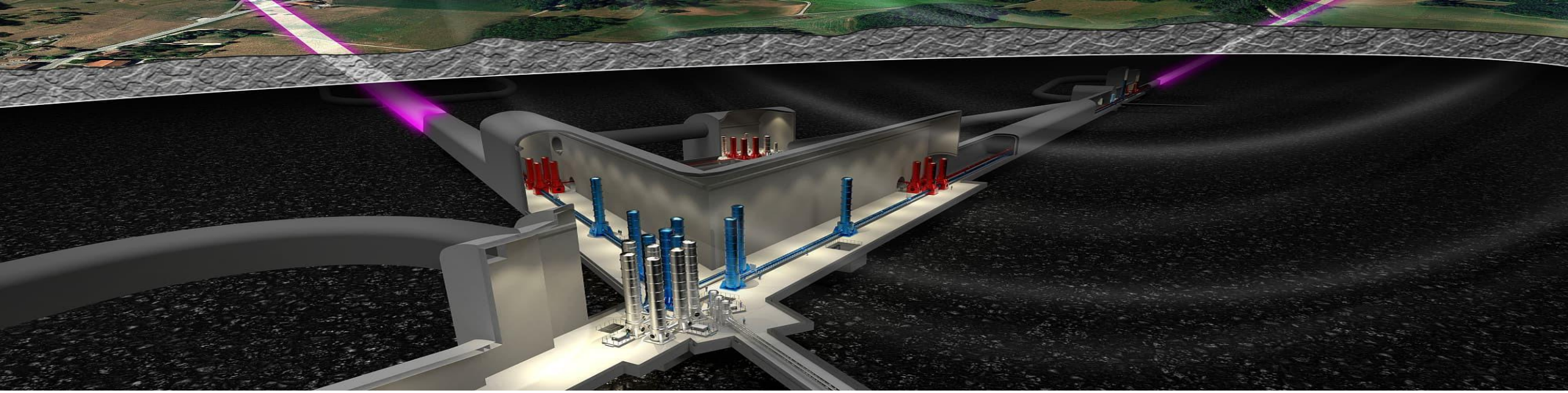


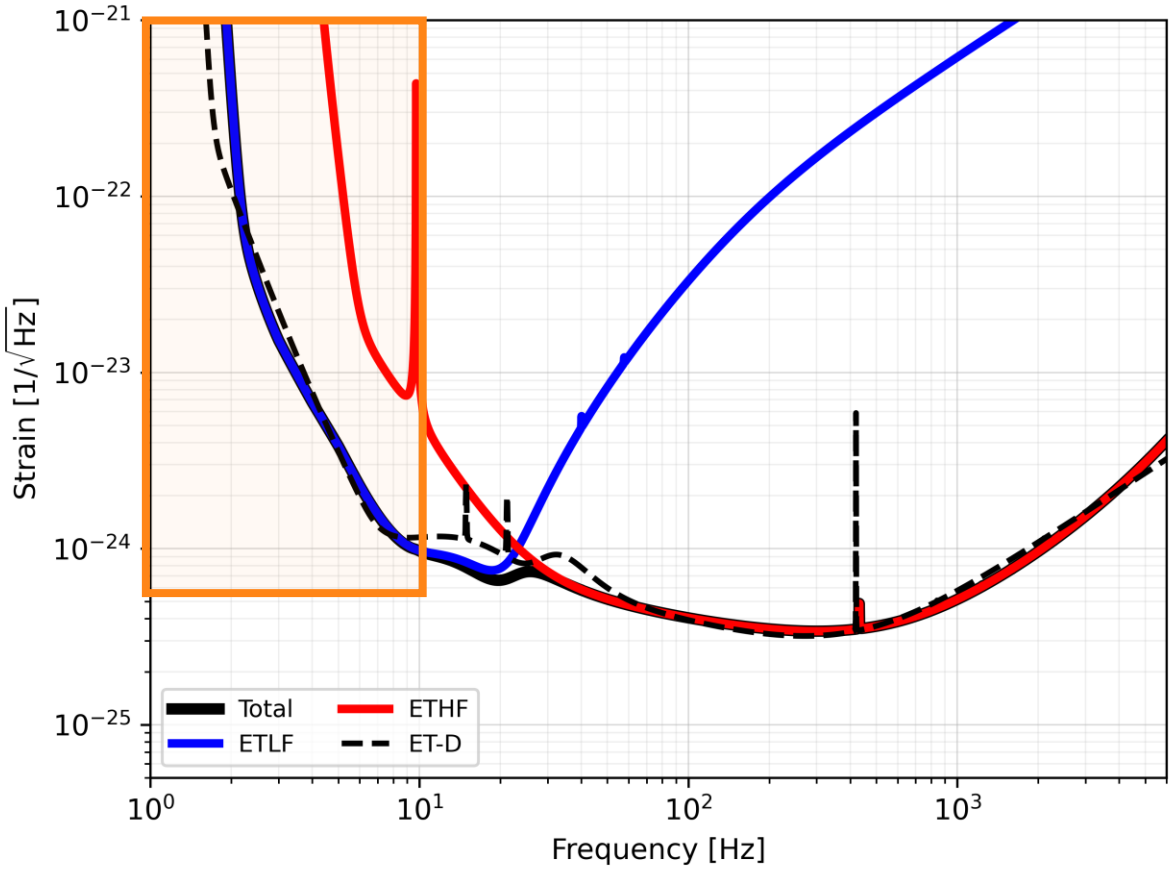
Newtonian Noise – Numerical Approach Validation Program

