BINARY BLACK HOLES:
A PERSPECTIVE AND AN OUTLOOK ABOUT NUMERICAL RELATIVITY

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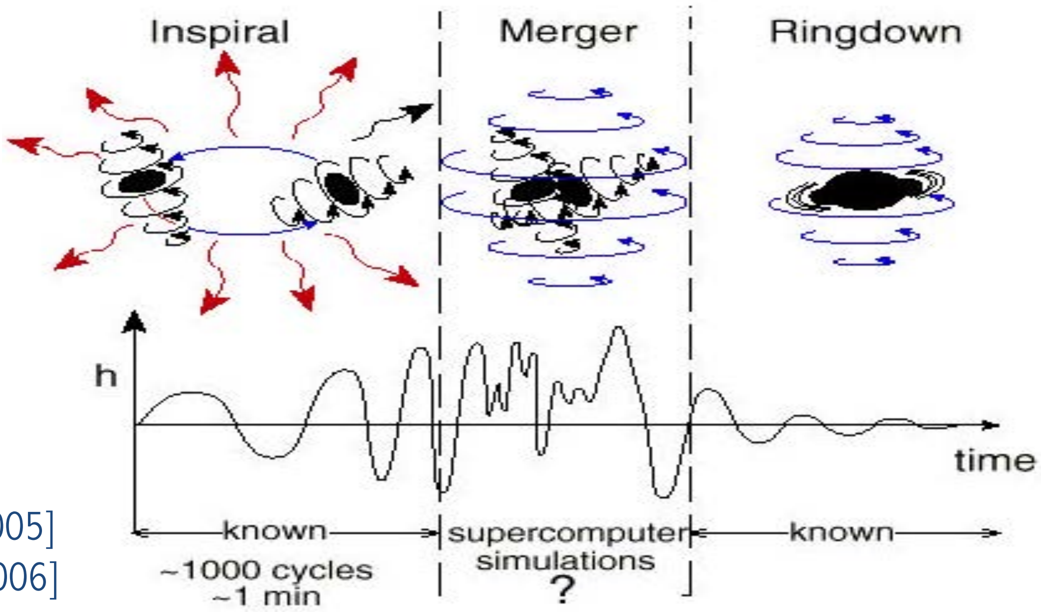
"NUMERICAL RELATIVITY IS CRUCIAL TO INTERPRETING THE OBSERVED WAVES"

"I have bet **these numerical relativists** that gravitational waves will be detected from black-hole collisions before their computations are sophisticated enough to simulate them. I expect to win…"

K.S. Thorne,
Spacetime Warps and the Quantum World: Speculations About the Future,"
in R.H. Price, ed., *The Future of Spacetime* (W.W. Norton, New York, 2002).

BBH Mergers, before *September 14, 2005*



- First successful inspiral and merger [Pretorius 2005]
- Moving punctures [Baker+2006; Campanelli +2006]
- Spectral Einstein Code (SXS) [Boyle+2006; Scheel+2008]

Image credits: Kip Thorne

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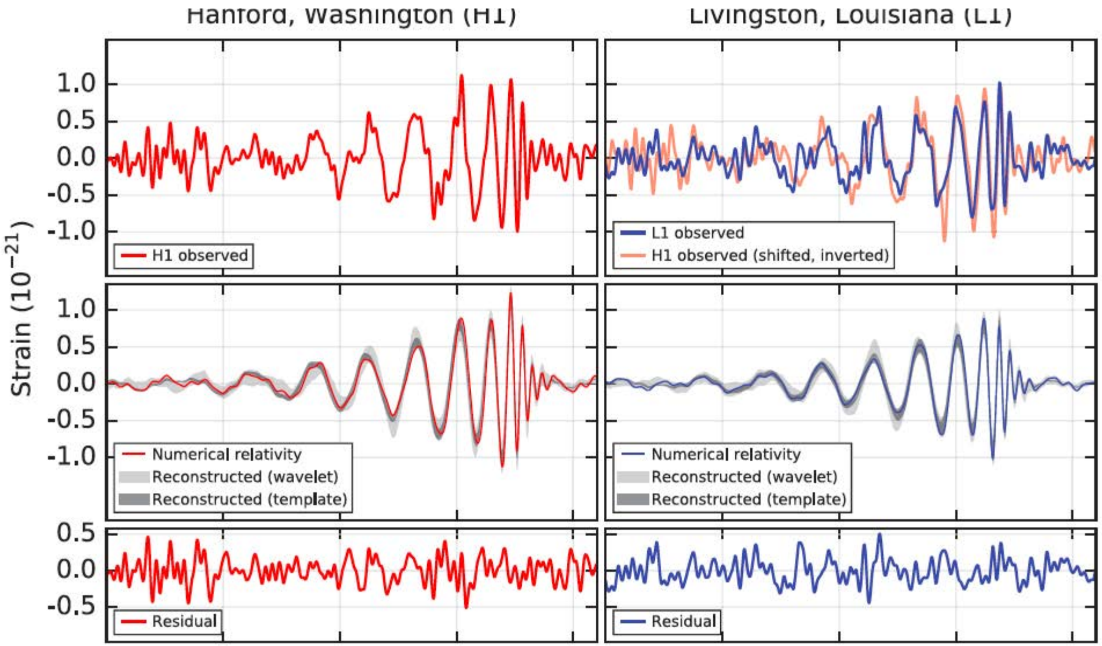
"…. but hope to lose,
because the simulation results are crucial to interpreting the observed waves."

