TrackHHL

A quantum algorithm for track reconstruction

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Theory Day in Maastricht

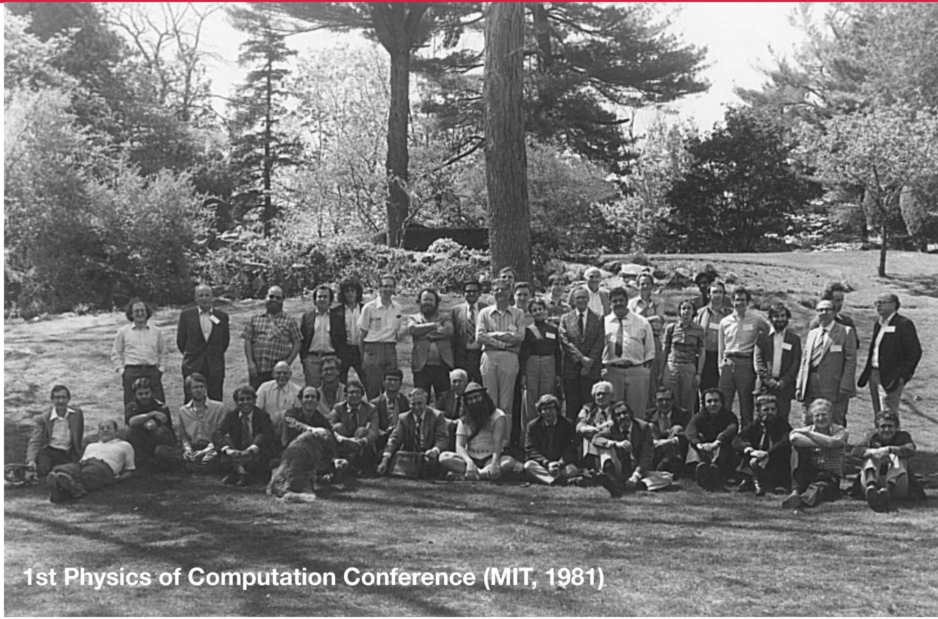
Outline

1. Introduction
    a. What is quantum computing?
    b. A trilogy of quantum algorithm
        i. Fourier Transform
        ii. Phase Estimation
        iii. Matrix inversion
2. TrackHHL
    a. The LHCb and Track Reconstruction
    b. A Global Tracking Algorithm
        i. Classical
        ii. HHL
        iii. 1-Bit Quantum Filter

Quantum Computing

> Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical.
>
> Richard Feynman

- An often used (and abused) quote…
- Keynote talk by Feynman
  Simulating Physics with Computers
- Can a computer **simulate** (quantum) physics in an **exact** and **efficient** way?
  - **IF** we accomplished that on a **classical computer** → violation of Bell's theorem
  - What about a **quantum computer**?



1st Physics of Computation Conference (MIT, 1981)

- Challenge of quantum computing

Can we use the same framework as a **general purpose** computation paradigm? (and how?)

Qubit - Quantum unit of information

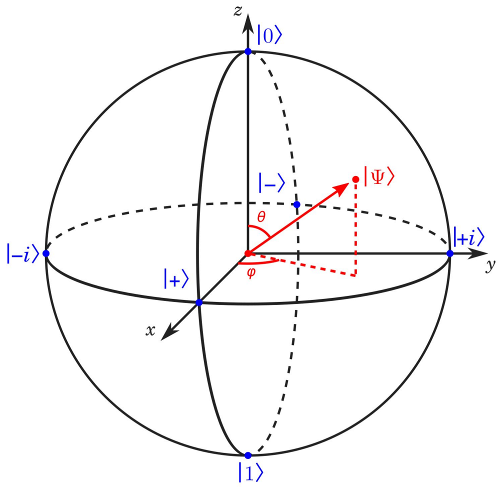
- Simple **2-level** quantum system

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$$

- Upon measurement on the **computational basis**
  - Found in state $|0\rangle$ ($|1\rangle$) with **probability** $|\alpha|^2$ ($|\beta|^2$)
- Quantum mechanics is invariant under a **global phase rotation**

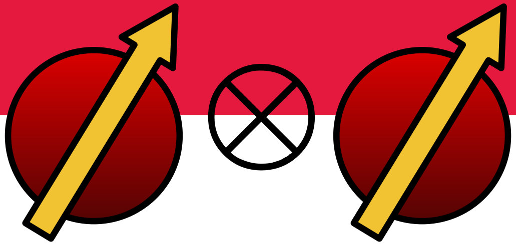
$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

- 1-to-1 match between a point on the surface of the (Bloch) sphere and 1-qubit states



- Sadly, this fails to visually generalize to multi-qubit systems

Multi-qubit systems



- An N qubit state is described by 2^N amplitudes
  - *simulation volume* scales faster than *system volume*

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

$$|\psi\rangle = \begin{pmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{pmatrix}$$

- Notation

$$|12\rangle = |1100\rangle = |1\rangle \otimes |1\rangle \otimes |0\rangle \otimes |0\rangle$$

- Entangled states
  - Alice and Bob prepare many 2-qubit systems in a Bell state, then take a qubit each
  - Independent local measurements give uniformly random outcomes
  - Perfectly correlated when compared

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\rho_B = \mathrm{Tr}_A \rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rho_A = \mathrm{Tr}_B \rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|\overset{AB}{00}\rangle + |\overset{AB}{11}\rangle)$$

(this doesn't really prove the quantumness of entanglement → CHSH game)

Operations on qubits

- Computation via **quantum gates**
  - Unitary operators
  - Reversible
  - 1-qubit gates → 2^1 x 2^1 unitaries
  - 2-qubit gates → 2^2 x 2^2 unitaries
- Universal gate set
  - Can approximate any unitary via an arbitrarily long chain of gates from the set
- For example… to construct the Bell state from the all-0 state $|00\rangle$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & X \end{pmatrix}$$

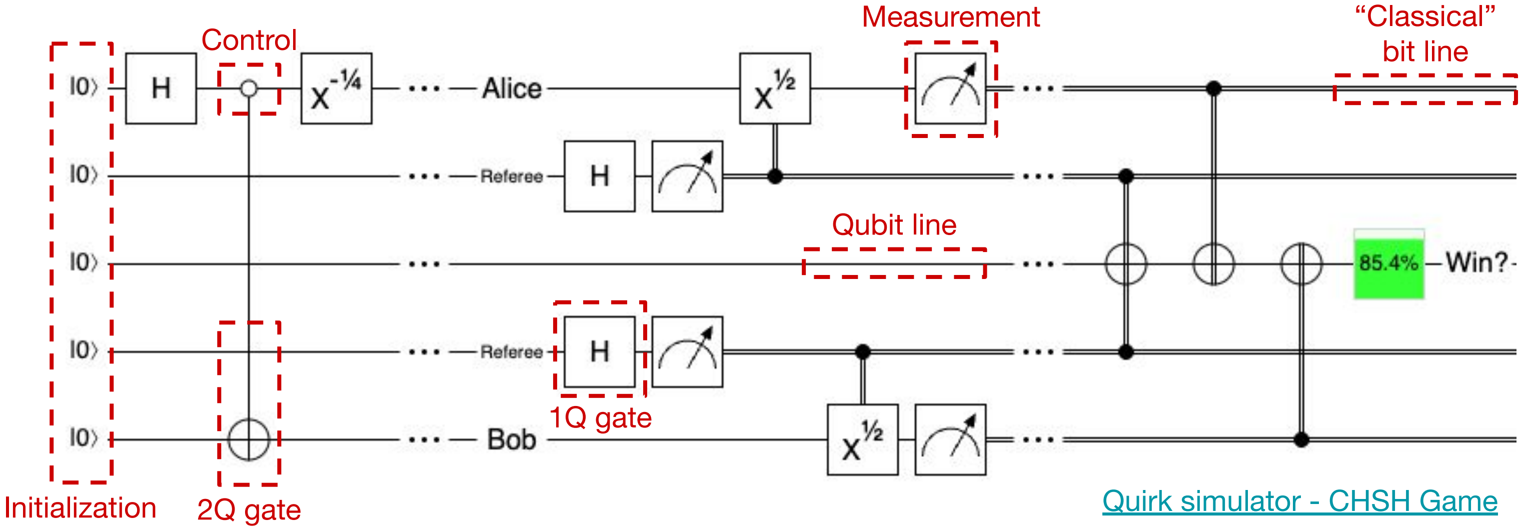
$$R_x(\theta) = e^{-i\frac{\theta}{2}X} \quad R_y(\theta) = e^{-i\frac{\theta}{2}Y} \quad R_z(\theta) = e^{-i\frac{\theta}{2}Z}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\overset{AB}{|00\rangle} \xrightarrow{\text{Apply } H \text{ on first qubit}} \frac{1}{\sqrt{2}}\left(\overset{AB}{|00\rangle} + \overset{AB}{|10\rangle}\right) \xrightarrow{\text{Apply a CNOT (controlled by q1)}} \frac{1}{\sqrt{2}}\left(\overset{AB}{|00\rangle} + \overset{AB}{|11\rangle}\right)$$

Quantum circuit

- Convenient schematic way of representing quantum algorithms



[Quirk simulator - CHSH Game]

A trilogy of quantum algorithms

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$U|\psi\rangle = e^{2\pi i\phi}|\psi\rangle$$

$$Ax = b$$

Quantum Fourier Transform

$N = 2^n$

- Given a quantum state $|x\rangle = \sum_{j=0}^{N-1} x_j |j\rangle$, produce a state $|y\rangle = \sum_{k=0}^{N-1} y_k |k\rangle$ such that

$$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{2\pi ijk/N}$$

- The QFT algorithm achieves this in ***O(log^2 N)***
  - For comparison, the classical FFT takes *O(N* log *N)*
- Great! Should we replace FFT with QFT? **No!**
  - **Exact arbitrary state preparation** requires at least *O(N)*
  - **Quantum state tomography** requires at least *O(N)*

How does QFT work?

- Like any quantum algorithm, QFT is a unitary transformation
  - Fully determined by its action on the computational basis vectors
- Let's see how QFT acts on a $|j\rangle \mapsto \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ijk/N} |k\rangle$
- We can rewrite this expression in a waaay more useful form using the **binary fraction notation**

$$0.b_1 b_2 \ldots b_n = \frac{b_1}{2^1} + \frac{b_2}{2^2} + \ldots + \frac{b_n}{2^n}$$

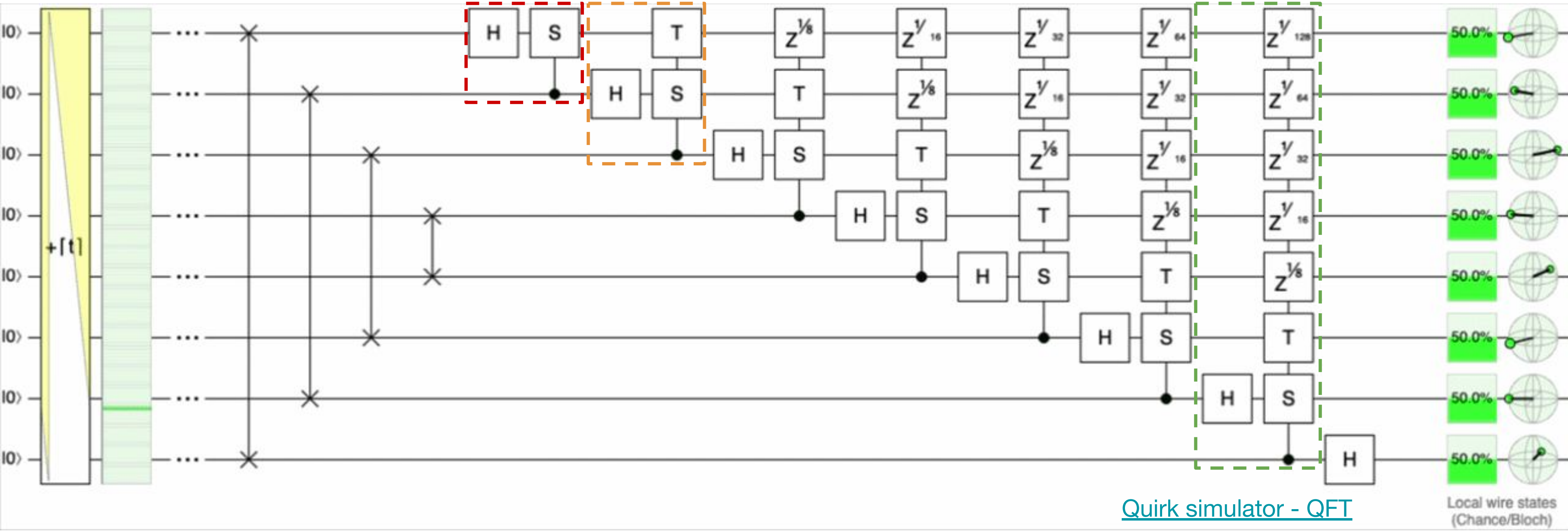
Action on qubit 1 — Action on qubit 2 — Action on qubit n

$$\frac{\left(|0\rangle + e^{2\pi i 0.j_n}|1\rangle\right)\left(|0\rangle + e^{2\pi i 0.j_{n-1}j_n}|1\rangle\right)\ldots\left(|0\rangle + e^{2\pi i 0.j_1 j_2 \ldots j_n}|1\rangle\right)}{2^{n/2}}$$

