CP violation in the quark sector

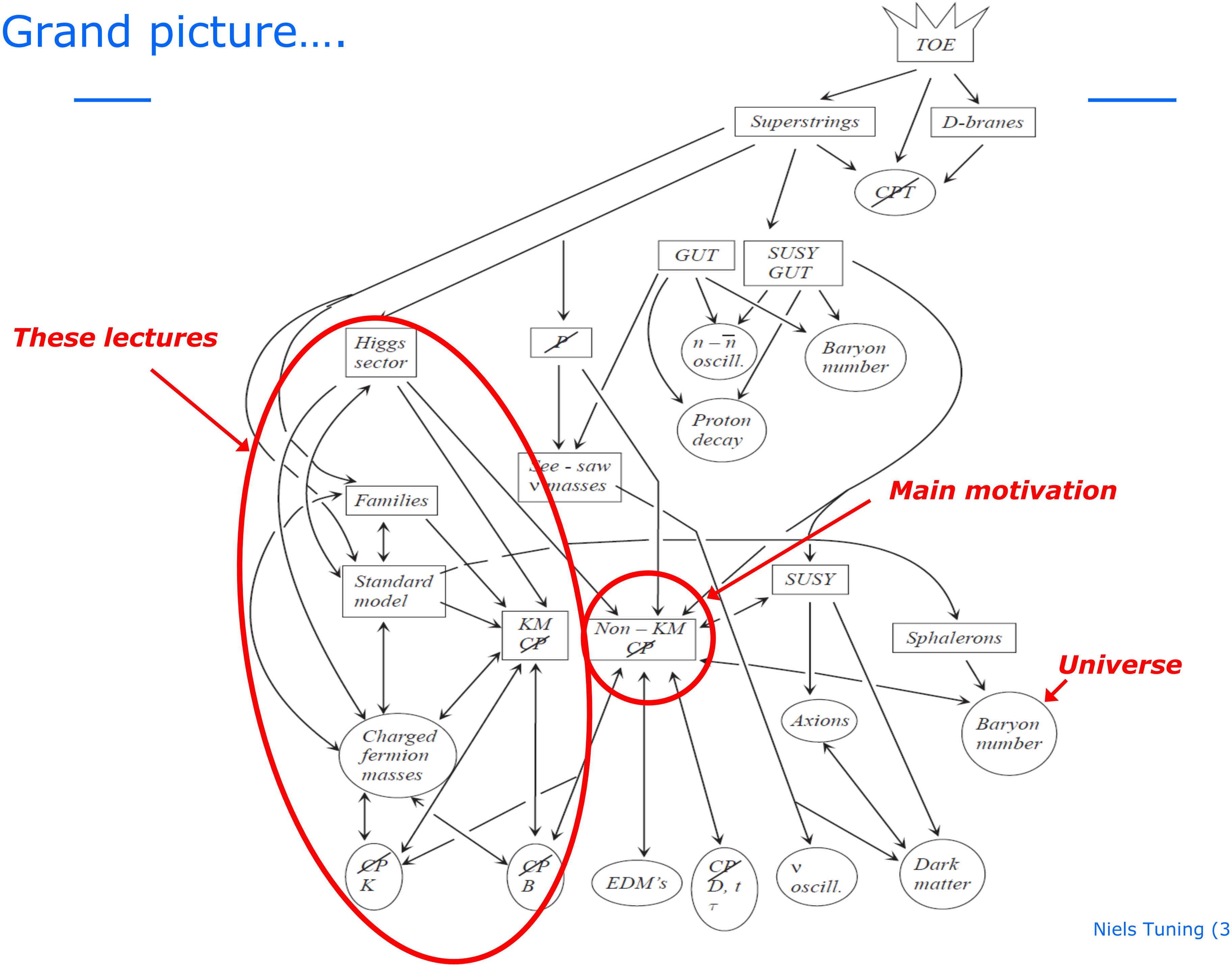
N. Tuning

P. Koppenburg

CP violation in the quark sector

1)	Quarks:	SM, CKM, UT	Niels, 30’
2)	Mesons:	Mixing and CPV types	Niels, 30’
3)	B-mesons:	Decays and Experiment	Patrick, 60’

Grand picture….



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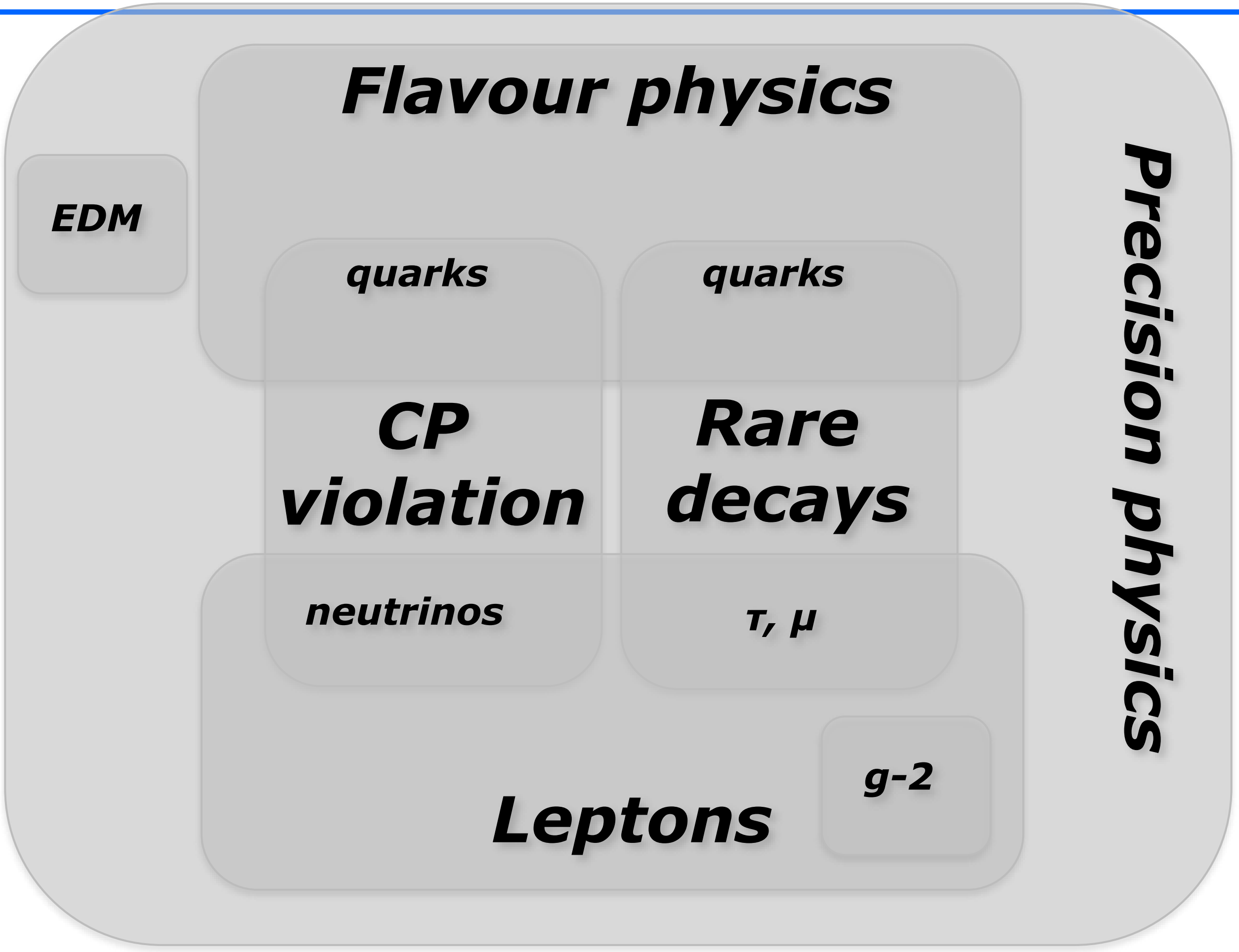
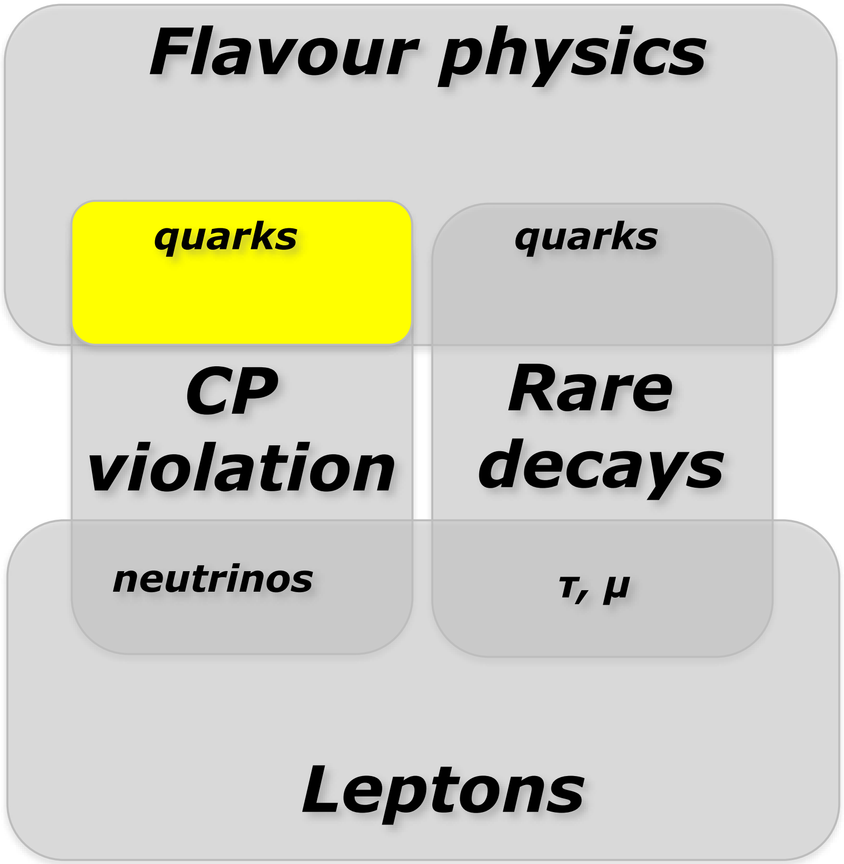

Flavour physics
EDM
quarks
quarks
CP violation
Rare decays
neutrinos
τ, μ
g-2
Leptons
Precision physics

Today



Flavour physics
quarks
quarks
CP violation
Rare decays
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τ, μ
Leptons

Flavour physics has a track record…

GIM mechanism in $K^0 \to \mu\mu$

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Milis theory is discussed.

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We wish to propose a simple model in which the divergences are properly ordered. Our model is founded in a quark model, but one involving four, not three, fundamental fermions; the weak interactions are medi-

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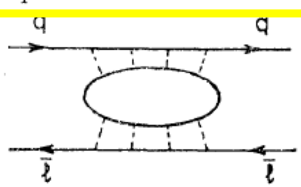
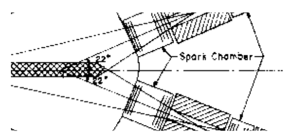


Glashow, Iliopoulos, Maiani,
Phys.Rev. D2 (1970) 1285

CP violation, $K_L^0 \to \pi\pi$



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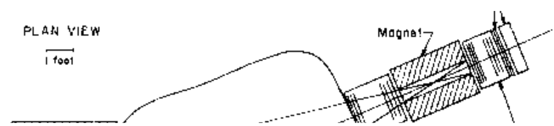

W. Chinowsky, Ann. Phys. (N.Y.) 5, 156 (1958).
[2]D. Neagu, E. O. Okonov, N. I. Petrov, A. M. Rosanova, and V. A. Rusakov, Phys. Rev. Letters 6, 552 (1961).
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Christenson, Cronin, Fitch, Turlay,
Phys.Rev.Lett. 13 (1964) 138-140

$B^0 \leftrightarrow \bar{B}^0$ mixing

DESY 87-029
April 1987

OBSERVATION OF B^0 - $\bar{B}^0$ MIXING

The ARGUS Collaboration

In summary, the combined evidence of the investigation of B^0 meson pairs, lepton pairs and B^0 meson-lepton events on the $\Upsilon(4S)$ leads to the conclusion that B^0-$\bar{B}^0$ mixing has been observed and is substantial.

Parameters	Comments
$r > 0.09$ 90%CL	This experiment
$x > 0.44$	This experiment
$B^{\frac{1}{2}} f_B \approx f_\pi < 160$ MeV	B meson ($\approx$ pion) decay constant
$m_b < 5\text{GeV}/c^2$	b-quark mass
$\tau_b < 1.4 \cdot 10^{-12}$s	B meson lifetime
$\lvert V_{td} \rvert < 0.018$	Kobayashi-Maskawa matrix element
$\eta_{QCD} < 0.86$	QCD correction factor [17]
$m_t > 50\text{GeV}/c^2$	t quark mass

ARGUS Coll.
Phys.Lett.B192:245,1987

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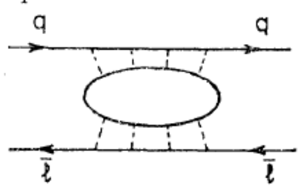

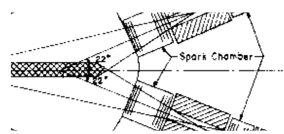

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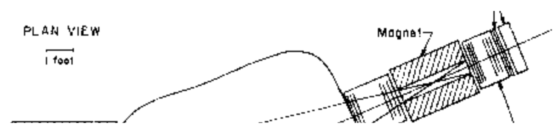

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Rare decay implies 2nd up-quark
"discovery" of charm?

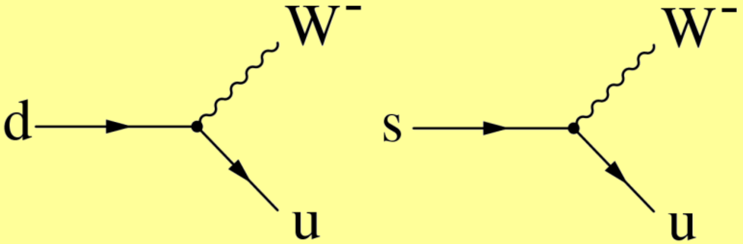
CP violation implies 3rd family
"discovery" of bottom?

Mixing implies heavy quark
"discovery" of top?

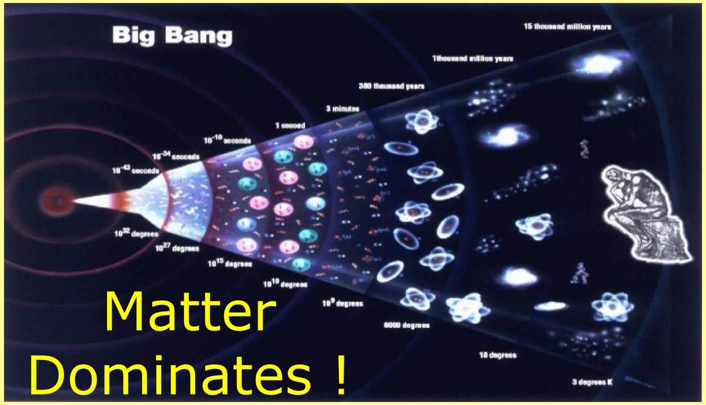
Motivation

- CP-violation (or flavour physics) is about charged current interactions

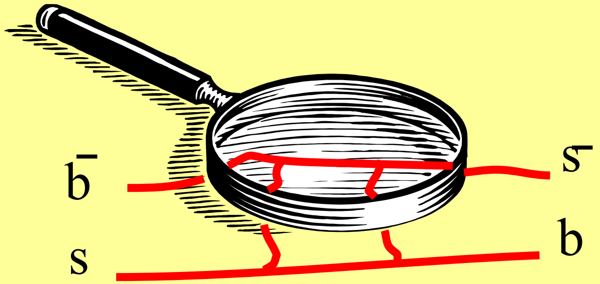
- Interesting because:
  1) Standard Model:
     in the heart of quark interactions



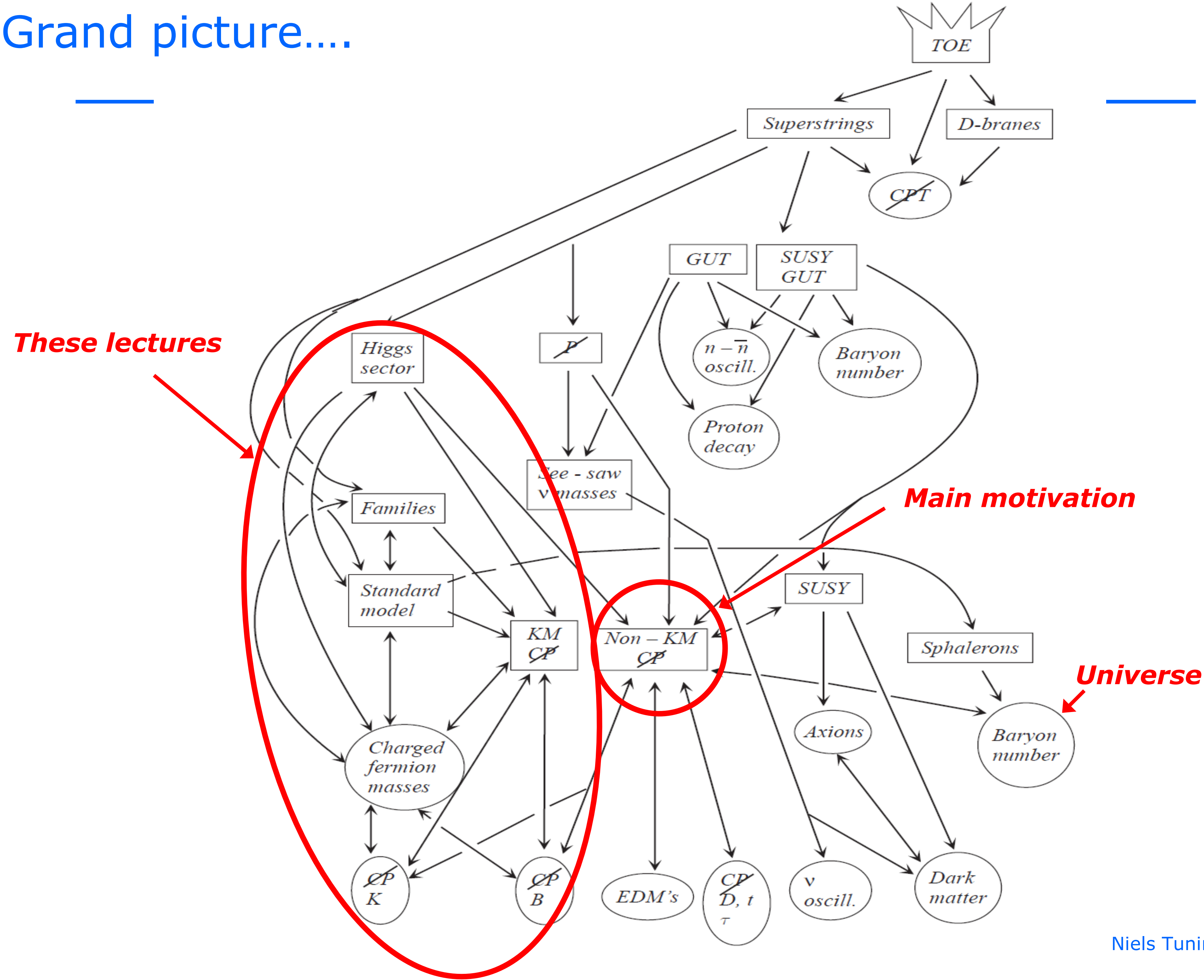
  2) Cosmology:
     related to matter – anti-matter asymetry



  3) Beyond Standard Model:
     measurements are sensitive to new particles



Grand picture….