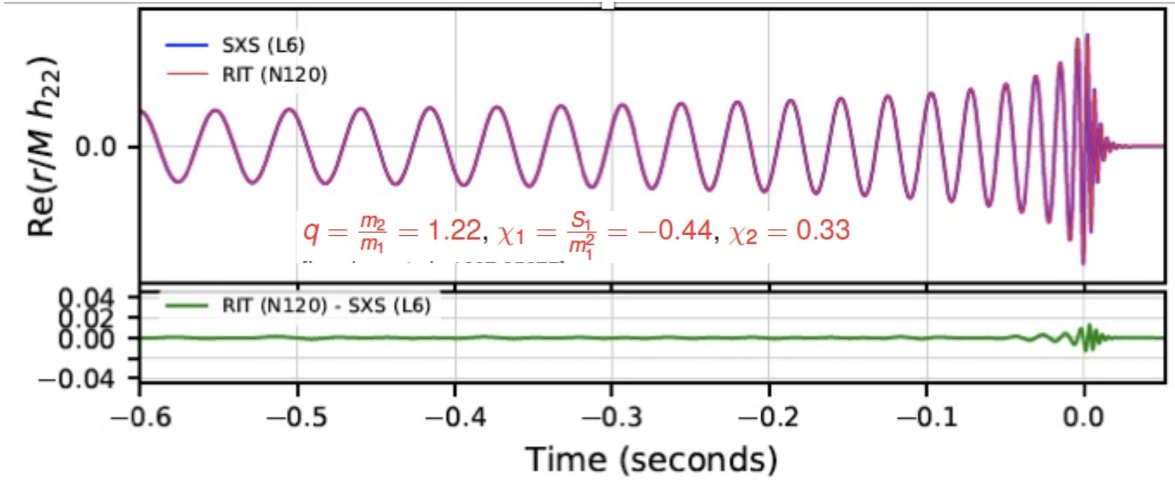
GW150914, September 14, 2015

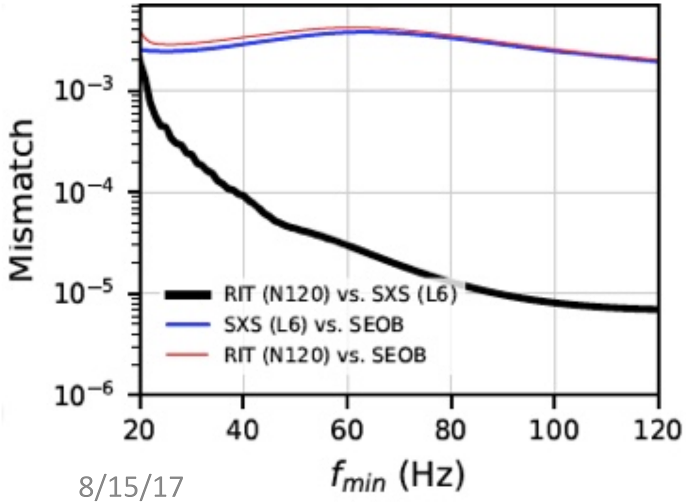


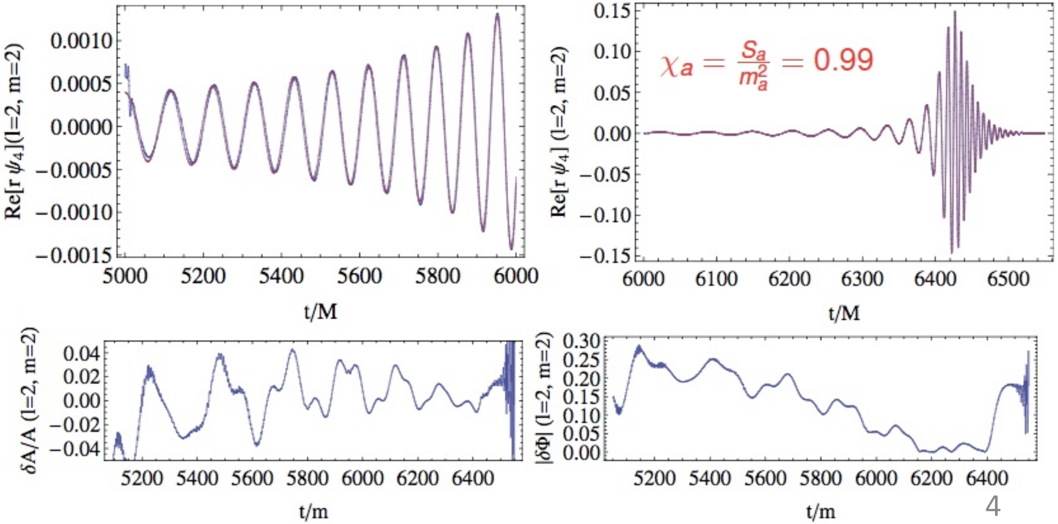
Abbott *et al.* (LVC) PRL. 116, – February 11, 2016

HOW WELL NR DO AT PRESENT?

