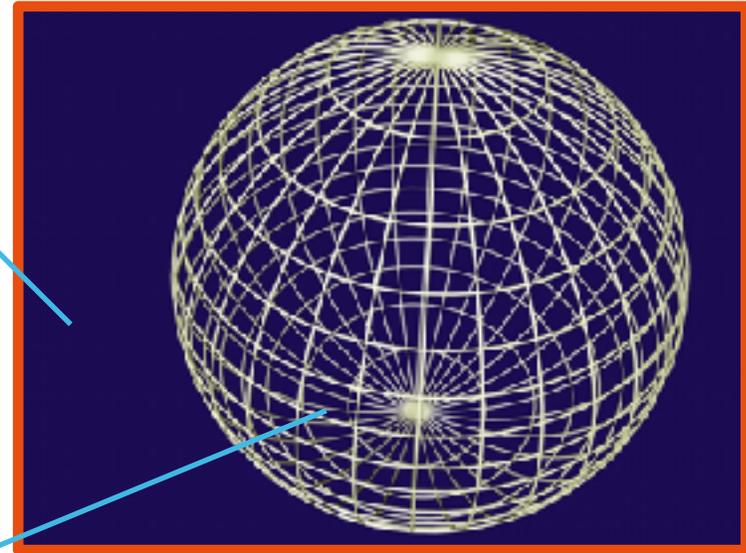


# Signature flip in metrics using hypersurface theory



$$g_{\alpha\beta}^+$$

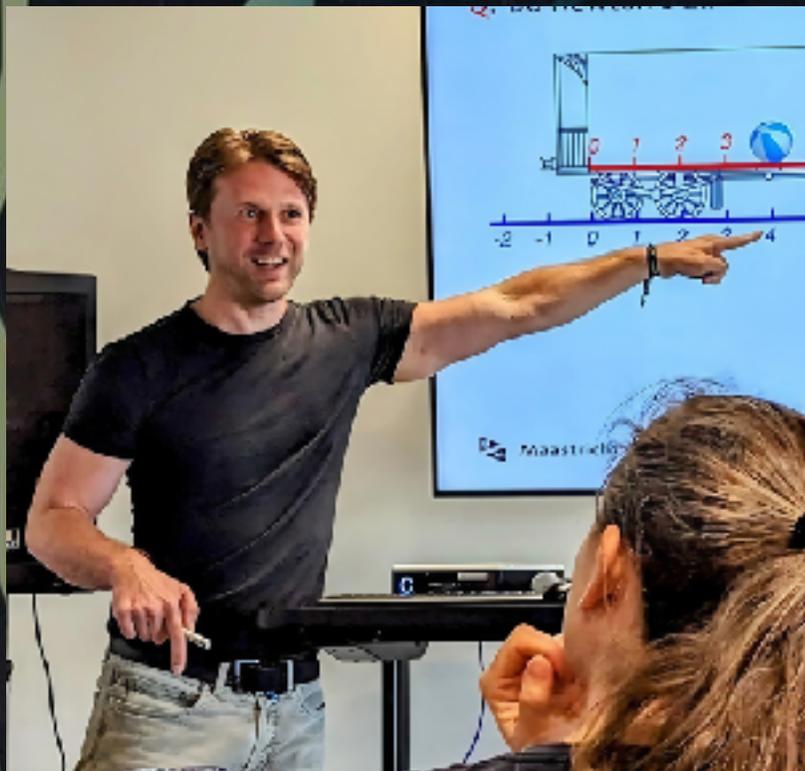
$$g_{\alpha\beta}^-$$



Friday 27th of March 2026,  
Nikhef Theory Day

Gideon Koekoek

# Gideon Koekoek



**Position:** Associate professor at Maastricht University

**At Nikhef since:** 2006 (PhD studies), 2017 (staff member)

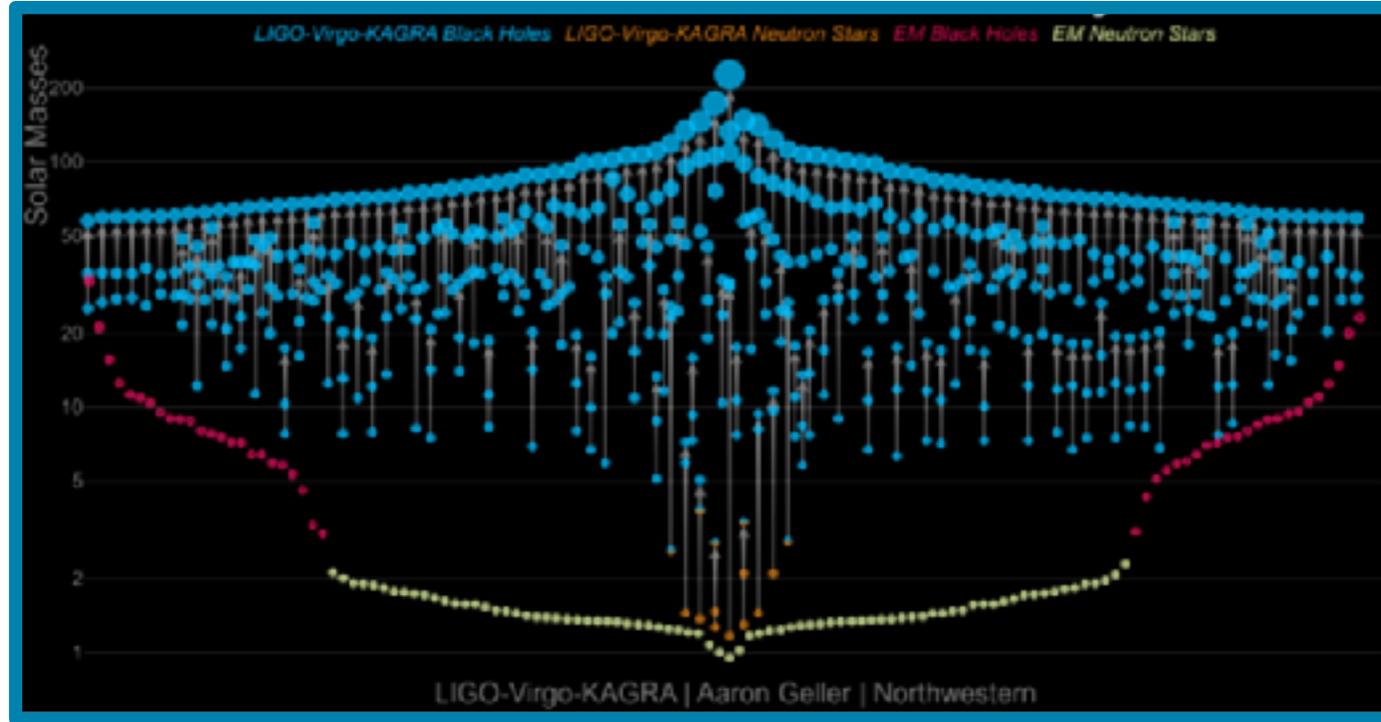
**Based at:** Maastricht University

**Working for:** I divide my time equally between research in GW/GR, curriculum building and teaching, and the political/public/educational outreach of fundamental physics with GW in particular. In the latter, I (co-)coordinate regional, national, and international initiatives, such as the Einstein Telescope Education Centre and the MaGIC Programme for teacher professionalisation. My overarching ambition is to make the Netherlands an internationally recognised central hub for education of fundamental physics.

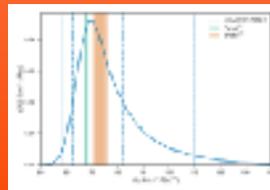