Personal impression:

- People think (?) it is a complicated part of the SM. Why?

1) Non-intuitive concepts?
   - *Imaginary phase* in transition amplitude, T ~ $e^{i\varphi}$
   - *Different bases* to express quark states, d' =0.97 d + 0.22 s + 0.003 b
   - *Oscillations* (mixing) of mesons: $|K^0\rangle \leftrightarrow |\bar{K}^0\rangle$

2) Complicated calculations?

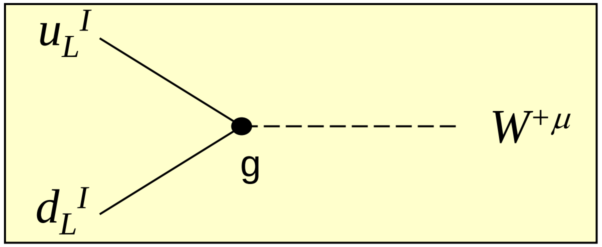
$$\Gamma\left(B^0 \to f\right) \propto \left|A_f\right|^2 \left[\left|g_+(t)\right|^2 + |\lambda|^2 \left|g_-(t)\right|^2 + 2\Re\left(\lambda g_+^*(t) g_-(t)\right)\right]$$

$$\Gamma\left(\bar{B}^0 \to f\right) \propto \left|\bar{A}_f\right|^2 \left[\left|g_+(t)\right|^2 + \frac{1}{|\lambda|^2}\left|g_-(t)\right|^2 + \frac{2}{|\lambda|^2}\Re\left(\lambda^* g_+^*(t) g_-(t)\right)\right]$$

3) Many decay modes? *"Beetopaipaigamma…"*
   - PDG reports 400+ decay modes of the B^0-meson:
     - Γ_1 $l^+ \nu_l$ *anything* $(10.33 \pm 0.28) \times 10^{-2}$
     - Γ_{347} $\nu \nu \gamma$ $<4.7 \times 10^{-5}$ *CL=90%*
   - And for one decay there are often more than one decay *amplitudes*…

CP violation in the SM Lagrangian

- Focus on charged current interaction ($W^{\pm}$): let's trace it



The Standard Model Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- $\mathcal{L}_{Kinetic}$: • Introduce the massless fermion fields
  • Require local gauge invariance ➔ gives rise to existence of gauge bosons
- $\mathcal{L}_{Higgs}$: • Introduce Higgs potential with $<\phi> \neq 0$
  • Spontaneous symmetry breaking

  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$

  The W^+, W^-, Z^0 bosons acquire a mass
- $\mathcal{L}_{Yukawa}$: • Ad hoc interactions between Higgs field & fermions

Fields: Notation

Y = Q - T_3

Fermions:
$$\psi_L = \left(\frac{1-\gamma_5}{2}\right)\psi \quad ; \quad \psi_R = \left(\frac{1+\gamma_5}{2}\right)\psi$$
with $\psi = Q_L, u_R, d_R, L_L, l_R, \nu_R$

Quarks:

Under SU2:
Left handed doublets
Right hander singlets

- $$\begin{pmatrix} u^I(3,2,1/6) \\ d^I(3,2,1/6) \end{pmatrix}_{L\,i} \equiv Q^I_{L\,i}\,(3,2,1/6)$$

Interaction rep.
Left-handed
generation index
SU(3)$_C$
SU(2)$_L$
Hypercharge Y
(=avg el.charge in multiplet)

- $u^I_{Ri}\,(3,1,2/3)$
- $d^I_{Ri}\,(3,1,-1/3)$

Leptons:

- $$\begin{pmatrix} \nu^I(1,2,-1/2) \\ l^I(1,2,-1/2) \end{pmatrix}_{L\,i} \equiv L^I_{L\,i}\,(1,2,-1/2)$$
- $l^I_{Ri}\,(1,1,-1)$
- $\left(\nu^I_{Ri}\right)$

Scalar field:

- $$\phi\,(1,2,1/2) = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

Note:
Interaction representation: standard model interaction is independent of generation number

Fields: Notation

Y = Q - T_3

Explicitly:

• The left handed quark doublet :

$$Q^I_{Li}(3,2,1/6) = \begin{pmatrix} u^I_r, u^I_g, u^I_b \\ d^I_r, d^I_g, d^I_b \end{pmatrix}_L, \begin{pmatrix} c^I_r, c^I_g, c^I_b \\ s^I_r, s^I_g, s^I_b \end{pmatrix}_L, \begin{pmatrix} t^I_r, t^I_g, t^I_b \\ b^I_r, b^I_g, b^I_b \end{pmatrix}_L \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = 1/6)$$

• Similarly for the quark singlets:

$$u^I_{Ri}(3,1,\ 2/3) = \left(u^I_r, u^I_r, u^I_r\right)_R, \left(c^I_r, c^I_r, c^I_r\right)_R, \left(t^I_r, t^I_r, t^I_r\right)_R \quad \left(Y = 2/3\right)$$

$$d^I_{Ri}(3,1,-1/3) = \left(d^I_r, d^I_r, d^I_r\right)_R, \left(s^I_r, s^I_r, s^I_r\right)_R, \left(b^I_r, b^I_r, b^I_r\right)_R \quad \left(Y = -1/3\right)$$

• The left handed leptons: $L^I_{Li}(1,2,-1/2) = \begin{pmatrix} \nu^I_e \\ e^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\mu \\ \mu^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\tau \\ \tau^I \end{pmatrix}_L \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad \left(Y = -1/2\right)$

• And similarly the (charged) singlets: $l^I_{Ri}(1,1,-1) = e^I_R, \mu^I_R, \tau^I_R \quad \left(Y = -1\right)$

$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$ **:The Kinetic Part**

$\mathcal{L}_{Kinetic}$: Fermions + gauge bosons + interactions

Procedure:
Introduce the Fermion fields and *demand* *that the theory is local gauge invariant under* $SU(3)_C xSU(2)_L xU(1)_Y$ *transformations.*

Start with the Dirac Lagrangian: $\mathcal{L} = i\bar{\psi}(\partial^\mu \gamma_\mu)\psi$

Replace: $\partial^\mu \to D^\mu \equiv \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig' B^\mu Y$

Fields:
- G_a^μ : 8 gluons
- W_b^μ : weak bosons: W_1, W_2, W_3
- B^μ : hypercharge boson

Generators:

L_a	: Gell-Mann matrices:	$\frac{1}{2}\lambda_a$	(3x3)	$SU(3)_C$
T_b	: Pauli Matrices:	$\frac{1}{2}\tau_b$	(2x2)	$SU(2)_L$
Y	: Hypercharge:			$U(1)_Y$

For the remainder we only consider Electroweak: $SU(2)_L \times U(1)_Y$

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Kinetic Part

$$L_{kinetic} : i\overline{\psi}(\partial^{\mu}\gamma_{\mu})\psi \rightarrow i\overline{\psi}(D^{\mu}\gamma_{\mu})\psi$$

$$with \quad \psi = Q_{Li}^{I}, \quad u_{Ri}^{I}, \quad d_{Ri}^{I}, \quad L_{Li}^{I}, \quad l_{Ri}^{I}$$

For example, the term with $Q_{Li}{}^{I}$ becomes:

$$L_{kinetic}(Q_{Li}^{I}) = i\overline{Q_{Li}^{I}}\gamma_{\mu}D^{\mu}Q_{Li}^{I}$$

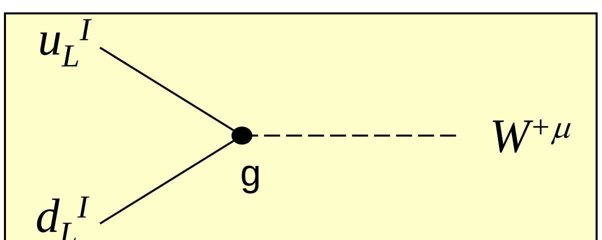
$$= i\overline{Q_{Li}^{I}}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g_s G_a^{\mu}\lambda_a + \frac{i}{2}gW_b^{\mu}\tau_b + \frac{i}{6}g'B^{\mu}\right)Q_{Li}^{I}$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Writing out only the weak part for the quarks:

$$L_{kinetic}^{Weak}(u,d)_L^I = i\overline{(u,d)_L^I}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g\left(W_1^{\mu}\tau_1 + W_2^{\mu}\tau_2 + W_3^{\mu}\tau_3\right)\right)\begin{pmatrix} u \\ d \end{pmatrix}_L^I$$

$$= i\overline{u_L^I}\gamma_{\mu}\partial^{\mu}u_L^I + i\overline{d_L^I}\gamma_{\mu}\partial^{\mu}d_L^I - \frac{g}{\sqrt{2}}\overline{u_L^I}\gamma_{\mu}W^{-\mu}d_L^I - \frac{g}{\sqrt{2}}\overline{d_L^I}\gamma_{\mu}W^{+\mu}u_L^I - \ldots$$



$L = J_{\mu}W^{\mu}$

$W^{+} = (1/\sqrt{2})\,(W_1 + i\,W_2)$

$W^{-} = (1/\sqrt{2})\,(W_1 - i\,W_2)$

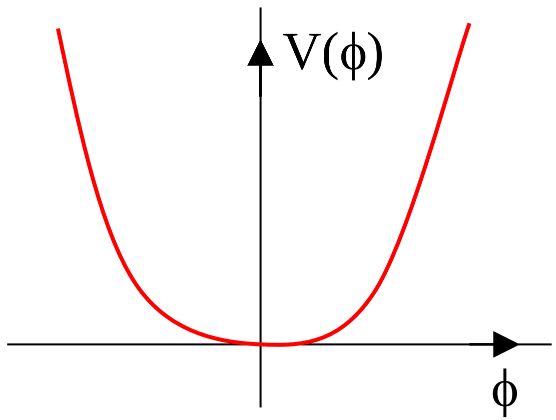
$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$: The Higgs Potential

$$\mathcal{L}_{Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_{Higgs} \qquad V_{Higgs} = \frac{1}{2}\mu^2\left(\phi^\dagger\phi\right) + |\lambda|\left(\phi^\dagger\phi\right)^2$$

Symmetry

$\mu^2 > 0:$

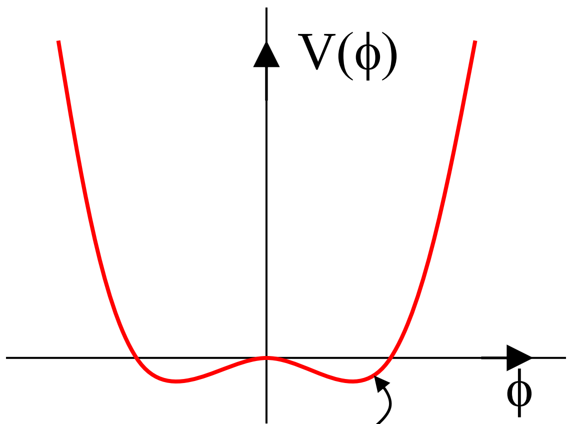
$<\varphi> = 0$



Broken Symmetry

$\mu^2 < 0:$

$<\varphi> = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$



$v = \sqrt{-\mu^2/|\lambda|} \sim 246$ GeV

Spontaneous Symmetry Breaking: The Higgs field adopts a non-zero vacuum expectation value

Procedure: $\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \Re e\,\varphi^+ + i\Im m\,\phi^+ \\ \Re e\,\varphi^0 + i\Im m\,\phi^0 \end{pmatrix}$

Substitute: $\Re e\,\varphi^0 = \dfrac{v + H^0}{\sqrt{2}}$

And rewrite the Lagrangian (tedious):

(The other 3 Higgs fields are “eaten” by the W, Z bosons)

1. $G_{SM} : \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right) \rightarrow \left(SU(3)_C \times U(1)_{EM}\right)$
2. The W^+, W^-, Z^0 bosons acquire mass
3. The Higgs boson *H* appears

$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$: The Yukawa Part

Since we have a Higgs field we can (should?) add (ad-hoc) interactions between ϕ and the fermions in a gauge invariant way.

The result is:

(doublets: $\overline{\psi}_{Li}$, ϕ; singlet: ψ_{Rj})

$$-\mathcal{L}_{Yukawa} = Y_{ij}\left(\overline{\psi}_{Li}\,\phi\right)\psi_{Rj} + h.c.$$

$$= Y_{ij}^{d}\left(\overline{Q_{Li}^{I}}\,\phi\right)d_{Rj}^{I} + Y_{ij}^{u}\left(\overline{Q_{Li}^{I}}\,\tilde{\phi}\right)u_{Rj}^{I} + Y_{ij}^{l}\left(\overline{L_{Li}^{I}}\,\phi\right)l_{Rj}^{I} + h.c.$$

i, j : indices for the 3 generations!

With: $\tilde{\phi} = i\sigma_2\,\phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}$

(The CP conjugate of ϕ
To be manifestly invariant under SU(2))

Y_{ij}^{d} , Y_{ij}^{u} , Y_{ij}^{l}

are arbitrary complex matrices which operate in family space (3x3)
➔ Flavour physics!

$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$: The Yukawa Part

Writing the first term explicitly:

$$Y^d_{ij}\,(\overline{u^I_L}, \overline{d^I_L})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d^I_{Rj} \quad =$$

$$\begin{pmatrix} Y^d_{11}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{12}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{13}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \\ Y^d_{21}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{22}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{13}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \\ Y^d_{31}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{32}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{33}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \end{pmatrix} \bullet \begin{pmatrix} d^I_R \\ s^I_R \\ b^I_R \end{pmatrix}$$

$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$: The Yukawa Part

There are 3 Yukawa matrices (in the case of massless neutrino's):

$$Y_{ij}^{d} \quad , \quad Y_{ij}^{u} \quad , \quad Y_{ij}^{l}$$

Each matrix is 3x3 complex:

- 27 real parameters
- 27 imaginary parameters ("phases")

➢ many of the parameters are equivalent, since the physics described by one set of couplings is the same as another

➢ It can be shown (see ref. [Nir]) that the independent parameters are:

  - 12 real parameters
  - 1 imaginary phase

➢This single phase is the source of all CP violation in the Standard Model

……Revisit later

$L_{Yukawa} \xrightarrow{S.S.B} L_{Mass}$: The Fermion Masses

Start with the Yukawa Lagrangian

$$-L_{Yuk} = Y_{ij}^{d}\,(\overline{u_L^I}, \overline{d_L^I})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + Y_{ij}^{u}\,(\ldots) + Y_{ij}^{l}\,(\ldots)$$

$$S.S.B. \;:\; \Re e\left(\varphi^0\right) \rightarrow \frac{v+H}{\sqrt{2}}$$

After which the following mass term emerges:

$$-L_{Yuk} \rightarrow -L_{Mass} = \overline{d_{Li}^I}\, M_{ij}^d\, d_{Rj}^I + \overline{u_{Li}^I}\, M_{ij}^u\, u_{Rj}^I + \overline{l_{Li}^I}\, M_{ij}^l\, l_{Rj}^I + h.c.$$

with $M_{ij}^d \equiv \frac{v}{\sqrt{2}} Y_{ij}^d \;,\; M_{ij}^u \equiv \frac{v}{\sqrt{2}} Y_{ij}^u \;,\; M_{ij}^l \equiv \frac{v}{\sqrt{2}} Y_{ij}^l$

L_{Mass} is CP violating in a similar way as L_{Yuk}

$$\mathcal{L}_{Yukawa} \xrightarrow{S.S.B} \mathcal{L}_{Mass}$$

: The Fermion Masses

Writing in an explicit form:

$$-\mathcal{L}_{Mass} = \left(\overline{d^I}, \overline{s^I}, \overline{b^I}\right)_L \left(M^d\right) \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R + \left(\overline{u^I}, \overline{c^I}, \overline{t^I}\right)_L \left(M^u\right) \begin{pmatrix} u^I \\ c^I \\ t^I \end{pmatrix}_R + \left(\overline{e^I}, \overline{\mu^I}, \overline{\tau^I}\right)_L \left(M^l\right) \begin{pmatrix} e^I \\ \mu^I \\ \tau^I \end{pmatrix}_R + h.c.$$

The matrices M can always be diagonalised by unitary matrices V_L^f and V_R^f such that:

$$V_L^f \, M^f \, V_R^{f\dagger} = M^f_{diagonal}$$

$$\left[\left(\overline{d^I}, \overline{s^I}, \overline{b^I}\right)_L V_L^{f\dagger} \, V_L^f \, M^f \, V_R^{f\dagger} \, V_R^f \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R \right]$$

Then the real fermion mass eigenstates are given by:

$$d_{Li} = \left(V_L^d\right)_{ij} \cdot d^I_{Lj} \qquad d_{Ri} = \left(V_R^d\right)_{ij} \cdot d^I_{Rj}$$

$$u_{Li} = \left(V_L^u\right)_{ij} \cdot u^I_{Lj} \qquad u_{Ri} = \left(V_R^u\right)_{ij} \cdot u^I_{Rj}$$

$$l_{Li} = \left(V_L^l\right)_{ij} \cdot l^I_{Lj} \qquad l_{Ri} = \left(V_R^l\right)_{ij} \cdot l^I_{Rj}$$

$d_L{}^I$, $u_L{}^I$, $l_L{}^I$ are the weak interaction eigenstates
d_L, u_L, l_L are the mass eigenstates ("physical particles")

$L_{Yukawa} \xrightarrow{S.S.B} L_{Mass}$: The Fermion Masses

In terms of the mass eigenstates:

$$-L_{Mass} = \left(\bar{d},\bar{s},\bar{b}\right)_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u},\bar{c},\bar{t}\right)_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

$$+ \left(\bar{e},\bar{\mu},\bar{\tau}\right)_L \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R + h.c.$$

$$-L_{Mass} = m_u \bar{u}u + m_c \bar{c}c + m_t \bar{t}t$$
$$+ m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b$$
$$+ m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau$$

In flavour space one can choose:

Weak basis: The gauge currents are diagonal in flavour space, but the flavour mass matrices are non-diagonal

Mass basis: The fermion masses are diagonal, but some gauge currents (charged weak interactions) are not diagonal in flavour space

In the weak basis: L_{Yukawa} = CP violating

In the mass basis: $L_{Yukawa} \rightarrow L_{Mass}$ = CP conserving

➔ What happened to the charged current interactions (in $L_{Kinetic}$) ?