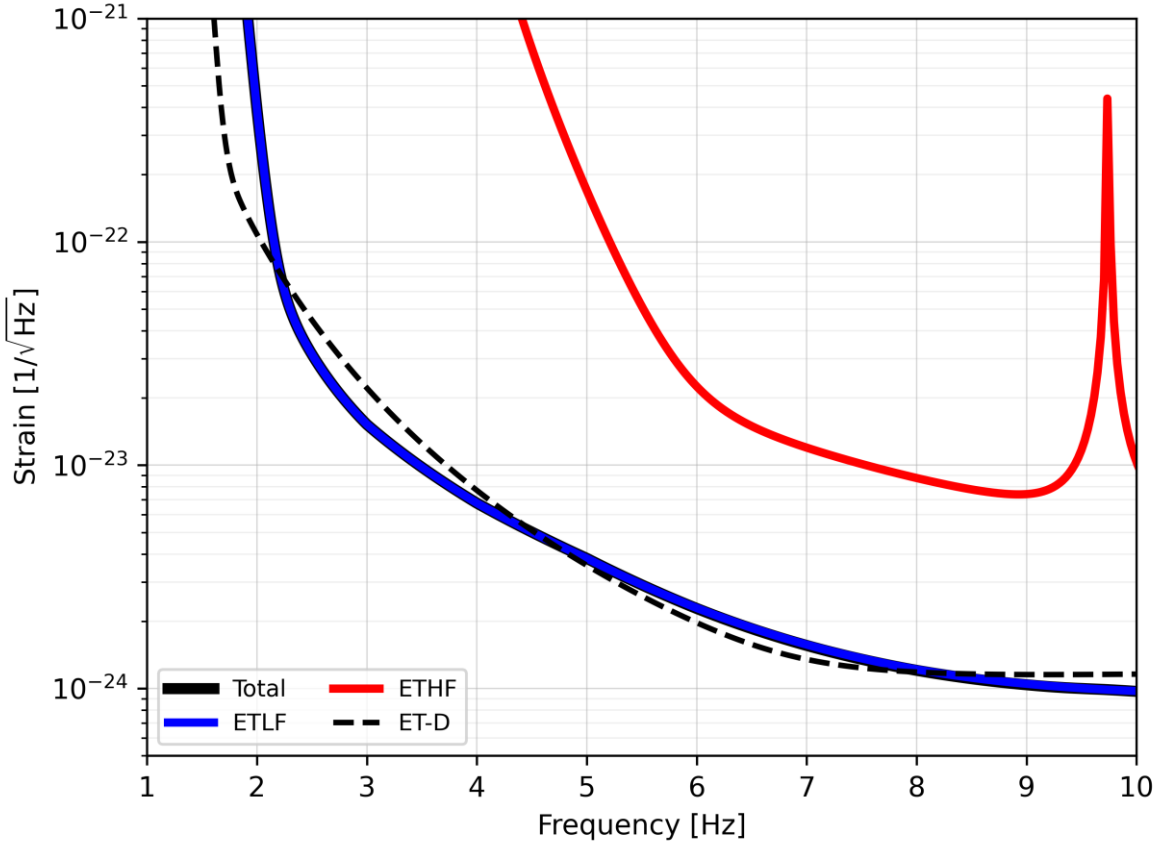
13 January 2026 | Newtonian Noise Meeting | Valentin Tempel

III. Physikalisches Institut
RWTH AACHEN UNIVERSITY

Reminder: Newtonian Noise according to ISB (ET Collaboration)

Current model of ET Noise Budget (ET / ISB / Interferometer / ET-NoiseBudget · GitLab)



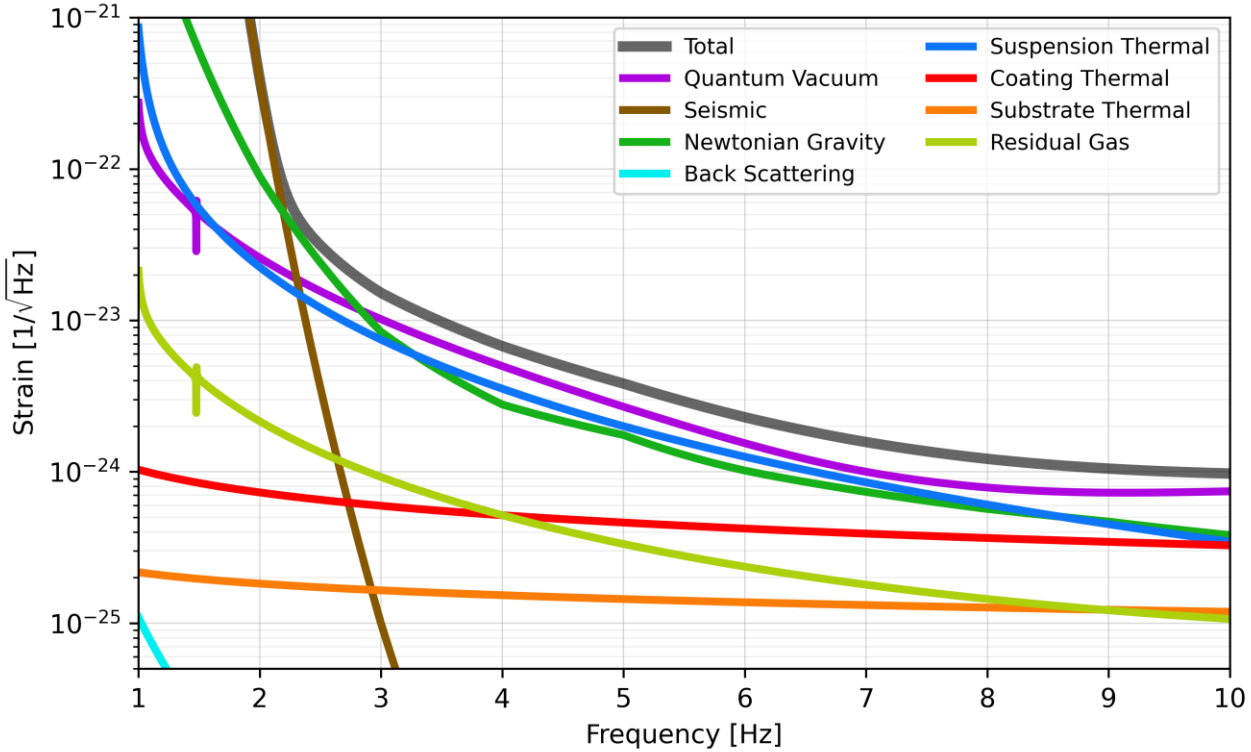


Note: strain/sensitivities in the following for single interferometer (“L”-shape)

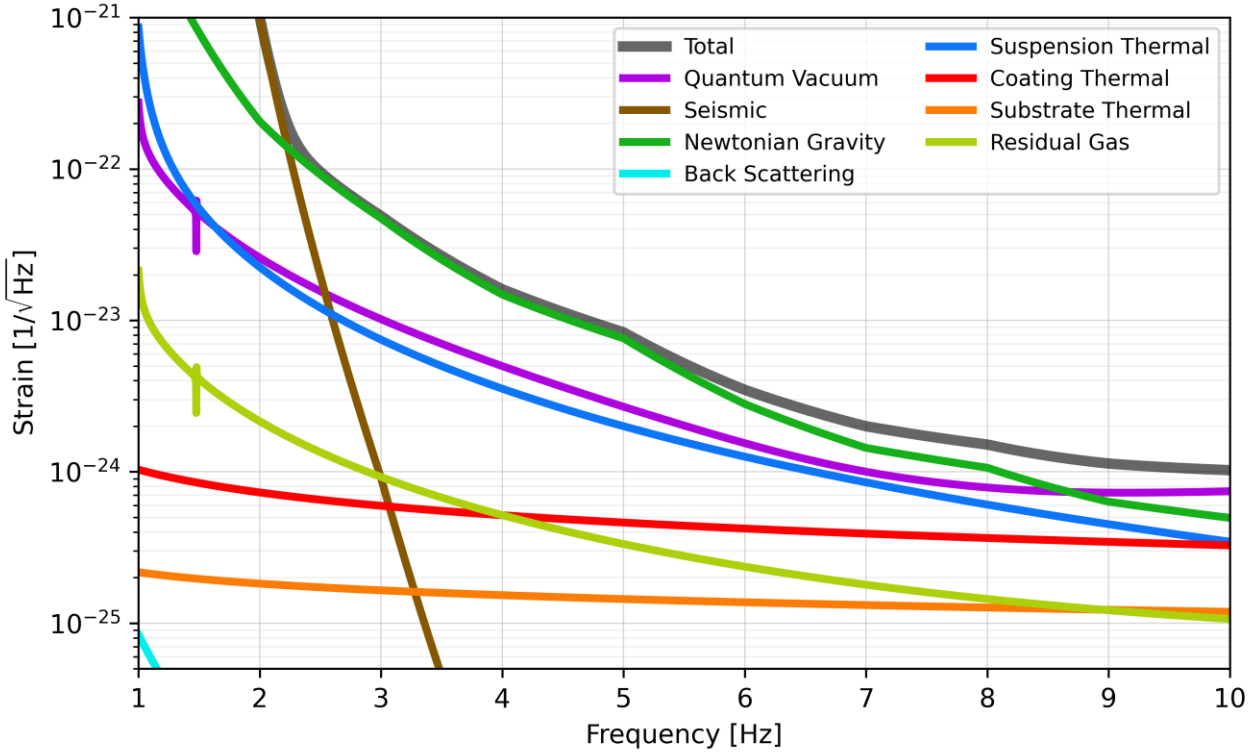
ET-LF noise for Sardinia vs. EMR according to ISB