- NR models for GW150914 show overlap 99.9% [Lovelace+2016]
- Even for very large spins of 0.99 [Zlochower+2016]
- Mismatch [Lange+2017]: High Res (0) Med Res (3.90E-05) Low Res (5.27E-05)

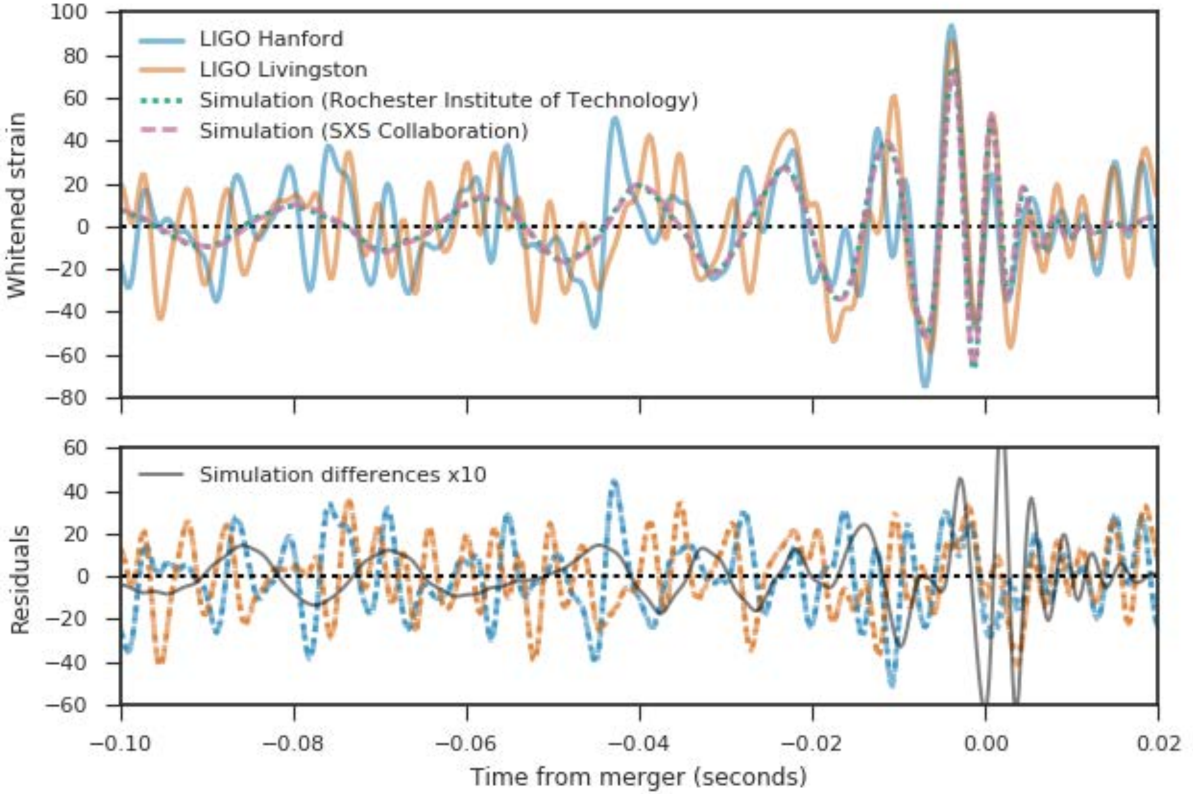






NR FOLLOW-UPS OF GW170104:

Images credits: Andrew Williamson (RIT)



q=0.5246, χ_1=(0.1607, -0.1023, -0.0529) , χ_2=(-0.3623, 0.5679, -0.3474)

Even for the low resolution, precessing, run the NR accuracy is 10 times better than residuals!

