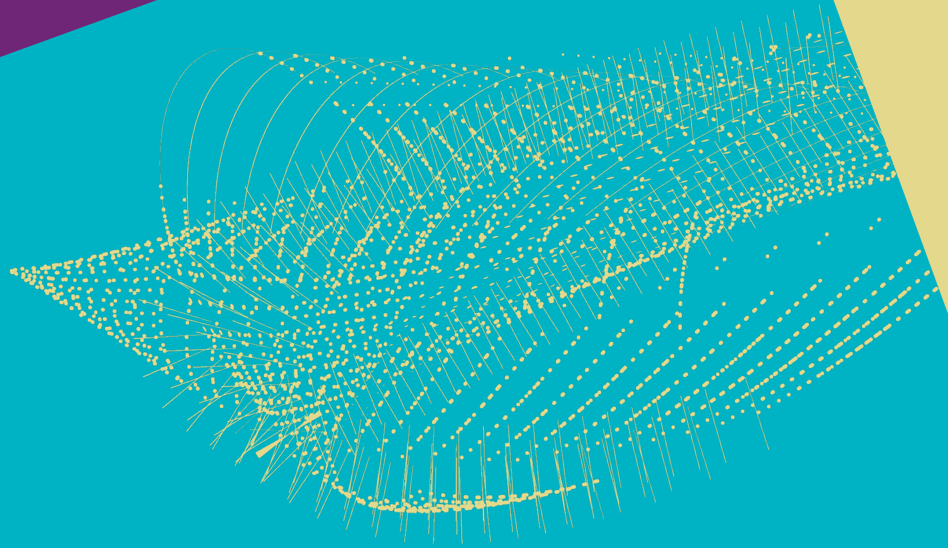
Nikhef

Maastricht University

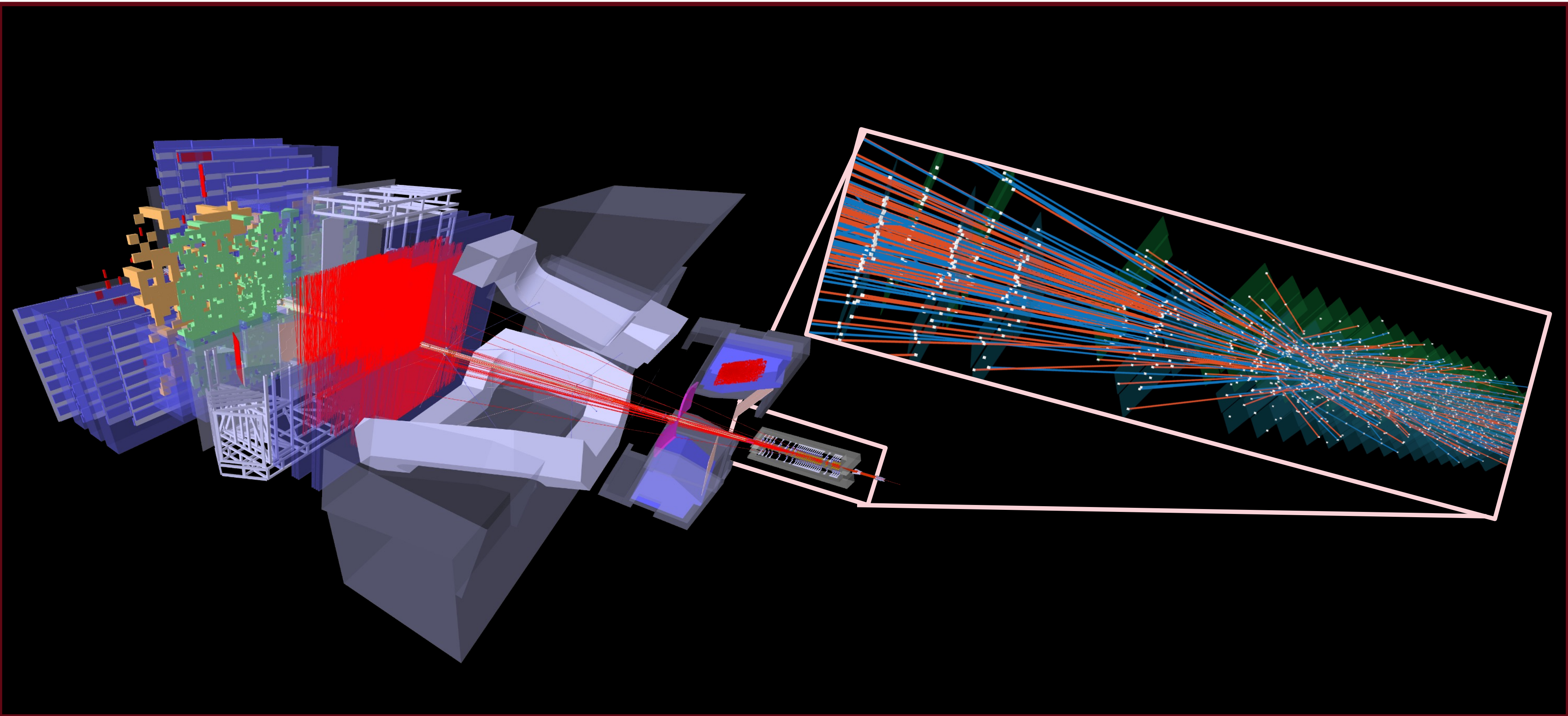
LHCb

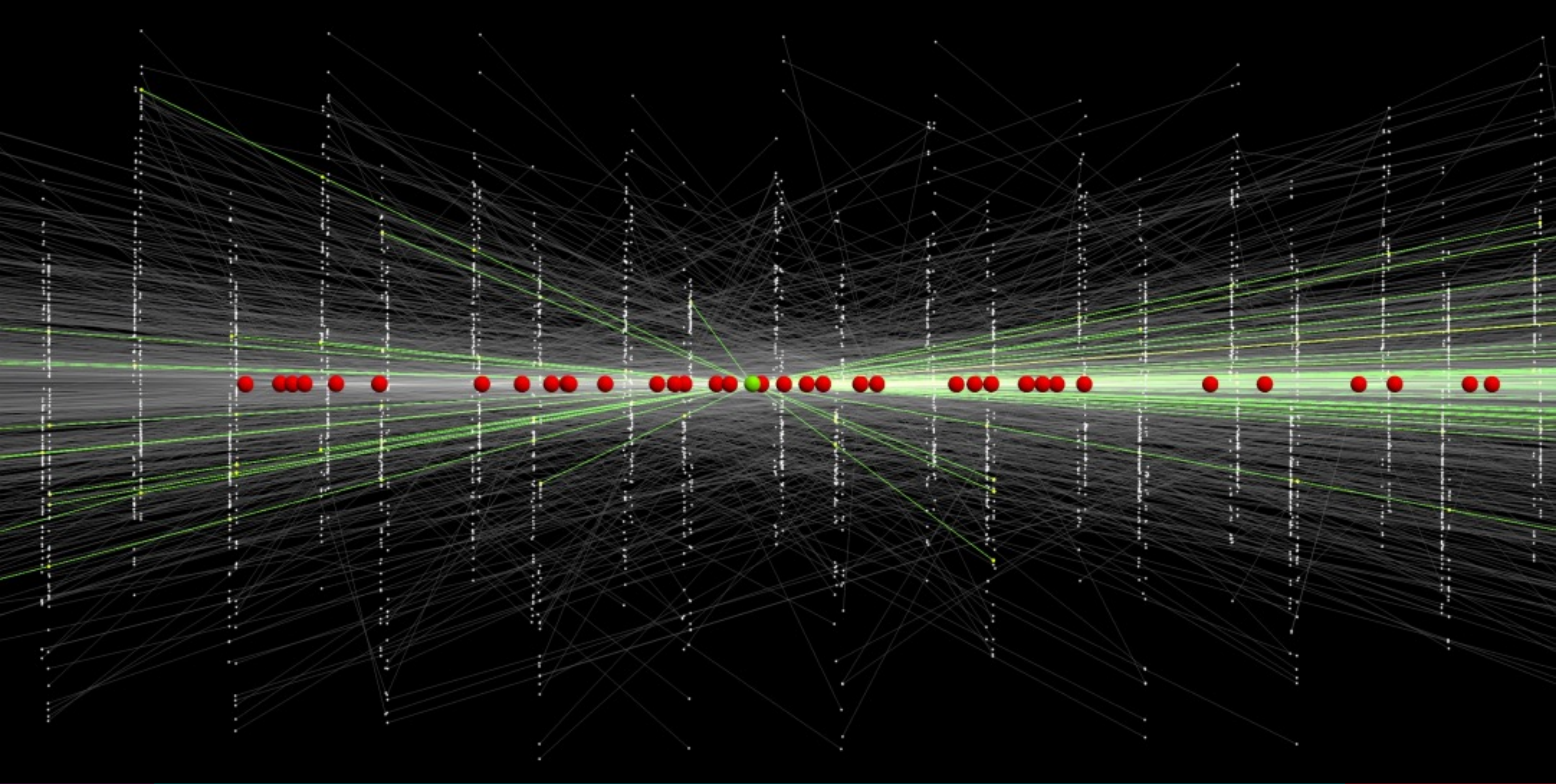


The 1-Bit Quantum Filter for particle track reconstruction

Xenofon Chiotopoulos
Maastricht University (DACS/GWFP) and Nikhef
Quantum Tracking Mini Workshop Maastricht 2026

R&D: LHCb Velo detector





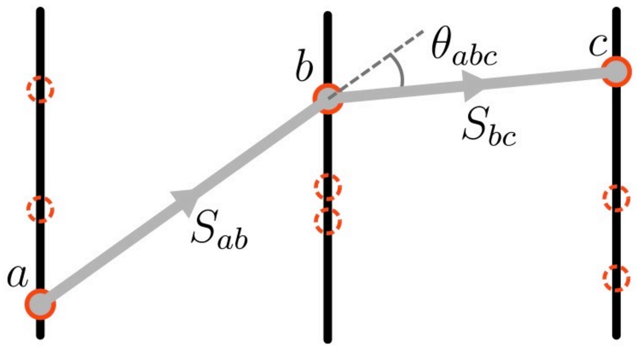
Method: “Global” algorithm for track finding

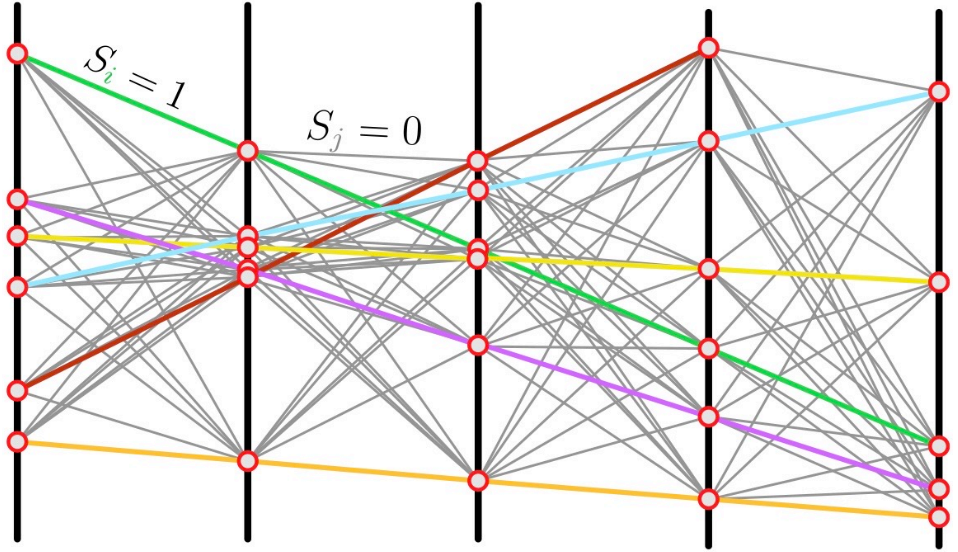
$$\mathcal{H}(S) = \sum_{abc} f(\theta) S_{ab} S_{bc} + \alpha \sum_{ab} S_{ab}^2 + \beta \sum_{ab} (1 - 2S_{ab})^2 \quad \Longleftrightarrow \quad \mathcal{H}(S) = \sum_{ij} A_{ij} S_i S_j + \sum_i b_i S_i \quad S_i \in \{0,1\}$$

Angular Term

Gap Term

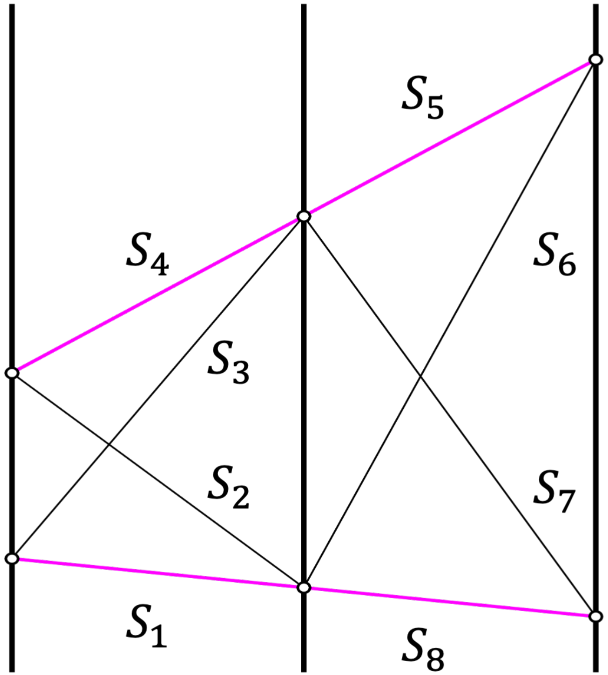
Spectral Term



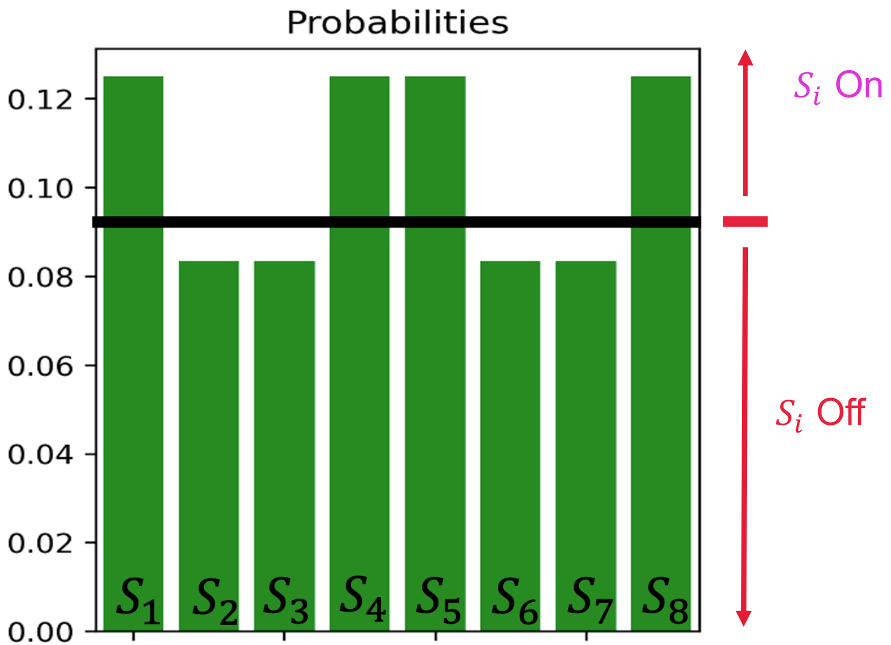


Method: "Global" algorithm for track finding

$$\mathcal{H}(S) = \sum_{abc} f(\theta) S_{ab} S_{bc} + \alpha \sum_{ab} S_{ab}^2 + \beta \sum_{ab} (1 - 2S_{ab})^2 \quad \Leftrightarrow \quad \mathcal{H}(S) = \sum_{ij} A_{ij} S_i S_j + \sum_i b_i S_i \qquad S_i \in \{0,1\}$$