QFT in action

$$R_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{pmatrix}$$

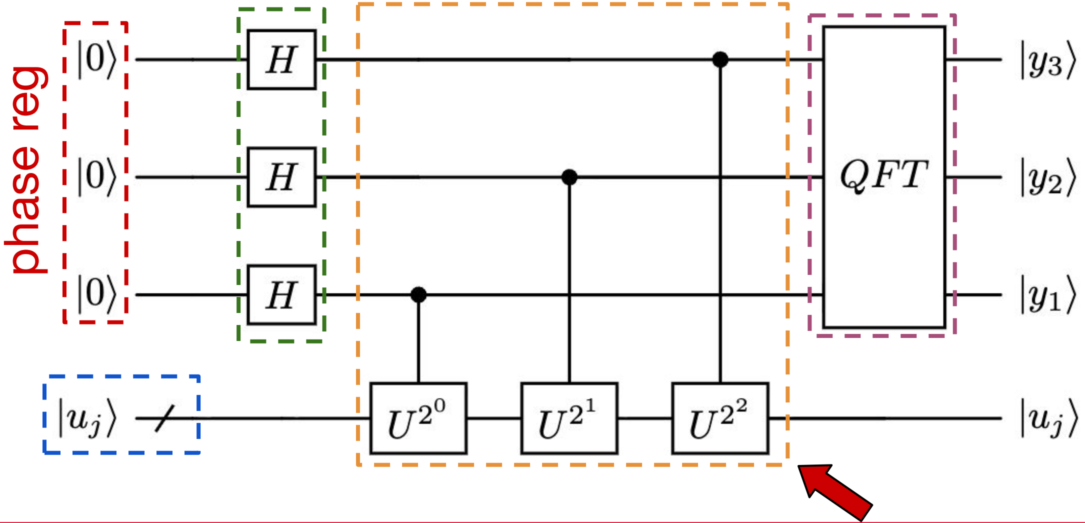
$$|0\rangle + |1\rangle \longrightarrow |0\rangle + e^{2\pi i 0.j_{n-1}j_n} |1\rangle$$



Quirk simulator - QFT

Quantum Phase Estimation

- Given a unitary operator *U* and one of its eigenvector $|u\rangle$ with eigenvalue $e^{2\pi i\phi}$ QPE stores an *m*-bit approximation of ϕ in a *m*-qubit register (phase)
  - The eigenvector is loaded an *n*-qubit register (system)
  - Phase register in uniform superposition via Hadamard
  - Repeated controlled application of powers of U
  - Inverse QFT



How does QPE work?

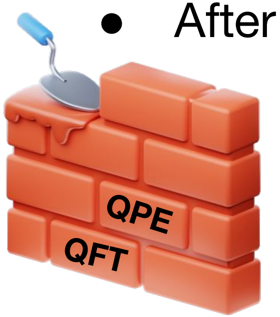
- U and U^{2^k} share the same eigenvector $U^{2^k}|u\rangle = e^{2\pi i 2^k \phi}|u\rangle$
- What happens when the application of U^{2^k} is controlled by a qubit in a uniform superposition state?
  - "nothing" on the 0 subspace
  - we accumulate a phase on the 1 subspace

$$|0\rangle + |1\rangle \mapsto |0\rangle + e^{2\pi i 2^k \phi}|1\rangle$$

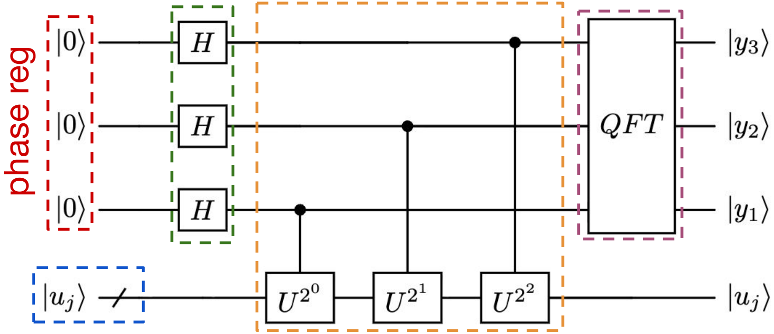
- After *m* repetitions, and using the binary fraction notation…

$$\frac{\left(|0\rangle + e^{2\pi i 0.\phi_m}|1\rangle\right)\left(|0\rangle + e^{2\pi i 0.\phi_{m-1}\phi_m}|1\rangle\right)\ldots\left(|0\rangle + e^{2\pi i 0.\phi_1\phi_2\ldots\phi_m}|1\rangle\right)}{2^{m/2}}$$

- After an iQFT



$$\frac{1}{2^m}\sum_{j=0}^{2^m-1}\left(\sum_{k=0}^{2^m-1} e^{2\pi i k(\phi - j/2^m)}\right)|j\rangle$$



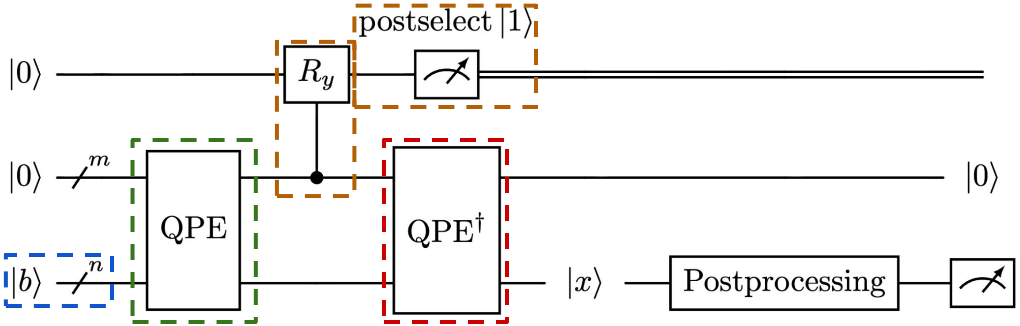
Harrow-Hassadim-Lloyd (HHL) algorithm

- HHL takes advantage of QPE to solve a system of linear equations $Ax = b$
  - The vector *b* is normalized and **embedded in the amplitudes** of an n-qubit register
  - QPE is applied to the unitary $U = e^{iA}$
  - An R_y rotation applied to an **ancilla qubit** inverts matrix in its own eigenbasis
    This transformation $x \to 1/x$ is not unitary. Correctly executed if ancilla is left in state 1
  - QPE is **uncomputed** to disentangle phase and system registers

$$|x\rangle = \frac{A^{-1}|b\rangle}{\|A^{-1}|b\rangle\|}$$

$$O(\kappa^2 \log N)$$

$$\kappa = \frac{|\lambda_{\max}|}{|\lambda_{\min}|}$$

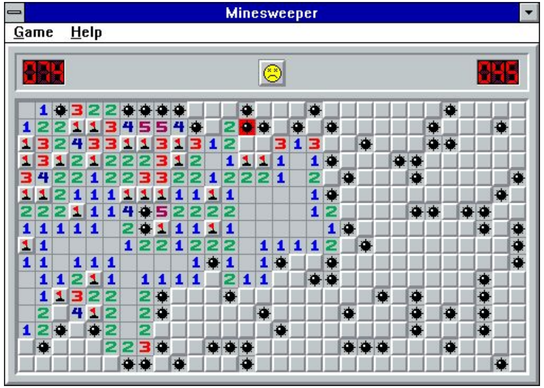


Requirements for HHL

$$O(\kappa^2 \log N)$$

1. The **preparation** of $|b\rangle$ can be done *efficiently*
   a. "Efficient" arbitrary state preparation is impossible → *b* cannot be an arbitrary vector
   b. Potential penalty: ***O(N)***

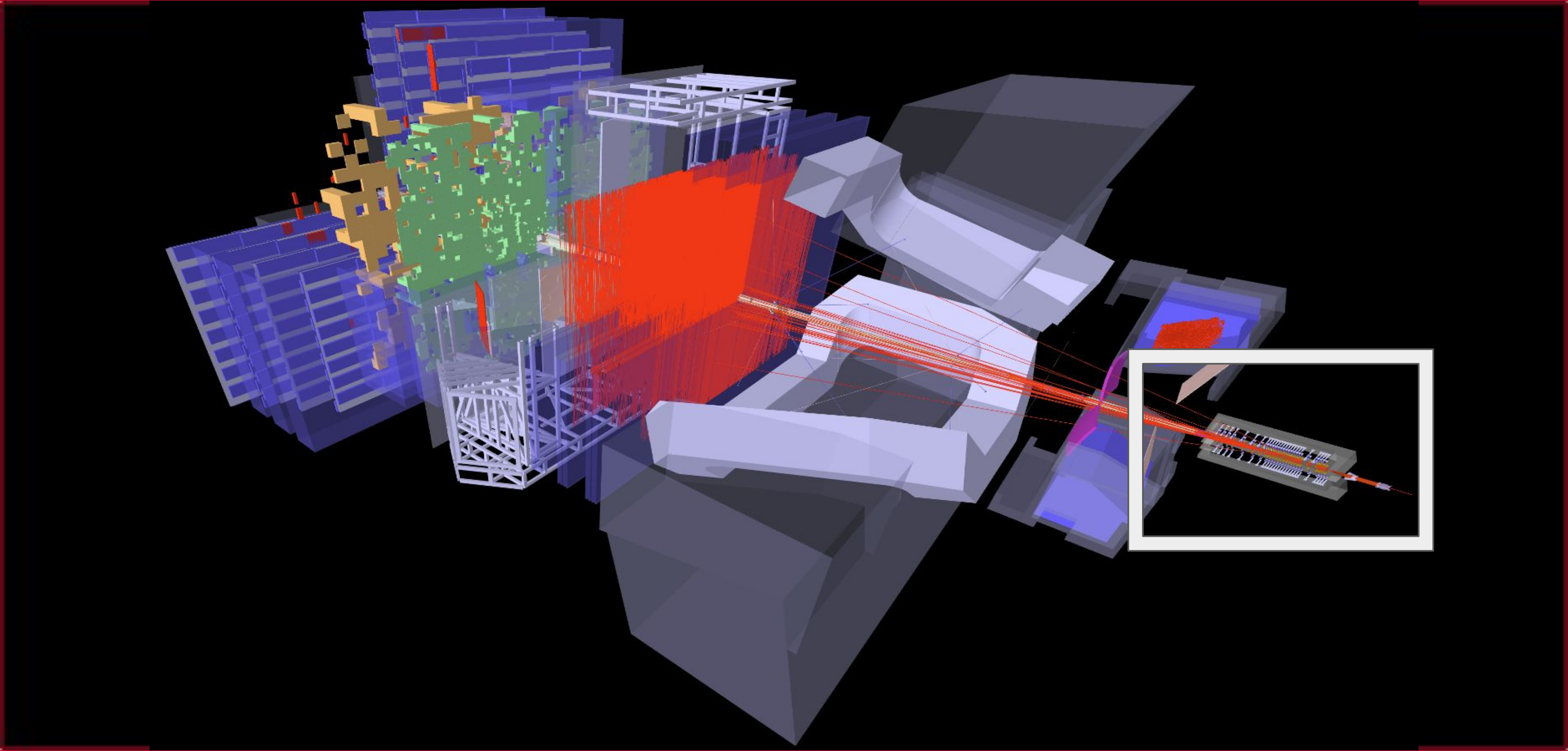
2. The **unitaries** e^{iA}, e^{2iA}, e^{4iA}… can be prepared *efficiently*
   a. This is in general very hard → Many gates are required
   b. Possible for sparse matrices
   c. Potential penalty: $\boldsymbol{O(N^2)}$