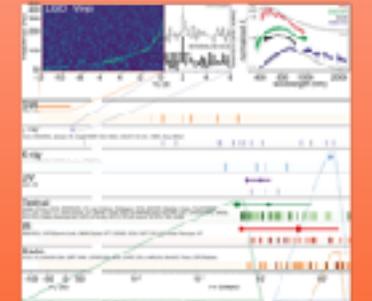
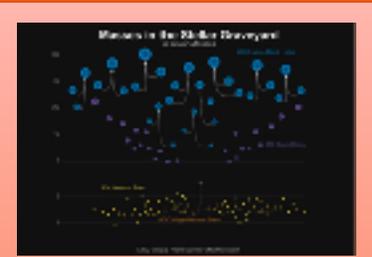
**Background:** I did my studies in physics and astronomy at VU University and Nikhef specialising in theoretical physics, obtaining my PhD in 2011 in the then newly founded gravitational wave group. I already then had an interest in educational aspects as well, picking up some awards and nominations, nudging me to take an eclectic turn of career by doing a second MSc degree in Physics Education and being a professional educator at all levels for a few years (VU, U.Leiden, secondary education). When Maastricht started their new GW-group and joined Nikhef, I joined the staff to help start research in GW physics, build curriculum, and spread the gospel of GW physics, and have been there since.

**Hobbies and fun facts:** I have an endless amount of black Hema shirts, that I wear all year.

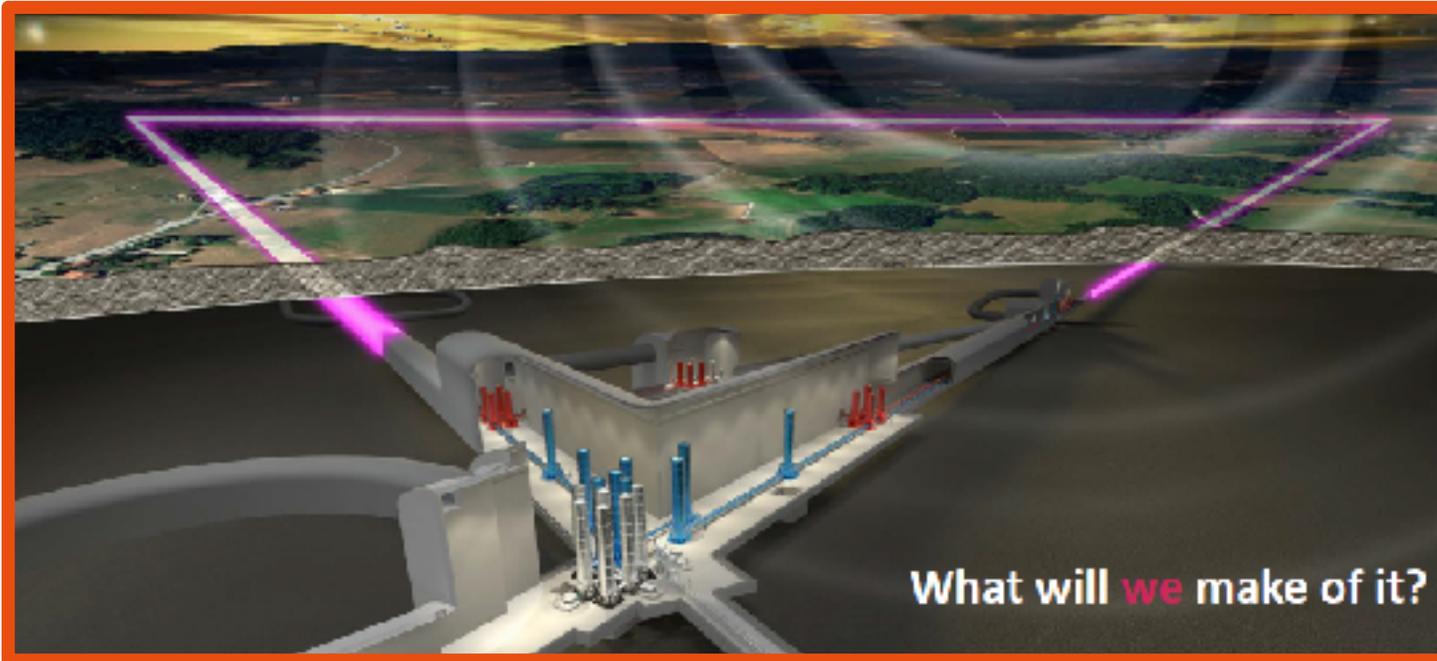
# Exotic compact objects and gravitational waves



