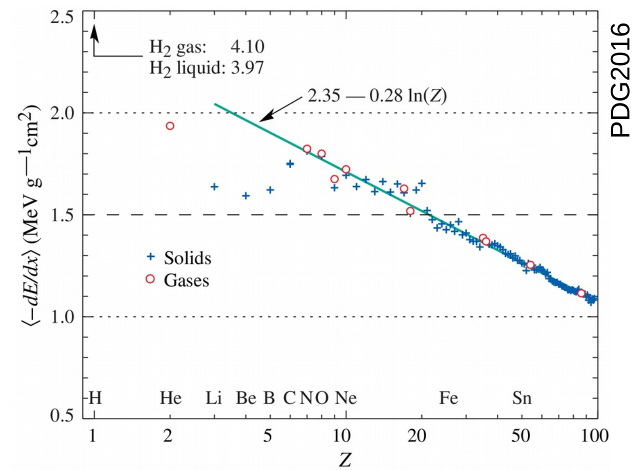
Bethe-Bloch Formula



-dE/dx / ρ

Minimum is for all particles at 3-4 times $\beta\gamma$

Minimum is for all materials 1-2 MeV cm^2/g



Rise is not large: 10-70%
=> call all particles above minimum: mips (Minimum Ionizing Particles)

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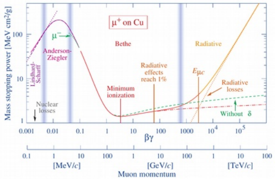
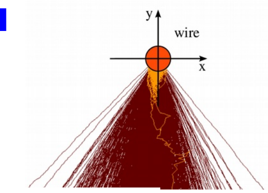
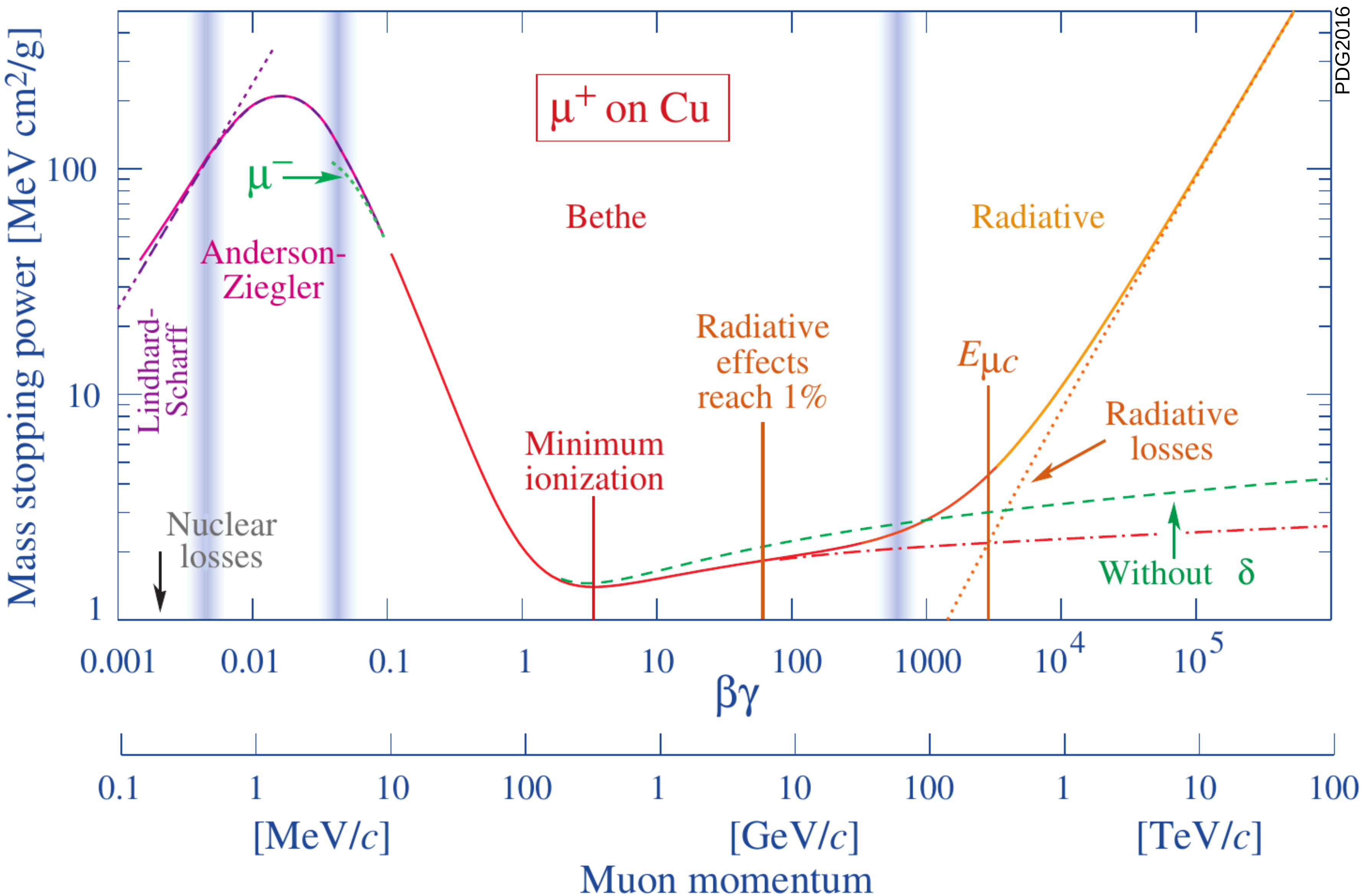


Stopping Power





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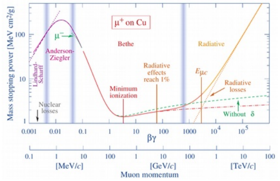
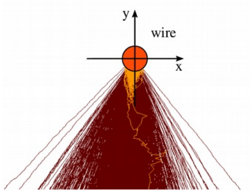
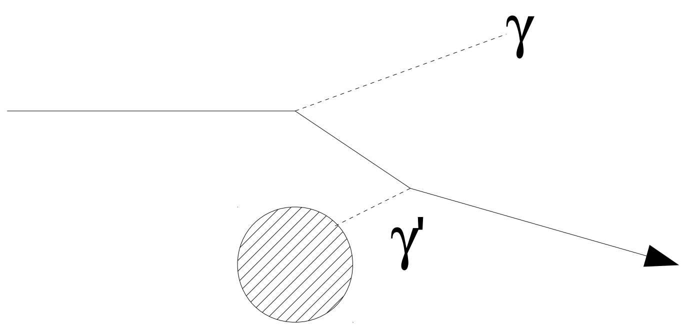


Bremsstrahlung



For high energies, particles can radiate of photons in the electrical field of a nucleus.

Energy dependence $dE/dx \propto Z^2 E/M^2$

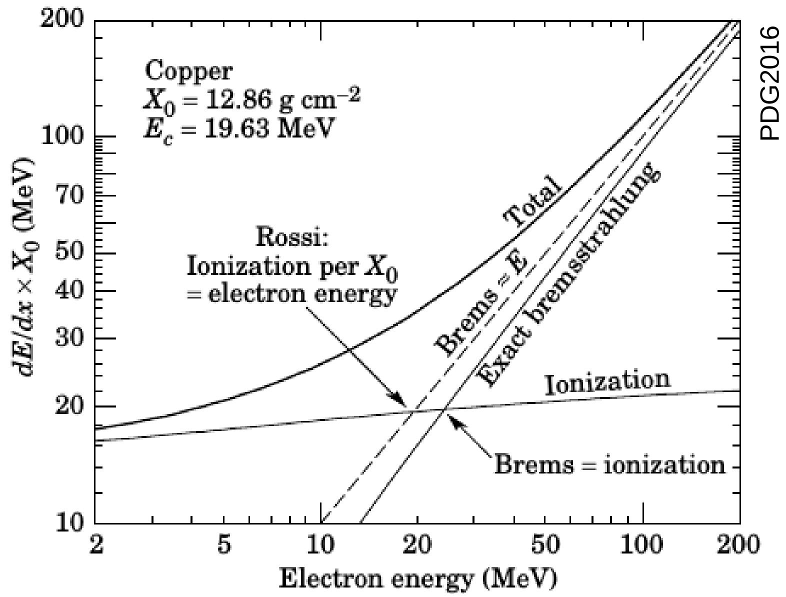


Previous definition of the critical energy:

$$\left(\left.\frac{dE}{dx}\right|_{E_C}\right)_{ion} = \left(\left.\frac{dE}{dx}\right|_{E_C}\right)_{brems}$$

New definition according to Rossi

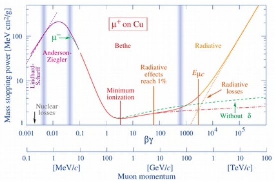
$$\left(\left.\frac{dE}{dx}\right|_{E_C}\right)_{ion} \cdot X_0 = -E_c$$



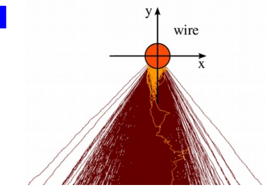
For this approximation numbers can be given:

in liquids and solids $E_c \approx \dfrac{610\ \text{MeV}}{Z + 1.24}$

in gases $E_c \approx \dfrac{710\ \text{MeV}}{Z + 0.92}$



Energy deposition of Electrons



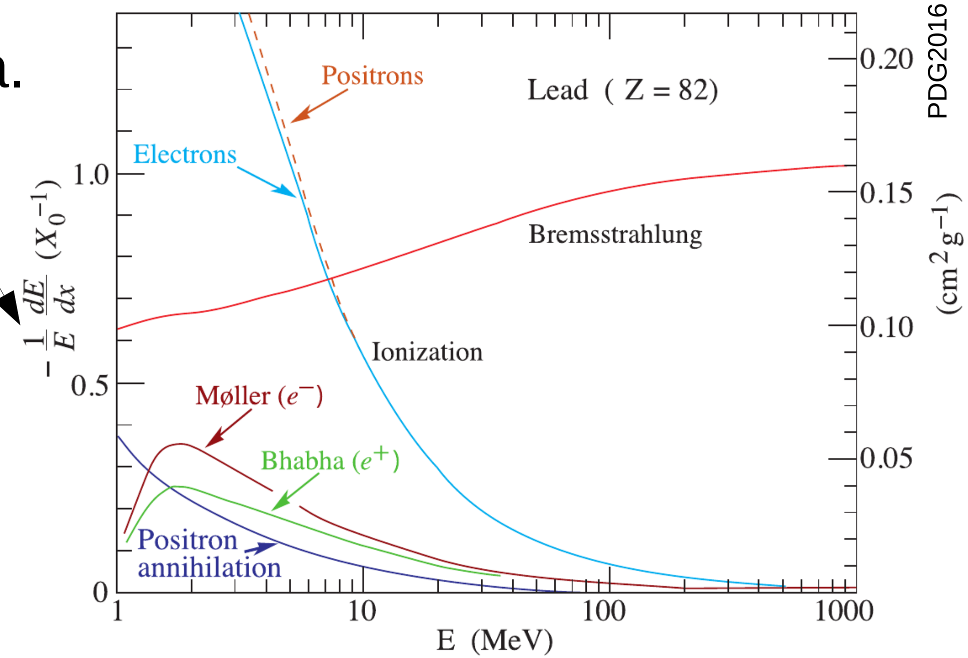
Described by Berger-Seltzer Formula.
In picture: fractional energy loss.

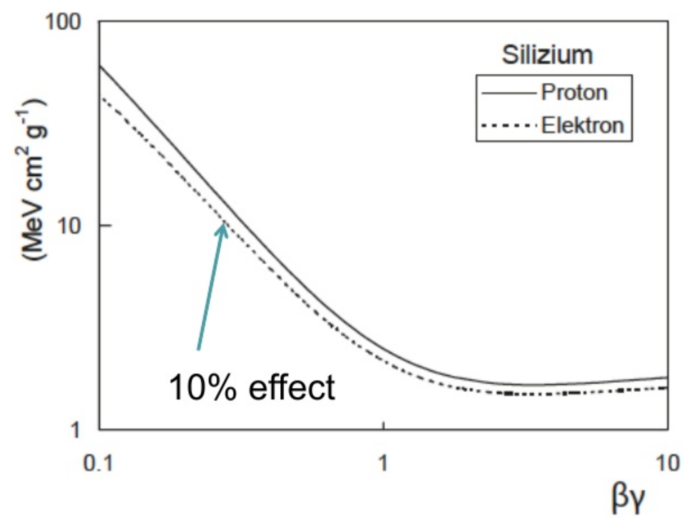
Important differences:

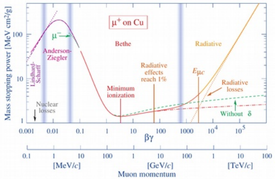
- Bremsstrahlung
- Kinematics ($M = m_e$)
- Spin
- Identical particles (2 outgoing electrons can not be distinguished).
  → the lower energetic particle is the scattered one: $T_{max} = E/2$

For positrons also annihilation processes:

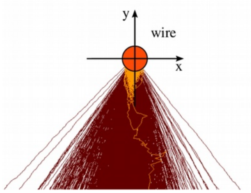
$e^+ + e^- \rightarrow \gamma\gamma$







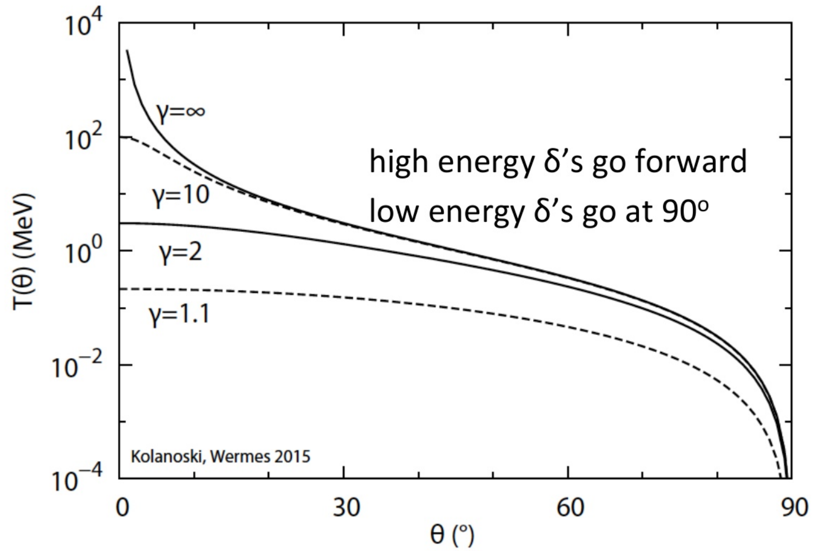
Delta-Electrons (I)



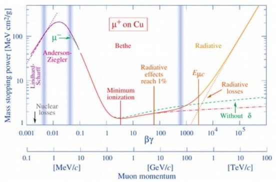
High energetic particles can transfer a lot of energy in some collisions to the electrons. This creates δ-electrons, which have a lot of kinetic energy and can ionize atoms themselves.

Sorting out the 4-momenta, one gets a relation of the kinetic energy and the angle of emission:

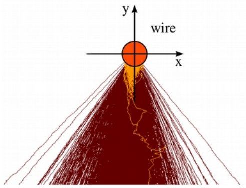
$$T(\theta) = \frac{2m_e c^2 \beta^2 \gamma^2 \cos^2\theta}{\gamma^2\left(1-\beta^2\cos^2\theta\right) + 2\gamma\frac{m_e}{M} + \frac{m_e^2}{M^2}} \xrightarrow{\gamma \gg 1} \frac{2m_e c^2}{\tan^2\theta}$$



δ-electrons deposit energy some distance from the original track → degrade performance of tracking detectors.