$L_W \rightarrow L_{CKM}$: The Charged Current

The charged current interaction for quarks in the ***interaction*** basis is:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \overline{u_{Li}^I} \gamma^\mu d_{Li}^I W_\mu^+$$

The charged current interaction for quarks in the ***mass*** basis is:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \overline{u_{Li}} V_L^u \gamma^\mu V_L^{d\dagger} d_{Li} W_\mu^+$$

The unitary matrix: $V_{CKM} = \left(V_L^u \cdot V_L^{d\dagger}\right)$ With: $V_{CKM} \cdot V_{CKM}^\dagger = 1$

is the Cabibbo Kobayashi Maskawa mixing matrix:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \left(\bar{u}, \bar{c}, \bar{t}\right)_L \left(V_{CKM}\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^\mu W_\mu^+$$

Lepton sector: similarly $V_{MNS} = \left(V_L^\nu \cdot V_L^{l\dagger}\right)$

However, for massless neutrino's: $V_L{}^\nu$ = arbitrary. Choose it such that $V_{MNS} = 1$

➔ There is no mixing in the lepton sector

Charged Currents

The charged current term reads:

$$L_{CC} = \frac{g}{\sqrt{2}} \overline{u_{Li}^{I}} \gamma^{\mu} W_{\mu}^{-} d_{Li}^{I} \quad + \quad \frac{g}{\sqrt{2}} \overline{d_{Li}^{I}} \gamma^{\mu} W_{\mu}^{+} u_{Li}^{I} \quad = \quad J_{CC}^{\mu -} W_{\mu}^{-} + J_{CC}^{\mu +} W_{\mu}^{+}$$

$$= \frac{g}{\sqrt{2}} \overline{u}_i \left(\frac{1-\gamma^5}{2} \right) \gamma^{\mu} W_{\mu}^{-} V_{ij} \left(\frac{1-\gamma^5}{2} \right) d_j \quad + \quad \frac{g}{\sqrt{2}} \overline{d}_j \left(\frac{1-\gamma^5}{2} \right) \gamma^{\mu} W_{\mu}^{+} V_{ji}^{\dagger} \left(\frac{1-\gamma^5}{2} \right) u_i$$

$$= \frac{g}{\sqrt{2}} \overline{u}_i \gamma^{\mu} W_{\mu}^{-} V_{ij} \left(1-\gamma^5\right) d_j \quad + \quad \frac{g}{\sqrt{2}} \overline{d}_j \gamma^{\mu} W_{\mu}^{+} V_{ij}^{*} \left(1-\gamma^5\right) u_i$$

Under the CP operator this gives: (Together with (x,t) -> (-x,t))

$$L_{CC} \xrightarrow{CP} \frac{g}{\sqrt{2}} \overline{d}_j \gamma^{\mu} W_{\mu}^{+} V_{ij} \left(1-\gamma^5\right) u_i \quad + \quad \frac{g}{\sqrt{2}} \overline{u}_i \gamma^{\mu} W_{\mu}^{i} V_{ij}^{*} \left(1-\gamma^5\right) d_j$$

A comparison shows that CP is conserved only if $V_{ij} = V_{ij}^{*}$

In general the charged current term is CP violating

The Standard Model Lagrangian (recap)

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- $\mathcal{L}_{Kinetic}$: •Introduce the massless fermion fields
  •Require local gauge invariance → gives rise to existence of gauge bosons
  ➔ CP Conserving

- $\mathcal{L}_{Higgs}$: •Introduce Higgs potential with $<\phi> \neq 0$
  •Spontaneous symmetry breaking
  ➔ CP Conserving

  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$
  The W^+, W^-, Z^0 bosons acquire a mass

- $\mathcal{L}_{Yukawa}$: •Ad hoc interactions between Higgs field & fermions
  ➔ CP violating with a single phase

- $\mathcal{L}_{Yukawa} \rightarrow \mathcal{L}_{mass}$:
  - fermion weak eigenstates:
    - mass matrix is (3x3) non-diagonal ➔ CP-violating
  - fermion mass eigenstates:
    - mass matrix is (3x3) diagonal ➔ CP-conserving!

- $\mathcal{L}_{Kinetic}$ in mass eigenstates: CKM – matrix ➔ CP violating with a single phase

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

Recap

$$-L_{Yuk} = Y_{ij}^{d}\,(\overline{u_L^I}, \overline{d_L^I})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + \ldots$$

$$L_{Kinetic} = \frac{g}{\sqrt{2}}\overline{u_{Li}^I}\gamma^\mu W_\mu^- d_{Li}^I + \frac{g}{\sqrt{2}}\overline{d_{Li}^I}\gamma^\mu W_\mu^+ u_{Li}^I + \ldots$$

Diagonalize Yukawa matrix Y_{ij}

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$-L_{Mass} = \left(\bar{d}, \bar{s}, \bar{b}\right)_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u}, \bar{c}, \bar{t}\right)_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \ldots$$

$$L_{CKM} = \frac{g}{\sqrt{2}}\overline{u_i}\gamma^\mu W_\mu^- V_{ij}\left(1-\gamma^5\right)d_j + \frac{g}{\sqrt{2}}\overline{d_j}\gamma^\mu W_\mu^+ V_{ij}^*\left(1-\gamma^5\right)u_i + \ldots$$

$$L_{SM} = L_{CKM} + L_{Higgs} + L_{Mass}$$

Ok…. We've got the CKM matrix, now what?

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- It's *unitary*
  - *"probabilities add up to 1":*
  - d'=0.97 d + 0.22 s + 0.003 b ($0.97^2+0.22^2+0.003^2=1$)
- How many free parameters?
  - How many real/complex?
- How do we normally visualize these parameters?

What do we know about the CKM matrix?

- Magnitudes of elements have been measured over time
  - Result of a *large* number of measurements and calculations

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.99911 \end{pmatrix} \pm \begin{pmatrix} 0.00010 & 0.00044 & 0.00012 \\ 0.00044 & 0.00011 & 0.00076 \\ 0.00024 & 0.00974 & 0.00003 \end{pmatrix}$$

Magnitude of elements shown only, no information of phase

What do we know about the CKM matrix?

- Magnitudes of elements have been measured over time
  - Result of a *large* number of measurements and calculations

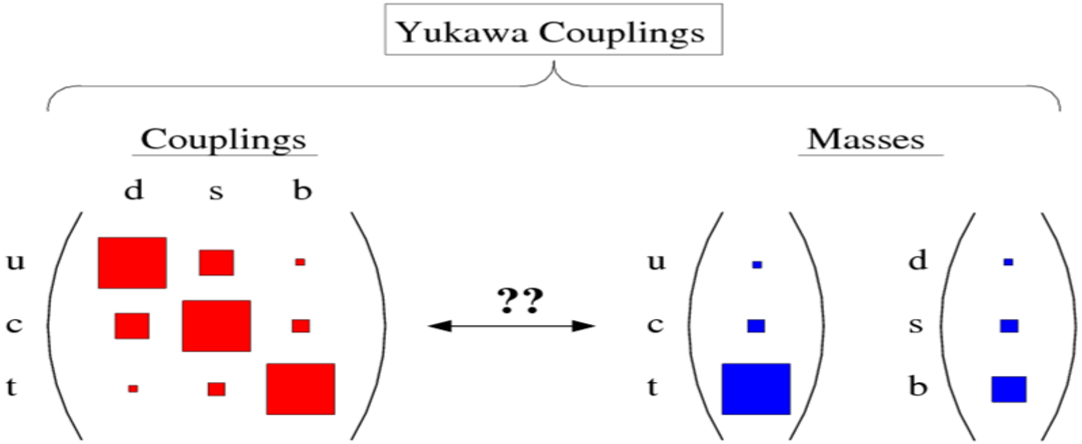
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\lambda \approx \sin\theta_C = \sin\theta_{12} \approx 0.24$$

Magnitude of elements shown only, no information of phase

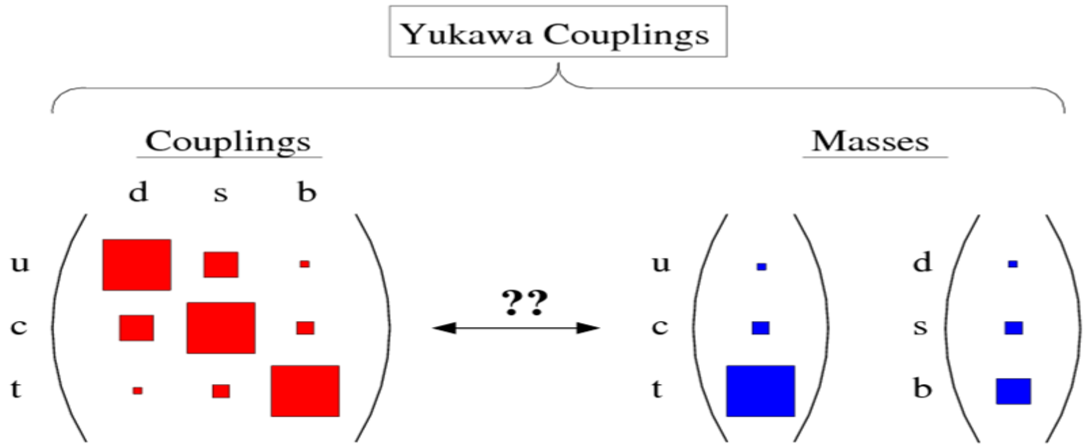
What's going on??



- ??? Edward Witten, 17 Feb 2009…

- See "*From F-Theory GUT's to the LHC*" by Heckman and Vafa (arXiv:0809.3452)

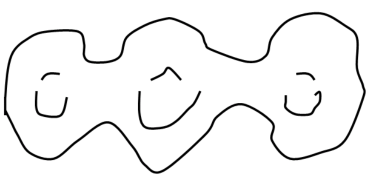
What's going on??



In 2004, Time magazine stated that Witten was widely thought to be the world's greatest living theoretical physicist.

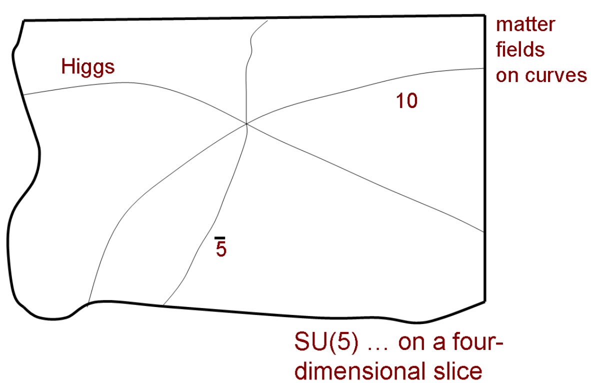
- ??? Edward Witten, 17 Feb 2009…

In this approach, the ordinary Higgs field is a wavefunction on K, as are the quark and lepton fields



Quark and lepton masses and the CKM matrix are determined by the overlaps of these wavefunctions.

The picture is a little like this:



Higgs fields and quarks and leptons are supported on the three curves, and the Yukawa couplings that gives masses to down quarks and charged leptons come from the intersection drawn. (Up quark masses come from a similar intersection.)

In the leading approximation, only one particle of each type (i.e. the third generation particles – top, bottom, tau) get masses. The others have wavefunctions that vanish at the intersection point.

- See "*From F-Theory GUT's to the LHC*" by Heckman and Vafa (arXiv:0809.3452)

Intermezzo: How about the leptons?

- We now know that neutrinos also have flavour oscillations
  - Neutrinos have mass
  - Diagonalizing Y^l_{ij} doesn't come for free any longer

$$\begin{aligned} \mathcal{L}_{Yukawa} &= Y_{ij}\overline{\psi_{Li}}\,\phi\,\psi_{Rj} + h.c. \\ &= Y^d_{ij}\overline{Q^I_{Li}}\,\phi\,d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\,\tilde{\phi}\,u^I_{Rj} + Y^l_{ij}\overline{L^I_{Li}}\,\phi\,l^I_{Rj} \end{aligned}$$

- thus there is the equivalent of a CKM matrix for them:
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

vs

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}.$$

Intermezzo: How about the leptons?

- the equivalent of the CKM matrix
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \begin{bmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

vs

$$\begin{bmatrix}|d'\rangle\\ |s'\rangle\\ |b'\rangle\end{bmatrix} = \begin{bmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix}\begin{bmatrix}|d\rangle\\ |s\rangle\\ |b\rangle\end{bmatrix}.$$

- a completely different hierarchy!

$$U_{MNSP} \approx \begin{pmatrix} 0.82 & 0.55 & 0.15\\ 0.37 & 0.57 & 0.70\\ 0.39 & 0.59 & 0.69 \end{pmatrix}$$

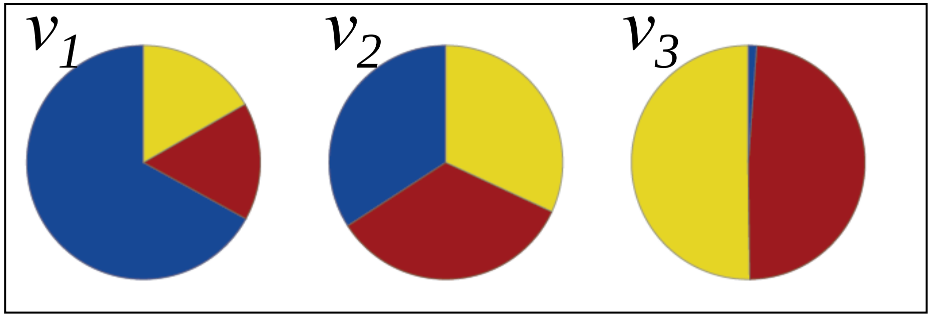
$$V_{CKM} = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365\\ 0.22438 & 0.97359 & 0.04214\\ 0.00896 & 0.04133 & 0.99911 \end{pmatrix}$$

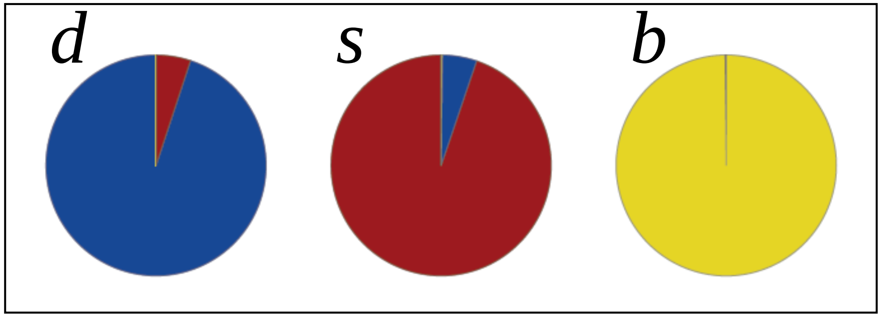
Intermezzo: How about the leptons?

- the equivalent of the CKM matrix
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} \quad \textbf{vs} \quad \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}.$$

- a completely different $\begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2 \\ |U_{\tau 1}|^2 & |U_{\tau 2}|^2 & |U_{\tau 3}|^2 \end{pmatrix} \approx \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{pmatrix}$





See eg. PhD thesis R. de Adelhart Toorop

From 2 to 3 generations

- 2 generations: d'=0.97 d + 0.22 s   (θ_c=13°)

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- 3 generations: d'=0.97 d + 0.22 s + 0.003 b

Parameterization used by Particle Data Group (3 Euler angles, 1 phase):

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

Possible forms of 3 generation mixing matrix

→ Different parametrizations! It's about phase *differences!*

Re-phasing V:

$$
\overline{(u,\ c,\ t)}_L \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,
$$

$$=$$

$$
\overline{(u,\ c,\ t)}_L \begin{pmatrix} V_{ud}\, e^{-i\phi} & V_{us} & V_{ub} \\ V_{cd}\, e^{-i\phi} & V_{cs} & V_{cb} \\ V_{td}\, e^{-i\phi} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d\, e^{i\phi} \\ s \\ b \end{pmatrix}_L
$$

