were carried out for wide energy intervals (through 0·5 MeV). At the same time, the larger the charge and the mass of the nucleus, the smaller must be the energy intervals taken, since with increase of the compound nucleus the number and density of the levels are also increased.

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56. ON THE ENERGY LOSS OF FAST PARTICLES BY IONISATION

The energy distribution function has been determined for fast particles which have traversed a layer of matter of a given thickness and lost energy in the latter as a result of ionisation collisions.

If a fast particle traverses a layer of matter its energy on exit will be less than the initial value due to ionisation losses. For a given thickness of the layer this energy loss is not constant, the probability of considerable fluctuations being quite appreciable.

By fast particles we mean here particles (electrons, mesotrons, etc.) which possess sufficiently large energies so that the usual ionisation theory may be applied. The layer of matter is considered not to be very thick so that the mean total energy loss is small compared with the initial energy E_0.

The unknown distribution function will be denoted by $f(x, \Delta)$; this is the probability that a particle of a given initial energy E_0 on traversing a layer x will lose an amount of energy lying between Δ and $\Delta + \mathrm{d}\Delta$ (the function f is normalised so that $\int f \,\mathrm{d}\Delta = 1$). We shall write the kinetic equation which defines this function. Let $w(E, \varepsilon)$ be the probability (per unit length of the path) of an energy loss ε for a particle of energy E. As the ionisation losses are considered to be small compared with E_0, instead of E we may write E_0 in $w(E, \varepsilon)$; we shall write $w(E_0, \varepsilon)$ simply as $w(\varepsilon)$. As usual the kinetic equation can be obtained by equating the change of the distribution function $(\mathrm{d}f/\mathrm{d}x)\mathrm{d}x$ on a length $\mathrm{d}x$ to the "collision integral" which expresses the difference in the number of particles which acquire, due to ionisation losses along $\mathrm{d}x$, a given energy E, and the number of particles, which leave the given energy interval. We then obtain the following equation:

$$\frac{\partial f}{\partial x} = \int_0^\infty w(\varepsilon)\,[f(x, \Delta - \varepsilon) - f(x, \Delta)]\,\mathrm{d}\varepsilon. \tag{1}$$

For the upper limit of integration we may write ∞ as $f(x, \Delta) = 0$ for $\Delta < 0$ and $w(\varepsilon) = 0$ for $\varepsilon > E_0$. Equation (1) does not contain explicitly the independent variables x and Δ. This permits us to find the solution by applying the Laplace transformation. Carrying out this transformation with respect to the

L. Landau, On the energy loss of fast particles by ionisation, *J. Phys. U.S.S.R.* **8**, 201 (1944).

independent variable Δ we write

$$\varphi(p, x) = \int_0^\infty f(\Delta)\, \mathrm{e}^{-p\Delta}\, \mathrm{d}\Delta . \tag{2}$$

Then $f(\Delta)$ can be expressed in terms of $\varphi(p)$ as follows:

$$f(x, \Delta) = \frac{1}{2\pi \mathrm{i}} \int_{-\mathrm{i}\infty + \sigma}^{+\mathrm{i}\infty + \sigma} \mathrm{e}^{p\Delta}\, \varphi(p, x)\, \mathrm{d}p, \tag{3}$$

where the integration is carried out over a straight line parallel to the imaginary axis and shifted to the right of the latter ($\sigma > 0$).

Multiplying both sides of the equation (1) by $\mathrm{e}^{-p\Delta}$ and integrating with respect to $\mathrm{d}\Delta$ we easily obtain

$$\frac{\partial \varphi\,(p, x)}{\partial x} = -\varphi(p, x) \int_0^\infty w(\varepsilon)(1 - \mathrm{e}^{-p\varepsilon})\, \mathrm{d}\varepsilon \tag{4}$$

For $x = 0$, i.e. on the incidence surface, we must have $f(0, \Delta) = \delta(\Delta)$, which means that on this surface there is a single particle of energy $E = E_0$. We then find from (2) that for $x = 0$, $\varphi(p) = 1$. Integrating equation (4), with this initial condition we get

$$\varphi\,(p, x) \exp\left[-x \int_0^\infty w(\varepsilon)(1 - \mathrm{e}^{-p\varepsilon})\, \mathrm{d}\varepsilon\right].$$

Inserting this in (3) we obtain the following general expression for the distribution function f in terms of the probability $w(\varepsilon)$

$$f(x, \Delta) = \frac{1}{2\pi \mathrm{i}} \int_{+\mathrm{i}\infty + \sigma}^{-\mathrm{i}\infty + \sigma} \mathrm{e}^{p\Delta - x\int_0^\infty w(\varepsilon)(1 - \mathrm{e}^{-p\varepsilon})\mathrm{d}\varepsilon}\, \mathrm{d}p . \tag{5}$$

In principle formula (5) is the solution of our problem in the general case. In order to be able to apply it, the function $w(\varepsilon)$ must be known; generally speaking, the form of this function has been determined only for energies which are large compared with the energy of the atomic electrons. It can be shown, however, that in case the energy losses are not too small a complete knowledge of the function $w(\varepsilon)$ is not necessary. The condition of applicability (see equation (20)) of the corresponding formulae will be deduced below.

