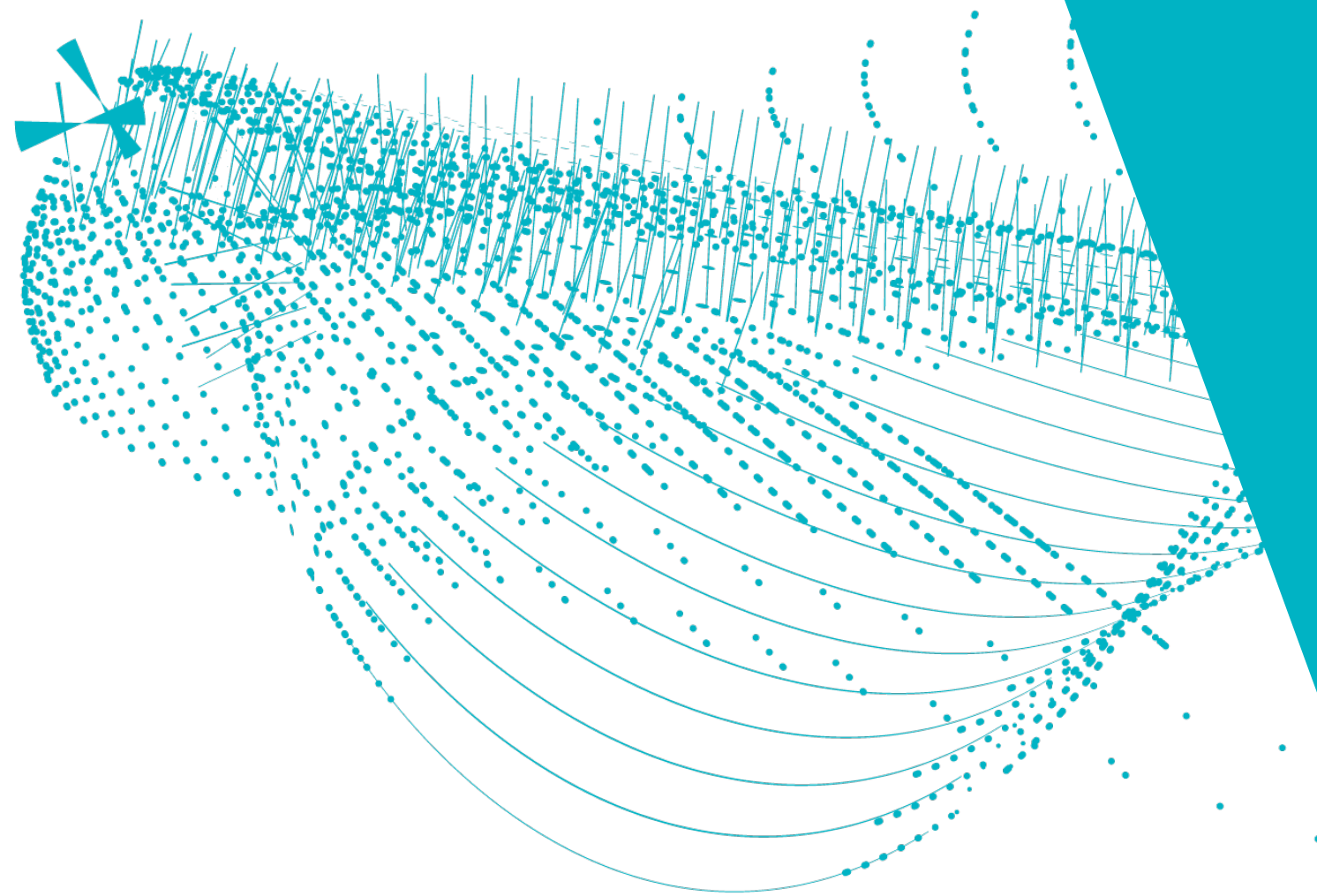




Neutrino Oscillations

Historical Overview and Determination
of the Neutrino Mass Ordering

Daan van Eijk

Topical Lectures
9 December 2025
Nikhef

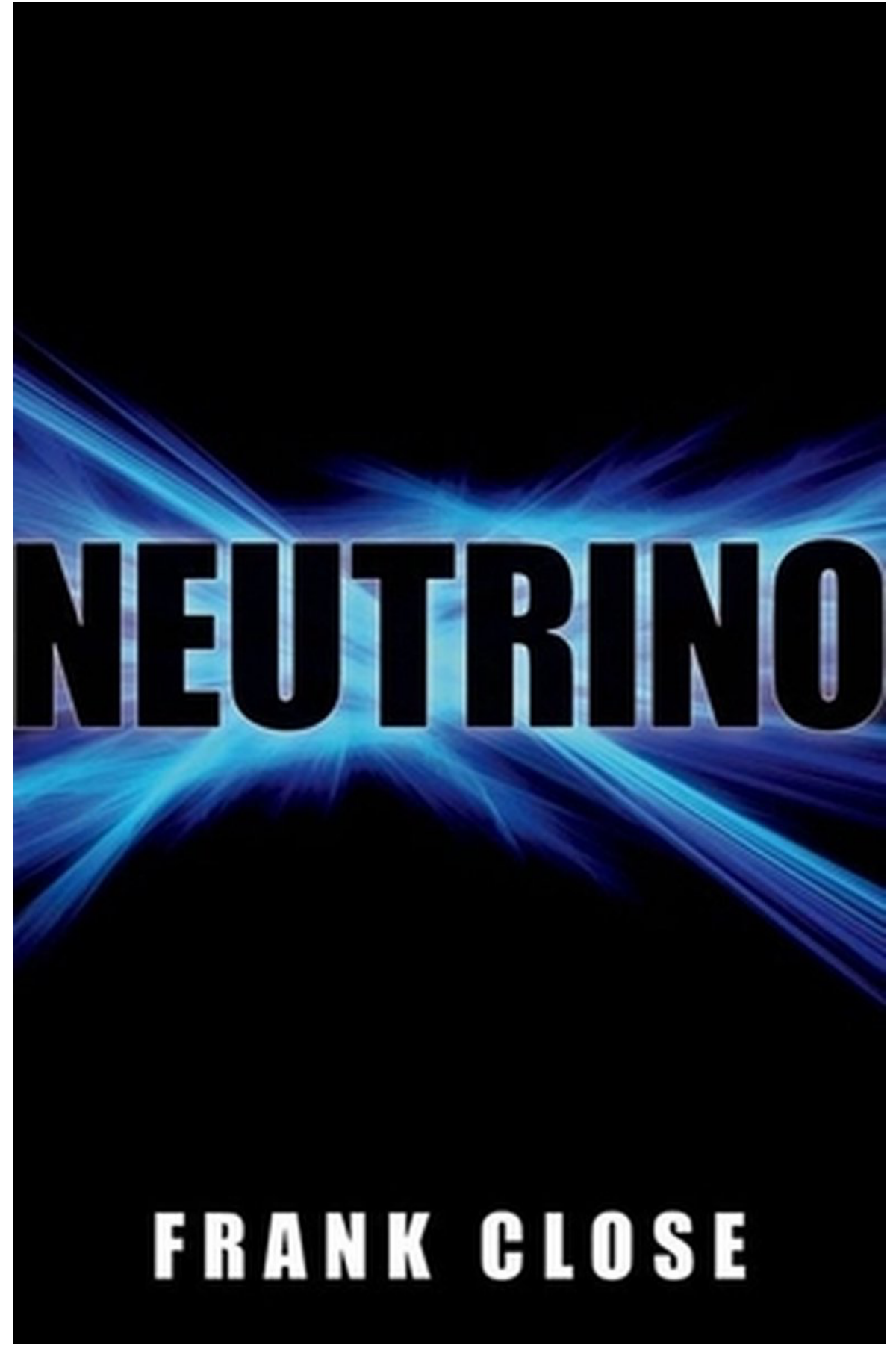
Programme

- Lecture 1 (today):
 - A bit of history and recap
 - A bit (more) of neutrino oscillations
- Lecture 2 (tomorrow):
 - Determination of the neutrino mass ordering

About Me

- PhD@Nikhef, 2012 for LHCb
- TNO
- Postdoc@Nikhef for KM3NeT
- Postdoc@UW Madison for IceCube
- Staff physicist@Nikhef for KM3NeT

Reading Tip

The book cover for 'Neutrino' by Frank Close features a black background. In the center, there is a bright blue, starburst-like pattern of light rays. The word 'NEUTRINO' is written in large, bold, black capital letters across the middle of the cover, superimposed on the light pattern. At the bottom, the author's name 'FRANK CLOSE' is written in white, bold, capital letters.

NEUTRINO

FRANK CLOSE

Listening Tip



<https://www.preposterousuniverse.com/podcast/2025/12/08/228-ryan-patterson-on-the-physics-of-neutrinos/>

Questions

- Why would one need a neutrino?
- How would you discover a neutrino?
- How would you figure out that a neutrino is different from an anti-neutrino?
- How would you figure out that a muon neutrino is different from an electron neutrino?
- How would you figure out that a tau neutrino is different from an electron and a muon neutrino?
- How would you solve the solar neutrino puzzle?

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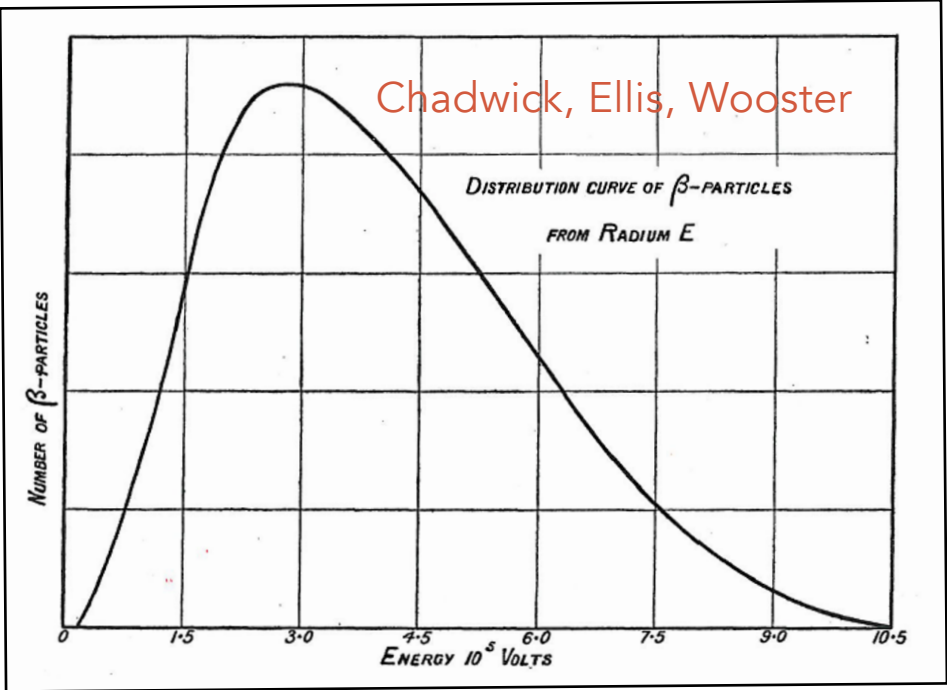
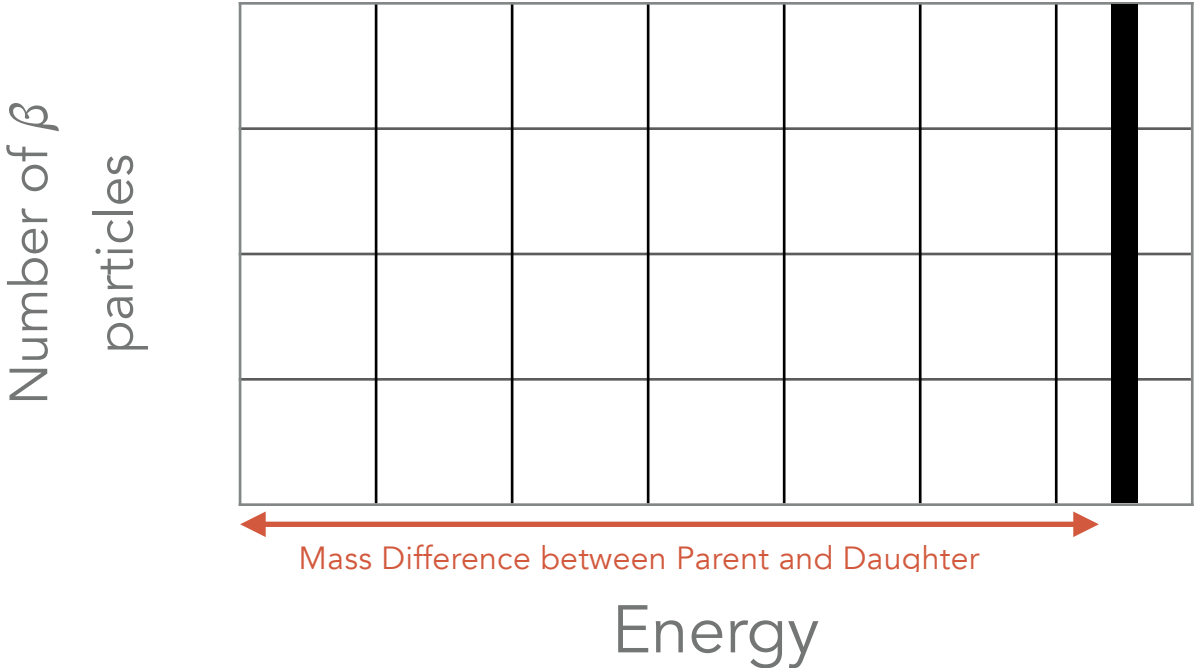
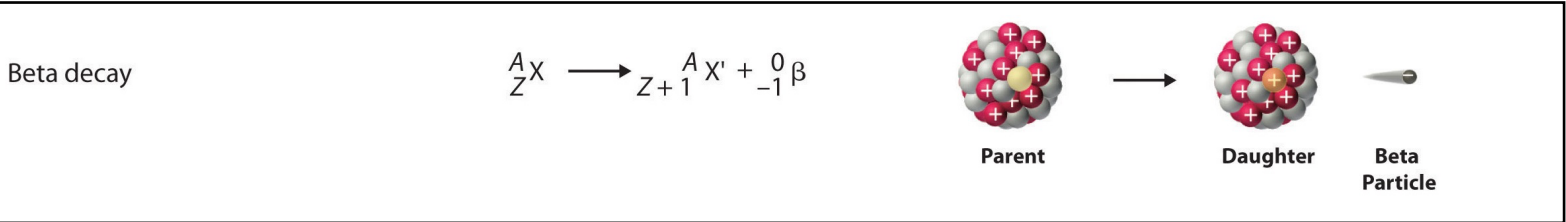
Back to Basics: Back to Early 20th Century

Three known types of radioactivity:

- Alpha radiation (positive)
- Beta radiation (negative)
- Gamma radiation (neutral)

1902: Marie and Pierre Curie discover that the negative particles in beta decay/'radiation' are electrons. (Electrons are already known and named at that time.)

Question: if beta decay is like the image below, what is the energy of the electron?



One of the Proposed Remedies

Original - Photocopy of PLC 0393
Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der
Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst
ansuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich
angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie
des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg
verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz
zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale
Teilchen, die ich Neutronen nennen will, in den Kernen existieren,
welche den Spin $1/2$ haben und das Ausschlussprinzip befolgen und
sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie
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"O, daran soll man am besten gar nicht denken, sowie an die neuen
Steuern." Darum soll man jeden Weg zur Rettung ernstlich diskutieren.
Also, liebe Radioaktive, prüfet, und richtet. Leider kann ich nicht
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vom 6. zum 7. Dez. in Zürich stattfindenden Balles hier unabkömmlich
bin. Mit vielen Grüssen an Euch, sowie an Herrn Baek, Euer
untertänigster Diener

ges. W. Pauli

Open letter to the group of radioactive people at the
Gauverein meeting in Tübingen.

Copy

Physics Institute
of the ETH
Zürich

Zürich, Dec. 4, 1930
Gloriastrasse

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, because of the "wrong" statistics of the N- and Li-6 nuclei and the continuous beta spectrum, I have hit upon **a desperate remedy** to save the "exchange theorem" (1) of statistics and the law of conservation of energy. Namely, the possibility that in the nuclei **there could exist electrically neutral particles, which I will call neutrons**, that have spin 1/2 and obey the exclusion principle and that further differ from light quanta in that they do not travel with the velocity of light. **The mass of the neutrons should be** of the same order of magnitude as the electron mass and in any event **not larger than 0.01 proton mass**. - The continuous beta spectrum would then make sense with the assumption that in beta decay, in addition to the electron, a neutron is emitted such that the sum of the energies of neutron and electron is constant.

Now it is also a question of which forces act upon neutrons. For me, the most likely model for the neutron seems to be, for wave-mechanical reasons (the bearer of these lines knows more), that the neutron at rest is a magnetic dipole with a certain moment μ . The experiments seem to require that the ionizing effect of such a neutron can not be bigger than the one of a gamma-ray, and then μ is probably not allowed to be larger than $e \cdot (10^{-13} \text{ cm})$.

But so far I do not dare to publish anything about this idea, and trustfully turn first to you, dear radioactive people, with the question of how likely it is to find experimental evidence for such a neutron if it would have the same or perhaps a 10 times larger ability to get through [material] than a gamma-ray.

I admit that my remedy may seem almost improbable because one probably would have seen those neutrons, if they exist, for a long time. **But nothing ventured, nothing gained**, and the seriousness of the situation, due to the continuous structure of the beta spectrum, is illuminated by a remark of my honored predecessor, Mr Debye, who told me recently in Bruxelles: "Oh, It's better not to think about this at all, like new taxes." Therefore one should seriously discuss every way of rescue. Thus, dear radioactive people, scrutinize and judge. - Unfortunately, **I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball on the night from December 6 to 7**. With my best regards to you, and also to Mr. Back, your humble servant

signed W. Pauli

One of the Proposed Remedies

- Invent a new particle: the [neutron]
 - Spin 1/2
 - Electrically neutral
 - Massive (this distinguishes it from a photon)
 - Small mass, not bigger than 0.01 x proton mass
 - The sum of the emitted [neutron] and electron is constant and explains the continuous electron spectrum

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Neutron discovery and Introduction of the 'Undiscoverable' Neutrino

- 1932:
 - Chadwick discovers the neutron, which bears the same mass as the proton
 - Rutherford had conjectured this neutron as early as 1920, but this was not picked up somehow
- 1933:
 - Fermi renamed Pauli's [neutron] to neutrino (i.e. 'little one') and develops his (effective) theory to explain the beta spectrum
 - Closer examination of beta spectra reveals that the neutrino mass must be VERY tiny indeed:
'Possibilité d'émission de particules neutres **de masse intrinsèque nulle** dans les radioactivités beta', Perrin, Comptes-Rendus 197 (1933) 1625
- 1934:
 - Bethe and Peierls conclude from Fermi's theory that the neutrino cross section is so small that
'there is **no practical way of observing the neutrino**'

Or is it?

- Bruno Pontecorvo: Bethe's and Peierls's opinion 'too drastic'
 - Pontecorvo's idea (1946): a huge neutrino flux can overcome the extremely tiny cross section

Questions

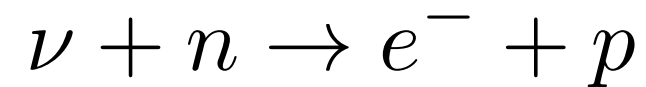
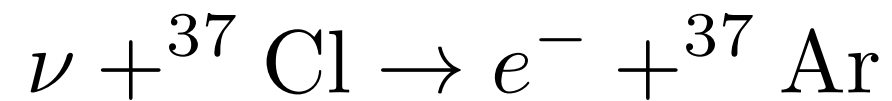
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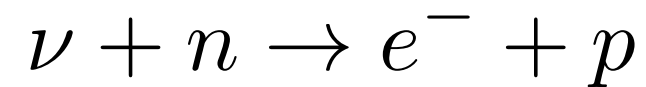
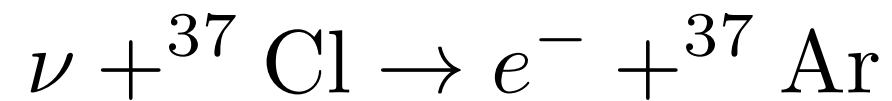
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- Extract the radioactive argon atoms, and count the number of decays (half-life of 34 days), this equals the number of detected neutrinos
- Raymond Davis took up the experimental challenge
 - Brookhaven (1955): ?

Or is it?