Delta-Electrons (II)

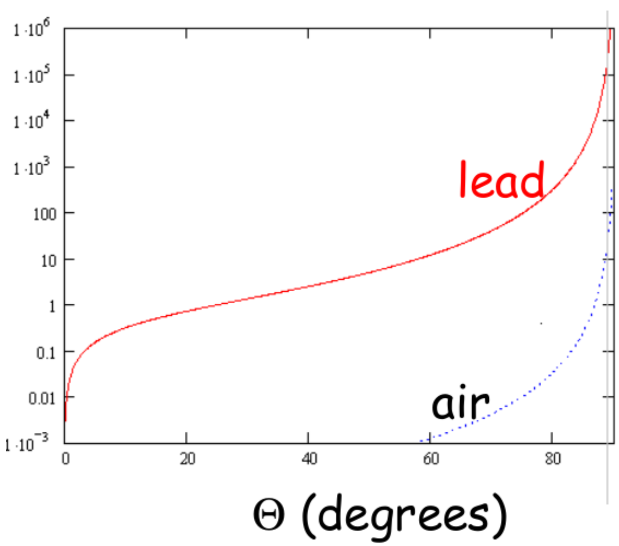


Important is the number of the electrons:

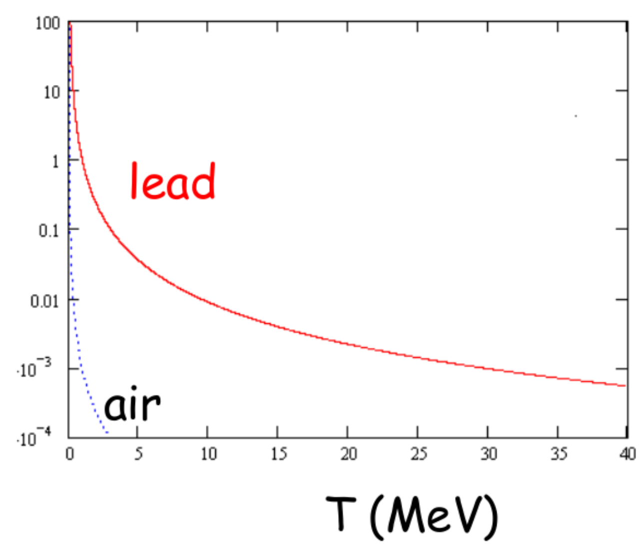
$$\frac{d^2N}{dxdT} = n_e \frac{d\sigma}{dT} = \frac{1}{2}K\frac{Z}{A}\rho\frac{z^2}{\beta^2}\frac{F(T)}{T^2} \xrightarrow{T(\theta)} \frac{d^2N}{dxd\cos\theta} = \frac{1}{2}K\frac{Z}{A}\rho z^2\frac{1}{\cos^3\theta}\frac{1}{m_ec^2} \approx 0.15\frac{\mathrm{cm}^2}{\mathrm{g}}z^2\rho\frac{1}{\cos^3\theta}$$

$$\xrightarrow{\text{integrating}} N = \frac{1}{2}K\frac{Z}{A}\rho\frac{z^2}{\beta^2}\left(\frac{1}{T_{min}} - \frac{1}{T_{max}}\right) \approx 0.077\,\frac{\mathrm{MeV\,cm}^2}{\mathrm{g}}z^2\rho\Delta x\frac{1}{T_{min}}$$

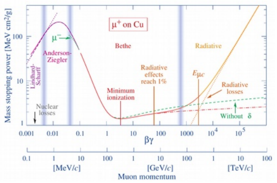
$d^2N/d\theta\, dx$



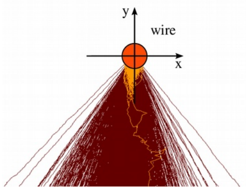
$d^2N/dT\, dx$



- Rate changes rapidly over many orders of magnitude
- 1-1 relation between T and θ
- Almost all δs at an angle of 90°

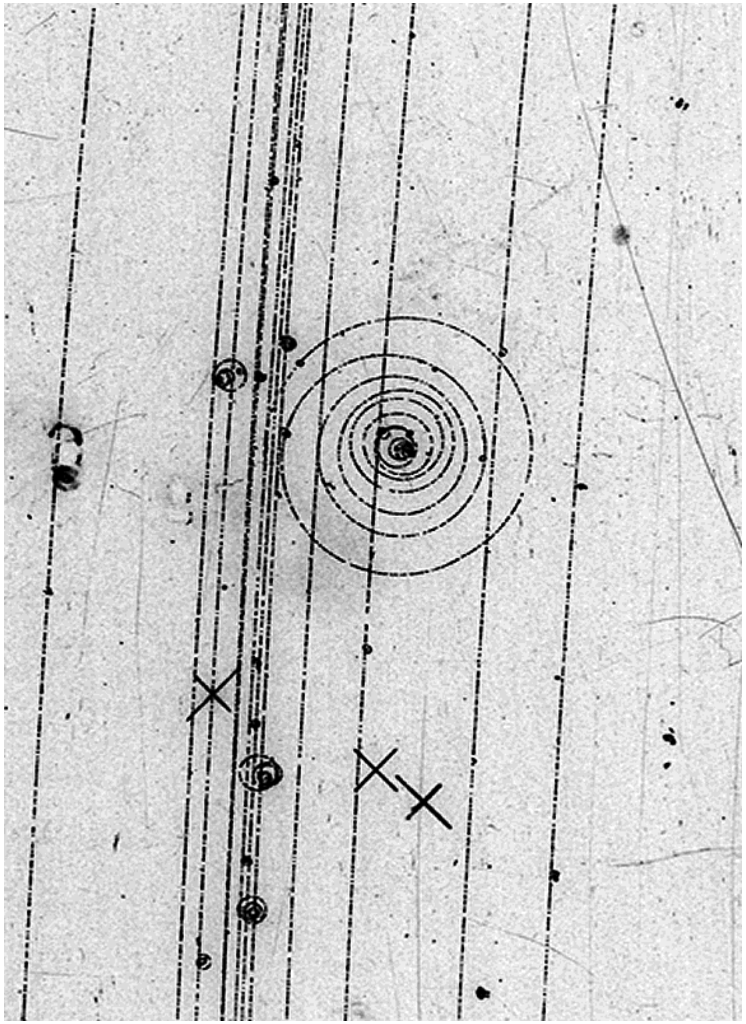


Pictures of δ-electrons

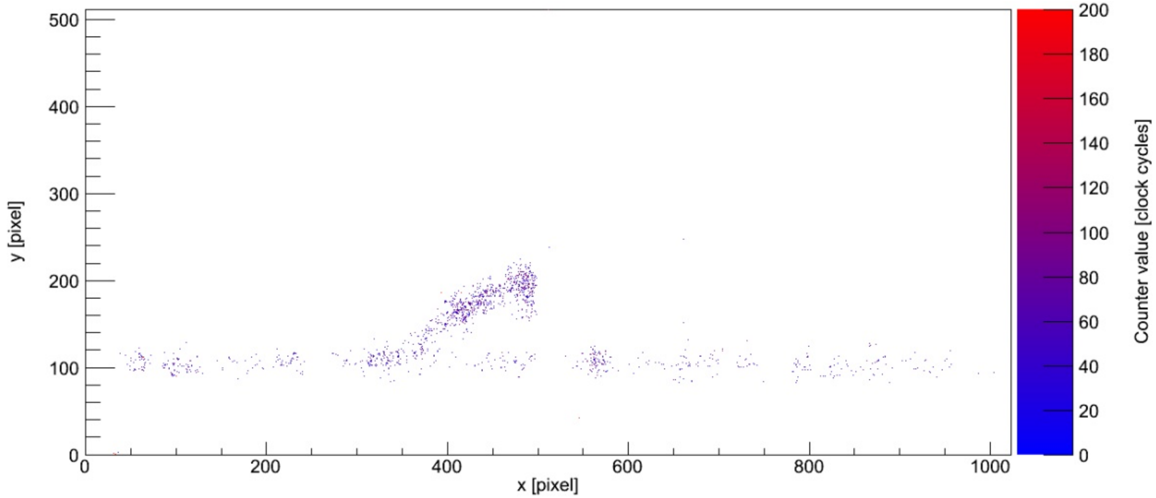


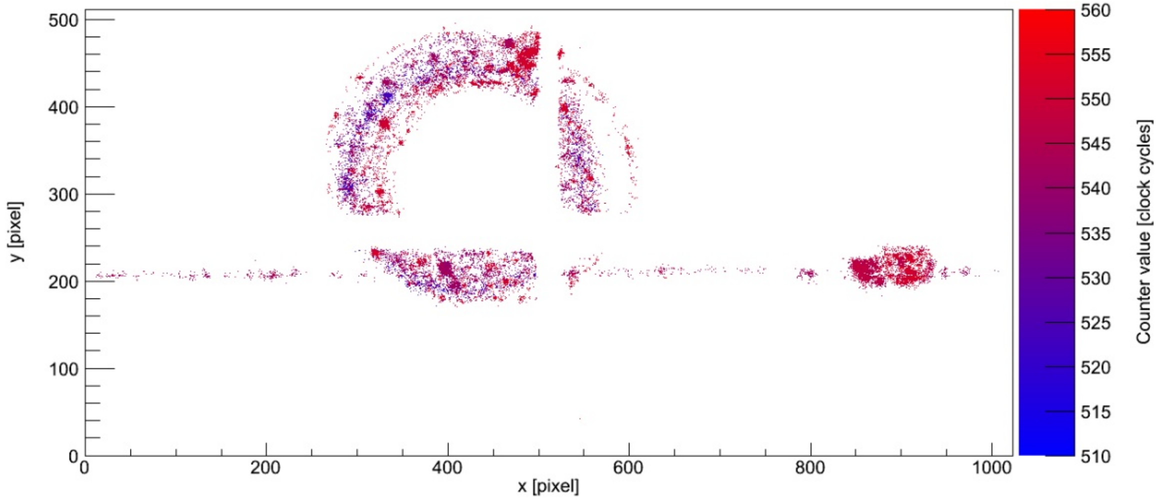
Bubble chamber

http://hst-archive.web.cern.ch/archiv/HST2005/bubble_chambers/BCwebsite/index.htm



GridPix





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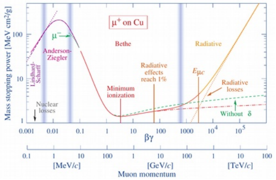
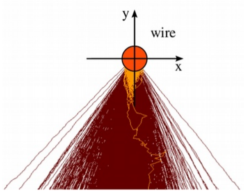
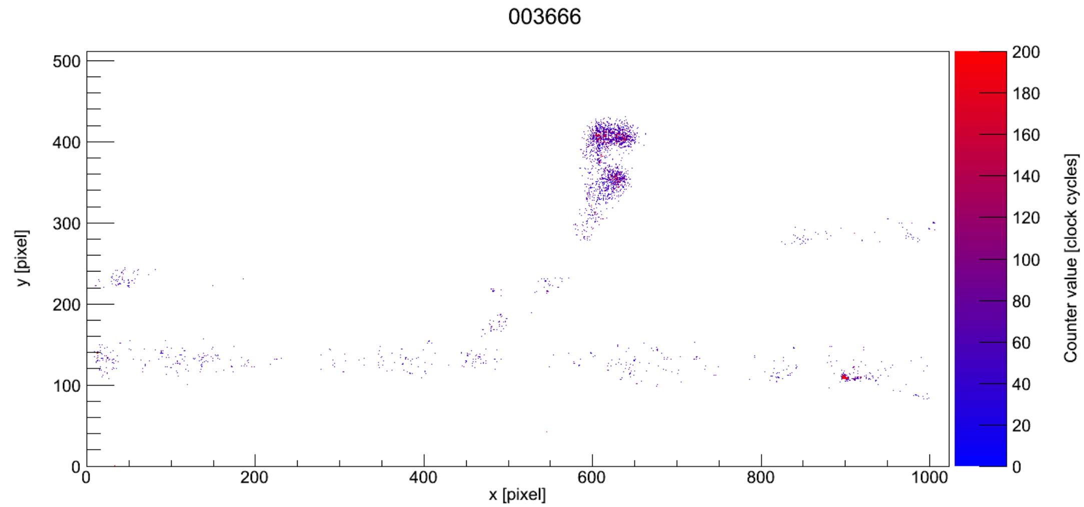


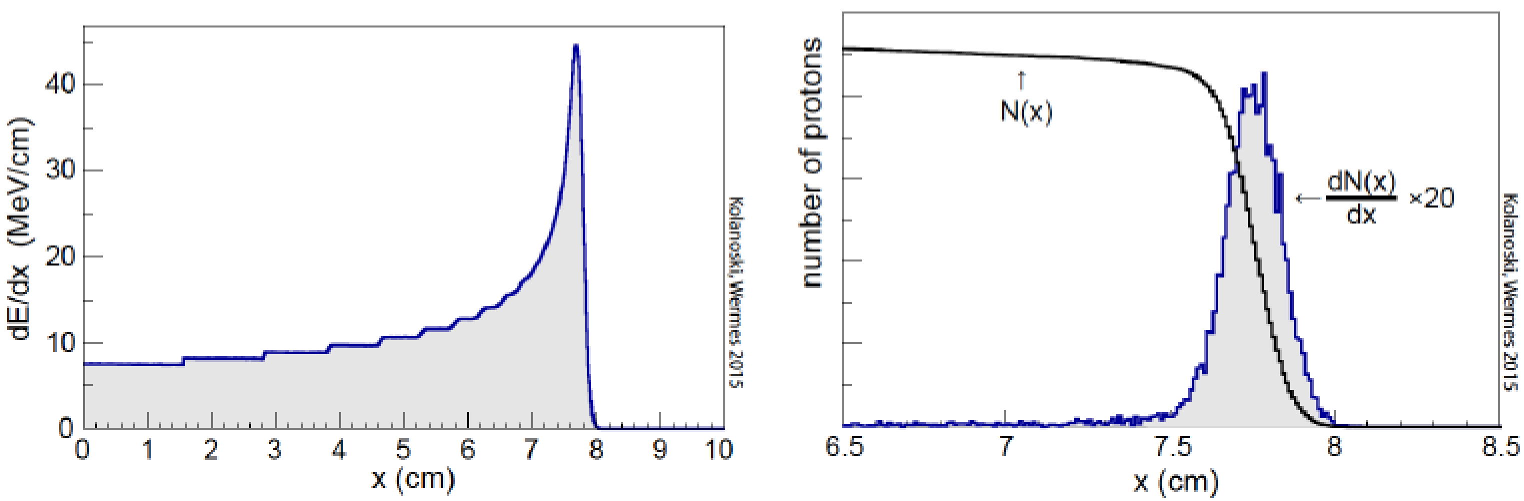


Range of Particles

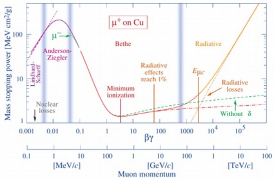
Because slow particles deposit a lot of energy, tracks have a characteristic large 'blob' at their end.



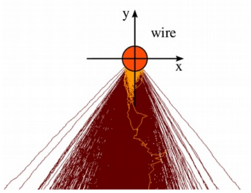
This is called the Bragg peak. It is used for hadron therapy in cancer treatment.



(a) (b)



Energy Fluctuations (I)



The Bethe-Bloch formula gives the average energy loss over a short distance Δx.

The actual energy loss over Δx is random, since it is the sum of single energy transfers.

$$\Delta E = \sum_{n=1}^{N} T_n$$

Number of interactions *N*: For thin absorbers N follows the Poisson distribution

$$\frac{\sigma_N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

Energy transfer δE_n : The distribution follows $1/T^2$

$$\frac{d^2N}{dx\,dT} = n_e \frac{d\sigma}{dT} = \frac{1}{2} K \frac{Z}{A} \rho \frac{z^2}{\beta^2} \frac{F(T)}{T^2}$$

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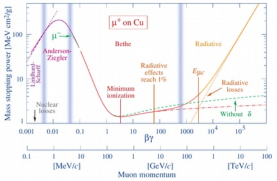
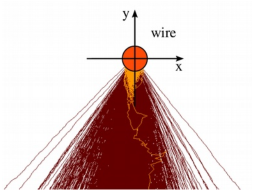


Energy Fluctuations (II)



Combining both effects, the distribution of the resulting energy loss varies strongly with the detector thickness and the particle energy: The parameter

$$\kappa = \frac{\xi}{T_{max}} = \frac{\frac{1}{2} K \frac{Z}{A} \rho \frac{z^2}{\beta^2} \Delta x}{T_{max}}$$ determines the shape.