In order to evaluate the integral in the exponent of the integrand we shall proceed as follows. Let ε_0 be a certain characteristic atomic energy (of the order of the mean binding energy of the atomic electrons). Further, let $\varepsilon_{\max}$ be the maximum energy which can be transferred to an electron by the particle during ionisation. We assume that in integral (5) only those values of p are important for which

$$p\varepsilon_0 \ll 1, \quad p\varepsilon_{\max} \gg 1 . \tag{6}$$

The limitations which this assumption imposes on the region of applicability of the results will be examined below. We now introduce a certain energy ε_1 such that $\varepsilon_1 \gg \varepsilon_0$ and $p\,\varepsilon \ll 1$; this can always be done due to (6). We shall split the integral over $\mathrm{d}\,\varepsilon$ into two integrals with limits from 0 to ε_1 and ε_1 to ∞, respectively. In the first of them we may (due to $p\varepsilon_1 \ll 1$) write $\mathrm{e}^{-ps} \cong 1 - p\,\varepsilon$. Thus

$$\int_0^\infty w(\varepsilon)(1 - \mathrm{e}^{-p\varepsilon})\, \mathrm{d}\varepsilon = p \int_0^{\varepsilon_1} \varepsilon\, w\ (\varepsilon)\, \mathrm{d}\varepsilon + \int_{\varepsilon_1}^\infty w(\varepsilon)(1 - \mathrm{e}^{-p\varepsilon})\, \mathrm{d}\varepsilon . \tag{7}$$

For $\varepsilon_0 \ll \varepsilon \ll \varepsilon_{\max}$ the following well known formula holds (see, for instance ref. 1):

$$w(\varepsilon) = \frac{2\pi N e^4 \varrho \sum Z}{m v^2 \sum A} \frac{1}{\varepsilon^2} \tag{8}$$

(m—mass of electron, e—charge of electron coinciding with that of the incident particle; v—velocity of the particle with the energy E_0; N is Avogadro's number, ϱ—density of the substance; $\sum Z$—sum of the atomic numbers in the molecule of the substance; $\sum A$—sum of the atomic weights). In particular, it can be seen that for $p\,\varepsilon \gg 1$ the second integral in (7) rapidly converges so that these intervals of energy are not important. As $p\,\varepsilon_{\max} \gg 1$ it follows that for this integral expression (8) may be applied over the complete region of integration.

As is well known[1] for the integral $\int_0^{\varepsilon_1} w(\varepsilon)\, \varepsilon\, \mathrm{d}\varepsilon$, i.e. for the energy lost by a particle (per unit path) and averaged over the interval from 0 to ε_1, the following formula holds

$$\int_0^{\varepsilon_1} w(\varepsilon)\, \varepsilon\, \mathrm{d}\varepsilon = \frac{2\pi N e^4 \varrho \sum Z}{m v^2 \sum A} \ln \frac{\varepsilon_1}{\varepsilon'},$$

$$\ln \varepsilon' = \ln \frac{\left(1 - \dfrac{v^2}{c^2}\right) I^2}{2m v^2} + \frac{v^2}{c^2} \tag{9}$$

where I is a certain ionisation potential of the atom, the value of which is usually accepted to equal $I = I_0 Z$ where $I_0 = 13{\cdot}5$ eV.