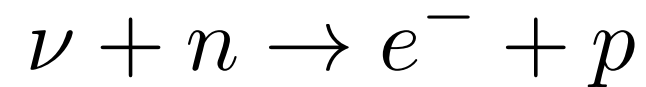
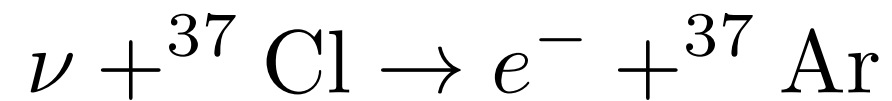
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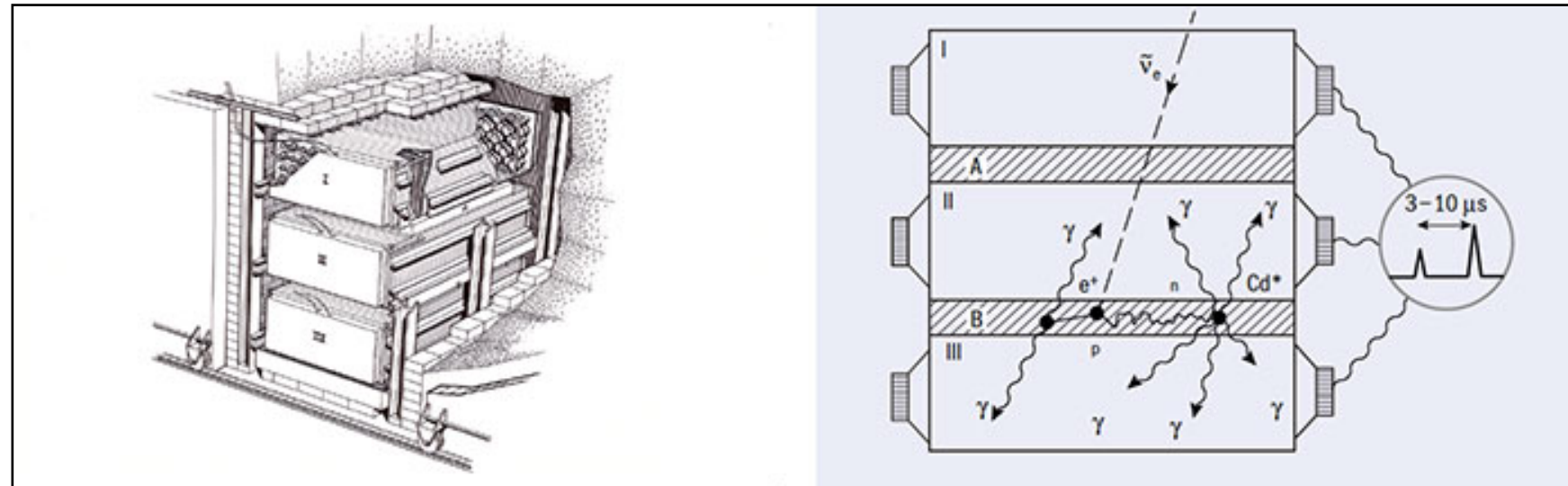
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 - Savannah River (1955): nothing!

Neutrino Discovery

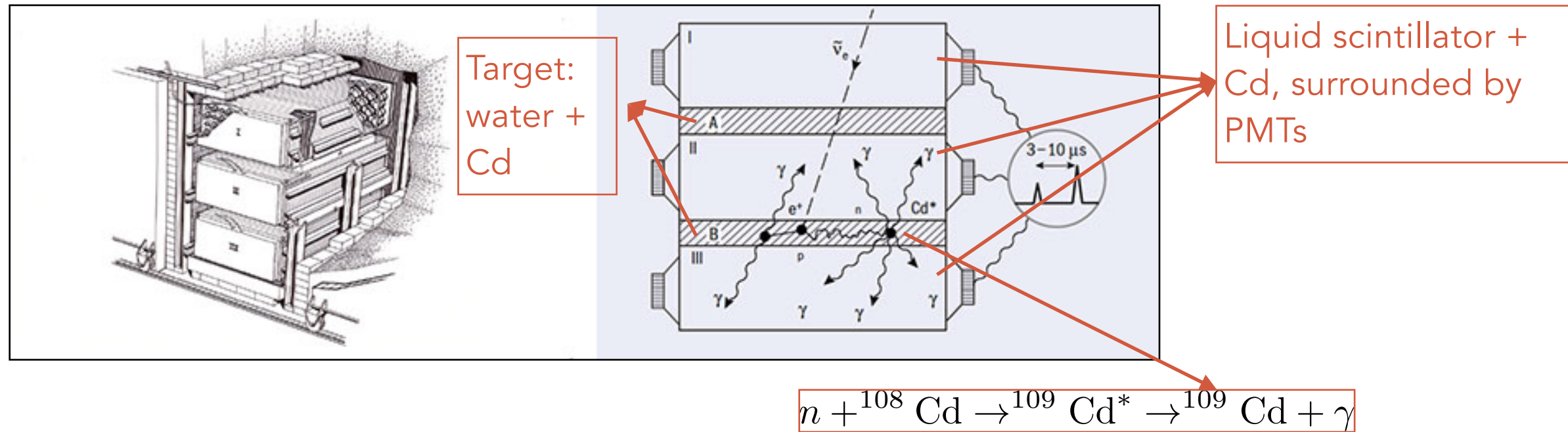
- Cowan and Reines had a similar but slightly different idea:
 - Do you remember what?
 - (Hint: how does it differ from $\nu + n \rightarrow e^- + p$)?



Neutrino Discovery

- Cowan and Reines had a similar but slightly different idea:

$$\nu + p \rightarrow e^+ + n$$

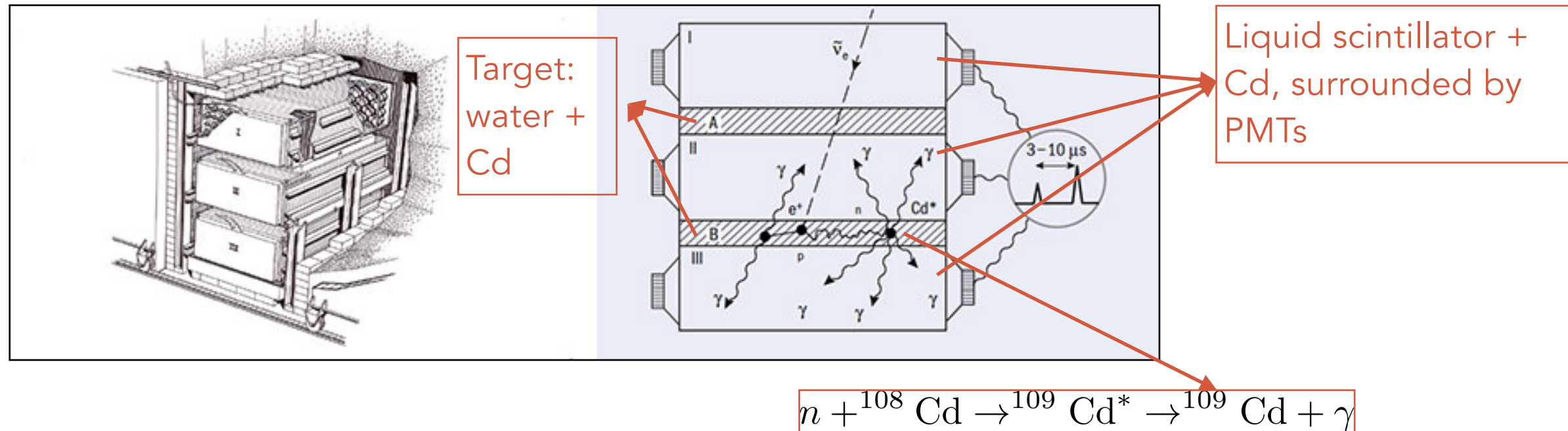


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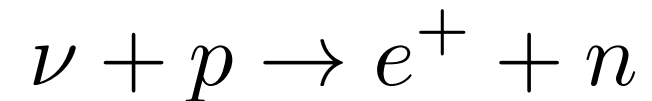
$$\nu + p \rightarrow e^+ + n$$

Something is wrong here, can you see what?

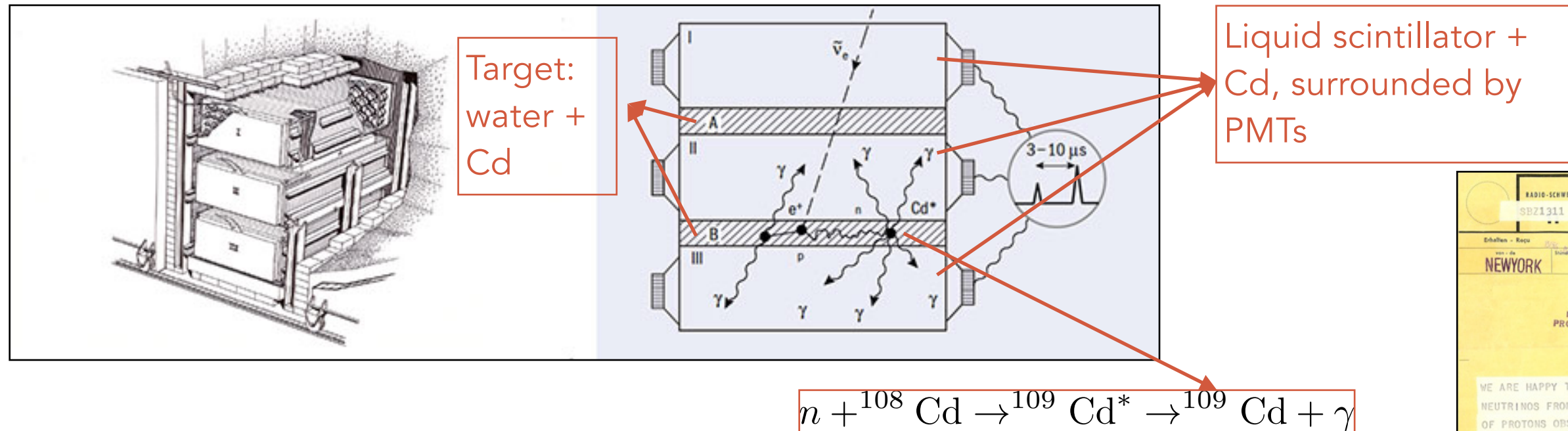


Neutrino Discovery

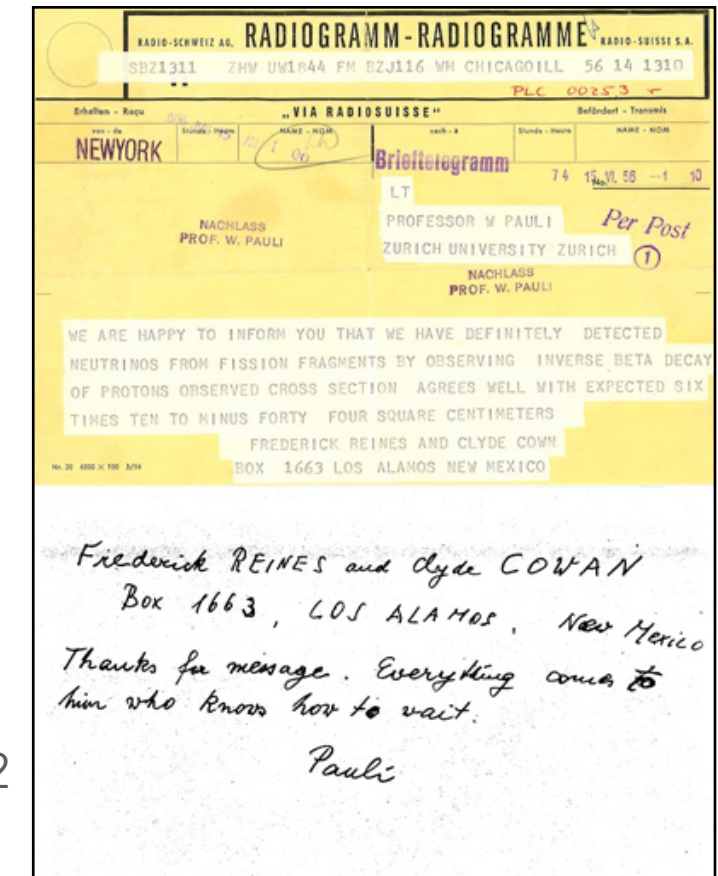
- Cowan and Reines had a similar but slightly different idea:



Something is wrong here, can you see what?



- Hanford: nothing, due to overwhelming background
- Savannah River (1956): DISCOVERY!
 - Cross section as tiny as Bethe and Peierls had predicted: $6.3 \times 10^{-44} \text{ cm}^2$



Questions

- Why would one need a neutrino?
- How would you discover a neutrino?
- **How would you figure out that a neutrino is different from an anti-neutrino?**
- How would you figure out that a muon neutrino is different from an electron neutrino?
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Then Why Didn't Davis See Anything?

Then Why Didn't Davis See Anything?

- We know now that nuclear reactors actually produce anti-neutrinos, not neutrinos.
- So really what Cowan and Reines were seeing was

$$\bar{\nu} + p \rightarrow e^{+} + n$$

- And Davis's sought after

$$\bar{\nu} + n \rightarrow e^{-} + p$$

- Simply doesn't happen
- This shows there is a difference between neutrinos and antineutrinos and calls for the introduction of a quantity called lepton number

Intermezzo: Neutrino Oscillations to Explain a Rumour

- Pontecorvo (now in Russia, but that's a different story...) heard a rumour that Davis *did* observe some events.
- Based on the rumour, using similar mechanisms as in the kaon-antikaon system, he publishes (in 1957 and 1958) his first ideas for neutrino oscillations:
 - 'If the conservation law for the neutrino charge [*that's lepton number*] does not hold, **neutrino to antineutrino transitions in vacuum in principle are possible**'
 - 'In the inclusive experiment of Reines and Cowan **due to neutrino oscillations a deficit of antineutrino events could be observed.**'
 - 'The cross section of the process [*that Cowan and Reines observed*] with the antineutrinos coming from the reactor must be smaller than expected. This is due to the fact that the neutral lepton beam [*i.e. the neutrino beam*] which at the source is capable of inducing the reaction **changes its composition** on the way from the reactor to the detector.'
 - 'Effects of **transformation of neutrino into antineutrino and vice versa** may be unobservable in the laboratory but **will certainly occur, at least, on an astronomical scale.**'

Questions

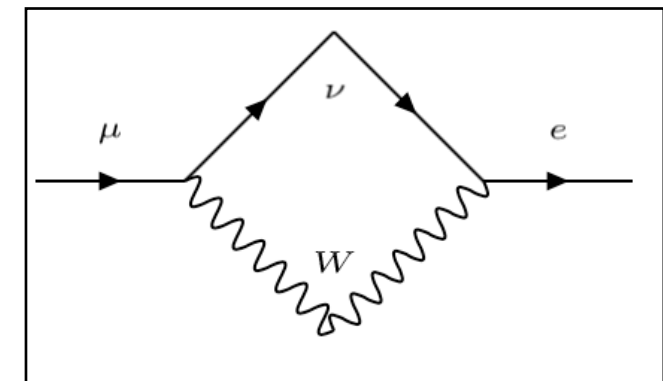
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Lederman Experiment (1957)

- Cross section rises with energy: use (NEW!) particle accelerators
- First time an accelerator was used to study neutrinos
- At Brookhaven, protons on target gives intense beams of pions, decaying into muon and neutrino (with opposite spin as the pion is spinless):
 $\pi^+ \rightarrow \mu^+ + \nu_{(\mu)}$
 - At that time, it's still thought that this is 'the' neutrino
- The muon then decays into an electron and two neutrinos: $\mu^+ \rightarrow e^+ + \nu_{(e)} + \bar{\nu}_{(\mu)}$

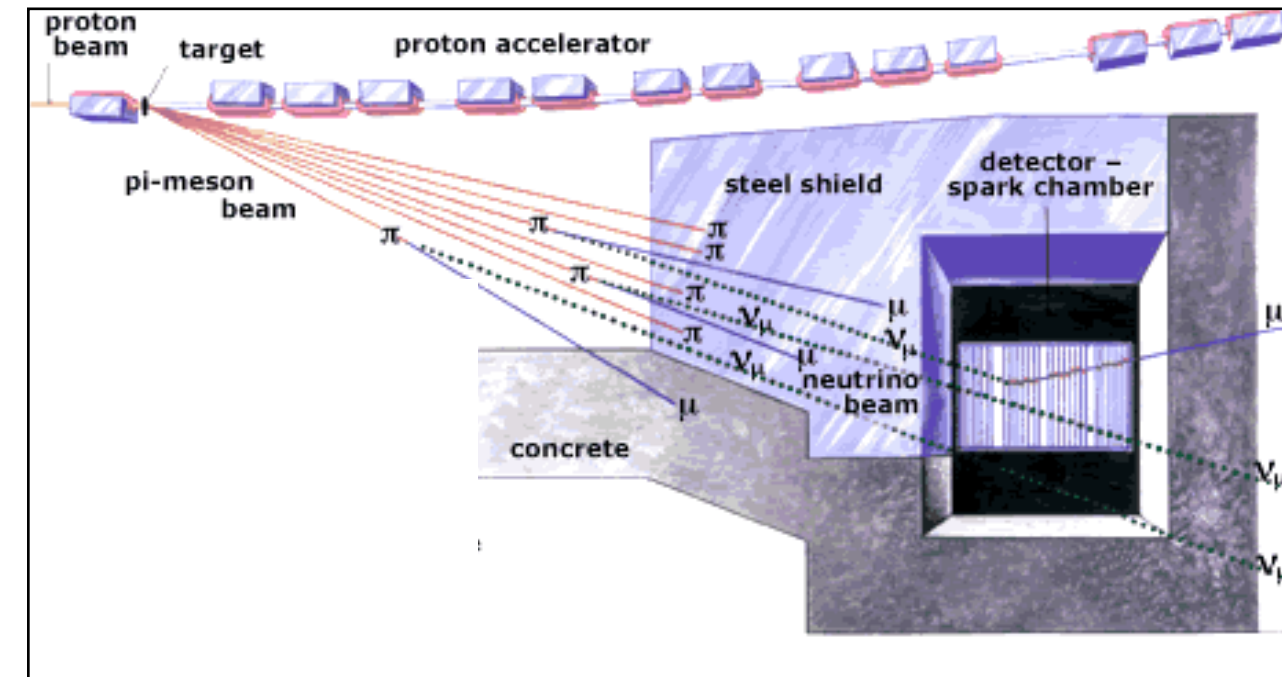
Muon Neutrino?

- 'Traditional' beta decays of nuclear particles produce electrons (or positrons) and neutrinos.
- For pion decays, only one in ten thousand decays is to electrons and neutrinos. Overwhelmingly though, a pion decays to muons (or anti-muons) and neutrinos (Steinberger, 1958)
- This leads Pontecorvo (there he is again!) to ask in his 1959 Russian paper:
 - Are those neutrinos that are produced with muons the same as those emitted in conventional beta decays?
- Additional evidence for muon neutrinos:
 - $\mu^- \rightarrow e^- + \gamma$ is **never** observed, while it is allowed (but suppressed) if there is only one neutrino:



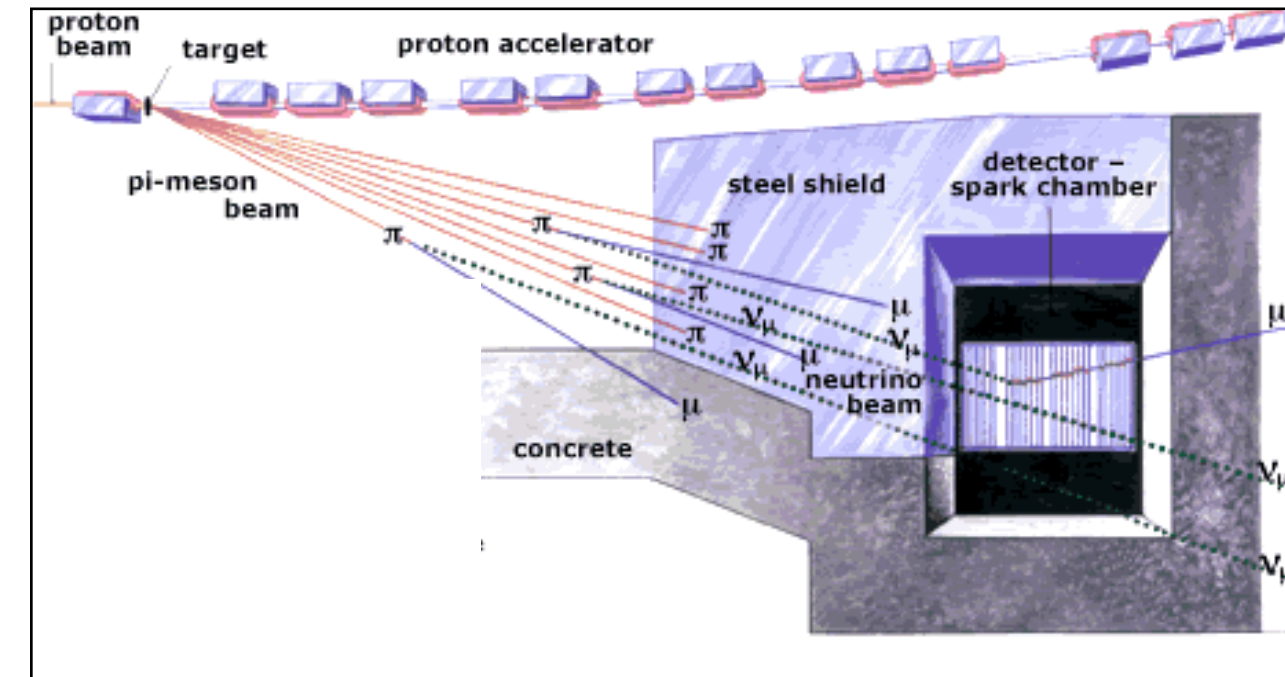
Muon Neutrino!