With medium, high resolutions runs, and improved extraction we can easily get an **extra factor x5**

Top panel shows the whitened data (with Livingston data shifted by -2.93ms and sign flipped), and the whitened strain from the two simulations overlaid. Bottom panel shows the **residuals, and the difference between the two simulations multiplied by 10 in grey.**

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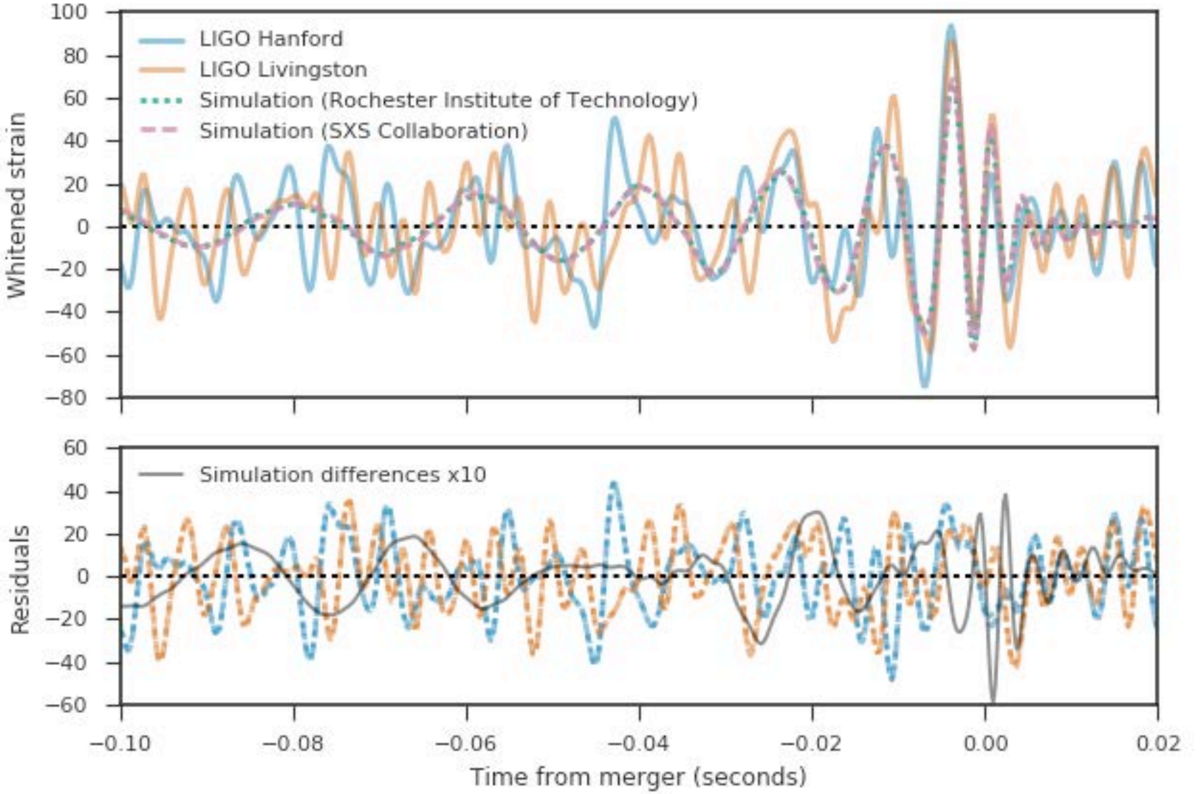

$q=0.7147$, $\chi_1=(0,0, 0.2205)$, $\chi_2=(0,0, -0.7110)$.

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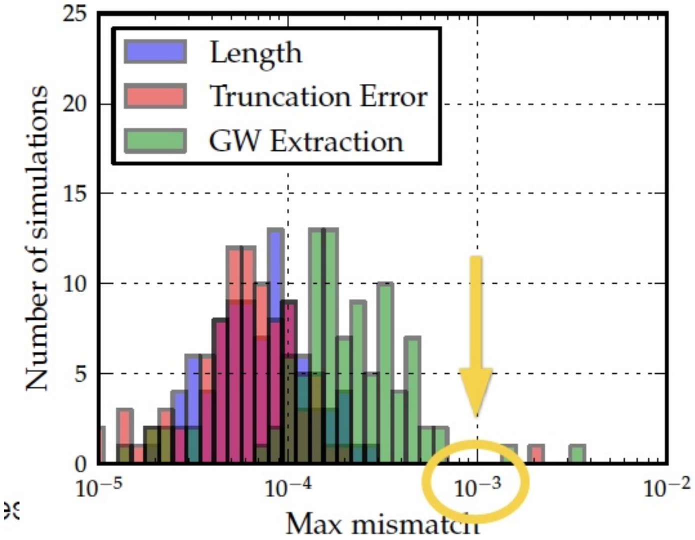
TOTALLY INDEPENDENT APPROACHES AND CODES!!!