Substituting (8) in the second term of the right-hand side of (7) we obtain an integral of the form

$$\int_{\varepsilon_1}^\infty \frac{1 - \mathrm{e}^{-p\varepsilon}}{\varepsilon^2}\, \mathrm{d}\varepsilon = \frac{1}{\varepsilon_1}(1 - \mathrm{e}^{-p\varepsilon_1}) + p \int_{\varepsilon_1}^\infty \frac{\mathrm{e}^{-p\varepsilon}}{\varepsilon}\, \mathrm{d}\varepsilon .$$

Remembering that $\varepsilon_1 p \ll 1$ and introducing $z = p\,\varepsilon$ as the integration variable, we write:

$$\frac{1}{p} \int_{\varepsilon_1}^\infty \frac{1 - \mathrm{e}^{-p\varepsilon}}{\varepsilon^2}\, \mathrm{d}\varepsilon = 1 + \int_{\varepsilon_1 p}^\infty \frac{\mathrm{e}^{-z}}{z}\, \mathrm{d}z = 1 + \int_{\varepsilon_1 p}^1 \frac{\mathrm{d}z}{z} + \int_0^1 \frac{\mathrm{e}^{-z} - 1}{z}\, \mathrm{d}z + \int_1^\infty \frac{\mathrm{e}^{-z}}{z}\, \mathrm{d}z .$$

The sum of two latter integrals in the right-hand side of the equality, as is well known, is $-C$ where $C = 0{\cdot}577\ldots$ is Euler's constant (see, for instance ref. 2). Thus

$$\int_{\varepsilon_1}^{\infty} \frac{1 - \mathrm{e}^{-p\varepsilon}}{\varepsilon^2}\,\mathrm{d}\varepsilon = p(1 - C - \ln p\varepsilon_1).$$

Substituting this expression in equations (9) and (7) we obtain:

$$x\int_0^{\infty} w(\varepsilon)\,(1 - \mathrm{e}^{-p\varepsilon})\,\mathrm{d}\varepsilon = \xi p(1 - C - \ln p\varepsilon'), \tag{10}$$

where instead of the co-ordinate x we have introduced the quantity

$$\xi = x\frac{2\pi N e^4 \varrho \sum Z}{m v^2 \sum A}. \tag{11}$$

Finally, inserting (10) in (5) and introducing a new integration variable $u = \xi p$ we obtain the following integral representation of the distribution function:

$$f(x, \Delta) = \frac{1}{\xi}\varphi(\lambda), \tag{12}$$

where

$$\varphi(\lambda) = \frac{1}{2\pi \mathrm{i}}\int_{-\mathrm{i}\infty+\sigma}^{+\mathrm{i}\infty+\sigma} \mathrm{e}^{u\ln u + \lambda u}\,\mathrm{d}u, \tag{13}$$

$$\lambda = \frac{\Delta - \xi\left(\ln\dfrac{\xi}{\varepsilon'} + 1 - C\right)}{\xi}. \tag{14}$$

Thus the function of two variables $f(\Delta, x)$ turns out to equal the product of $1/\xi$ by a universal function $\varphi(\lambda)$ of a non-dimensional variable λ. This function was computed by the Calculation Bureau of the Mathematical Institute of the Academy of Sciences of the U.S.S.R., to whom the author acknowledges his gratitude. The function $\varphi(\lambda)$ has a maximum at $\lambda = -0{\cdot}05$. Thus the most probable value of the energy loss is given by the expression

$$\Delta_0 = \xi\left(\ln\frac{\xi}{\varepsilon'} + 0{\cdot}37\right), \tag{15}$$

which is more precise than the usual formula for this quantity.

Let us introduce the notation

$$\eta = \frac{2\pi V e^4 \varrho \sum Z}{m c^2 \sum A} x = \frac{1{\cdot}54 \times 10^3 \mu \sum Z}{\sum A}, \tag{16}$$

where μ is the mass of the layer of matter per cm² of its surface and η is measured in electron volts. Then $\xi = \eta/(v/c)^2$ and we have

$$\Delta_0 = \frac{\eta}{\left(\dfrac{v}{c}\right)^2}\left[\ln\frac{3 \times 10^3 \eta}{Z^2\left(1 - \dfrac{v^2}{c^2}\right)} + 1 - \frac{v^2}{c^2}\right]. \tag{17}$$

The probability of an energy loss lying between Δ and $\Delta + \mathrm{d}\Delta$ is

$$f(x, \Delta)\,\mathrm{d}\Delta = \varphi\left(\frac{\Delta - \Delta_0}{\xi}\right)\mathrm{d}\left(\frac{\Delta - \Delta_0}{\xi}\right), \tag{18}$$

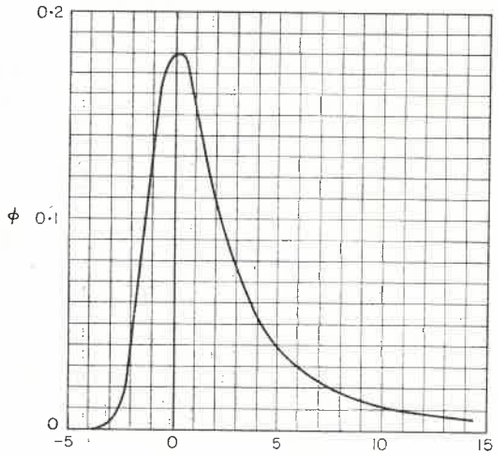

FIG. 1.

and the integral probability for an energy loss exceeding Δ is

$$\int_{\Delta}^{\infty} f(x, \Delta)\,\mathrm{d}\Delta = \psi\left[\frac{\Delta - \Delta_0}{\xi}\right], \tag{19}$$

where φ and ψ are two universal functions which are shown in Figs. 1 and 2.