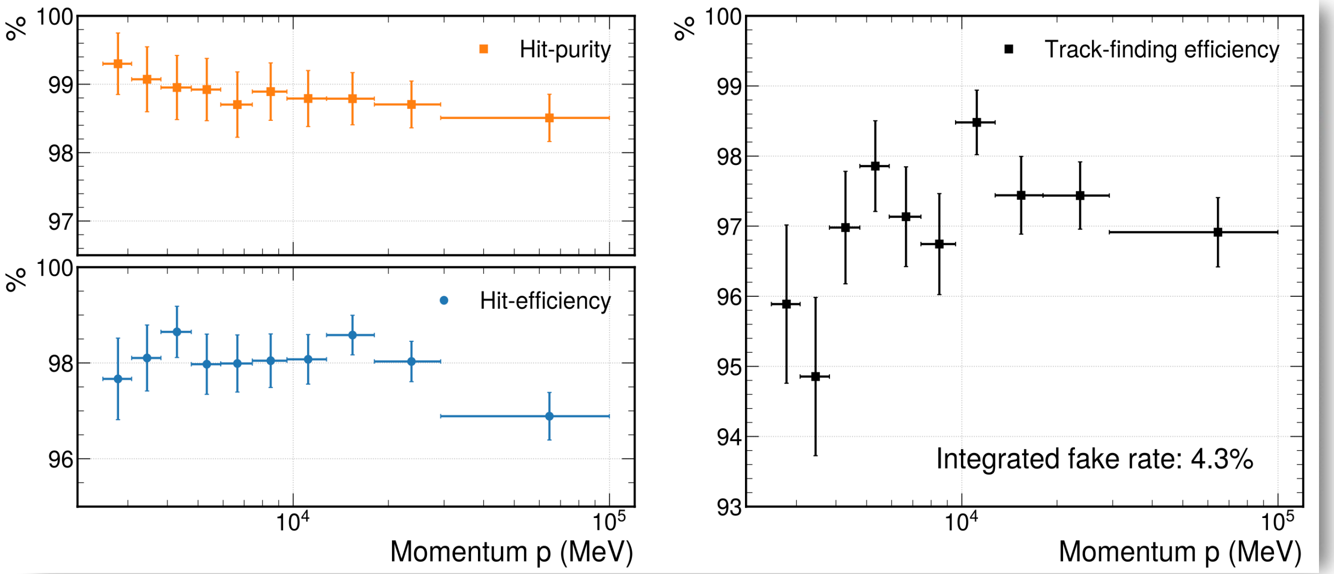
$$\nabla_S \mathcal{H} = 0 \Rightarrow \boldsymbol{S} = A^{-1} b$$



Method: "Global" algorithm for track finding

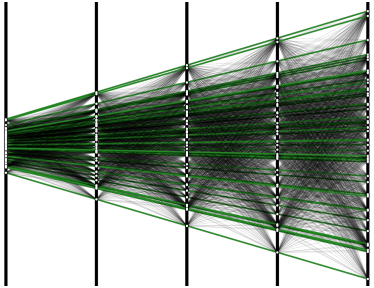
$$\mathcal{H}(S) = \sum_{abc} f(\theta) S_{ab} S_{bc} + \alpha \sum_{ab} S_{ab}^2 + \beta \sum_{ab} (1 - 2S_{ab})^2 \quad \Longleftrightarrow \quad \mathcal{H}(S) = \sum_{ij} A_{ij} S_i S_j + \sum_i b_i S_i \quad S_i \in \{0,1\}$$

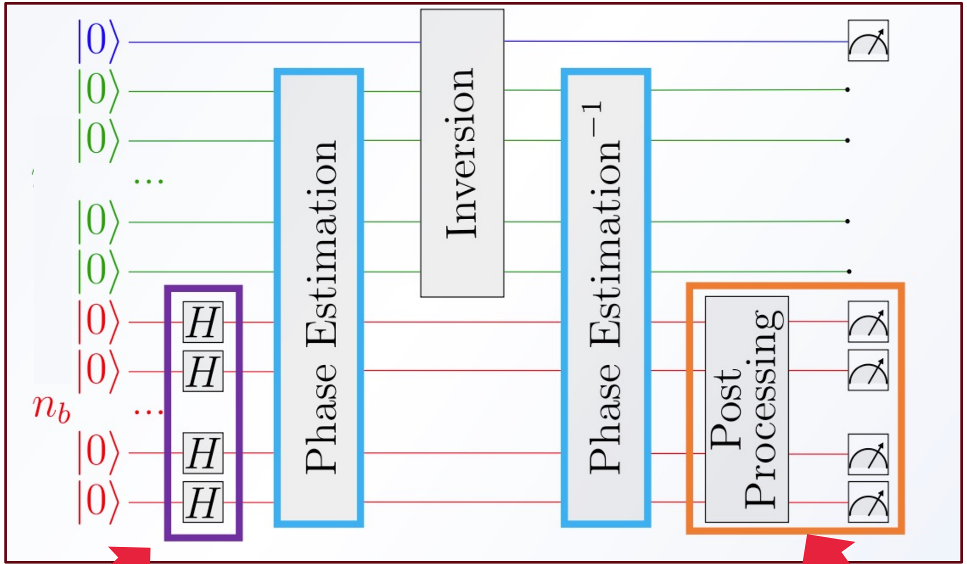
Tracking performance on LHCb simulated events

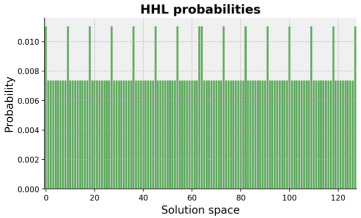


https://arxiv.org/pdf/2308.00619 [1]

The Quantum Algorithm: Harrow-Hassadim-Lloyd



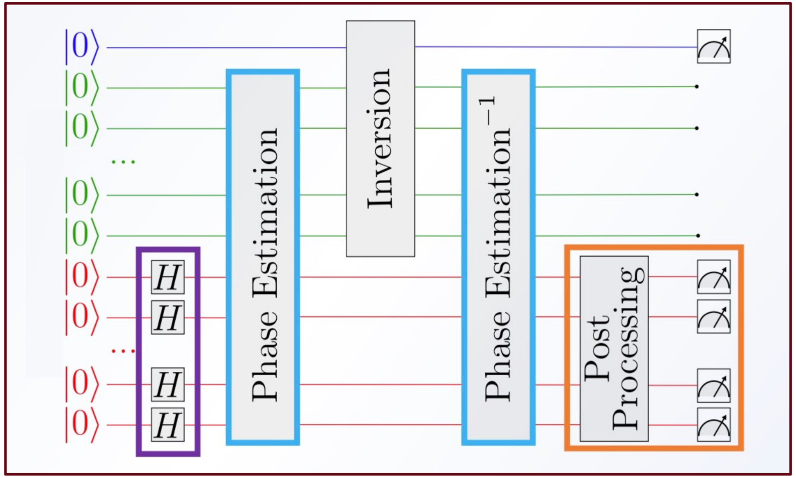


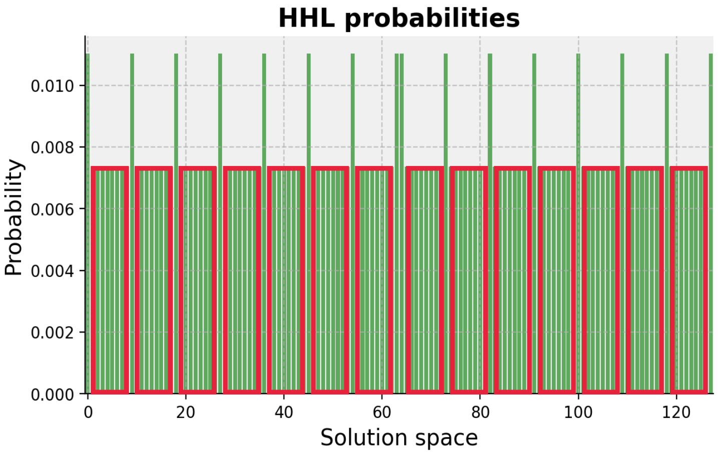


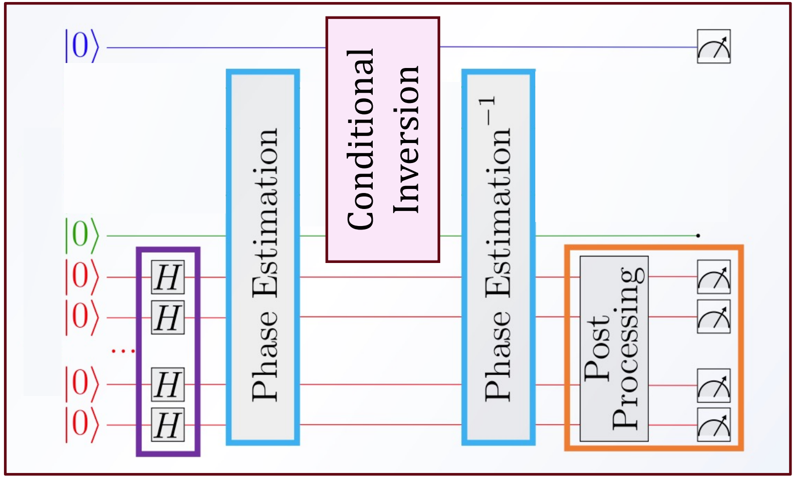
State Preparation $U(\mathbf{b})|0\rangle$
constant input vector

Phase Estimation e^{iAt}
extremely sparse matrix

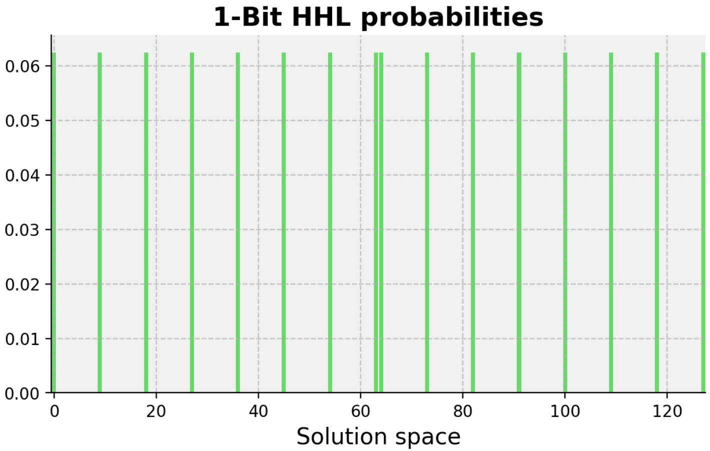
Read-Out $\langle S|M|S\rangle$
track parameters
collision vertices


|0⟩
Phase Estimation
Inversion
Phase Estimation^{-1}
Post Processing
H

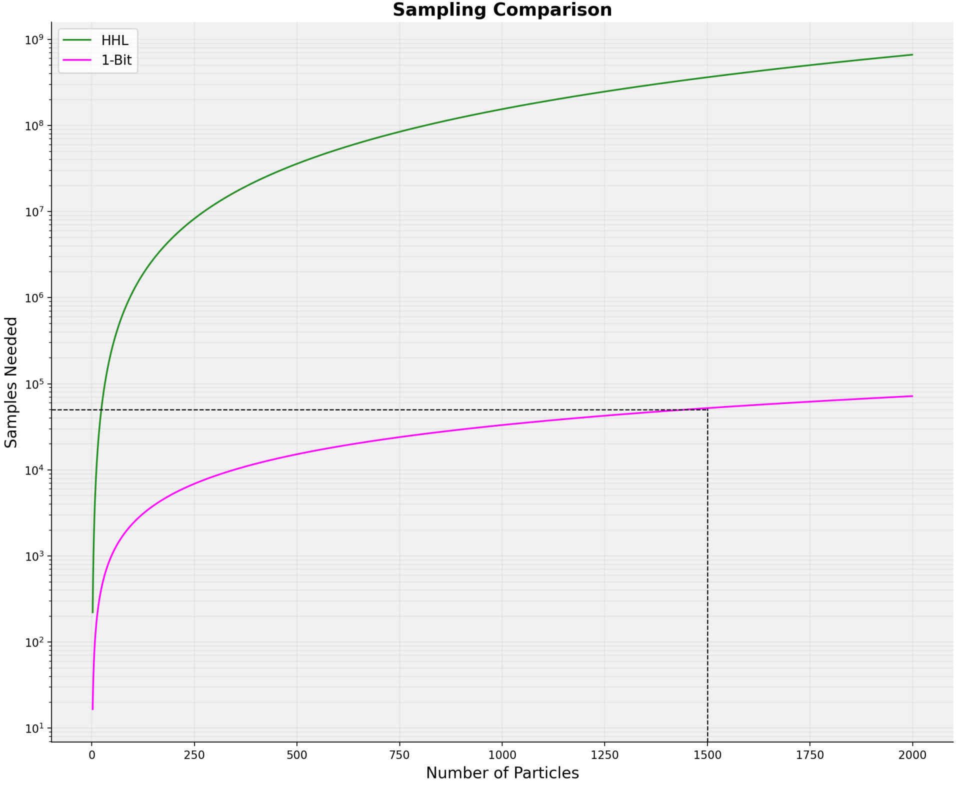

HHL probabilities
Probability
0.000
0.002
0.004
0.006
0.008
0.010
0
20
40
60
80
100
120
Solution space

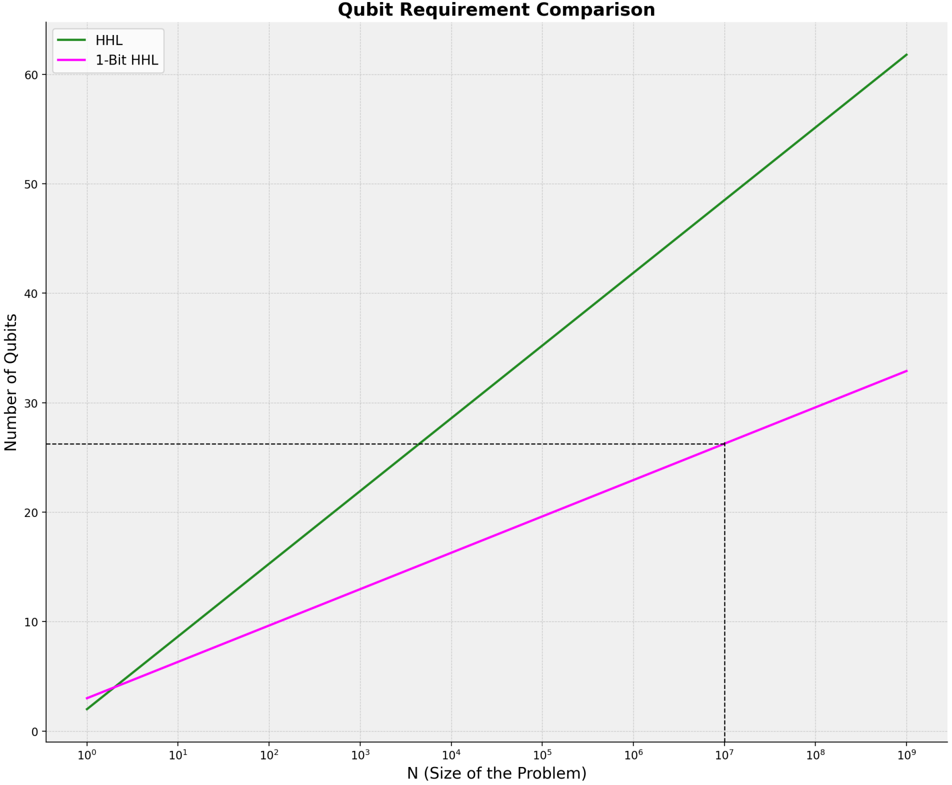

|0⟩
Phase Estimation
Conditional Inversion
Phase Estimation^{-1}
Post Processing
H

[2]

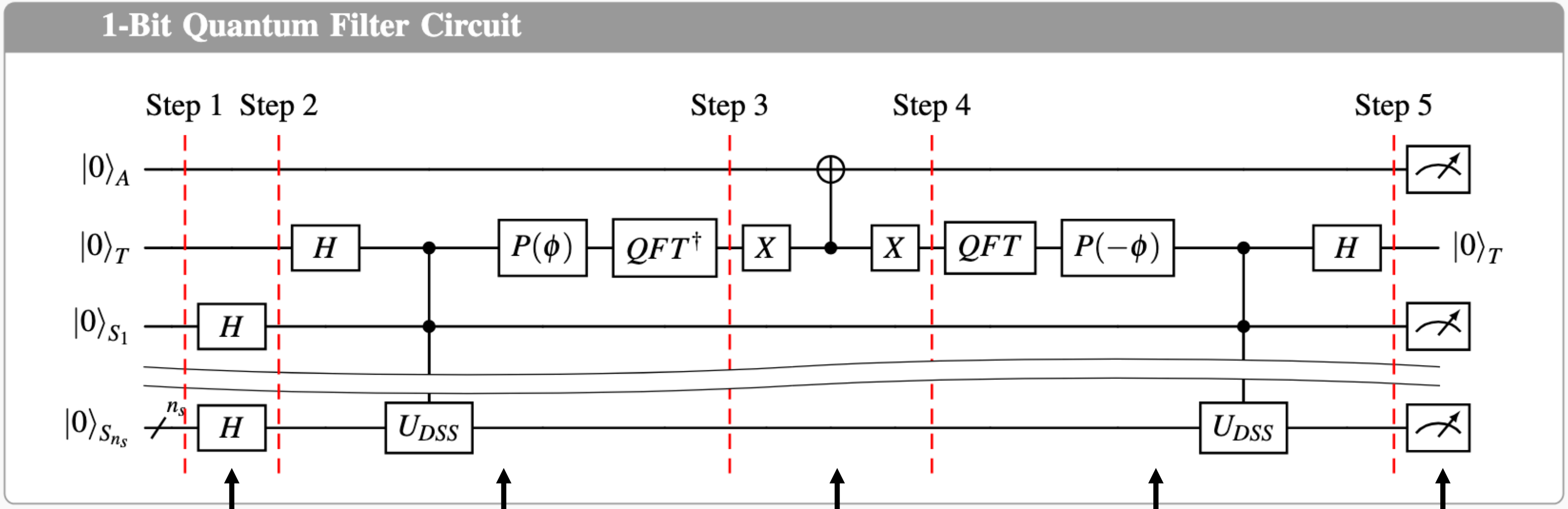

1-Bit HHL probabilities
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0
20
40
60
80
100
120
Solution space