	LazEv	SpEC
Initial data		
Formulation of Einstein constraint equations	conformal method using Bowen-York solutions [37–39]	conformal thin sandwich [38, 40]
Singularity treatment	puncture data [41]	quasi-equilibrium black-hole excision [42–44]
Numerical method	pseudo-spectral [45]	pseudo-spectral [46]
Achieving low orbital eccentricity	post-Newtonian inspiral [47]	iterative eccentricity removal [48, 49]
Evolution		
Formulation of Einstein evolution equations	BSSNOK [50–52]	first-order generalized harmonic with constraint damping [11, 53–55]
Gauge conditions	evolved lapse and shift [56–58]	damped harmonic [59]
Singularity treatment	moving punctures [12, 13]	excision [60]
Outer boundary treatment	Sommerfeld	minimally-reflective, constraint-preserving [53, 61]
Discretization	high-order finite-differences [62, 63]	pseudo-spectral methods
Mesh refinement	adaptive mesh refinement [64]	domain decomposition with spectral adaptive mesh refinement [46, 59]

Image from [Lovelace+2016];
See also Larry Kidder's talk at IAP 2017 to appreciate the meaning of this!

NR ERRORS – WAVEFORM EXTRACTION

Image credits: Chu+2015

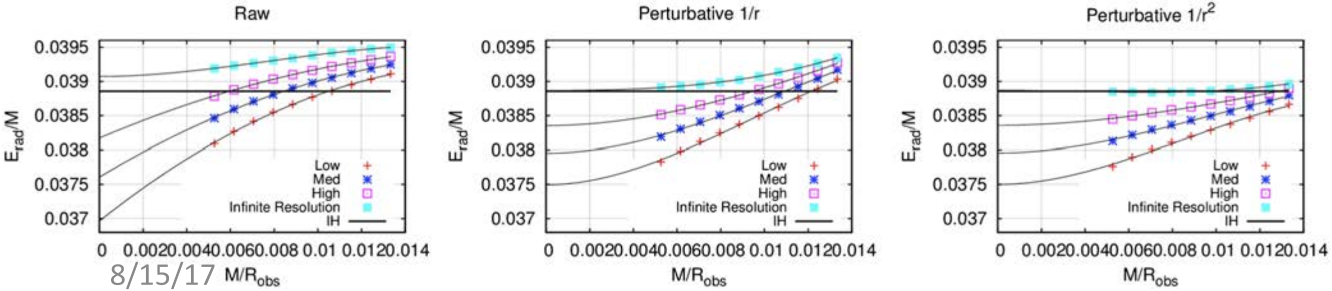


- NR source of errors, mostly due to finite extraction radius, resolution and sum over modes [Chu+2015]

- Extract information at finite radii:
  - Newman-Penrose scalar: $\lim_{r\to\infty} r\psi_4 = \lim_{r\to\infty} r(\ddot{h}_+ - i\ddot{h}_\times).$
  - Extrapolate to infinity via perturbative expansion

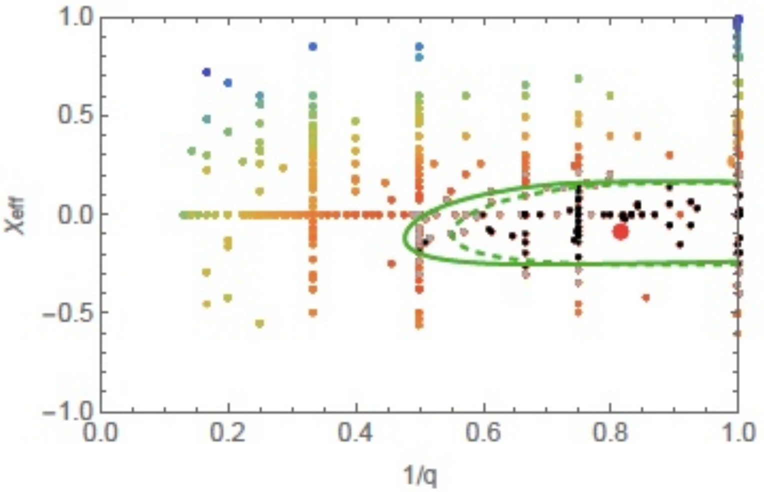
- Now improved to error ≲1E-4 [Nakano+2015] with new more accurate extraction to order $1/r^2$ (including spins)

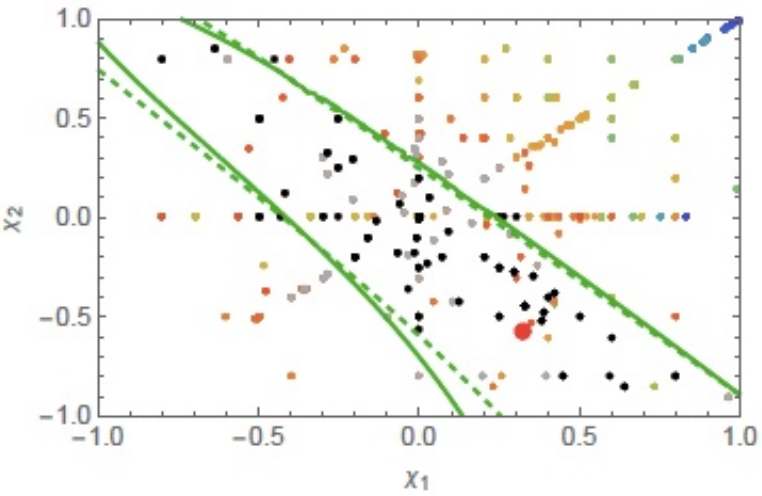
$$r\psi_4^{\ell m}|_{r=\infty} = r\psi_4^{\ell m}(t,r) - \frac{(\ell-1)(\ell+2)}{2r}\int dt[r\psi_4^{\ell m}(t,r)]$$
$$+\frac{(\ell-1)(\ell+2)(\ell^2+\ell-4)}{8r^2}\int\int dtdt[r\psi_4^{\ell m}(t,r)]$$
$$-\frac{3M}{2r^2}\int dt[r\psi_4^{\ell m}(t,r)] + \mathcal{O}(1/r^3).$$