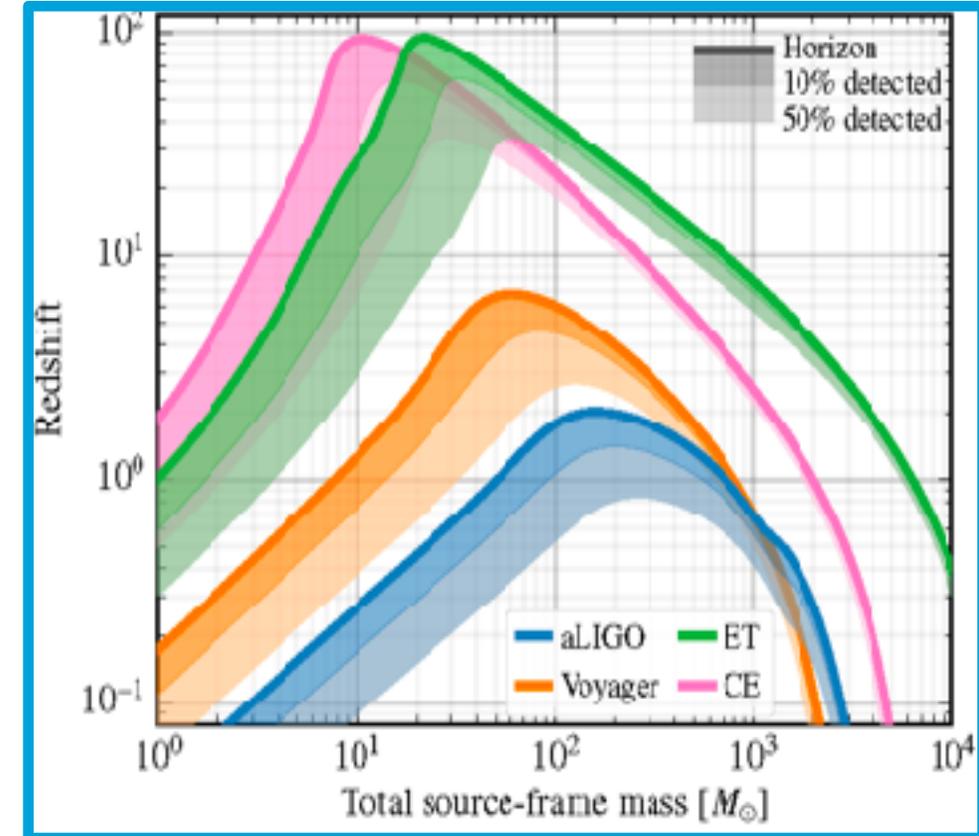
[www.ligo.caltech.edu/news/ligo20250826](http://www.ligo.caltech.edu/news/ligo20250826)