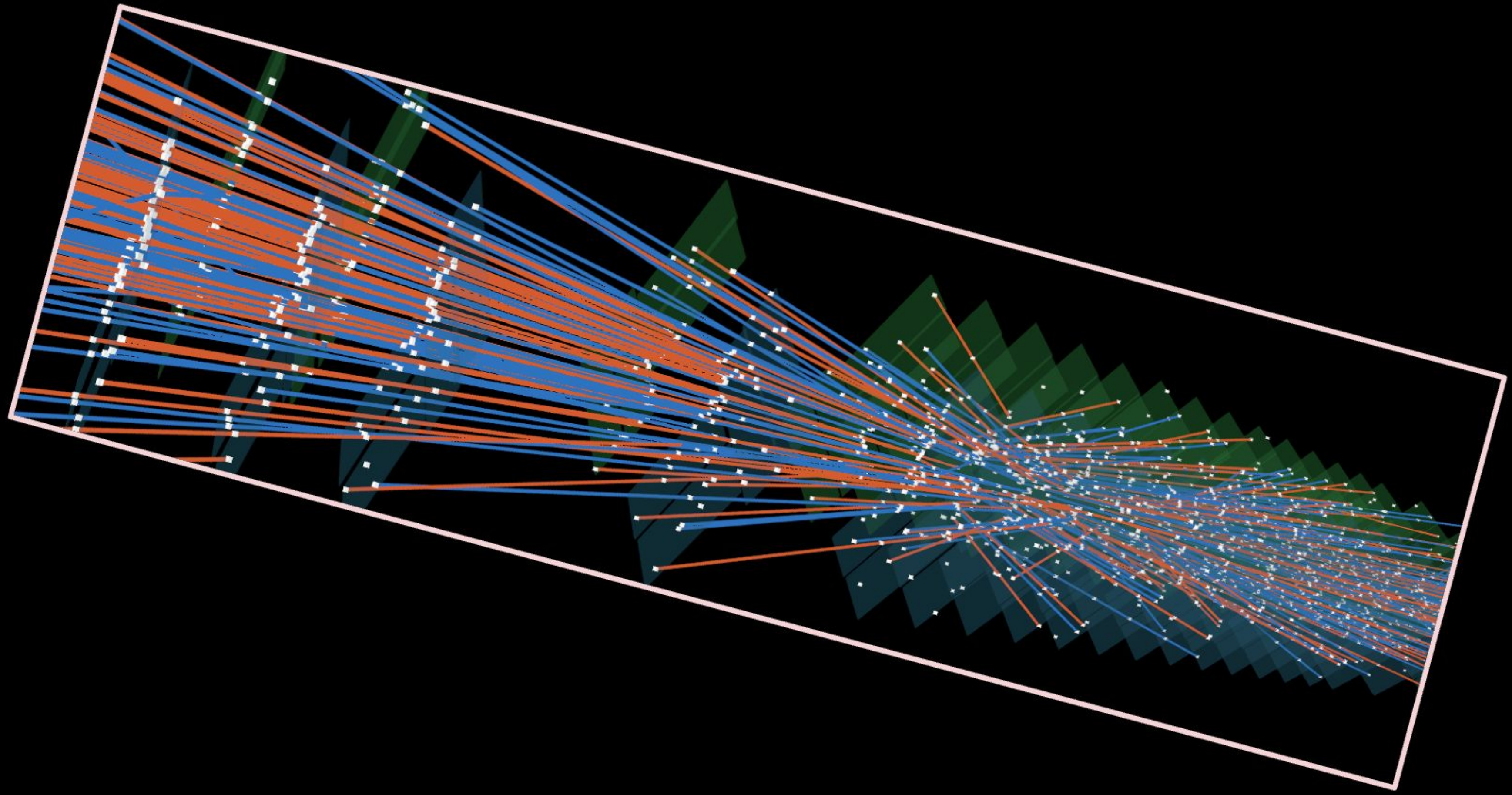
3. The output state does not need to be **fully read-out**
   a. Solution *x* is encoded in the amplitudes → not directly observables
   b. Quantum State Tomography
   c. Potential penalty: ***O(N)***

Can we use it for tracking?

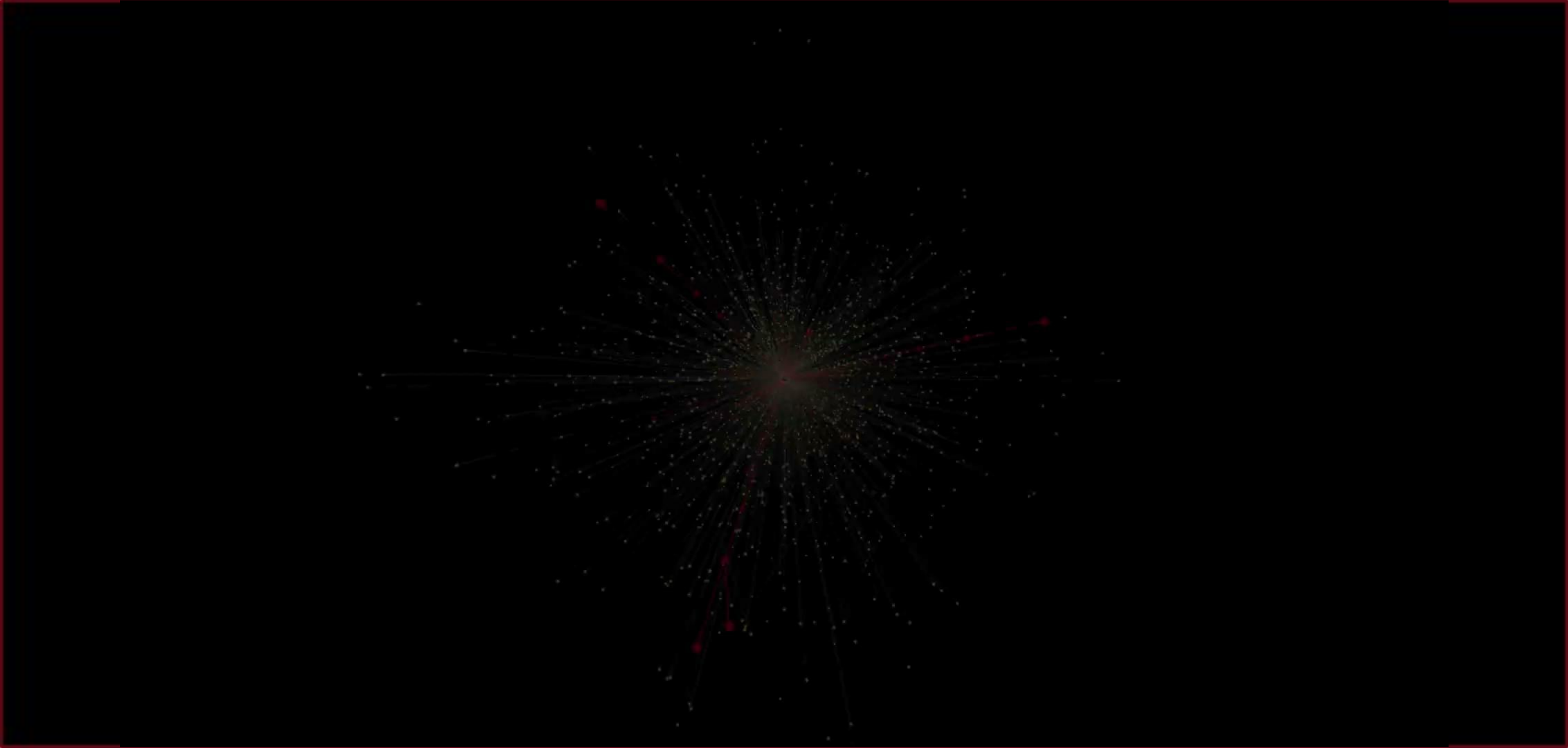
The LHCb VeLo (Vertex Locator)



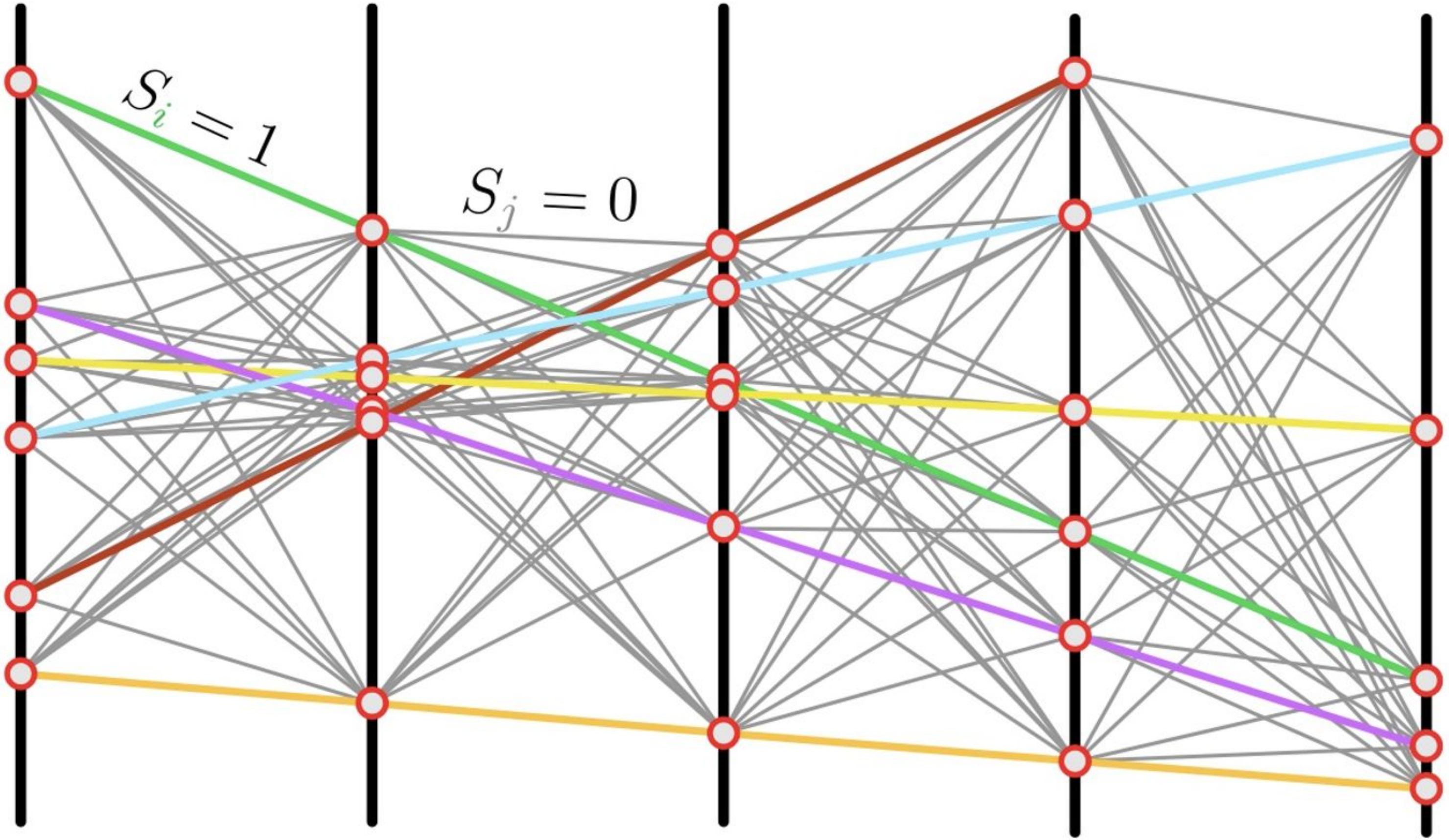
The LHCb VeLo (Vertex Locator)