HIGHER WAVEFORM MODES CAN HELP CONSTRAINING PARAMETERS

- Important for both PE [Lange+2017]
  - Bayesian method that directly compares GW data to NR simulations
  - Using l= 3 modes gain more information from the signal and can better constrain the parameters





- Also important to test GR: mode mixing unique to GR vs non-GR

Images credits: J. Lange, RIT.

HOW WELL NR DO AT EXTRACTING HIGHER WAVEFORM MODES?

Matching for various waveform modes for GW150914

ℓ	m	$N100$	$N110$	$N120$	$\langle h^{L6}_{\ell m} \| h^{L6}_{\ell m} \rangle$
2	0	0.8854	0.8863	0.8870	9.82
2	1	0.9905	0.9914	0.9908	16.78
2	2	0.9980	0.9980	0.9980	927.74
3	0	0.7822	0.8146	0.8356	1.02
3	1	0.9517	0.9569	0.9582	1.52
3	2	0.9978	0.9980	0.9981	28.59
3	3	0.9927	0.9933	0.9933	42.17
4	0	0.3603	0.3581	0.3554	0.05
4	1	0.7910	0.8348	0.8616	0.17
4	2	0.9074	0.9425	0.9562	1.79
4	3	0.9844	0.9909	0.9938	2.50
4	4	0.9863	0.9886	0.9901	40.95
5	0	0.3638	0.4050	0.4458	0.01
5	1	0.2994	0.3652	0.4227	0.01
5	2	0.6108	0.6176	0.6392	0.14
5	3	0.7813	0.8709	0.9197	0.32
5	4	0.9705	0.9815	0.9879	2.49
5	5	0.9315	0.9552	0.9696	4.94

Table from Lovelace+2016];

For nearly equal mass, NR do quite well already …

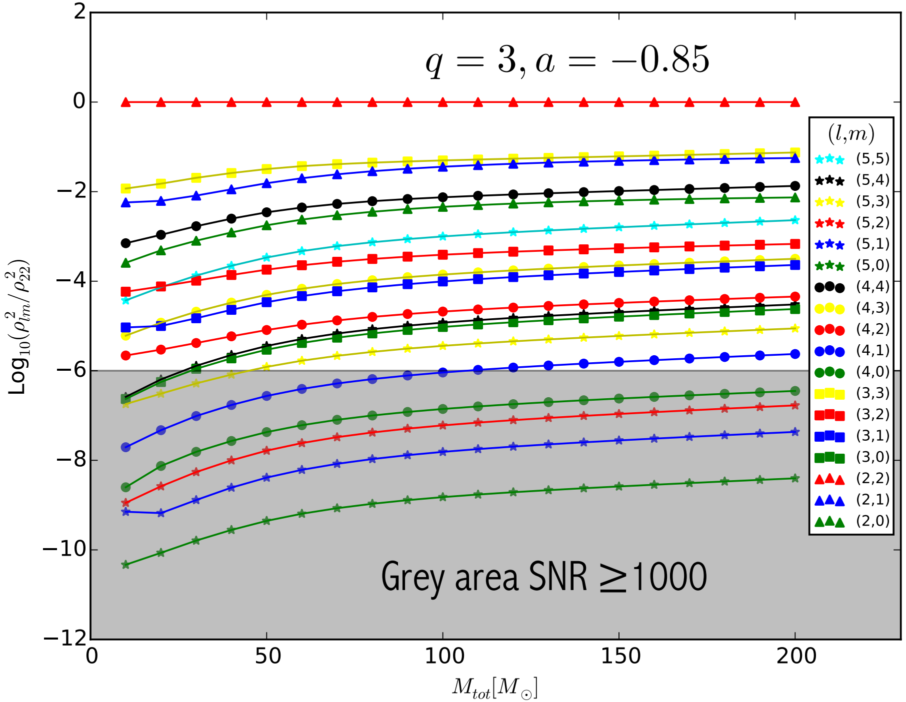
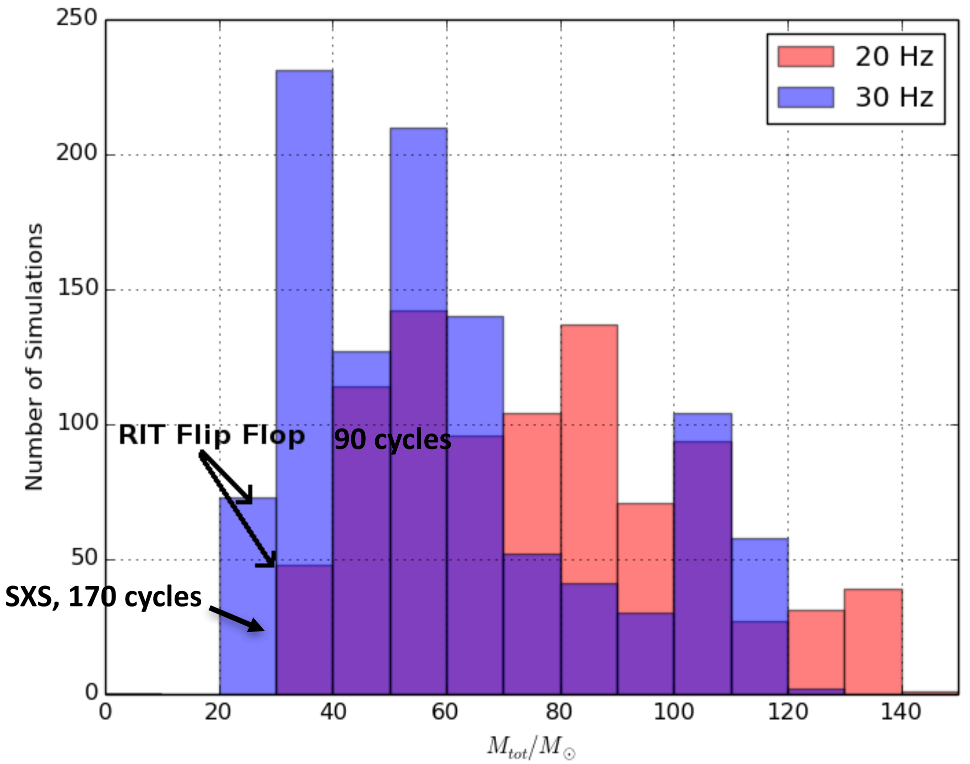