- Lederman, Schwartz and Steinberger (1962) use Brookhaven's accelerator again (used in 1957 for establishing that muon is spin 1/2 and violates parity)
- Schwartz's bright idea: use the neutrinos themselves! Higher energy means higher cross section means an actual chance to see the (muon?) neutrinos interacting
- Intense proton beams on beryllium gives large numbers of pions
- Pions decay to muons and (muon?) neutrinos.
- 'Use the neutrinos themselves' -> How? Discuss!



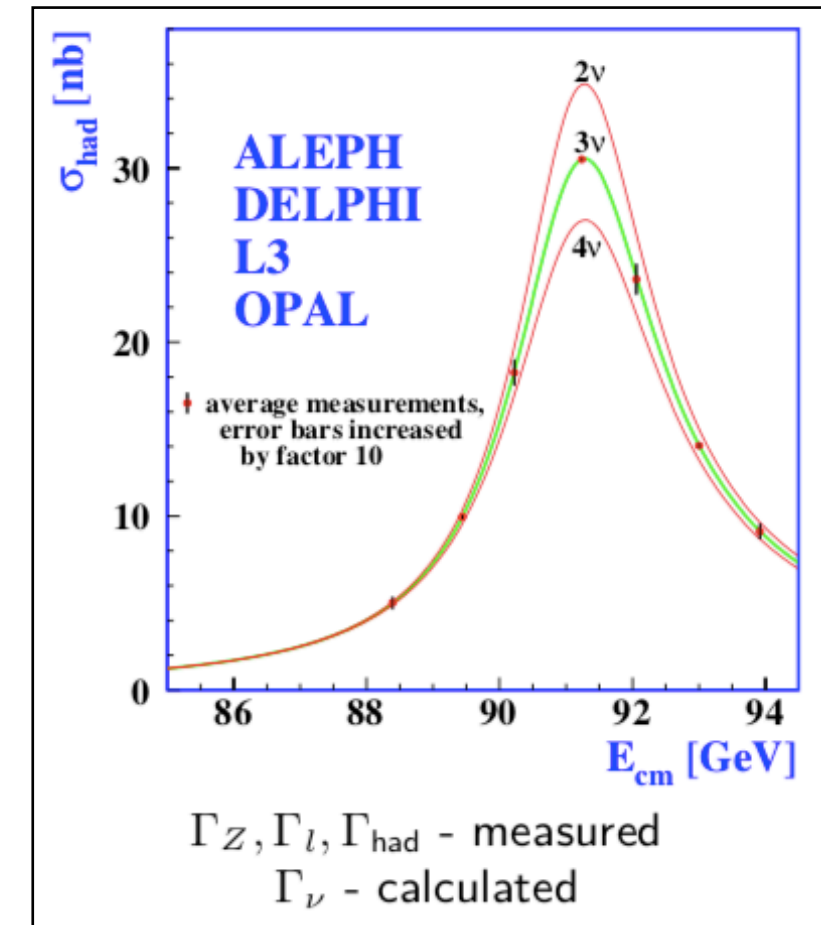
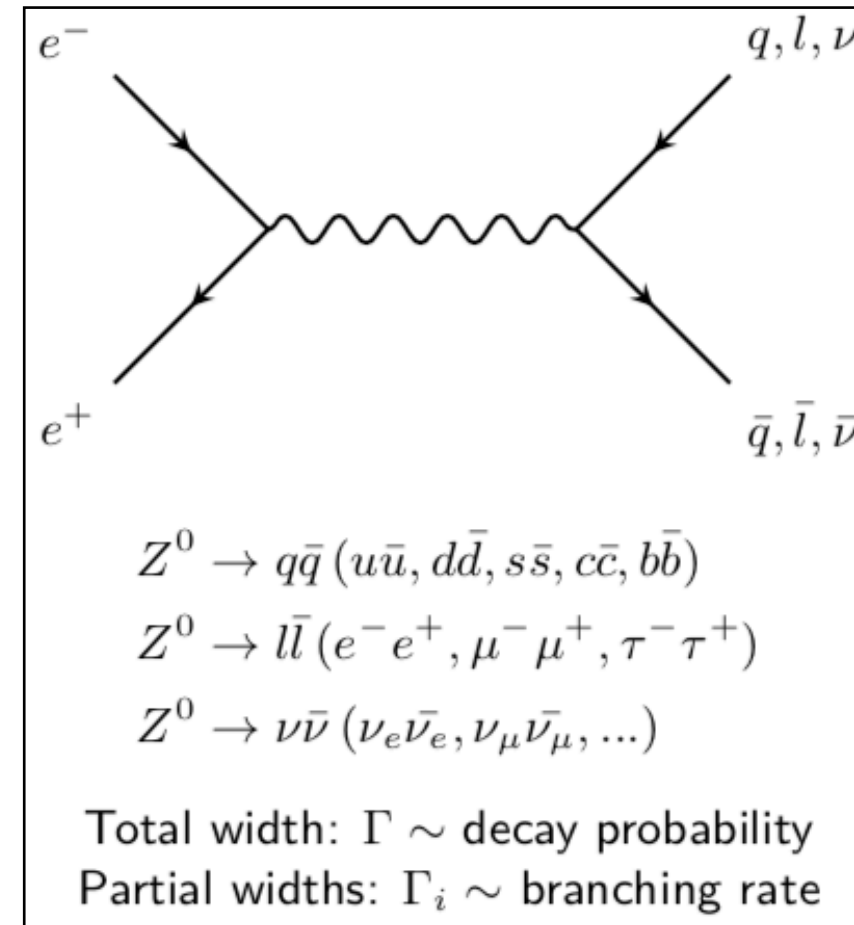
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- Intense proton beams on beryllium gives large numbers of pions
- Pions decay to muons and (muon?) neutrinos.
- Filter out the muons with thick steel plates and see how every now and then a neutrino creates **either a muon or an electron**.
- Ten days and 10^{14} neutrinos later, 51 **muons were detected and no electrons**.
- Conclusion: there is a muon-neutrino and it's completely different from the electron-neutrino.



Number of Neutrino Species

- Electroweak theory developed by Glashow, Weinberg and Salam (GWS) in the sixties predicts the existence of $W^{+/-}$ and Z^0
 - Discovered in the early eighties at CERN in the UA1 and UA2 experiment
- Z^0 implies neutral current interactions for neutrinos
 - Found in early seventies in Gargamelle at CERN
- Tau discovered in 1975, predicting the associated tau neutrino.
- 1989: LEP experiments measure the 'invisible' width of the Z^0



$$\Gamma_Z = \Gamma_{\text{hadronic}} + 3\Gamma_{\text{leptonic}} + N_\nu \Gamma_\nu$$

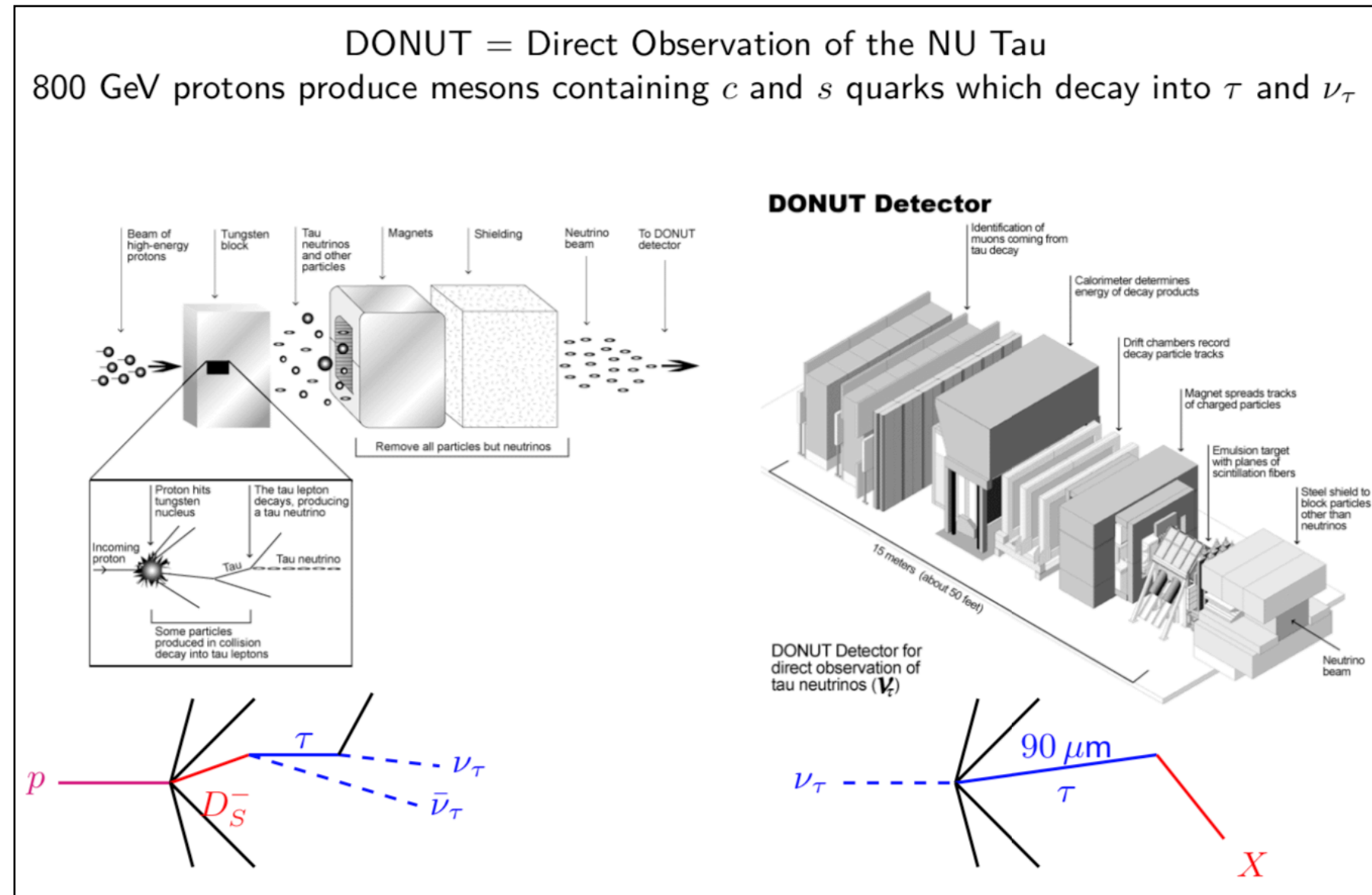
$$N_\nu = 2.99 \pm 0.02$$

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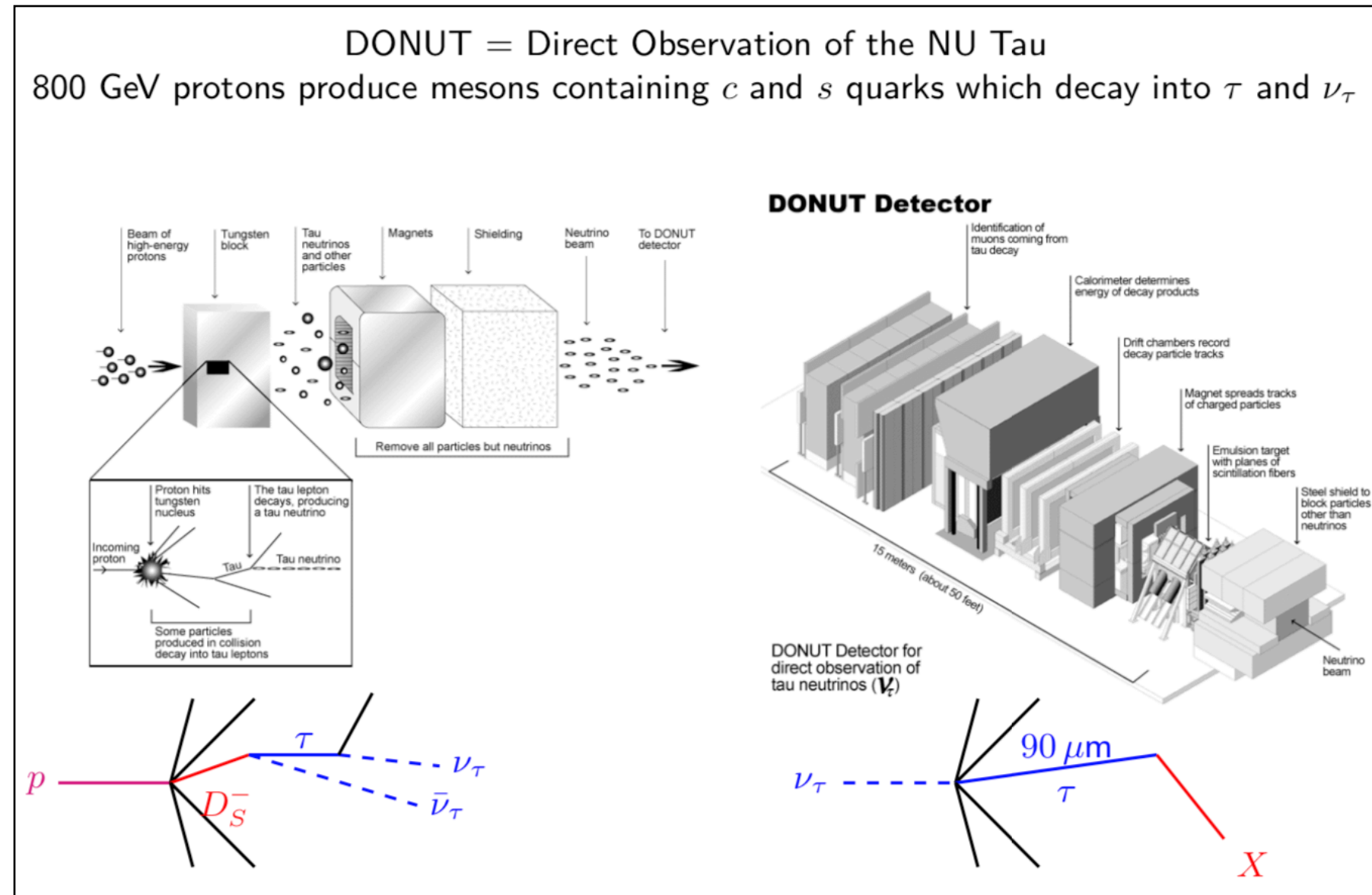
Tau Neutrino Discovery

- Detecting a ν_τ is a challenge:
 - ν_τ has to produce a τ lepton
 - One has to track the τ
 - τ lifetime is 3×10^{-13} s (ct = 90 μm)
- How do you measure such a short lifetime
- Hint: in LHC, you know where and when to look, here, you don't!



Tau Neutrino Discovery

- Detecting a ν_τ is a challenge:
 - ν_τ has to produce a τ lepton
 - One has to track the τ
 - τ lifetime is 3×10^{-13} s (ct = 90 μ m)
- Use nuclear emulsion
- 2000: DONUT experiment at Fermilab
 - DONUT searched for decays into 1 charged particle (86% of τ decays)
 - Out of 10^{13} neutrinos, only 1000 ν interactions recorded out of which 4 (!) were identified as ν_τ



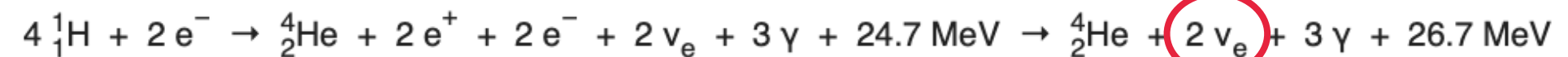
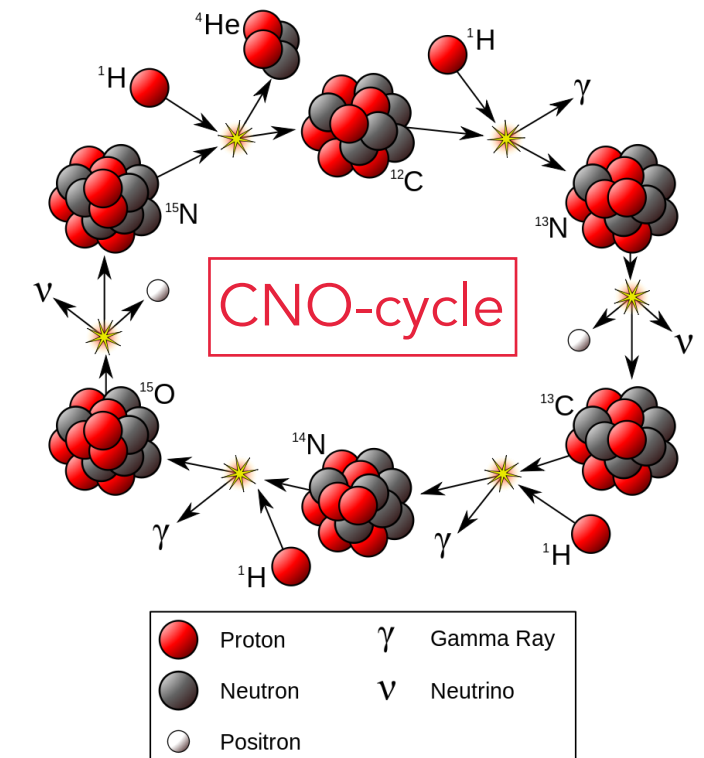
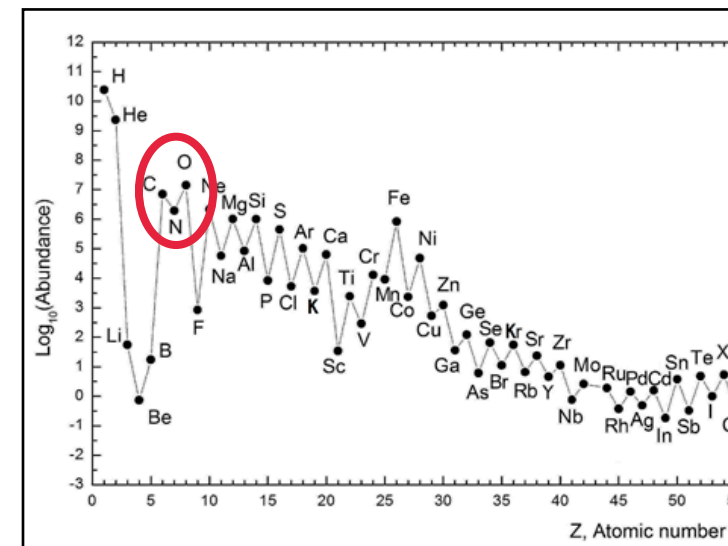
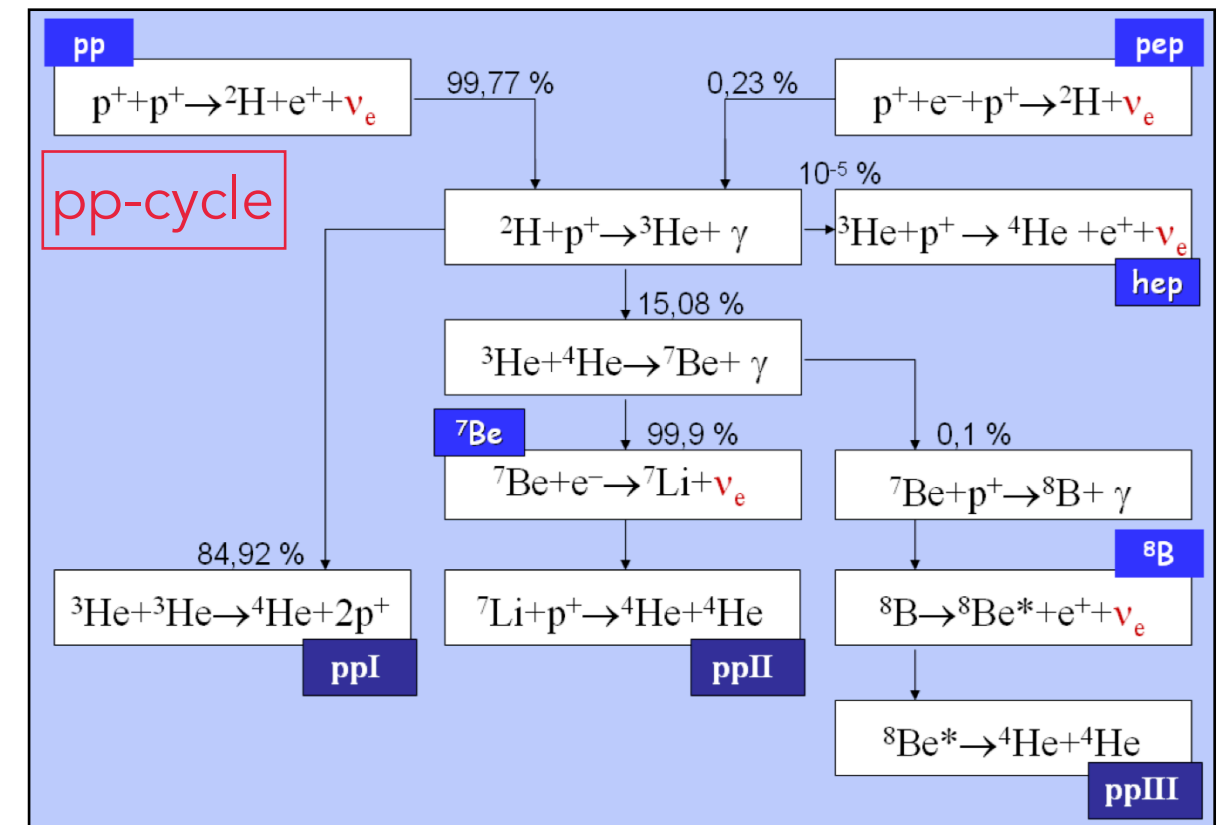
Questions