# Exotic compact objects and gravitational waves



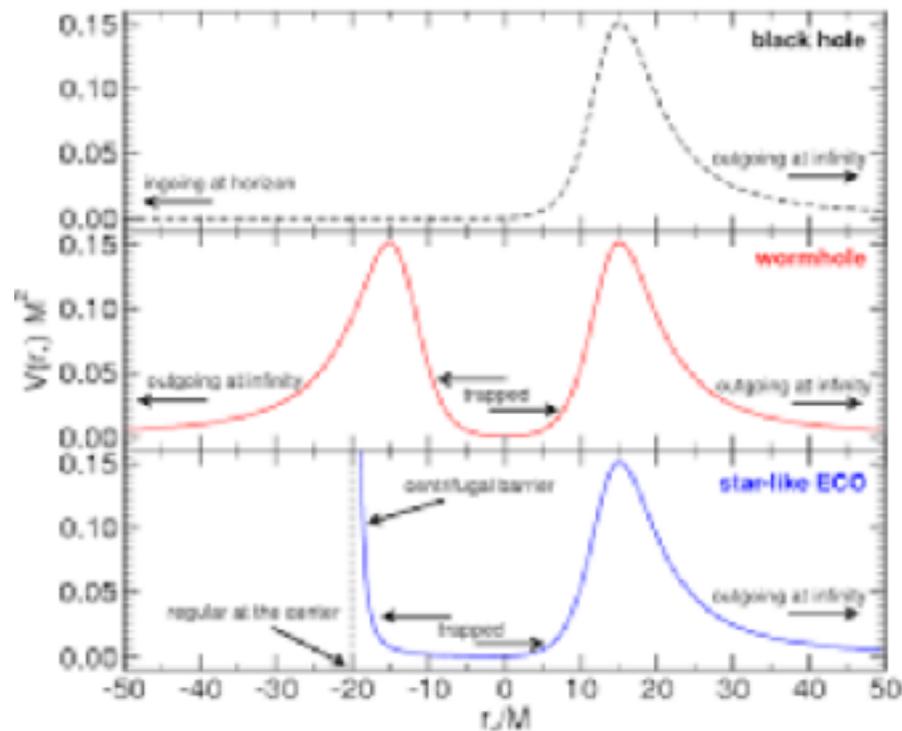
- Underground laboratory
- Three laser beams, 10 km long
- European collaboration, Limburg is one of the three candidate sites
- $O(10^5-10^6)$  more signals



# Exotic compact objects and gravitational waves

Gravitational waves are a powerful tool to explore black holes, GR, and cosmology.  
But also to explore *black hole mimickers*.

A key example: ECO with reflective layer



A key example: the gravastar



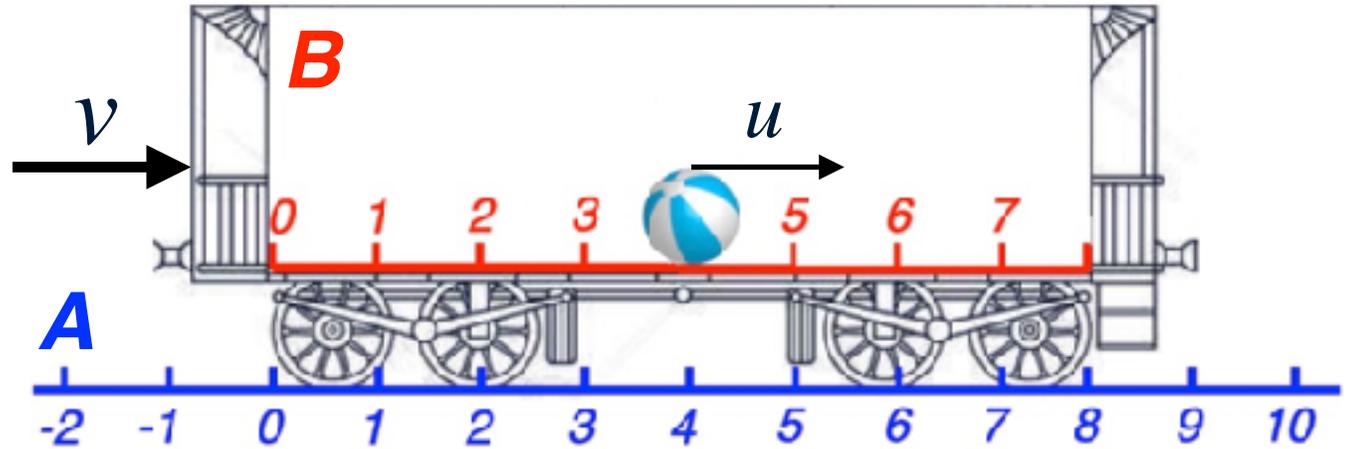
# A second copy of special relativity



Albert Einstein  
1879-1955

All inertial systems  
give the same laws of  
physics!

~~The speed of light is  
the same in all inertial  
systems!~~



$$\Delta t_B = \gamma \Delta t_A + \kappa \Delta x_A$$

$$\Delta x_B = \beta \Delta x_A + \sigma \Delta t_A$$

four unknowns:  $\gamma, \kappa, \beta, \sigma$

+

four conditions on  
the two equations

=

Gives expressions  
for  $\gamma, \kappa, \beta, \sigma$ .

<https://arxiv.org/pdf/2109.11925>

## Results in two copies of flat spacetime

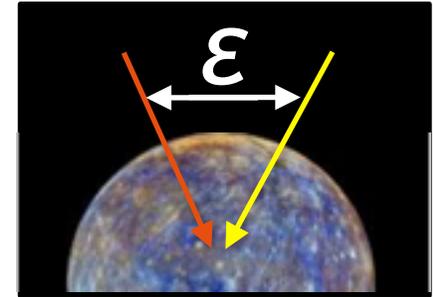
One has a metric tensor with *Lorentzian signature*: time and space differ by a minus sign

The other has a metric tensor with a *Euclidian signature*: time and space have the same sign

# A second copy of general relativity

It is straightforward to show that the usual Riemann tensor is still a measure of curvature:

$$\frac{D^2 \epsilon^\mu}{D\tau^2} = R^\mu_{\alpha\nu\beta} \epsilon^\nu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau},$$



and that the invariant integration measure is

Invariant action then is



$$\sqrt{|g'|} d^4x'$$

$$S = \int \sqrt{|g|} R d^4x,$$



$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$$

Another method is by the Lovelock Theorem: a second rank tensor containing first and second derivatives of  $g$  is of the form

$$\left. \begin{aligned} G_{\alpha\beta} &\propto R_{\alpha\beta} + b \cdot g_{\alpha\beta} R, \\ D_\alpha T^{\alpha\beta} &= 0. \end{aligned} \right\}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$$

Coupling to matter?