- According to current ISB budget, NN is the dominating noise contribution for EMR ET LF between approx. 2-8 Hz even after NN suppression by a factor of 3!
- consider that Terziet is maybe not the noisiest corner point

  → **need for underground data from all ET corners!**

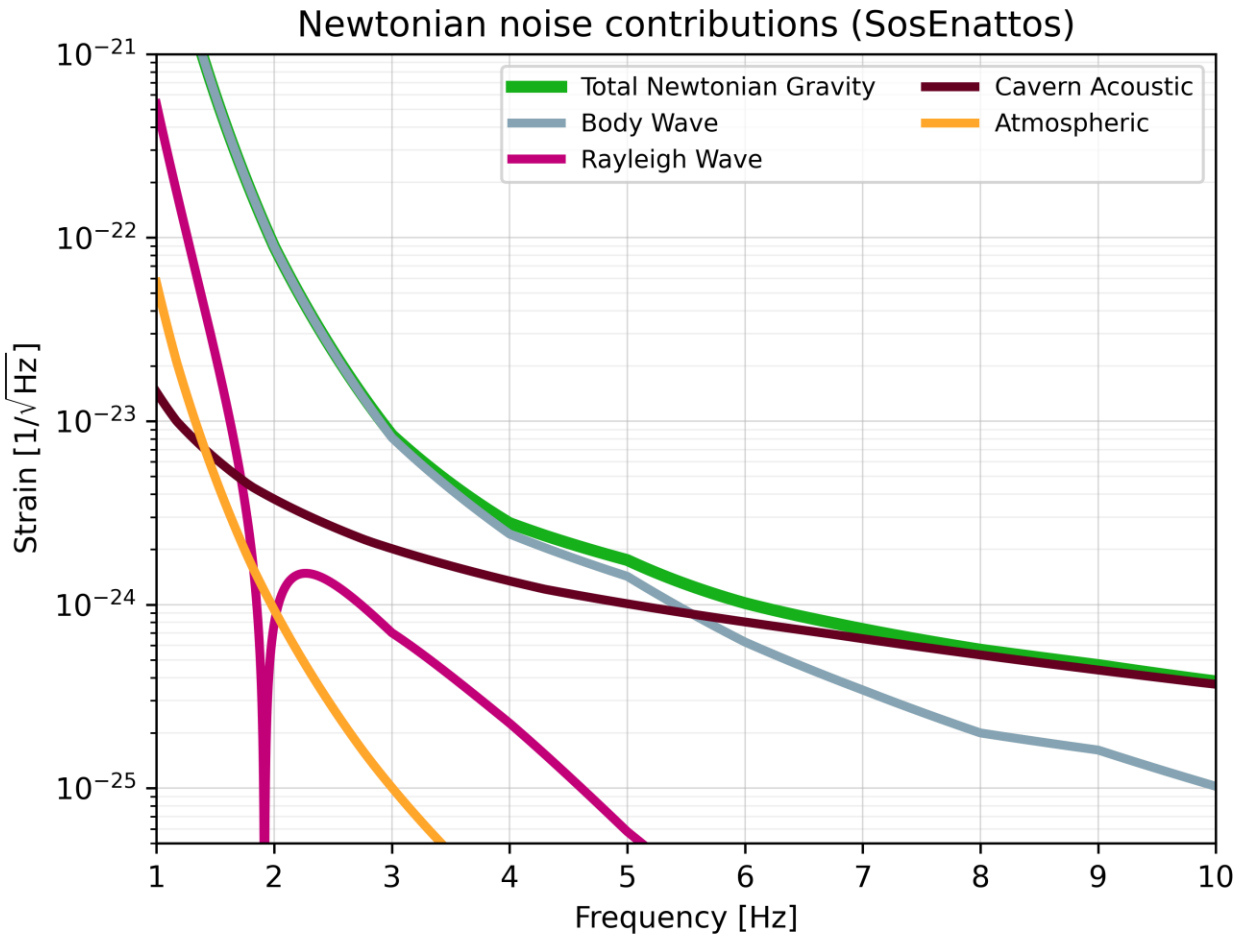


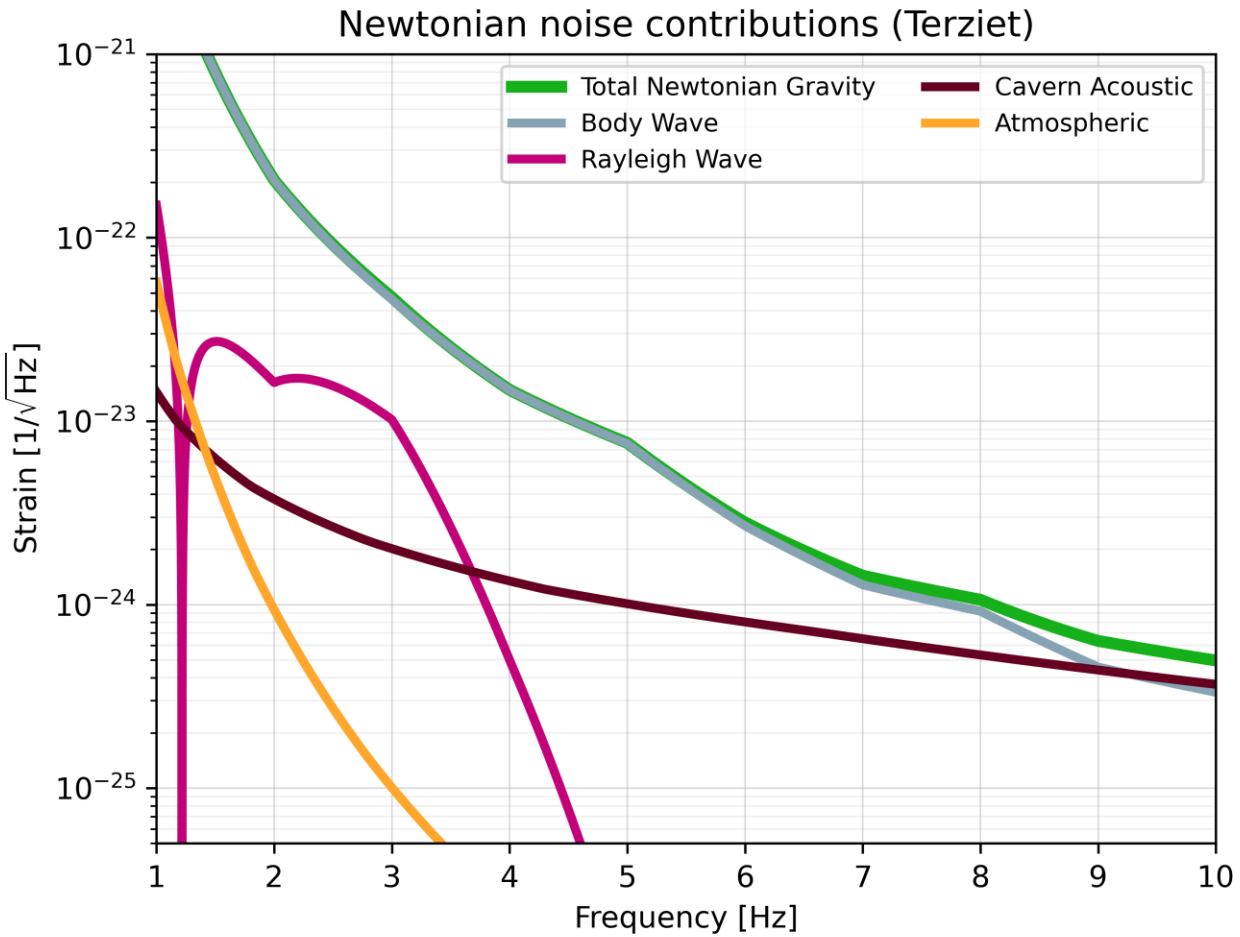