- Why would one need a neutrino?
- How would you discover a neutrino?
- How would you figure out that a neutrino is different from an anti-neutrino?
- How would you figure out that a muon neutrino is different from an electron neutrino?
- How would you figure out that a tau neutrino is different from an electron and a muon neutrino?
- **How would you solve the solar neutrino puzzle?**

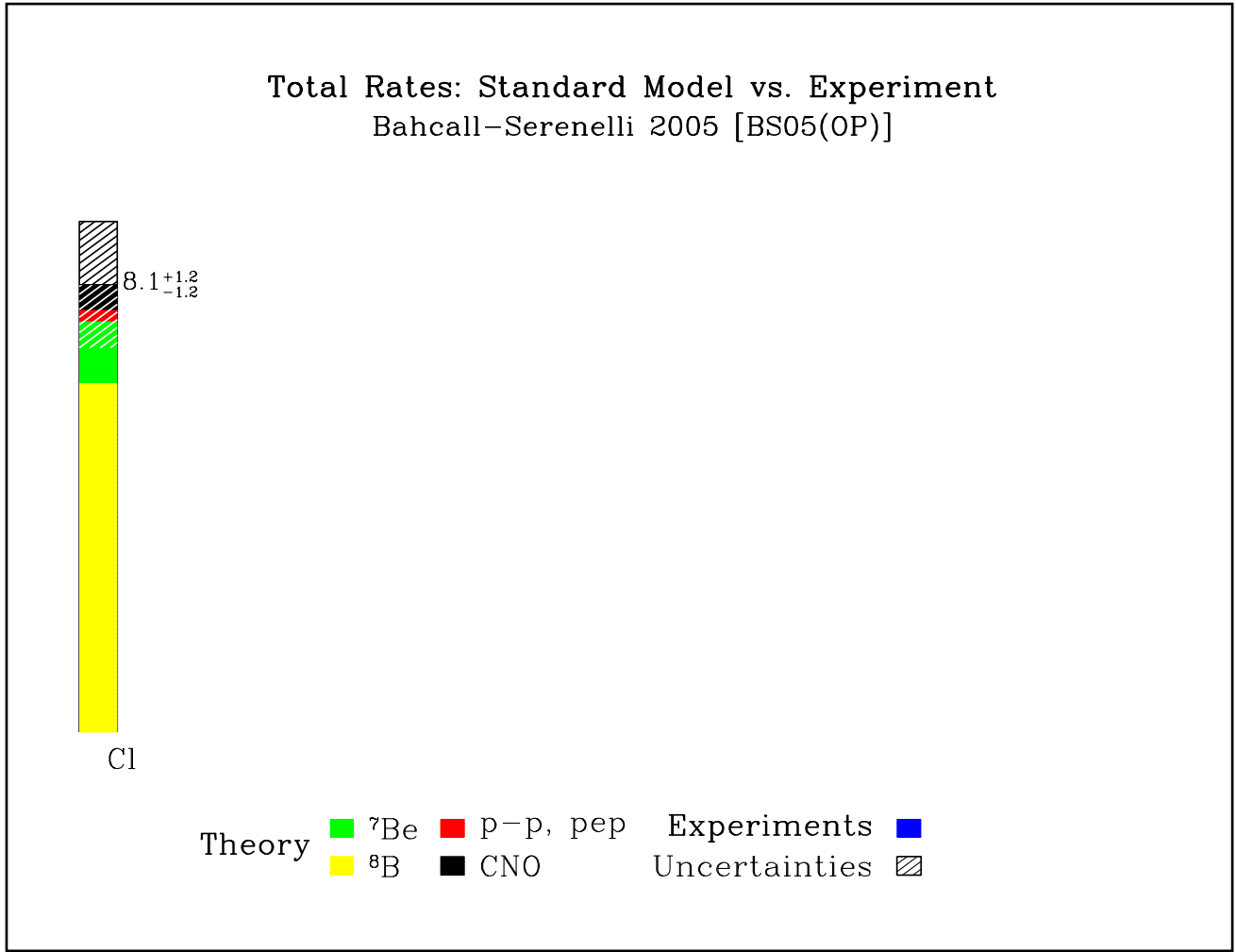
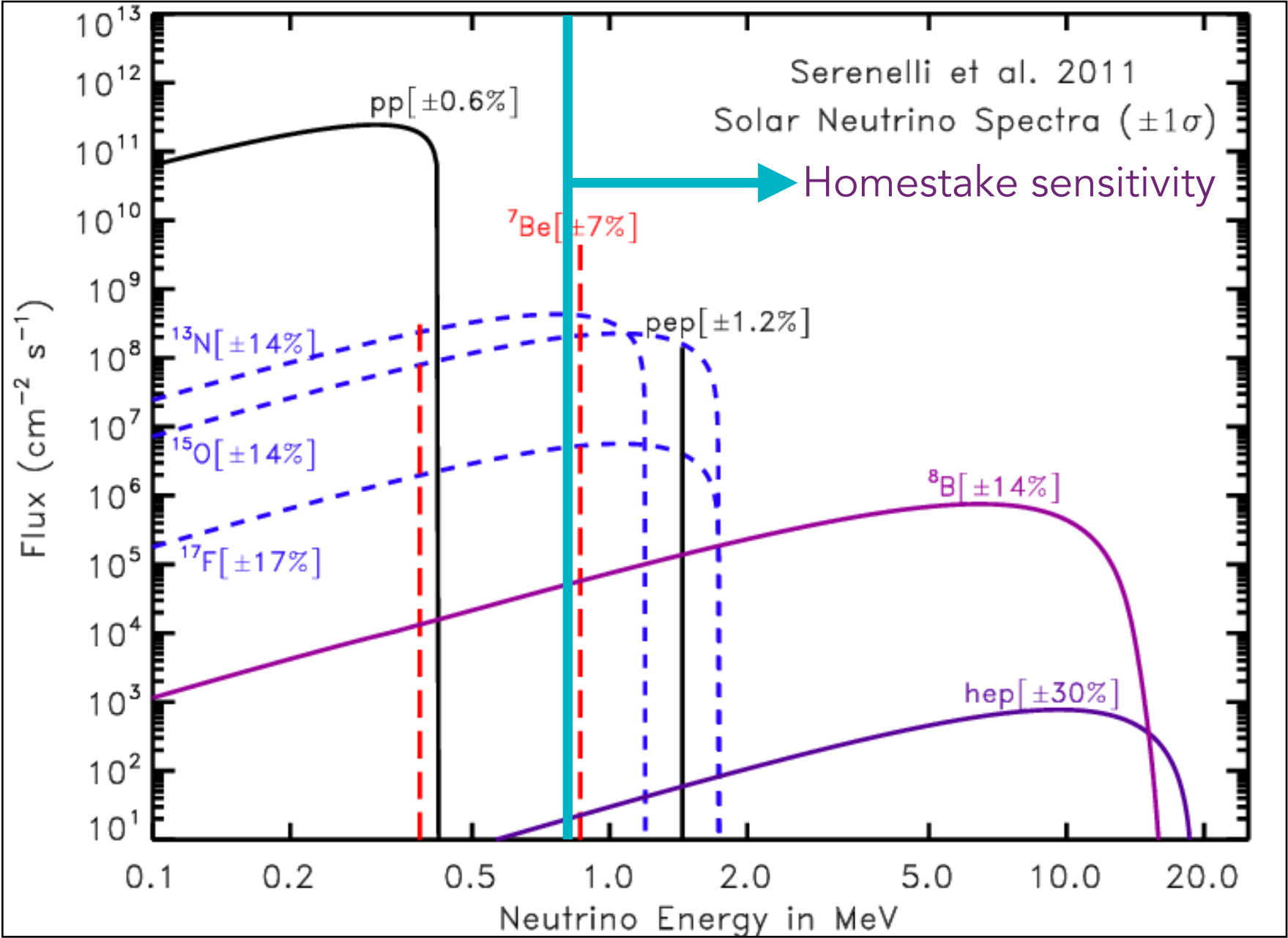
What Davis and Bahcall were Doing in the Meantime...

- Remember Raymond Davis?

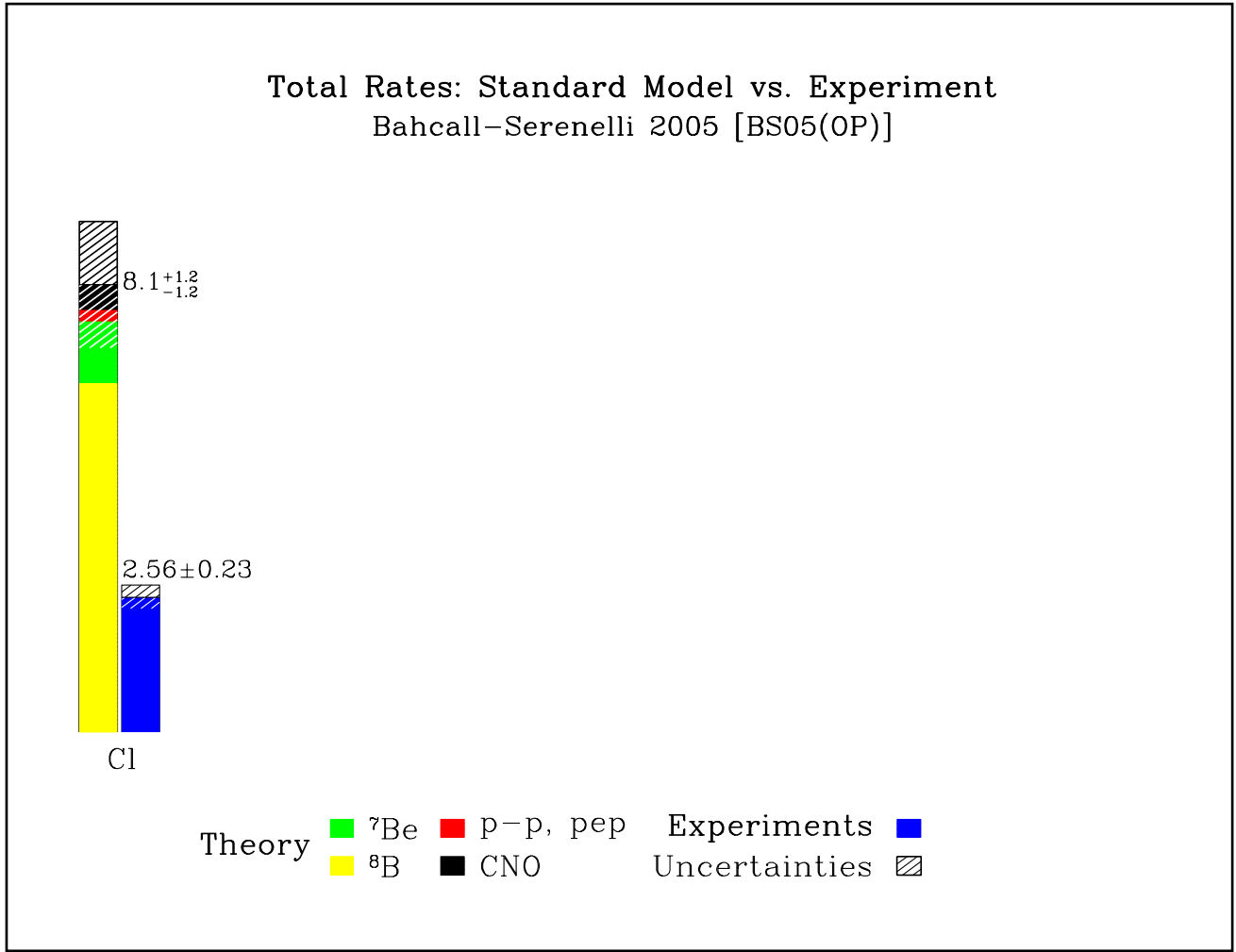
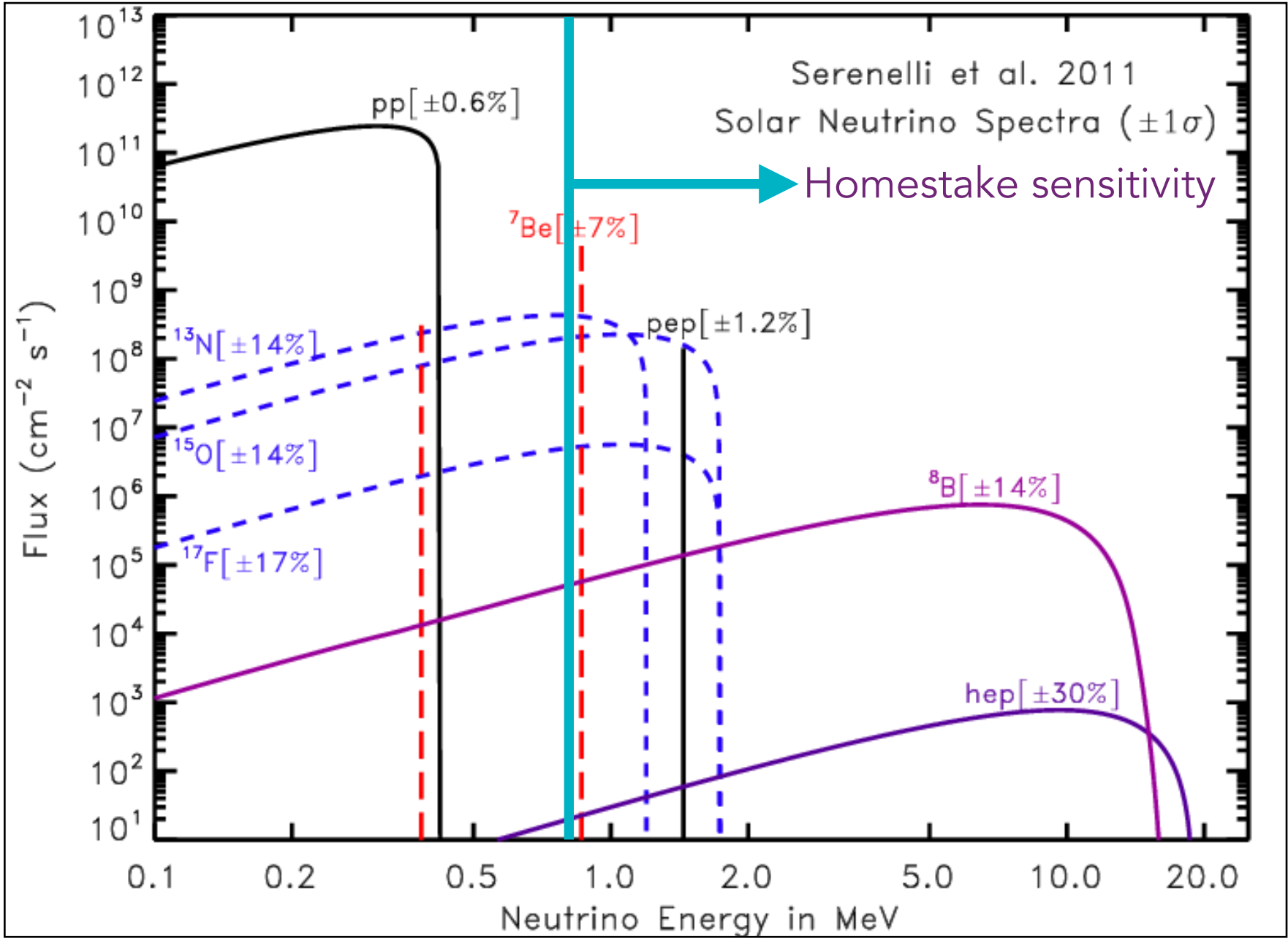
$$\nu + {}^{37}\text{Cl} \rightarrow e^{-} + {}^{37}\text{Ar}$$
- After not finding anything close to reactors, he turned to the Sun as a neutrino source:
 - "to see into the interior of a star and thus verify directly the hypothesis of nuclear energy generation in stars"
- Solar neutrino fluxes calculated by Bethe and later Bahcall
 - Pure ν_e
 - Flux on Earth: $\sim 10^{11}/\text{cm}^2/\text{sec}$
 - Energy: $< 10 \text{ MeV}$



The Homestake experiment: 1965-1994 (!!!)