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = \kappa T_{\alpha\beta}$$

# Trap one type into the other?

## Lorentzian signature:

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta} R = +\frac{8\pi G}{c^4} T_{\alpha\beta}$$

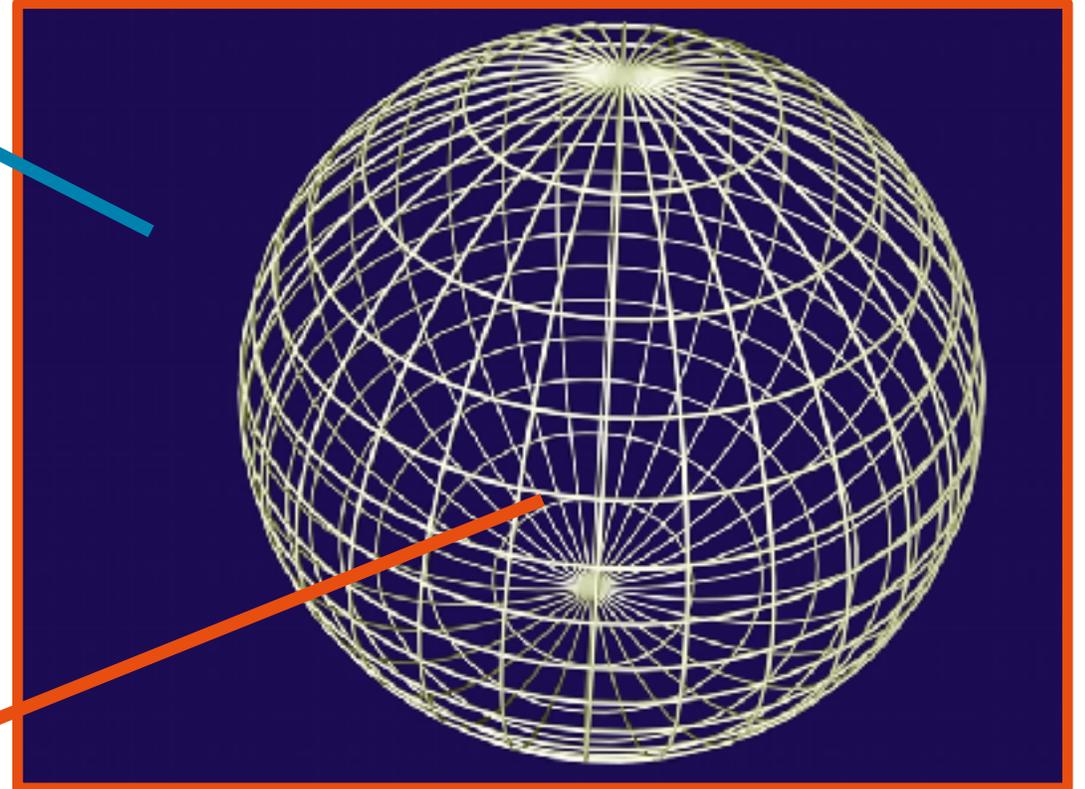
$$g_{\alpha\beta}^+ = \text{diag}\left\{-\left(1 - \frac{2M}{r}\right), \left(1 - \frac{2M}{r}\right)^{-1}, r^2, r^2 \sin^2 \theta\right\}$$

## Euclidian signature:

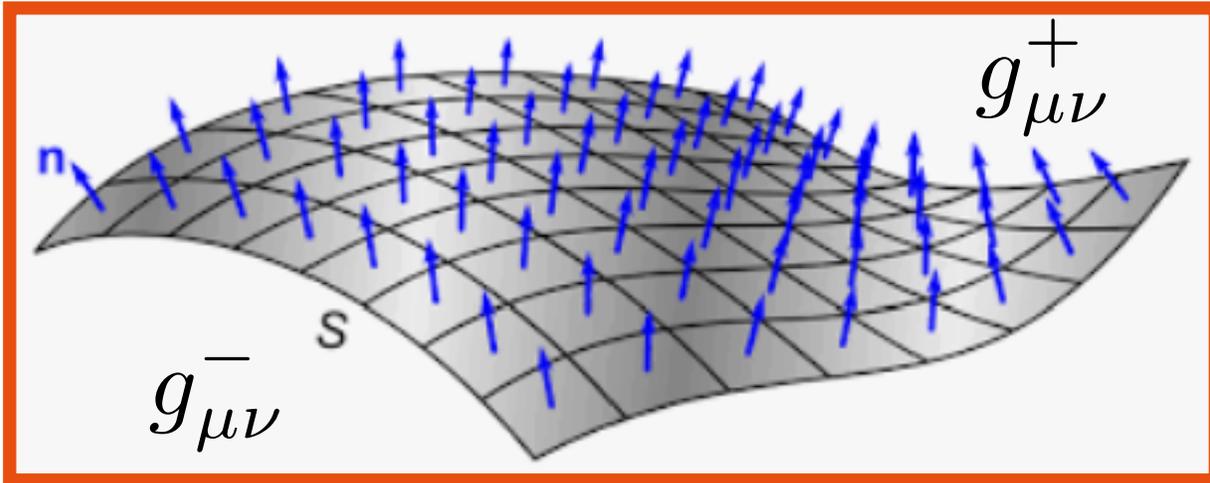
$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta} R = -\frac{8\pi G}{c^4} T_{\alpha\beta}$$

$$g_{\alpha\beta}^- = \text{diag}\left\{\left(1 - \frac{2M}{r}\right), \left(1 - \frac{2M}{r}\right)^{-1}, r^2, r^2 \sin^2 \theta\right\}$$

Conventions chosen such that it approximates to Newtonian gravity at low  $v$  and low  $G$ .