For $\kappa \gtrsim 1$ symmetric, for $\kappa \ll 1$ strongly asymmetric

The latter case can be described by the Landau distribution.

$$f_L(\lambda) = \frac{1}{\pi} \int_0^\infty e^{-t \ln t - \lambda t} \sin(\pi t) dt$$

It assumes:

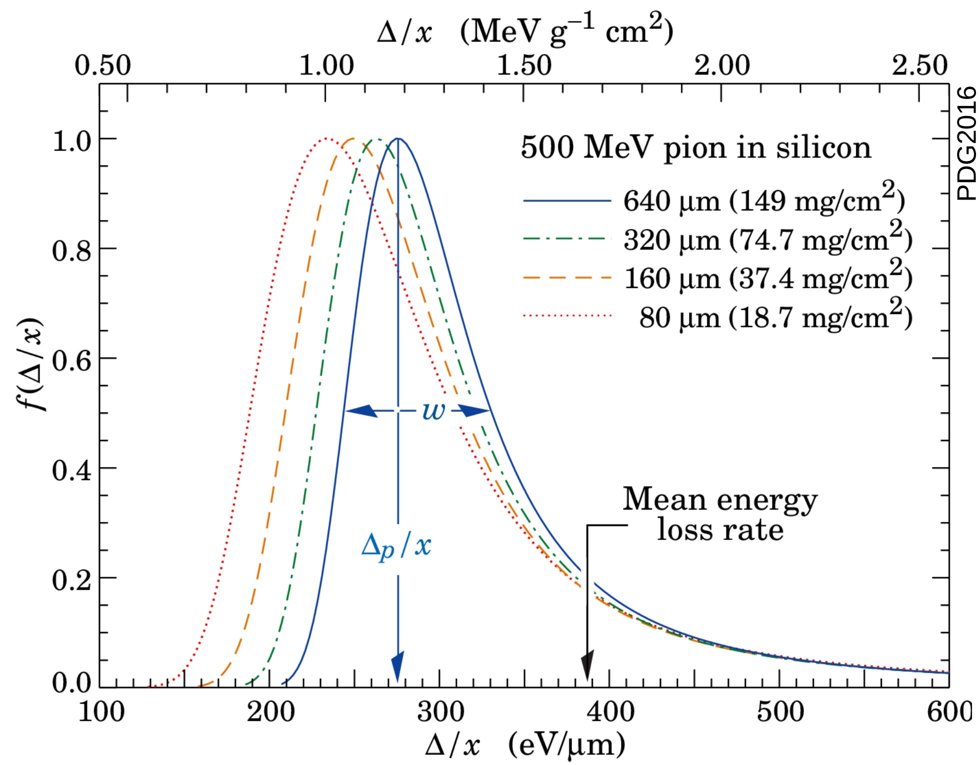
1) $T_{max} \rightarrow \infty$

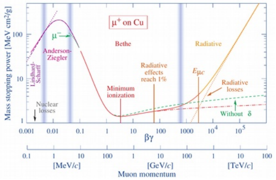
2) Electrons are free (no shell effects)

3) $\Delta E \ll E$

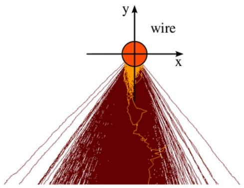
=> λ_{max}=-0.22278, $FWHM_{\lambda}$=4.018

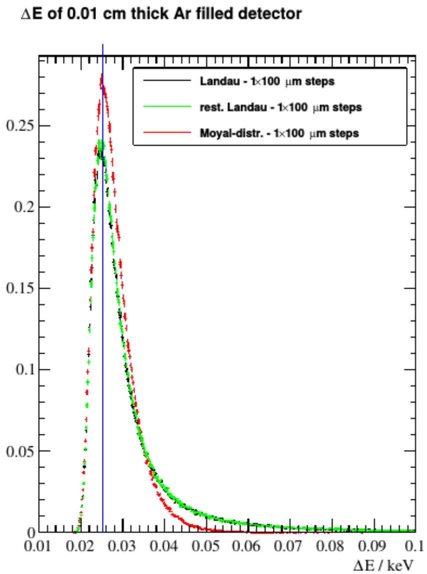
$$\lambda = \lambda(\Delta E_w, \xi) = \frac{\Delta E - \Delta E_w}{\xi} - 0.22278$$

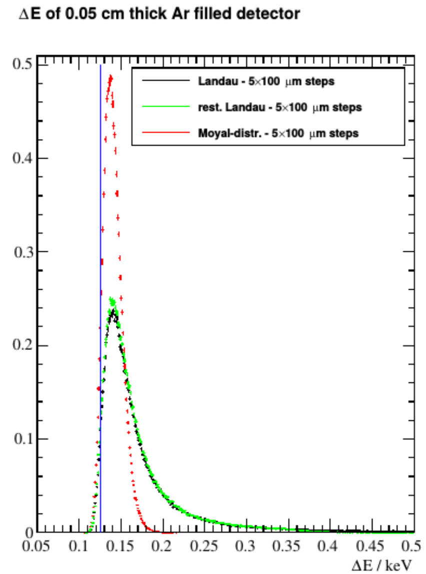


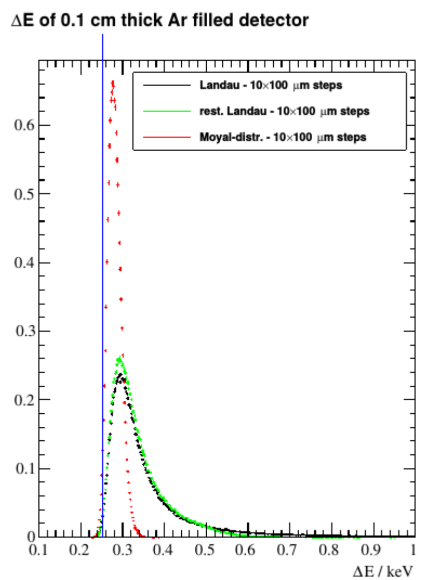


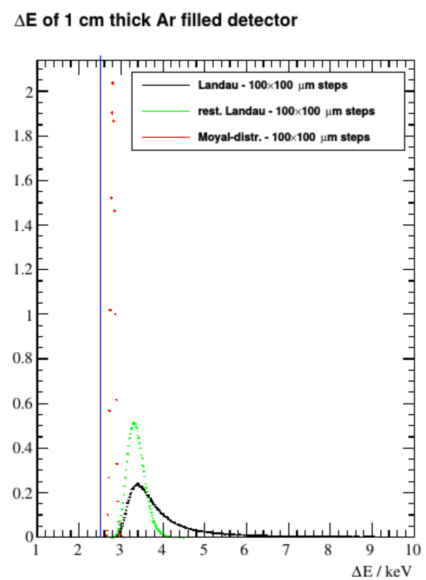
Energy fluctuations (III)











Landau distribution assumes $T_{max} \to \infty$

=> problematic

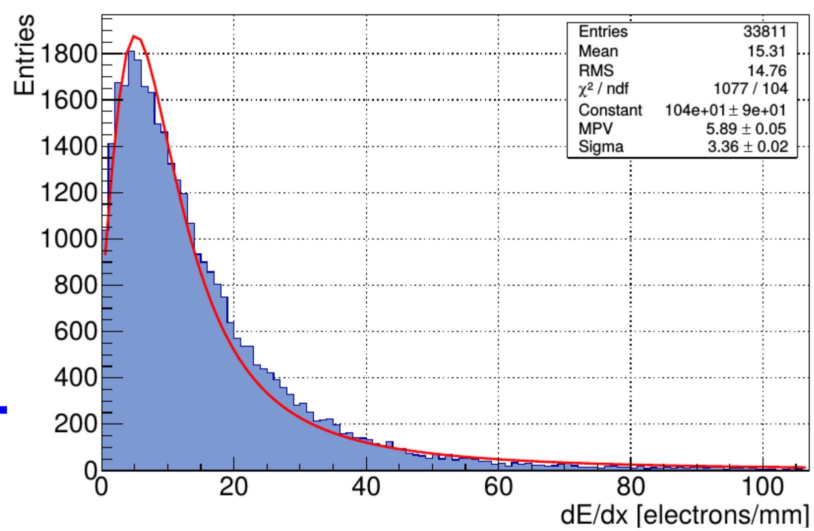
Vavilov developed a generalization.

Also Moyal distribution is often used as approximation:

$$f_M(\lambda) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\lambda + e^{-\lambda})}$$

None of those agree perfectly, because shell effects are neglected.

5 GeV electrons in Ar-based gas mixture



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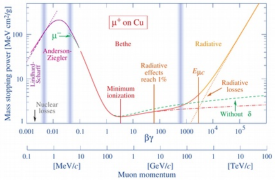
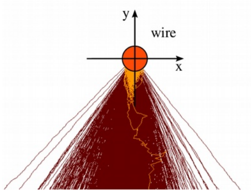


Radiation Length



Integrating the *dE*/*dx* of the Bremsstrahlung spectrum over all energies yields:

$$\left(\frac{dE}{dx}\right)_{brems} = -\frac{E}{X_0}$$

where X_0 is call radiation length

$$\rho X_0 = \frac{716.408 \text{ g cm}^{-2} A}{Z(Z+1)\ \ln\frac{287}{\sqrt{Z}}}$$

Integrating over *x* gives

$$E(x) = E_0 e^{-\frac{x}{X_0}}$$

which indicates one of the definitions of X_0: the path length, until electrons have only 1/*e* of the original energy.
However, X_0 is an important measure for many em. processes that take place in the electric field of nuclei, e.g. multiple scattering or pair production from photons, etc.

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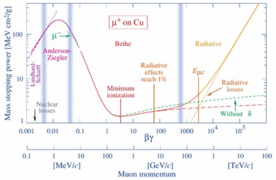
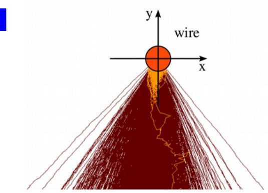


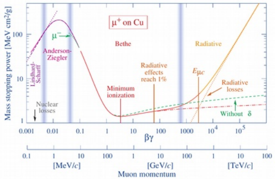
Impact on Tracking Detectors



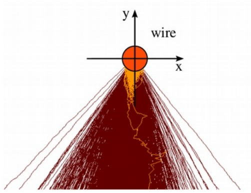
Energy loss is the most important process for detecting particles, since it generates electrical charges, which can be amplified, readout and digitized.

Things to keep in mind:

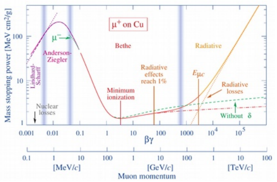
- A higher *N* is better for a precise position measurement (statistics)
- Actually it is not the number of electrons, but the number of primary interactions
- Av. ionization energy higher than shell levels (Ar: E_I=11-15 eV, W_I=26 eV)
- Particle identification can be done by momentum and *dE/dx*
- δ-electrons degrade the position resolution of tracking detectors
- Multiple scattering and Bremsstrahlung degrade performance of tracking detectors
  => use a short radiation length of detector in particular low *Z* material

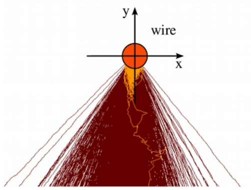


Numbers for Standard Detectors



	Z	A	ρ	$\frac{dE}{dx}\Big\vert_{min}$	X_0	E_C
argon	18	40	$1.66\ \frac{\mathrm{mg}}{\mathrm{cm}^3}$	$2.5\ \frac{\mathrm{keV}}{\mathrm{cm}}$	11763 cm	38 MeV
silicon	14	28	$2.33\ \frac{\mathrm{g}}{\mathrm{cm}^3}$	$3870\ \frac{\mathrm{keV}}{\mathrm{cm}}$	9.36 cm	40.2 MeV





Drift and Diffusion

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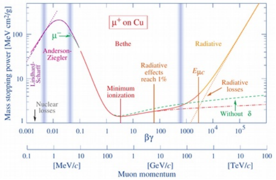
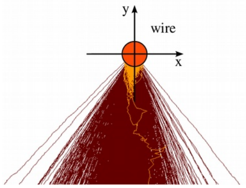
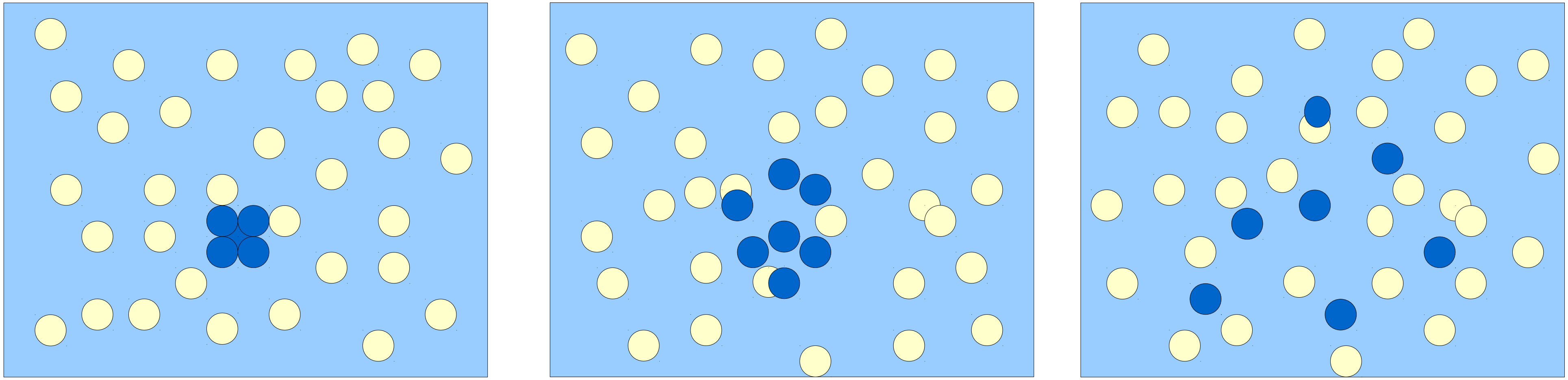


Diffusion



$t_0 = 0$ $t_1 > t_0$ $t_2 > t_1$



Areas with different concentrations of some type of gas aim for equilibrium => diffusion

Same happens if a number of electrons/ions is released in the gas

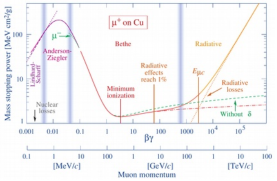
<u>Requirements:</u>

a) $T>0$ →

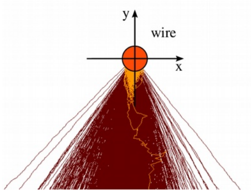
$$E_{th} = \frac{3}{2}kT = \frac{1}{2}m\langle v_{th}\rangle^2 \quad \Rightarrow \quad \langle v_{th}\rangle = \sqrt{\frac{3kT}{m}}$$

For Ar atoms at 300 K this gives $<v_{th}>$~5 mm/µs

b) conc. gradient *dn/dx*



Diffusion Equation



For a certain distribution *f(r,v,t)*, the density *ρ* and number *N* is given by

$$\rho(\vec{r},t) = \int f(\vec{r},\vec{v},t)d^3v \qquad\qquad N(t) = \int \rho(\vec{r},t)d^3r$$

If *N* is constant, then the continuity equation holds for any area:

$$\frac{\partial\rho}{\partial t} + \nabla\vec{j}_D = 0$$

The diffusion current density j_D is proportional to ∇*ρ*: $\vec{j}_D = -D\vec{\nabla}\rho$

This gives the diffusion equation (Fick's law)

$$\frac{\partial\rho}{\partial t} - D\Delta\rho = 0$$

with the solution $$\rho(\vec{r},t)) = \frac{N}{(4\pi Dt)^{\frac{3}{2}}}e^{-\frac{\vec{r}^2}{4Dt}}$$

This is a Gaussian distribution with the width *D,* which is the diffusion constant.