Resource Complexity





The 1-Bit Quantum Filter



State Preparation

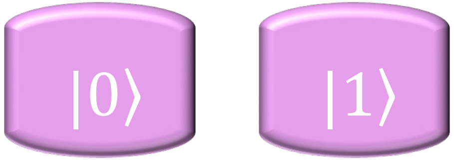
Phase Estimation

Controlled Inversion

Uncomputation

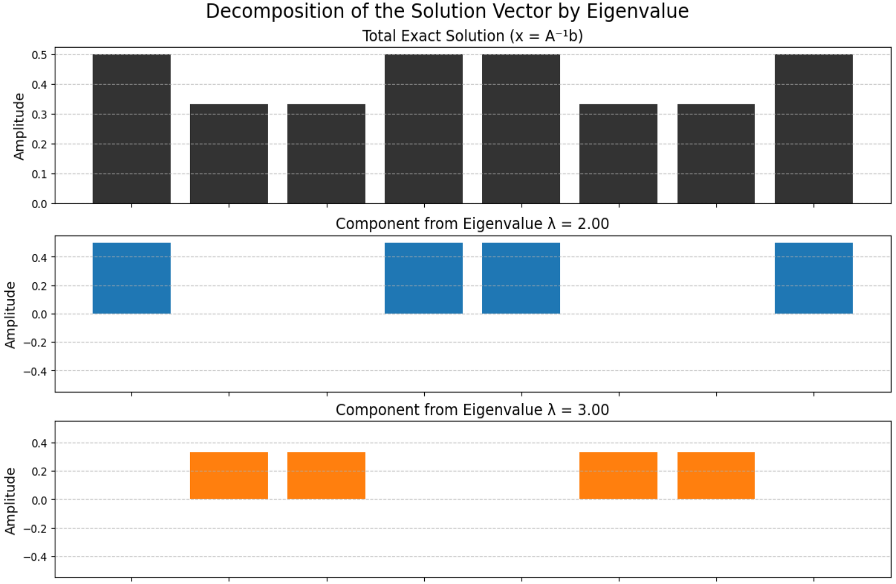
Measurement

QPE BINS

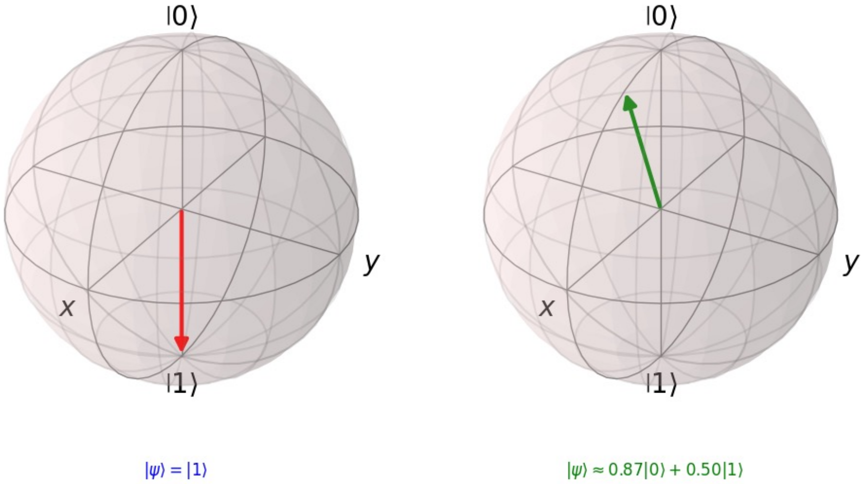


Time Evolution

$$e^{-iAt} \qquad t = \frac{\pi}{\lambda_c} \qquad P(0|\lambda_j) = \cos^2\left(\frac{\lambda_k t}{2}\right)$$



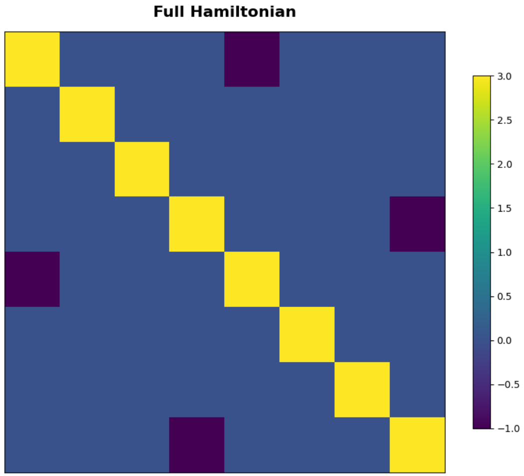
$\lambda = 3$ $\qquad$ $\lambda \neq 3$



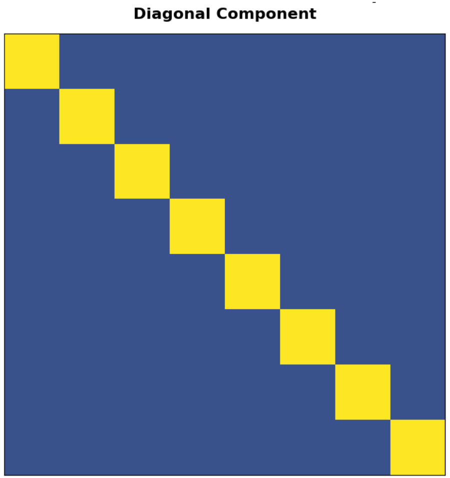
$$\mathcal{H}(S) = \sum_{abc} f(\theta) S_{ab} S_{bc} + \alpha \sum_{ab} S_{ab}^2 + \beta \sum_{ab} (1 - 2S_{ab})^2 \longrightarrow \lambda_c = \alpha + \beta$$