Sardinia (SosEnattos)



EMR (Terziet)

ET-LF NN for Sardinia (SosEnattos) vs. EMR (Terziet) according to ISB





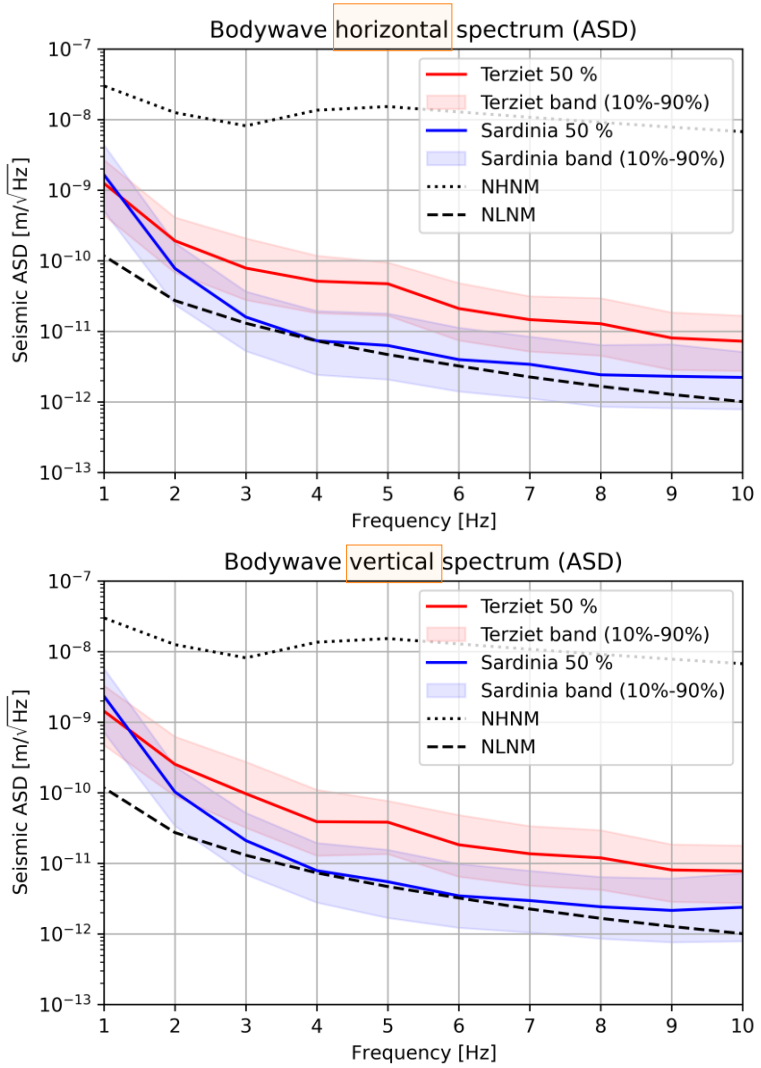
According to ISB **body wave NN is the dominating NN source** for all sites, Rayleigh waves relevant for EMR

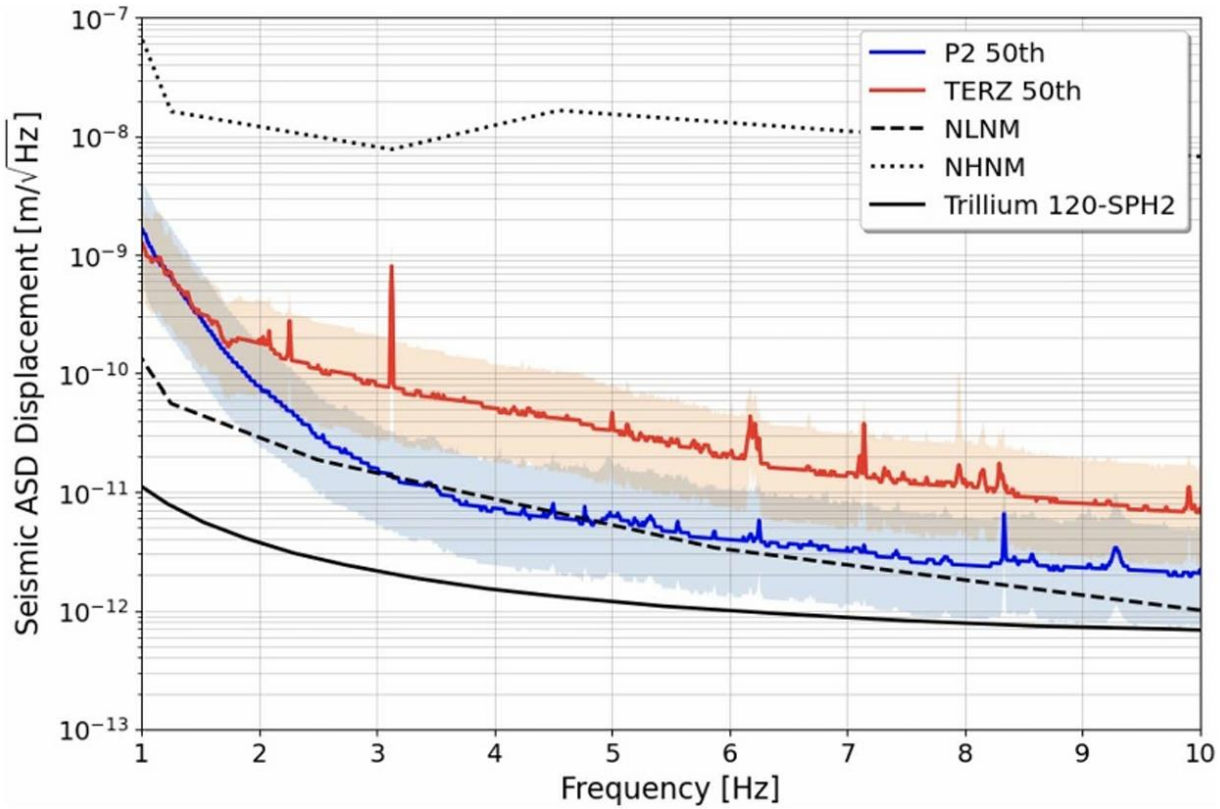
Note: both sites have a cancellation factor of 3 included for these plots!