To the left of the maximum of Fig. 1 ($E < E_0$, i.e. the energy loss is less than the most probable) the curve decreases very rapidly; to the right of the maximum ($E > E_0$) the curve decreases at a considerably slower rate.

It can be seen from Fig. 2 that the median of the probability distribution (i.e. the vertical line which divides the area of this curve into two equal parts) lies approximately by one unit to the right of the axis of ordinates.

Let us determine what limitations on the region of applicability of the obtained results are imposed by the assumptions (6) made above. Evidently, in integral (13) the region $u \sim 1$ of the integration variable is significant. As

$u = \xi p$ assumption (6) reduces to the conditions

$$\xi \gg \varepsilon, \tag{20}$$

$$\xi \ll \varepsilon_{\max}. \tag{21}$$

The first condition means that the observed energies must be sufficiently large in comparison with the atomic energies. This is the main limitation of applicability of the results obtained here. If the fast particle is an electron or positron then $\varepsilon_{\max} = E_0$; condition (21) $\xi \ll E_0$ is at any rate fulfilled as at the very start it was assumed that the total energy loss of the particle is small compared to E_0. However, if the mass of the fast particle is large in comparison with the

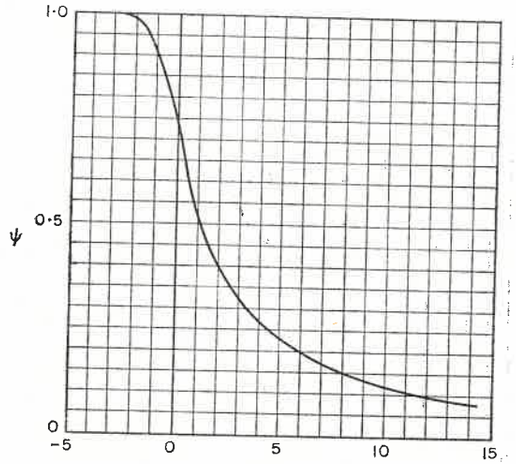

Fig. 2.

mass of the electron (mesotrons, protons), $\varepsilon_{\max}$ may be considerably less than E_0 and then relation (21) may limit the validity of the formulae. It is easy to show that in this case the condition

$$\xi \ll \frac{2m v^2}{1 - \frac{v^2}{c^2}}, \qquad \eta \ll 2mc^2 \frac{\left(\frac{v}{c}\right)^4}{1 - \frac{v^2}{c^2}} \tag{22}$$

must be fulfilled. It should be mentioned that if large fluctuations, i.e. large values of $(\Delta - \Delta_0)/\varepsilon$ interest us then instead of (22) we must write

$$\Delta - \Delta_0 \ll \frac{2m v^2}{1 - \frac{v^2}{c^2}},$$

as can be shown in an elementary way.

The opposite limiting case to (22) can also easily be examined with aid of (5); as has repeatedly been shown it leads to a gaussian distribution of the fluctuations.

We shall now derive the asymptotic formula for the function $\varphi(\lambda)$ at large values of λ. We shall first consider negative λ of large absolute values. We use the "saddle point method" to compute integral (13). The exponent $u \ln u - |\lambda| u$ in the integrand of (13) is minimum at $u = e^{|\lambda|-1}$. Let us use this value as the quantity σ, which defines the line of integration. Writing $u = e^{|\lambda|-1} + i\zeta$ we reduce the integral over du to an integral of $d\zeta$ from $-\infty$ to $+\infty$, in this integral only small values of ζ of importance. Correspondingly, expanding the expression in the exponent into a series in powers of ζ, we obtain

$$\varphi(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp\left\{-e^{|\lambda|-1} - \frac{\eta^2}{2e^{|\lambda|-1}}\right\} d\eta,$$

or, finally

$$\varphi(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left\{\frac{|\lambda| - 1}{2} - e^{|\lambda|-1}\right\}, \tag{23}$$

which shows that $\varphi(\lambda)$ very rapidly decreases with increasing $|\lambda|$.