3 parameters: θ, τ, σ
1 phase: φ

Parametrization		Useful relations
P1: $V = R_{12}(\theta)\, R_{23}(\sigma,\varphi)\, R_{12}^{-1}(\theta')$ $\begin{pmatrix} s_\theta s_{\theta'} c_\sigma + c_\theta c_{\theta'} e^{-i\varphi} & s_\theta c_{\theta'} c_\sigma - c_\theta s_{\theta'} e^{-i\varphi} & s_\theta s_\sigma \\ c_\theta s_{\theta'} c_\sigma - s_\theta c_{\theta'} e^{-i\varphi} & c_\theta c_{\theta'} c_\sigma + s_\theta s_{\theta'} e^{-i\varphi} & c_\theta s_\sigma \\ -s_{\theta'} s_\sigma & -c_{\theta'} s_\sigma & c_\sigma \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta s_{\theta'} c_{\theta'} s_\sigma^2 c_\sigma \sin\varphi$; $\tan\theta = \lvert V_{ub}/V_{cb}\rvert$; $\tan\theta' = \lvert V_{td}/V_{ts}\rvert$; $\cos\sigma = \lvert V_{tb}\rvert$
P2: $V = R_{23}(\sigma)\, R_{12}(\theta,\varphi)\, R_{23}^{-1}(\sigma')$ $\begin{pmatrix} c_\theta & s_\theta c_{\sigma'} & -s_\theta s_{\sigma'} \\ -s_\theta c_\sigma & c_\theta c_\sigma c_{\sigma'} + s_\sigma s_{\sigma'} e^{-i\varphi} & -c_\theta c_\sigma s_{\sigma'} + s_\sigma c_{\sigma'} e^{-i\varphi} \\ s_\theta s_\sigma & -c_\theta s_\sigma c_{\sigma'} + c_\sigma s_{\sigma'} e^{-i\varphi} & c_\theta s_\sigma s_{\sigma'} + c_\sigma c_{\sigma'} e^{-i\varphi} \end{pmatrix}$	**KM**	$\mathcal{J} = s_\theta^2 c_\theta s_\sigma c_\sigma s_{\sigma'} c_{\sigma'} \sin\varphi$; $\cos\theta = \lvert V_{ud}\rvert$; $\tan\sigma = \lvert V_{td}/V_{cd}\rvert$; $\tan\sigma' = \lvert V_{ub}/V_{us}\rvert$
P3: $V = R_{23}(\sigma)\, R_{31}(\tau,\varphi)\, R_{12}(\theta)$ $\begin{pmatrix} c_\theta c_\tau & s_\theta c_\tau & s_\tau \\ -c_\theta s_\sigma s_\tau - s_\theta c_\sigma e^{-i\varphi} & -s_\theta s_\sigma s_\tau + c_\theta c_\sigma e^{-i\varphi} & s_\sigma c_\tau \\ -c_\theta c_\sigma s_\tau + s_\theta s_\sigma e^{-i\varphi} & -s_\theta c_\sigma s_\tau - c_\theta s_\sigma e^{-i\varphi} & c_\sigma c_\tau \end{pmatrix}$	**PDG**	$\mathcal{J} = s_\theta c_\theta s_\sigma c_\sigma s_\tau c_\tau^2 \sin\varphi$; $\tan\theta = \lvert V_{us}/V_{ud}\rvert$; $\tan\sigma = \lvert V_{cb}/V_{tb}\rvert$; $\sin\tau = \lvert V_{ub}\rvert$
P4: $V = R_{12}(\theta)\, R_{31}(\tau,\varphi)\, R_{23}^{-1}(\sigma)$ $\begin{pmatrix} c_\theta c_\tau & c_\theta s_\sigma s_\tau + s_\theta c_\sigma e^{-i\varphi} & c_\theta c_\sigma s_\tau - s_\theta s_\sigma e^{-i\varphi} \\ -s_\theta c_\tau & -s_\theta s_\sigma s_\tau + c_\theta c_\sigma e^{-i\varphi} & -s_\theta c_\sigma s_\tau - c_\theta s_\sigma e^{-i\varphi} \\ -s_\tau & s_\sigma c_\tau & c_\sigma c_\tau \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta s_\sigma c_\sigma s_\tau c_\tau^2 \sin\varphi$; $\tan\theta = \lvert V_{cd}/V_{ud}\rvert$; $\tan\sigma = \lvert V_{ts}/V_{tb}\rvert$; $\sin\tau = \lvert V_{td}\rvert$
P5: $V = R_{31}(\tau)\, R_{12}(\theta,\varphi)\, R_{31}^{-1}(\tau')$ $\begin{pmatrix} c_\theta c_\tau c_{\tau'} + s_\tau s_{\tau'} e^{-i\varphi} & s_\theta c_\tau & -c_\theta c_\tau s_{\tau'} + s_\tau c_{\tau'} e^{-i\varphi} \\ -s_\theta c_{\tau'} & c_\theta & s_\theta s_{\tau'} \\ -c_\theta s_\tau c_{\tau'} + c_\tau s_{\tau'} e^{-i\varphi} & -s_\theta s_\tau & c_\theta s_\tau s_{\tau'} + c_\tau c_{\tau'} e^{-i\varphi} \end{pmatrix}$		$\mathcal{J} = s_\theta^2 c_\theta s_\tau c_\tau s_{\tau'} c_{\tau'} \sin\varphi$; $\cos\theta = \lvert V_{cs}\rvert$; $\tan\tau = \lvert V_{ts}/V_{us}\rvert$; $\tan\tau' = \lvert V_{cb}/V_{cd}\rvert$
P6: $V = R_{12}(\theta)\, R_{23}(\sigma,\varphi)\, R_{31}(\tau)$ $\begin{pmatrix} -s_\theta s_\sigma s_\tau + c_\theta c_\tau e^{-i\varphi} & s_\theta c_\sigma & s_\theta s_\sigma c_\tau + c_\theta s_\tau e^{-i\varphi} \\ -c_\theta s_\sigma s_\tau - s_\theta c_\tau e^{-i\varphi} & c_\theta c_\sigma & c_\theta s_\sigma c_\tau - s_\theta s_\tau e^{-i\varphi} \\ -c_\sigma s_\tau & -s_\sigma & c_\sigma c_\tau \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta s_\sigma c_\sigma^2 s_\tau c_\tau \sin\varphi$; $\tan\theta = \lvert V_{us}/V_{cs}\rvert$; $\sin\sigma = \lvert V_{ts}\rvert$; $\tan\tau = \lvert V_{td}/V_{tb}\rvert$
P7: $V = R_{23}(\sigma)\, R_{12}(\theta,\varphi)\, R_{31}^{-1}(\tau)$ $\begin{pmatrix} c_\theta c_\tau & s_\theta & -c_\theta s_\tau \\ -s_\theta c_\sigma c_\tau + s_\sigma s_\tau e^{-i\varphi} & c_\theta c_\sigma & s_\theta c_\sigma s_\tau + s_\sigma c_\tau e^{-i\varphi} \\ s_\theta s_\sigma c_\tau + c_\sigma s_\tau e^{-i\varphi} & -c_\theta s_\sigma & -s_\theta s_\sigma s_\tau + c_\sigma c_\tau e^{-i\varphi} \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta^2 s_\sigma c_\sigma s_\tau c_\tau \sin\varphi$; $\sin\theta = \lvert V_{us}\rvert$; $\tan\sigma = \lvert V_{ts}/V_{cs}\rvert$; $\tan\tau = \lvert V_{ub}/V_{ud}\rvert$
P8: $V = R_{31}(\tau)\, R_{12}(\theta,\varphi)\, R_{23}(\sigma)$ $\begin{pmatrix} c_\theta c_\tau & s_\theta c_\sigma c_\tau - s_\sigma s_\tau e^{-i\varphi} & s_\theta s_\sigma c_\tau + c_\sigma s_\tau e^{-i\varphi} \\ -s_\theta & c_\theta c_\sigma & c_\theta s_\sigma \\ -c_\theta s_\tau & -s_\theta c_\sigma s_\tau - s_\sigma c_\tau e^{-i\varphi} & -s_\theta s_\sigma s_\tau + c_\sigma c_\tau e^{-i\varphi} \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta^2 s_\sigma c_\sigma s_\tau c_\tau \sin\varphi$; $\sin\theta = \lvert V_{cd}\rvert$; $\tan\sigma = \lvert V_{cb}/V_{cs}\rvert$; $\tan\tau = \lvert V_{td}/V_{ud}\rvert$
P9: $V = R_{31}(\tau)\, R_{23}(\sigma,\varphi)\, R_{12}^{-1}(\theta)$ $\begin{pmatrix} -s_\theta s_\sigma s_\tau + c_\theta c_\tau e^{-i\varphi} & -c_\theta s_\sigma s_\tau - s_\theta c_\tau e^{-i\varphi} & c_\sigma s_\tau \\ s_\theta c_\sigma & c_\theta c_\sigma & s_\sigma \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta s_\sigma c_\sigma^2 s_\tau c_\tau \sin\varphi$; $\tan\theta = \lvert V_{cd}/V_{cs}\rvert$; $\sin\sigma = \lvert V_{cb}\rvert$

Wolfenstein parameterization

$$\boxed{\sin\theta_{12} = \lambda} \tag{2.7}$$
$$\sin\theta_{23} = A\lambda^2 \tag{2.8}$$
$$\sin\theta_{13}e^{-i\delta_{13}} = A\lambda^3(\rho - i\eta) \tag{2.9}$$

where A, ρ and η are numbers of order unity. The CKM matrix then becomes $\mathcal{O}(\lambda^3)$:

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \delta V \tag{2.10}$$

3 real parameters: A, λ, ρ
1 imaginary parameter: η

Wolfenstein parameterization

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The higher order terms in the Wolfenstein parametrization are of particular importance for the B_s-system, as we will see in chapter 4, because the phase in $|V_{ts}|$ is only apparent at $\mathcal{O}(\lambda^4)$:

$$\delta V = \begin{pmatrix} -\frac{1}{8}\lambda^4 & 0 & 0 \\ \frac{1}{2}A^2\lambda^5(1-2(\rho+i\eta)) & -\frac{1}{8}\lambda^4(1+4A^2) & 0 \\ \frac{1}{2}A\lambda^5(\rho+i\eta) & \frac{1}{2}A\lambda^4(1-2(\rho+i\eta)) & -\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6) \tag{2.11}$$

3 real parameters: A, λ, ρ

1 imaginary parameter: η

Deriving the triangle interpretation

- Starting point: the 9 unitarity constraints on the CKM matrix

$$V^{+}V = \begin{pmatrix} V^{*}_{ud} & V^{*}_{cd} & V^{*}_{td} \\ V^{*}_{us} & V^{*}_{cs} & V^{*}_{ts} \\ V^{*}_{ub} & V^{*}_{cb} & V^{*}_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

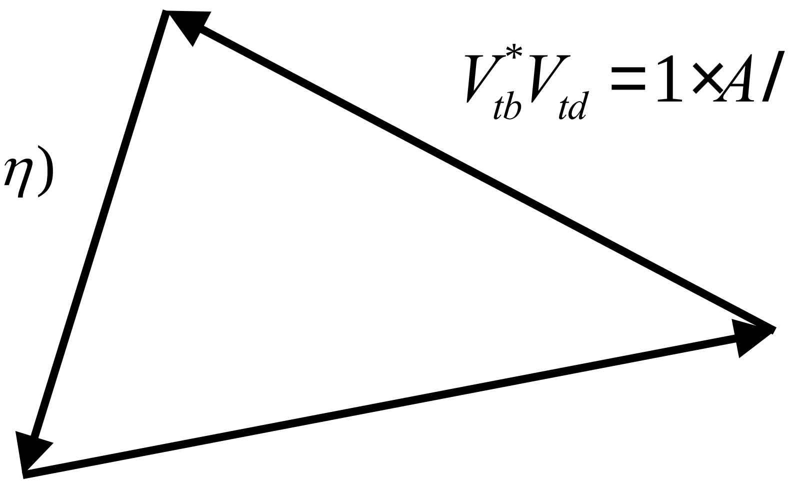
- Pick (arbitrarily) orthogonality condition with (i,j)=(3,1)

$$V^{*}_{ub}V_{ud} + V^{*}_{cb}V_{cd} + V^{*}_{tb}V_{td} = 0$$

Visualizing the unitarity constraint

- Sum of three complex vectors is zero → Form triangle when put head to tail

(Wolfenstein params to order λ^4)



$$V_{tb}^*V_{td} = 1 \times A\lambda^3(1-\rho-i\eta)$$

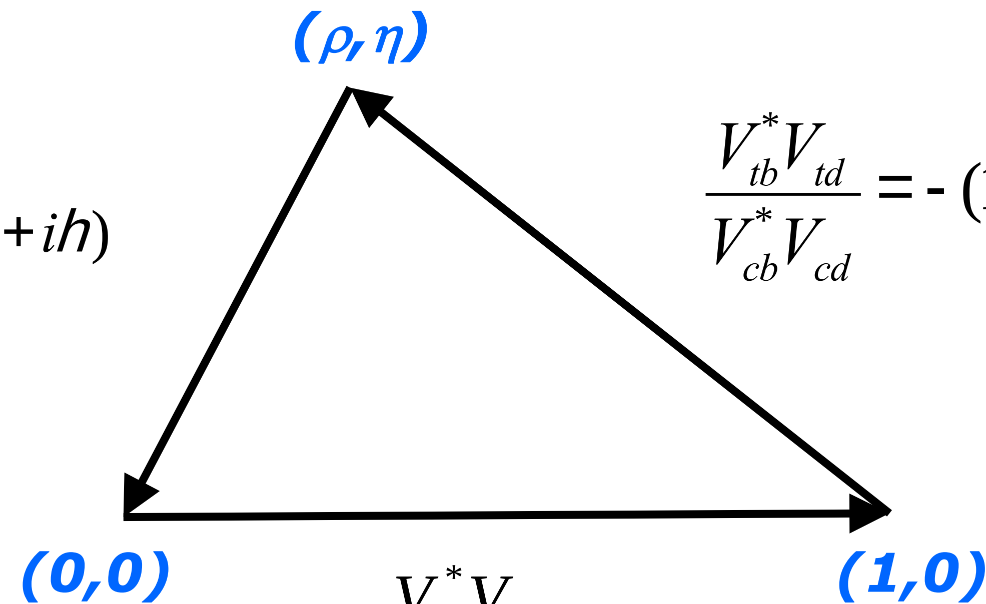
$$V_{ub}^*V_{ud} = A\lambda^3(\rho+i\eta)$$

$$V_{cb}^*V_{cd} = A\lambda^2\cdot(-\lambda)$$

Visualizing the unitarity constraint

- Divide all sides by length of base

$$\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = -(r + ih)$$



$$\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = -(1 - r - ih)$$

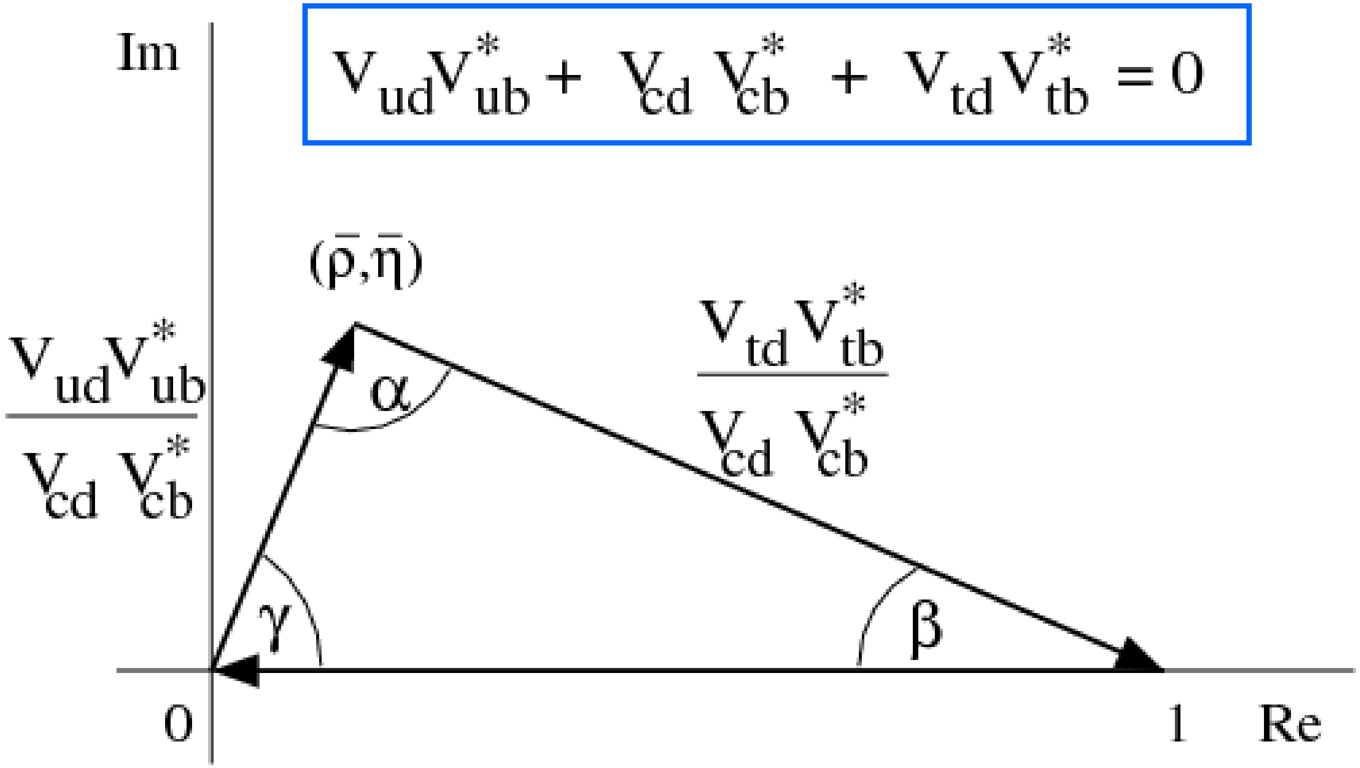
$$\frac{V_{cb}^* V_{cd}}{V_{cb}^* V_{cd}} \equiv 1$$

- Constructed a triangle with apex (ρ,η)

“The” Unitarity triangle

- We can visualize the CKM-constraints in (ρ,η) plane

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

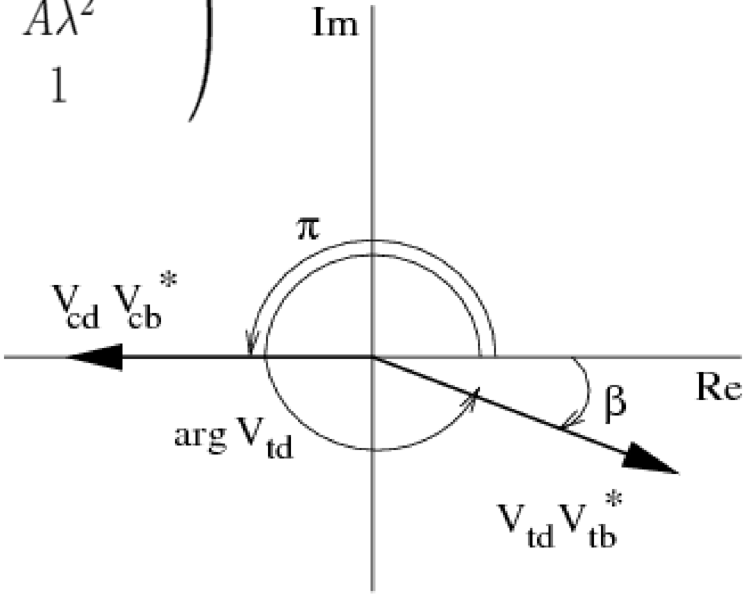


β

- We can correlate the angles β and γ to CKM elements:

$$\beta = \arg\left[-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right] = \pi + \arg\left[V_{cb}^* V_{cd}\right] - \arg\left[V_{tb}^* V_{td}\right] = 2\pi - \arg\left[V_{td}\right]$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



The phases in the Wolfenstein parameterization

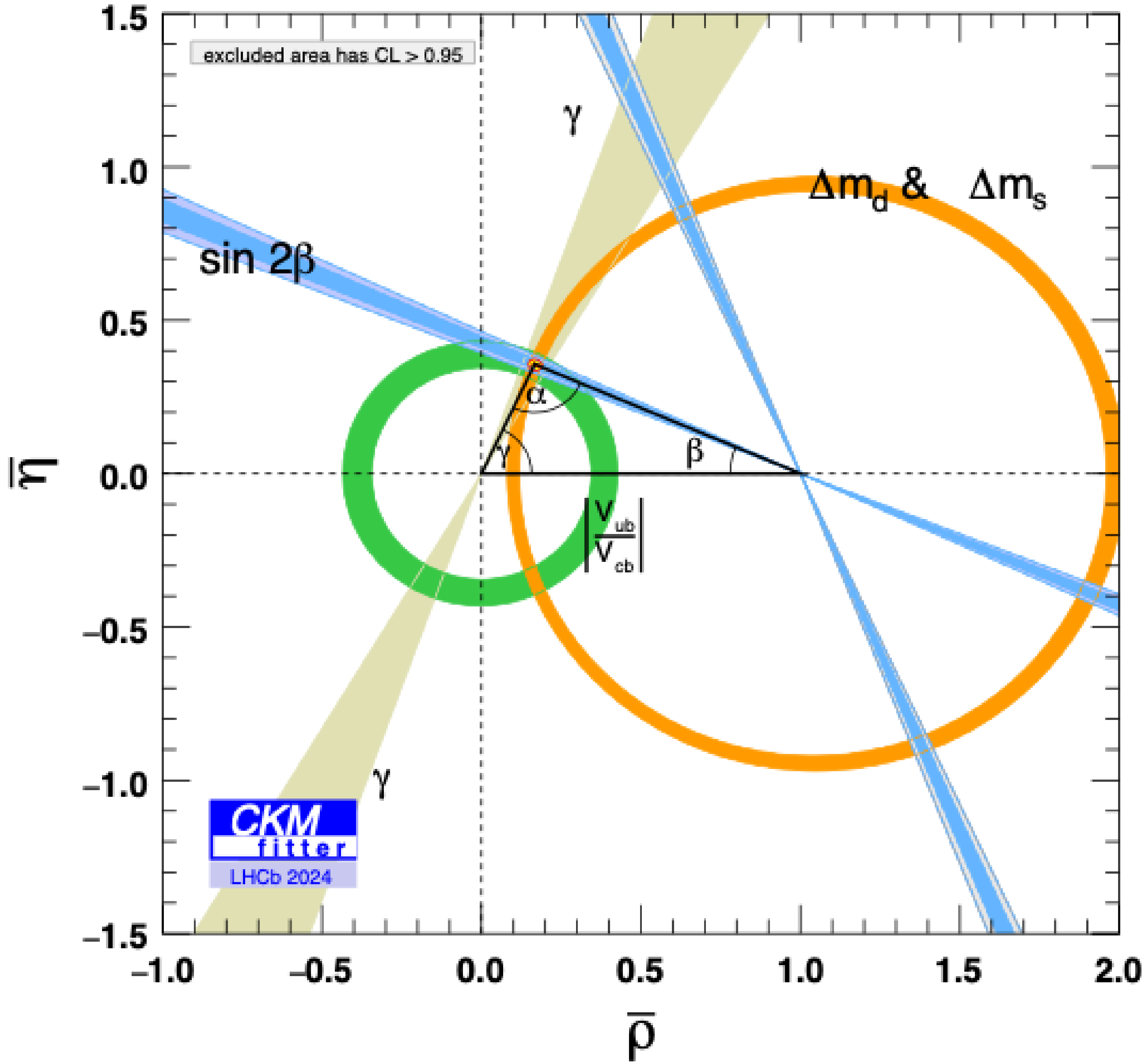
$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] \qquad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] \qquad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] \qquad \beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right]$$

$$\begin{aligned}
\beta \approx \quad & \pi + \arg(V_{cd}V_{cb}^*) - \arg(V_{td}V_{tb}^*) = \pi + \pi - \arg(V_{td}) & = -\arg(V_{td}) \\
\gamma \approx \quad & \pi + \arg(V_{ud}V_{ub}^*) - \arg(V_{cd}V_{cb}^*) = \pi - \arg(V_{ub}) - \pi & = -\arg(V_{ub}) \\
\beta_s \approx \quad & \pi + \arg(V_{ts}V_{tb}^*) - \arg(V_{cs}V_{cb}^*) = \pi + \arg(V_{ts}) - 0 & = \arg(V_{ts}) + \pi
\end{aligned}$$