Crucial input Data for Newtonian Noise: Underground Seismic ASD

… no matter what approach (numerical/analytical/…) we use!

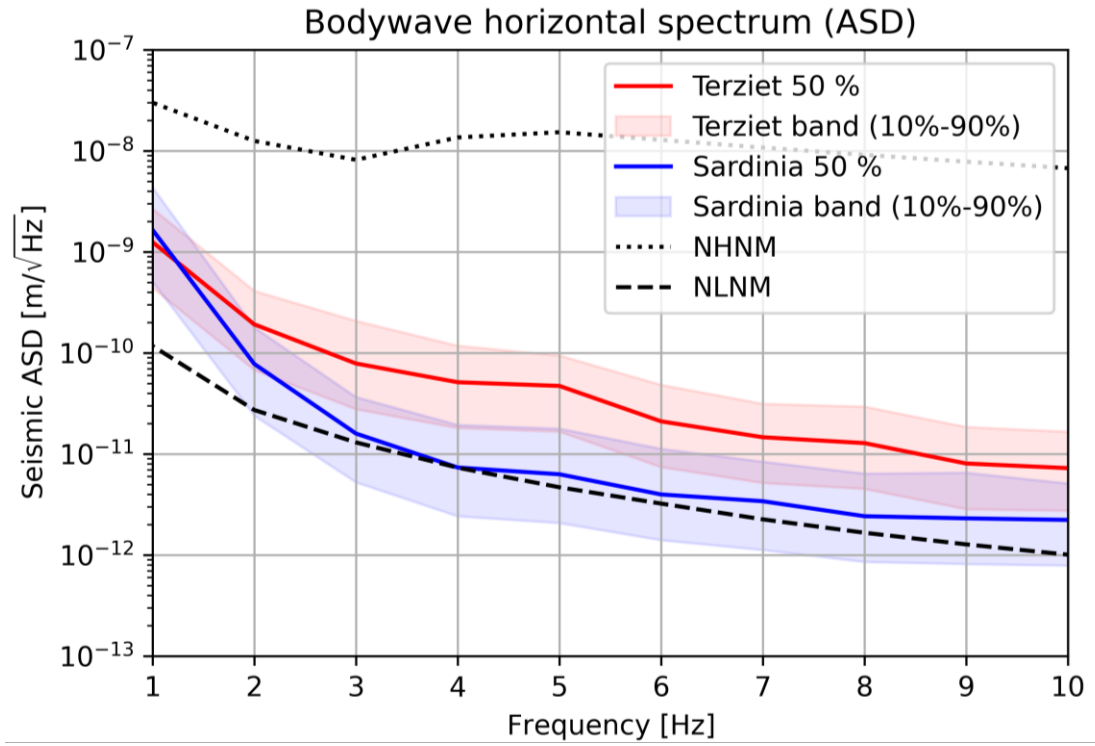
Current comparison of seismic “bodywave-spectra” according to ISB and Italian publication

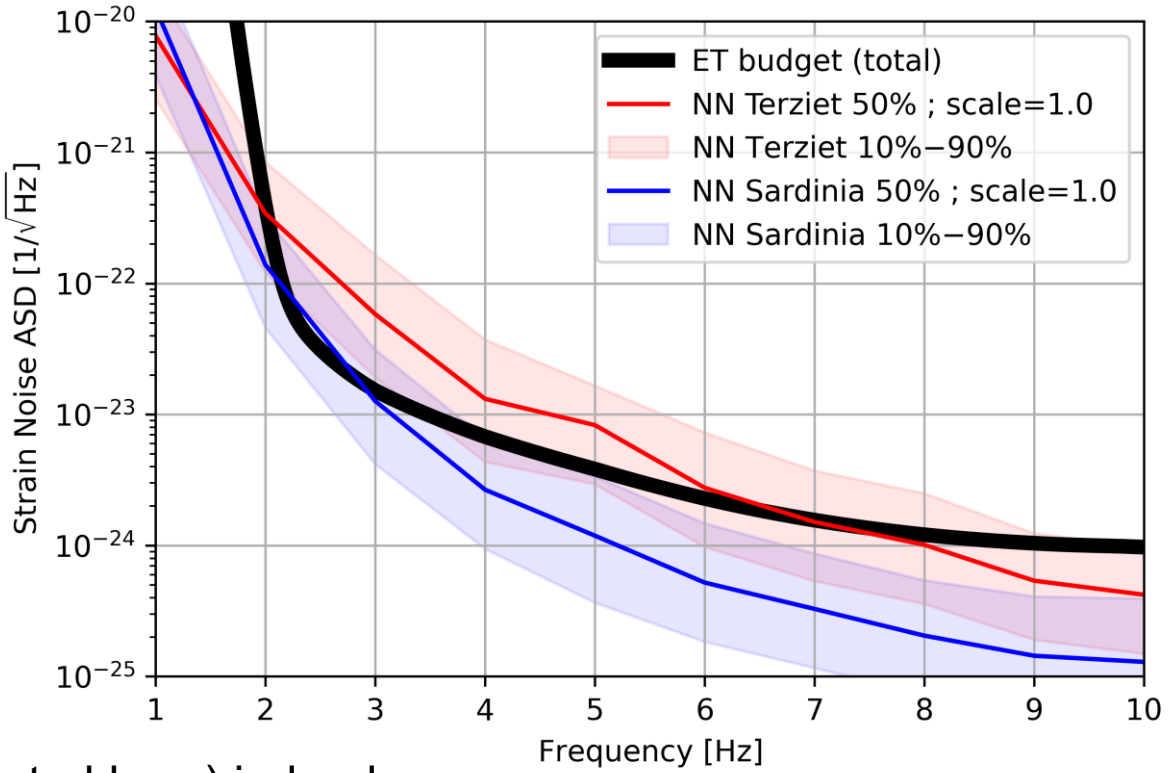




Plot taken from: **The impact of local noise recorded at the ET candidate sites on the signal to noise ratio of CBC gravitational wave signals for the ET triangle configuration (2025)**