Nikhef

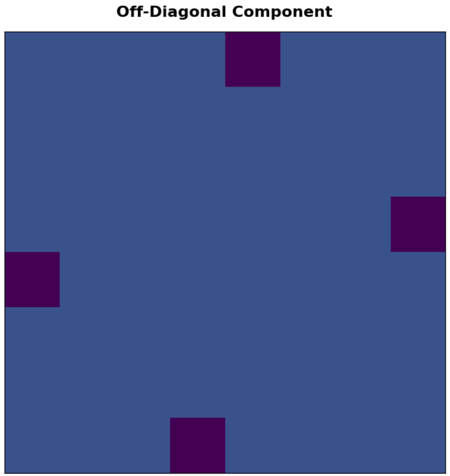
Matrix Decomposition



=

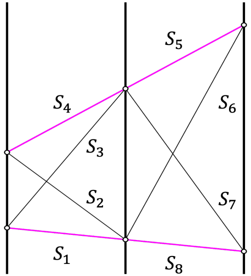


+



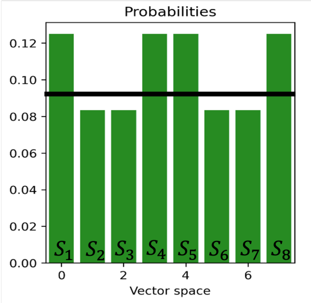
e^{-iAt}

Time Evolution
e.g Suzuki Trotter



$e^{-ic\mathbb{I}t}$

$P(\phi)$



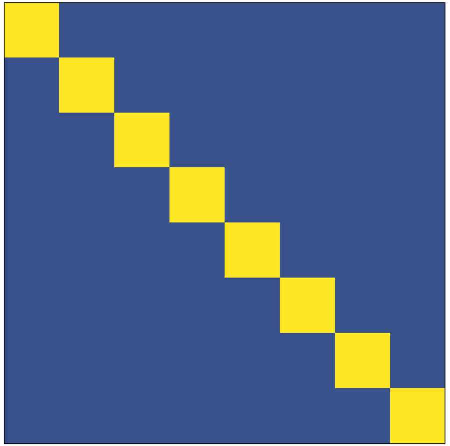
e^{-iBt}

U_{DSS}

Nikhef

Direct Structural Synthesis

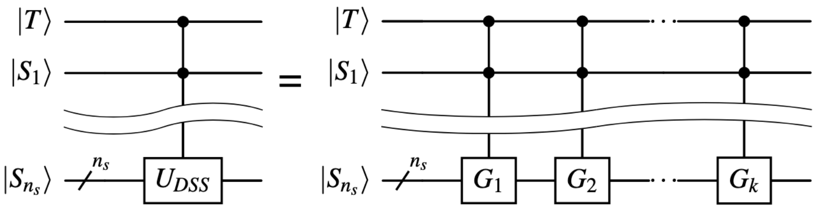
Diagonal Component



$e^{-ic\mathbb{I}t}$

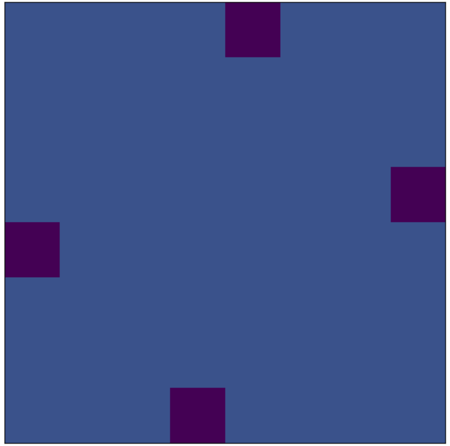
$P(\phi)$

Direct Structural Synthesis (DSS) Decomposition

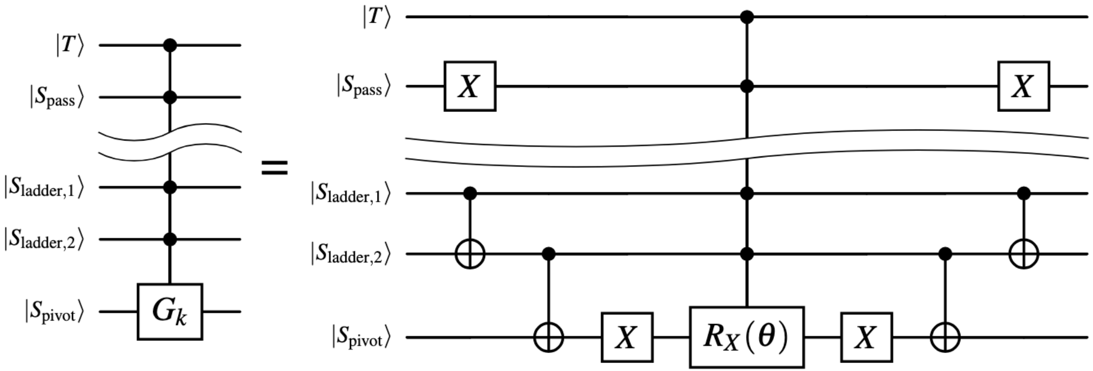


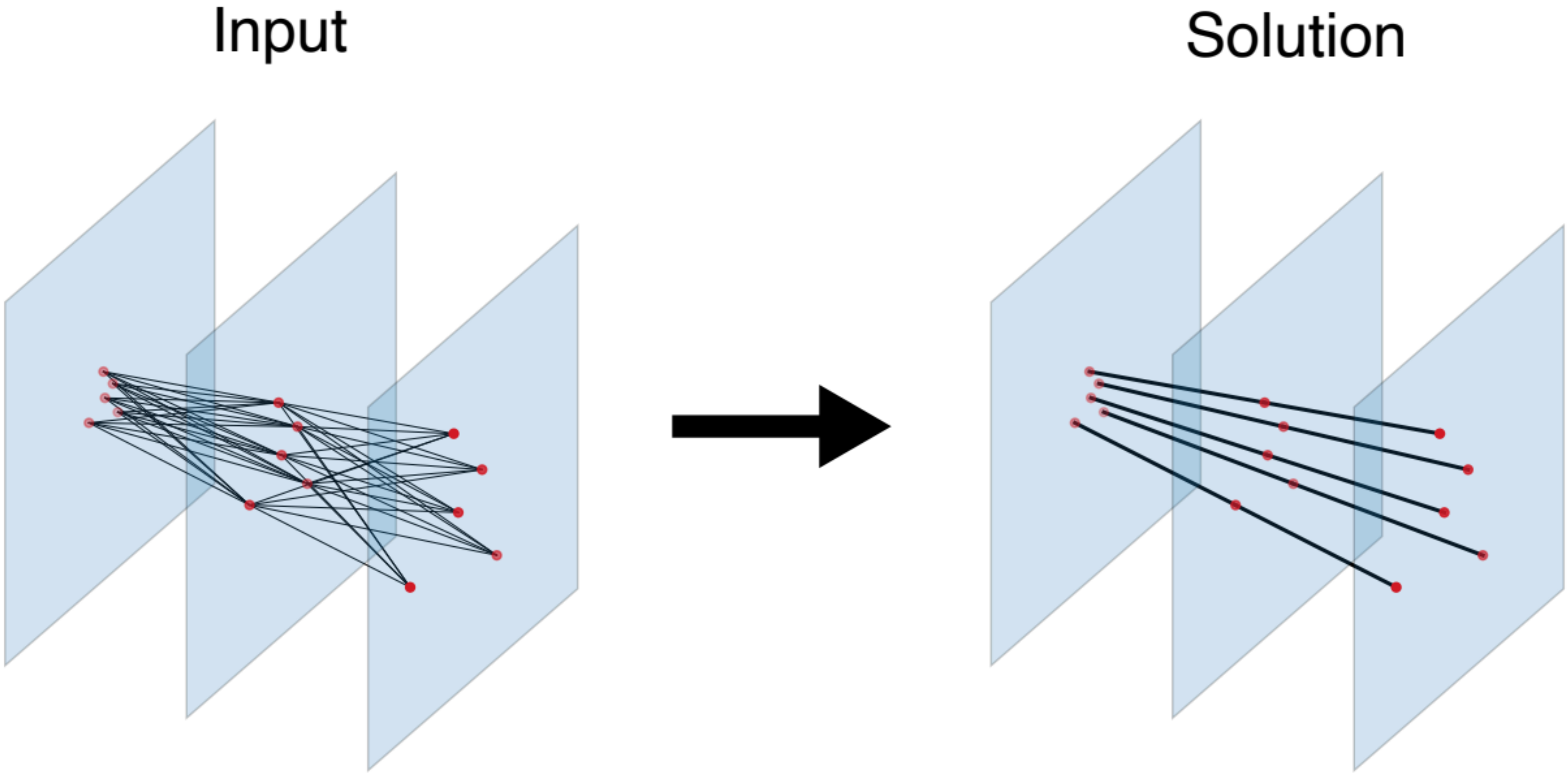
Off-Diagonal Component

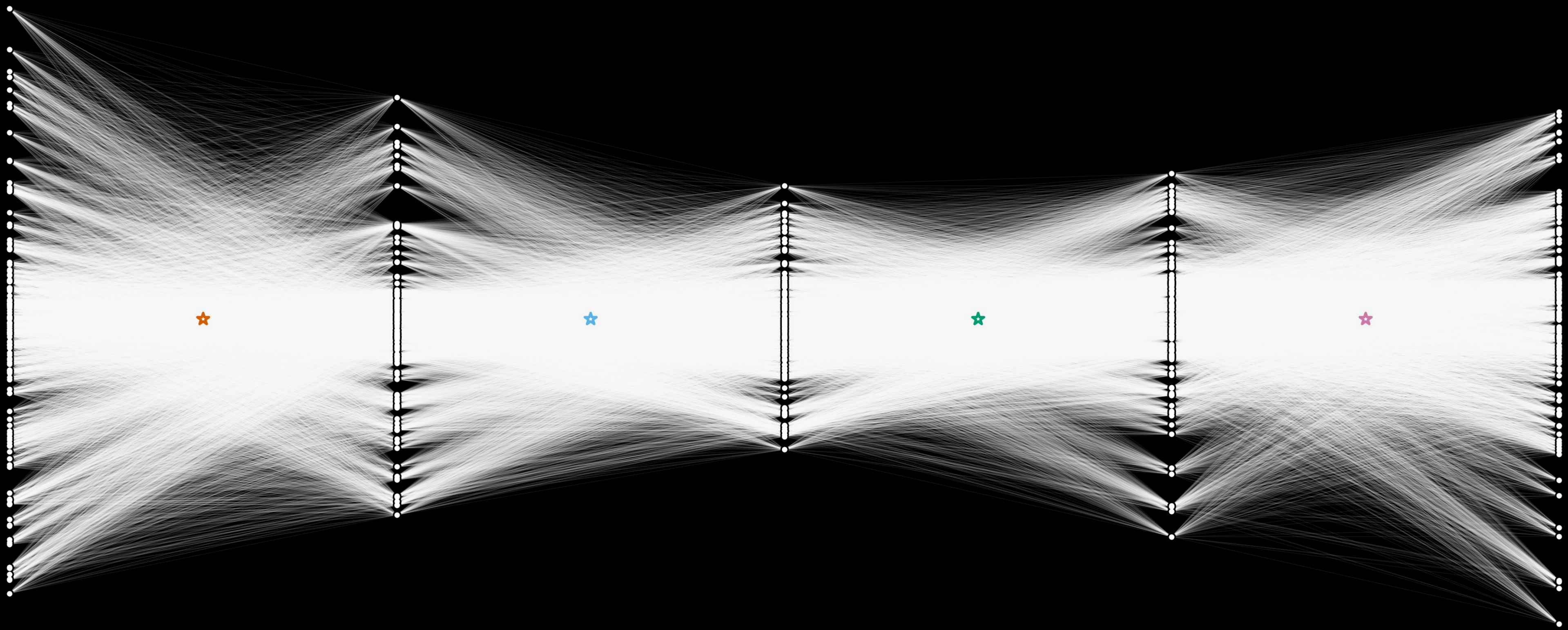
e^{-iBt}

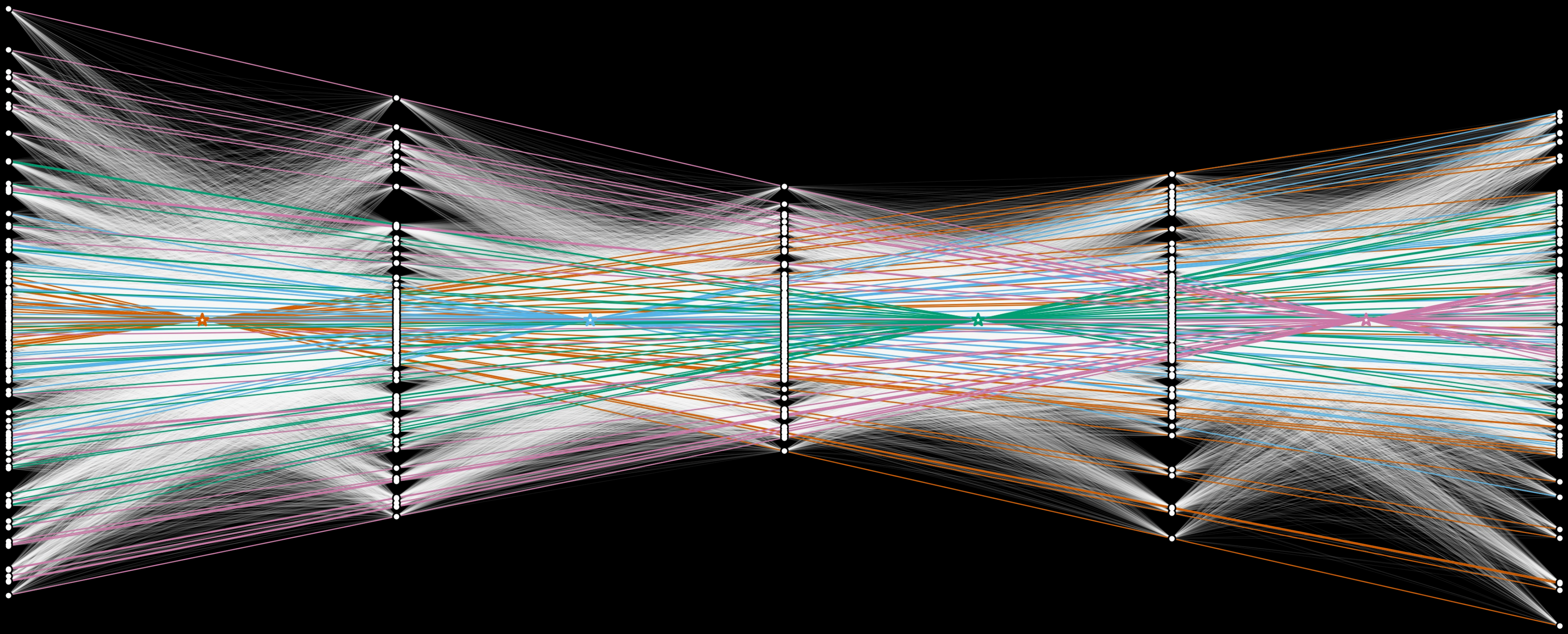


Generalized Two-Level Unitary Structure (G_k)

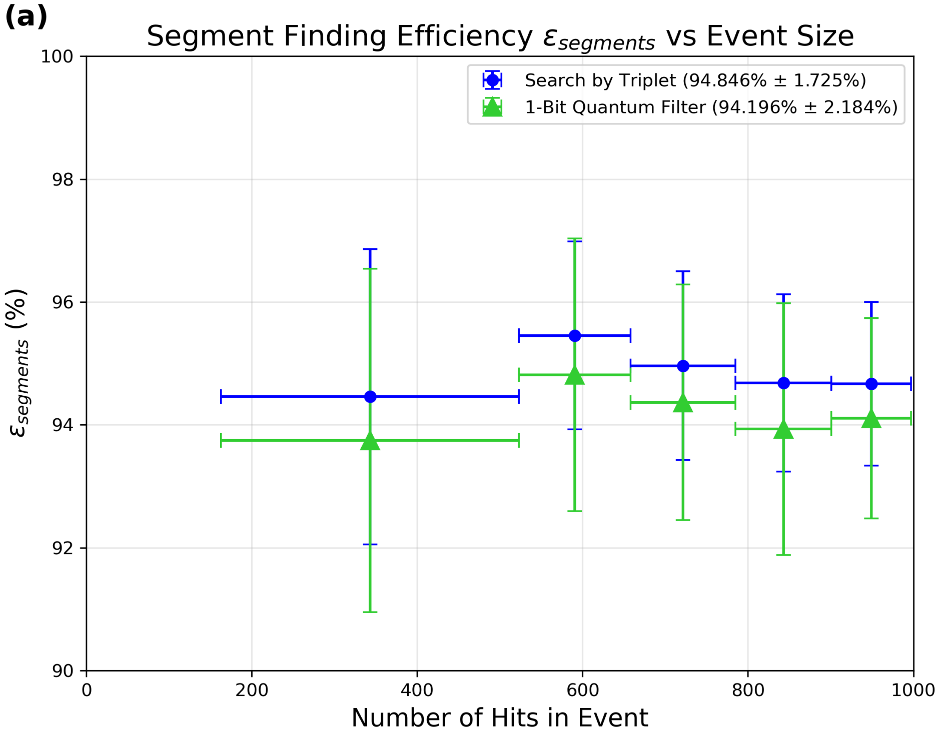


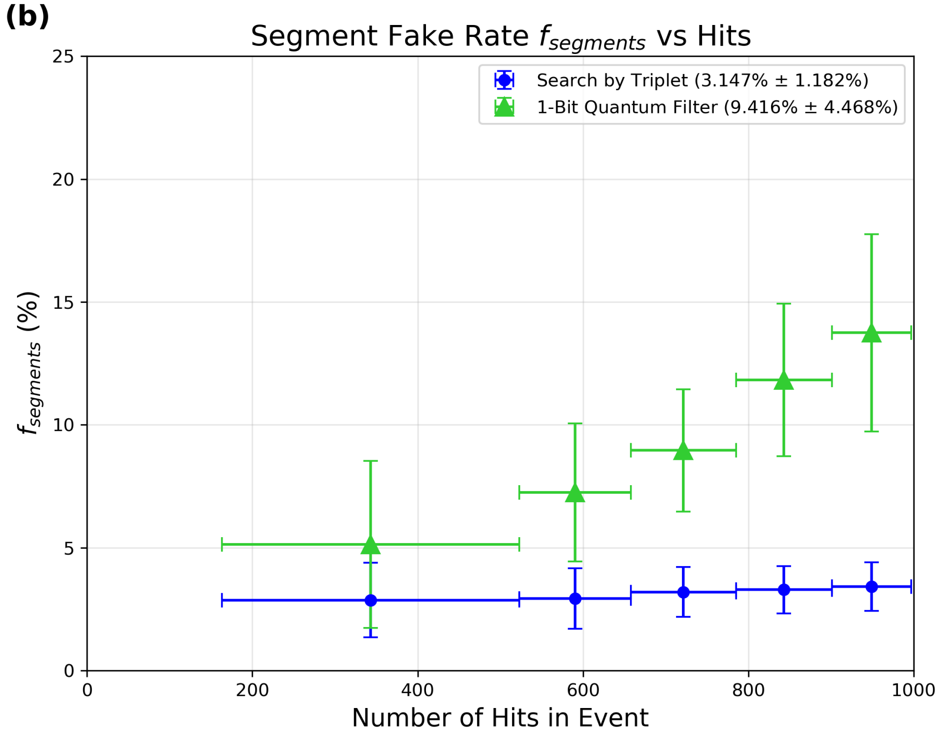

Input
Solution





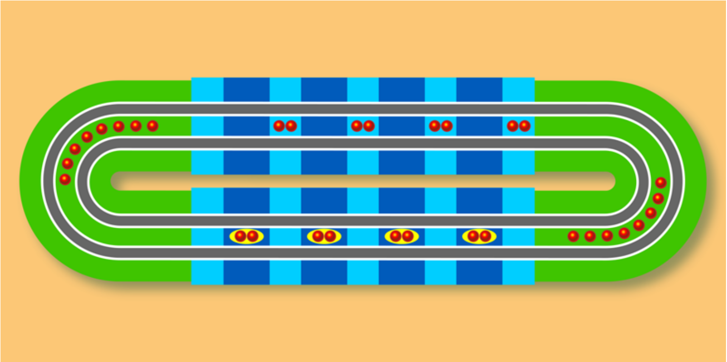
Testing Realistic Events

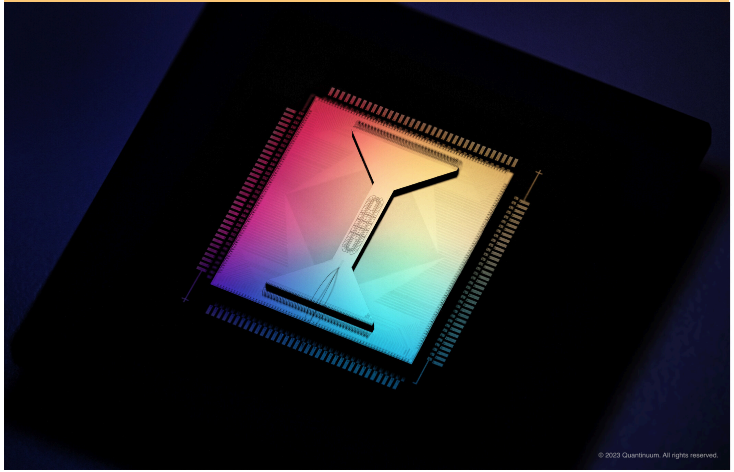




Trapped Ions (Quantinuum)

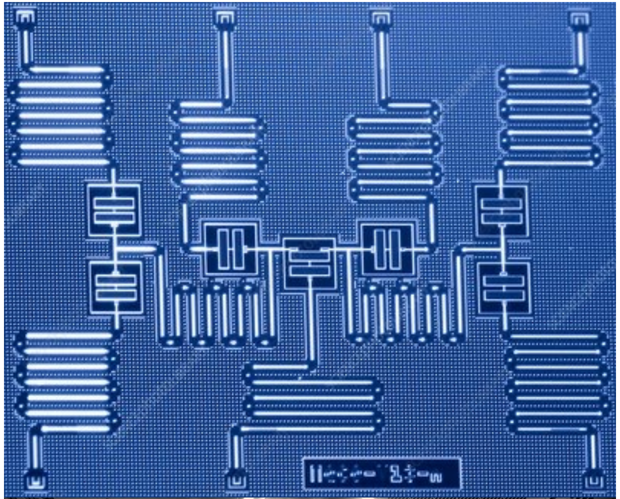
[3]

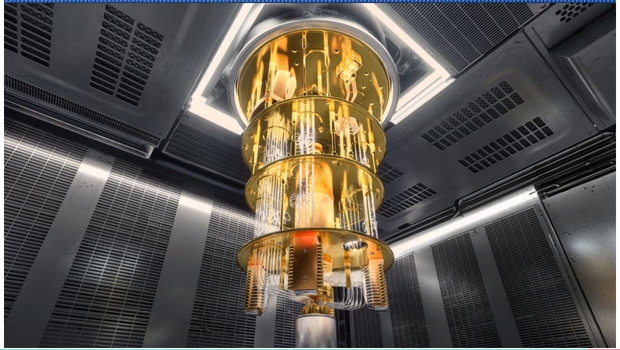




Superconducting (IBM)

[4]

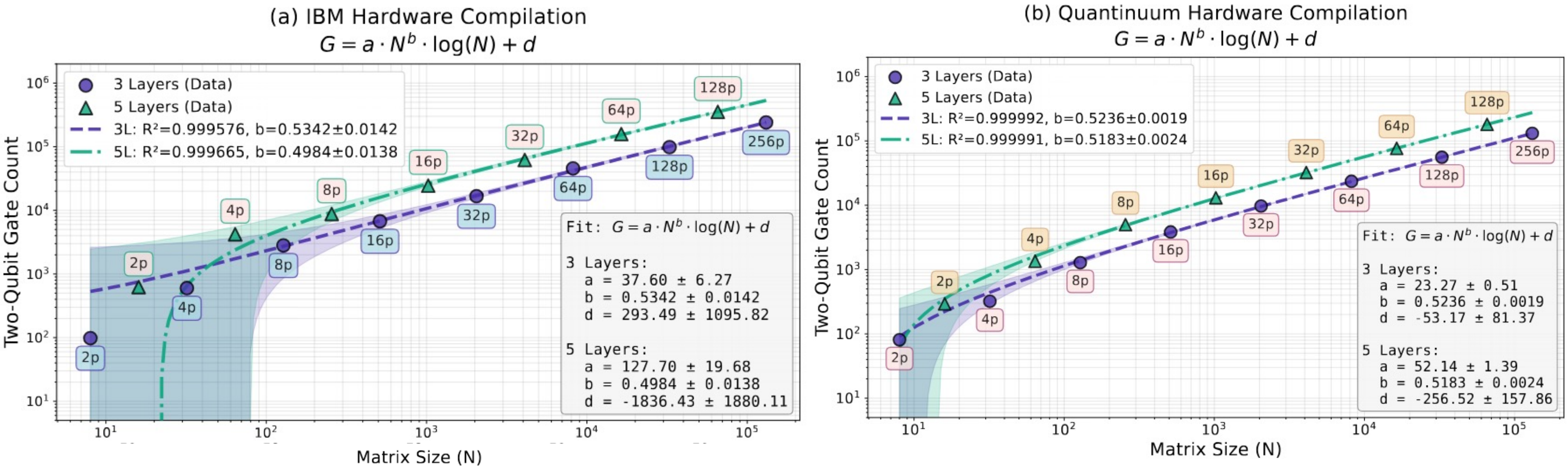




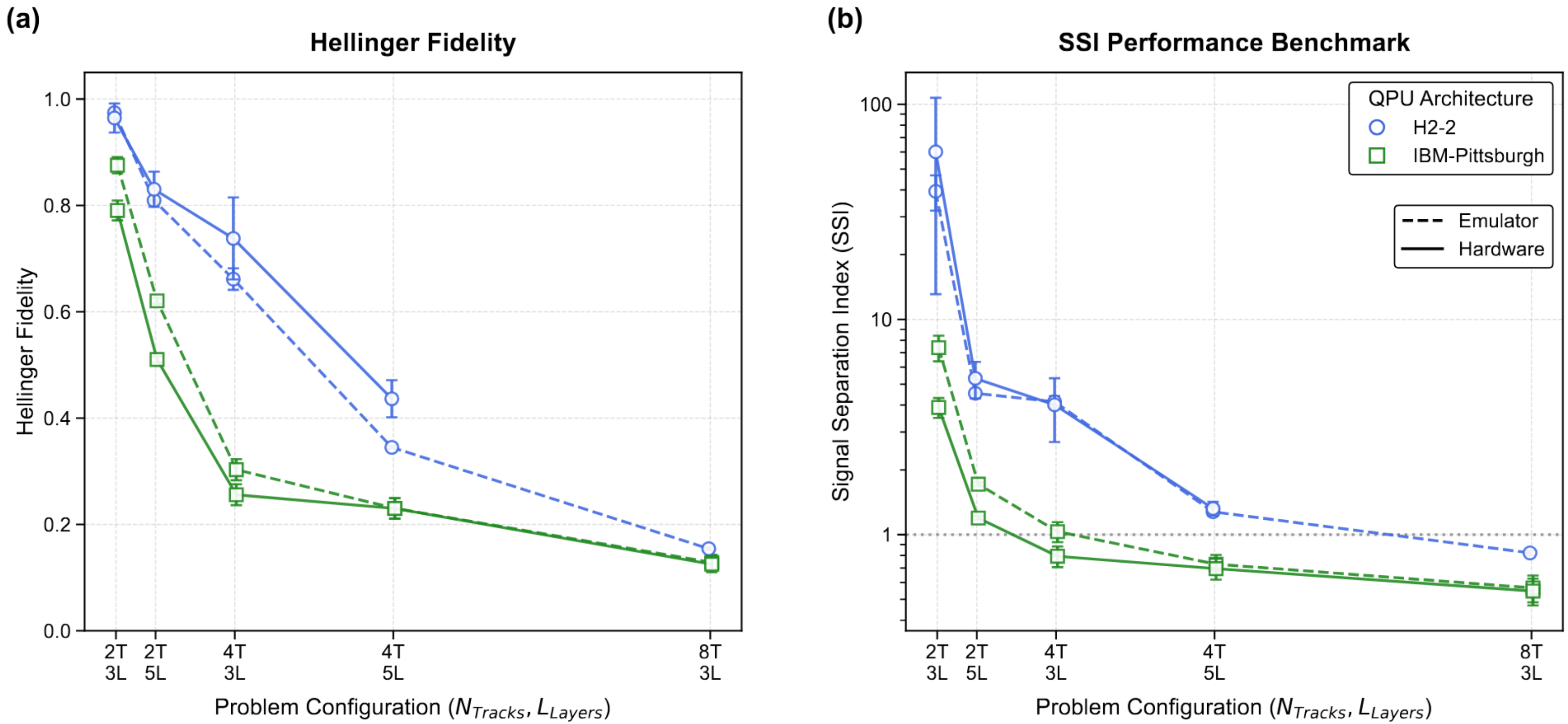
Testing Theoretical Complexity Fit

https://arxiv.org/abs/2601.07766 [5]