Lets become a particle



Constructing a Global Solution



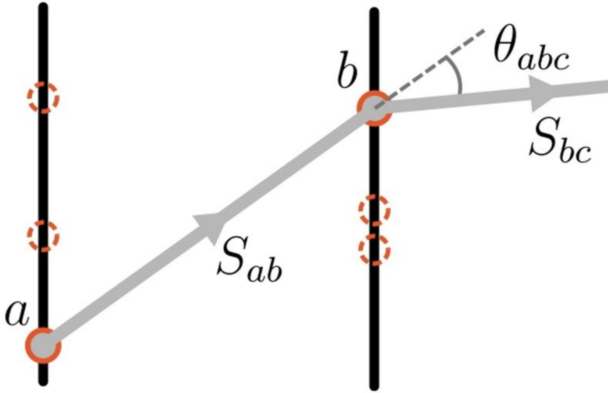
Constructing a Global Solution

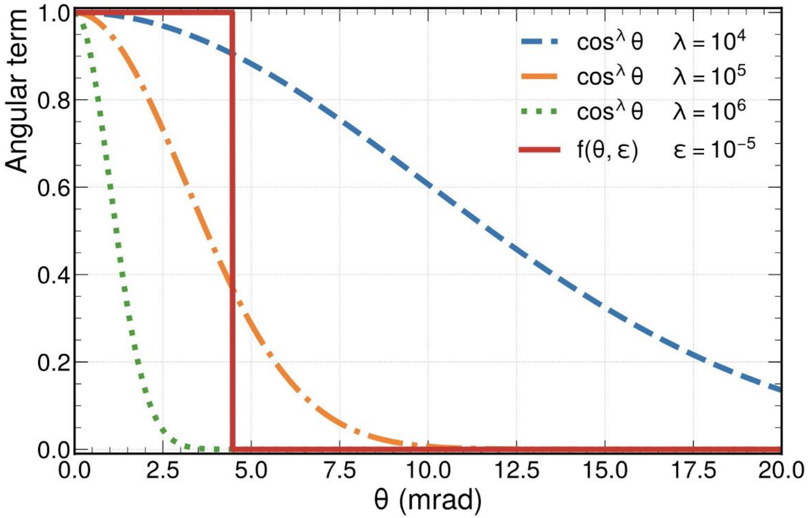
$$\mathcal{H}(S) = \sum_{abc} f(\theta) S_{ab} S_{bc} + \alpha \sum_{ab} S_{ab}^2 + \beta \sum_{ab} (1 - 2S_{ab})^2$$

Angular Term

Gap Term

Spectral Term





Formulating a QUBO

$$\mathcal{H}(S) = \sum_{abc} f(\theta) S_{ab} S_{bc} + \alpha \sum_{ab} S_{ab}^2 + \beta \sum_{ab} (1 - 2S_{ab})^2$$

$$\mathcal{H}(S) = \sum_{i,j} A_{i,j} S_i S_j + \sum_i b_i S_i \qquad S_i \in \{0, 1\}$$

Relaxation

$$\mathcal{H}(S) = \sum_{i,j} A_{i,j} S_i S_j + \sum_i b_i S_i \qquad S_i \in \{0,1\}$$

1

Lets Relax the Problem:

$$S_i \in \{0,1\} \Rightarrow S_i \in \mathbb{R}$$

2

Now we can minimize:

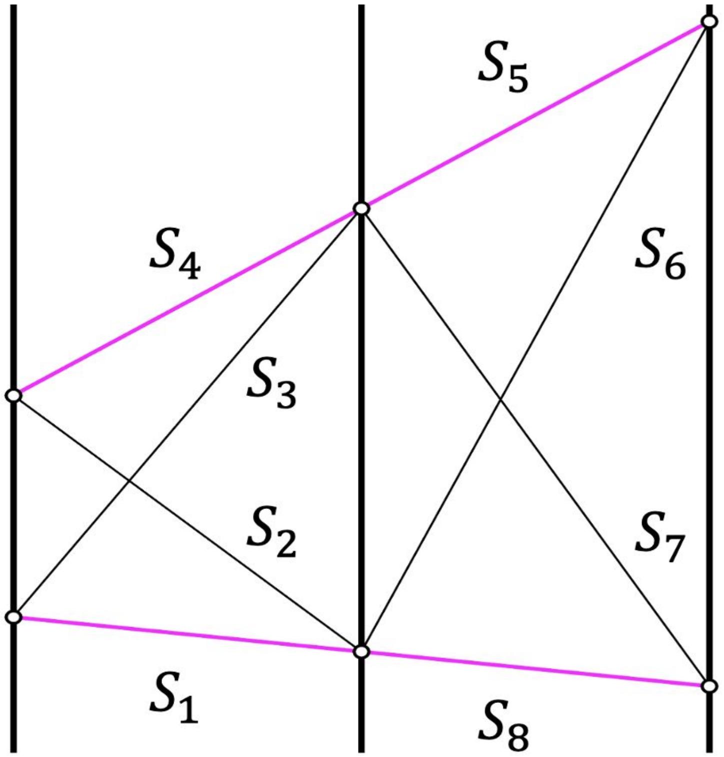
$$\nabla_S \mathcal{H} = 0 \Rightarrow \mathbf{AS} = b$$

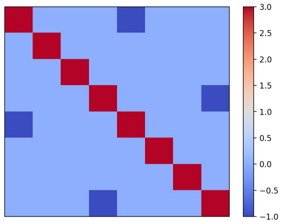
3

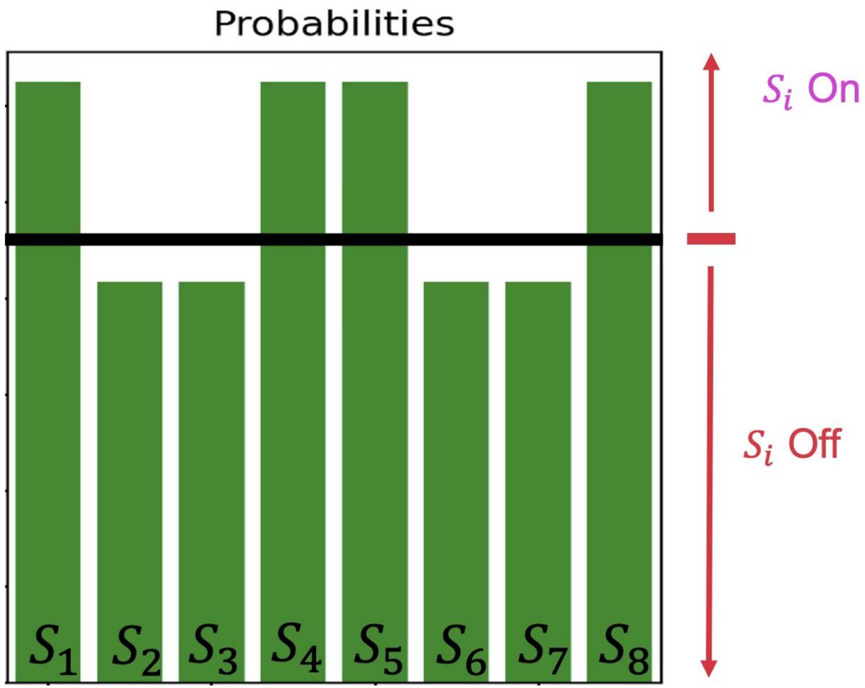
Discretization:

$$S_i \in \mathbb{R} \Rightarrow S_i \in \{0,1\}$$

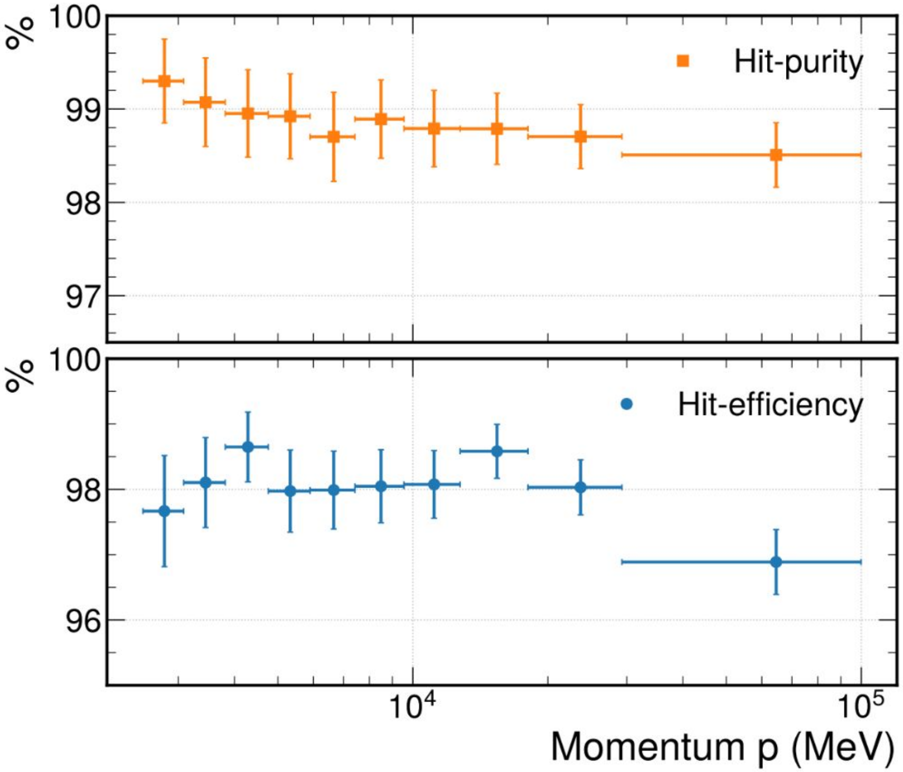
The Simplest Case

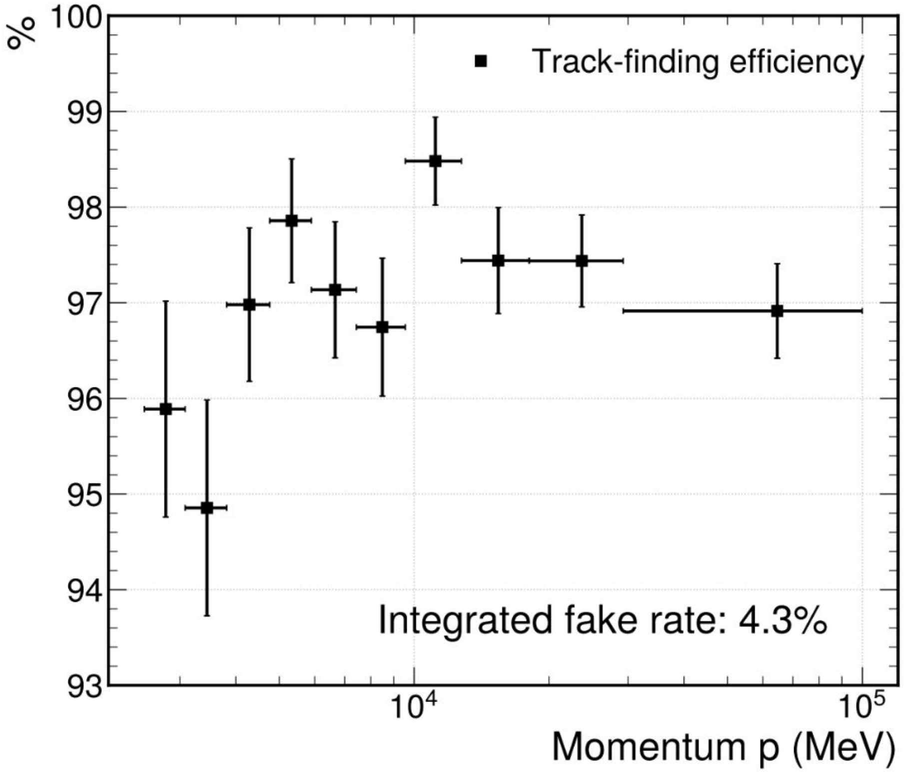






It Works! Classically

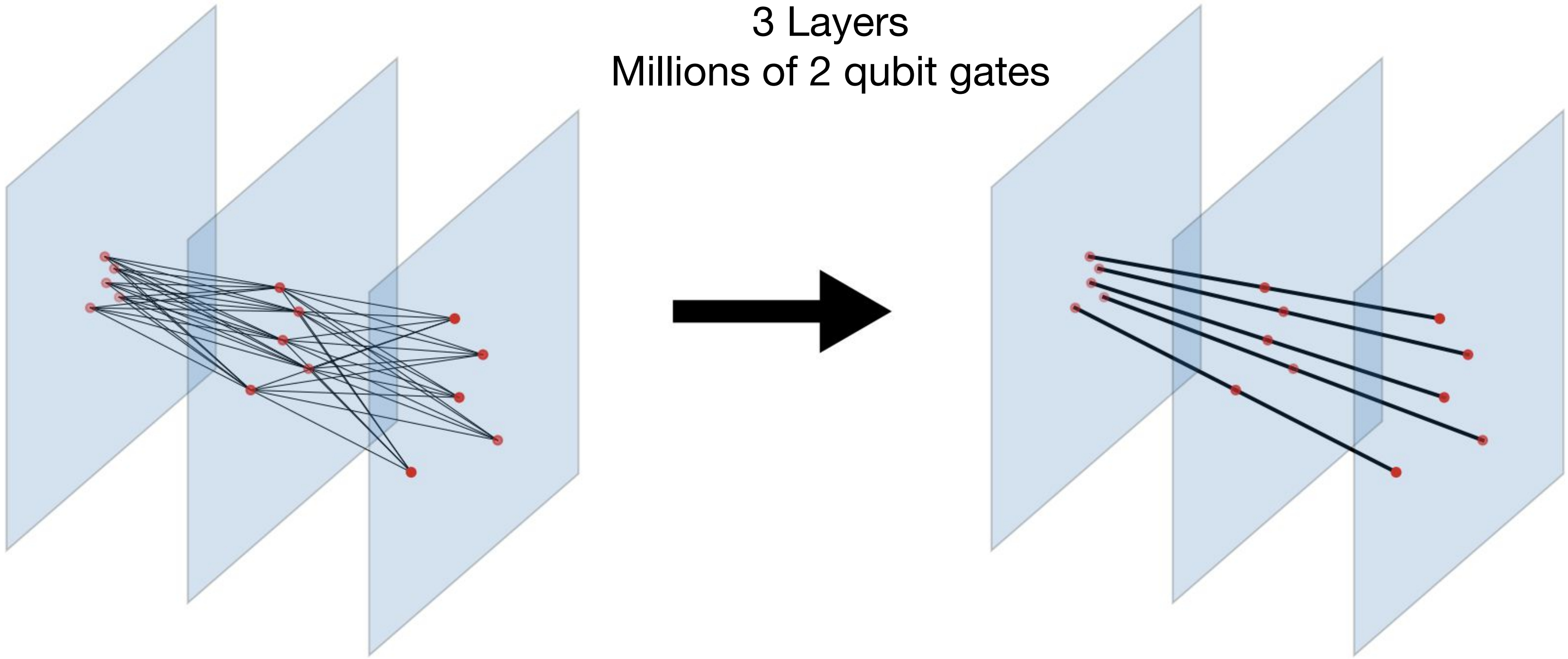




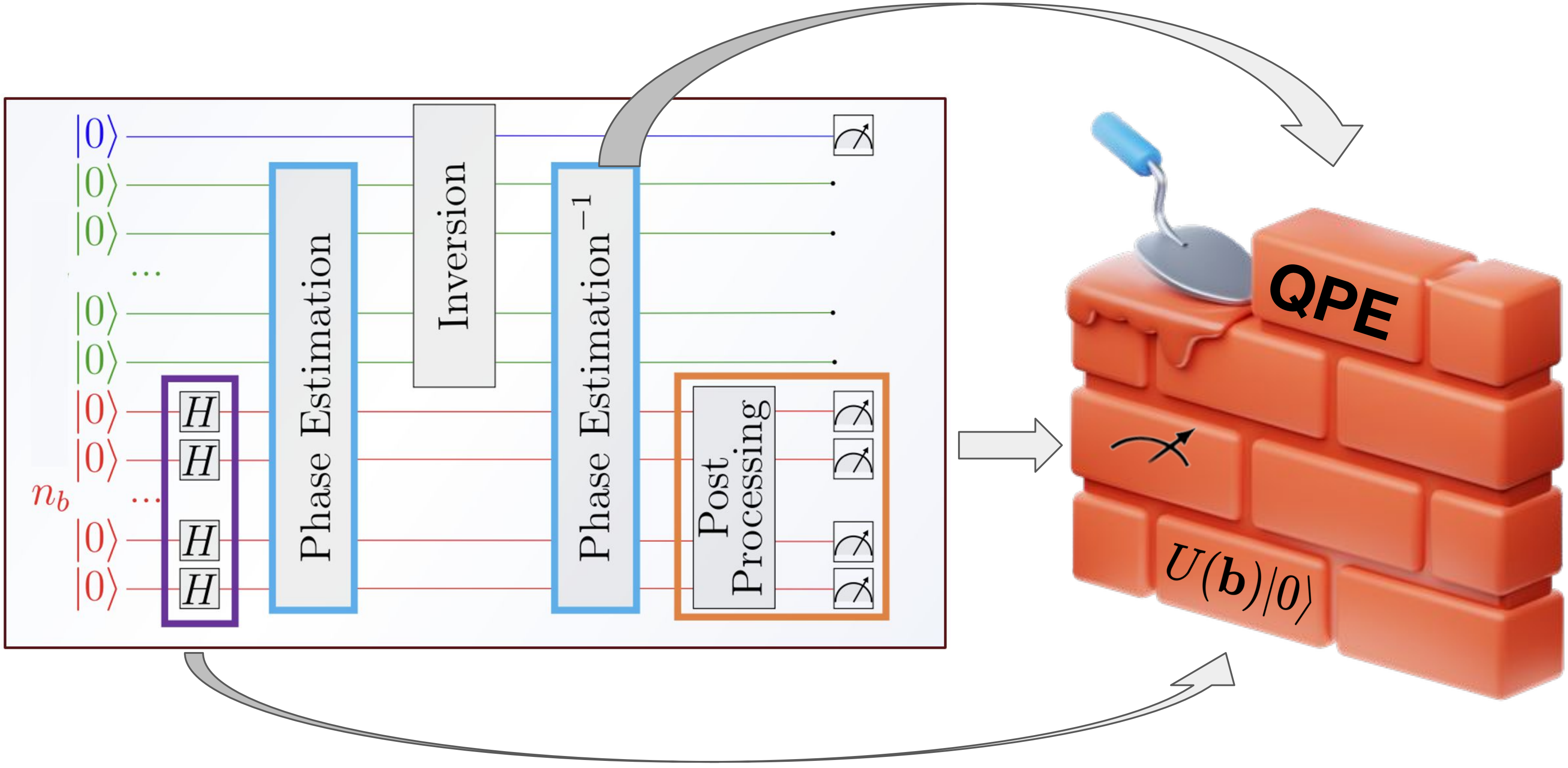
https://arxiv.org/pdf/2308.00619

HHL biggest event

5 Tracks
3 Layers
Millions of 2 qubit gates

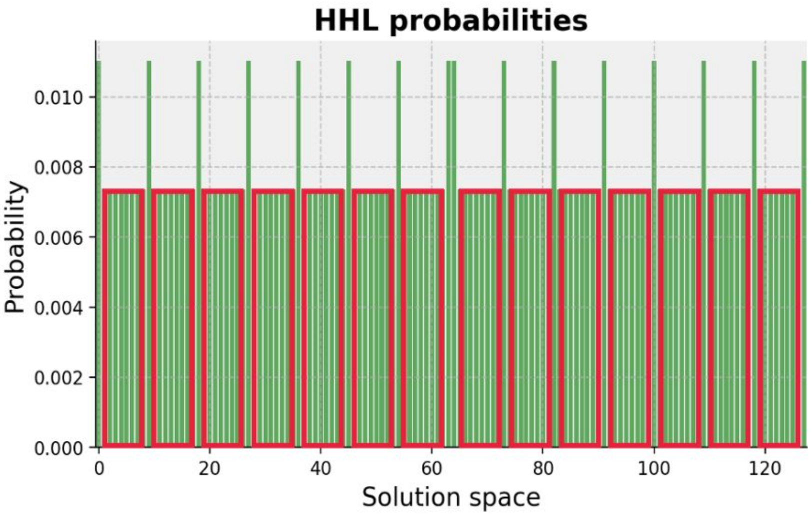


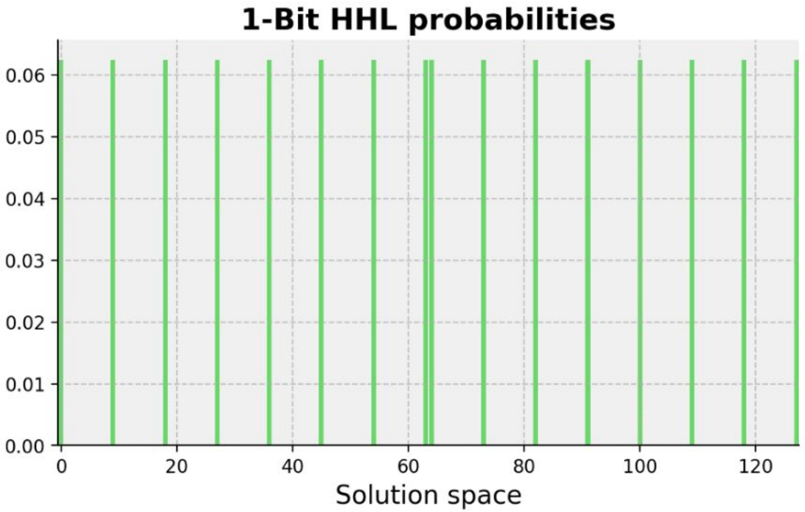
Solving in a Quantum way



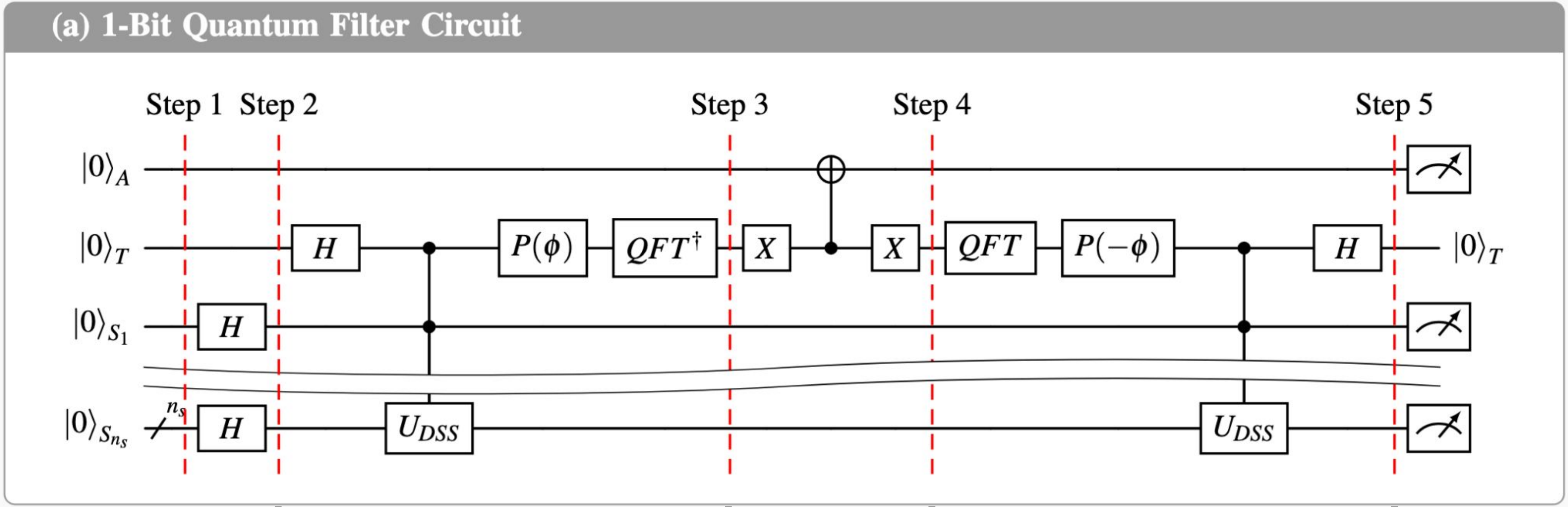

Quantum
Tracking

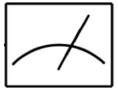
Moving the threshold