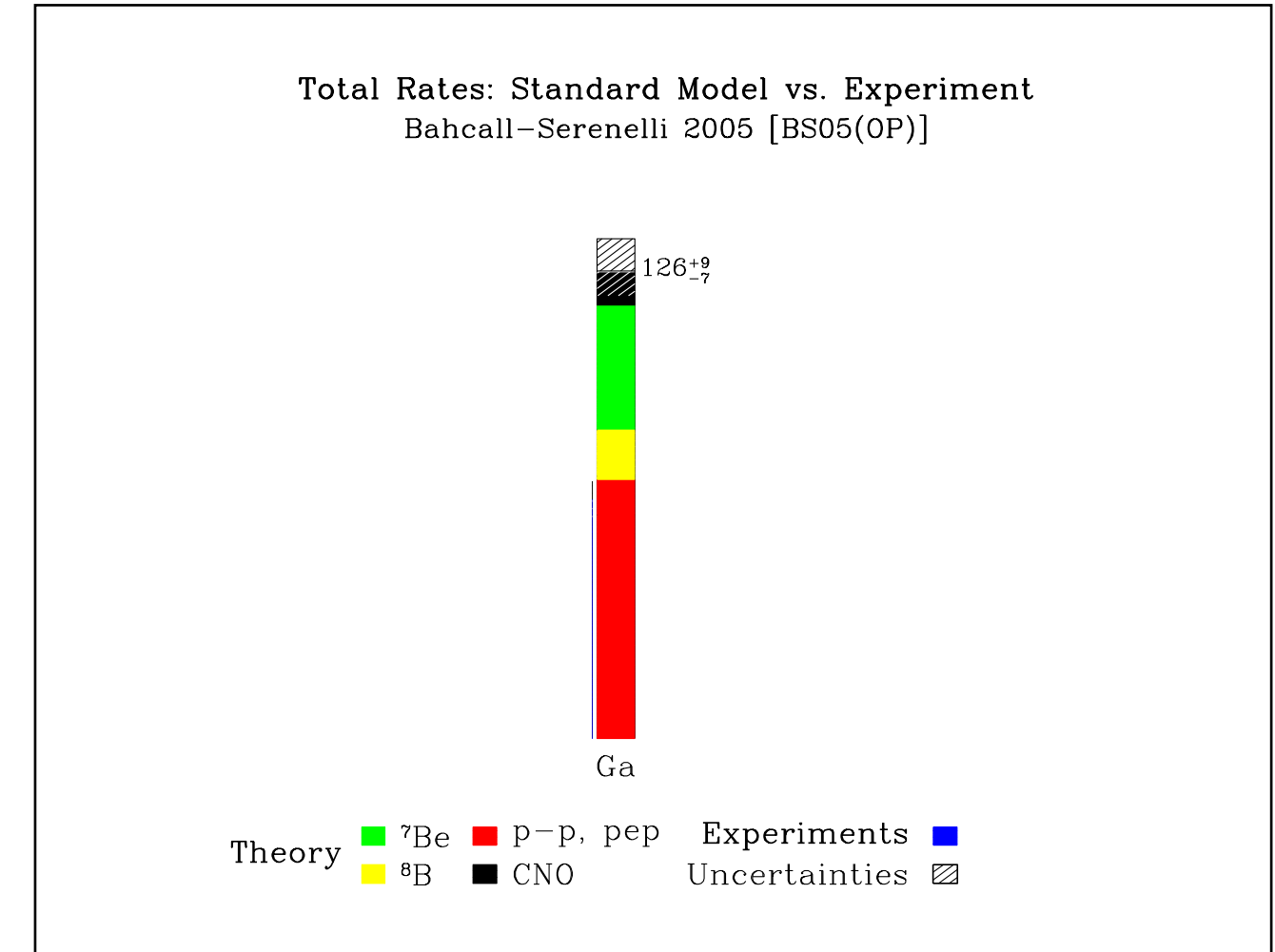
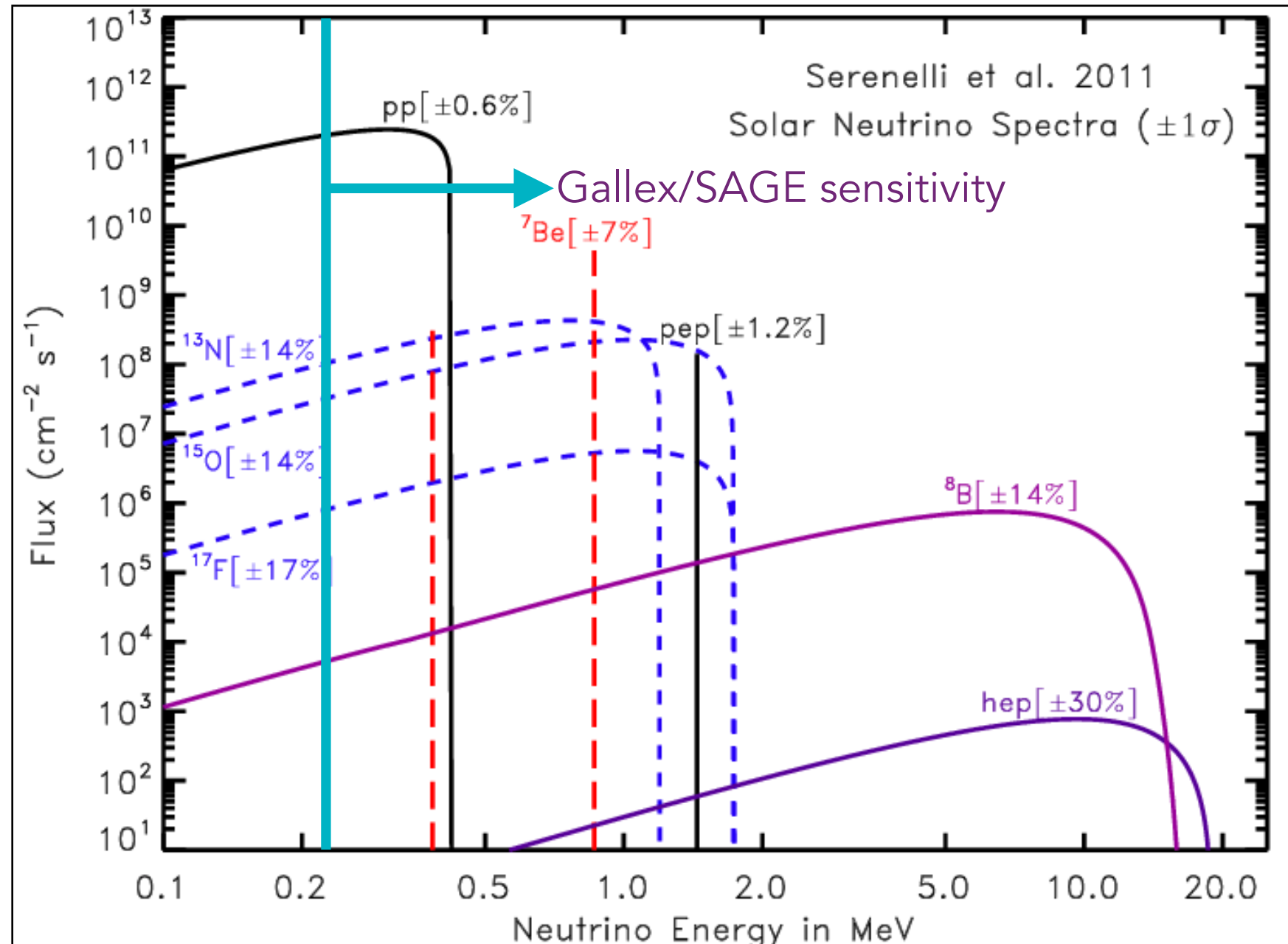


The Homestake experiment: 1965-1994 (!!!)



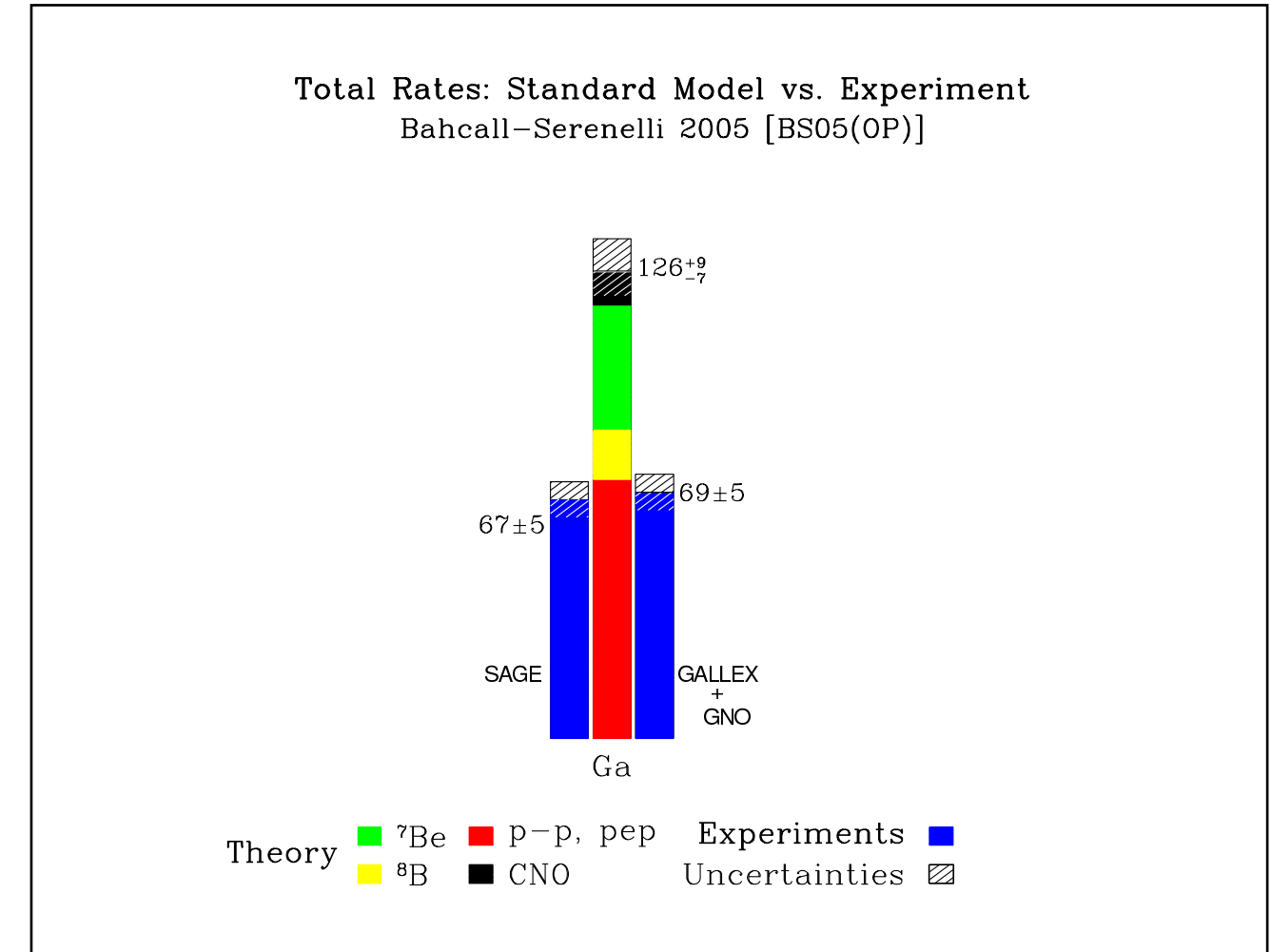
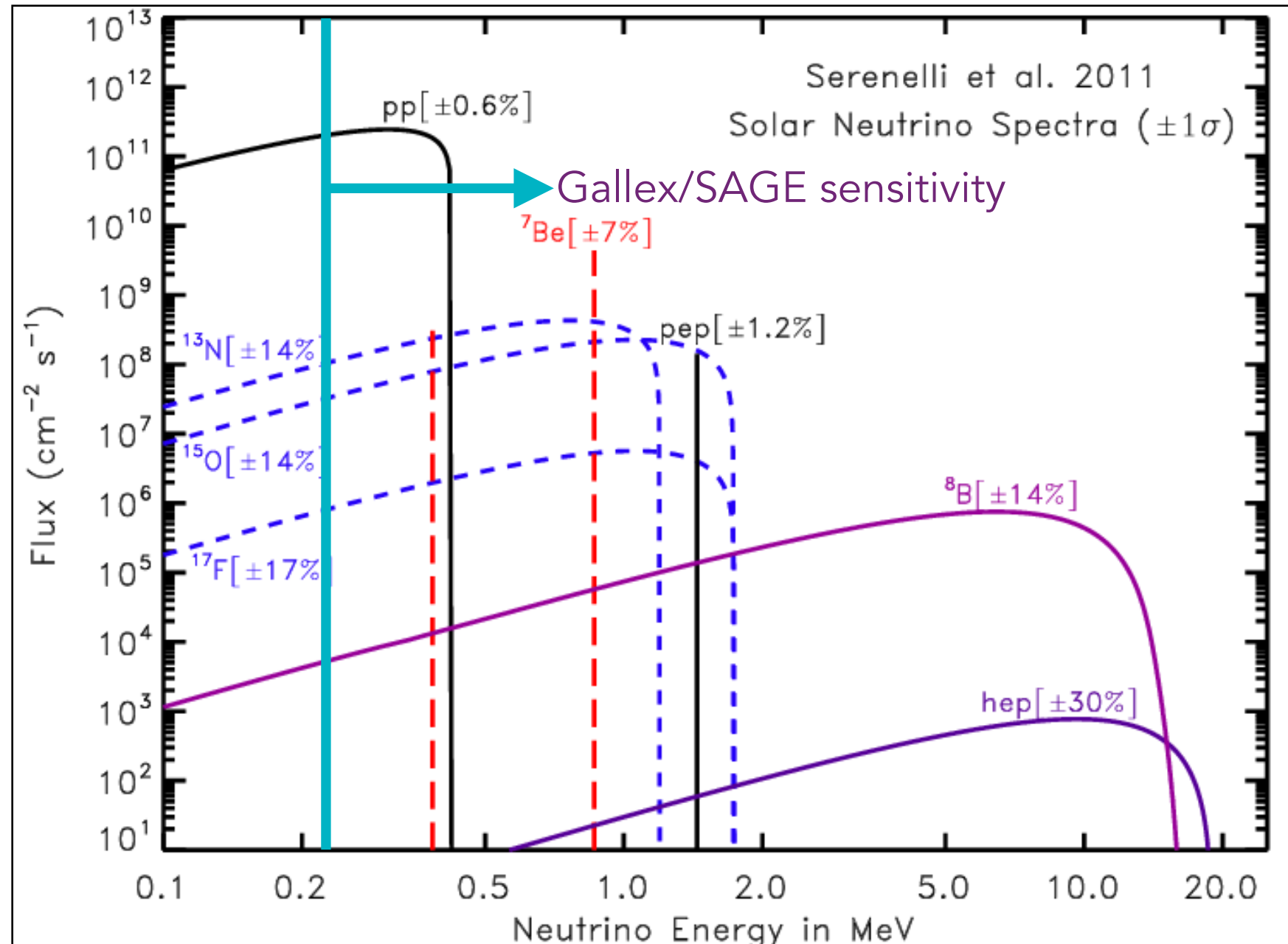
GALLEX (1991-1998) and SAGE (1990-)

- $\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$, Gallium is expensive: SAGE and GALLEX held world's entire supply

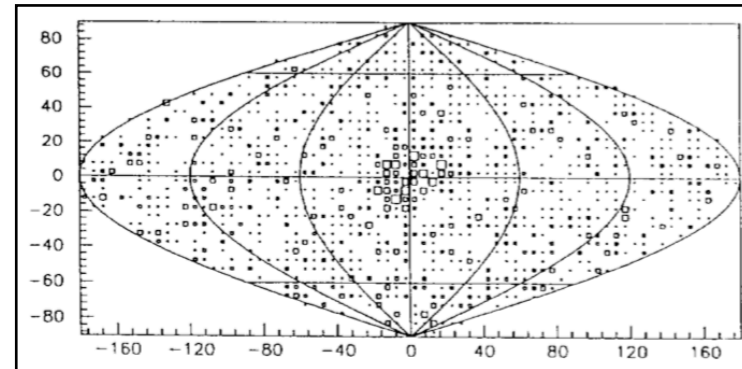
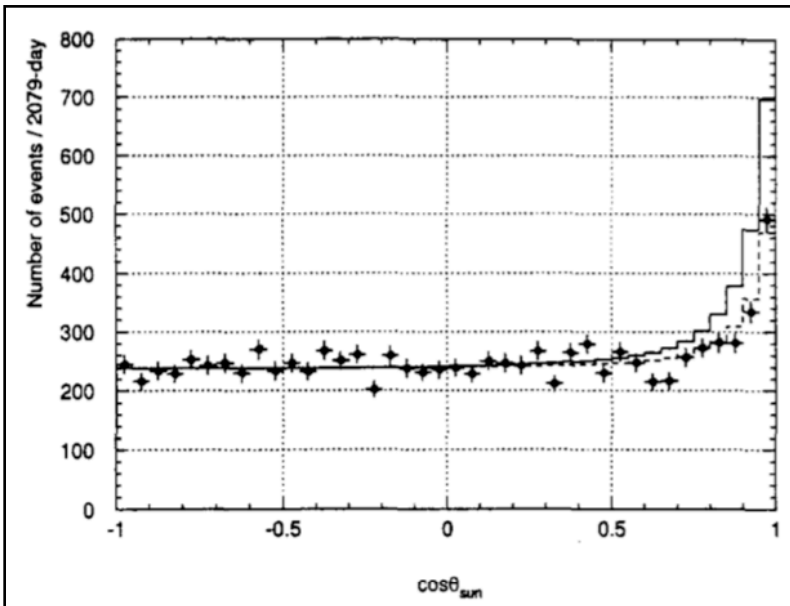


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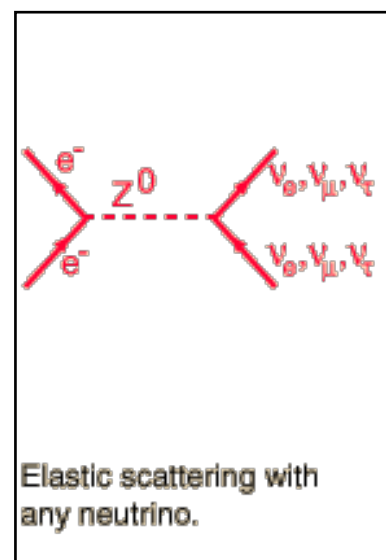
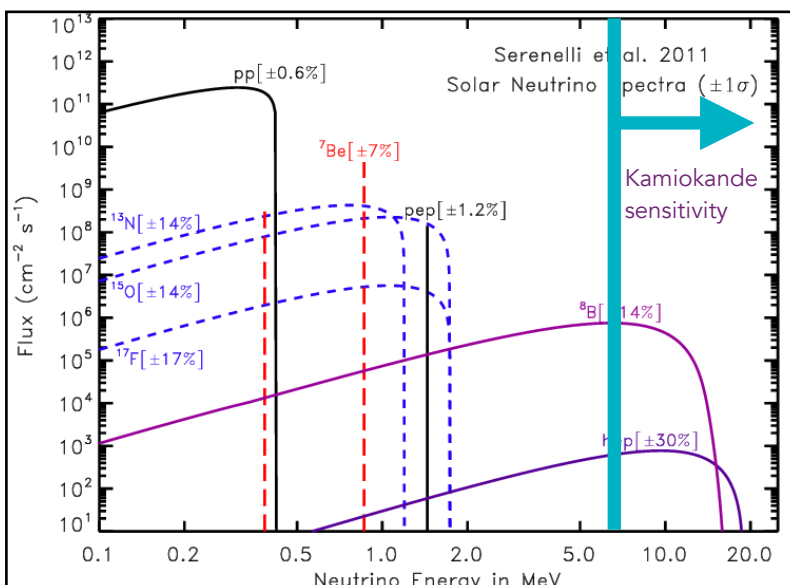
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Kamiokande (1987-1995)