From seismic ASD (input) to Strain Noise ASD (output)





- Under a very long list of assumptions (will not be repeated here) indeed:
  rescale Seismic ASD into analytic Strain Noise ASD (Harms model, compare plots above)
- However, many assumptions are certainly not realistic → large systematic uncertainties are possible

- **How large are the deviations from this model when increasing complexity of (numerical) model?**
- **What are the most important modifications to arrive at a realistic result?**

NN numerical modeling validation program (my proposal from last year)

The simulated mirror configuration (schematic)

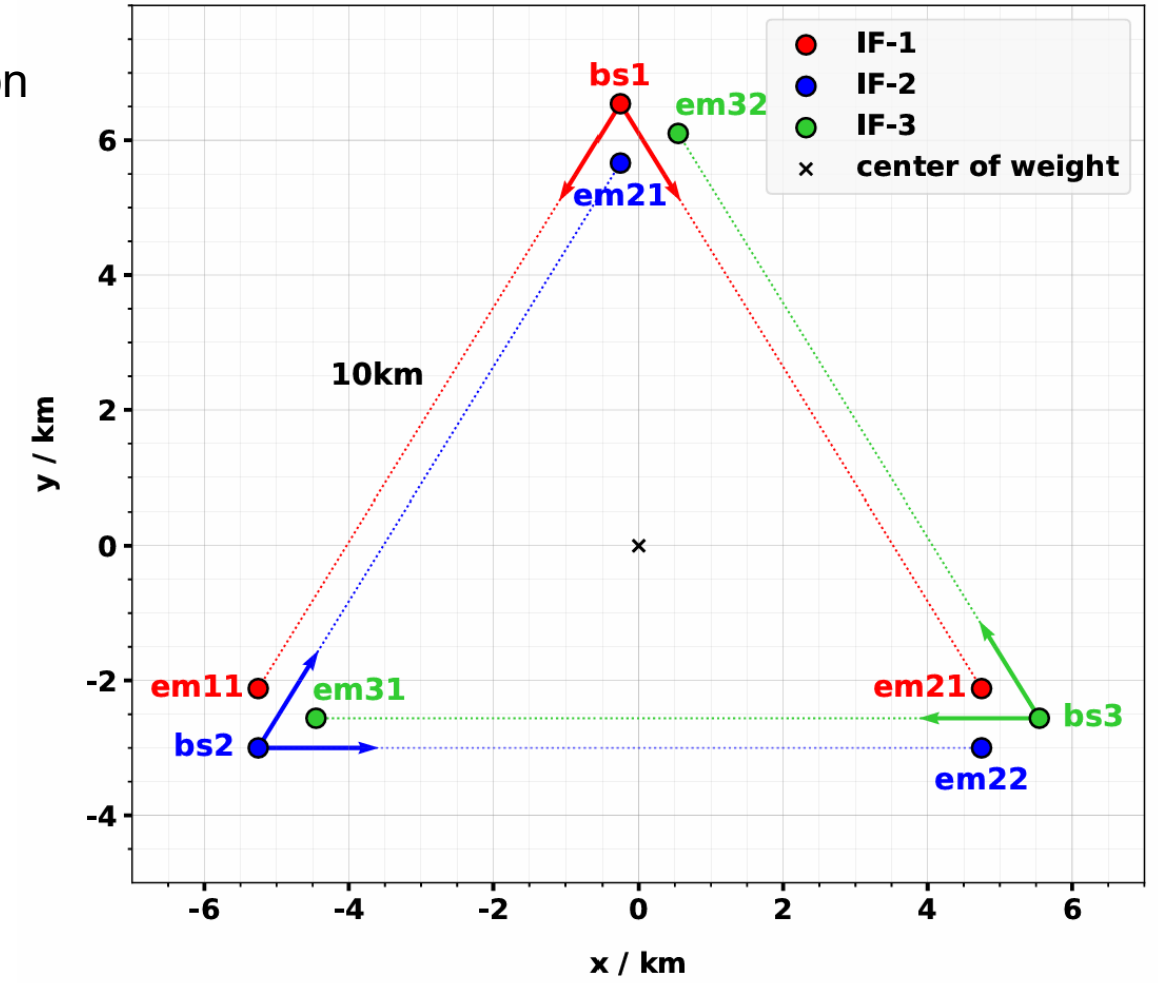
- Start with one interferometer (IF) in triangle configuration
  → already seismic data from all corners needed

- "**horizontal**" means in the xy-plane
- "**vertical**" means in z-direction

- ETM for one IF (end test mass)
  → probably mostly uncorrelated regarding NN
- ITM for one IF (input test mass, near beam splitter)
  → probably highly correlated regarding NN

- NN correlation simulation not possible?
  → assume all 4 mirrors uncorrelated

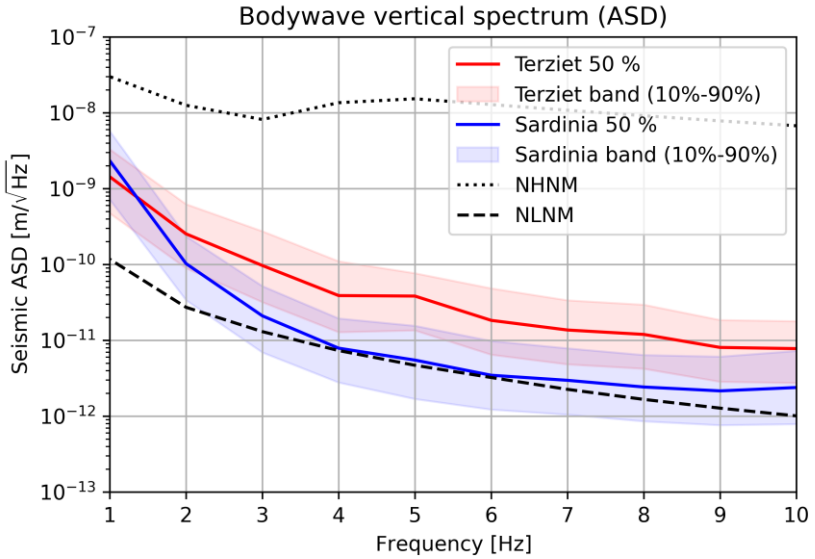
- data from corner is not available?
  → use Terziet seismic data as default



Desired quantities and plots (input and output)

- **for all three corner points**
- **for all three directions x,y [2 horizontal] and z [vertical]**
- both **measurement** and **reproduction in simulation**
  - seismic ASD underground (depth approx. 250m)
  - seismic ASD at ground level

→ In total 2x3x3 = 18 plots, with 10%,50%,90% percentiles
→ example for scaling (ISB data, without any simulation)