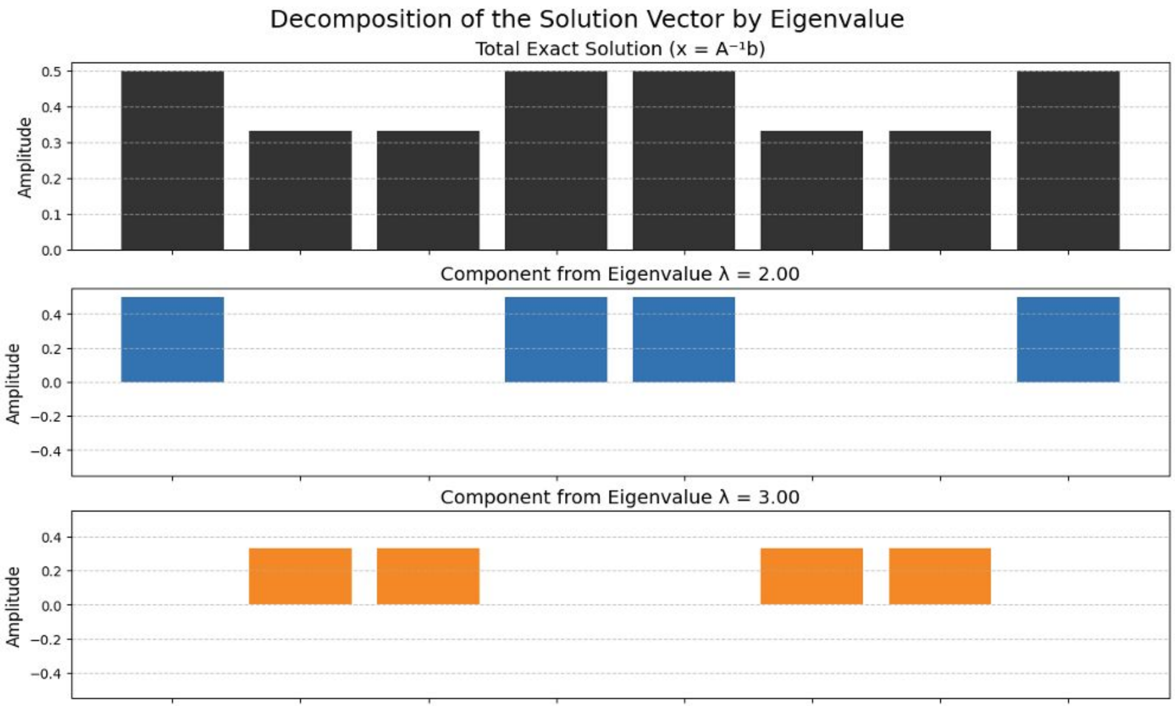


The 1-Bit Quantum Filter

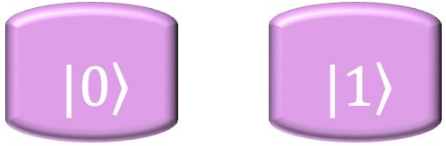


State Preparation	QPE	Inversion	$QPE^\dagger$	Measure
$U(\mathbf{b})\|0\rangle$	Ue^{iAt}		Ue^{-iAt}	

The Quantum Filter



QPE BIN



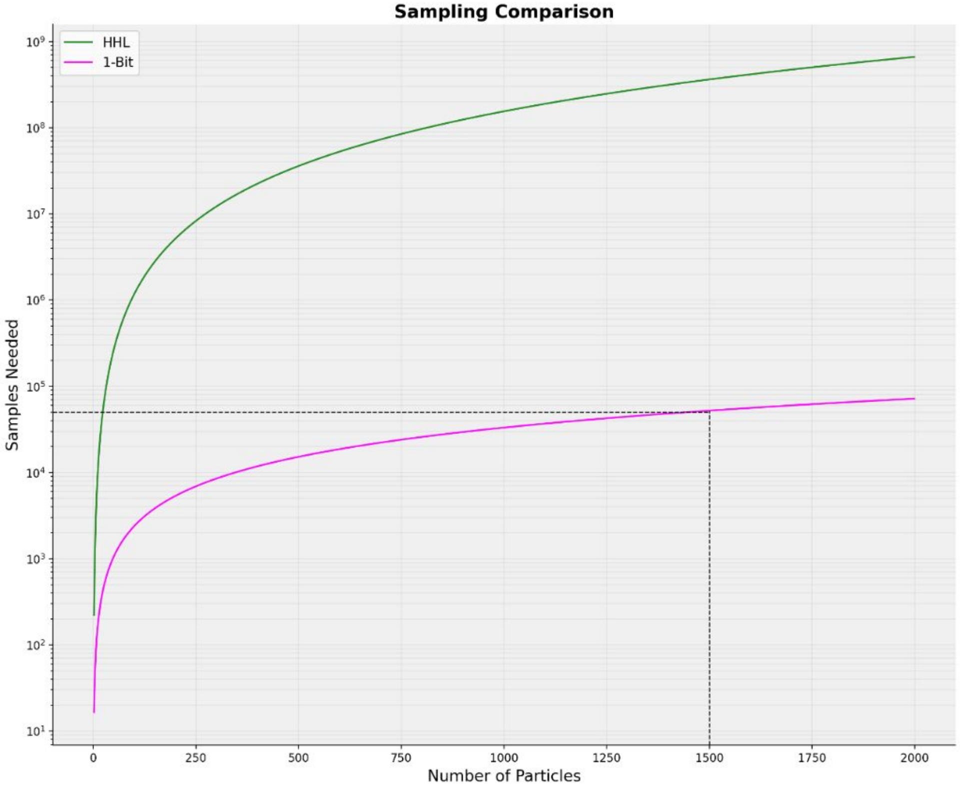
$$P(0|\lambda_j) = \cos^2\left(\frac{\lambda_k t}{2}\right)$$

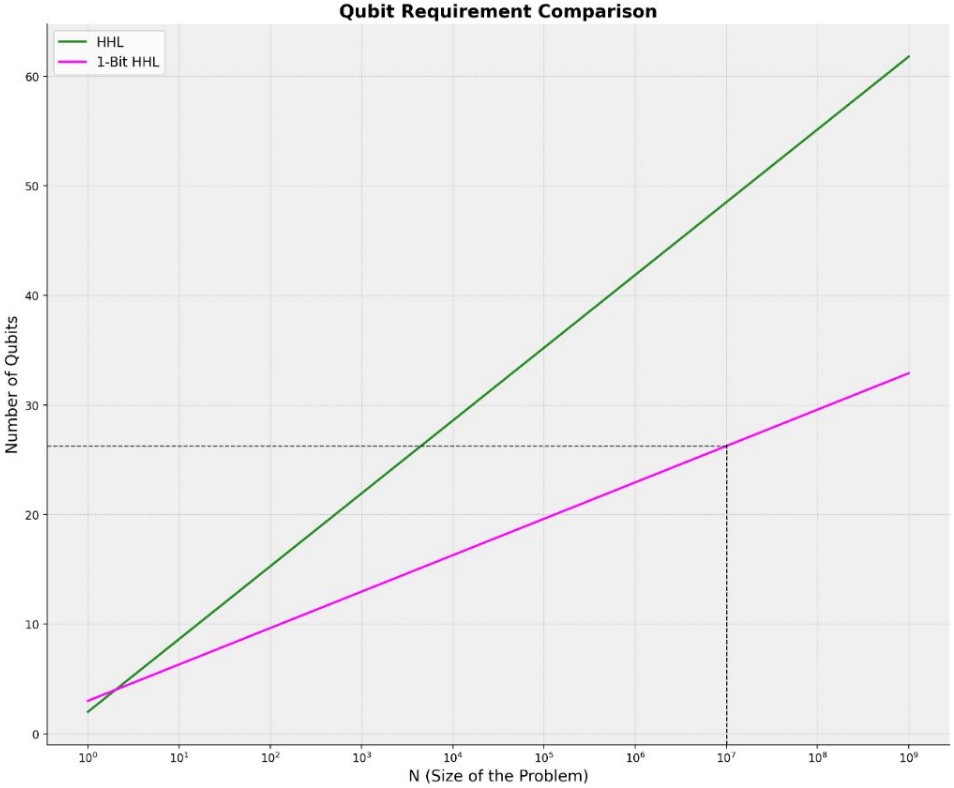
$$\lambda_c = \alpha + \beta \quad \Longrightarrow \quad t = \frac{\pi}{\lambda_c}$$

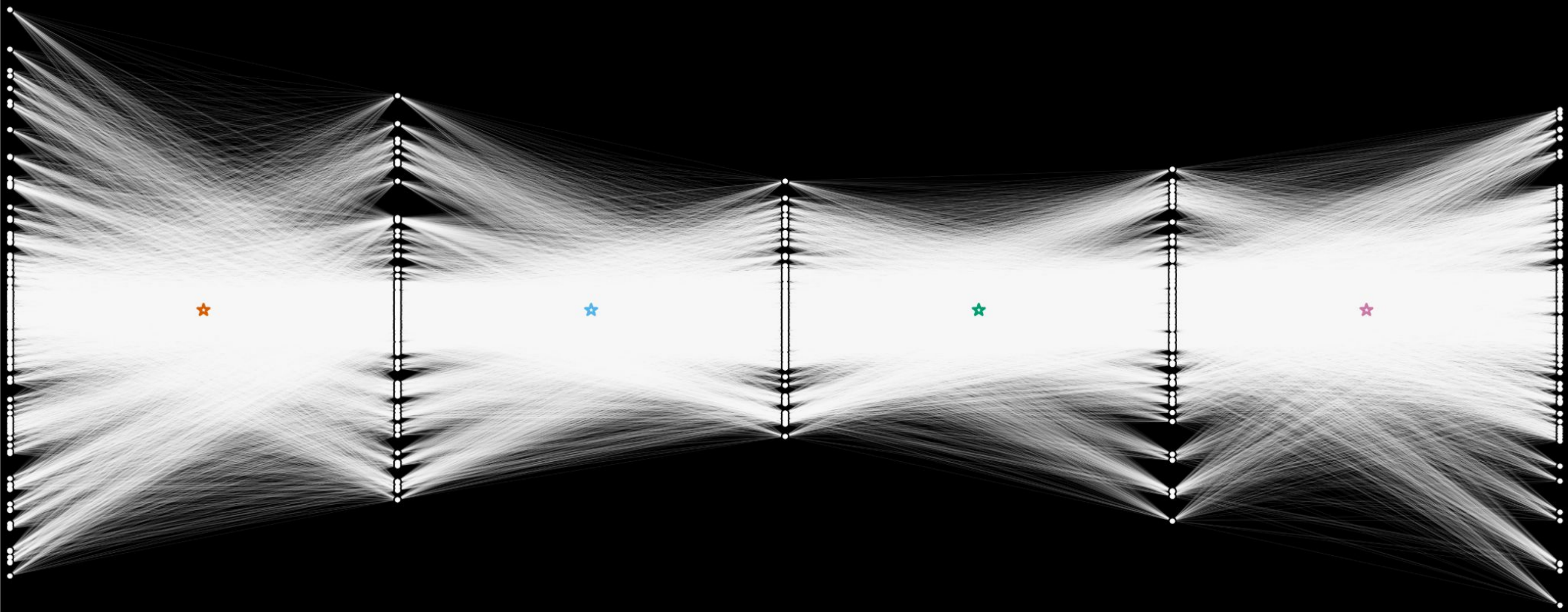
$$P(0|\lambda_j) = \cos^2\left(\frac{\lambda_c \pi}{2\lambda_c}\right) = \cos^2\left(\frac{\pi}{2}\right) = 0$$