Alternatively, the Wolfenstein phase convention in the CKM-matrix elements can be shown as:

$$V_{CKM,\text{Wolfenstein}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5) \qquad (2.16)$$

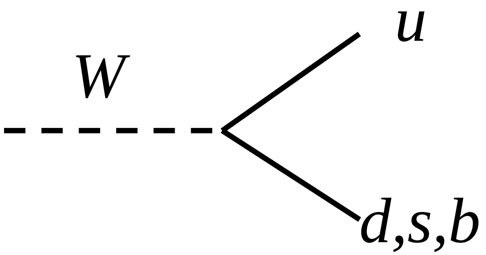
"The" Unitarity triangle



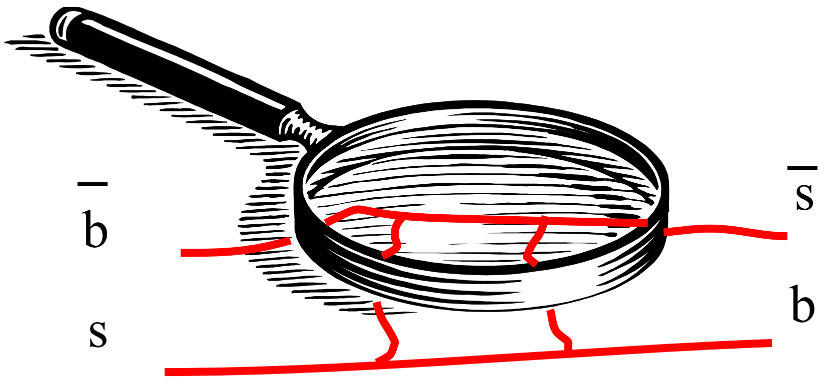
Quarks → Mesons

- Quarks:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



- Mesons
  - "Oscillations" important ingredient!



CP violation in the quark sector

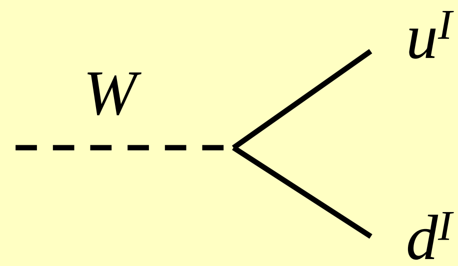
1)	Quarks:	SM, CKM, UT	Niels, 30'
2)	Mesons:	Mixing and CPV types	Niels, 30'
3)	B-mesons:	Decays and Experiment	Patrick, 60'

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

Recap

$$-L_{Yuk} = Y_{ij}^{d} (\overline{u_L^I}, \overline{d_L^I})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + \ldots$$

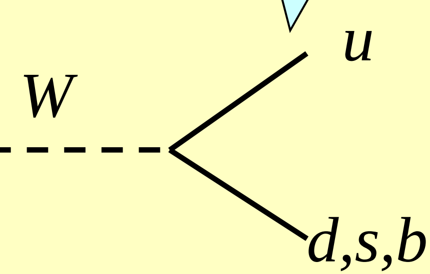
$$L_{Kinetic} = \frac{g}{\sqrt{2}} \overline{u_{Li}^I} \gamma^\mu W_\mu^- d_{Li}^I + \frac{g}{\sqrt{2}} \overline{d_{Li}^I} \gamma^\mu W_\mu^+ u_{Li}^I + \ldots$$



Diagonalize Yukawa matrix Y_{ij}

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$$-L_{Mass} = (\bar{d}, \bar{s}, \bar{b})_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + (\bar{u}, \bar{c}, \bar{t})_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \ldots$$

$$L_{CKM} = \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W_\mu^- V_{ij} \left(1-\gamma^5\right) d_j + \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W_\mu^+ V_{ij}^* \left(1-\gamma^5\right) u_i + \ldots$$

$$L_{SM} = L_{CKM} + L_{Higgs} + L_{Mass}$$

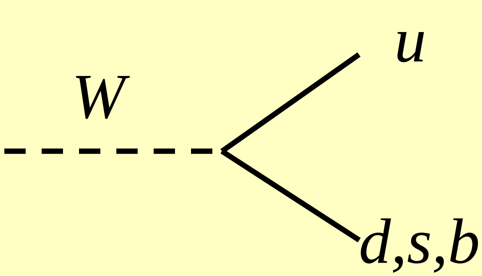
CKM-matrix: where are the phases?

- Possibility 1: simply 3 'rotations', and put phase on smallest:

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

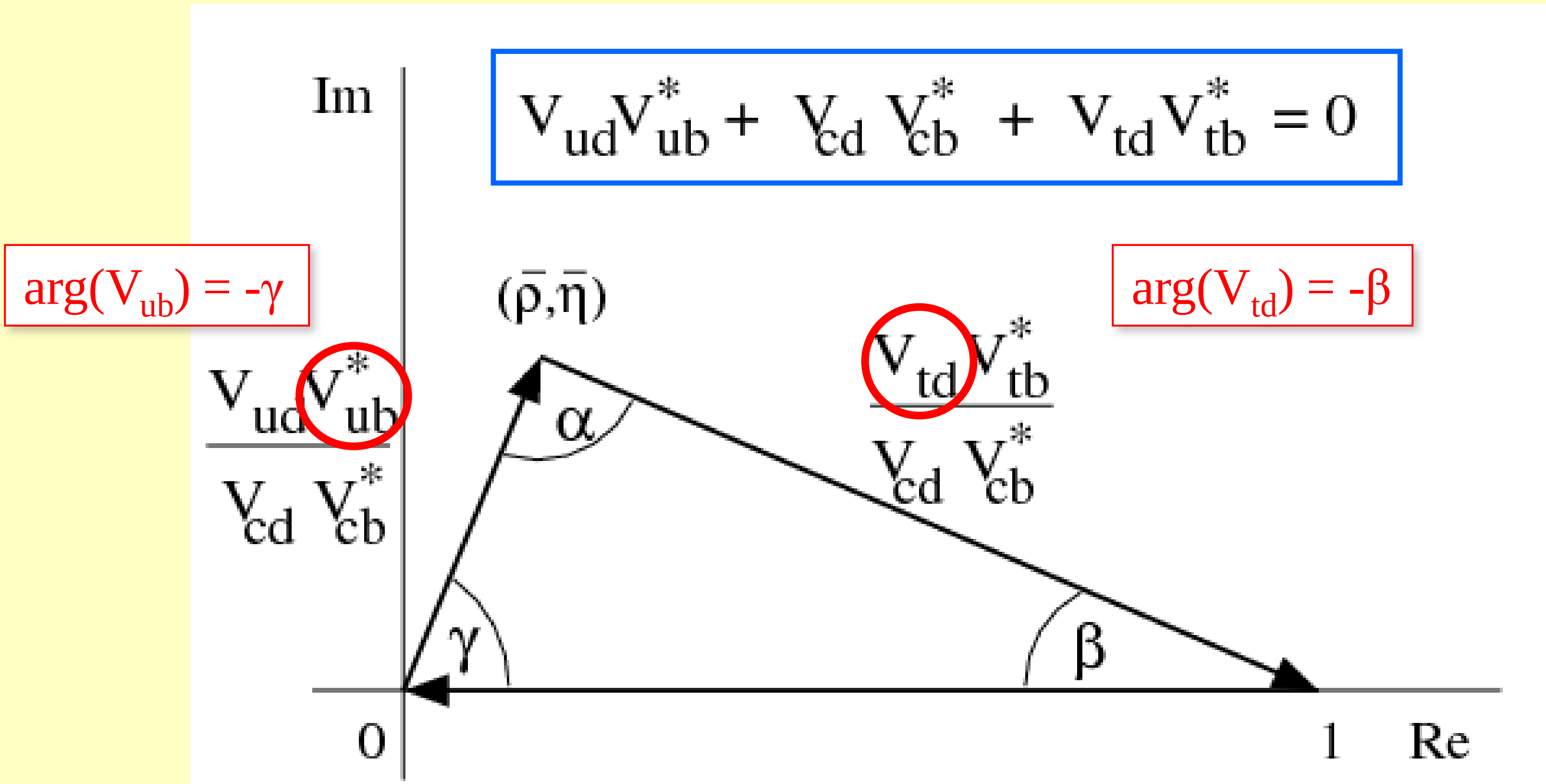
- Possibility 2: parameterize according to magnitude, in O(λ):



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

"The" Unitarity triangle

- We can visualize the CKM-constraints in (ρ,η) plane

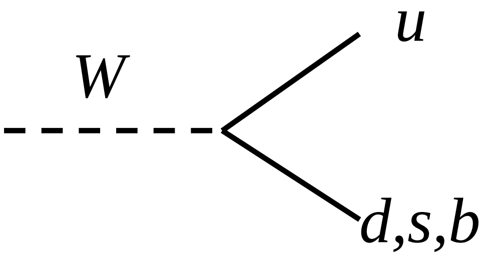


$$V_{CKM,\text{Wolfenstein}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

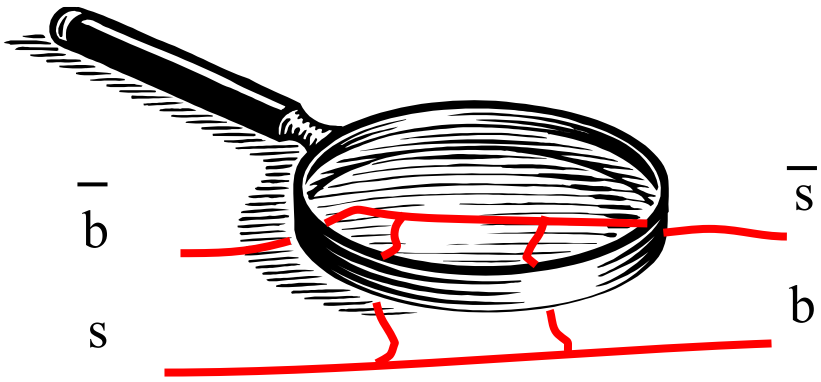
Quarks → Mesons

- Quarks:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



- Mesons
  - "Oscillations" important ingredient!



Dynamics of Neutral B (or K) mesons…

Time evolution of B^0 and $\overline{B}^0$ can be described by an *effective* Hamiltonian:

$$i\frac{\partial}{\partial t}\Psi = H\Psi \qquad \Psi(t) = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B}^0\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}}$$

No mixing, no decay…

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

No mixing, but with decays…
(i.e.: H is not Hermitian!)

➔With decays included, probability of observing *either* B^0 *or* B^0 must go down as time goes by:

$$\frac{d}{dt}\left(|a(t)|^2 + |b(t)|^2\right) = -\begin{pmatrix} a(t)^* & b(t)^* \end{pmatrix}\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \qquad \Rightarrow \Gamma > 0$$

Describing Mixing…

Time evolution of B^0 and $\overline{B}^0$ can be described by an *effective* Hamiltonian:

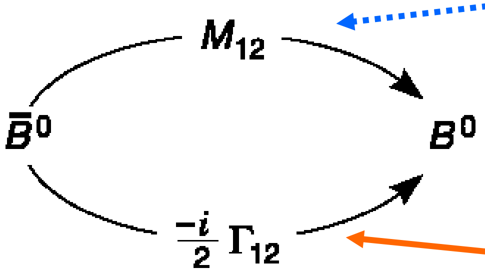
$$i\frac{\partial}{\partial t}\Psi = H\Psi \qquad \Psi(t) = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B}^0\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

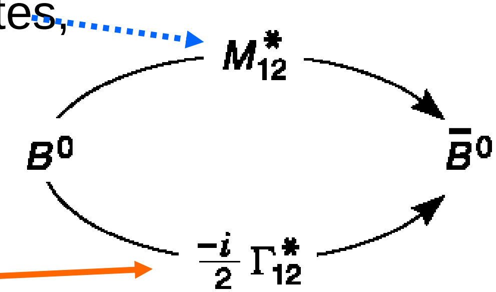
Where to put the mixing term?

$$H = \underbrace{\begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

Now with mixing – but what is the difference between M_{12} and Γ_{12}?



M_{12} describes $B^0 \leftrightarrow B^0$ via *off-shell* states, e.g. the weak box diagram



Γ_{12} describes $B^0 \leftrightarrow f \leftrightarrow B^0$ via *on-shell* states, eg. $f=\pi^+\pi^-$

Solving the Schrödinger Equation

$$i\frac{\partial}{\partial t}\psi(t)=\begin{pmatrix} M-\frac{i}{2}\Gamma & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{12}^*-\frac{i}{2}\Gamma_{12}^* & M-\frac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Eigenvalues:

– Mass and lifetime of physical states: mass eigenstates

$$\begin{vmatrix} M-\frac{i}{2}\Gamma-\lambda & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{12}^*-\frac{i}{2}\Gamma_{12}^* & M-\frac{i}{2}\Gamma-\lambda \end{vmatrix}=0$$

$$\text{notation } F=\sqrt{(M_{12}-\tfrac{i}{2}\Gamma_{12})(M_{12}^*-\tfrac{i}{2}\Gamma_{12}^*)}$$

$$\begin{aligned} m_1+\frac{i}{2}\Gamma_1 &= M-\Re F-\frac{i}{2}\Gamma-\Im F \\ m_2+\frac{i}{2}\Gamma_2 &= M+\Re F-\frac{i}{2}\Gamma+\Im F \end{aligned}$$

$$\Delta m = 2\Re\sqrt{\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)}$$

$$\Delta\Gamma = 4\Im\sqrt{\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)}$$

Solving the Schrödinger Equation

$$i\frac{\partial}{\partial t}\psi(t) = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Eigenvectors:

– mass eigenstates

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \end{aligned}$$

find p and q by solving

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \lambda_{\pm}\begin{pmatrix} p \\ q \end{pmatrix}$$

$$|B_H\rangle = p|B\rangle + q|\overline{B}\rangle$$

$$|B_L\rangle = p|B\rangle - q|\overline{B}\rangle$$

$$q/p = \sqrt{\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right) \Big/ \left(M_{12} - \frac{i}{2}\Gamma_{12}\right)}$$

Time evolution

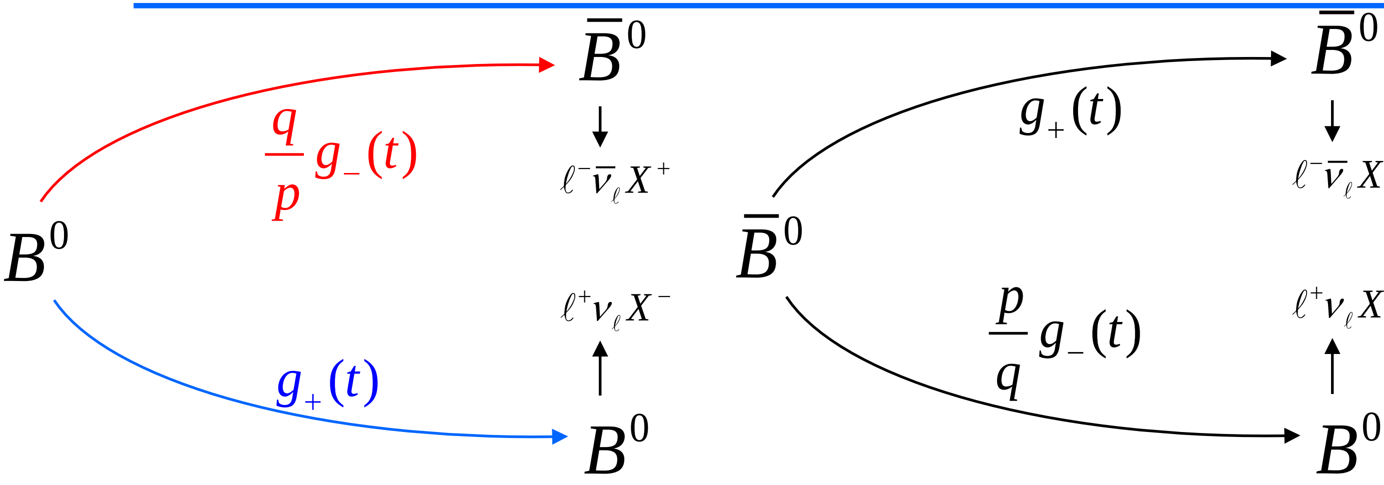
- With diagonal Hamiltonian, usual time evolution is obtained:

$$
\begin{aligned}
|P_H(t)\rangle &= e^{-im_H t - \frac{1}{2}\Gamma_H t}|P_H(0)\rangle \\
|P_L(t)\rangle &= e^{-im_L t - \frac{1}{2}\Gamma_L t}|P_L(0)\rangle
\end{aligned}
$$

$$
\begin{aligned}
|P^0\rangle &= \frac{1}{2p}\left[|P_H\rangle + |P_L\rangle\right] \\
|\bar{P}^0\rangle &= \frac{1}{2q}\left[|P_H\rangle - |P_L\rangle\right]
\end{aligned}
\qquad
\begin{aligned}
|P_H\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \\
|P_L\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle
\end{aligned}
$$