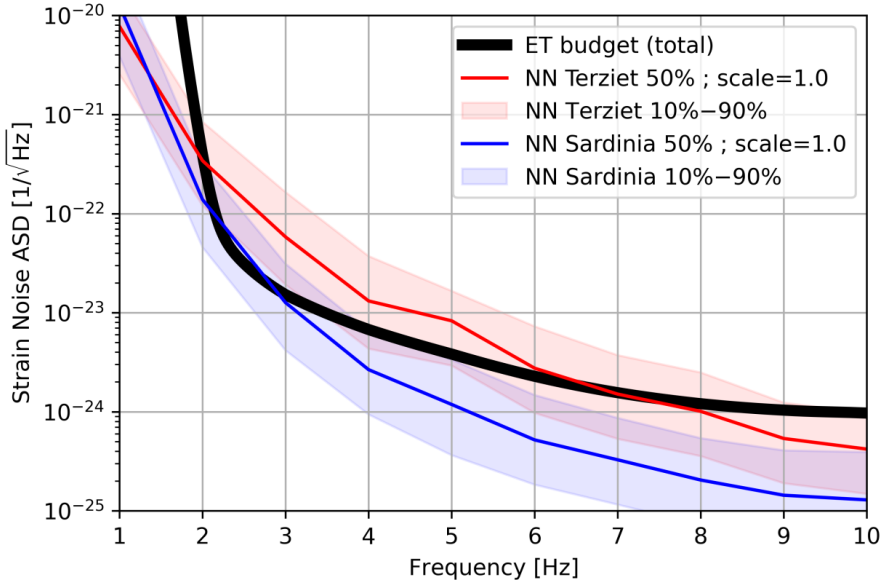


seismic ASD for NN estimation is given in $\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$

strain noise ASD for NN estimation is given in $\frac{1}{\sqrt{\mathrm{Hz}}}$

- **for all three interferometers**
  - simulated NN ASD in “mirror-movement-direction”
  - contributions from **individual mirrors, wave types and directions** might be interesting…
  - always compare with design budget

- example for scaling (ISB data, Harms model)



Proposed Progressive simulations:

Newtonian Noise Numerical Approach (v1) - Google Docs

1. homogenous flat half space with properties of layer including the mirror
2. layered flat half space with “1D-layers” from Terziet (as provided by Soumen)
3. layered flat half space with “1D-layers” from other corner points
4. layered half space with “3D-layers” and topography for all corner points (full EMR 3D model at best)

Proposed Progressive simulations:

Newtonian Noise Numerical Approach (v1) - Google Docs

1. homogenous flat half space with properties of layer including the mirror

    - a) simulate only incoming P-waves from lower half space (assume isotropy)
    - b) simulate only incoming S-waves from lower half space (assume isotropy)
    - c) simulate a random mixture of incoming P and S waves

    - each simulation should include mode conversion at earth surface z=0 and reflections
    - the resulting **underground seismic ASD has to be rescaled to the measured one**
    - to avoid missing important contribution: **match underground horizontal seismic spectrum first in scaling**

Proposed Progressive simulations:

Newtonian Noise Numerical Approach (v1) - Google Docs

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    - b) simulate only incoming S-waves from lower half space (assume isotropy)
    - c) simulate a random mixture of incoming P and S waves

    - each simulation should include mode conversion at earth surface z=0 and reflections
    - the resulting **underground seismic ASD has to be rescaled to the measured one**
    - to avoid missing important contribution: **match underground horizontal seismic spectrum first in scaling**

    - **compare** the **corresponding surface seismic ASD** to the measured surface seismic ASD
    - **compare the strain NN ASD to the one predicted by Harms analytical model**

→ As far as I understand:
Georgia has systematically calculated the NN accelerations for a single mirror in this scenario (December 2025), only rescaling to measured seismic ASD and conversion to strain NN are missing

Proposed Progressive simulations:

Newtonian Noise Numerical Approach (v1) - Google Docs

1. homogenous half space with properties of layer including the mirror

2. layered half space with “1D-layers” from Terziet (as provided by Soumen)

   a) simulate **sources at the flat top (and possibly also underground sources)** which produce
      - rayleigh waves and
      - some mixture of P- and S- waves

   b) **match seismic spectra measured at surface**

   → what fraction of underground seismic PSD is explained by the sources at top?

   c) fill up the rest of the underground spectra with “body waves” (compare 1. homogenous half space)

→ As far as I understand:

can be done in the simulations (in the near future), similar study - however with greatly simplified “body-wave-background” - was been performed for Terziet by Soumen and Maria Bader in 2021/2022

Proposed Progressive simulations: Summary and Outlook

Newtonian Noise Numerical Approach (v1) - Google Docs

1. homogenous flat half space with properties of layer including the mirror **(next week?)**

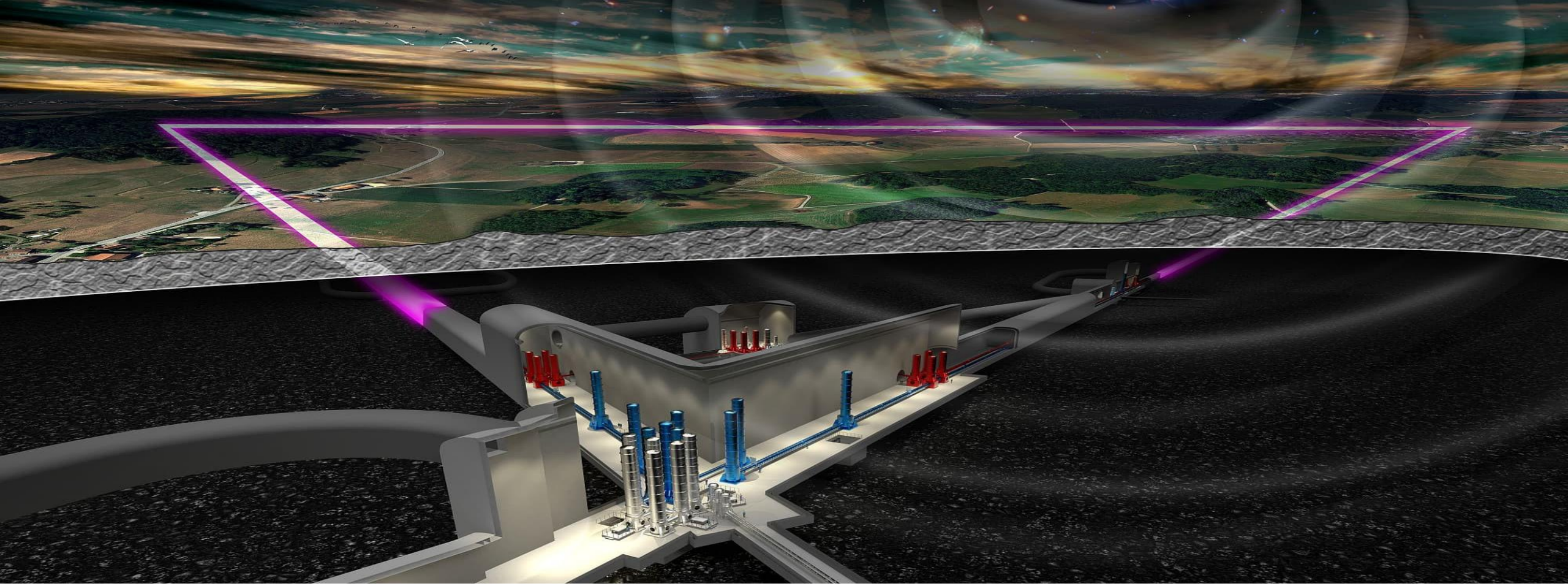
2. layered flat half space with “1D-layers” from **Terziet (soon?)**

3. layered flat half space with “1D-layers” from **other corner points (timeline?)**

    - I have no feeling how much work this is?
    - How far are we with investigation of other corner points / 3D underground model of EMR?