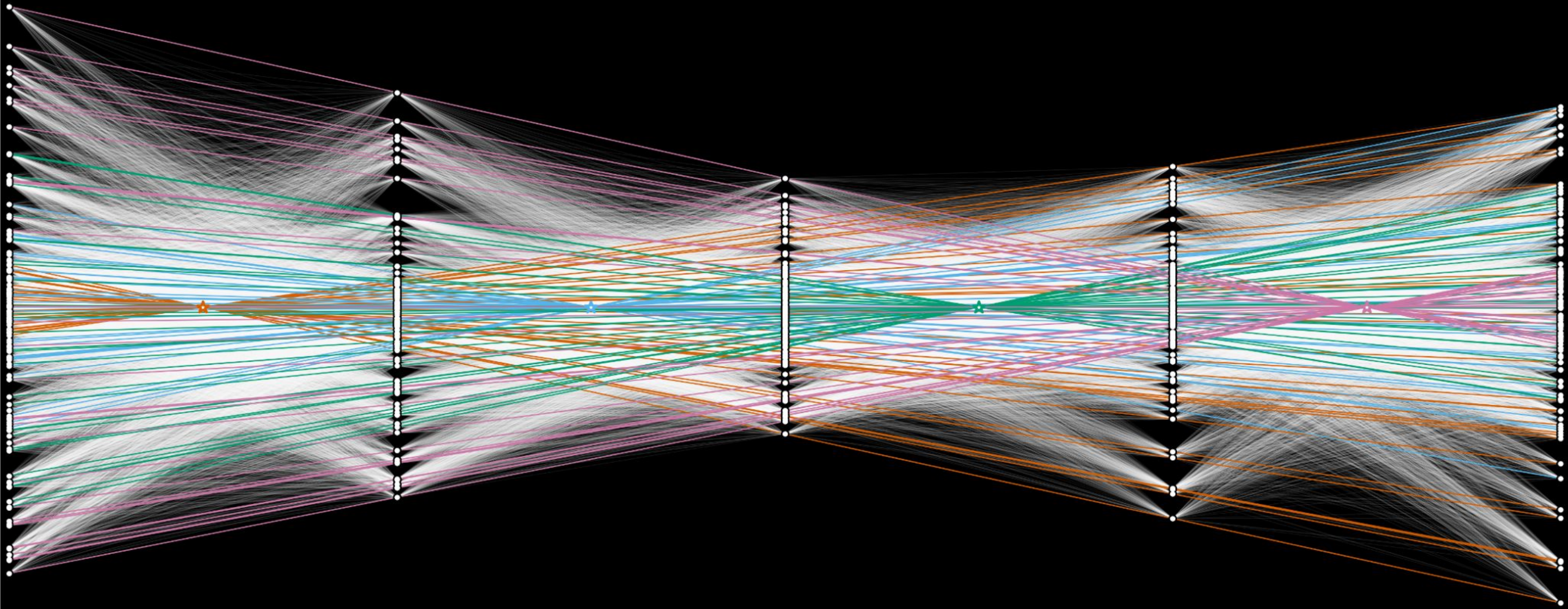
$$P(1|\lambda_j) = \sin^2\left(\frac{\lambda_c \pi}{2\lambda_c}\right) = \sin^2\left(\frac{\pi}{2}\right) = 1$$

Resource Complexity

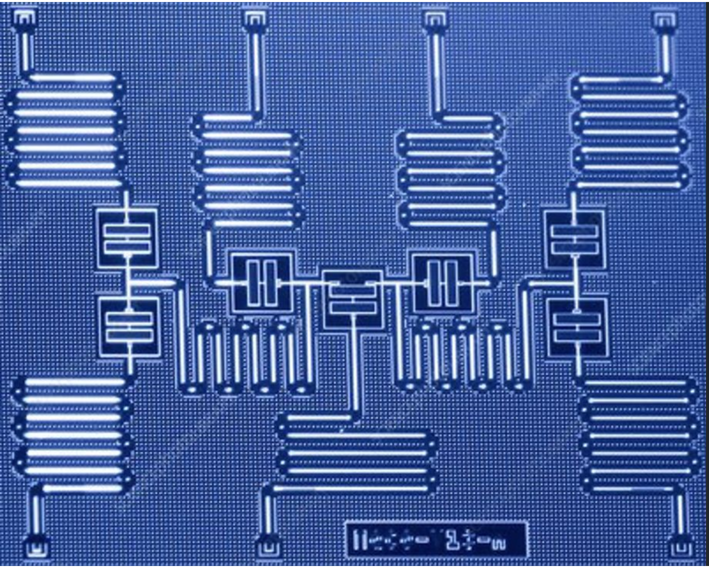


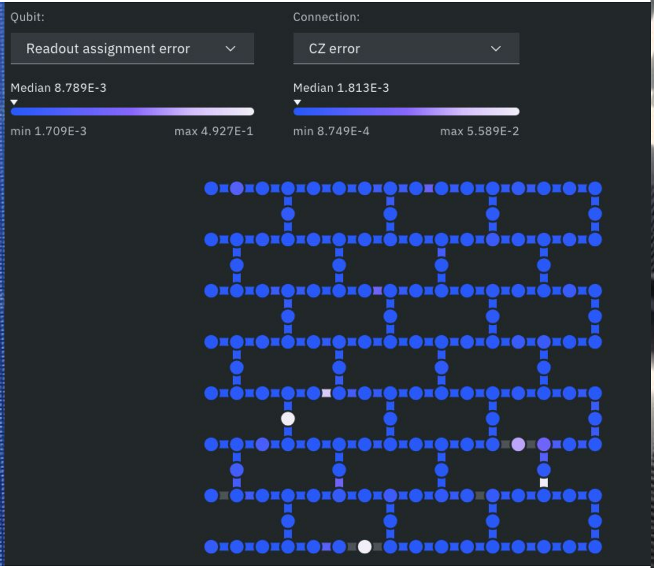






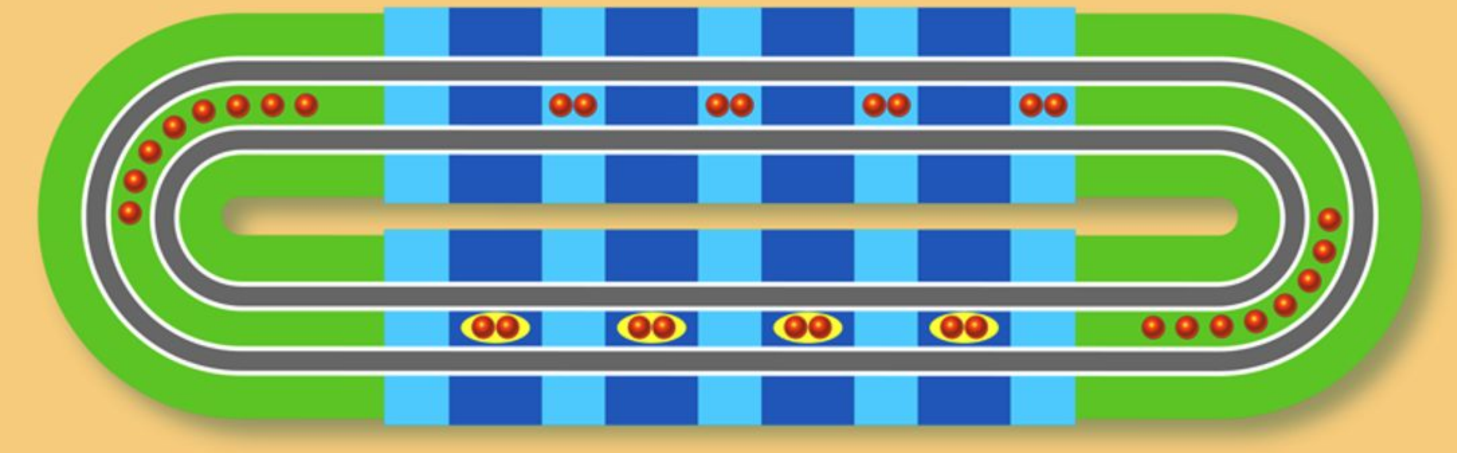
Hardware Intermezzo: Superconducting (IBM)

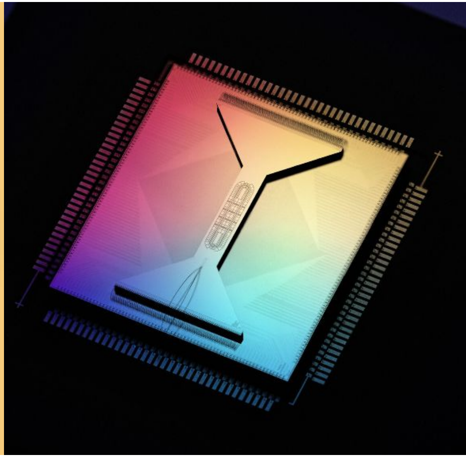




https://www.ibm.com/quantum

Hardware Intermezzo: Trapped Ions (Quantinuum)