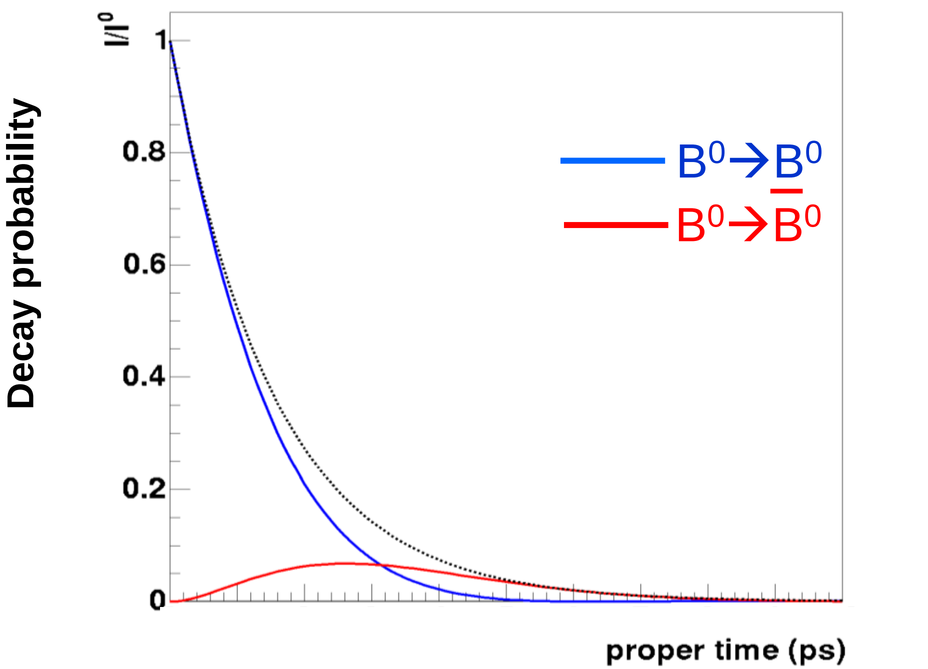
$$
\begin{aligned}
|P^0(t)\rangle &= \frac{1}{2p}\left\{e^{-im_H t - \frac{1}{2}\Gamma_H t}|P_H(0)\rangle + e^{-im_L t - \frac{1}{2}\Gamma_L t}|P_L(0)\rangle\right\} \\
&= \frac{1}{2p}\left\{e^{-im_H t - \frac{1}{2}\Gamma_H t}(p|P^0\rangle + q|\bar{P}^0\rangle) + e^{-im_L t - \frac{1}{2}\Gamma_L t}(p|P^0\rangle - q|\bar{P}^0\rangle)\right\} \\
&= \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|P^0\rangle + \frac{q}{2p}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle \\
&= g_+(t)|P^0\rangle + \left(\frac{q}{p}\right) g_-(t)|\bar{P}^0\rangle \qquad (3.6)
\end{aligned}
$$

Measuring B Oscillations



For B^0, expect:
$\Delta\Gamma \sim 0$,
$|q/p|=1$

$$\left|g_\pm(t)\right|^2 \sim \frac{e^{-\Gamma t}}{2}\left[1 \pm \cos(\Delta m \cdot t)\right]$$



$$x \equiv \frac{\Delta m}{\Gamma} \approx 1$$

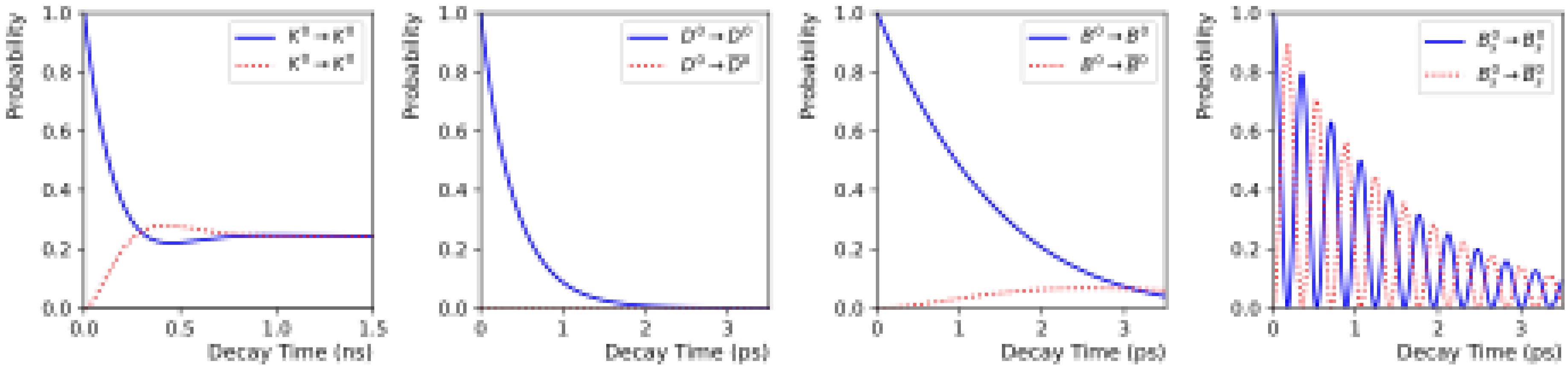
Proper Time →

Compare the mesons:

- Same concept, different phenomenology:

$P^0 \rightarrow P^0$
$P^0 \rightarrow \overline{P}^0$

K^0 D^0 B^0 B_s^0



	m	$\tau = 1/\Gamma$	$(c\tau)$	$\Gamma = (\Gamma_L + \Gamma_H)/2$	Δm (ps^{-1})	$\Delta\Gamma = \Gamma_L - \Gamma_H$	x	y
K^0-system	498 MeV	K_S: 0.09 ns (27mm), K_L: 51 ns (15m)		5.57 ns^{-1}	0.005			
D^0-system	1865 MeV	0.410 ps	(0.12 mm)	2.44 ps^{-1}	0.010	0.031 ps^{-1}	0.004	0.0064
B^0-system	5280 MeV	1.517 ps	(0.46 mm)	0.659 ps^{-1}	0.507	0 ps^{-1}	0.77	0.0005
B_s^0-system	5367 MeV	1.527 ps	(0.46 mm)	0.655 ps^{-1}	17.77	0.087 ps^{-1}	27.0	0.068

By the way, $\hbar$=6.58 10^{-22} MeVs

$x = \Delta m/\Gamma$: avg nr of oscillations before decay

Oscillations (1)

- Start with Schrodinger equation:

$$i\frac{\partial\psi}{\partial t} = H\psi = (M - \frac{i}{2}\Gamma)\psi = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\psi$$

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

(2-component state in P^0 and $\overline{P^0}$ subspace)

- Find eigenvalue:

$$\begin{vmatrix} M - \frac{i}{2}\Gamma - \lambda & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma - \lambda \end{vmatrix} = 0$$

- Solve eigenstates:

$$\psi_\pm = \begin{pmatrix} p \\ \pm q \end{pmatrix}$$

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \end{aligned}$$

we find p and q by solving

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \lambda_\pm \begin{pmatrix} p \\ q \end{pmatrix} \longrightarrow \frac{q}{p} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}$$

- Eigenstates have diagonal Hamiltonian: mass eigenstates!

Oscillations (2)

- Two mass eigenstates

$$|P_H\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$$
$$|P_L\rangle = p|P^0\rangle - q|\bar{P}^0\rangle$$

- Time evolution:

$$|P_H(t)\rangle = e^{-im_H t - \frac{1}{2}\Gamma_H t}|P_H(0)\rangle$$
$$|P_L(t)\rangle = e^{-im_L t - \frac{1}{2}\Gamma_L t}|P_L(0)\rangle$$

$$|P^0\rangle = \frac{1}{2p}\left[|P_H\rangle + |P_L\rangle\right]$$
$$|\bar{P}^0\rangle = \frac{1}{2q}\left[|P_H\rangle - |P_L\rangle\right]$$

$$\left|P^0(t)\right\rangle = \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|P^0\rangle + \frac{q}{2p}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle$$

- Probability for $|P^0\rangle \rightarrow |\overline{P^0}\rangle$!
- Express in $M = m_H + m_L$ and $\Delta m = m_H - m_L$ → Δm dependence

Oscillations: summary

- p, q: $|B_H\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$

  $|B_L\rangle = p|B^0\rangle - q|\overline{B}^0\rangle$

- Δm, ΔΓ: $\Delta m = 2\Re\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$

  $\Delta\Gamma = 4\Im\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$

q,p,M_{ij},Γ_{ij} related through:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}$$

- x,y: mixing often quoted in *scaled* parameters: $x = \frac{\Delta m}{\Gamma}$ $y = \frac{\Delta\Gamma}{2\Gamma}$

$$\cos(\Delta m t) = \cos\left(\frac{\Delta m}{\Gamma}\frac{t}{\tau}\right) = \cos\left(x\frac{t}{\tau}\right)$$

Time dependence (if ΔΓ~0, like for B^0):

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\overline{B^0}\rangle$$

$$|\overline{B^0}(t)\rangle = g_+(t)|\overline{B^0}\rangle + \frac{p}{q}g_-(t)|B^0\rangle$$

with

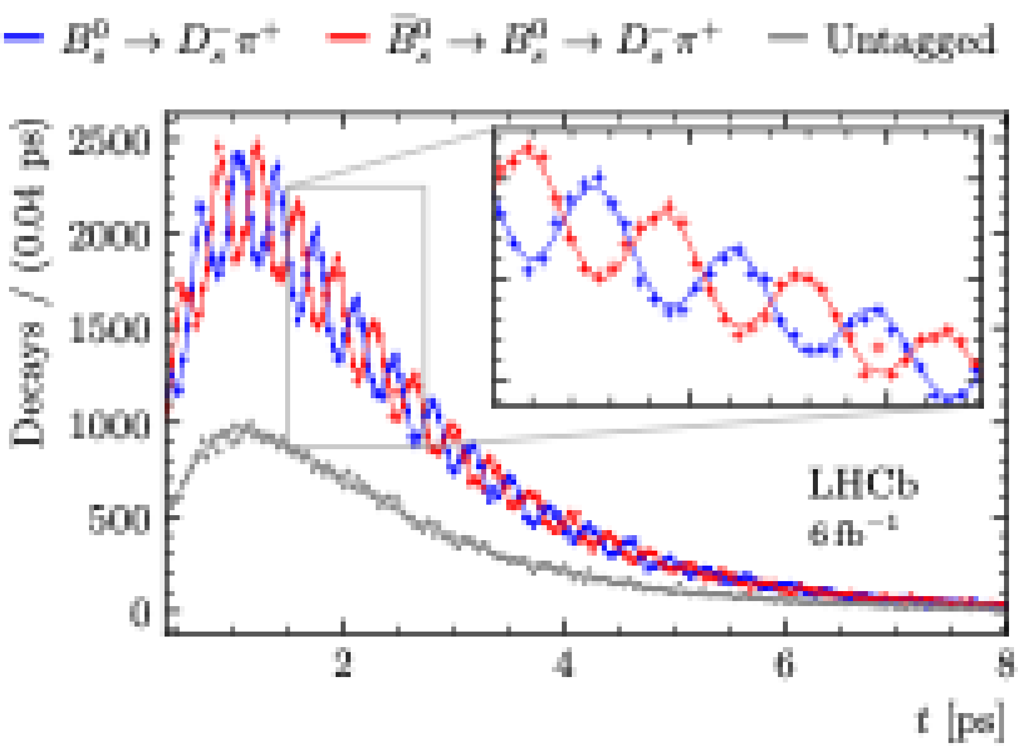
$$g_+(t) = e^{-imt}e^{-\Gamma t/2}\cos\frac{\Delta m t}{2}$$

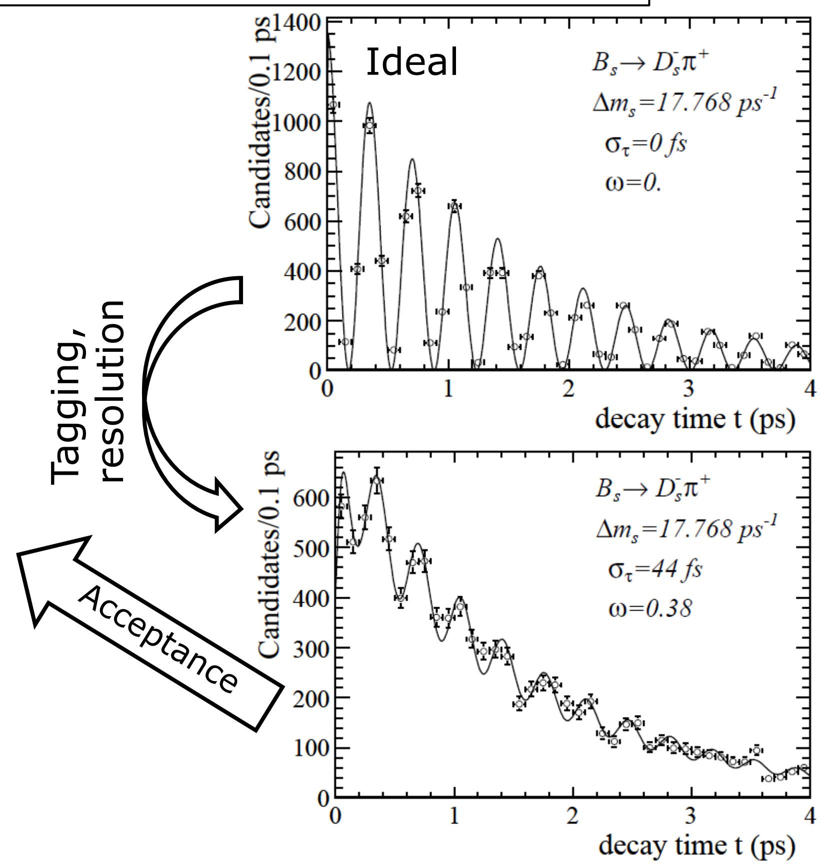
$$g_-(t) = e^{-imt}e^{-\Gamma t/2}\,i\sin\frac{\Delta m t}{2}$$

B_s^0 mixing (Δm_s)

$$\frac{N_{B^0 \to B^0}(t) - N_{B^0 \to \bar{B}^0}(t)}{N_{B^0 \to B^0}(t) + N_{B^0 \to \bar{B}^0}(t)} = \cos(\Delta m \cdot t)$$

$$17.7683 \pm 0.0051\,(\mathrm{stat}) \pm 0.0032\,(\mathrm{syst})\,\mathrm{ps}^{-1}$$





LHCb, Nature Phys. 18 (2022) 1
2104.04421 [hep-ex]

Mixing → CP violation?

- NB: Just mixing is not necessarily CP violation!
- However, by studying certain *decays with and without mixing*, CP violation is observed

- Next: Measuring CP violation…

Meson Decays

- Formalism of meson *oscillations*:

$$\left|P^0(t)\right\rangle = \frac{1}{2}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}+e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|P^0\rangle+\frac{q}{2p}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}-e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle$$

$$|\langle\bar{P}^0(t)|P^0\rangle|^2=|g_-(t)|^2\left(\frac{p}{q}\right)^2$$

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t\pm\cos\Delta m t\right)$$

- Subsequent: *decay*

$$P^0 \longrightarrow f$$

$$P^0 \longrightarrow f$$

Notation: Define A_f and λ_f

$$\begin{aligned} A(f) &= \langle f|T|P^0\rangle & \bar{A}(f) &= \langle f|T|\bar{P}^0\rangle \\ A(\bar{f}) &= \langle \bar{f}|T|P^0\rangle & \bar{A}(\bar{f}) &= \langle \bar{f}|T|\bar{P}^0\rangle \end{aligned}$$

and define the complex parameter λ_f (not be confused with the Wolfenstein parameter λ !):

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}, \qquad \bar{\lambda}_f = \frac{1}{\lambda_f}, \qquad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}, \qquad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}} \tag{3.14}$$

The general expression for the time dependent decay rates, $\Gamma_{P^0\rightarrow f}(t) = |\langle f|T|P^0(t)\rangle|^2$,

Some algebra for the decay $P^0 \to f$

$$\left|P^0(t)\right\rangle = \frac{1}{2}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}+e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|P^0\rangle + \frac{q}{2p}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}-e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle$$

$$\Gamma_{P^0\to f}(t) = |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2|g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)]\right)$$

$$A(f) = \langle f|T|P^0\rangle$$

$$\bar{A}(f) = \langle f|T|\bar{P}^0\rangle$$

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

Interference

— $P^0 \to f$

— $P^0 \to \bar{P}^0 \to f$

Some algebra for the decay $P^0 \rightarrow f$

$$
\begin{aligned}
\Gamma_{P^0\to f}(t) &= |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2|g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t)g_-(t)]\right) \\
\Gamma_{P^0\to \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left|\frac{q}{p}\right|^2 \left(|g_-(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2|g_+(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+(t)g_-^*(t)]\right) \\
\Gamma_{\bar{P}^0\to f}(t) &= |A_f|^2 \left|\frac{p}{q}\right|^2 \left(|g_-(t)|^2 + |\lambda_f|^2|g_+(t)|^2 + 2\Re[\lambda_f g_+(t)g_-^*(t)]\right) \\
\Gamma_{\bar{P}^0\to \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left(|g_+(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2|g_-(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+^*(t)g_-(t)]\right) \qquad (3.15)
\end{aligned}
$$

$$
\begin{aligned}
|g_\pm(t)|^2 &= \frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t \pm \cos\Delta m t\right) \\
g_+^*(t)g_-(t) &= \frac{e^{-\Gamma t}}{2}\left(\sinh\frac{1}{2}\Delta\Gamma t + i\sin\Delta m t\right) \\
g_+(t)g_-^*(t) &= \frac{e^{-\Gamma t}}{2}\left(\sinh\frac{1}{2}\Delta\Gamma t - i\sin\Delta m t\right) \qquad (3.16)
\end{aligned}
$$

The 'master equations'

('direct') Decay — *Interference*

$$
\begin{aligned}
\Gamma_{P^0\to f}(t) &= |A_f|^2 \frac{e^{-\Gamma t}}{2} \\
&\left((1+|\lambda_f|^2)\cosh\frac{1}{2}\Delta\Gamma t + 2\Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t + (1-|\lambda_f|^2)\cos\Delta m t - 2\Im\lambda_f \sin\Delta m t \right) \\
\Gamma_{\bar{P}^0\to f}(t) &= |A_f|^2 \left|\frac{p}{q}\right|^2 \frac{e^{-\Gamma t}}{2} \qquad (3.17)\\
&\left((1+|\lambda_f|^2)\cosh\frac{1}{2}\Delta\Gamma t + 2\Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t - (1-|\lambda_f|^2)\cos\Delta m t + 2\Im\lambda_f \sin\Delta m t \right)
\end{aligned}
$$

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

The ‘master equations’

(‘direct’) Decay ***Interference***

$$
\begin{aligned}
\Gamma_{P^0\to f}(t) &= |A_f|^2 \frac{e^{-\Gamma t}}{2} \left((1+|\lambda_f|^2)\cosh\frac{1}{2}\Delta\Gamma t + 2\Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t + (1-|\lambda_f|^2)\cos\Delta m t - 2\Im\lambda_f \sin\Delta m t \right) \\
\Gamma_{\bar{P}^0\to f}(t) &= |A_f|^2 \left|\frac{p}{q}\right|^2 \frac{e^{-\Gamma t}}{2} \left((1+|\lambda_f|^2)\cosh\frac{1}{2}\Delta\Gamma t + 2\Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t - (1-|\lambda_f|^2)\cos\Delta m t + 2\Im\lambda_f \sin\Delta m t \right)
\end{aligned} \quad (3.17)
$$

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

$$
\begin{aligned}
\Gamma_{P^0\to f}(t) &= |A_f|^2 (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2} \left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t + C_f \cos\Delta m t - S_f \sin\Delta m t \right) \\
\Gamma_{\bar{P}^0\to f}(t) &= |A_f|^2 \left|\frac{p}{q}\right|^2 (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2} \left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t - C_f \cos\Delta m t + S_f \sin\Delta m t \right)
\end{aligned} \quad (3.18)
$$

with

$$
D_f = \frac{2\Re\lambda_f}{1+|\lambda_f|^2} \qquad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \qquad S_f = \frac{2\Im\lambda_f}{1+|\lambda_f|^2}. \quad (3.19)
$$