# A few-slides intro to hypersurface theory



Divide spacetime up into two parts:

$$g_{\mu\nu} = g_{\mu\nu}^+ \Theta(l) + g_{\mu\nu}^- \Theta(-l)$$

Tensors needed to describe the hypersurface  $S$ :

- \*\* its own metric tensor  $h_{ab}$ ,
- \*\* a normal vector  $n^\lambda$  and its tangent vector  $k^\lambda$ , that embed  $S$  into the surrounding spacetime

Continuity conditions:

No jump in tidal forces when crossing the surface:  
first derivative of metrics is continuous

No energy loss when crossing the surface:  
second derivative of metric is continuous

# Junction conditions

No jump in tidal forces when crossing the hypersurface

This corresponds to metric's **first** derivative being continuous

$$\partial_l g_{\mu\nu} = g_{\mu\nu}^+ \Theta(l) + g_{\mu\nu}^- \Theta(-l) + (g_{\mu\nu}^+ - g_{\mu\nu}^-) \delta(l)$$

in which the final term needs to vanish.

Projected onto the surface:

$$[h_{ab}] = 0$$

No energy loss when crossing the hypersurface

This corresponds to metric's **second** derivative being continuous:

Plugged into the field equations and setting;

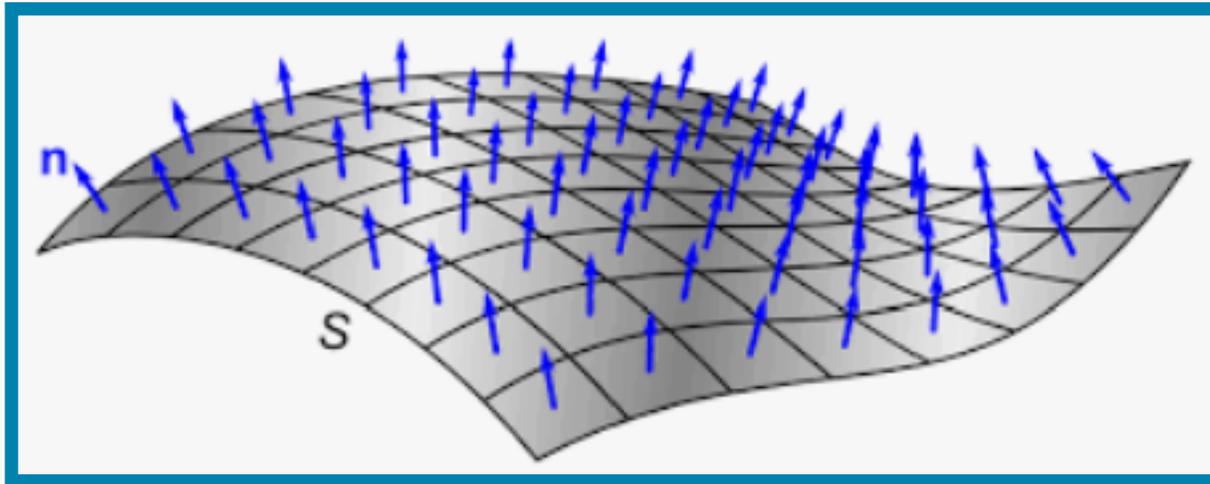
$$T_{\alpha\beta} = T_{\alpha\beta}^+ \Theta(l) + T_{\alpha\beta}^- \Theta(-l) + S_{\alpha\beta} \delta(l)$$

leads to the energy-momentum of the surface:

$$\begin{aligned} 8\pi S_{\alpha\beta} &= A_{\alpha\beta} - \frac{1}{2} A g_{\alpha\beta} \\ A_{\beta\gamma\delta}^{\alpha} &= [\Gamma_{\beta\delta}^{\alpha}] n_{\gamma} - [\Gamma_{\beta\gamma}^{\alpha}] n_{\delta} \end{aligned}$$

# Junction conditions

Junction conditions for **null surfaces** require a little more work..