Asymptotic analysis value = $\mathcal{O}\left(\sqrt{N}\ Log(N)\right)$

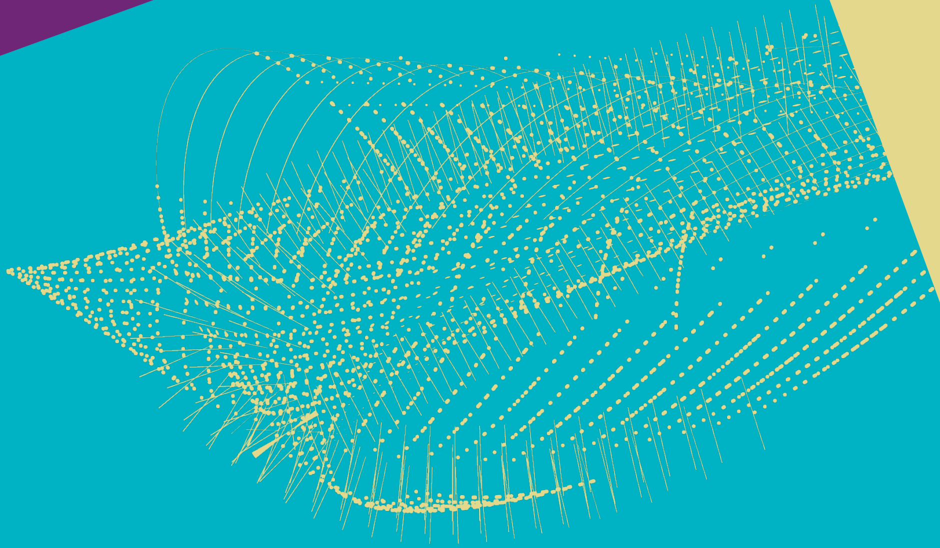


Testing Solution Fidelity



Nikhef

Maastricht University



Next Steps

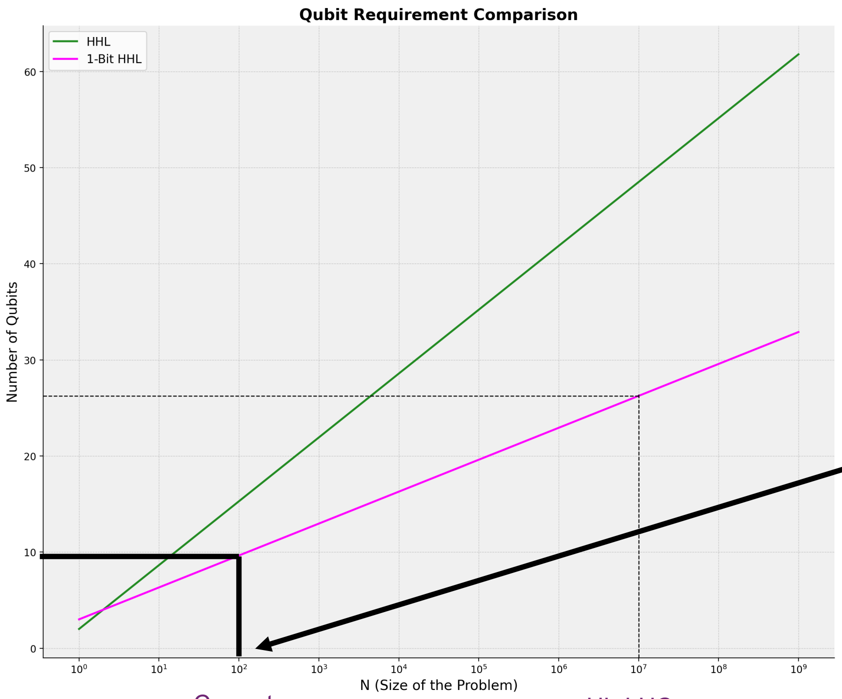
1. Upgrade The 1-Bit Quantum Filter

2. Extract Dense Physics Observables

1. Upgrade The 1-Bit Quantum Filter

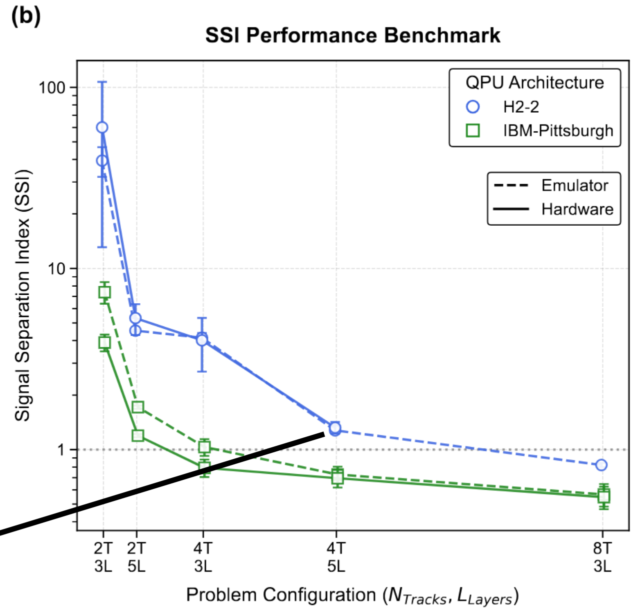
2. Extract Dense Physics Observables

Upgrading the 1-Bit Quantum Filter



Current Hardware

HL-LHC

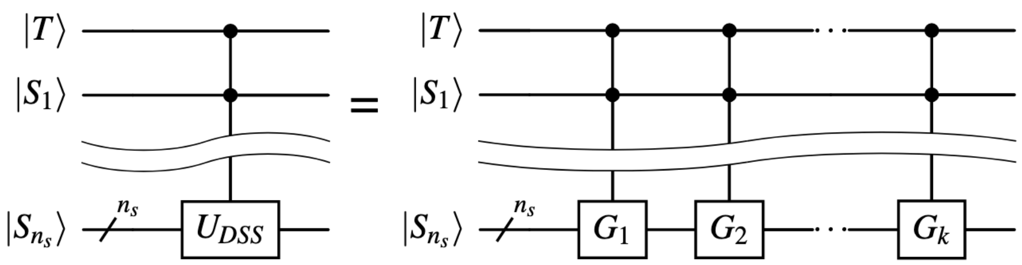


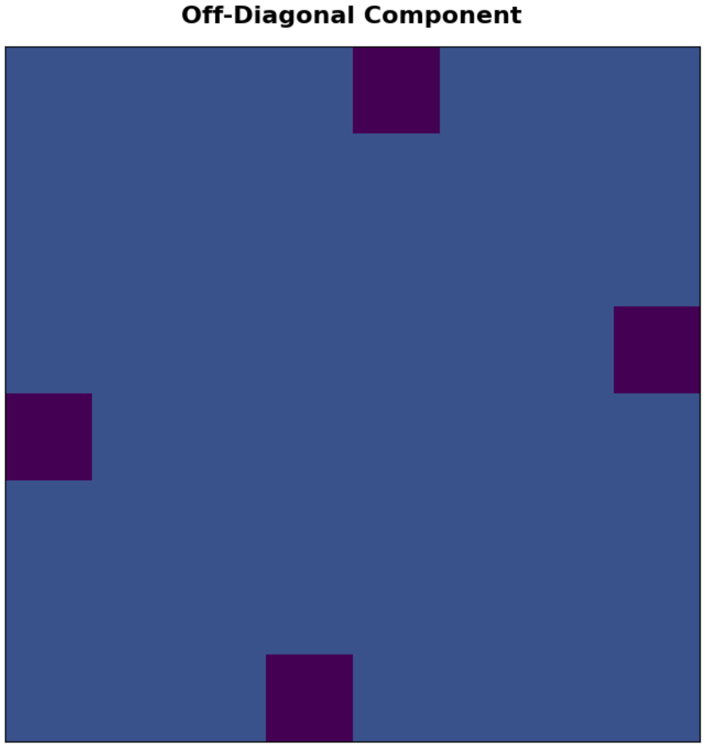
QPU	Qubit Count
Quantinuum System Model H2	56
IBM Heron R3 Pittsburgh	156

Nikhef

Qubit Fanout

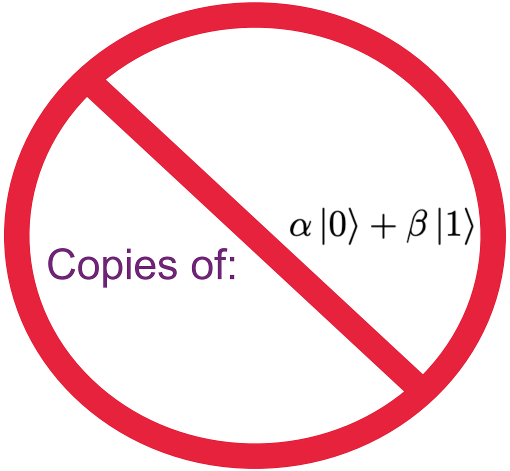
Direct Structural Synthesis (DSS) Decomposition





Parallelizing Phase Estimation via Qubit Fan-Out (Quantum SIMD)

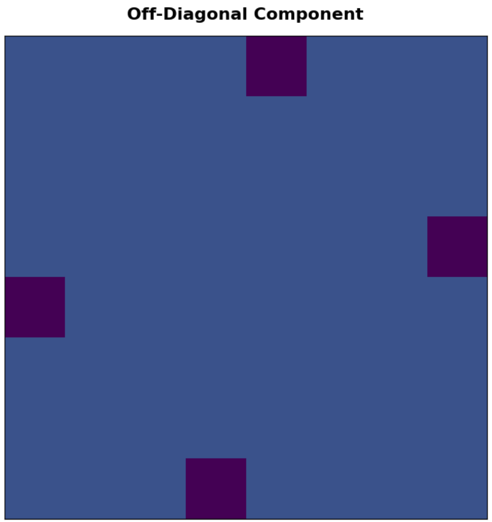
No Cloning Theorem: Prevents creating independent copies of an unknown clock state



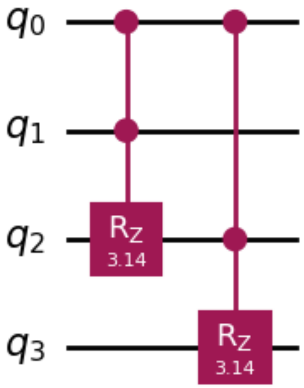
Fan-Out: Use extrac ancilla quibts to create a fully entangled control state

GHZ: $\alpha\left|00...0\right\rangle + \beta\left|11...1\right\rangle$

Exploiting Subspace Commutativity

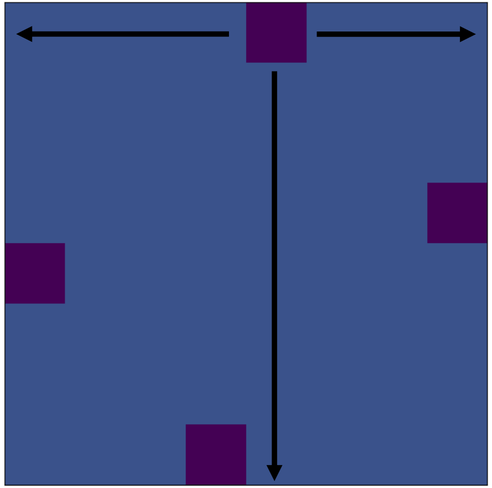


1-Bit Quantum Filter Circuit

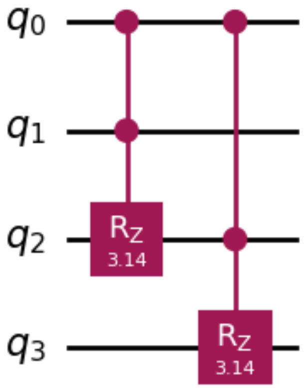


Decomposed Depth:
14 gates

Exploiting Subspace Commutativity

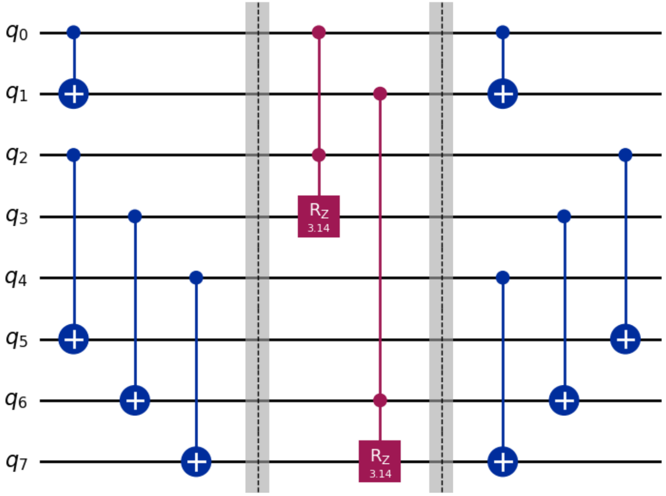


1-Bit Quantum Filter Circuit



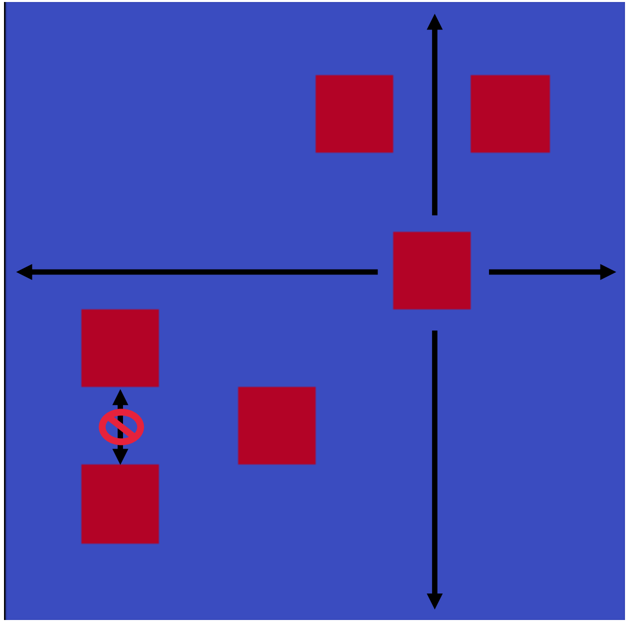
Decomposed Depth:
14 gates

Fan-Out Circuit

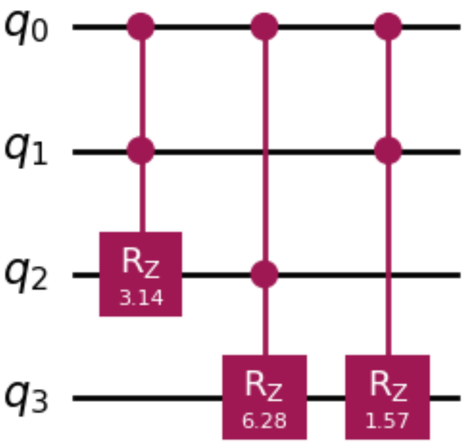


Decomposed Depth:
10 gates

Issues with Non-Commutativity



1-Bit Quantum Filter Circuit



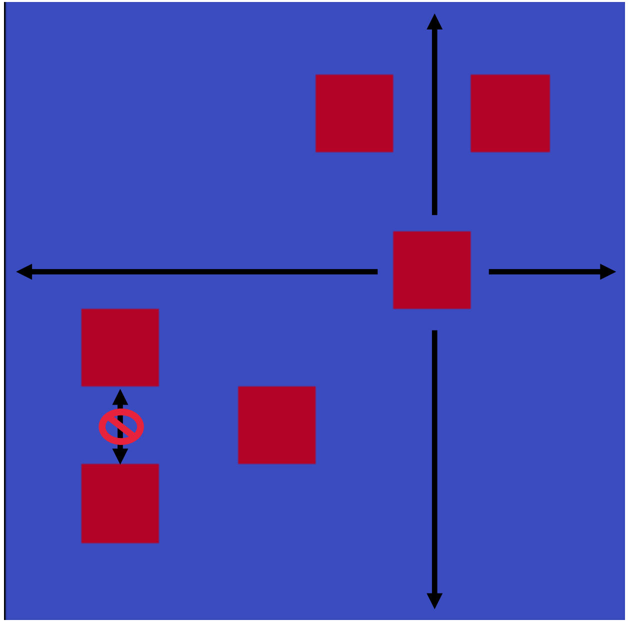
Algorithm 1: Greedy Commuting Layer Assignment