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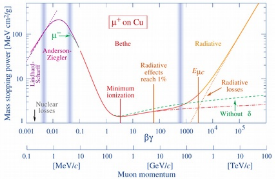
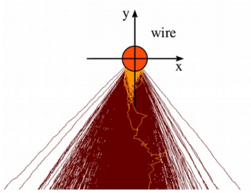


Diffusion Constant



D is the mean quadratic width of the charge cloud which spread with time

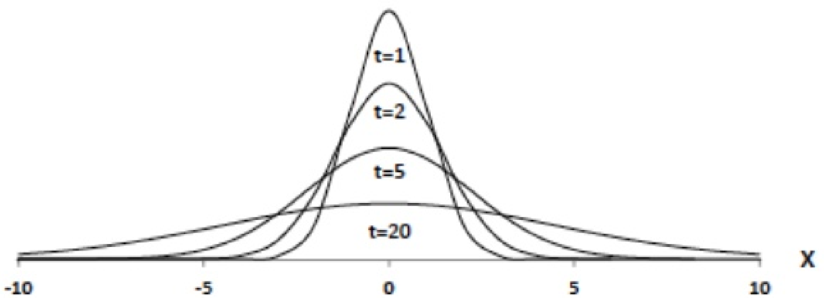
$$\frac{\partial \langle x^2 \rangle}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{N}\int x^2 \rho(\vec{r},t)d^3\vec{r}\right) = 2D$$

Integrating over time: $\langle x^2 \rangle = 2Dt \qquad \sigma_x = \sigma_y = \sigma_z = \sqrt{2Dt}$

$$D = \frac{1}{3}\langle \lambda \cdot v_{th} \rangle = \frac{1}{3}\frac{1}{n\sigma}\sqrt{\frac{3kT}{m}} \approx \frac{1}{3}\frac{kT}{p\sigma}\sqrt{\frac{3kT}{m}} = \frac{kT}{p\sigma}\sqrt{\frac{(kT)^3}{3m}}$$

<u>Ions:</u> Diffusion almost independent of gas $D \approx 0.1\,\mathrm{cm^2/s} \Rightarrow \sigma_x \approx 140\,\mu\mathrm{m}$ after 1ms

Kolanoski, Wermes 2015



<u>Electrons:</u> very gas dependent

$\sigma_x \approx 100\ \mu\mathrm{m}$ to 1 mm after 1 ms

Gas	λ_{ion} (cm)	D_{ion} (cm²/s)	μ_{ion} (cm²/Vs)
H_2	1.8×10^{-5}	0.34	13.0
He	2.8×10^{-5}	0.26	10.2
Ar	1.0×10^{-5}	0.04	1.7
O_2	1.0×10^{-5}	0.06	2.2
Si (holes)	2.4×10^{-6}	12	450

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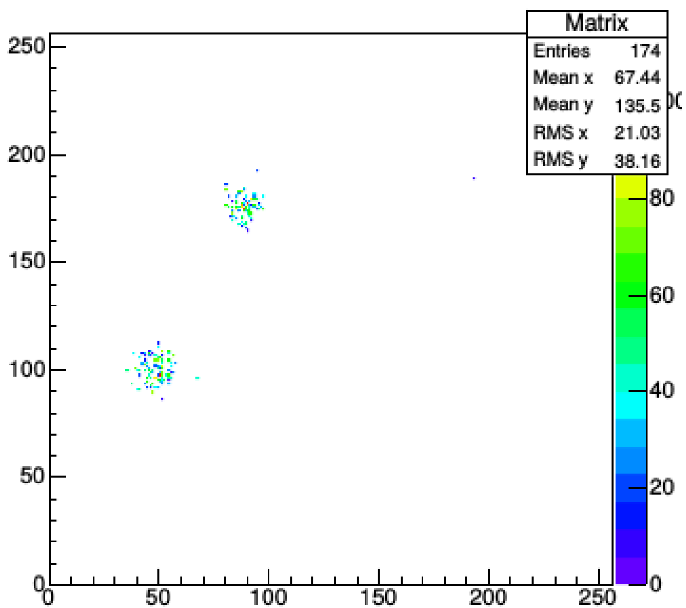
Example

$\mathrm{Ar/CO_2}\ (70:30)$

Drift field: 500 V/cm

Drift distance: $\sim$ 3 to 4 cm

$D \approx 140\ \mu\mathrm{m}/\sqrt{\mathrm{cm}}$

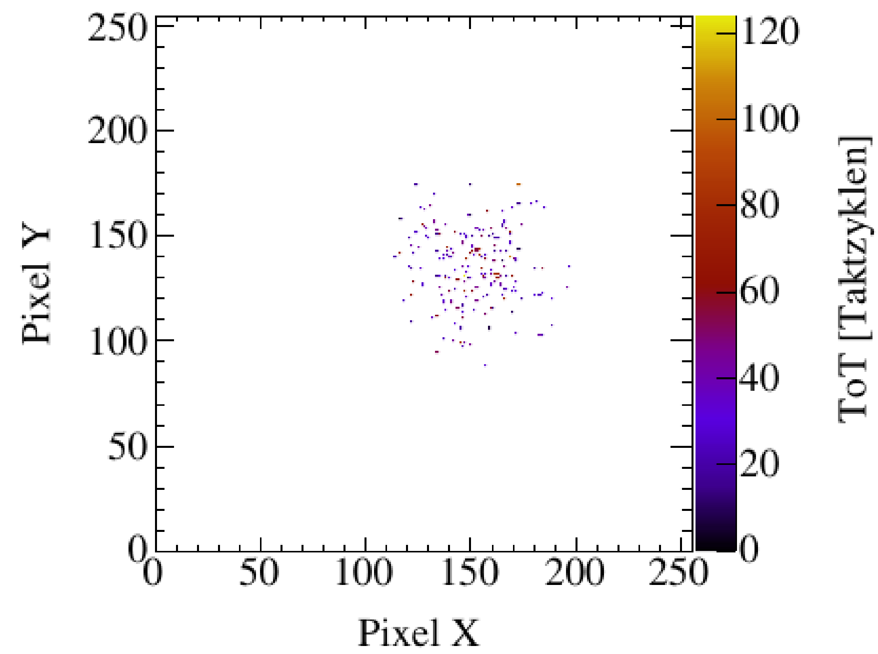


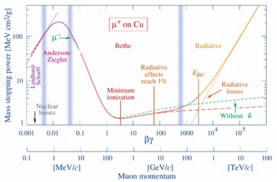
$\mathrm{Ar/iC_4H_{10}}\ (97.7:2.3)$

Drift field: 500 V/cm

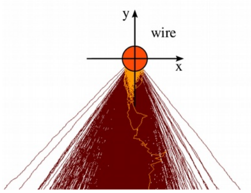
Drift distance: 3 cm

$D \approx 700\ \mu\mathrm{m}/\sqrt{\mathrm{cm}}$





Drift



If an electric field is applied, we have a superposition of a drift movement and diffusion.
Drift can be described by the following equation of motion (Langevin equation):

$$m\frac{d\vec{v}}{dt} = e\vec{E} + e\left(\vec{v}\times\vec{B}\right) - K\vec{v}$$

where *v* is the drift velocity. The stationary solution is:

$$\vec{v} = \frac{e}{m}\tau\left|\vec{E}\right|\frac{1}{1+\omega^2\tau^2}\left(\hat{E} + \omega\tau\left(\hat{E}\times\hat{B}\right) + \omega^2\tau^2\left(\hat{E}\cdot\hat{B}\right)\hat{B}\right)$$

with the collision time $\tau = m/K$ and the cyclotron frequency $\omega = eB/m$.

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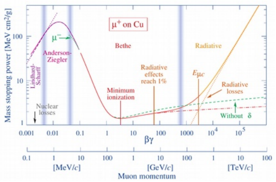
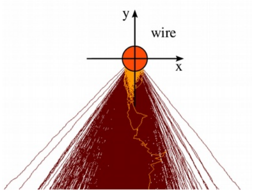


Drift in Electrical Fields



$$\vec{v} = \frac{e}{m}\tau \left|\vec{E}\right| \frac{1}{1+\omega^2\tau^2}\left(\hat{E} + \omega\tau\left(\hat{E}\times\hat{B}\right) + \omega^2\tau^2\left(\hat{E}\cdot\hat{B}\right)\hat{B}\right)$$

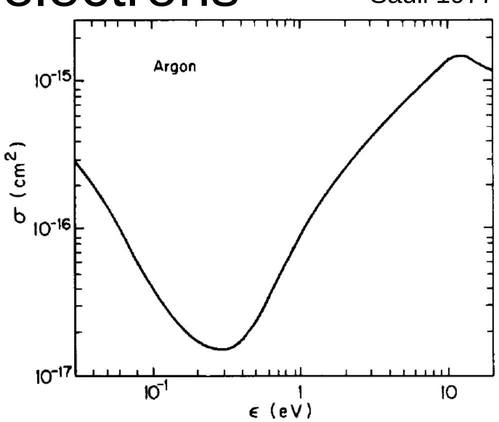
If B = 0 then ω=0, the equation then simplifies to

$$\vec{v} = \frac{e}{m}\tau \left|\vec{E}\right| \hat{E} \equiv \mu\vec{E}$$

where μ= $e\tau/m$ is called mobility.

The concept works reasonably good for ions (i.e. mobility is constant).
But it does not work for electrons (strongly gas and E dependent):
Elastic cross section exhibit maxima and minima because of quantum mechanical interferences with the wave functions of shell electrons (Ramsauer effect).
→ The cross section is large (resonant) when de Broglie wave length (~1/p) in the order of the size of the atom.

Sauli 1977



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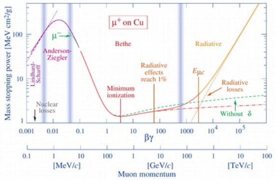
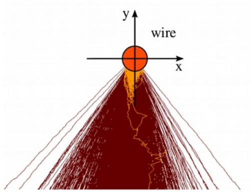
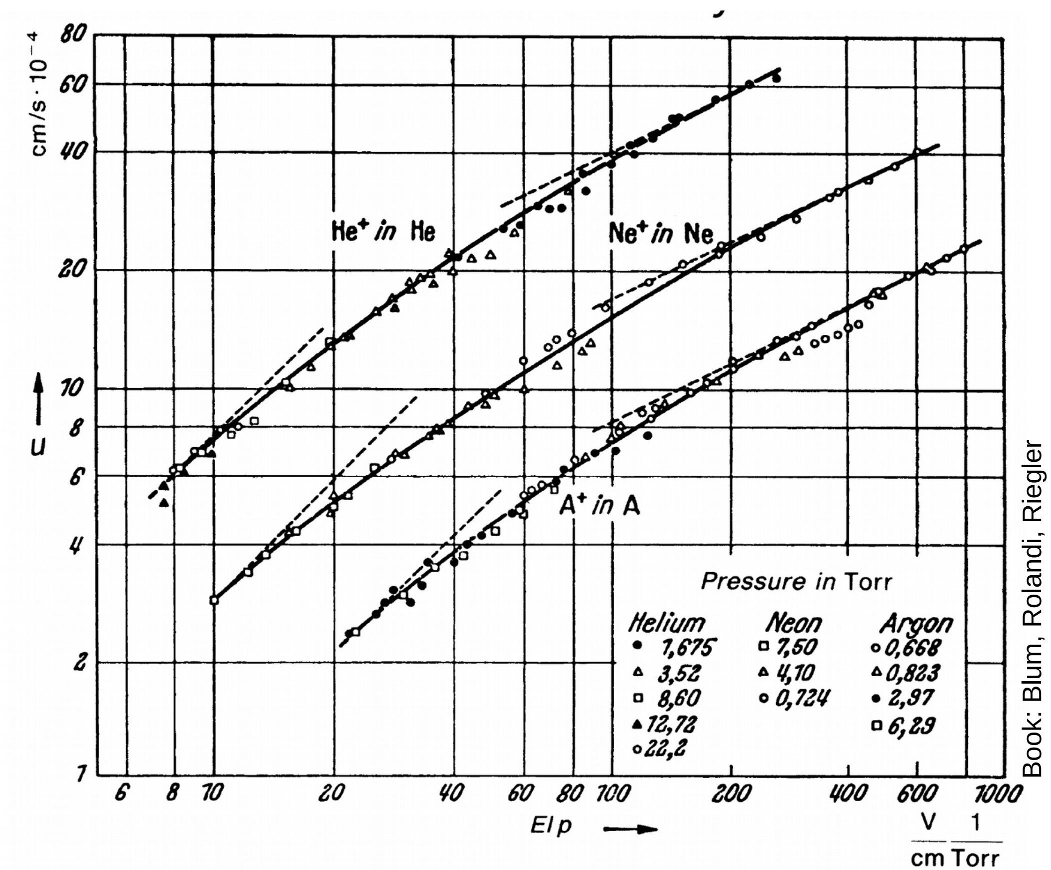


Examples

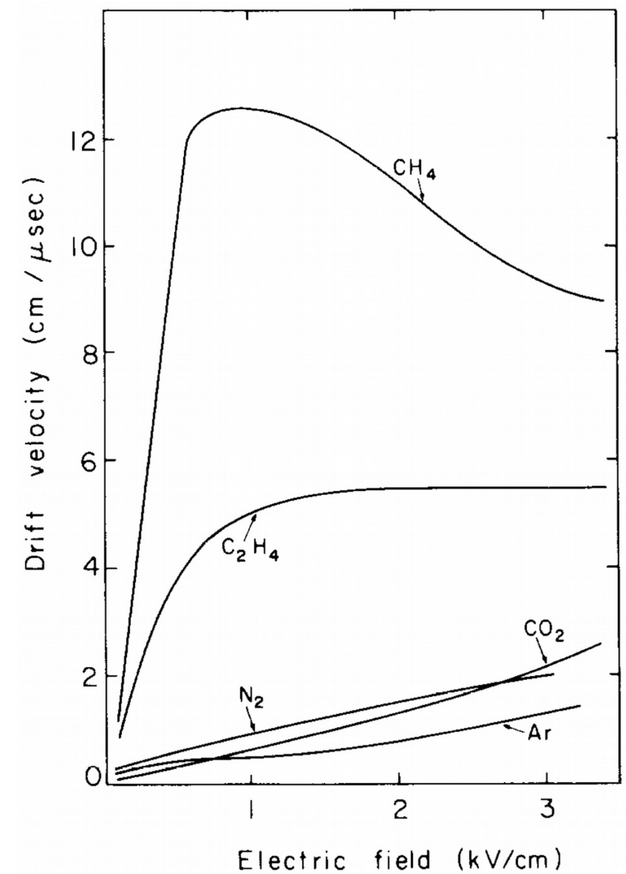


Mobility of ions



Book: Blum, Rolandi, Riegler

Drift velocity of electrons



Sauli 1977

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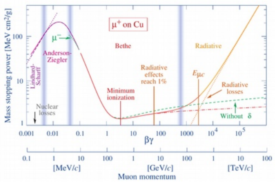
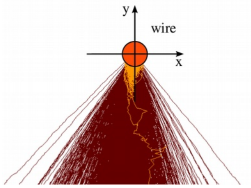


Drift in Electrical and Magnetic Fields



$$\vec{v} = \frac{e}{m}\tau \left|\vec{E}\right| \frac{1}{1+\omega^2\tau^2}\left(\hat{E} + \omega\tau\left(\hat{E}\times\hat{B}\right) + \omega^2\tau^2\left(\hat{E}\cdot\hat{B}\right)\hat{B}\right)$$

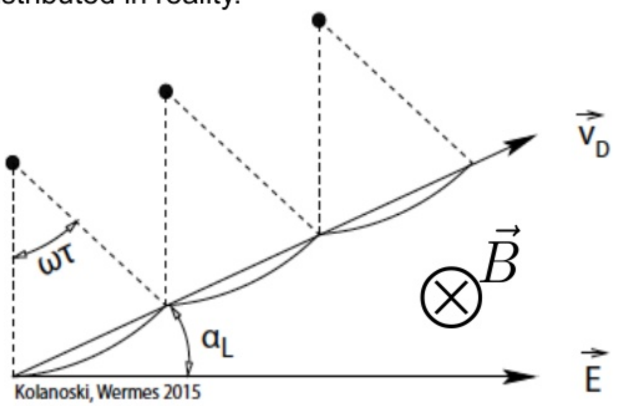
Motion difficult to describe in general, but in special cases:

1) $B||E$: second term is 0 → no change in direction of drift

2) $B\perp E$: third term is 0 → But direction of drift changes!
   Lorentz forces deviates particles during flight

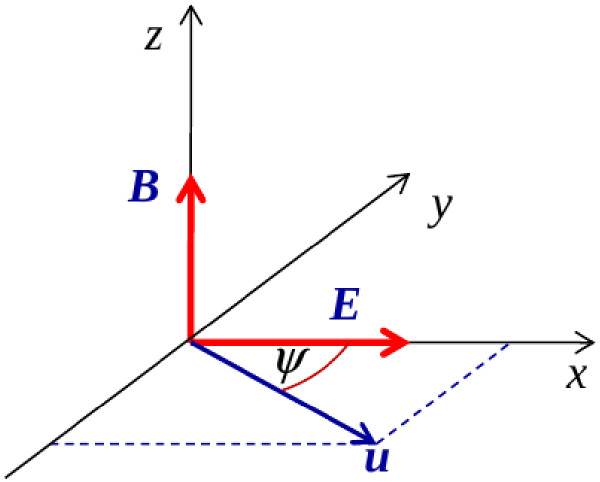
=> Lorentz angle: $\tan\psi \equiv \frac{v_y}{v_x} = -\omega\tau$

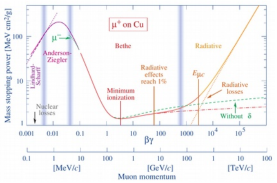
Simplified picture: segments are stochastically distributed in reality.