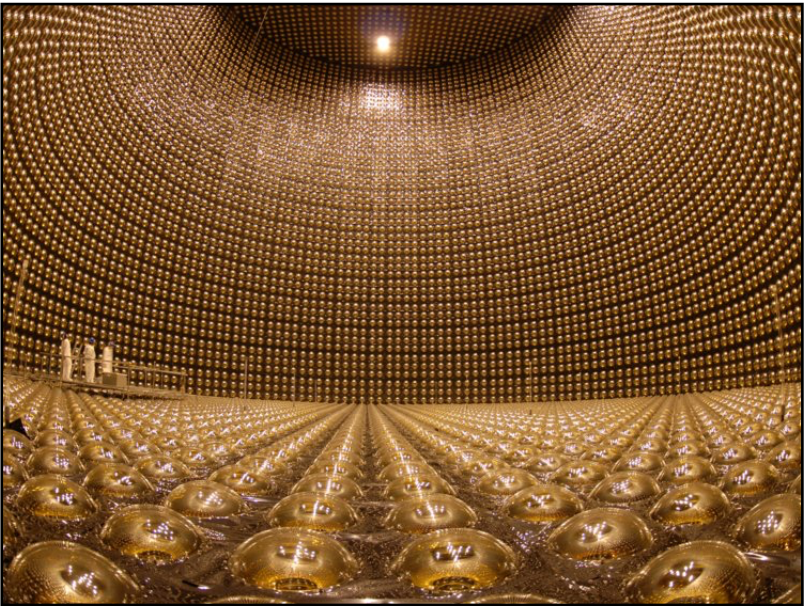
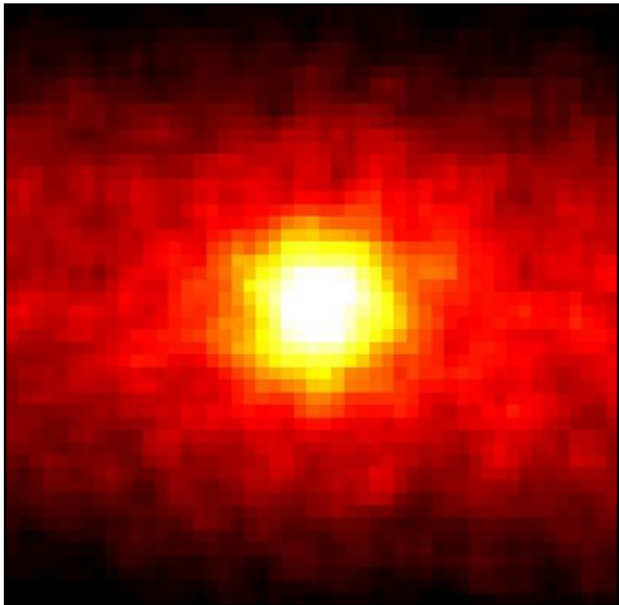
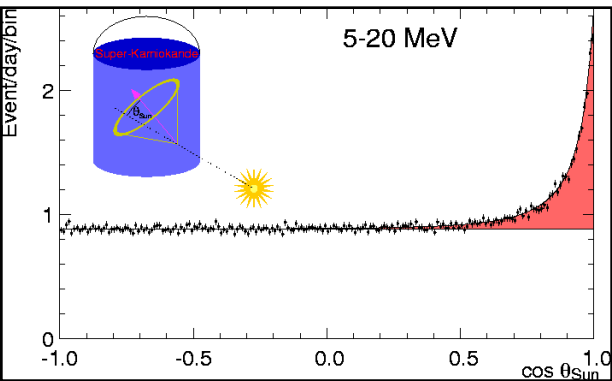


$$\frac{Data}{SSM_{BP92}} = 0.492 \pm 0.033(stat.) \pm 0.06(sys.)$$

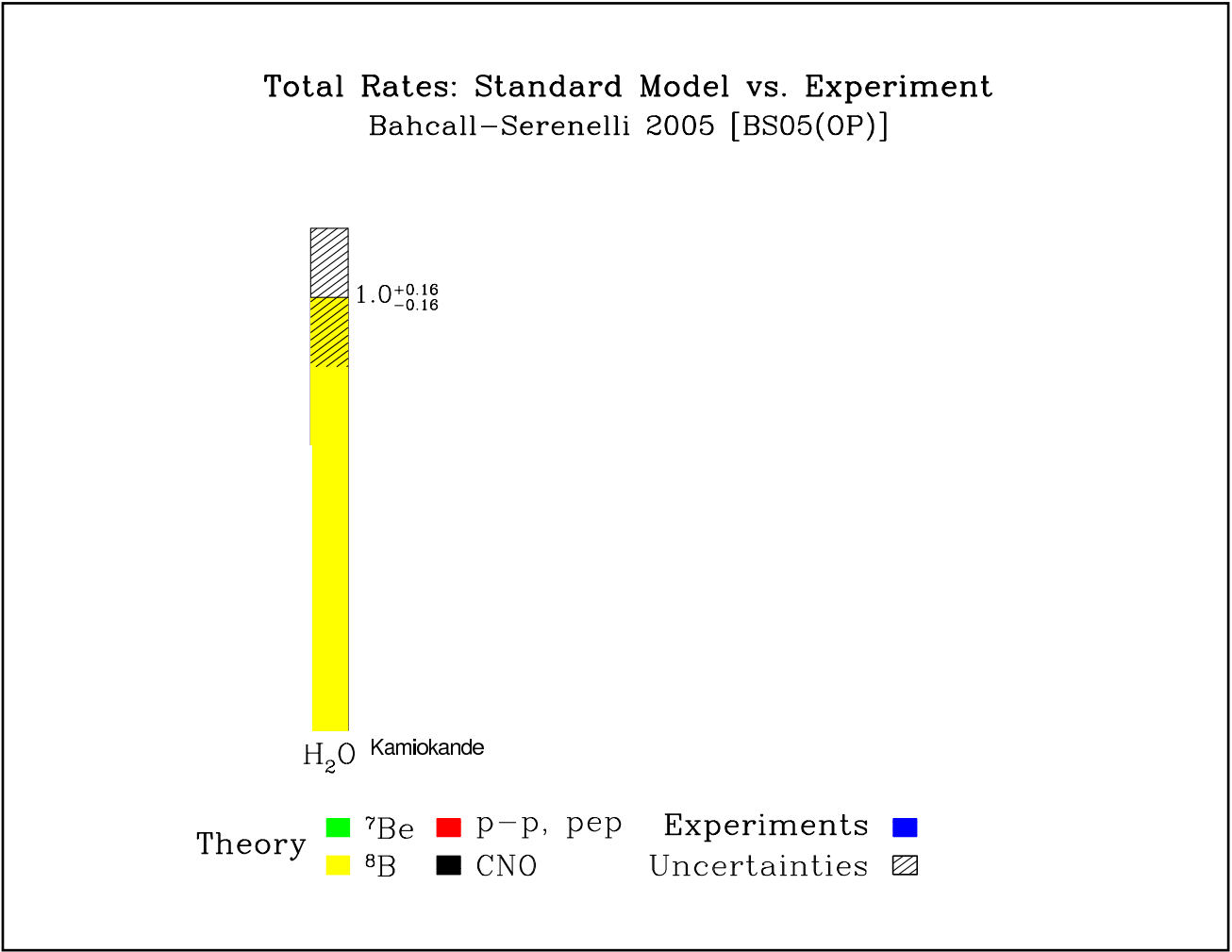
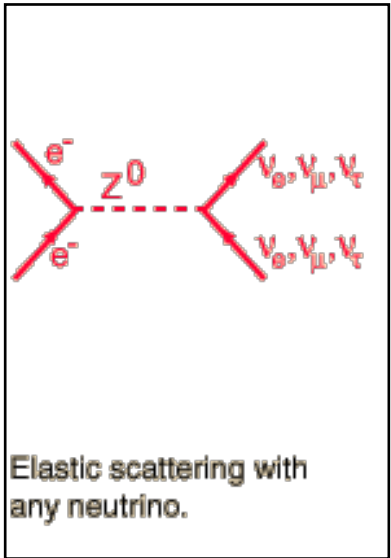
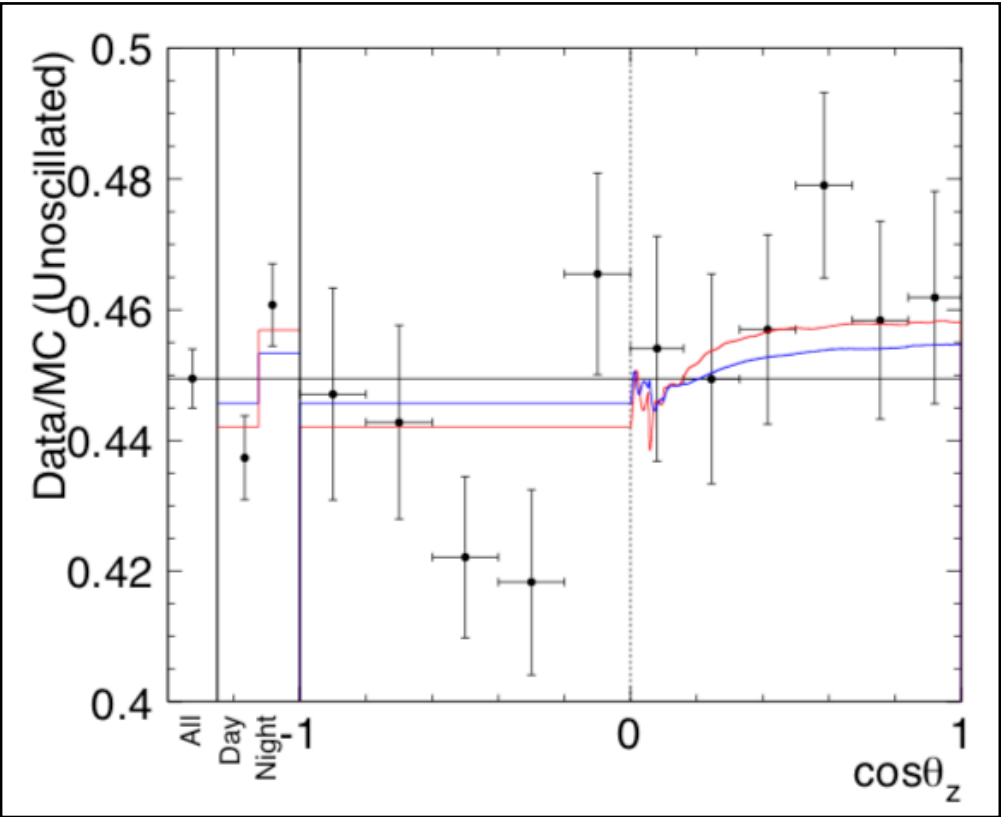


- Water-Cherenkov detector in Kamioka mine, Japan
- Neutron Decay Experiment, refurbished to Neutrino Detection Experiment, ready late 1986
 - Water purification, better electronics, installation of outer veto detector
- Time, direction and energy
- Beginning of neutrino astronomy
 - First directional information -> neutrinos actually come from the sun!
 - They got really lucky: SN1987A on 23 February 1987
- Hints for atmospheric neutrino deficit (more on that later)

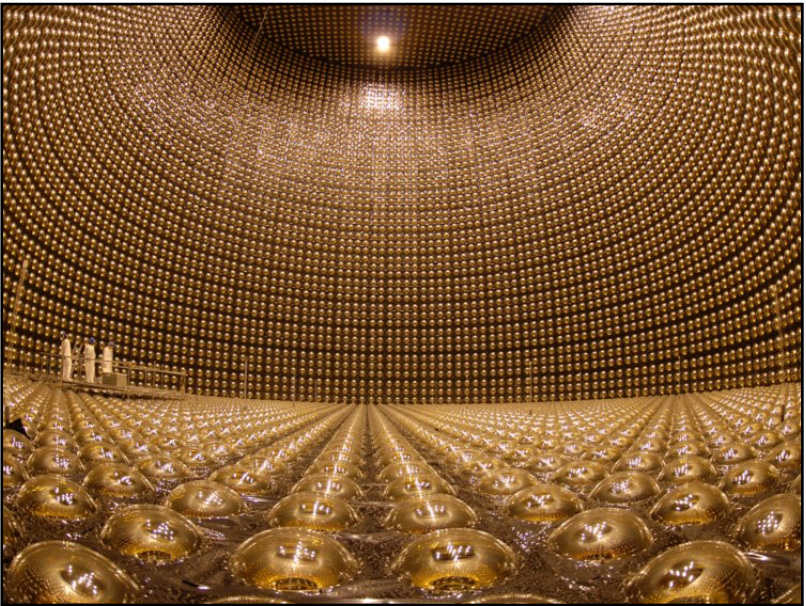
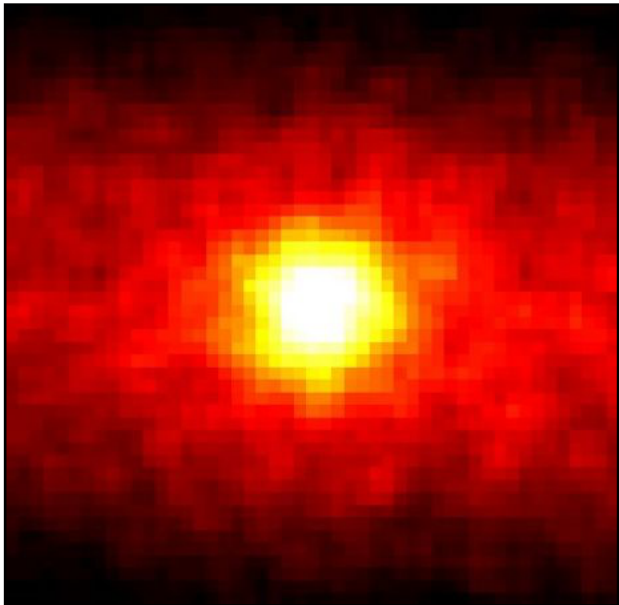
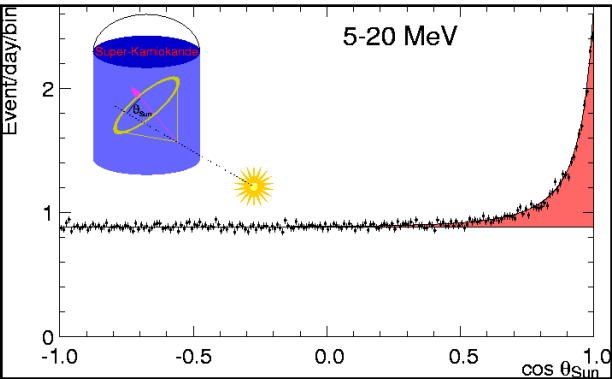
Super Kamiokande (SuperK, 1996-)



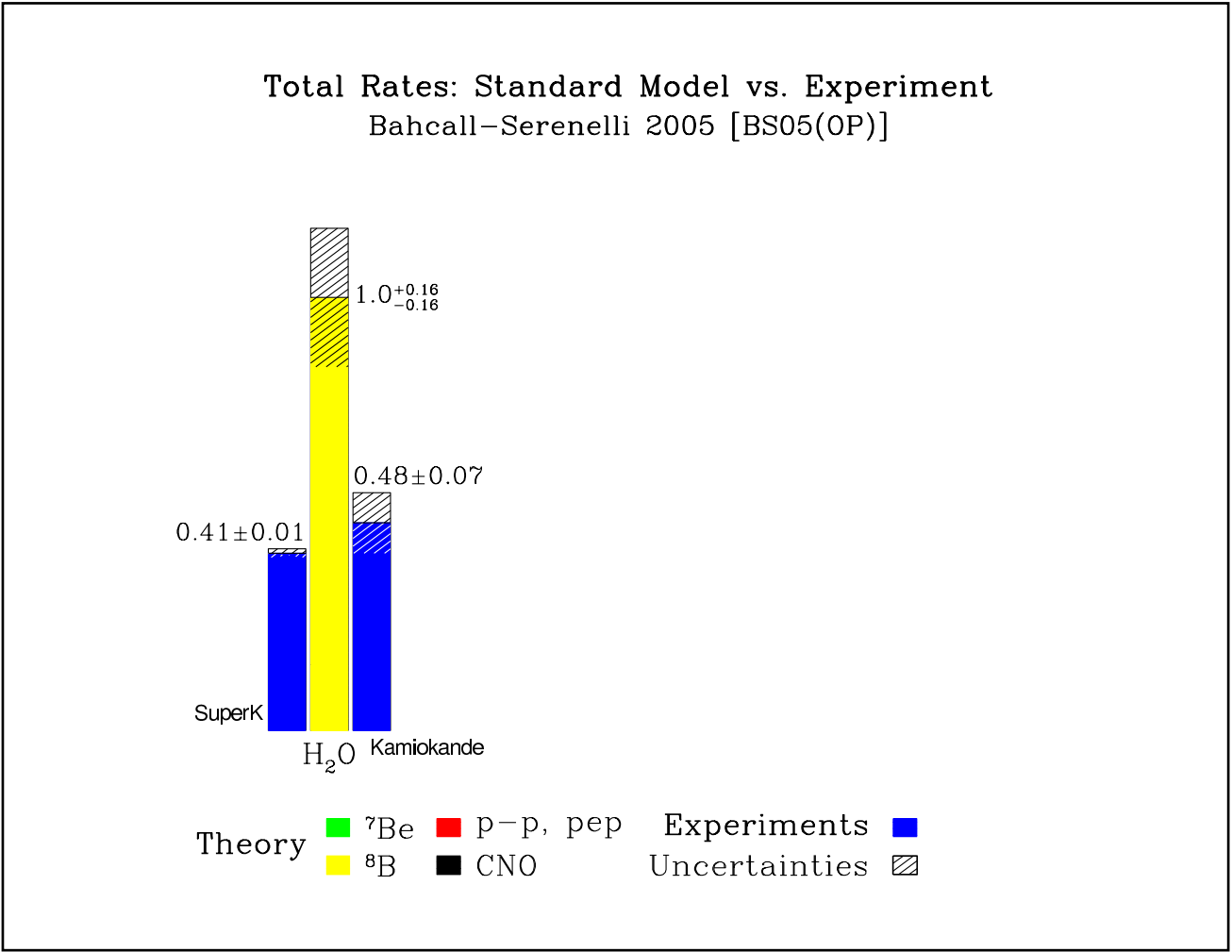
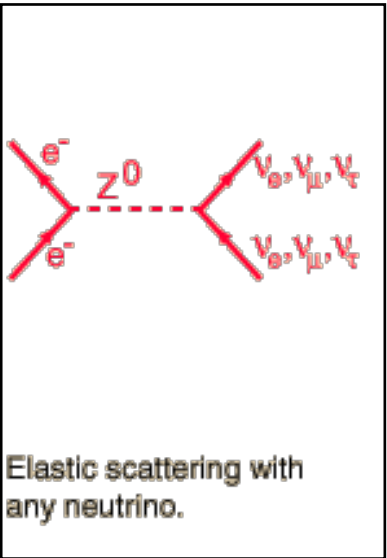
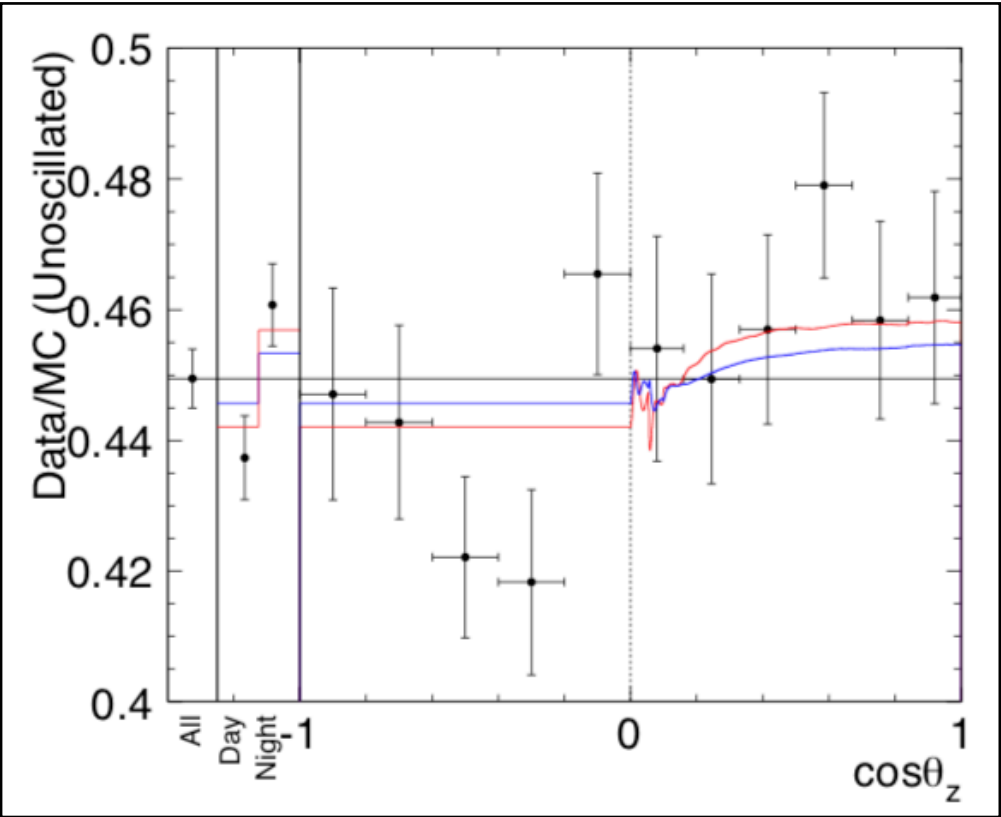
More on
atmospheric and
accelerator
neutrinos in
SuperK later....