Input: Sparse set of unique interaction pairs $E = \{(i, j)\}$
Output: Dictionary of commuting layers L

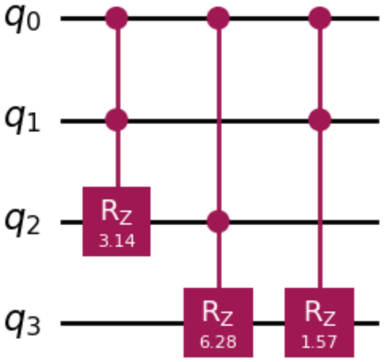
1. Initialize L as an empty dictionary.
2. Initialize `used_colors` as an empty hash map (with lazy $\mathcal{O}(1)$ set allocation).
3. **For each** interaction pair $(i, j) \in E$:
   - (a) *Lazily fetch or initialize active colors for states i and j*
   - (b) $C_i \leftarrow$ `used_colors`$[i]$
   - (c) $C_j \leftarrow$ `used_colors`$[j]$
   - (d) *Determine unavailable layers*
   - (e) `forbidden_colors` $\leftarrow C_i \cup C_j$
   - (f) *Greedily find the lowest available layer*
   - (g) `color` $\leftarrow 0$
   - (h) **While** `color` $\in$ `forbidden_colors`:
     - `color` $\leftarrow$ `color` $+ 1$
   - (i) *Assign interaction and update state trackers*
   - (j) Append (i, j) to L[`color`]
   - (k) `used_colors`$[i]$.add(`color`)
   - (l) `used_colors`$[j]$.add(`color`)
4. **Return** L

Average Grouping Complexity $\mathcal{O}(k)$

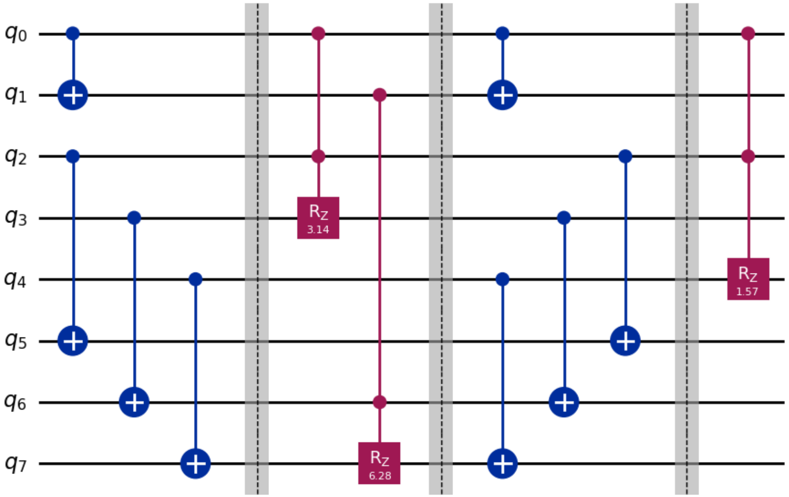
Issues with Non-Commutativity



1-Bit Quantum Filter Circuit

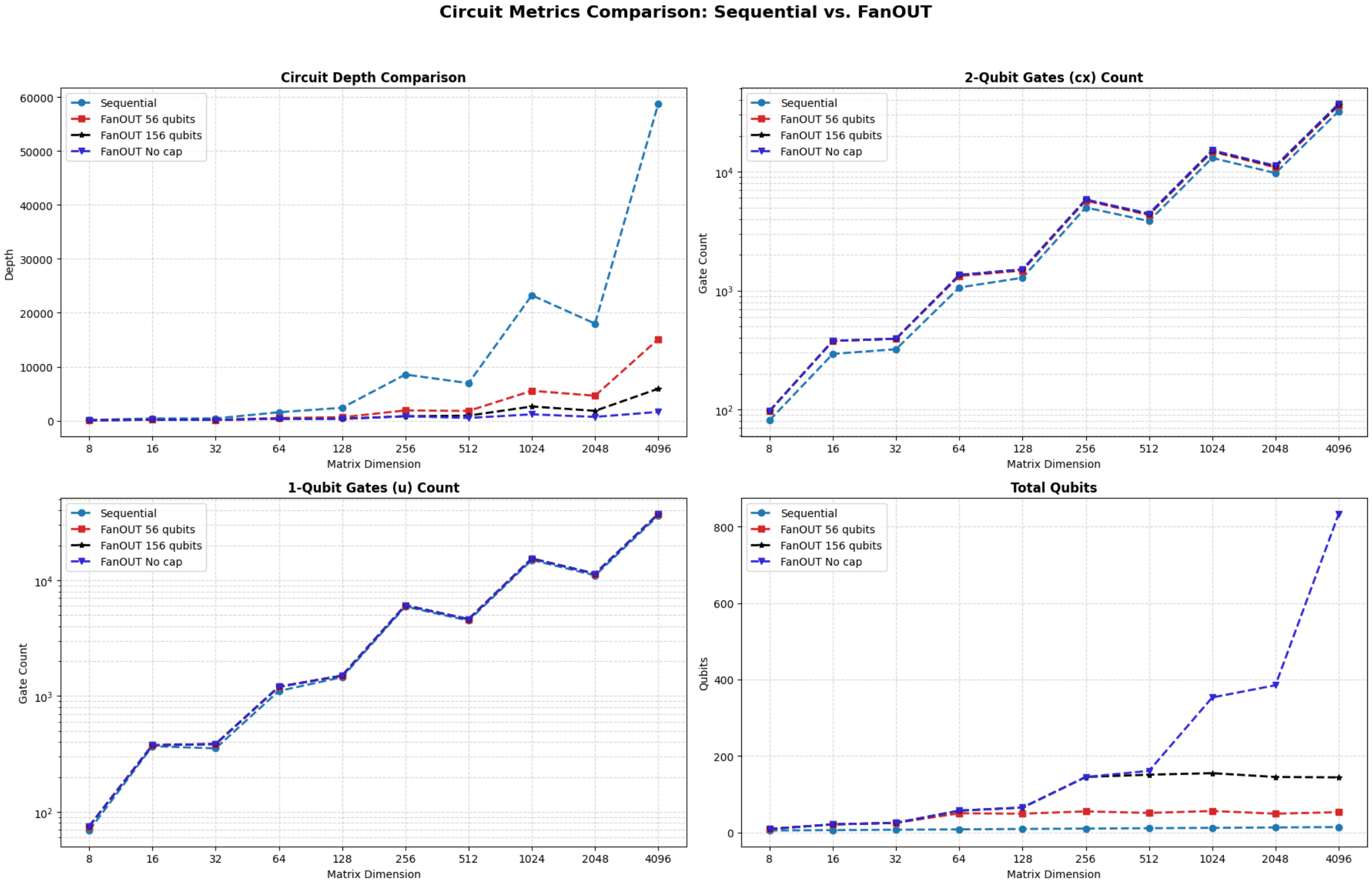


Fan-Out Circuit

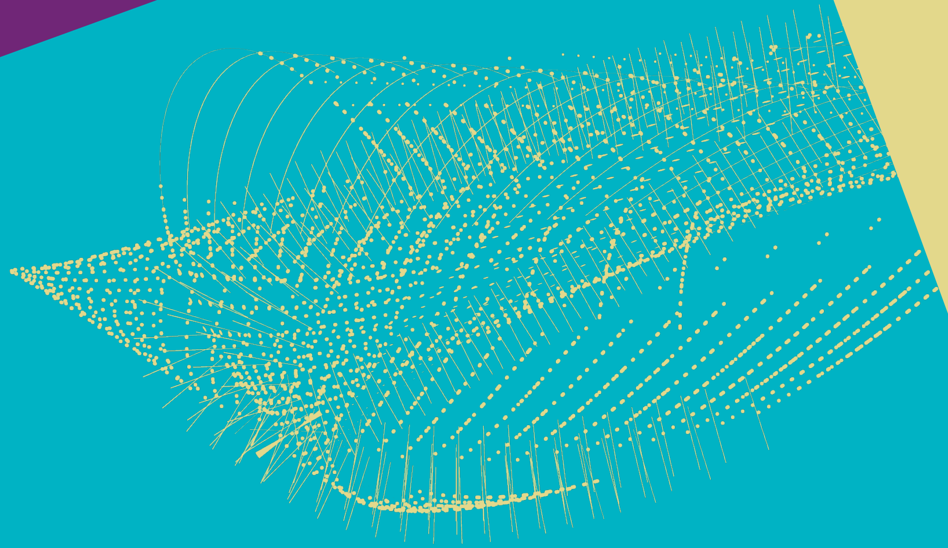


Nikhef

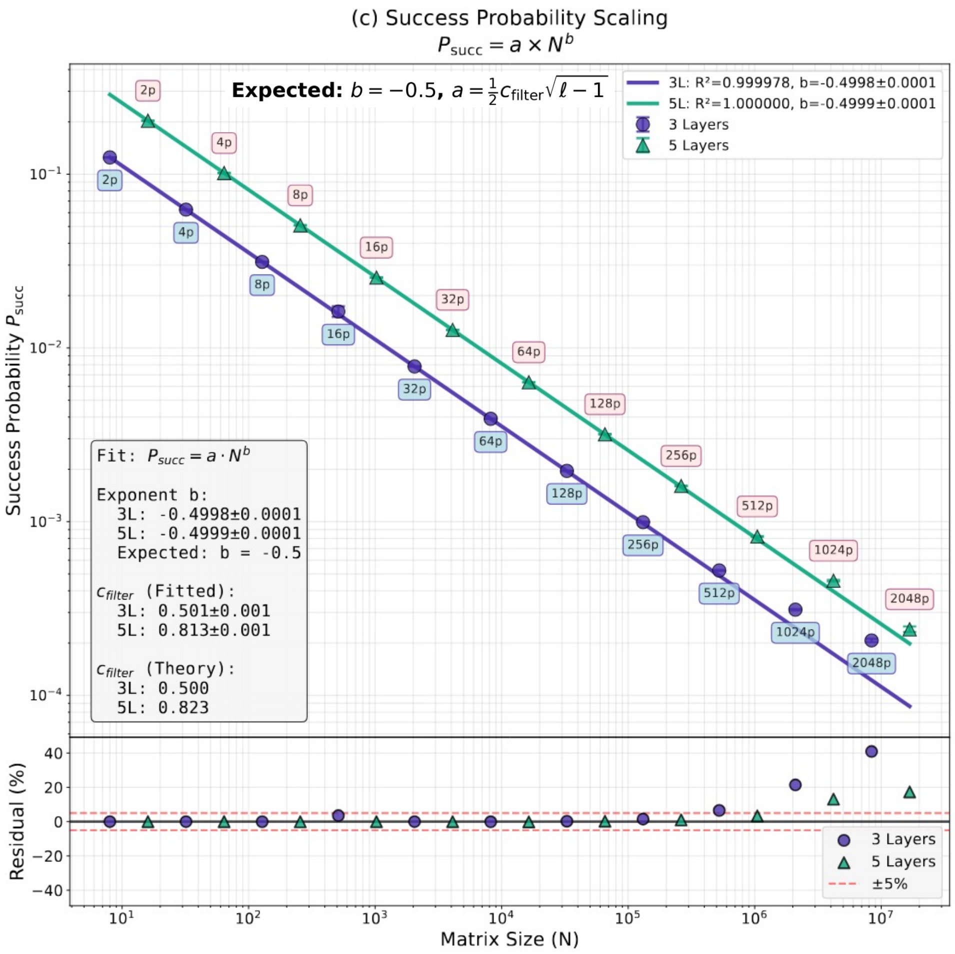
Tracking with Fan-Out



Nikhef



Thank You



Quantum Tracking: Success Probability

Bibliography

[1]: D. Nicotra et al., "A quantum algorithm for track reconstruction in the LHCb vertex detector”, *Journal of Instrumentation*, vol. 18, P11028, 2023. [arXiv:2308.00619]

[2]: X. Chiotopoulos et al., "TrackHHL: A Quantum Computing Algorithm for Track Reconstruction at the LHCb”, *EPJ Web of Conferences*, vol. 337, 01181, 2025. [arXiv:2511.11458]

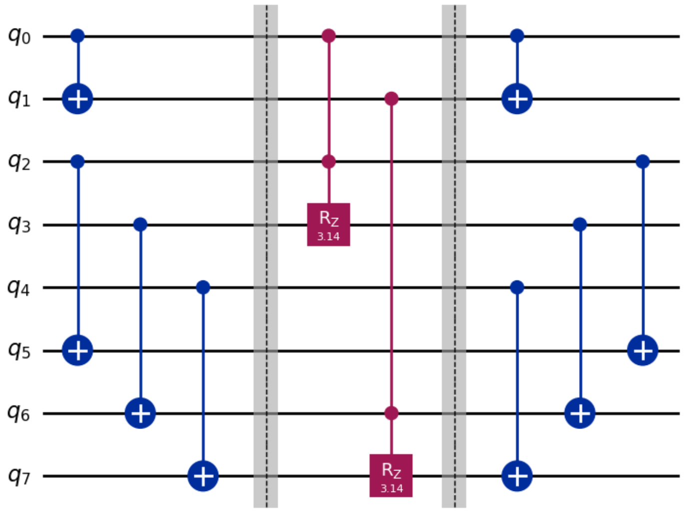
[3]: S. A. Moses et al., "A Race-Track Trapped-Ion Quantum Processor," *Physical Review X*, vol. 13, 041052, 2023. https://DOI:https://doi.org/10.1103/PhysRevX.13.041052

[4]: Kim, Y., Eddins, A., Anand, S. *et al.* Evidence for the utility of quantum computing before fault tolerance. *Nature* **618**, 500–505 (2023).