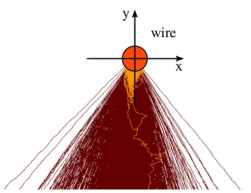
Since $m_{ion} \gg m_e$

→ $\psi_{ion} \ll \psi_e$





Diffusion in Magnetic Fields



Since $\omega = eB/m$ and $m_{ion} \approx 10^4 \cdot m_e$

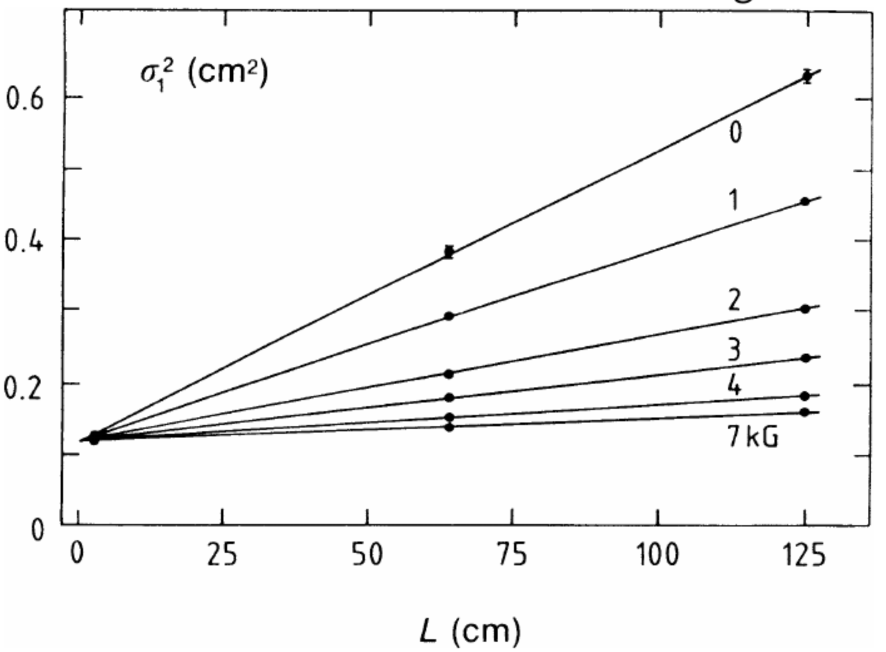
$$\omega_e \tau_e \approx 10^4 \cdot \omega_{ions} \tau_{ions}$$

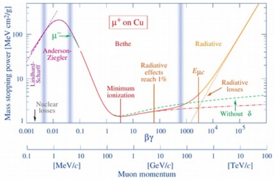
There is basically no impact on ions,
but significant impact on electron movement.
Diffusion is reduced:

$$D_T(\omega) = \frac{D_T(0)}{1 + \omega^2 \tau^2}$$

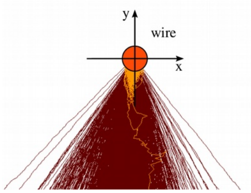
$$D_L(\omega) = D_L(0)$$

Reduction of electron diffusion due to magnetic field





Simulation



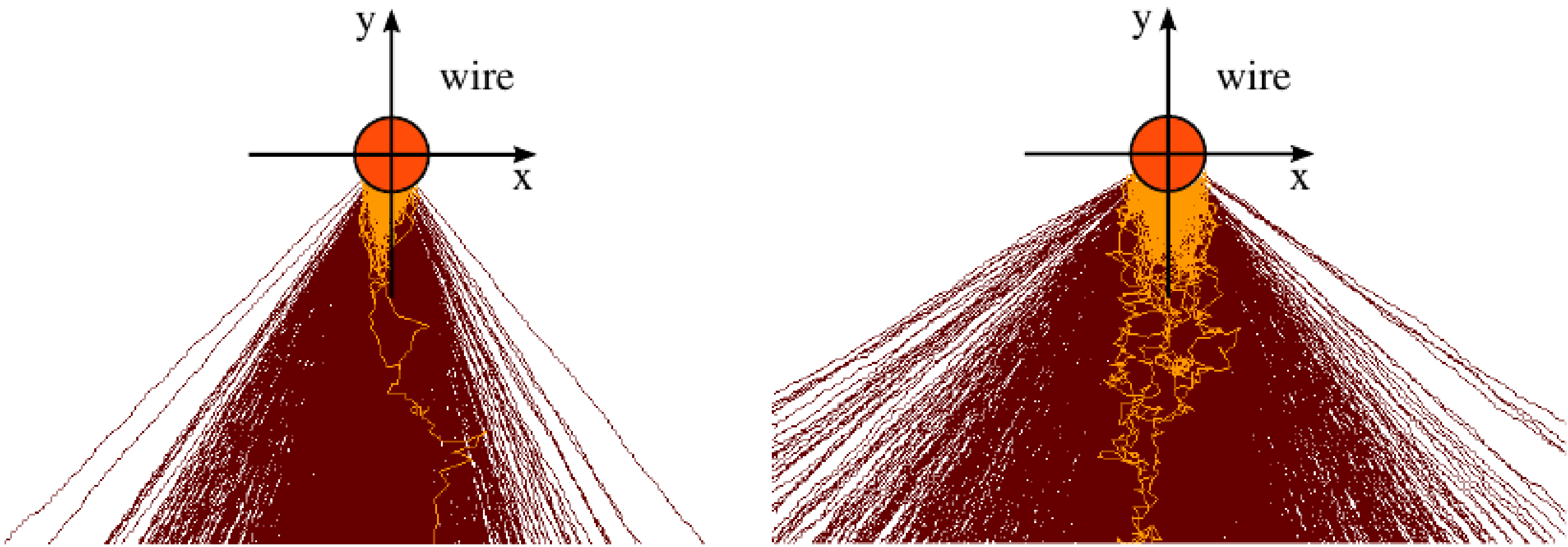
Transport properties (v_{drift}, D, attachment, gas amplification) can be very well simulated by dedicated simulation tools:

For gas properties: Magboltz (in fortran)

http://magboltz.web.cern.ch/magboltz/

For detector behavior: Garfield++ (in C++)

https://garfieldpp.web.cern.ch/garfieldpp/



(a) Ar/CO_2, U = 1250 V, avalanche size 1402 e (b) Ar/Isobutane 700V, U = 700 V, avalanche size 1637 e

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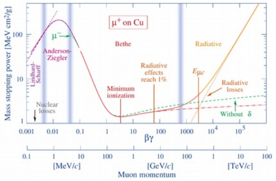
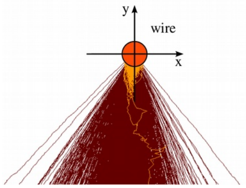
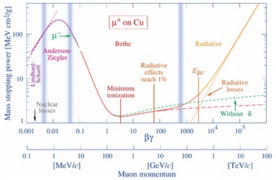
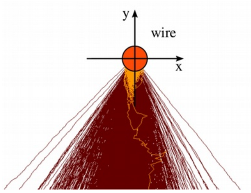




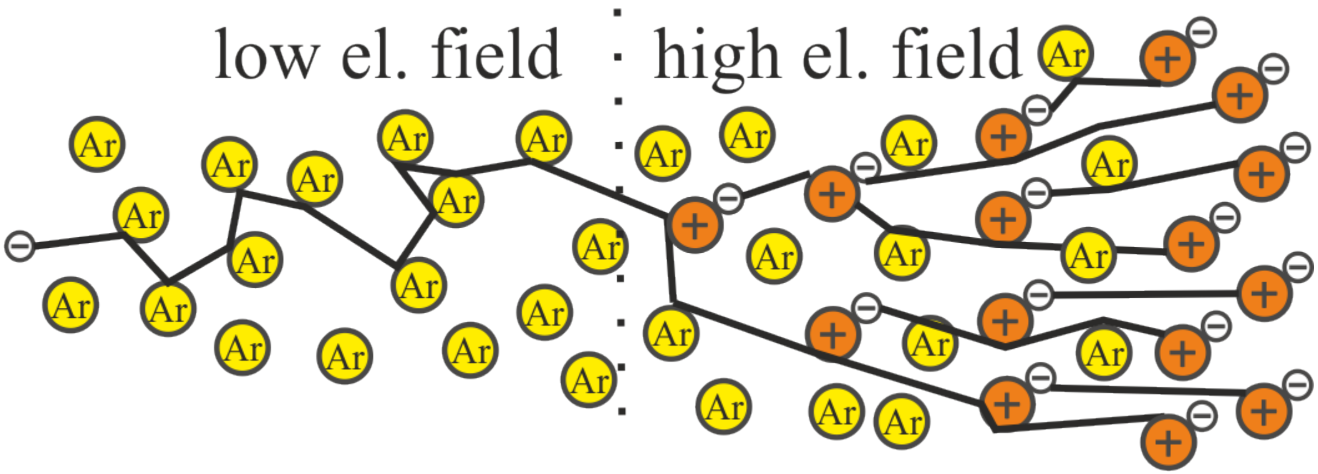
Gas Amplification



Simple Picture



If free electrons enter an area with high electrical fields, they can gain enough energy between two collisions with atoms to ionize them.



An avalanche of electrons is developed.

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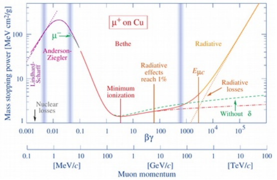
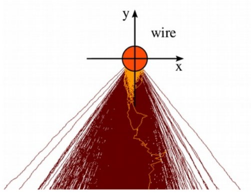
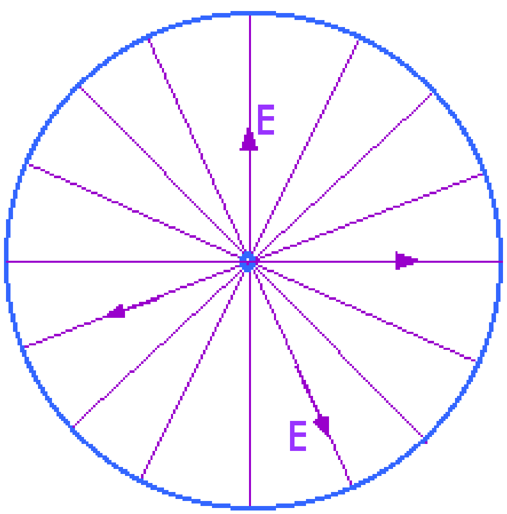


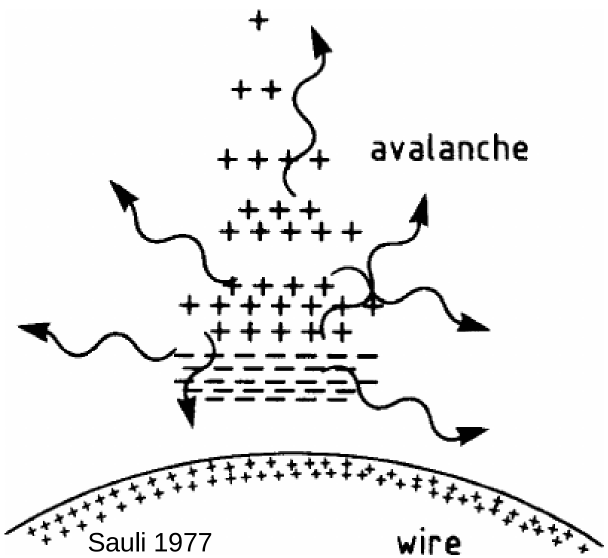
Wire-based Tubes

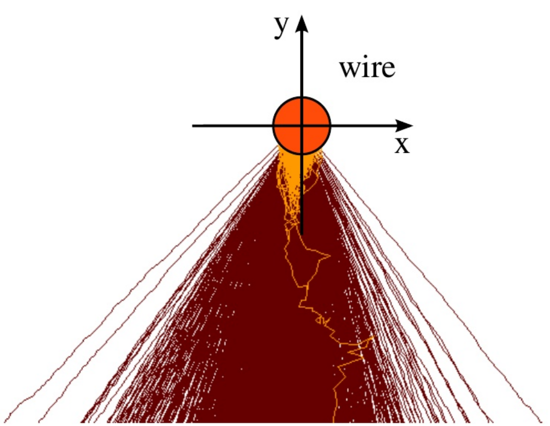


The cylindrical geometry of a wire-based tube detector is optimal:

1) Natural transition from a low electric field to a high electric field (geometry!)
2) Needs only one HV channel
3) Large volume read out by 1 single channel
4) Low capacitance (→ low noise)







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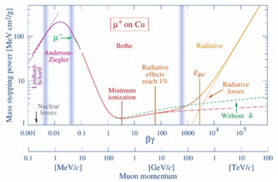
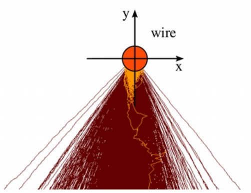