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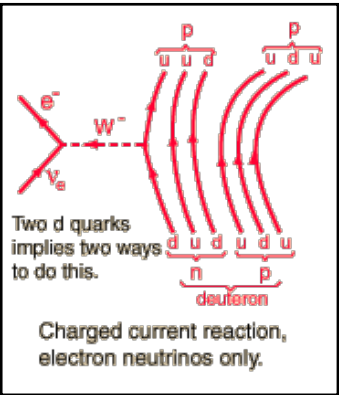
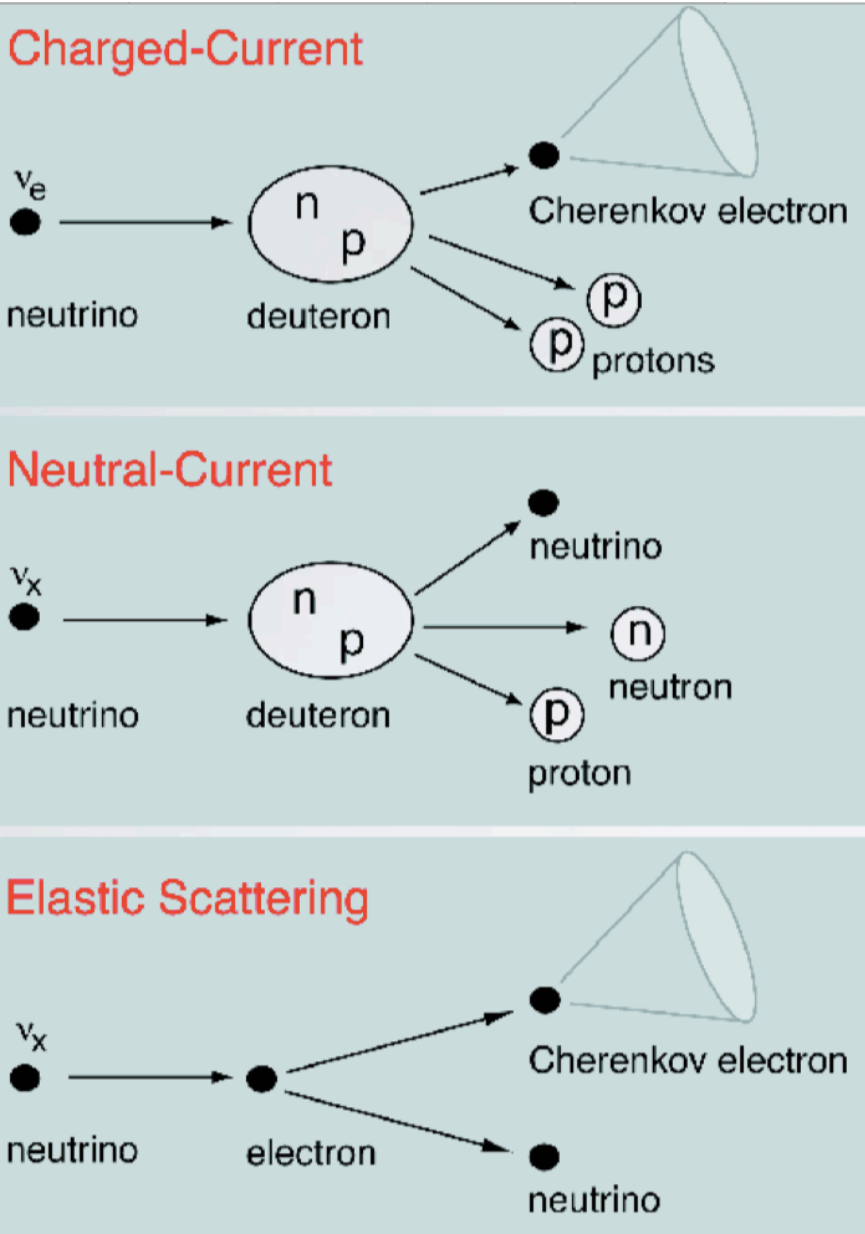
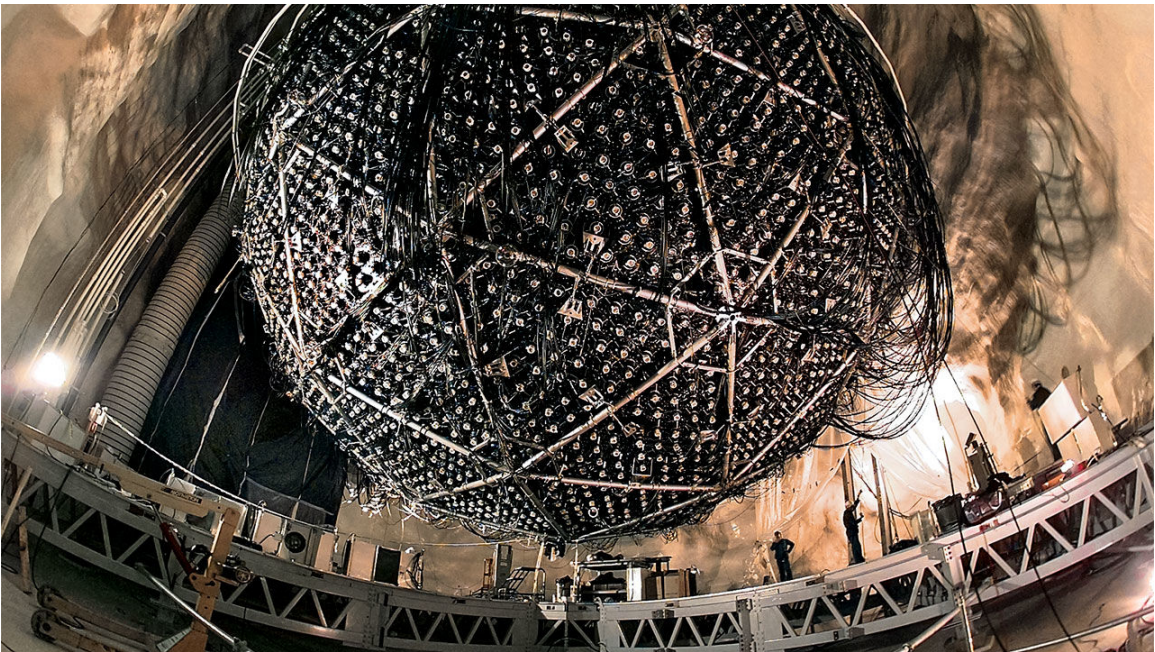


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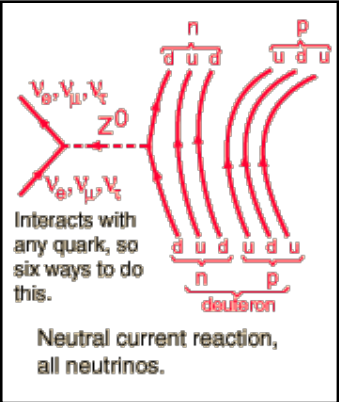


SNO

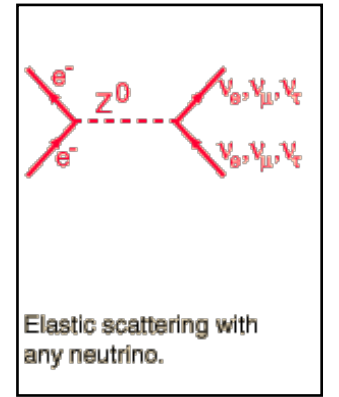
- Sudbury mine in Canada
- Heavy water Cherenkov detector
- Can distinguish ratio of electron (CC) to all neutrino flavours (NC) and solve the neutrino puzzle on its own



Does not happen for ν_μ and ν_τ due to final lepton mass threshold



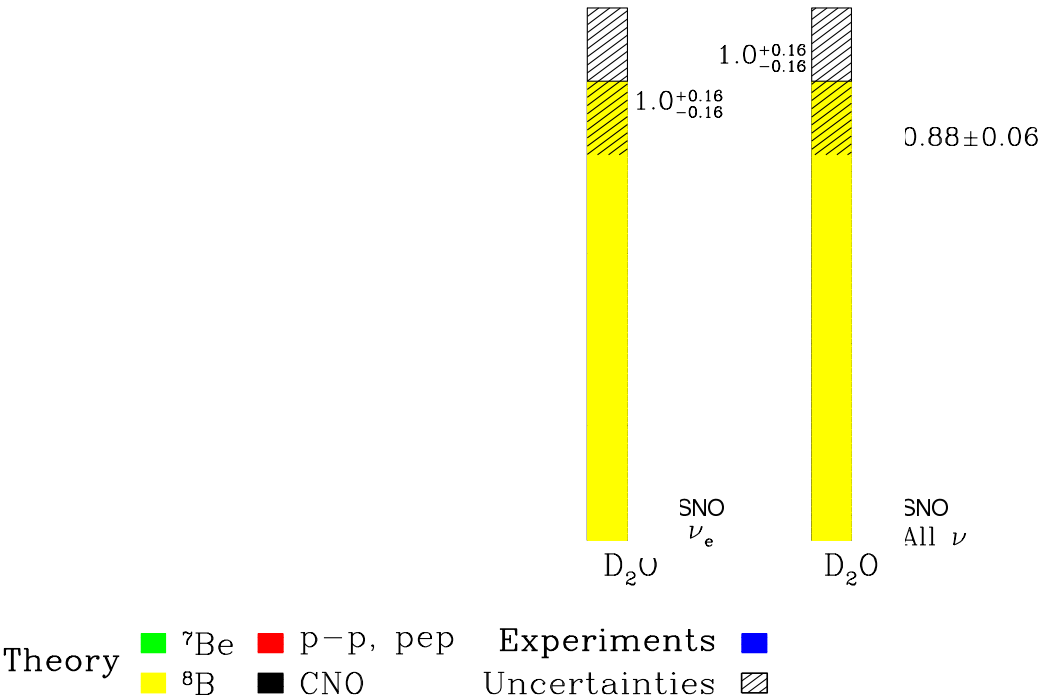
Treats all neutrino flavours the same!



Probes

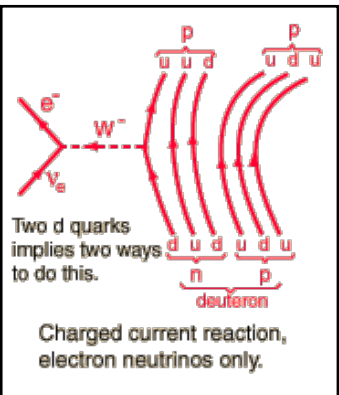
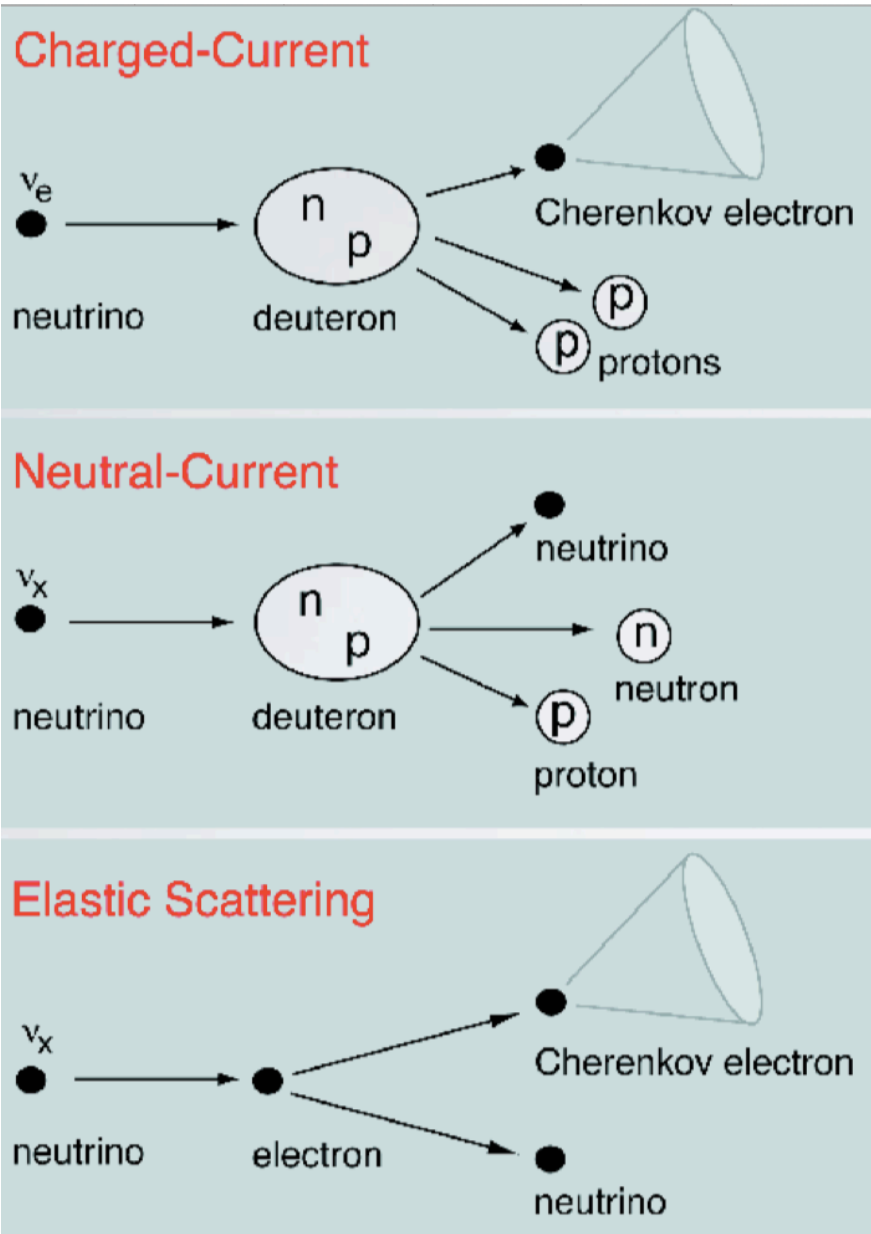
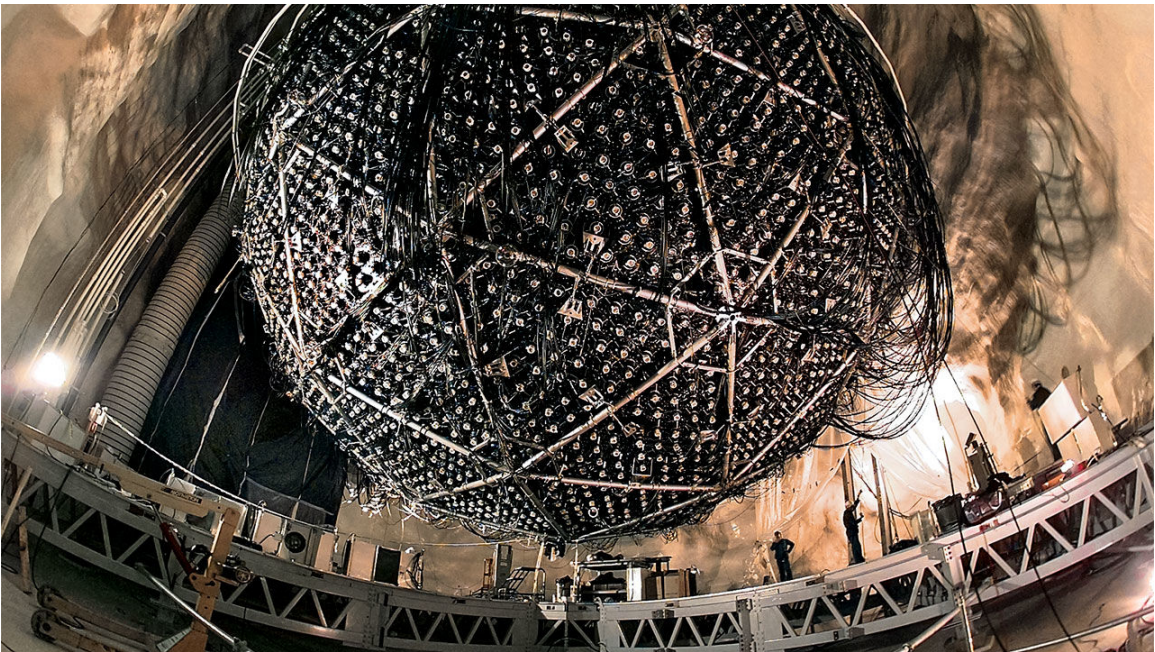
$$\phi(\nu_e) + 0.15(\phi(\nu_\mu) + \phi(\nu_\tau))$$

Total Rates: Standard Model vs. Experiment
Bahcall–Serenelli 2005 [BS05(OP)]

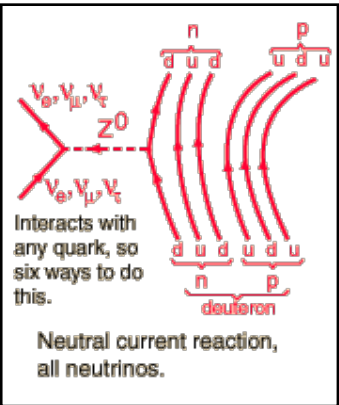


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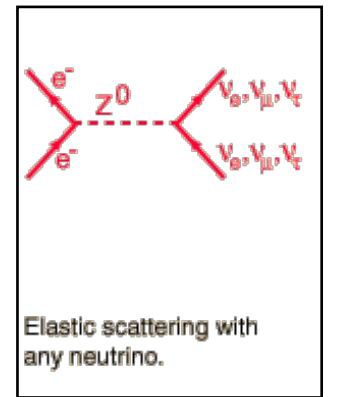
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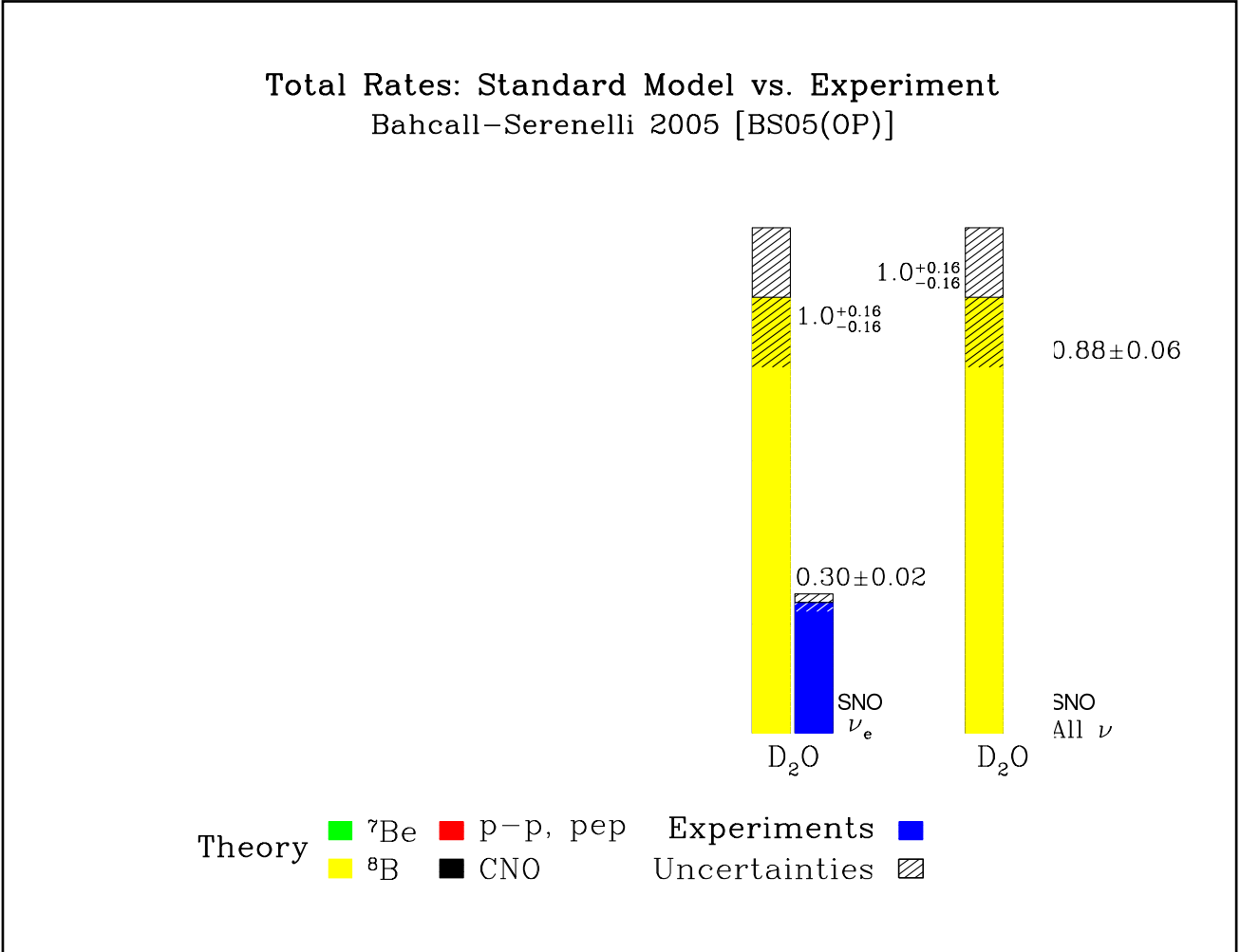