Image credits: Jacob Lange (RIT) from waveform extracted from the SXS Catalog: https://arxiv.org/pdf/1304.6077.pdf

HOW WELL NR WAVEFORMS DO IN LENGTH?

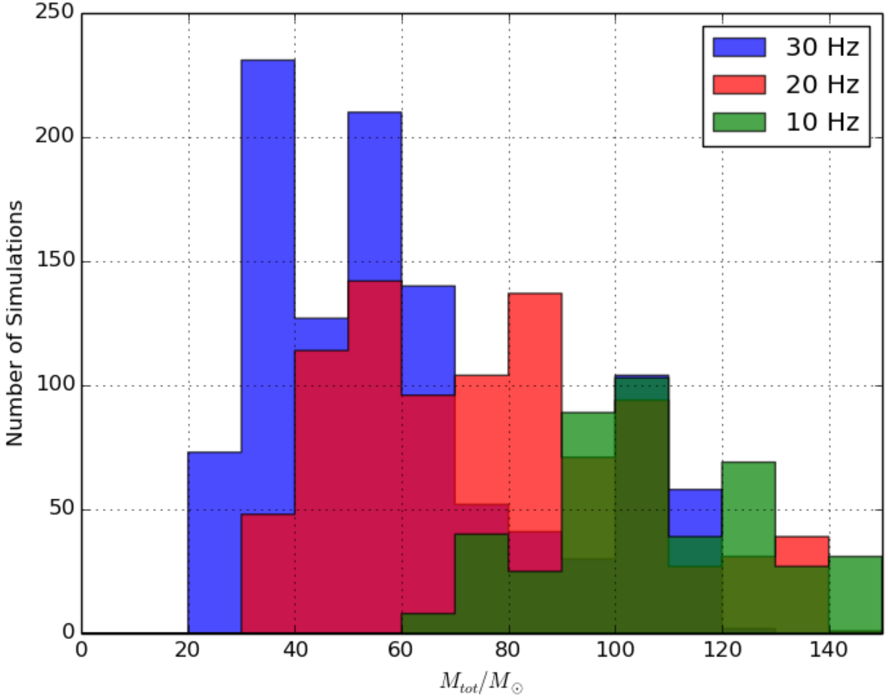


simulations that start at 20Hz or 30Hz for a given total mass

Images credits: James Healy (RIT)

- For comparable mass, $M_{tot}/M_{\odot} \gtrsim 50$ is covered by today's simulations, with ~20 orbits.
- As $M_{tot}/M_{\odot}$ becomes smaller, the duration of the signal increase very quickly, and for Mtot ~ 30 and below, one needs hybrids.
- Some high-mass ratio waveforms and long waveforms are now available, but they are still quite computationally challenging, so one needs hybrids.
- Little on eccentricity (without or with spin), but expect a lot of work in progress in 5 years!