Mathematical Description



Gas amplification is defined as an increase of charge by *dn*, while the charge *n* is passing through a layer *dx*:

$$dn = n\alpha\, dx$$

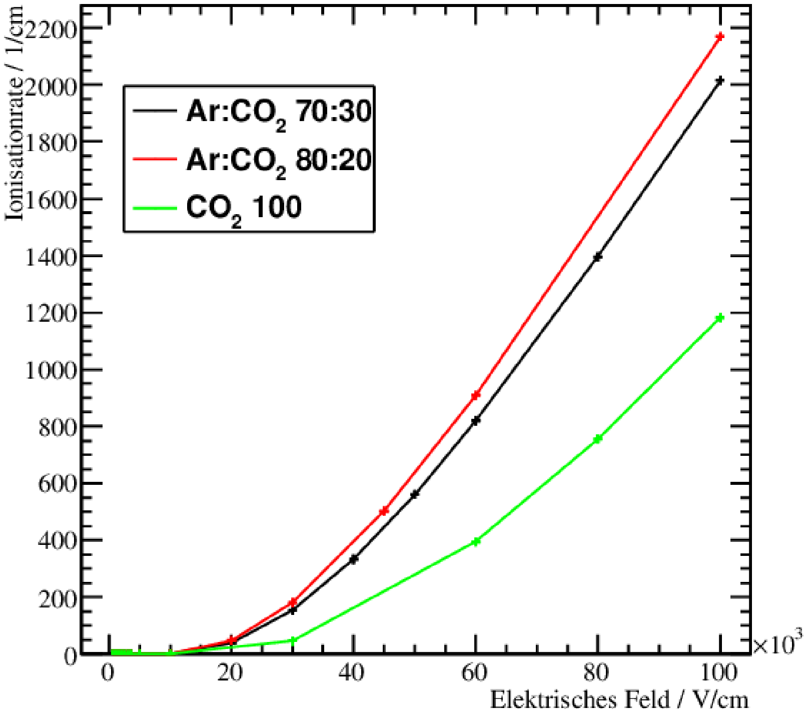
Integrating gives: $n = n_0\, e^{\alpha x}$

The definition of the gas gain is then $G = n/n_0 = e^{\alpha x}$

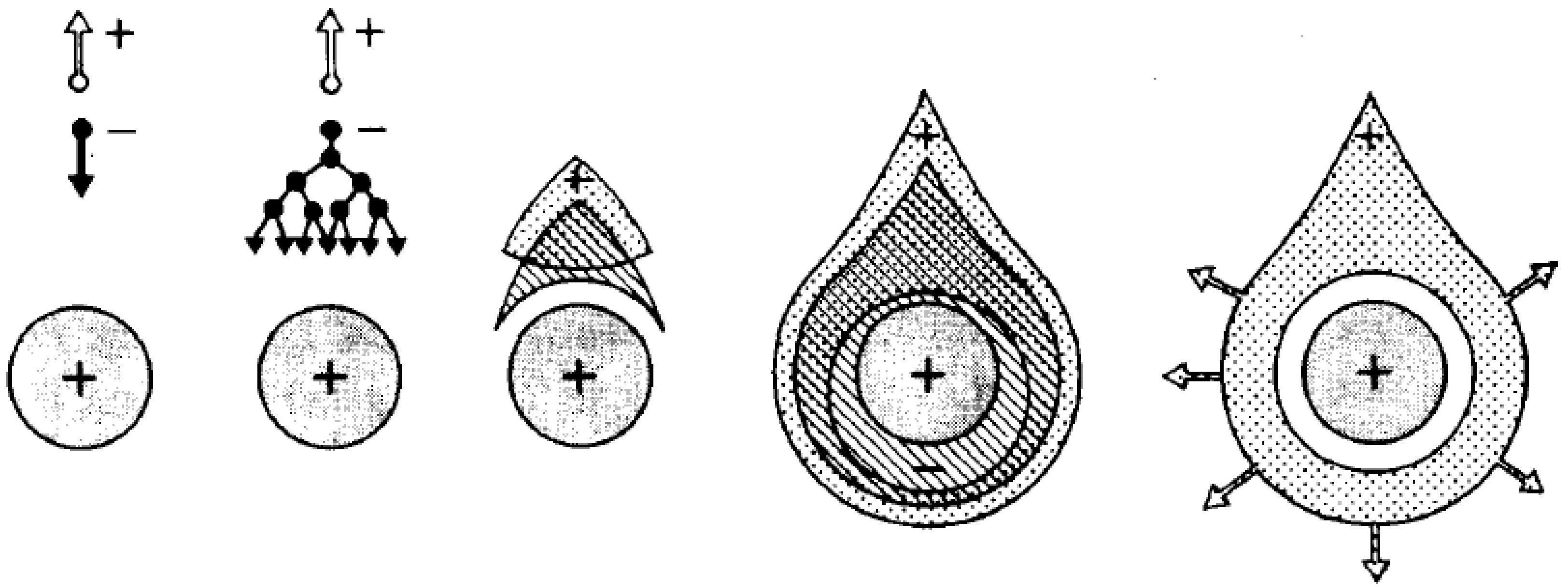
More general: $$G = \frac{n}{n_0} = e^{\int_{x_i}^{x_f} \alpha(E(x))dx}$$

where $\alpha = \sigma_{ion} n = 1/\lambda_{ion}$ is the first Townsend coefficient. It gives the number of generated free electrons/ions per unit of length.

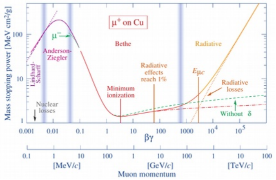
Typical values of *G* are 2000 – 10^6.



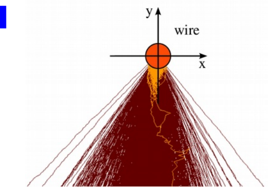
Schematic Sketch



Sauli 1977



Fluctuations of Gas Gain



Of course the gas gain of a single electron also varies because of statistical processes.

If a sufficiently high field is applied, the probability, that no ionization during the drift path *s* happens is: $P(1,s)=e^{-\alpha s}$ (only one electron after *s*)

for exactly one ionization: $P(2,s)=e^{-\alpha s}(1-e^{-\alpha s})$ (two electrons after s)

for exactly *n*-1 ionizations: $P(n,s)=e^{-\alpha s}(1-e^{-\alpha s})^{n-1}$ (*n* electrons after s)

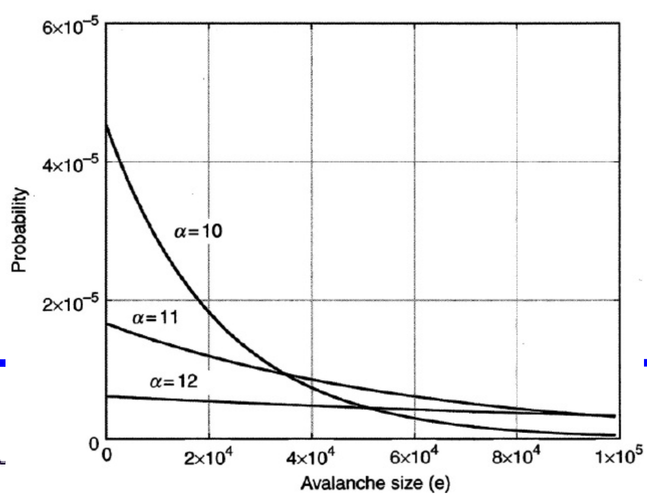
Redefining $e^{\alpha s}=\bar{n}$ as the average number of electrons, one gets

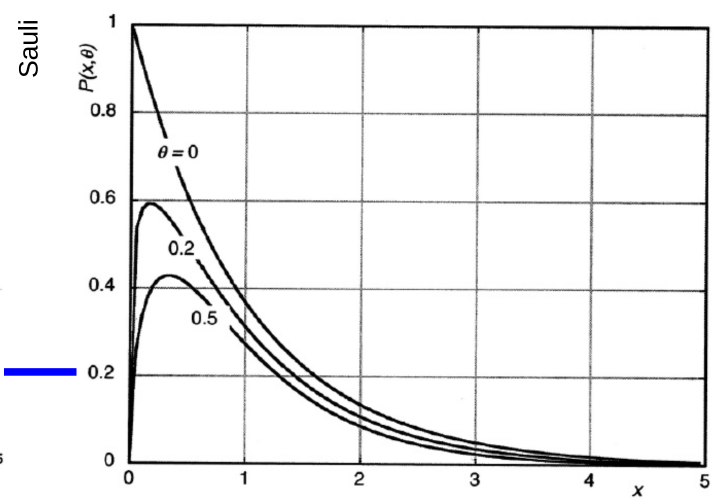
$P(n,s)=1/\bar{n}\,(1-1/\bar{n})^{n-1} \approx e^{-n/\bar{n}}/\bar{n}$ (Furry's law)

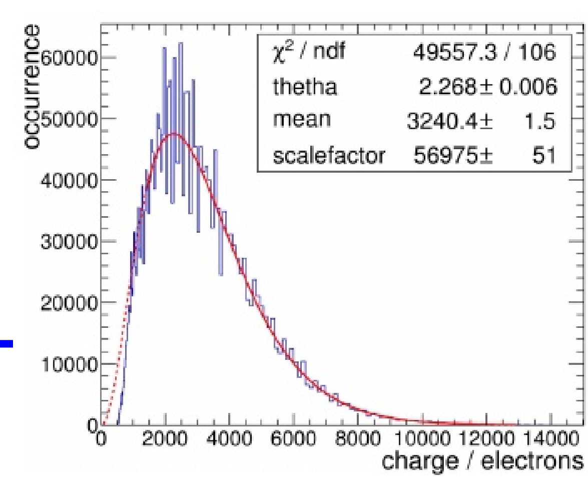
For high gas gains a peak shaped distribution develops:

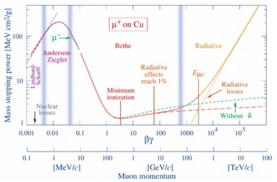
$$P(x,\theta) = [x(\theta+1)]^{\theta} e^{-x(\theta+1)}$$

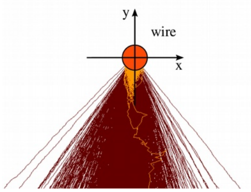
This is the Polya-distribution and in general describes the gas amplification.







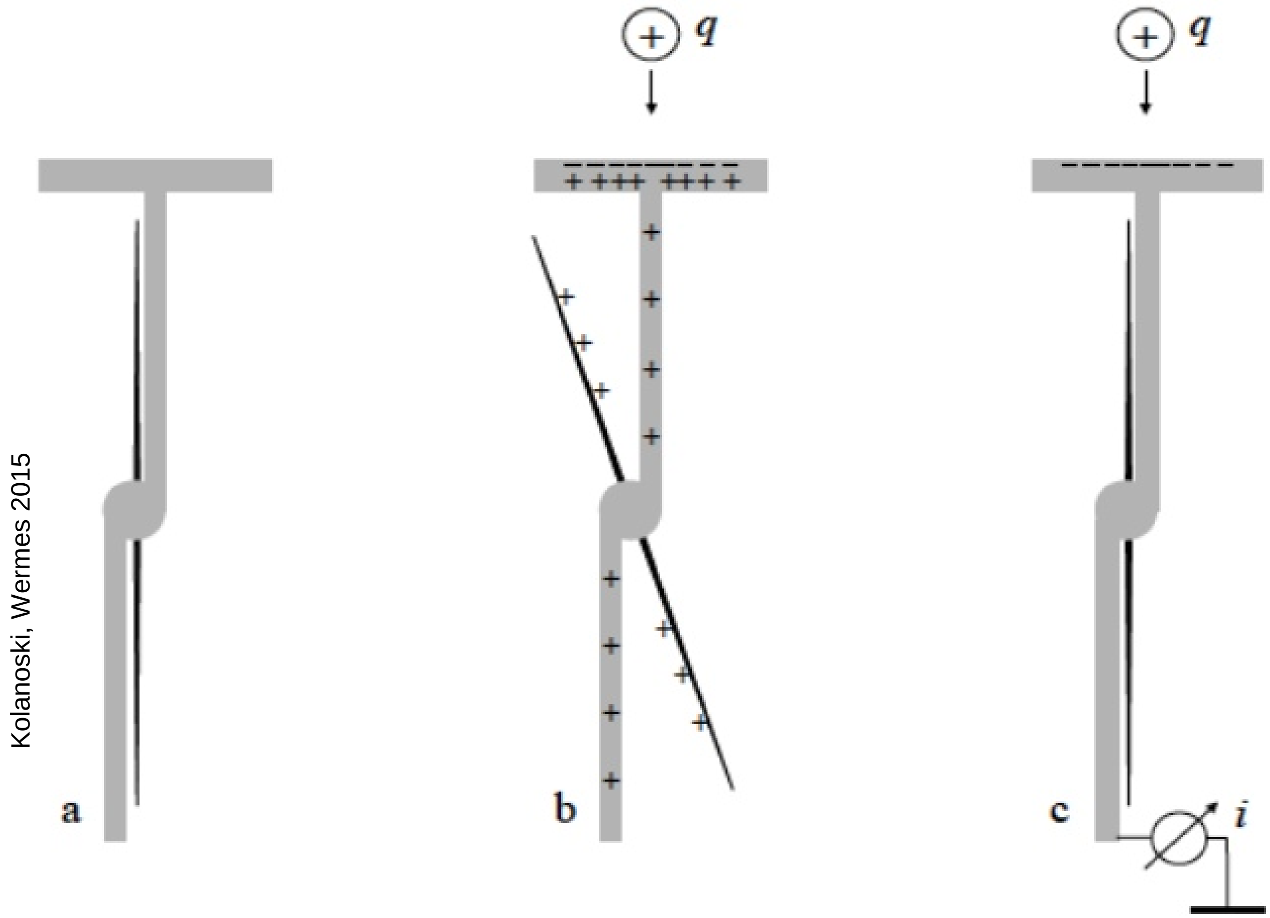




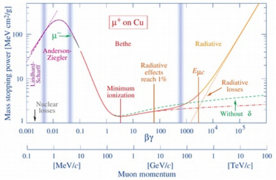
Signal Generation

Induction of Charge

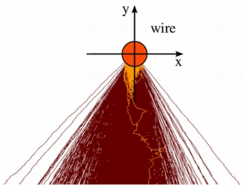
A charge moving towards a metal plane induces a current in the plane.



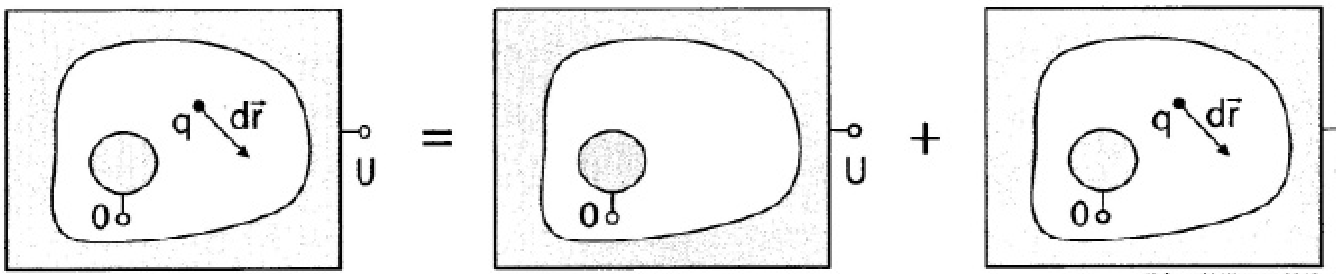
In the same way the signal of a detector starts as soon as charges are created and start to move to the readout electrodes.



Ramow's Theorem (I)



Setup, where inner conductor U=0 V, outer one connected to supply with U.
→ Charge q moves in the electric field.



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Work is given by $dW_q + dW_U + dW_E = 0$

→ power supply provides $dW_U = U\,dQ$,

The electrical field can be split in two components E_0 and E_q.

The work done on the q is given by its motion in E_0: $dW_q = qE_0\,dr$,

But the electric fields don't change their energy content: $dW_E = dW_{E0} + dW_{Eq} = 0$.

=> $dW_q + dW_U + dW_E = qE_0\,dr + U\,dQ = 0$

E_0 scales with U, but its form depends only on geometry: $\phi_w = \phi_0/U$ $\quad E_w = -\nabla\phi_w$.