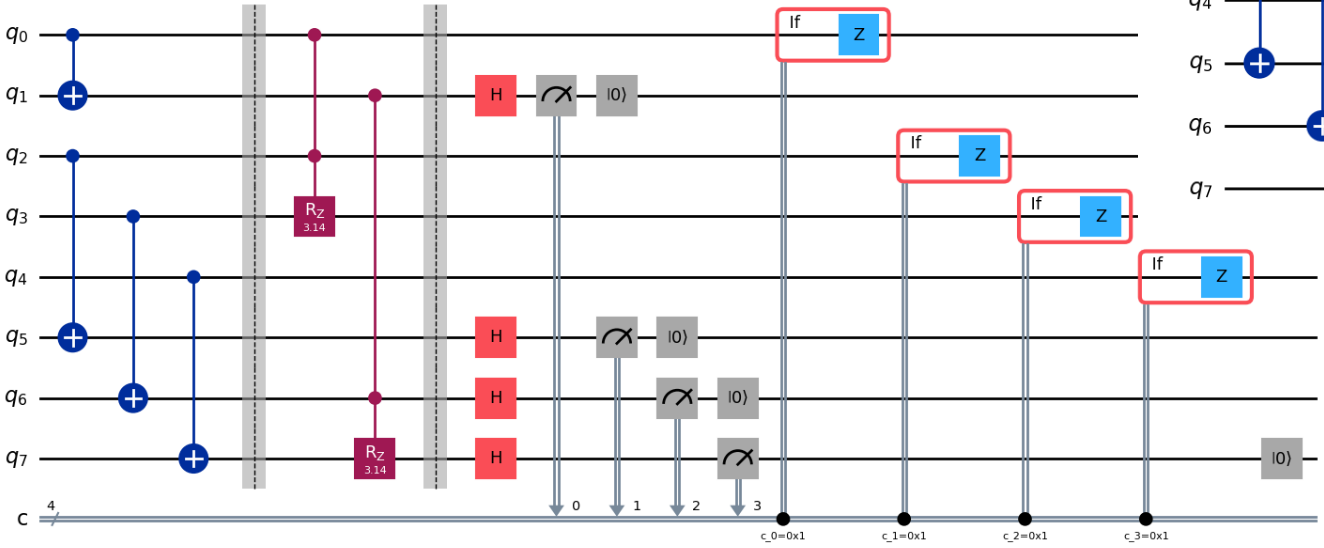
[5]: X. Chiotopoulos et al., "TrackHHL: The 1-Bit Quantum Filter for particle trajectory reconstruction”, Pre-Print [arXiv:2601.07766]

Uncomputation via Mid-Circuit Measurments

Fan-Out Circuit



Fan-Out Circuit with Mid-Circuit Measurements



Nikhef

Uncomputation via Mid-Circuit Measurments

$$|\psi\rangle = \alpha|00\rangle + \beta|11\rangle \tag{1}$$

To disentangle the copy qubit without a reverse CNOT, we apply a Hadamard gate to it. The Hadamard transforms:

$$|0\rangle \to \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad and \quad |1\rangle \to \frac{|0\rangle - |1\rangle}{\sqrt{2}} \tag{2}$$

If we apply this to the copy qubit in our entangled state, the math regroups like this:

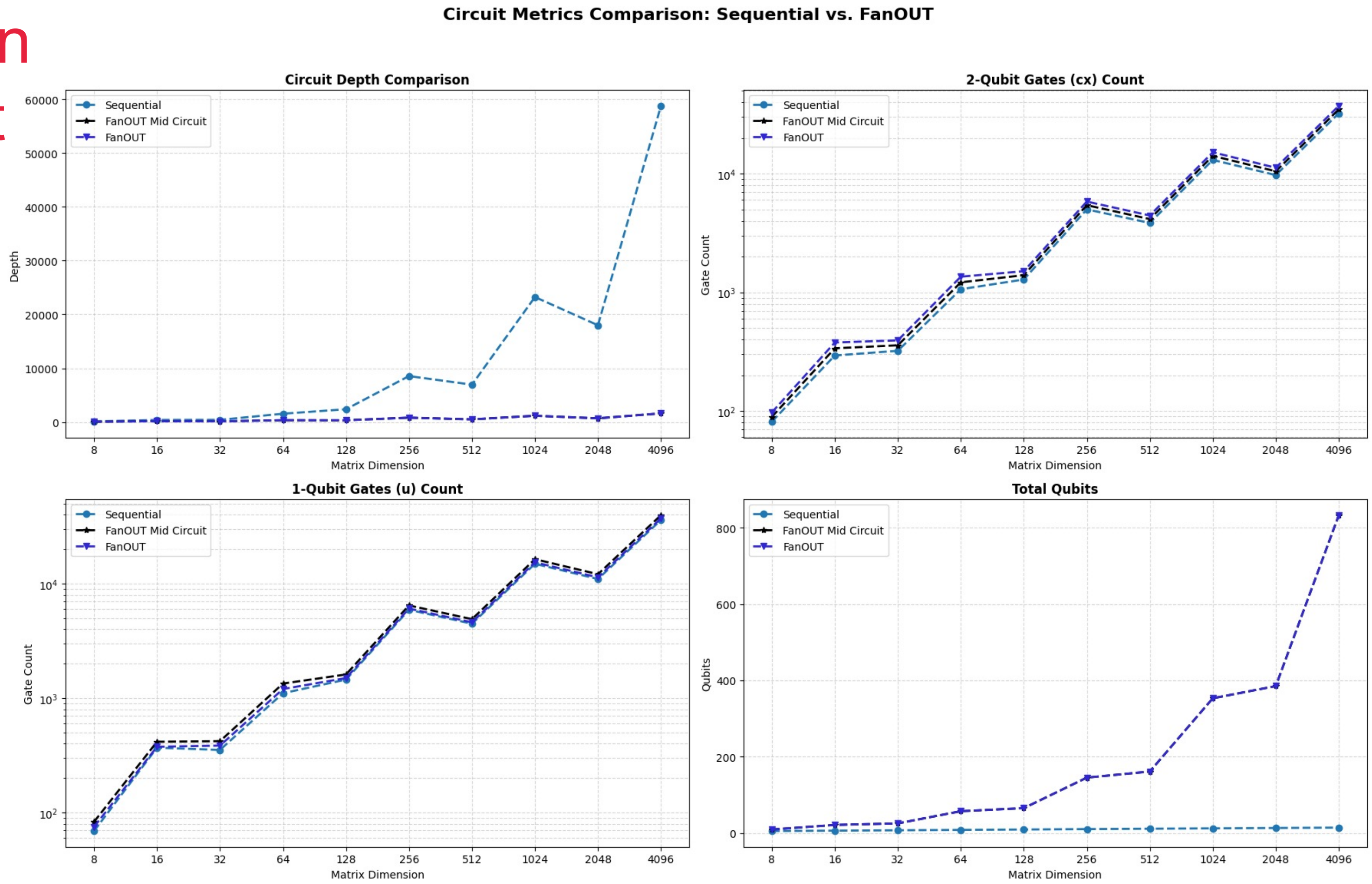
$$|\psi\rangle = |0\rangle_{\text{copy}} \otimes (\alpha|0\rangle + \beta|1\rangle)_{\text{master}} + |1\rangle_{\text{copy}} \otimes (\alpha|0\rangle - \beta|1\rangle)_{\text{master}} \tag{3}$$

Now, look at the two possible measurement outcomes for the copy qubit:

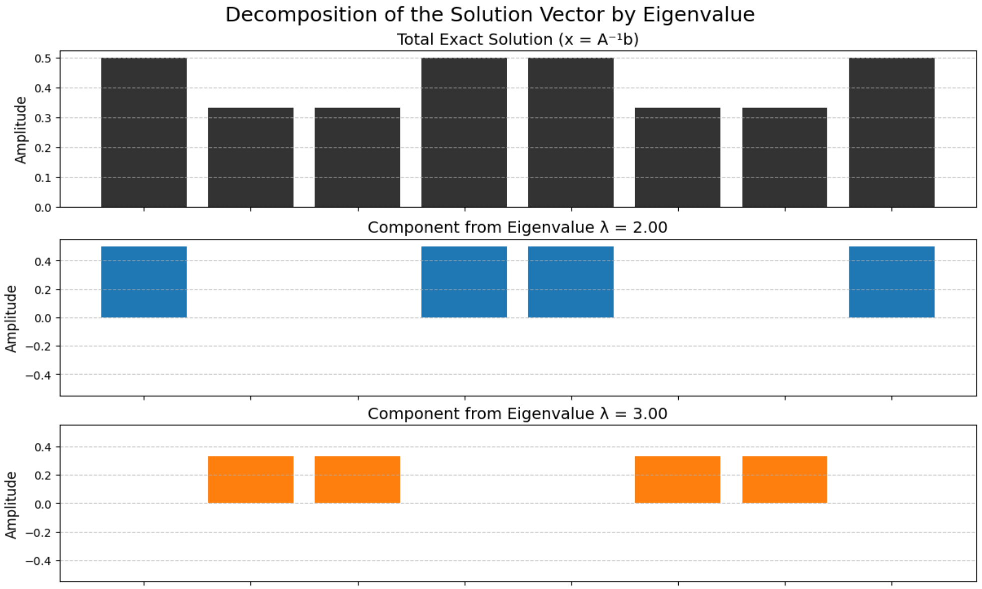
1. If you measure 0: The master qubit collapses to exactly $\alpha|0\rangle + \beta|1\rangle$. It is perfectly uncomputed and holds the correct phase.
2. If you measure 1: The master qubit collapses to $\alpha|0\rangle - \beta|1\rangle$. The state is uncomputed, but a minus sign (a Z-error) has been introduced to the $|1\rangle$ state.

Therefore, you just use an if statement. If the hardware reads a 1, you use Z gate at the master qubit to flip that minus sign back to a plus.

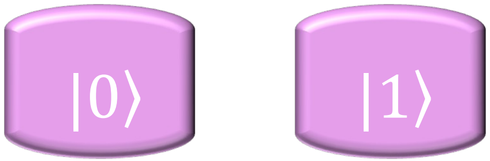
Uncomputation via Mid-Circuit Measurments



The Spectral Filter



QPE BIN



$$P(0|\lambda_j) = \cos^2\left(\frac{\lambda_k t}{2}\right)$$

$$\lambda_c = \alpha + \beta \implies t = \frac{\pi}{\lambda_c}$$

$$P(0|\lambda_j) = \cos^2\left(\frac{\lambda_c \pi}{2\lambda_c}\right) = \cos^2\left(\frac{\pi}{2}\right) = 0$$

$$P(1|\lambda_j) = \sin^2\left(\frac{\lambda_c \pi}{2\lambda_c}\right) = \sin^2\left(\frac{\pi}{2}\right) = 1$$

$$F(P,Q) = \left(\sum_i \sqrt{P(x_i) \cdot Q(x_i)} \right)^2$$

1. $P(x_i)$: The ideal probability of state x_i (from simulation).
2. $Q(x_i)$: The experimental probability of state x_i (from hardware).
3. Interpretation: 1.0 is a perfect match; 0.0 means no overlap.

Signal Seperation Index

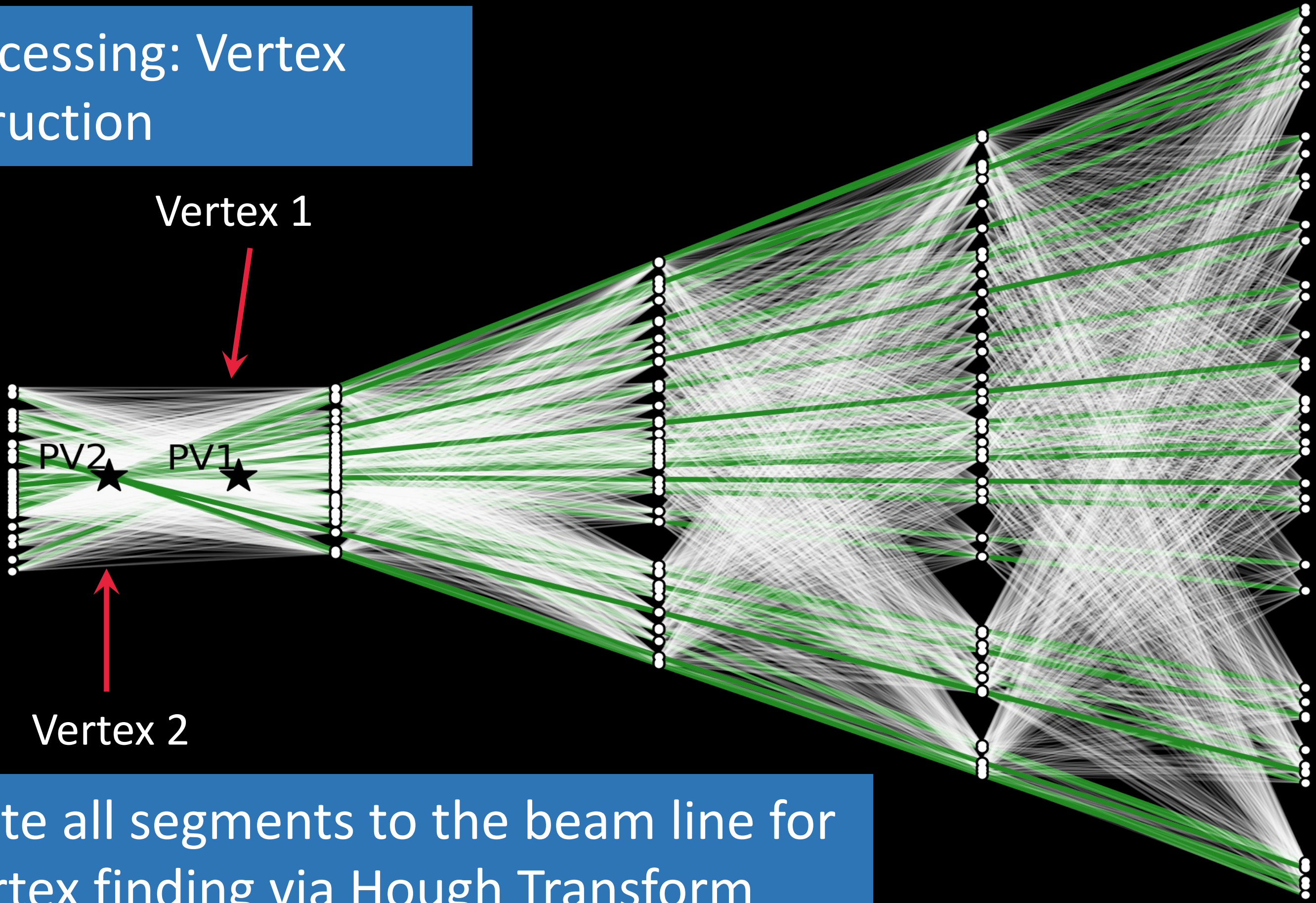
Hellinger Fidelity

$$\text{SSI} = \frac{\mu_{\text{signal}}}{\mu_{\text{noise}}} = \frac{\frac{1}{N_S} \sum_{s \in S} P(s)}{\frac{1}{N_B} \sum_{b \in B} P(b)}$$

1. S: The set of n correct "Signal" states (valid tracks).
2. B: The set of n incorrect "Background" states (the n noise states with the largest amplitudes).
3. Interpretation: Values > 1 mean the signal stands out above the noise floor.

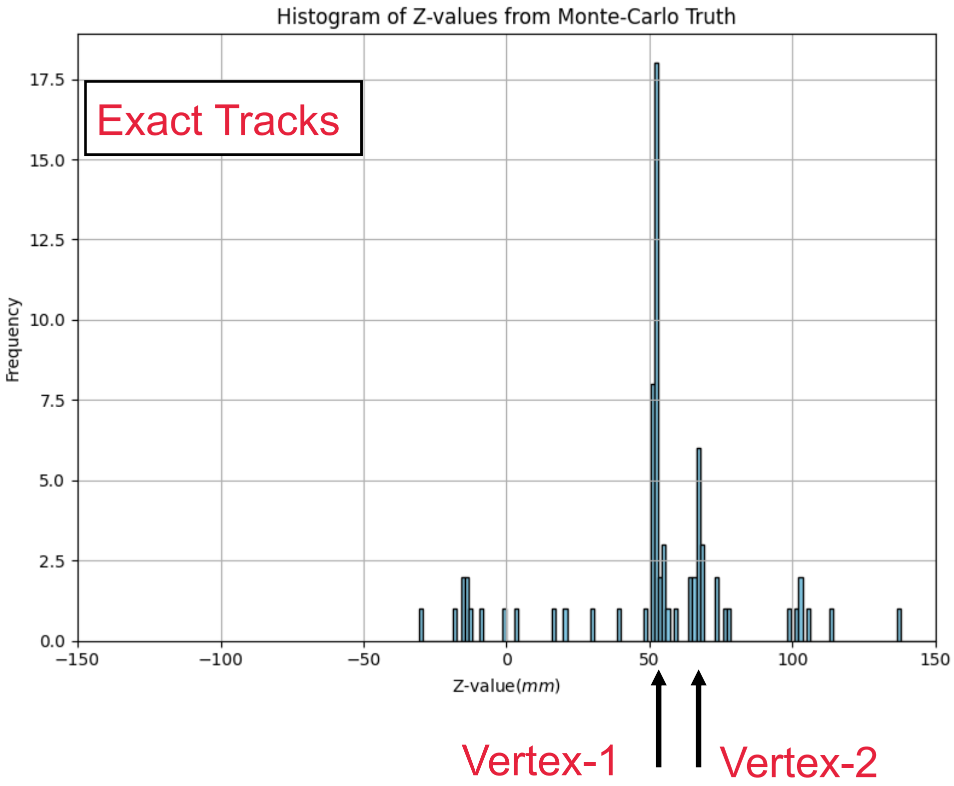
Nikhef

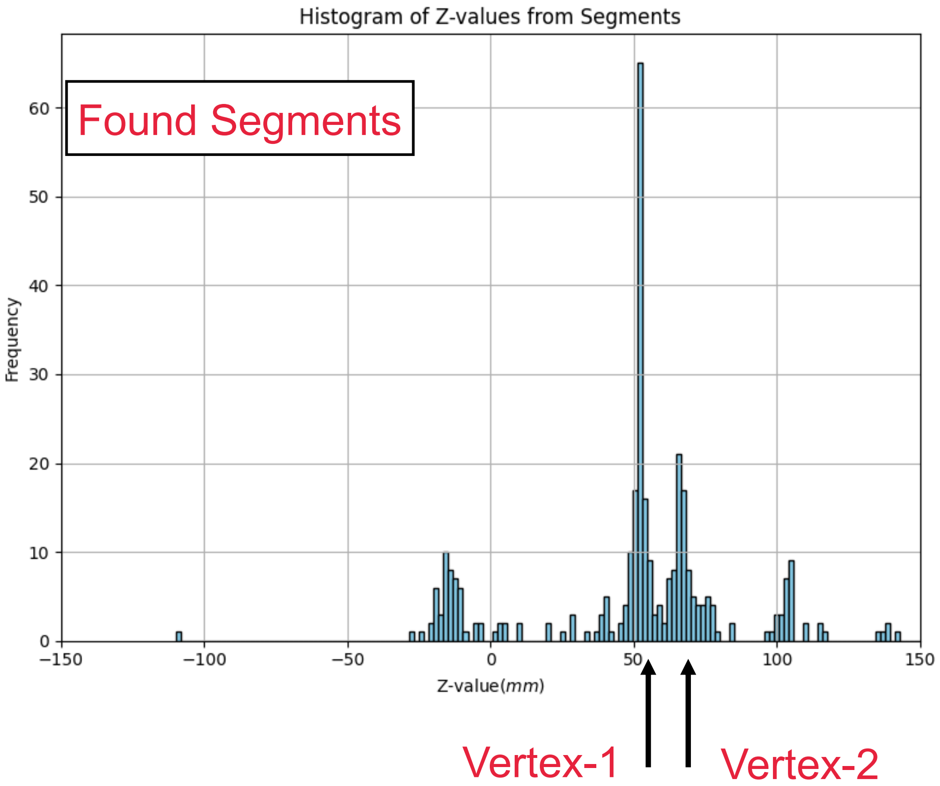
Post-processing: Vertex Reconstruction



Extrapolate all segments to the beam line for direct Vertex finding via Hough Transform