Surface intrinsic coordinates are  $y^a = (\lambda, \theta^A)$ ,

The tangents to the surface:  $e_{\lambda}^{\alpha} \equiv \frac{dx^{\alpha}}{d\lambda} \equiv k^{\alpha}$

A normal to the surface is degenerate, because they are themselves null. This gives freedom to define the auxiliary fields:

$$N_{\alpha} N^{\alpha} = 0, \quad k_{\alpha} N^{\alpha} = -1, \quad N_{\alpha} e_A^{\alpha} = 0.$$

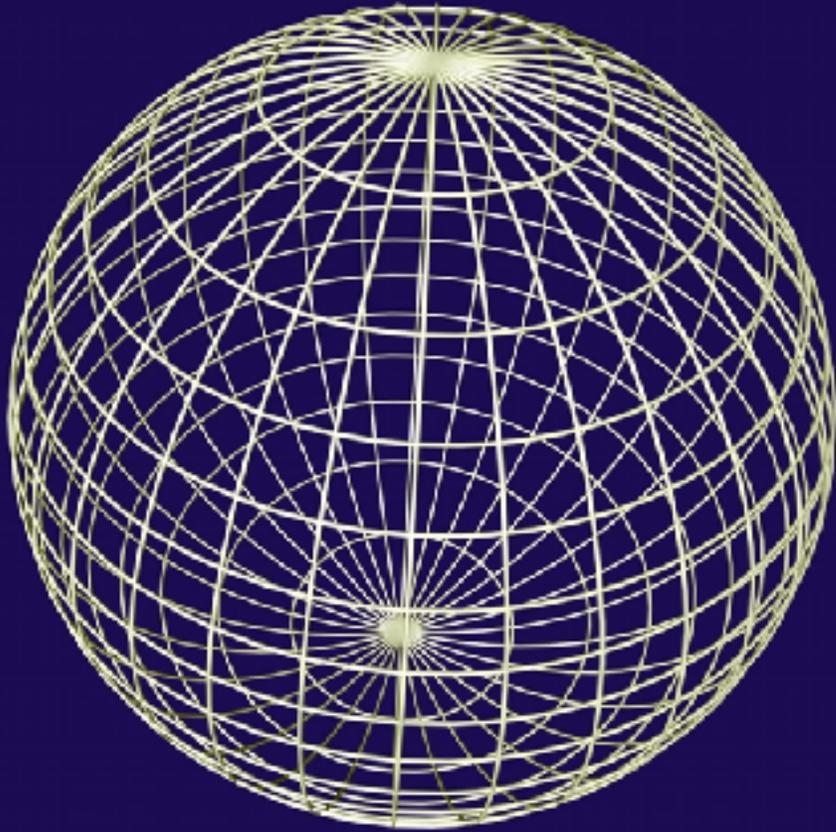
The ‘discontinuity coefficient’  $A$  can then be expressed in terms of the auxiliary field  $N$ , and projected on the surface to express the discontinuity in terms of intrinsic coordinates  $y$ .

$$C_{ab} \equiv (D_{\alpha} N_{\beta}) e_b^{\alpha} e_b^{\beta}.$$



$$\mu \propto \sigma^{AB} [C_{AB}] \quad j^A \propto \sigma^{AB} [C_{\lambda B}], \quad p \propto [C_{\lambda\lambda}],$$

# Can hypersurfaces trap the two different relativities?

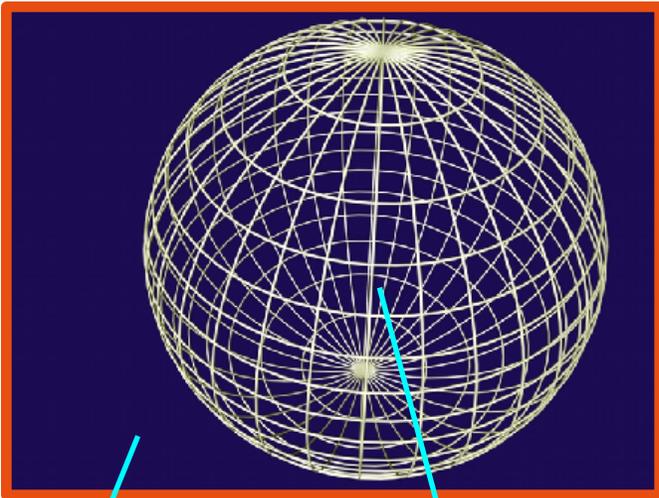


**YES**

**TL;DR:**

The hypersurface must be spherical, the surface must not have any material properties, and the radius  $R$  of the surface comes out as  $2M$ : the event horizon of a black hole.

# The technical slide



$g_{\alpha\beta}^+$

$g_{\alpha\beta}^-$

Describe inside and outside manifolds in terms of ingoing Eddington coordinates  $v, r$  :

$$v = r^* + t$$

..then the metric tensors become:

$$g_{\alpha\beta}^L = \begin{pmatrix} -f & 1 \\ 1 & 0 \end{pmatrix} \quad g_{\alpha\beta}^E = \begin{pmatrix} f & -1 \\ -1 & 2/f \end{pmatrix}$$

in which  $f = 1 - 2M/R$ . The tangent and auxiliary fields are:

$$N_{\alpha}^L = (-1, 0, 0, 0), \quad N_{\alpha}^E = (\pm i, (-1 \mp i)/f, 0, 0),$$

The **first junction condition** gives  $R = 2M$ . This is a null shell.

For null shells, the **second junction condition** gives the material properties as:

$$C_{ab}^{L,E} = (D_{\alpha} N_{\beta}^{L,E}) \delta_a^{\alpha} \delta_b^{\beta} = -\Gamma_{ab}^{\rho} N_{\rho}^{L,E} = -2M \sin^2 \theta.$$

$$C_{\lambda\lambda}^{L,E} = (D_{\alpha} N_{\beta}) k^{\alpha} k^{\beta} = \frac{1}{4M}.$$



Same on both sides!