The induced charge is then $dQ = -\frac{\vec{E}}{U}d\vec{r} = -q\vec{E}_w d\vec{r}$ $\qquad i_S = -\frac{dQ}{dt} = q\vec{E}_w\vec{v}$ $\quad$ with $\quad \vec{v} = \frac{d\vec{r}}{dt}$

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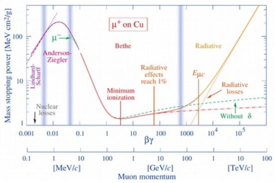
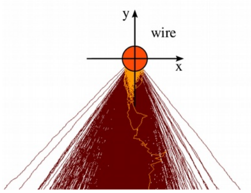
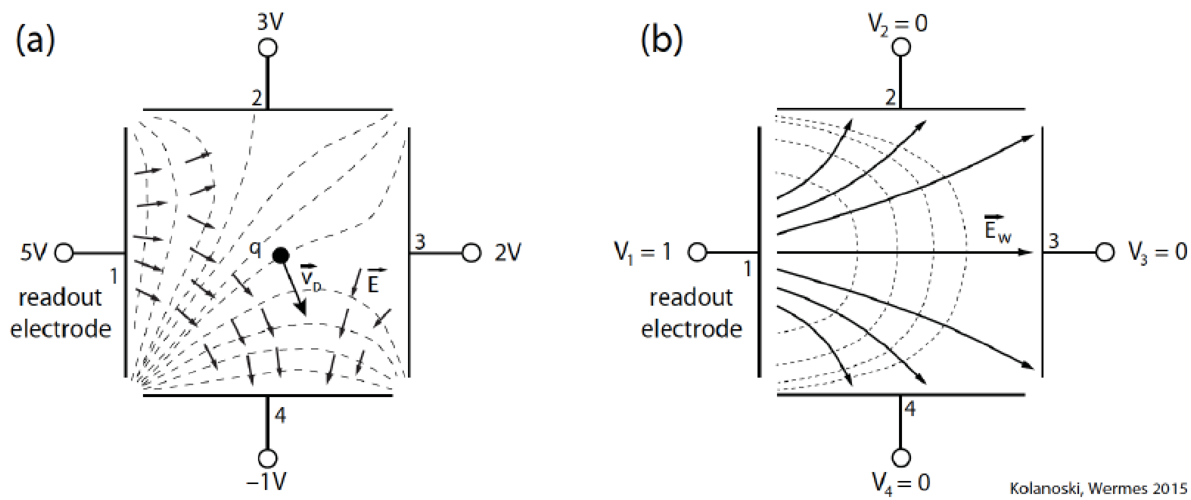




Multi-Electrode Problem

If we are interested in reading out U_1 only

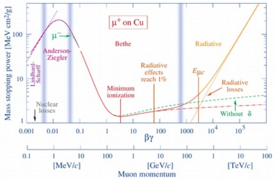
→ use superposition principle for the potentials of all electrodes



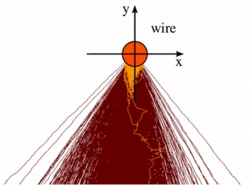
$$\phi_0(\vec{r}) = \sum_{i=1}^{k} \phi_i(\vec{r})$$

Potential ϕ_i of each electrode can be calculated independently by setting the voltage of all electrodes to 0 V, except the one under investigation: U

This can be done independently of U~1 → weighting potential ϕ_w.



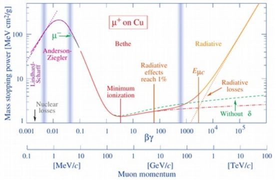
Summary



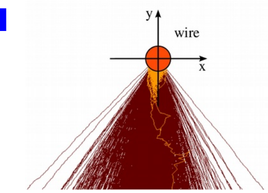
- Induced charge $$dQ = -q\vec{E}_w d\vec{r}$$

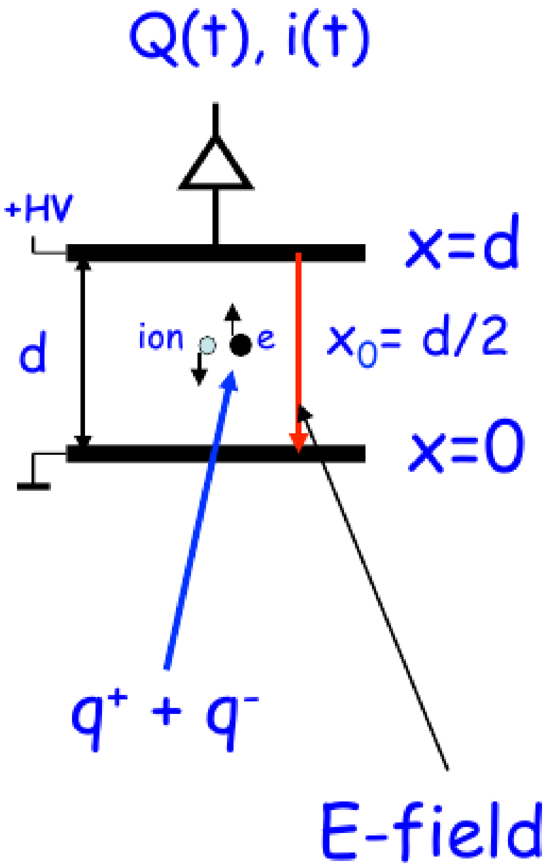
- Induced signal current $$i_S = q\vec{E}_w \vec{v}$$

- Weighting field E_w tells how $q(t)$ couples to an electrode and is independent of U or E_0, but depends only on the electrode geometry $[E_w]$=1/m
- Weighting field is derived by putting all electrodes to 0V except one → 1
- $v(t)$ describes the movement of q and thus the time dependence of the pulse shape; it depends on U or E_0.
- Both charges induce signal an both/all electrodes
- On every electrode the total charge $Q_S^{tot} = Q_S^- + Q_S^+ = -Ne$ is induced.



Example: Two Parallel Plates (I)





Assuming infinitely long plates, the electric field is homogenous

$$\vec{E} = -\frac{U}{d}\vec{e}_x \quad C = \frac{\epsilon\epsilon_0 A}{d}$$

And the weighting field is $\vec{E}_w = -\frac{1}{d}\vec{e}_x$

Resulting in a current for both charges:

$$i_S^{\pm} = q^{\pm}\vec{E}_w\vec{v}^{\pm} = -\frac{q^{\pm}}{d}\frac{\vec{x}}{x}\vec{v}^{\pm} = \frac{e}{d}v^{\pm}$$

Both charges contribute with the same polarity to the signal on the readout electrode.

Integrating over the total drift time: $T^+ = x_0/v^+$ and $T^- = (d-x_0)/v^-$ gives the total charge:

$$Q_S^{tot} = Q_S^- + Q_S^+ = -\frac{e}{d}\left(\int_0^{T^-} v^- dt + \int_0^{T^+} v^+ dt\right) = -\frac{e}{d}v^-\left(\frac{d-x_0}{v^-}\right) - \frac{e}{d}v^+\left(\frac{x_0}{v^+}\right) = -e$$

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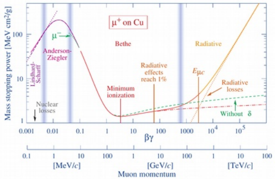
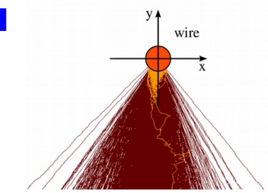


Example: Two Parallel Plates (II)



The voltage signal can be derived from

$$du_S^{\pm} = \frac{1}{C_D} dQ_S^{\pm} = -\frac{e}{C_D}\vec{E}_w d\vec{r} = -\frac{e}{C_D d} v^{\pm} dt$$

Which gives:

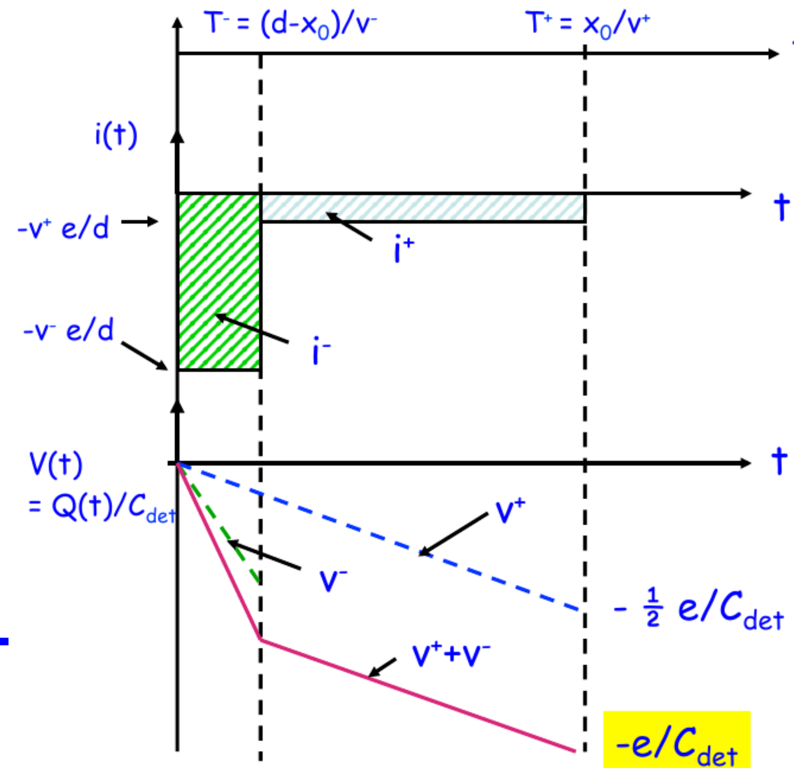
$$\text{for } 0 < t < T^- : \qquad u_s(t) = -\frac{e}{C_D d}\left(v^- + v^+\right) t$$

$$\text{for } T^- < t < T^+ : \qquad u_s(t) = -\frac{e}{C_D d}\left(d - x_o + v^+ t\right)$$

$$\text{for } t > T^+ : \qquad u_s(t) = -\frac{e}{C_D d}$$

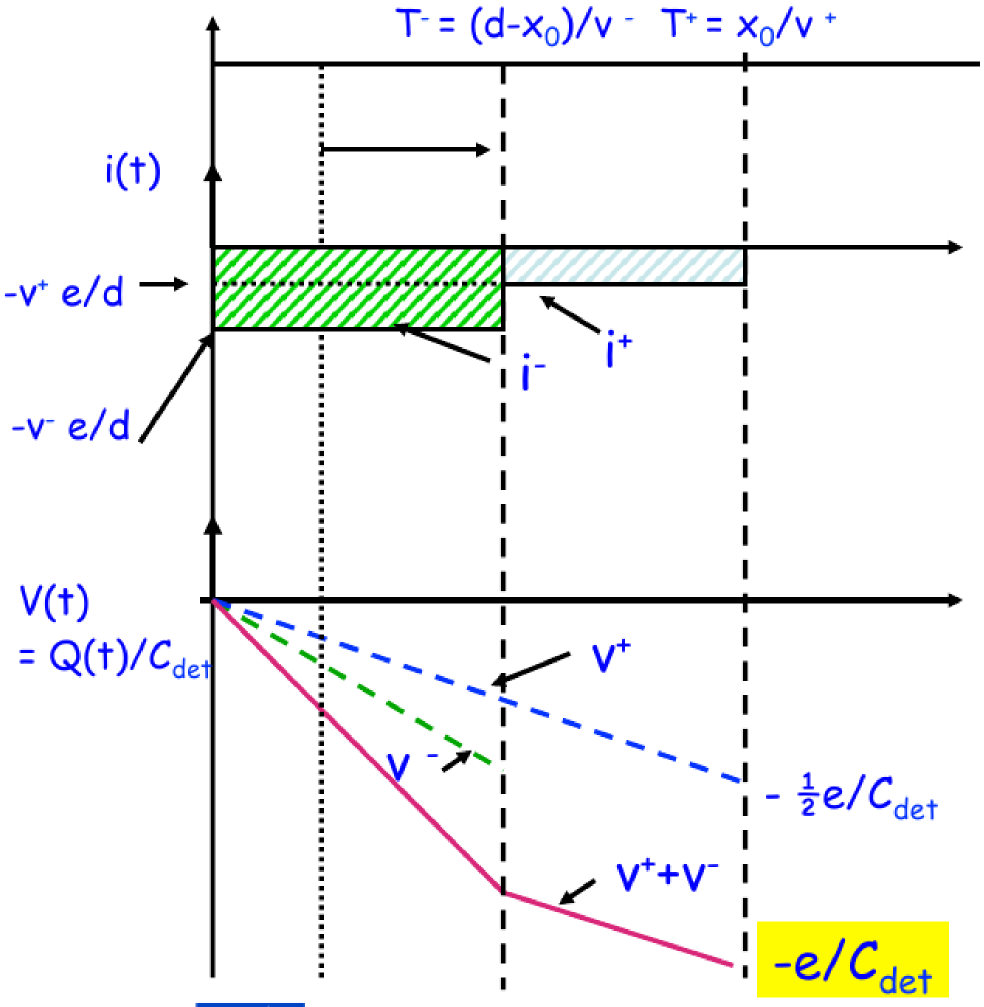
Signal depends on the relative drift velocities v^+ and v^- and the position of creation x_0.

1.) $v^- \approx 4v^+$, $x_0 = d/2$:

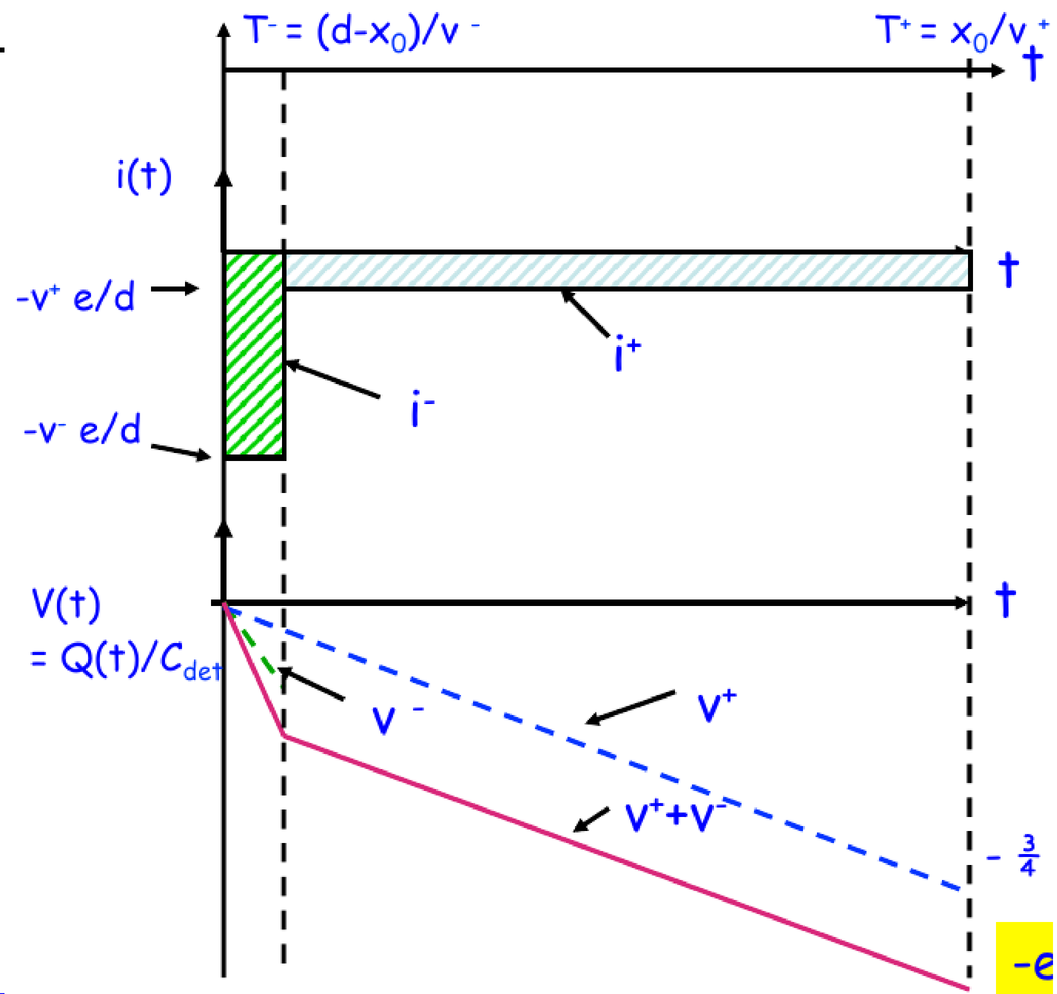


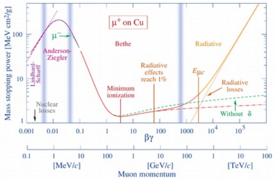
Example: Two Parallel Plates (III)