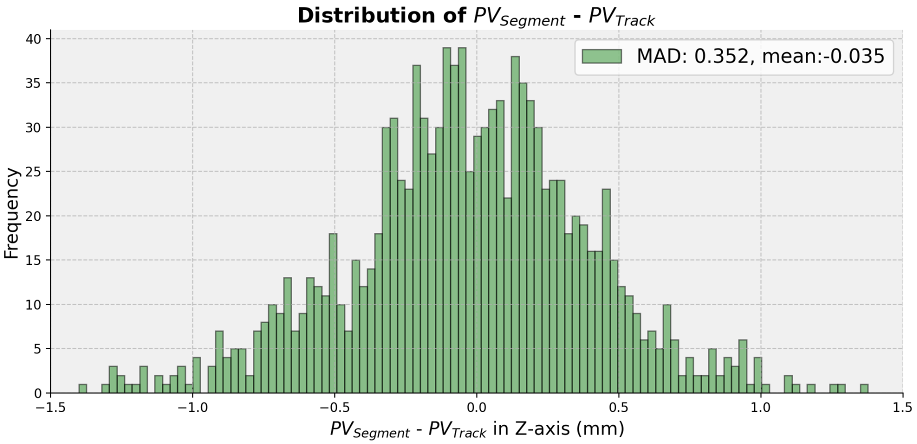
Vertex finding precision



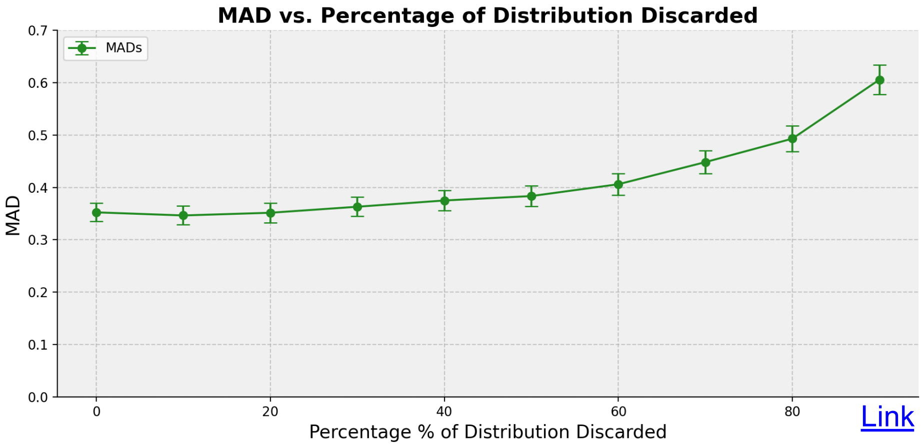


Vertex precision with fraction of segments

Distribution of precision of the extrapolated vertex point



Mean Average Distance (MAD) of recontructed vertex versus the fraction of segments obtained in readout

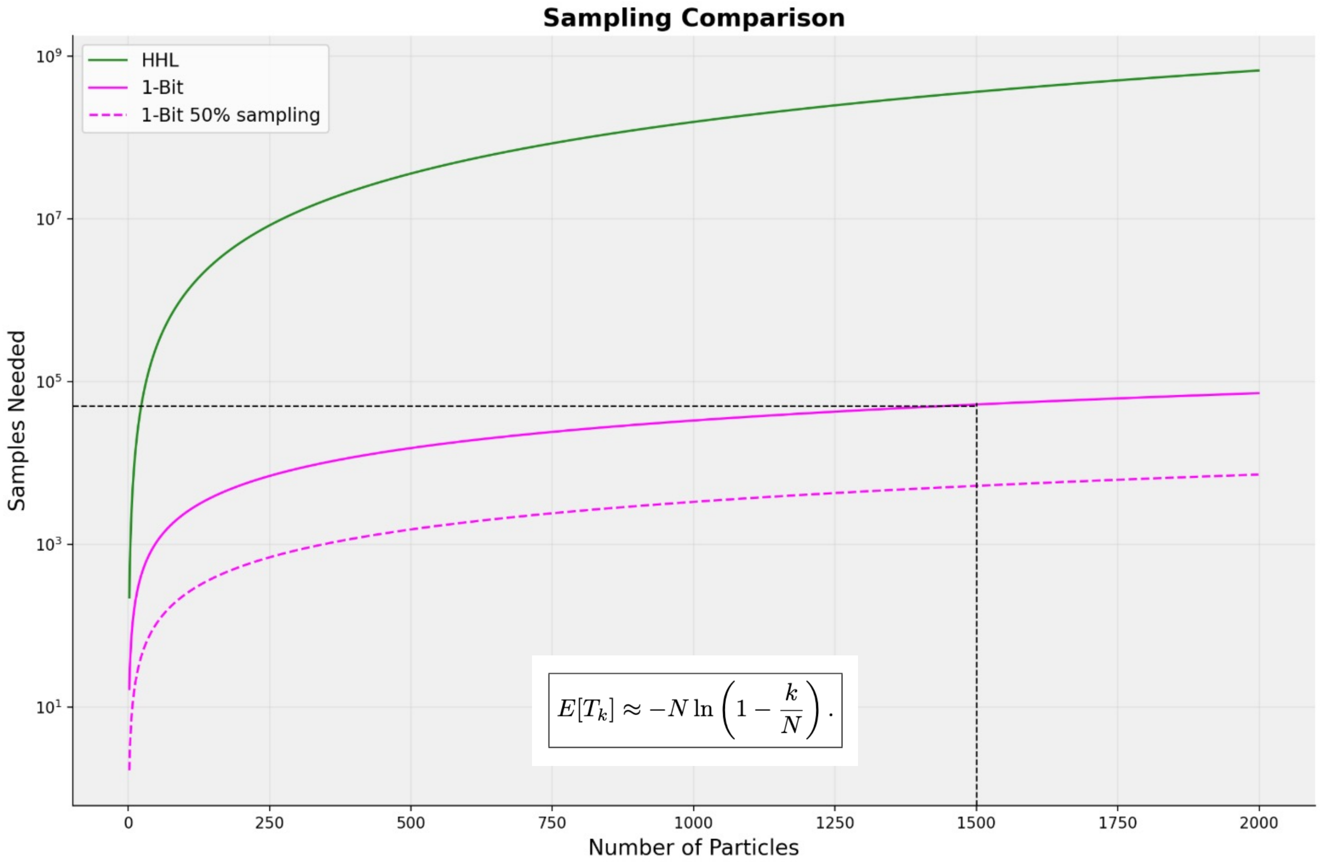


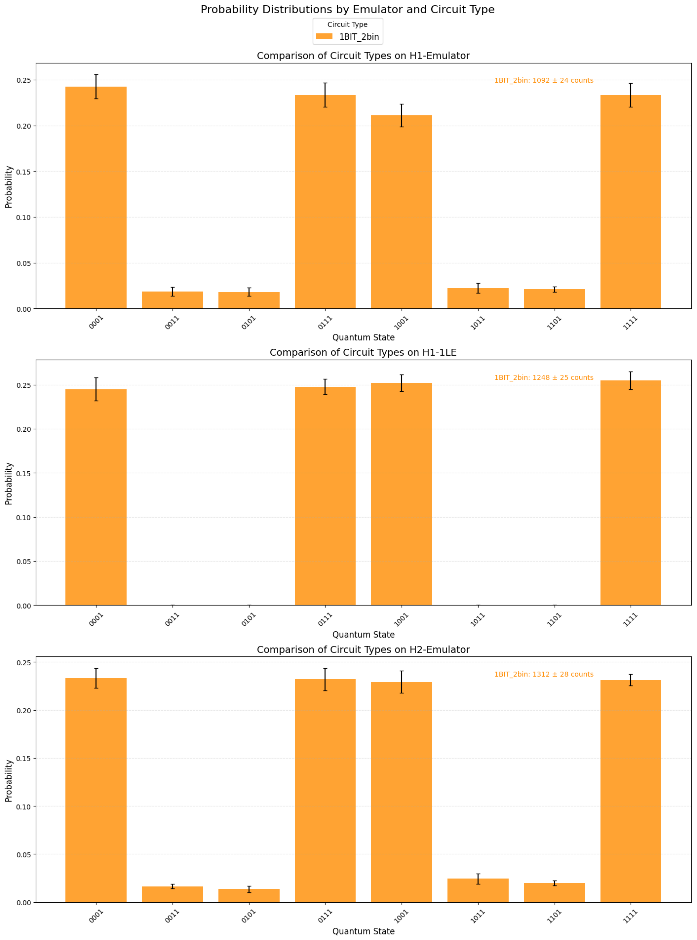
Link

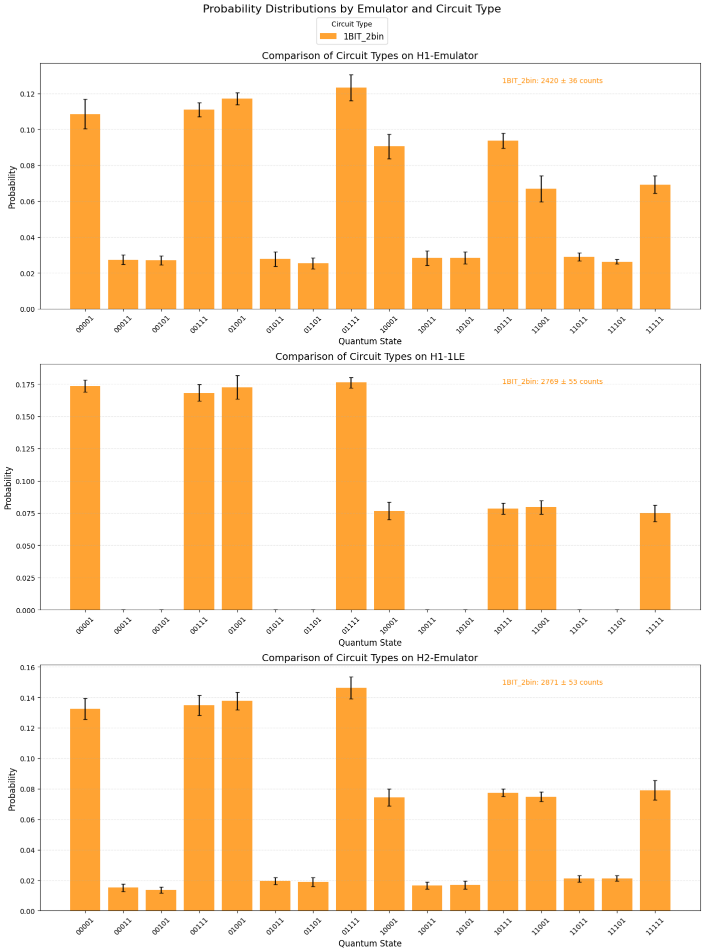
Requiring **only fraction (~50%) of all states** → strong reduction of number of readouts

https://arxiv.org/abs/2511.11458

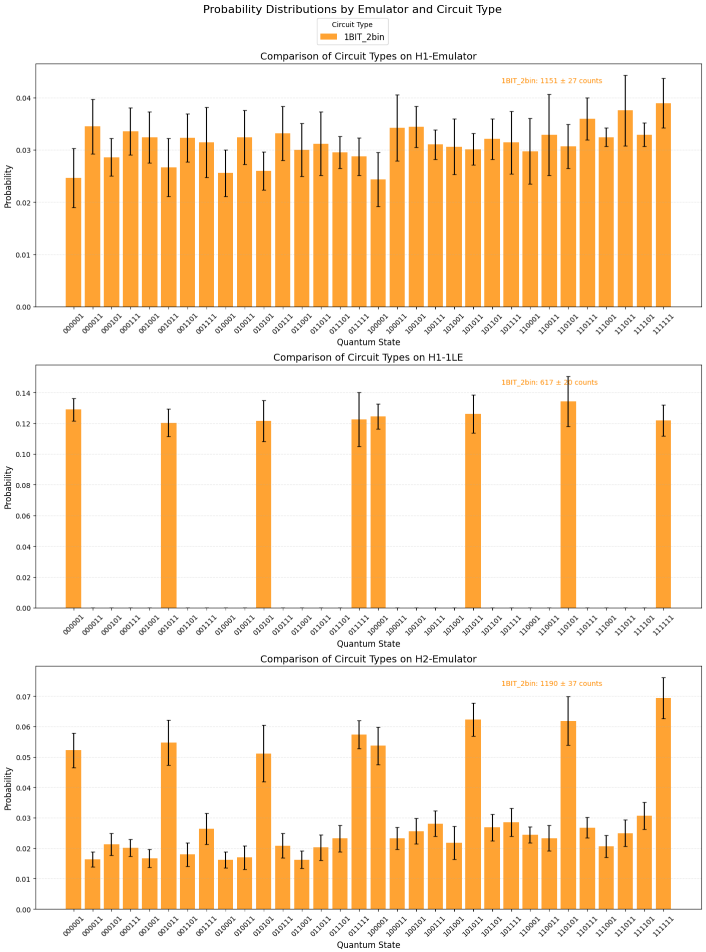
Readout issue: 1-bit QPE

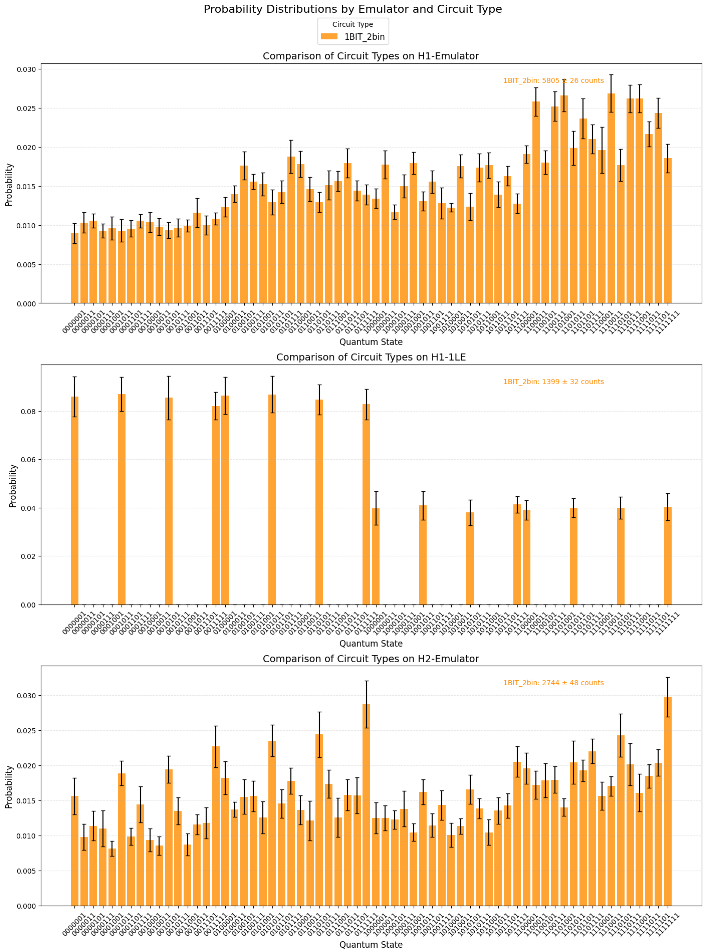






Quantum Tracking: Emulator Results





Quantum Tracking: Emulator Results