Hypersurface has no material properties!

# Can it be measured?

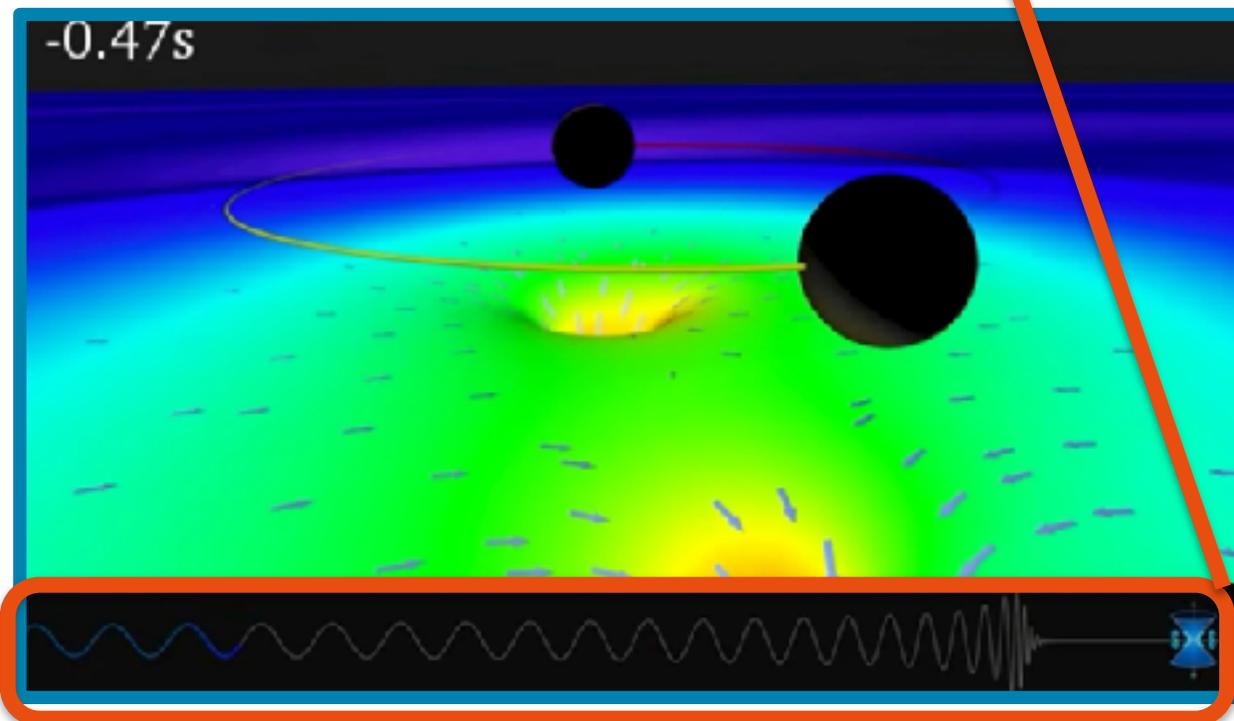
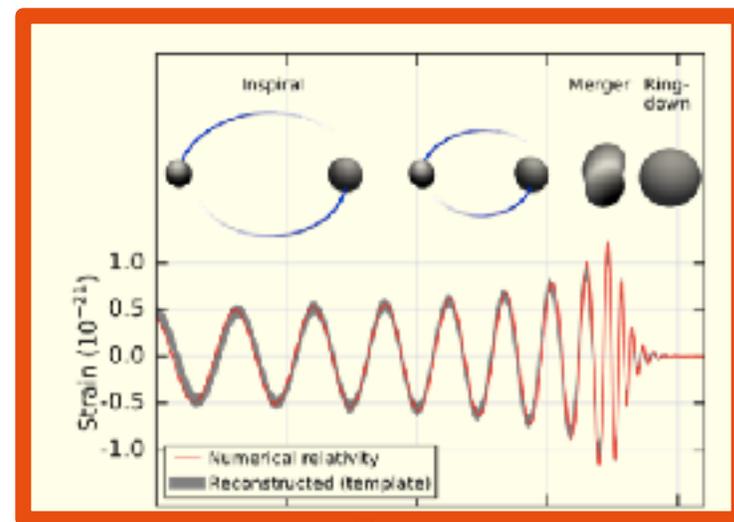
## Upshot:

The event horizon of a black hole can organically transition between the two types of relativities.

We can not peer inside, and a stationary black hole looks the same regardless of contents (Birkhoff/Wald uniqueness theorems).

But a *perturbed* black hole does have a specific gravitational wave frequency spectrum: the *Quasi-Normal Modes*.

Measuring QNM by LVK?  
See: arXiv:2509.08099v1 [gr-qc]



# Parting slide

## What the model has going for itself:

- \* All its relativity is based **exclusively on first principles**.
- \* Junction is a pure spacetime effect: **no exotic matter** need to be postulated.
- \* **Also works for Kerr spacetimes**, with the same conclusions.

## Open avenues to be explored:

- \* Constructs currently are **not yet dynamical**. (Due to the null-shell requirement, the system does not allow a Snyder-Oppenheimer collapse.)
- \* Dynamics/collapse **requires a a moving null-boundary**. These are provided by Vaidya metrics.
- \* **Goal:** calculate the black hole perturbation equations and quasi-normal mode spectrum.



Thank you!