2) $v^{-} \approx v^{+}$, $x_0 = d/2$:

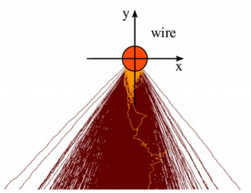


3) $v^{-} \approx 4v^{+}$, $x_0 = 3d/4$:





Example: Two Parallel Plates (IV)



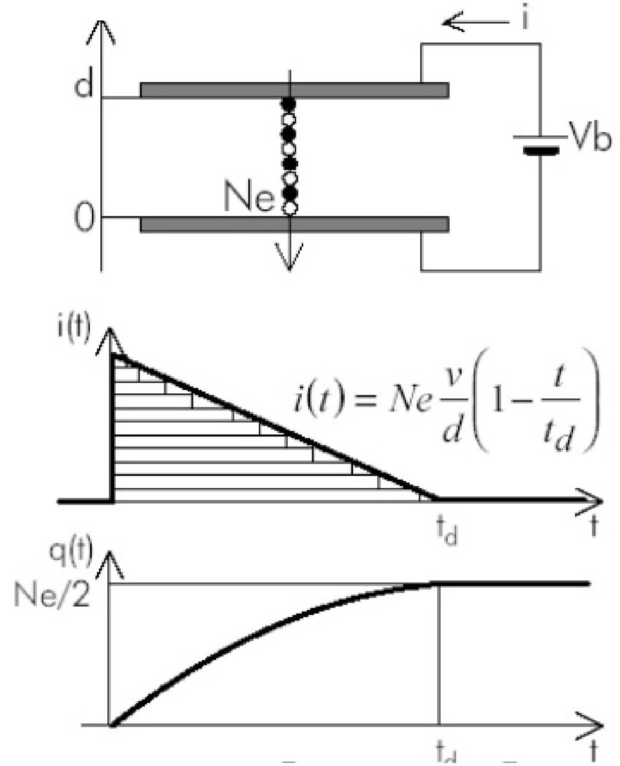
A track gives a continuous distribution of N charges: $\rho(x) = Ne/d\, dx$

$$i_S^-(t) = \frac{v^-}{d}\int_{v^-t}^{d}\frac{Ne}{d}dx = \frac{v^-}{d}\frac{Ne}{d}d - \frac{v^-}{d}\frac{Ne}{d}v^-t = \frac{1}{T^-}Ne - \frac{1}{T^-}Ne\frac{v^-t}{d} = \frac{Ne}{T^-}\left(1-\frac{t}{T^-}\right)$$

$$i_S^+(t) = \frac{v^+}{d}\int_0^{d-v^+t}\frac{Ne}{d}dx = \frac{1}{T^+}Ne\left(\frac{d-v^+t}{d}-0\right) = \frac{Ne}{T^+}\left(1-\frac{t}{T^+}\right)$$

The voltage signals is then

$$u_S^\pm(t) = \frac{Q_S^\pm}{C} = -\frac{1}{C}\int_0^t i_S^\pm(t')dt' = -\frac{Ne}{C}\left(\frac{t}{T^\pm}-\frac{1}{2}\left(\frac{t}{T^\pm}\right)^2\right)$$



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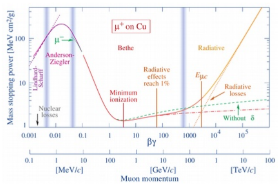
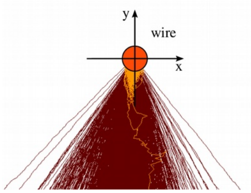
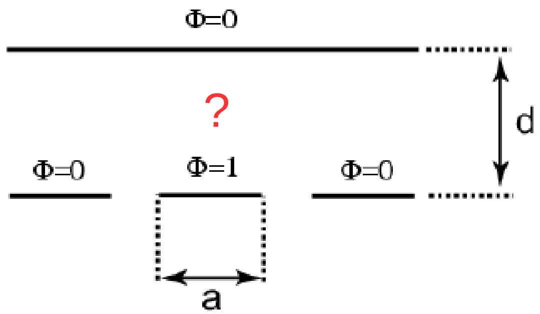


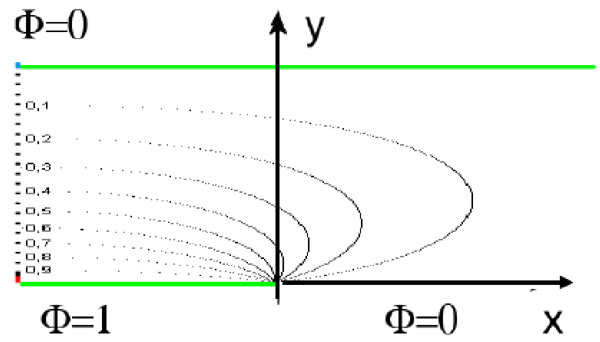
Segmented Electrode (I)



For segmented electrodes, the calculation of the weighting field is not trivial:

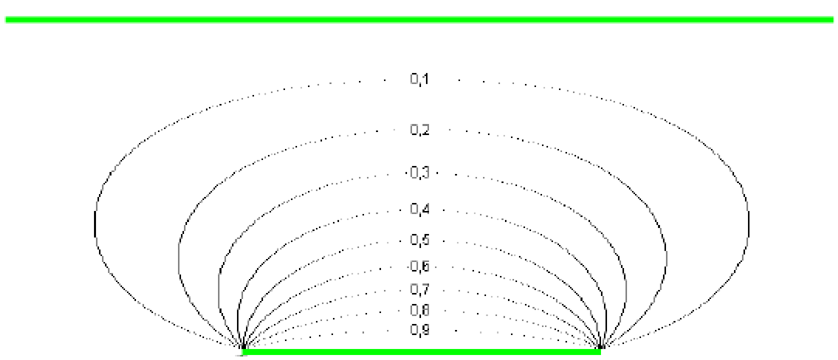


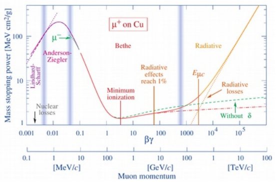
Some complicated math (conformal mapping….)



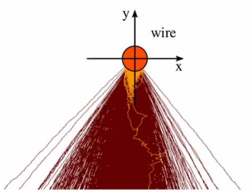
And in total:

$$\boxed{\Phi(x,y) = \frac{1}{\pi}\arctan\frac{\sin(\pi y)\cdot\sinh(\pi\frac{a}{2})}{\cosh(\pi x)-\cos(\pi y)\cosh(\pi\frac{a}{2})}}$$

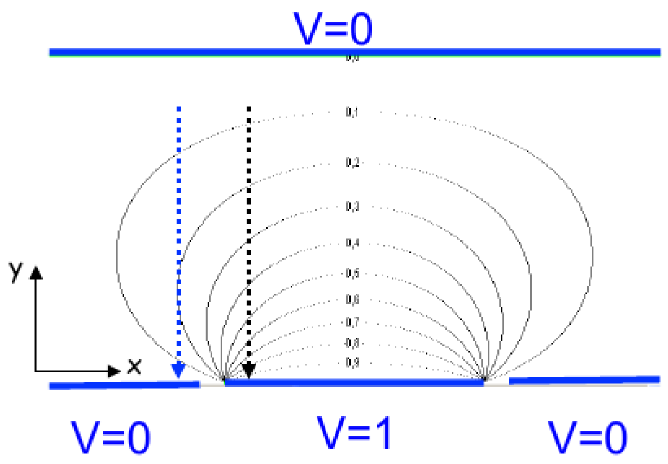


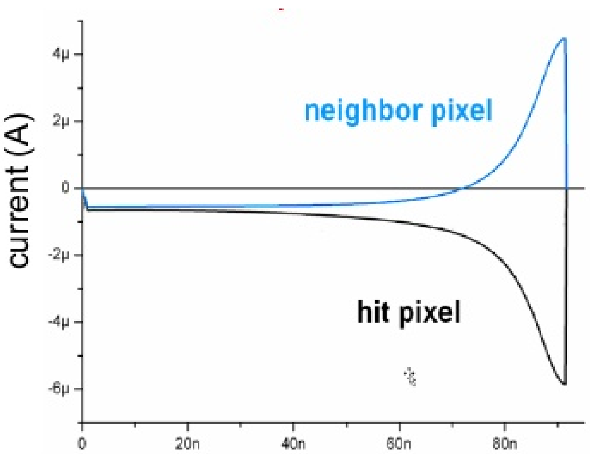


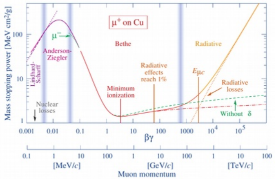
Segmented Electrode (II)



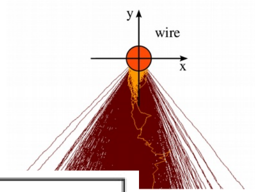
- For small pixels/thin strips, the weighting field reaches over the next electrode. Therefore, also neighboring electrodes also see some induced signal, even if they are not hit.
- At the beginning (charge far away), neighboring pixel sees almost as much signal as the central pixel
- When charge is close to readout plane, the induced signal on neighboring electrodes decreases.
- Most of the charge is induced, when pixel is near to the pixel because the weighting field is most dense there. → Small pixel effect.

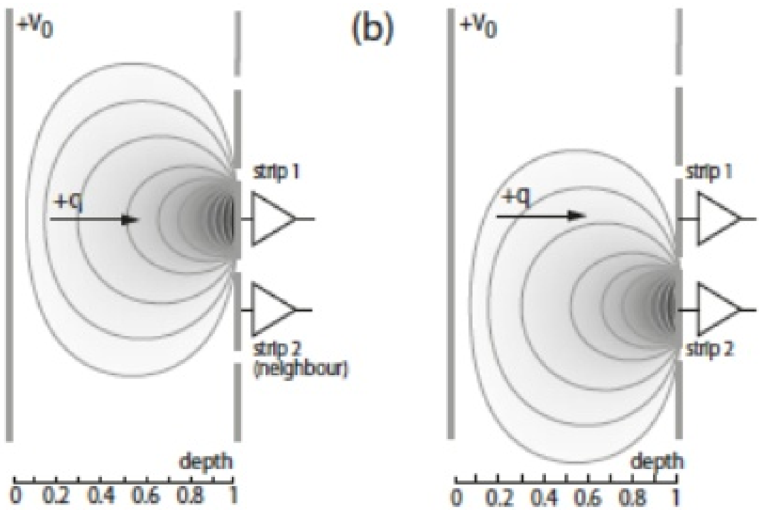


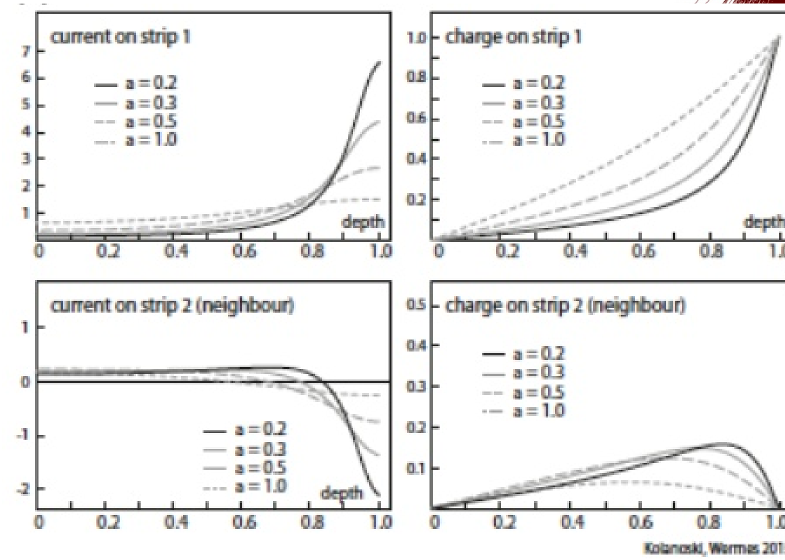


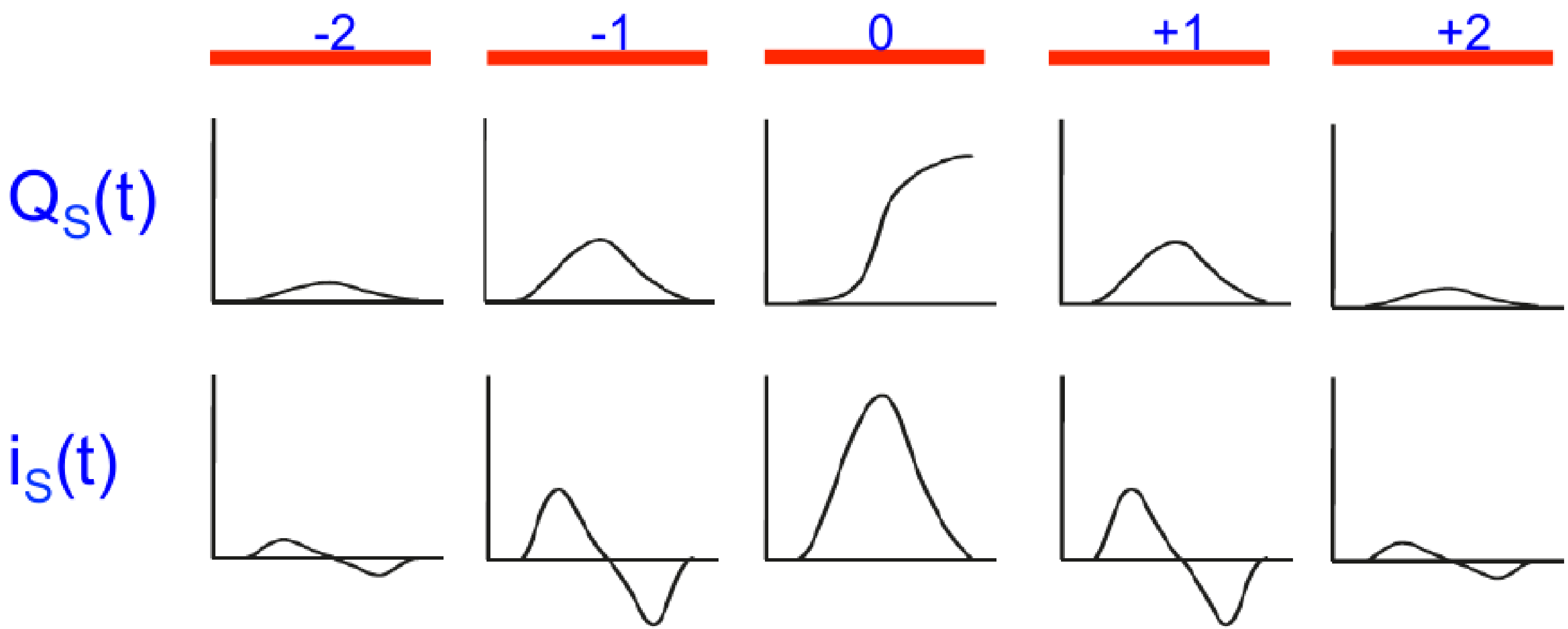


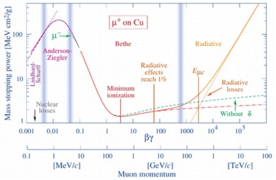
Segmented Electrode (III)



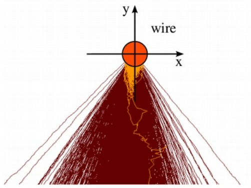








Literature



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7.) lecture notes of Prof. N. Wermes