For large positive λ it is convenient to take the integral long the contour shown in Fig. 3. Then after simple transformations it takes the form

$$\varphi(\lambda) = \frac{1}{\pi} \int_0^\infty e^{-u \ln u - \lambda u} \sin \pi u \, du. \tag{24}$$

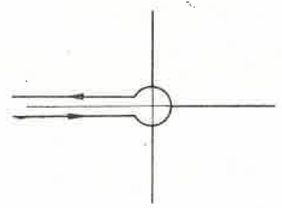

Fig. 3.

For large λ this integral tends to $1/\lambda^2$ and correspondingly $\int_\lambda^\infty \varphi(\lambda)\, d\lambda$ tends to $1/\lambda$. It will easily be seen that this means that the probability of such a fluctuation corresponds chiefly to the production of a single particle of the energy $\Delta - \Delta_0$. In order to obtain a more precise formula we introduce instead of λ the quantity ω which is related to λ by the equation

$$\lambda = \omega + \ln \omega + A,$$

where the value of the constant A is chosen below. Evidently,

$$\int_\lambda^\infty \varphi(\lambda)\, d\lambda = \int_0^\infty e^{-u \ln u - \lambda u} \frac{\sin \pi u}{\pi u} du = \int_0^\infty e^{-u \ln \omega u - \omega u - A u} \frac{\sin \pi u}{\pi u} du$$

or denoting ωu by U:

$$\int_{\lambda}^{\infty} \varphi(\lambda)\,\mathrm{d}\lambda = \frac{1}{\omega}\int_{0}^{\infty} \mathrm{e}^{-U-\frac{U}{\omega}(\ln U + A)} \frac{\sin\dfrac{\pi U}{\omega}}{U/\omega}\,\mathrm{d}U$$

This integral can be expanded in a series of powers of $1/\omega$. Then

$$\int_{\lambda}^{\infty} \varphi(\lambda)\,\mathrm{d}\varphi = \frac{1}{\omega} - \frac{1}{\omega^2}\int_{0}^{\infty} \mathrm{e}^{-U}\, U(\ln U + A)\,\mathrm{d}U + \ldots.$$

The second term vanishes if $A = C - 1$. Hence the following parametric relation exists between ψ and $(\Delta - \Delta_0)/\xi$ with an accuracy involving terms of the order $1/\omega^2$

$$\frac{\Delta - \Delta_0}{\xi} = \omega + \ln\omega - 0{\cdot}37, \quad \psi = \frac{1}{\omega}, \tag{25}$$

and

$$\varphi = \frac{1}{\omega(\omega + 1)}. \tag{26}$$

These formulae are valid already for $(\Delta - \Delta_0)/\xi = 10$ with an accuracy of several per cent.

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57. ON A STUDY OF THE DETONATION OF CONDENSED EXPLOSIVES

The modern hydrodynamic theory of the detonation of condensed explosives developed by Becker[1] and Schmidt[2] is based on the application to the detonation products of the following equation of state:

$$p(v - \alpha) = RT \tag{1}$$

where P is the pressure, v is the volume, T is the temperature, and α is the co-volume, representing the incompressible part of the detonation products.

Application of this equation leads to numerous discrepancies, since for a number of explosives it is found that $\alpha \geqq v_i$ where v_i is the initial volume of the explosive. Various hypotheses on a decrease in α with a rise in pressure, though confirmed by the experimental data of Bridgman[3] and Schmidt[4], are nevertheless insufficient to determine with accuracy the function $\alpha = \alpha(p)$, and hence do not remove the above discrepancies.

Inasmuch as the density of an explosive after it has decomposed is sufficiently high, it is in general rather futile to use any equation of state for a gas, but it is more expedient to compare the decomposition products with some liquid whose particles are in a state of oscillation, thus conditioning the nature of the course of the process of expansion of these products, and only when the detonation products expand to a definite volume v_k, when $v > v_k$, is it possible to make use of the equation of state for an ideal gas

$$p\,v = RT. \tag{2}$$

It is most expedient to build up such elements of the theory of detonation that are based on experimental data determined with a great accuracy, and satisfy all that has been said above. For a number of local action explosives it is found possible to determine the dependence of the detonation velocity D upon the density of the explosive δ_0 over a wide range of values of the latter (from 0·5 to 1·8). This dependence is shown on the accompanying graph and may be approximated by the simple formula

$$D = B\,\delta_0^{\varkappa}. \tag{3}$$

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