Classification of CP Violating effects

1. CP violation in decay

$$\Gamma(P^0 \to f) \neq \Gamma(\bar{P}^0 \to \bar{f})$$

This is obviously satisfied (see Eq. (3.15)) when

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1.$$

2. CP violation in mixing

$$\text{Prob}(P^0 \to \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \to P^0)$$

$$\left|\frac{q}{p}\right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0{}_{(\leadsto \bar{P}^0)} \to f)(t) \neq \Gamma(\bar{P}^0{}_{(\leadsto P^0)} \to f)(t)$$

$$\Im \lambda_f = \Im\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

Meson Decays

- Formalism of meson *oscillations*:

$$\left|P^0(t)\right\rangle = \frac{1}{2}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}+e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|P^0\rangle+\frac{q}{2p}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}-e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle$$

- Subsequent: *decay*

$$\Gamma_{P^0\to f}(t) = |A_f|^2 \left(|g_+(t)|^2+|\lambda_f|^2|g_-(t)|^2+2\Re[\lambda_f g_+^*(t)g_-(t)]\right)$$

$P^0 \to f$

$A(f) = \langle f|T|P^0\rangle$

$P^0 \to \bar{P}^0 \to f$

$\bar{A}(f) = \langle f|T|\bar{P}^0\rangle$

$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$

Interference

$$\Gamma_{P^0\to f}(t) = |A_f|^2\frac{e^{-\Gamma t}}{2}\left(\left(1+|\lambda_f|^2\right)\cosh\frac{1}{2}\Delta\Gamma t+2\Re\lambda_f\sinh\frac{1}{2}\Delta\Gamma t+\left(1-|\lambda_f|^2\right)\cos\Delta m t-2\Im\lambda_f\sin\Delta m t\right)$$

('direct') Decay ***Interference***

CP violation: type 3

$$\Im\lambda_f = \Im\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

$$\boxed{\Gamma(P^0{}_{(\rightsquigarrow \bar{P}^0)} \to f)(t) \neq \Gamma(\bar{P}^0{}_{(\rightsquigarrow P^0)} \to f)(t)}$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t)\to f} - \Gamma_{\bar{P}^0(t)\to f}}{\Gamma_{P^0(t)\to f} + \Gamma_{\bar{P}^0(t)\to f}}$$

$$\begin{aligned}
\Gamma_{P^0\to f}(t) &= |A_f|^2 \qquad (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t + C_f \cos\Delta m t - S_f \sin\Delta m t\right) \\
\Gamma_{\bar{P}^0\to f}(t) &= |A_f|^2 \left|\frac{p}{q}\right|^2 (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t - C_f \cos\Delta m t + S_f \sin\Delta m t\right)
\end{aligned}$$

$$D_f = \frac{2\Re\lambda_f}{1+|\lambda_f|^2} \qquad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \qquad S_f = \frac{2\Im\lambda_f}{1+|\lambda_f|^2}.$$

$$\boxed{A_{CP}(t) = \frac{\Gamma_{P^0(t)\to f} - \Gamma_{\bar{P}^0(t)\to f}}{\Gamma_{P^0(t)\to f} + \Gamma_{\bar{P}^0(t)\to f}} = \frac{2C_f \cos\Delta m t - 2S_f \sin\Delta m t}{2\cosh\frac{1}{2}\Delta\Gamma t + 2D_f \sinh\frac{1}{2}\Delta\Gamma t}}$$

Classification of CP Violating effects - Nr. 3:

Consider $f=\bar{f}$:

$$A_{CP}(t) = \frac{\Gamma_{P^0(t)\to f} - \Gamma_{\bar{P}^0(t)\to f}}{\Gamma_{P^0(t)\to f} + \Gamma_{\bar{P}^0(t)\to f}} = \frac{2C_f \cos\Delta m t - 2S_f \sin\Delta m t}{2\cosh\frac{1}{2}\Delta\Gamma t + 2D_f \sinh\frac{1}{2}\Delta\Gamma t}$$

If one amplitude dominates the decay, then $A_f = \bar{A}_f$

$$A_{CP}(t) = \frac{-\Im\lambda_f \sin\Delta m t}{\cosh\frac{1}{2}\Delta\Gamma t + \Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t}$$

3. CP violation in interference

$$\Gamma(P^0{}_{(\leadsto\bar{P}^0)} \to f)(t) \neq \Gamma(\bar{P}^0{}_{(\leadsto P^0)} \to f)(t)$$

$$\Im\lambda_f = \Im\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

Relax: $B^0 \rightarrow J/\Psi K_s$ simplifies…

$$D_f = \frac{2\Re\lambda_f}{1+|\lambda_f|^2} \qquad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \qquad S_f = \frac{2\Im\lambda_f}{1+|\lambda_f|^2}.$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t)\to f} - \Gamma_{\bar{P}^0(t)\to f}}{\Gamma_{P^0(t)\to f} + \Gamma_{\bar{P}^0(t)\to f}} = \frac{2C_f\cos\Delta m t - 2S_f \sin\Delta m t}{2\cosh\frac{1}{2}\Delta\Gamma t + 2D_f\sinh\frac{1}{2}\Delta\Gamma t}$$

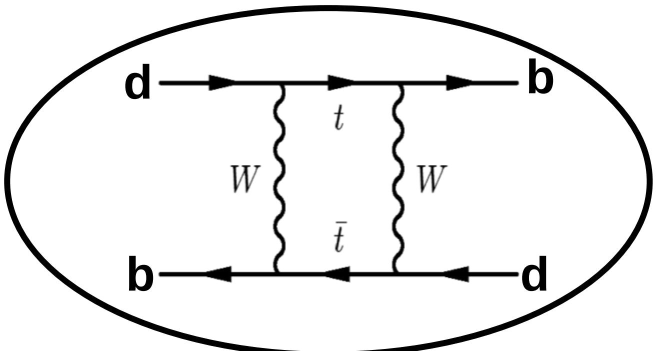
$|\lambda_f|=1$ $\qquad$ $\Delta\Gamma=0$

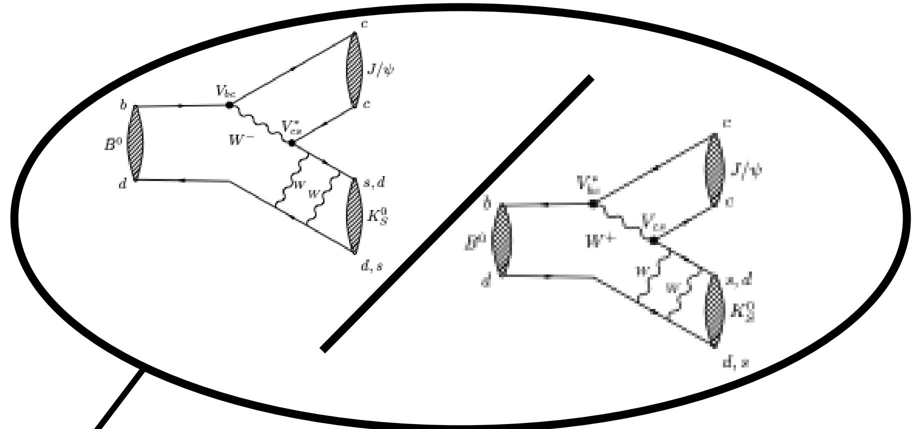
$$A_{CP}(t) = -\Im\lambda_f \sin(\Delta m t)$$

λ_f for $B^0 \to J/\psi K^0_S$

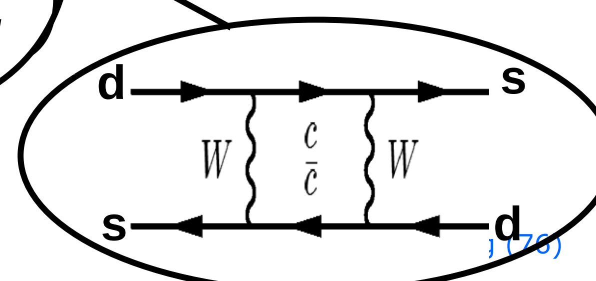
$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

$$\lambda_{J/\psi K_s} = \left(\frac{q}{p}\right)_{B^0} \frac{\bar{A}_{J/\psi K_s}}{A_{J/\psi K_s}} = \left(\frac{q}{p}\right)_{B^0} \frac{\bar{A}_{J/\psi K^0}}{A_{J/\psi K^0}} \left(\frac{p}{q}\right)_K$$





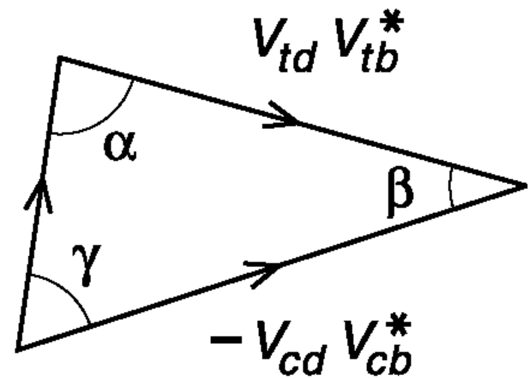
$$\lambda_{J/\psi K_s} = -\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)\left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right)\left(\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}\right)$$



λ_f for $B^0 \to J/\psi K^0_S$

$$\lambda_{J/\psi K_s} = -\left(\frac{V_{tb}^{*}V_{td}}{V_{tb}V_{td}^{*}}\right)\left(\frac{V_{cb}V_{cs}^{*}}{V_{cb}^{*}V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^{*}}{V_{cs}^{*}V_{cd}}\right)$$

$$= -e^{-2i\beta}$$



Time-dependent *CP* asymmetry

$$A_{CP}(t) = -\sin 2\beta \; \sin(\Delta m t)$$

- Theoretically clean way to measure β
- Clean experimental signature
- Branching fraction: O(10^{-4})
    - "Large" compared to other CP modes!

CP eigenvalue of final state $J/\psi K^0_S$

- CP $|J/\psi\rangle = +1\,|J/\psi\rangle$
- CP $|K^0_S\rangle = +1\,|K^0_S\rangle$
- CP $|J/\psi K^0_S\rangle = (-1)^l\,|J/\psi K^0_S\rangle$

($S(B)=0 \rightarrow L(J/\psi K^0_S)=1$!)

$$\lambda_{J/\psi K_s} = -\left(\frac{V_{tb}^{*}V_{td}}{V_{tb}V_{td}^{*}}\right)\left(\frac{V_{cb}V_{cs}^{*}}{V_{cb}^{*}V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^{*}}{V_{cs}^{*}V_{cd}}\right)$$

$$= -e^{-2i\beta}$$

Relative minus-sign between state and CP-conjugated state:

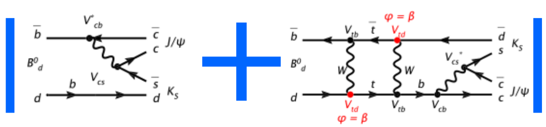
$$A_{CP}(t) = \frac{\Gamma_{\bar{B}\to f}(t) - \Gamma_{B\to f}(t)}{\Gamma_{\bar{B}\to f}(t) + \Gamma_{B\to f}(t)} = \mathrm{Im}(\lambda_f)\sin\Delta m t$$

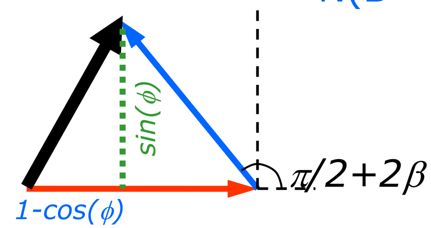
$$A_{CP}(t) = -\sin 2\beta \ \sin(\Delta m t)$$

Sum of 2 amplitudes: sensitivity to phase

- Now also look at CP-conjugate process
- Investigate situation at time t, such that $|A_1| = |A_2|$:

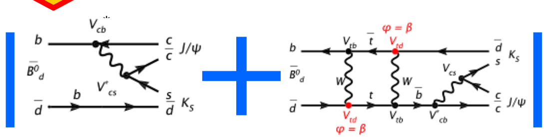
$\Gamma(B \rightarrow f) =$

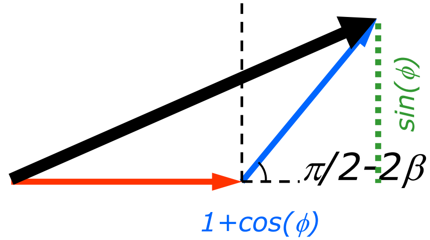




CP

$\Gamma(\overline{B} \rightarrow f) =$





$N(B^0 \rightarrow f) \propto |A|^2 \propto (1-\cos\phi)^2 + \sin^2\phi$
$= 1 - 2\cos\phi + \cos^2\phi + \sin^2\phi$
$= 2 - 2\cos(\pi/2 - 2\beta)$
$\propto 1 - \sin(2\beta)$

$$A_{CP} = \frac{N_{\overline{B^0}\rightarrow f} - N_{B^0 \rightarrow f}}{N_{B^0 \rightarrow f} + N_{\overline{B^0}\rightarrow f}} = \sin(2\beta)$$

$N(\overline{B^0} \rightarrow f) \propto (1+\cos\phi)^2 + \sin^2\phi$
$= 2 + 2\cos(\pi/2 - 2\beta)$
$\propto 1 + \sin(2\beta)$

- Directly observable result (essentially just from counting) measure CKM phase β directly!

$$A_{CP}(t = \pi / 2\Delta m) = \frac{N_{\overline{B^0}\rightarrow f} - N_{B^0 \rightarrow f}}{N_{B^0 \rightarrow f} + N_{\overline{B^0}\rightarrow f}} = \sin(2\beta)$$

Remember!

Necessary ingredients for CP violation:

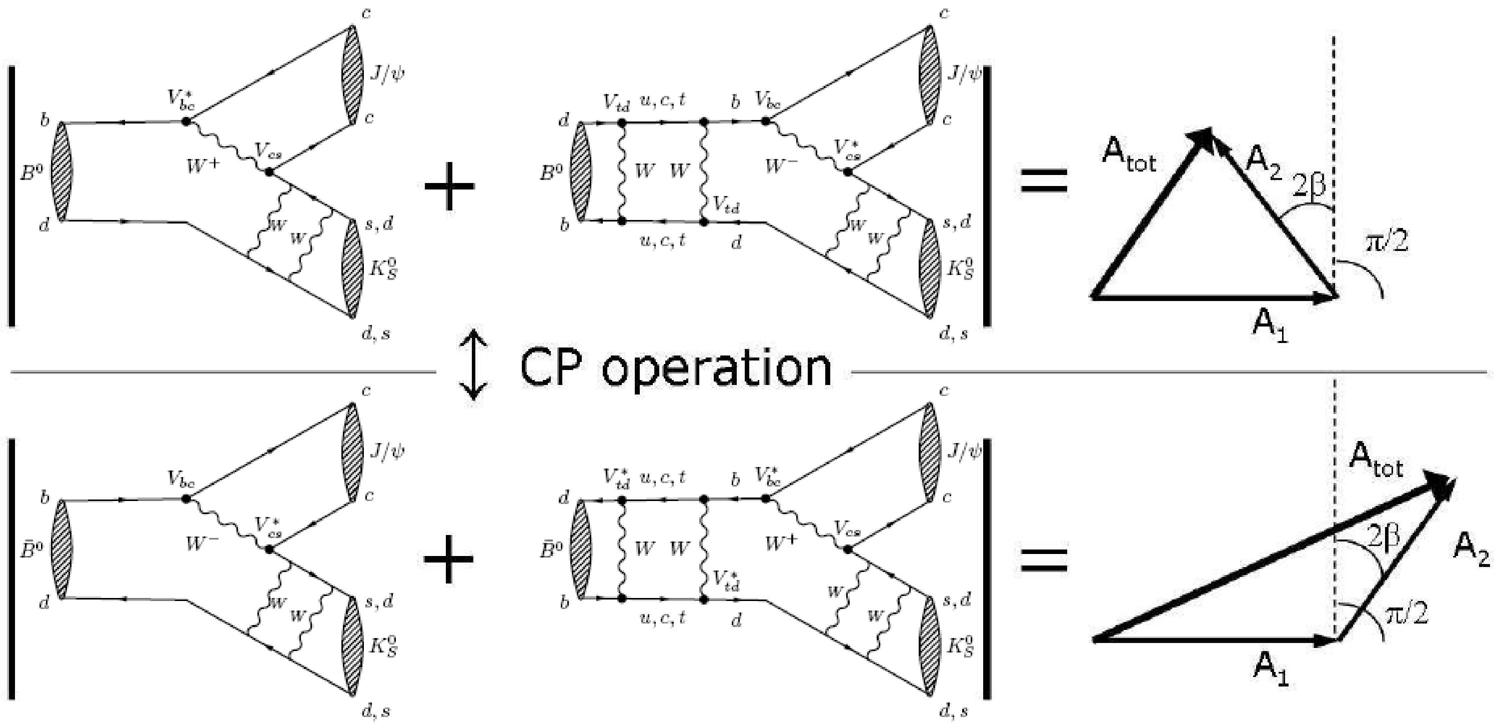
1) Two (interfering) amplitudes

2) Phase difference between amplitudes

- one CP conserving phase ('strong' phase)
- one CP violating phase ('weak' phase)

2 amplitudes
2 phases

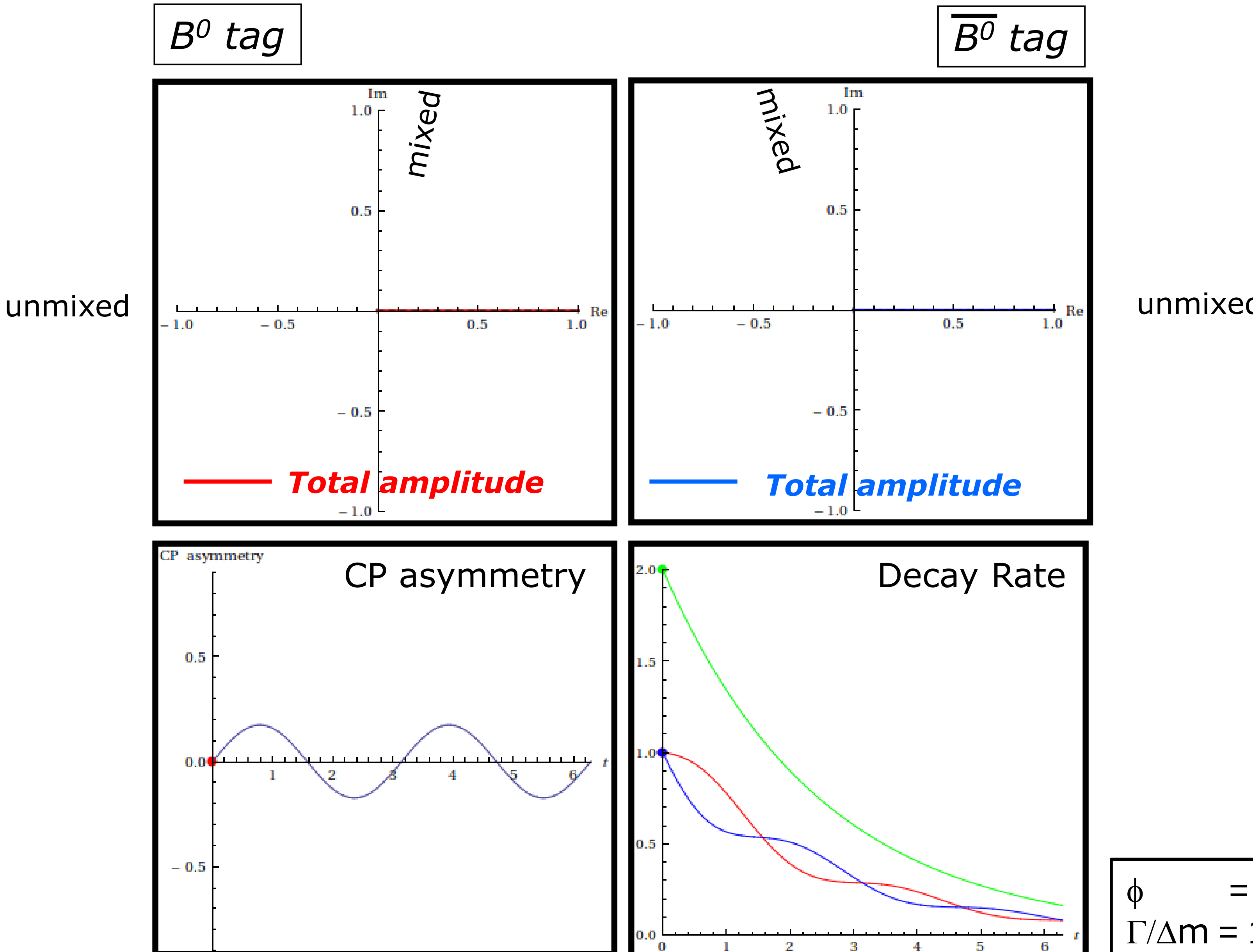
Remember!