4. layered half space with “3D-layers” and topography for all corner points (full EMR 3D model at best)

    - Maybe “body waves” are no longer needed to **simultaneously match measured underground and surface seismic spectra in realistic case** → not certain so far
    - **Anyway: I think we need intermediate results from 1., 2., 3. to convince ISB / ET collaboration of numerical approach and to replace Harms analytical model for EMR**



Backup

ET-NoiseBudget (ISB)

ET / ISB / Interferometer / ET-NoiseBudget · GitLab
https://gitlab.et-gw.eu/et/isb/interferometer/ET-NoiseBudget

- bodywave_spectrum_SosEnattos.txt
- bodywave_spectrum_SosEnattos_V.txt
- bodywave_spectrum_SosEnattos_old.txt
- bodywave_spectrum_Terziet.txt
- bodywave_spectrum_Terziet_V.txt
- bodywave_spectrum_Terziet_old.txt

```python
def spectrum_bodywave(f,Seismic):
    dataSosEnattos = np.loadtxt("acoustic_spectra/bodywave_spectrum_SosEnattos.txt")
    dataTerziet = np.loadtxt("acoustic_spectra/bodywave_spectrum_Terziet.txt")

    if Seismic.Site=='ET':
        bodywave=(5 * gwinc.noise.seismic.seismic_ground_NLNM(f))**2
    elif Seismic.Site =='SosEnattos':
        bodywave=loginterp(dataSosEnattos.T[0],dataSosEnattos.T[3],f)
    elif Seismic.Site =='Terziet':
        bodywave=loginterp(dataTerziet.T[0],dataTerziet.T[3],f)

    return bodywave
```

PSD: $S_{\mathrm{bw}}^{h} = \left(\frac{4\pi}{3} G\rho_0\right)^2 (3p+1) S_{\mathrm{bw}}(\xi_{\mathrm{TM}}, f) \frac{4}{L^2 (2\pi f)^4}$

```python
################################################################################
## Noise sources
## All equation numbers refer to those in https://arxiv.org/abs/2003.03434
################################################################################
def body_wave(f,Seismic):
    p = 0.33  # Fraction of body wave spectral density caused by compressional waves
    rock_density = Seismic.RhoR  # kg / m^3
    Sh =  (4/3 * np.pi * constants.G * rock_density)**2 * (3*p + 1) * spectrum_bodywave(f,Seismic) * 4 / (2*np.pi*f)**4  # Equation 7
    return np.sqrt(Sh)

def rayleigh_wave(f,Seismic):
    vr = spectrum_rayleigh_dispersion(f,Seismic)
    vs = 1.1 * vr  # Shear wave dispersion TODO: pulled from slide 16 of http://rses.anu.edu.au/~nick/teachdoc/lecture5.pdf, find better source
    vp = 2 * vr  # TODO: quick guess
    kr = 2 * np.pi * f / vr

    qp = 2 * np.pi * f * np.sqrt(1 / vr**2 - 1 / vp**2)
    qs = 2 * np.pi * f * np.sqrt(1 / vr**2 - 1 / vs**2)
    zeta = np.sqrt(qp / qs)

    h = -Seismic.Height  # Detector depth in m
    gamma = 0.8  # Factor quantifying cancellation of newtonian noise
    density_surface = Seismic.RhoS  # Density of surface in kg / m^3

    r0 = kr * (1 - zeta)  # eq. 3
    sh = -kr * (1 + zeta) * np.exp(-kr * h)  # eq. 4
    bh = 2/3 * (2 * kr * np.exp(-qp * h) + zeta * qs * np.exp(-qs * h))  # eq. 5
    R = np.abs((sh + bh) / r0)**2  # eq. 6
    SR = (2 * np.pi / np.sqrt(2) * gamma * constants.G * density_surface)**2 * R * spectrum_rayleigh_vertical(f,Seismic) * 4 / (2 * np.pi * f)**4

    return np.sqrt(SR)
```

Plane-wave NN ASD/PSD in homogenous media (according to Jan Harms)

$$\text{ASD:}\quad \tilde{h}_{\mathrm{NN}}(f) = \frac{4\pi}{3} G\rho_0 \frac{2\sqrt{2}}{L} \frac{1}{(2\pi f)^2} \tilde{x}(f) \qquad \text{PSD:}\quad S^h_{\mathrm{bw}} = \left(\frac{4\pi}{3} G\rho_0\right)^2 (3p+1) S_{\mathrm{bw}}(\xi_{\mathrm{TM}}, f) \frac{4}{L^2 (2\pi f)^4}$$

- $S_{bw}(\xi_{TM},f)$ is the power spectral density of body-wave displacement **along the direction of the arm** and the mirror is in a **spherical cavern** located in a quasi-infinite homogenous medium of density ρ_0
- Here $S_P(\xi_{TM}; f)/S_{bw}(\xi_{TM}; f)$ = p = 1/3 was assumed: $2 = 3p + 1 = p \cdot 4 + (1-p) \cdot 1 \text{ for } p = \frac{1}{3}$
- 4 (for P-waves) and 1 (for S-waves) come from the sum of squared coupling factors in homogenous full space for both wave types; no correlation assumed between displacement vectors from P- and S-waves.
- 4 comes from four mirrors forming an interferometer; where it is assumed that the noise in all mirrors is completely uncorrelated (we assume that this is NOT true)

What NN effects do we have in Jan Harms model?

- Jan Harms claims: spherical cavern walls produce the “lower-limit” term $\delta\vec{a}_{low}(\vec{r}_0, t) = -\frac{4}{3}\pi\, G\, \rho_0\, \vec{\xi}_{s,p}(\vec{r_0}, t)$.
- The mirror acceleration in quasi-infinite homogenous media, where the outer rock surface contributes nothing, is opposite to the displacement of the cavern walls (negative sign)
- The factor $-4\pi/3$ and strict anti-proportionality to displacement-vector at mirror $\vec{r}_0$ is only true in some cases, e.g. not for plane waves and elongated cylindrical caverns, it approaches 0 for elongated cavern (can be proven analytically)
- The body/bulk term is $-3\ \delta\vec{a}_{low}(\vec{r}_0, t)$ for plane p-waves in homogenous full space, compare:

$$\delta\vec{a}(\vec{r}_0, t) = \frac{4\pi}{3} G\rho_0 \left([+2] \cdot \vec{\xi}^{\,\mathrm{P}}(\vec{r}_0, t) + [-1] \cdot \vec{\xi}^{\,\mathrm{S}}(\vec{r}_0, t) \right)$$