AND, IF WE HAD ACCESS TO10HZ NOW . . .



simulations that start at 10Hz, 20Hz or 30Hz for a given total mass

Images credits: James Healy (RIT)

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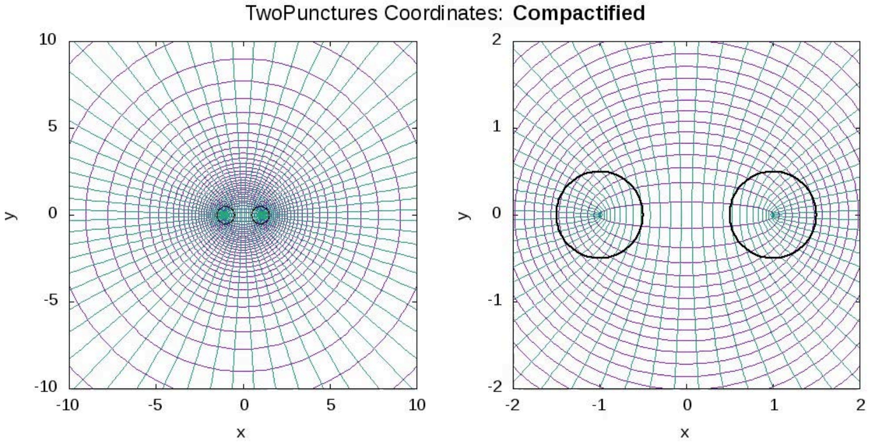
WE NEED A MORE EFFICIENT NUMERICAL RELATIVITY

Moore's Law is over, so new algorithmic developments are now the key for efficiency and accuracy!

- Load balancing and MPI vs task driven parallization (e.g. Charm++)
- Discontinuous Galarkin methods (SXS); see also [Miller&Schnetter, CQG, 2017] for moving punctures codes, such as ETK, etc.
- Moving Punctures in spherical/curvilinear coordinates Mewes+2017 (LazEv/ETK);
- New multipatch techniques? Shiokawa+, 2017, Avara+,2017

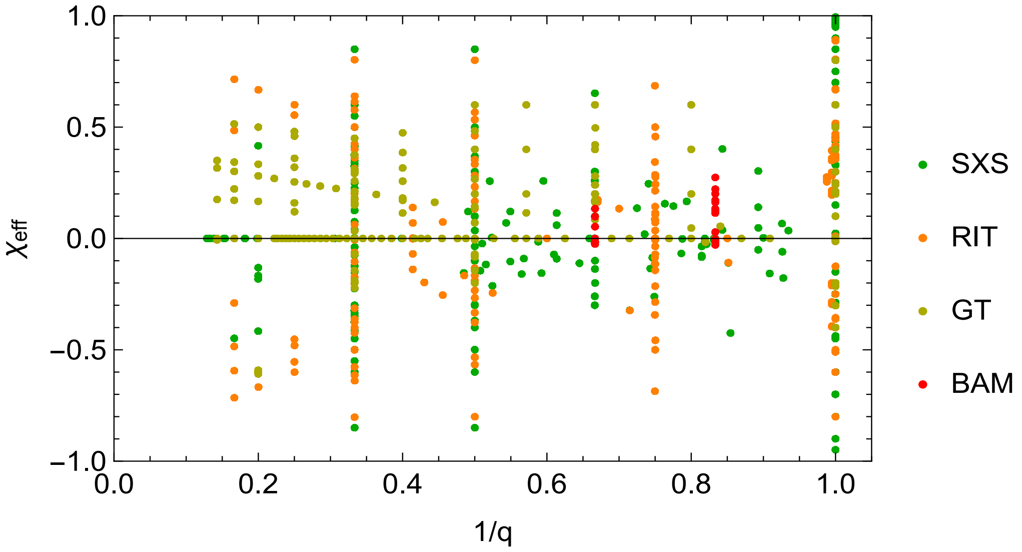


100—1,000x speed-up, supercomputer→desktop!
SENR code: http://tinyurl.com/senrcode
Zach Etienne, Ian Ruchlin (WVU), and Thomas Baumgarte (Bowdoin college)

FUTURE WORK IN NR SIMULATIONS OF BBH SYSTEMS

- Expand parameter space of catalogs
  - 8-dimensional parameter space: mass-ratio, spins, eccentricity
  - Need more waveforms for high (or small) q, high spin, eccentricity and many orbits
- Improve the accuracy and efficiency of simulations
  - High q, high spin, and many orbits are still too expensive
  - With better measurements, we need better accuracy
- Getting some remaining details correct
  - definitions of masses and spins NR vs PN/EOB
  - waveform extraction
- Explore non GR theories

8-dimensional parameter space: mass-ratio, spins, eccentricity



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