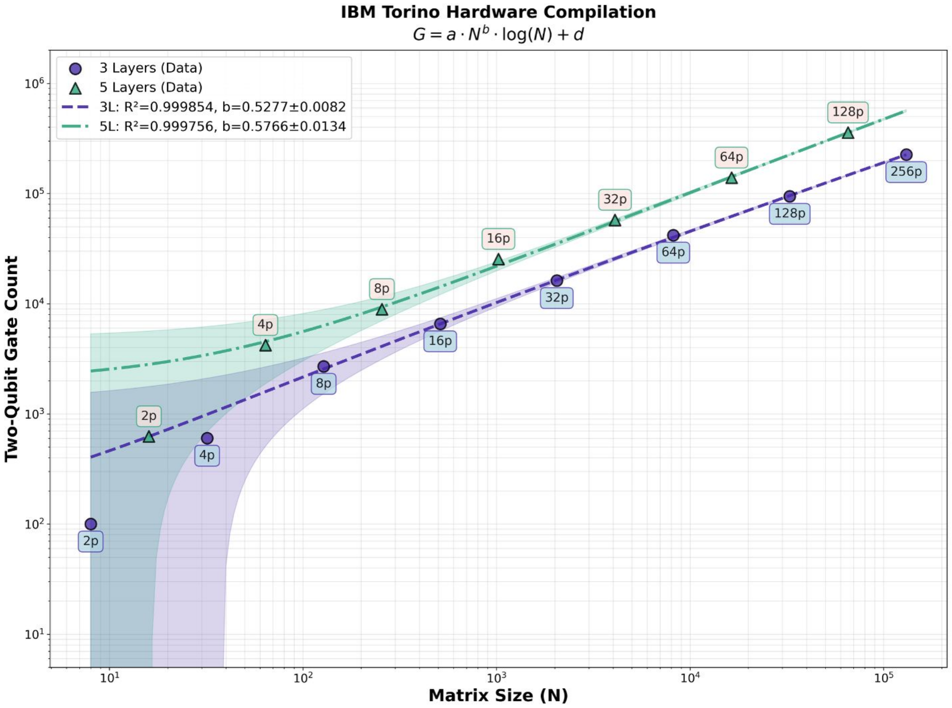


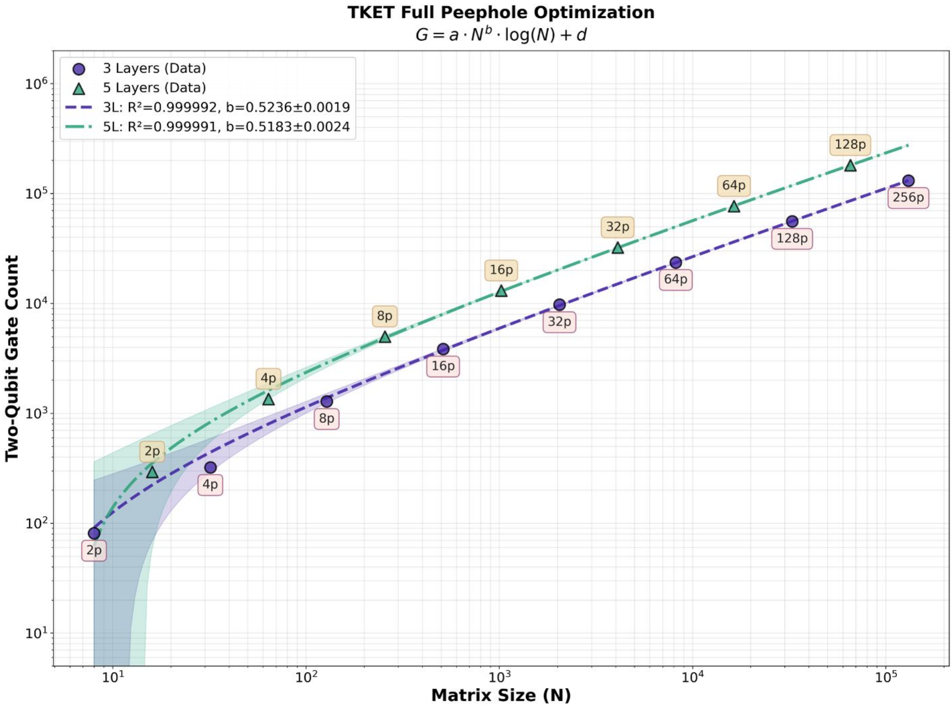


https://DOI:https://doi.org/10.1103/PhysRevX.13.041052

Asymptotic Complexity Fits: Gate Complexity

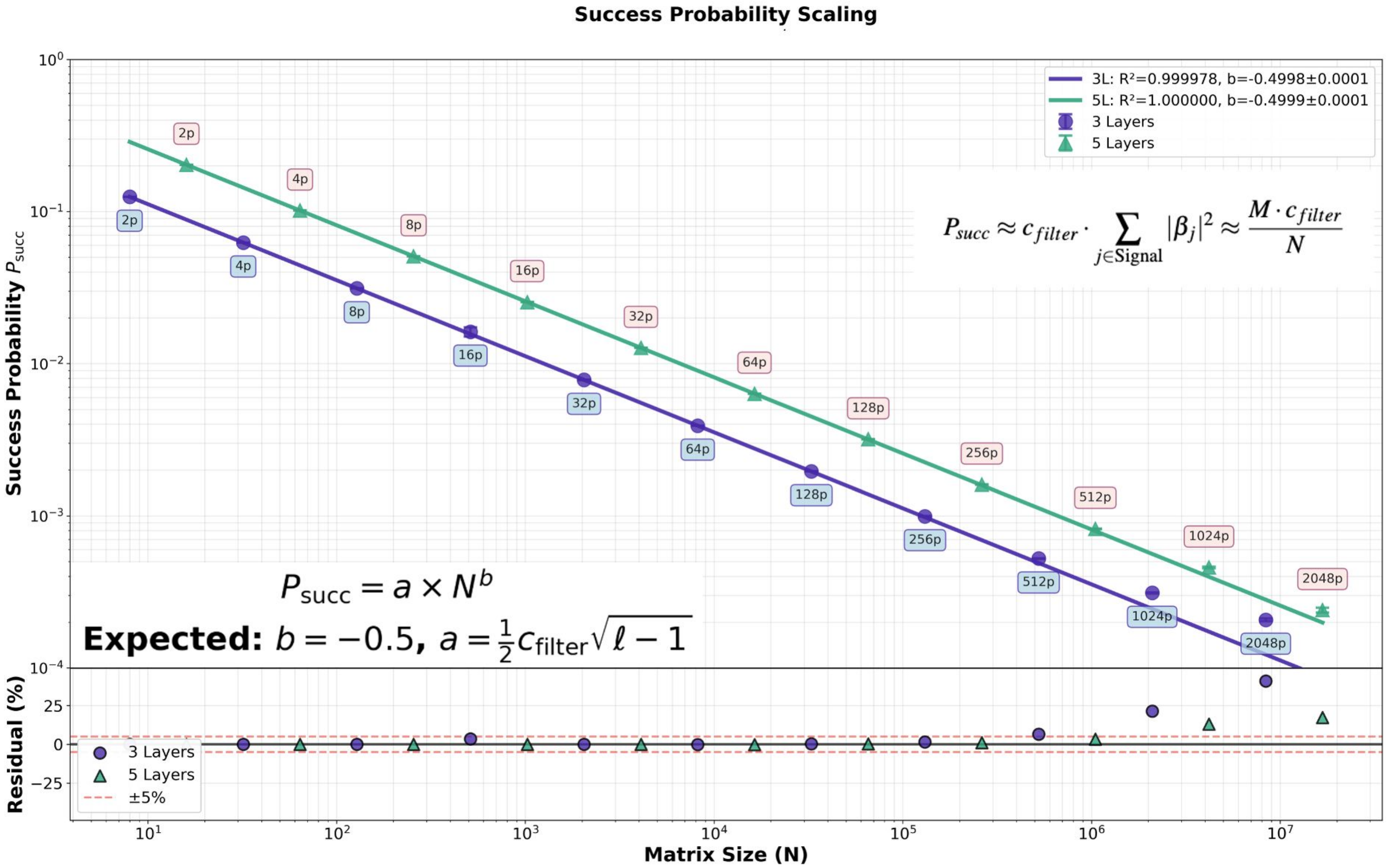
$$Derived\ Value = \mathcal{O}(\sqrt{N}\log N)$$



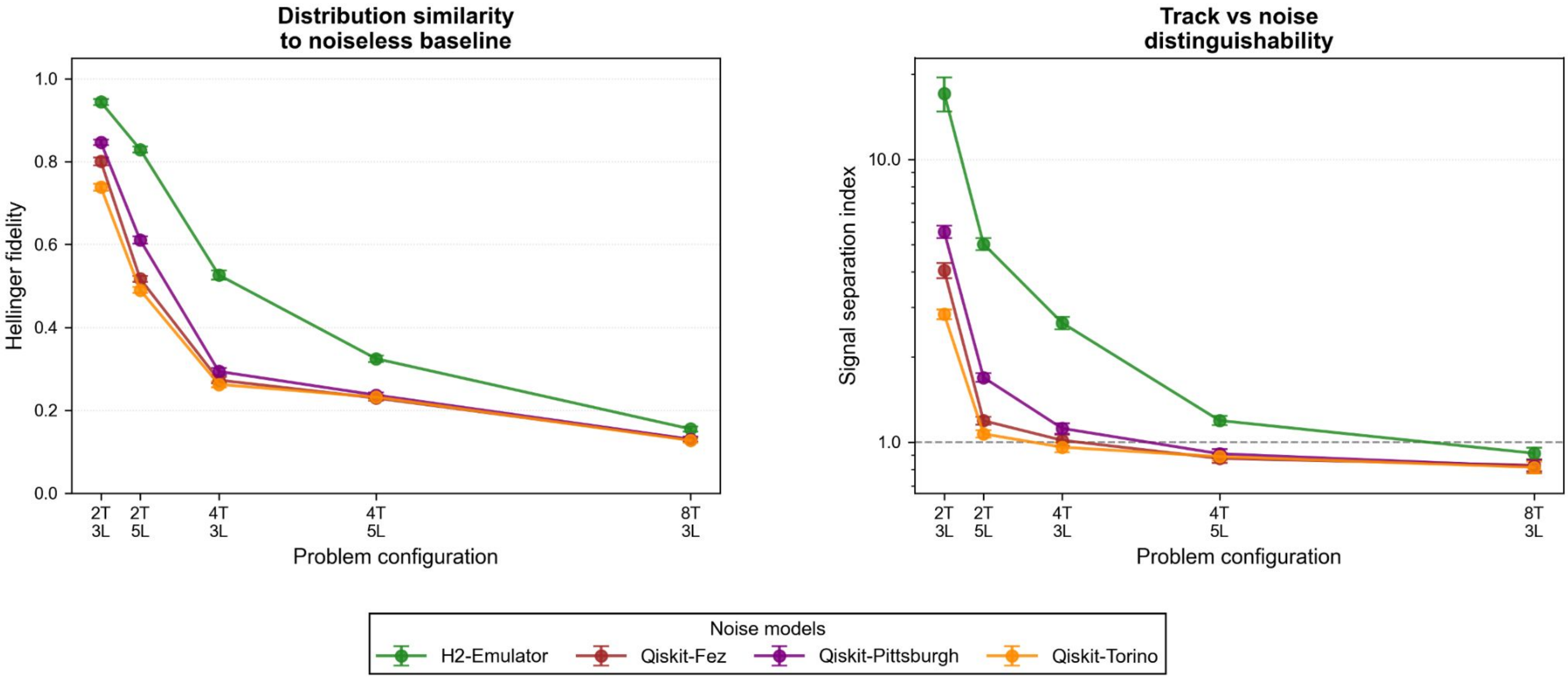


https://arxiv.org/abs/2601.07766

Asymptotic Complexity Fits: Gate Complexity



Hardware Noise Emulation Runs

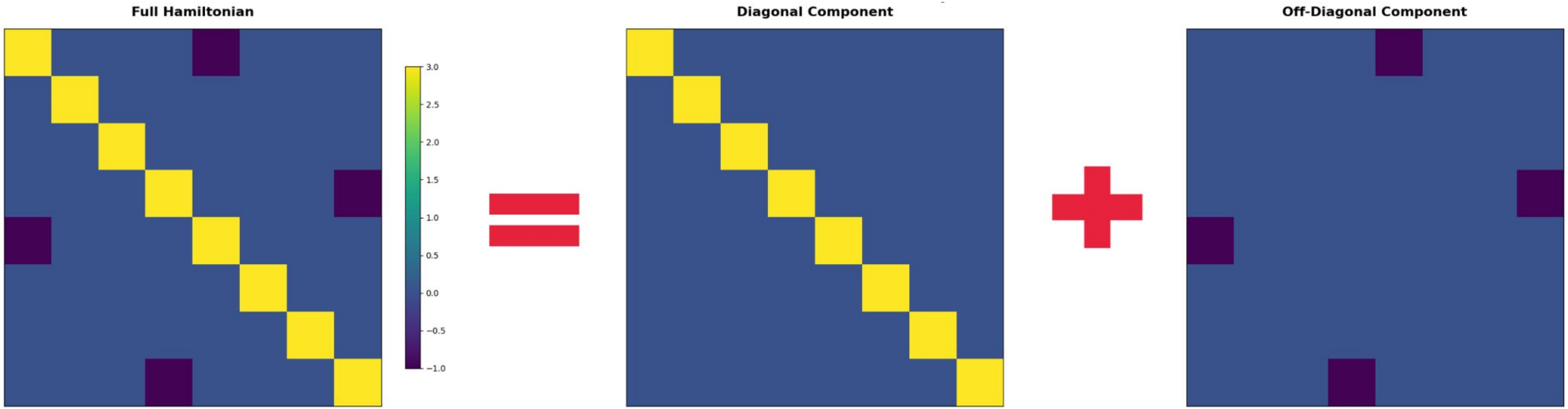


Can we use it for tracking?

Maybe

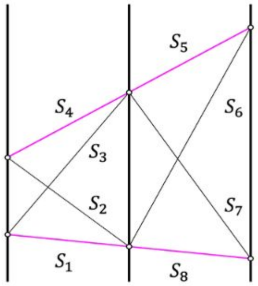
Backup

Matrix Decomposition



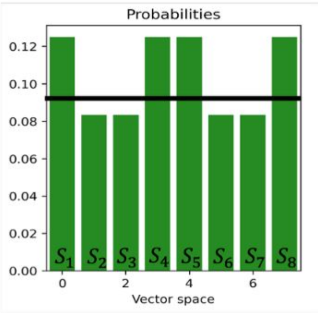
e^{-iAt}

Time Evolution
e.g Suzuki Trotter



$e^{-ic\mathbb{I}t}$

$P(\phi)$

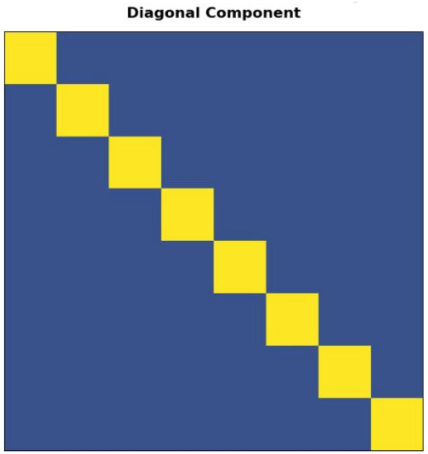


e^{-iBt}

U_{DSS}

Matrix Decomposition

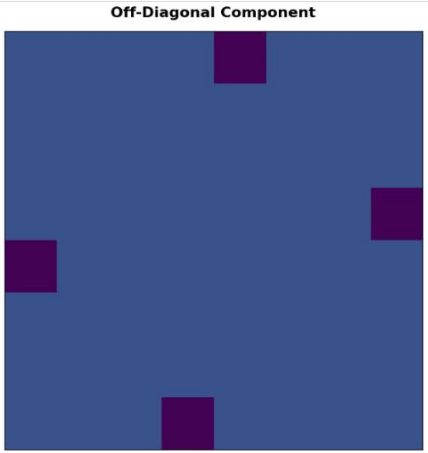
Direct Structural Synthesis



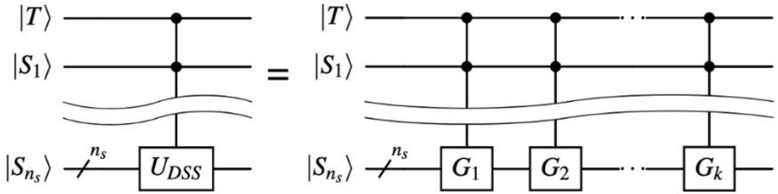
$e^{-ic\mathbb{I}t}$

$P(\phi)$

e^{-iBt}



Direct Structural Synthesis (DSS) Decomposition



Generalized Two-Level Unitary Structure (G_k)