BND SCHOOL 2017

Lecture 1 QFT

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QFT

- PESKIN SCHROEDER
- D. TONG LECTURES on QFT
- M. MAGGIORE (OXFORD)

PLAN of LECTURES

- CLASSICAL FIELDS
- QUANTUM FIELD THEORY (FREE)
- INTERACTION PICTURE
- ELEMENTARY PROCESSES

LECTURE 1

- BASICS LORENTZ INVARIANCE
- CLASSICAL FIELDS (SPIN 0, 1/2, 1)
- SYMMETRIES & CONS. LAWS

BASICS

- NATURAL UNITS $\hbar = c = 1$

- LORENTZ SYMMETRY

$$X^\mu = (t, x, y, z)$$

$$X^\mu \to X'^\mu = \Lambda^\mu{}_\nu X^\nu \qquad \mu = 0, .. 3$$

$$\eta_{\mu\nu} X^\mu X^\nu = t^2 - x^2 - y^2 - z^2 \qquad \eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\Rightarrow \quad \eta_{\mu\nu} (\Lambda^\mu{}_\rho X^\rho)(\Lambda^\nu{}_\sigma X^\sigma) = \eta_{\rho\sigma} X^\rho X^\sigma$$

$$\Rightarrow \quad \boxed{\eta_{\mu\nu} \Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma = \eta_{\rho\sigma}}$$

$$\Rightarrow \det \Lambda = \pm 1$$
$$\Lambda^0{}_0 \geq 1$$

$$\Lambda^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}$$

ANTYSYMMETRIC

6 INDEPENDENT D.O.F.

$$\Lambda = e^{-\frac{i}{2}\omega_{\mu\nu} J^{\mu\nu}}$$

$$\left[J^{\mu\nu}, J^{\rho\sigma}\right] = i\left[\eta^{\nu\rho} J^{\mu\sigma} - \eta^{\mu\rho} J^{\nu\sigma} - \eta^{\nu\sigma} J^{\mu\rho} + \eta^{\mu\sigma} J^{\nu\rho}\right]$$

LORENTZ TRASF OF FIELDS

$$\phi^a(x) \longrightarrow D[\Lambda]^a{}_b \, \phi^b(\Lambda^{-1}x)$$

$$D[\Lambda_1]D[\Lambda_2] = D[\Lambda_1\Lambda_2]$$

SCALAR FIELD

$$\phi(x) \to \phi'(x) = \phi(\Lambda^{-1}x)$$

$$D[\Lambda] = e^{\frac{1}{2}\omega_{\mu\nu}(J^{\mu\nu})_a{}^b}$$

ACTION PRINCIPLE

$q_i(t) \qquad i = 1 \ldots N$

$\mathcal{L}(q_i, \dot{q}_i) \qquad \dot{q}_i = \frac{\partial}{\partial t} q_i$

$S = \int dt\, \mathcal{L}(q_i, \dot{q}_i)$

$\delta S = 0 \implies$ EQUAT. OF MOTION (EL EQN)

$\mathcal{L}(\phi, \partial_\mu \phi) \qquad \underline{S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)}$

$\delta S = 0$ UNDER $\delta\phi$

$$\delta S = \int d^4x \left[\frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \underbrace{\delta(\partial_\mu \phi)}_{= \partial_\mu(\delta\phi)} \right] =$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\mu (\delta\phi) = \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi \right] - \left[\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right] \delta\phi$$

$$\delta S = \int d^4x \left\{ \underbrace{\left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right]}_{=0} \delta\phi + \underbrace{\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi \right]}_{\text{TOTAL DERIVATIVE}} \right\}$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \qquad \text{E.L. EQN}$$

REAL SCALAR FIELD

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} M^2 \phi^2 \qquad \text{(FREE)}$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -M^2 \phi \qquad \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi$$

$$\overset{\text{E.L. EQN}}{\Longrightarrow} \partial_\mu \partial^\mu \phi + M^2 \phi = 0 \iff \underset{\text{K.G.}}{(\Box + M^2)\phi = 0}$$

$$\Pi = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} = \partial^0 \phi = \dot{\phi}$$

$$\Rightarrow H = \Pi \dot{\phi} - \mathcal{L} = \frac{1}{2} \Pi^2 + \frac{1}{2}(\vec{\nabla} \phi)^2 + \frac{1}{2} M^2 \phi^2$$

COMPLEX SCALAR FIELD

$\phi(x)\,,\ \phi^*(x)$

$$\phi = \frac{1}{\sqrt{2}}\left(\varphi_1 + i\,\varphi_2\right)$$

REAL

$$S = \int d^4x \left\{ (\partial_\mu \phi^*)(\partial^\mu \phi) - M^2 \phi^* \phi \right\}$$

$\overset{\text{E.L.}}{\Longrightarrow}$

$$(\Box + M^2)\,\phi = 0$$

$$(\Box + M^2)\,\phi^* = 0$$

DIRAC FIELD

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \times \mathbb{1}_{n\times n} \qquad \text{C.A.}$$

$$S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$$

↳ SATISFY LORENTZ ALGEBRA

4x4 REPS of γ MATRICES

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\psi^\alpha(x) \longrightarrow \left(e^{-\frac{i}{2}\omega_{\mu\nu}S^{\mu\nu}}\right)^\alpha{}_\beta \, \psi^\beta(\Lambda^{-1}x) \qquad \alpha, \beta = 1..4$$

DIRAC LAGR.

$\cancel{\psi^{+}\psi}$

$\psi^{+}\gamma^{0}\psi$ L.I. SCALAR

$\psi^{+}\gamma^{0}\gamma^{\mu}\psi$ LORENTZ VECTOR

$$S = \int d^4x \; \psi^{+}(x)\gamma^{0}\left(i\gamma^{\mu}\partial_{\mu} - M\right)\psi(x) \qquad \boxed{\bar{\psi} = \psi^{+}\gamma^{0}}$$

$$\overset{E.L.}{\Longrightarrow} \quad \left(i\gamma^{\mu}\partial_{\mu} - M\right)\psi = 0 \quad \text{DIRAC EQN.}$$

E.M. FIELD

$$A^\mu = (\varphi, \vec{A}) \qquad \vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \partial_\mu F^{\mu\nu} = 0$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

$$A_\mu(x) \to \Lambda_\mu{}^\nu A_\nu(\Lambda^{-1}x)$$

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} \cancel{+ M^2 A_\mu A^\mu} \right]$$

$$\overset{E.L.}{\Longrightarrow} \quad -\partial_\mu F^{\mu\nu} = 0$$

GAUGE INVARIANCE

$$\boxed{\begin{array}{c}\text{MAXW EQN INVARIANT} \\ A_\mu(x) \to A_\mu(x) - \partial_\mu \Theta(x)\end{array}}$$

- LORENTZ GAUGE $\partial_\mu A^\mu = 0$

$$0 = \partial_\mu F^{\mu\nu} = \Box A^\nu = 0$$