2 amplitudes
2 phases

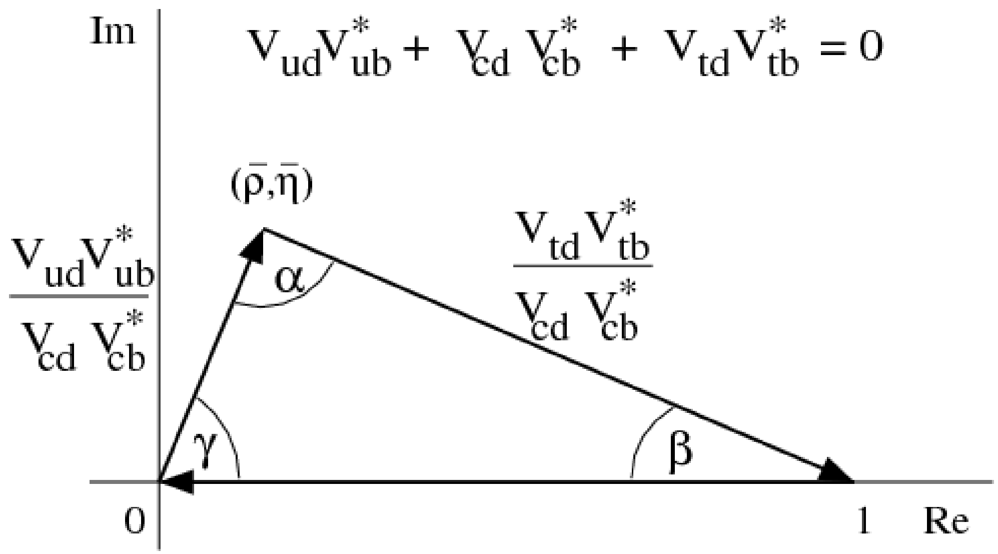
Courtesy Daan van Eijk

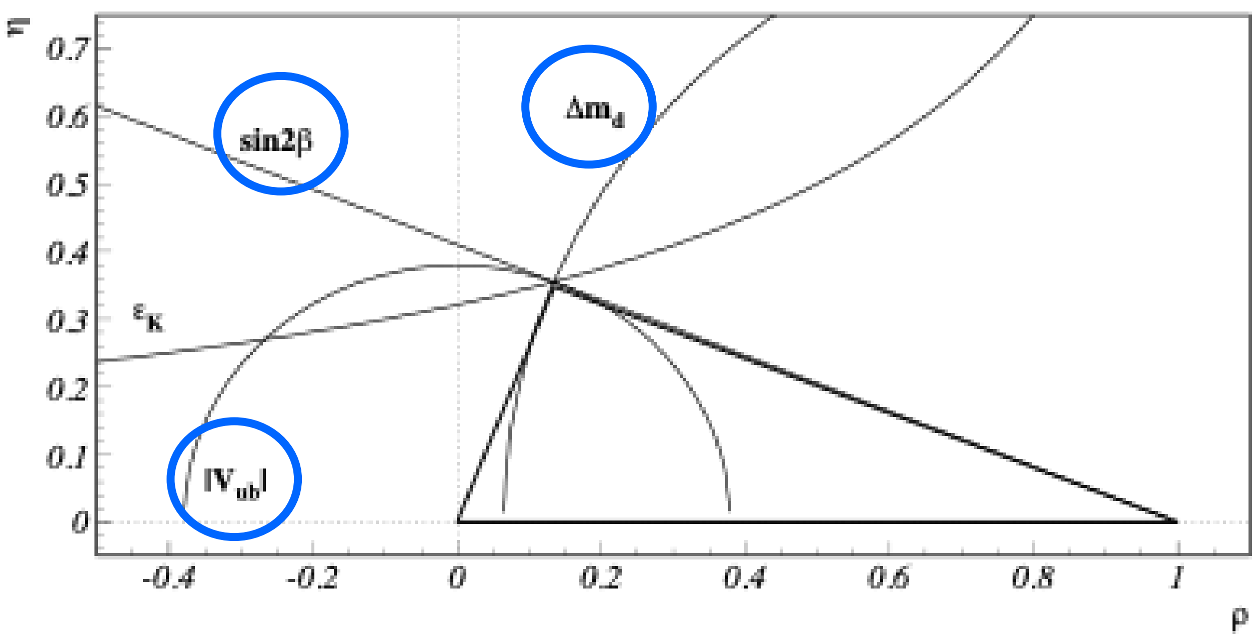
Time dependent CP violation



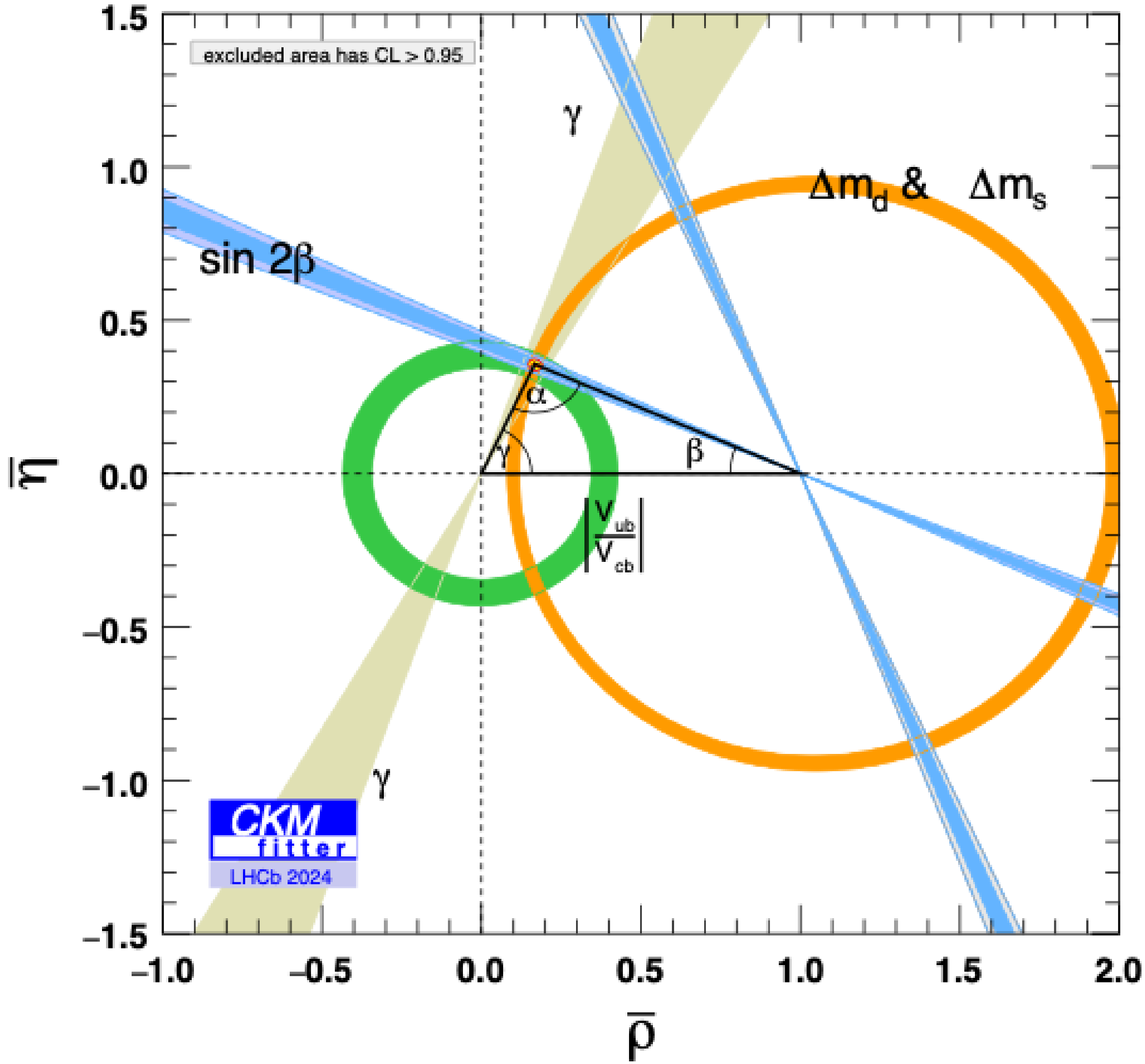
ϕ = 10 deg
$\Gamma/\Delta m$ = 1.3

"The" Unitarity triangle: consistency?!





"The" Unitarity triangle



Remember

Necessary ingredients for CP violation:

1) Two (interfering) amplitudes

2) Phase difference between amplitudes

   – one CP conserving phase ('strong' phase)
   – one CP violating phase ('weak' phase)

2 amplitudes
2 phases

Is there time?

Classification of CP Violating effects: where is discovery of CP violation with kaons from 1964?

1. CP violation in decay

$$\Gamma(P^0 \to f) \neq \Gamma(\bar{P}^0 \to \bar{f})$$

This is obviously satisfied (see Eq. (3.15)) when

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1.$$

2. CP violation in mixing

$$\text{Prob}(P^0 \to \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \to P^0)$$

$$\left|\frac{q}{p}\right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0{}_{(\leadsto \bar{P}^0)} \to f)(t) \neq \Gamma(\bar{P}^0{}_{(\leadsto P^0)} \to f)(t)$$

$$\Im \lambda_f = \Im\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

CP eigenvalue

- Remember:
  - $P^2 = 1$ $(x \rightarrow -x \rightarrow x)$
  - $C^2 = 1$ $(\psi \rightarrow \overline{\psi} \rightarrow \psi)$
  - $\Rightarrow CP^2 = 1$
- $CP \,|\, f> \; = \pm \,|\, f>$
- Knowing this we can evaluate the effect of CP on the K^0

- Mass/Lifetime eigenstates: *almost* CP eigenstates!

  $|K_1> \; = p|\, K^0> + q|\overline{K^0}>$

  $|K_2> \; = p|\, K^0> - q|\overline{K^0}>$

(S(K)=0 → L(ππ)=0)

$|K_1> (CP=+1) \longrightarrow \pi\,\pi \quad (CP = (-1)(-1)(-1)^{l=0} \qquad = +1)$

$|K_2> (CP=-1) \longrightarrow \pi\,\pi\,\pi \quad (CP = (-1)(-1)(-1)(-1)^{l=0} = -1)$

CP eigenvalue

- Remember:
  - $P^2 = 1$ ($x \rightarrow -x \rightarrow x$)
  - $C^2 = 1$ ($\psi \rightarrow \overline{\psi} \rightarrow \psi$)
  - $\rightarrow CP^2 = 1$
- $CP \,|\, f > = \pm \,|\, f >$
- Knowing this we can evaluate the effect of CP on the K^0

- Mass/Lifetime eigenstates: *almost* CP eigenstates!

  $|K_S> = p|K^0> + q|\overline{K^0}>$

  $|K_L> = p|K^0> - q|\overline{K^0}>$

(S(K)=0 → L(ππ)=0)

$|K_s>$ *(CP=+1)* $\rightarrow \pi\,\pi$ *(CP= (-1)(-1)(-1)*$^{l=0}$ *=+1)*

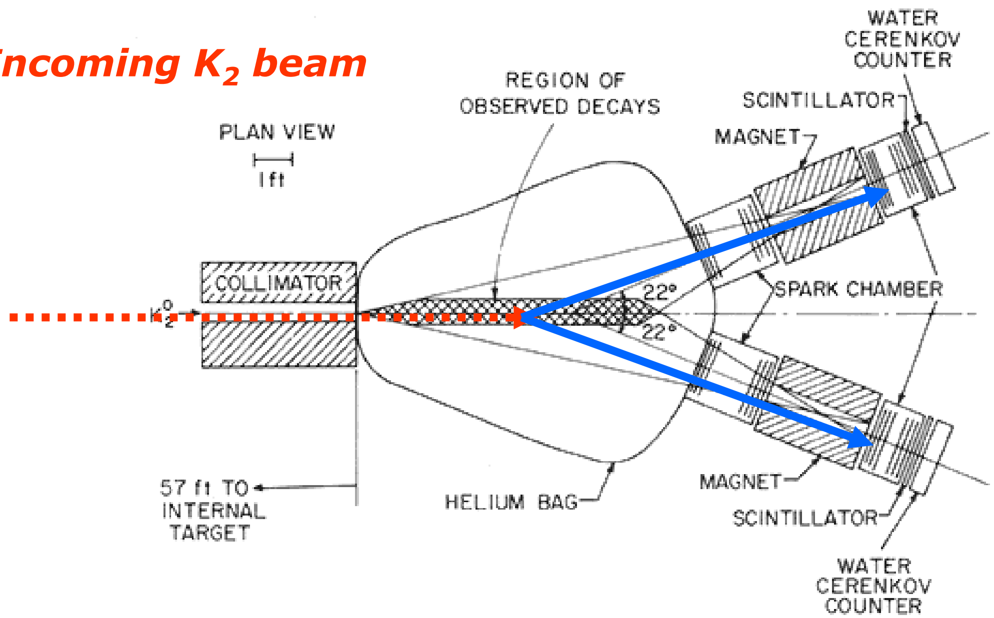
$|K_L>$ *(CP=-1)* $\rightarrow \pi\,\pi\,\pi$ *(CP = (-1)(-1)(-1)(-1)*$^{l=0}$ *= -1)*

The Cronin & Fitch experiment

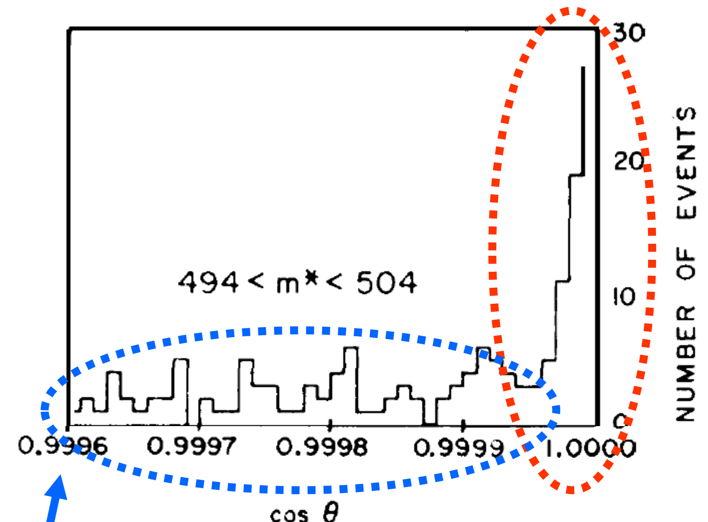
Essential idea: Look for $K_2 \rightarrow \pi\pi$ decays
20 meters away from K^0 production point

Decay pions

Incoming K_2 beam



$K_2 \rightarrow \pi\pi$ decays (CP Violation!)



$K_2 \rightarrow \pi\pi\pi$ decays

Result: an excess of events at Θ=0 degrees!

- CP violation, because K_2 (CP=-1) changed into K_1 (CP=+1)

Note scale: 99.99% of $K \rightarrow \pi\pi\pi$ decays are left of plot boundary

Kaons…

- Different notation: confusing!

$K_1, K_2, K_L, K_S, K_+, K_-, K^0$

$$|K_L\rangle = |K_2\rangle + \epsilon |K_1\rangle$$
$$|K_S\rangle = |K_1\rangle + \epsilon |K_2\rangle$$

$$|K_L\rangle = p\left|K^0\right\rangle - q\left|\overline{K^0}\right\rangle$$
$$|K_S\rangle = p\left|K^0\right\rangle + q\left|\overline{K^0}\right\rangle$$

- Smaller CP violating effects

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \delta V$$

$$\delta V = \begin{pmatrix} -\frac{1}{8}\lambda^4 & 0 & 0 \\ \frac{1}{2}A^2\lambda^5(1-2(\rho+i\eta)) & -\frac{1}{8}\lambda^4(1+4A^2) & 0 \\ \frac{1}{2}A\lambda^5(\rho+i\eta) & \frac{1}{2}A\lambda^4(1-2(\rho+i\eta)) & -\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

- But historically important!
  - Concepts same as in B-system, so you have a chance to understand…

Kaons…

- Different notation: confusing!

$K_1, K_2, K_L, K_S, K_+, K_-, K^0$

=

CP eigenstates

$$|K_L\rangle = |K_2\rangle + \epsilon |K_1\rangle$$

$$|K_S\rangle = |K_1\rangle + \epsilon |K_2\rangle$$

$$|K_L\rangle = p\left|K^0\right\rangle - q\left|\overline{K^0}\right\rangle$$

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$$|K_L\rangle = p\,|K^0\rangle - q\,|\overline{K^0}\rangle$$
$$|K_S\rangle = p\,|K^0\rangle + q\,|\overline{K^0}\rangle$$

- Smaller CP violating effects

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \delta V$$

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Kaons…

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$K_1, K_2, K_L, K_S, K_+, K_-, K^0$

Flavour eigenstates

$$|K_L\rangle = |K_2\rangle + \epsilon\,|K_1\rangle$$
$$|K_S\rangle = |K_1\rangle + \epsilon\,|K_2\rangle$$

$$|K_L\rangle = p\,|K^0\rangle - q\,|\overline{K^0}\rangle$$
$$|K_S\rangle = p\,|K^0\rangle + q\,|\overline{K^0}\rangle$$

- Smaller CP violating effects

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \delta V$$

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➢ But historically important!

- Concepts same as in B-system, so you have a chance to understand…

CP violation in the quark sector

1)	Quarks:	SM, CKM, UT	Niels, 30’
2)	Mesons:	Mixing and CPV types	Niels, 30’
3)	B-mesons:	Decays and Experiment	Patrick, 60’