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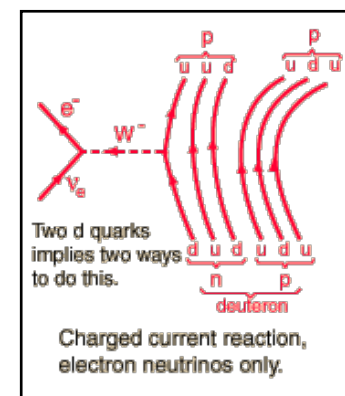
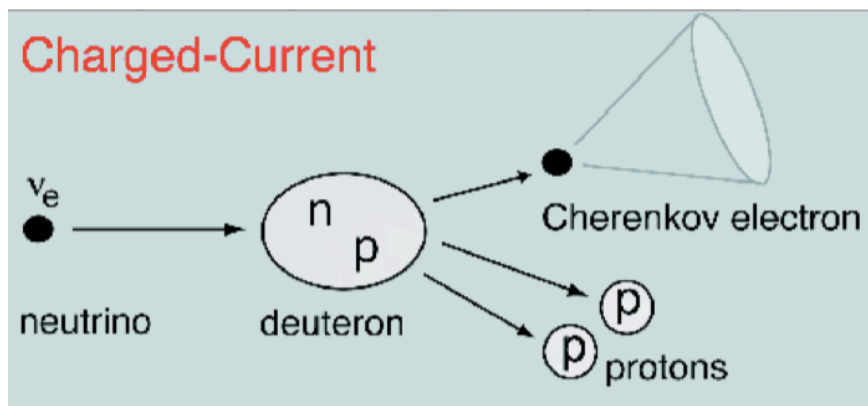
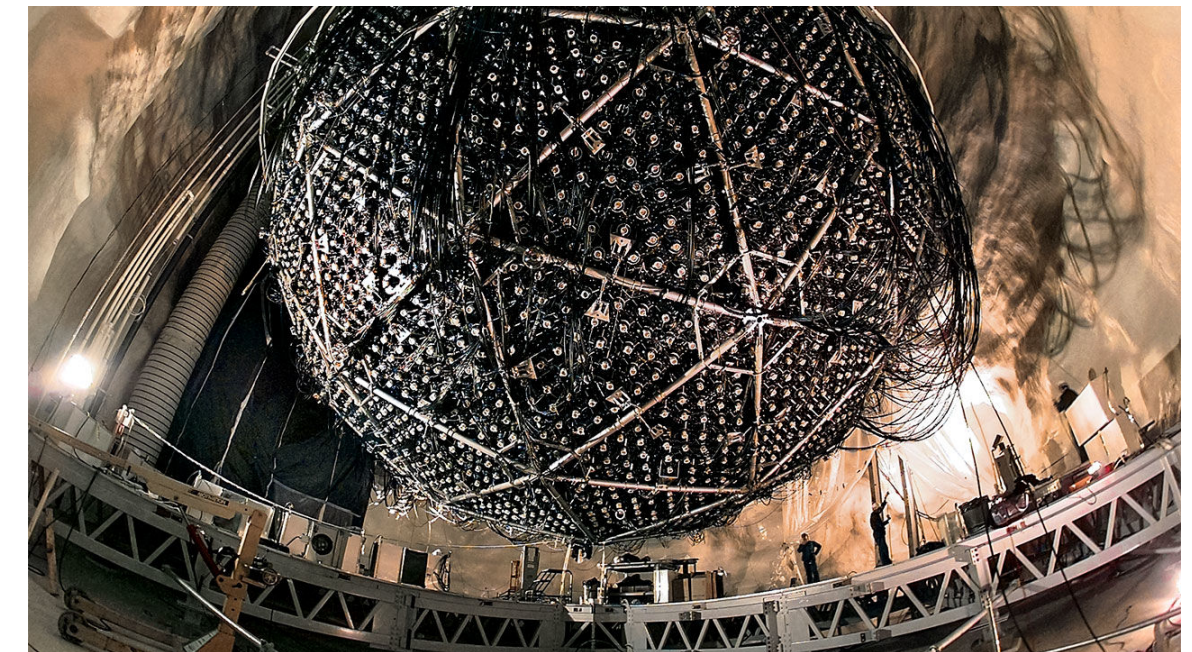
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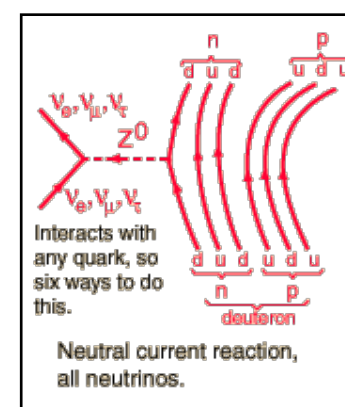
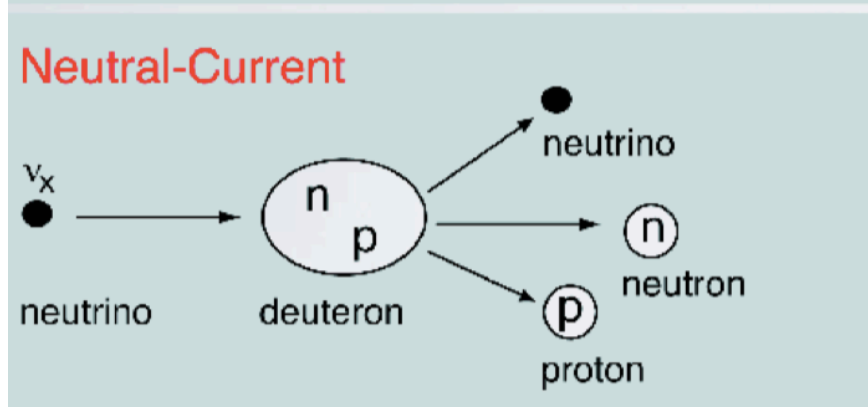


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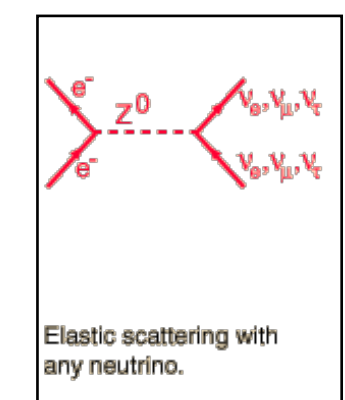
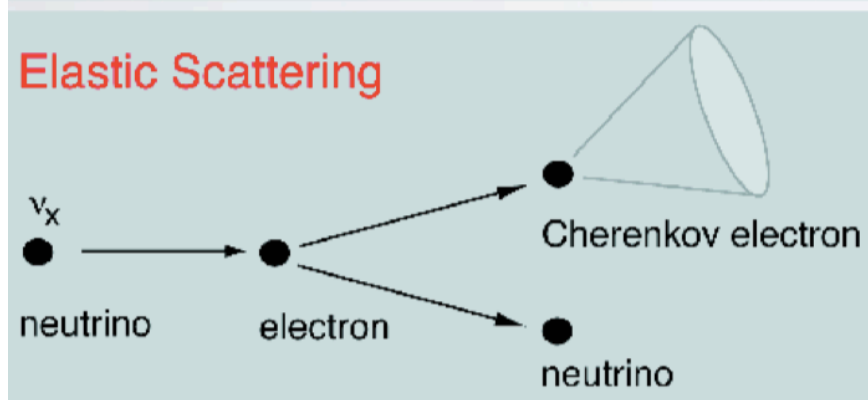
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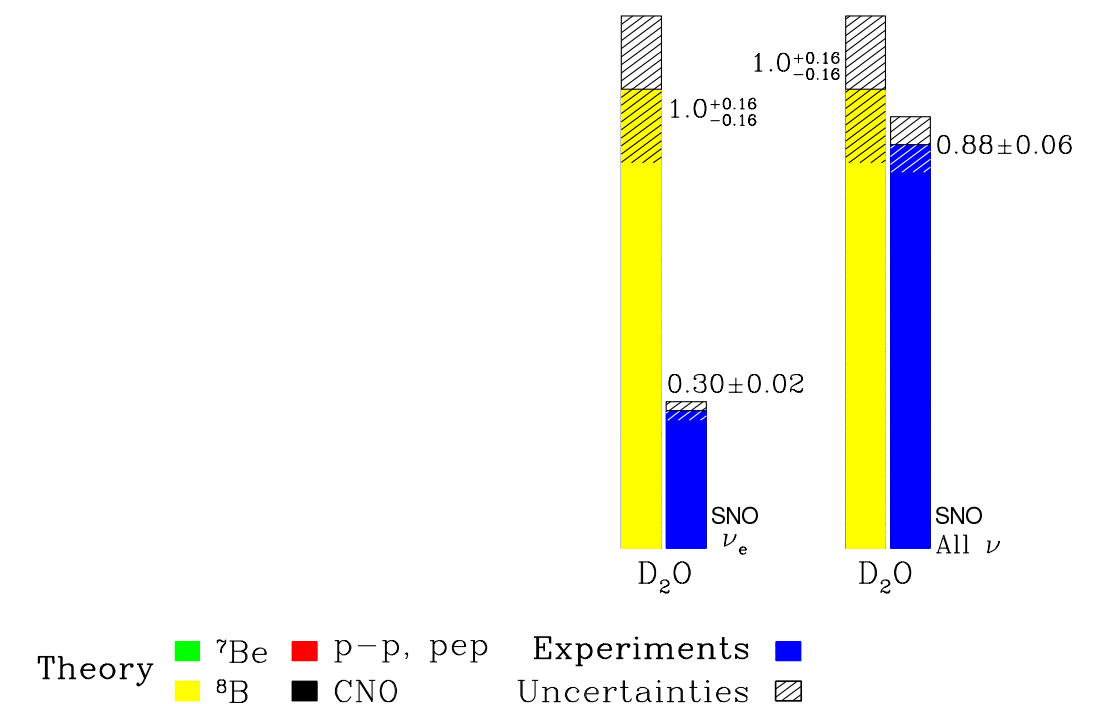
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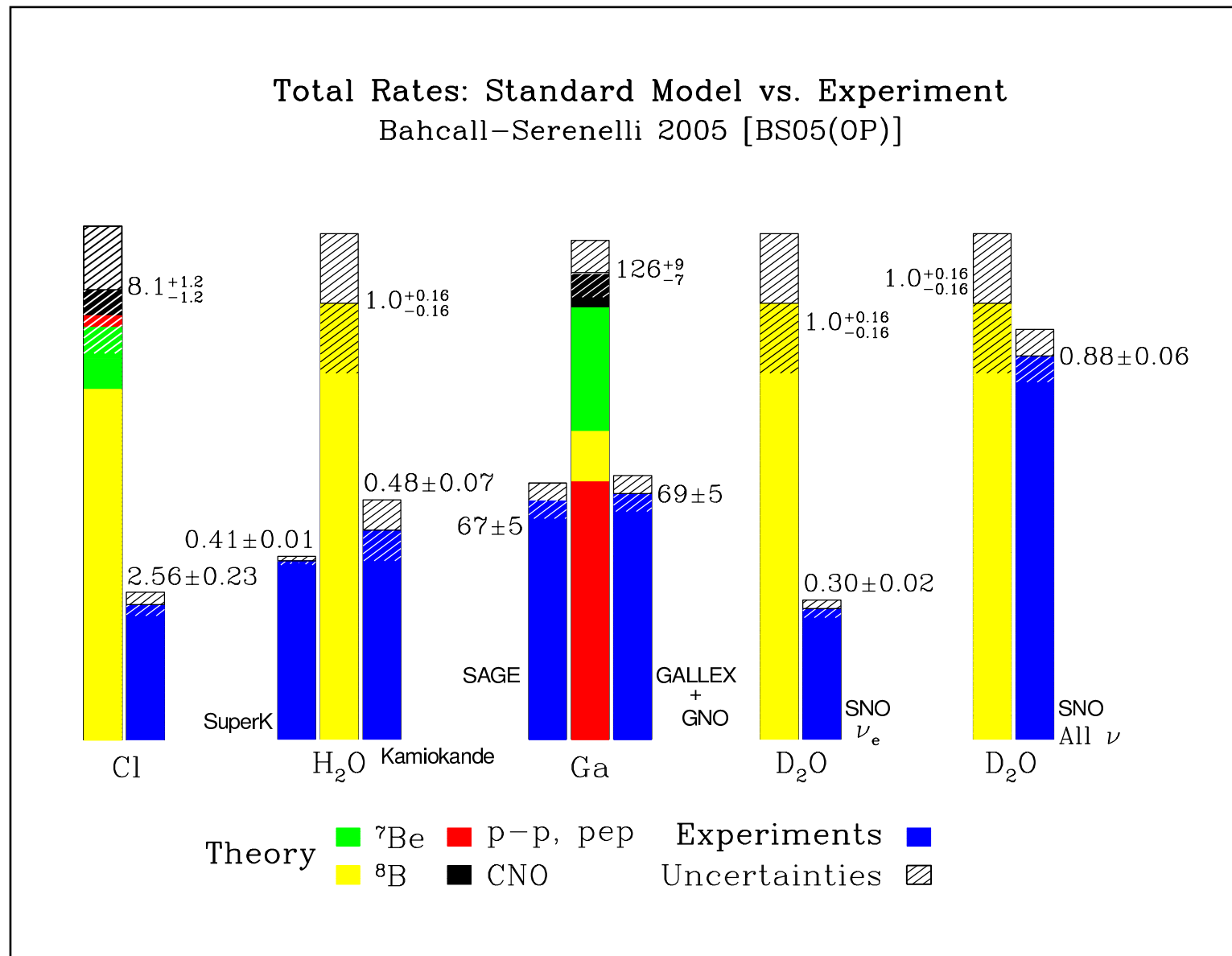
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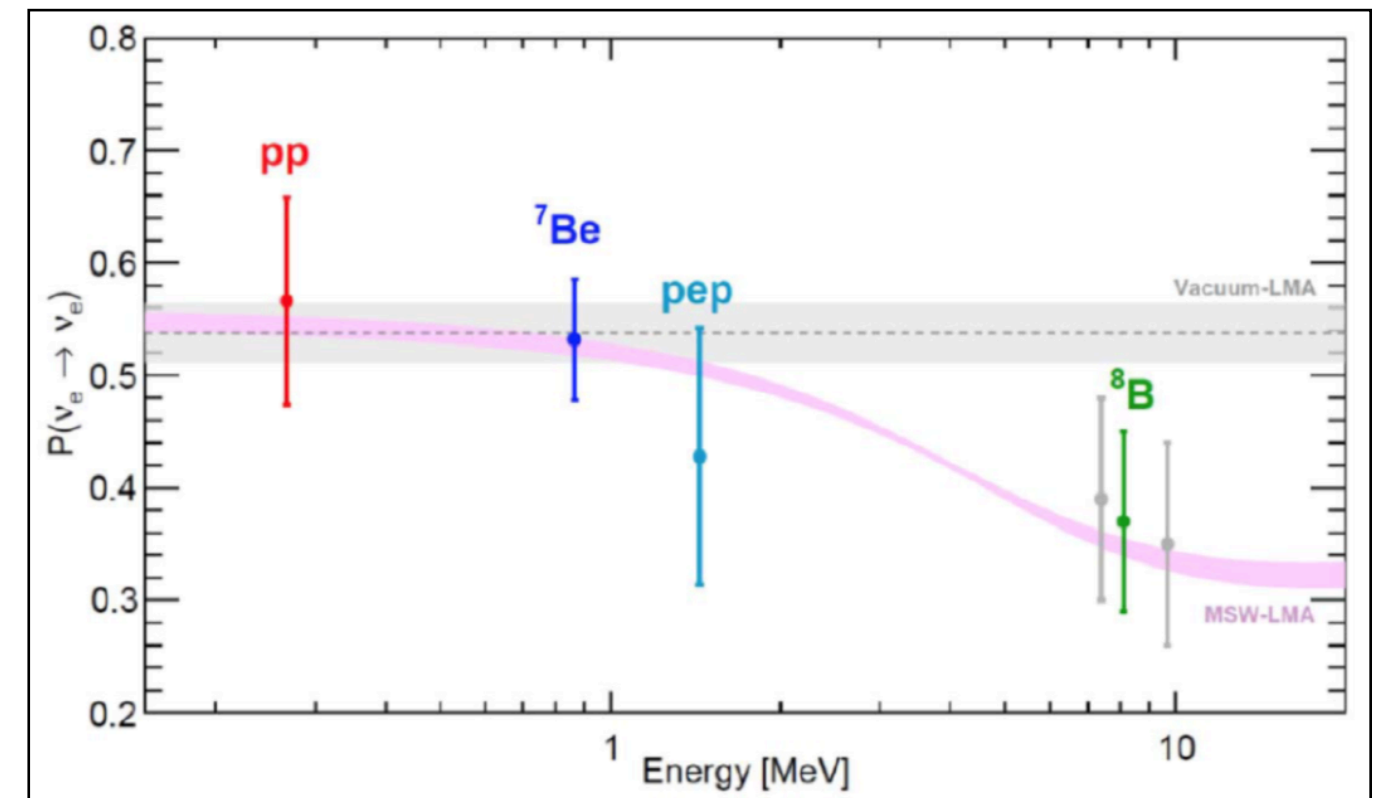
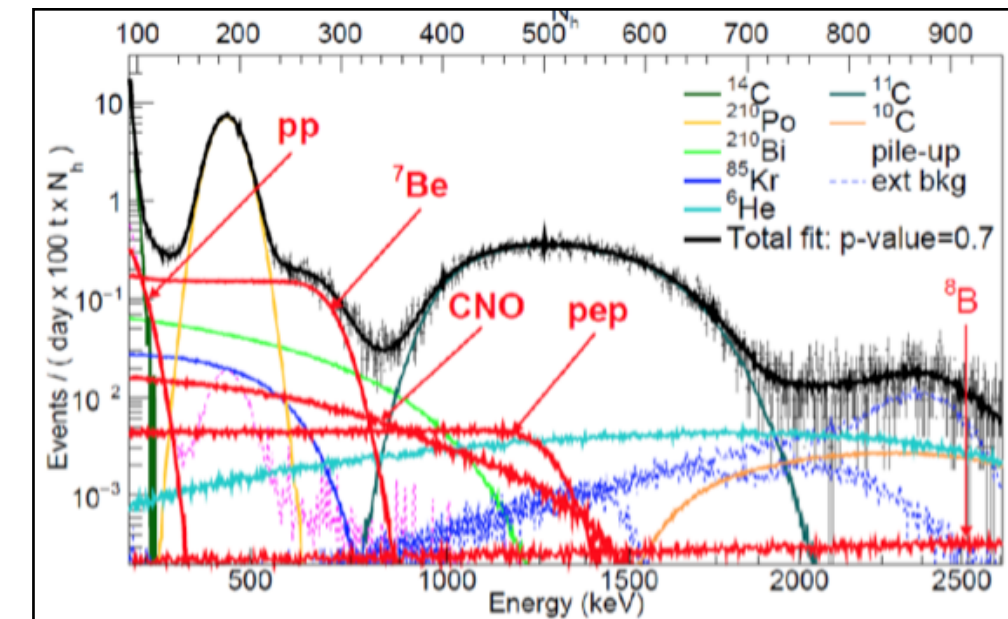
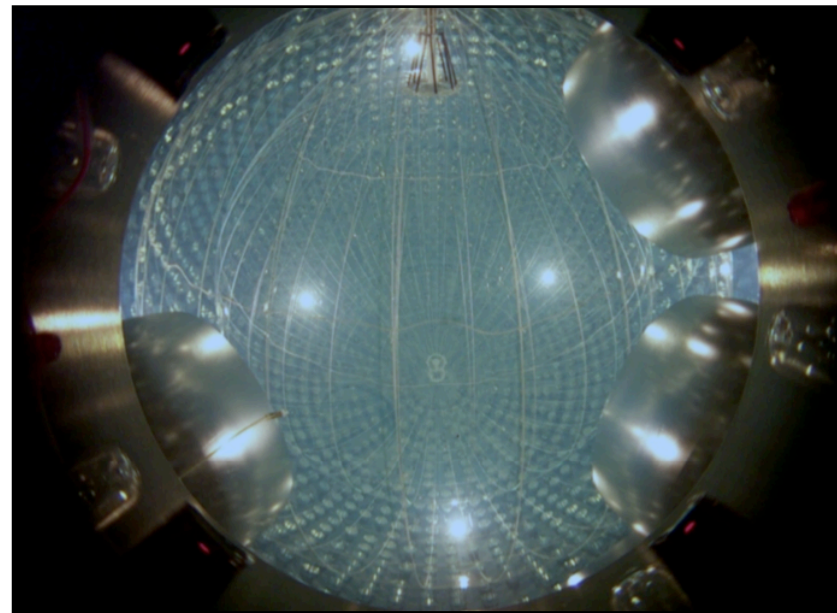
The Solar Neutrino Problem and its Resolution



- Bahcall's Solar Model had been right all the time!
- Pontecorvo had been right all the time, too: neutrinos oscillate!
- But why do the various experiments measure different deficits?
 - They're not all sensitive to the same channel
 - CC, NC, ES
 - They're not all sensitive to the same solar neutrino energies
- Intriguingly, the decades-old solar neutrino problem is very subtle:
 - Not just vacuum oscillations
 - Significant matter effects (MSW) in the interior of the Sun
 - And in Earth's interior (day-night asymmetry)

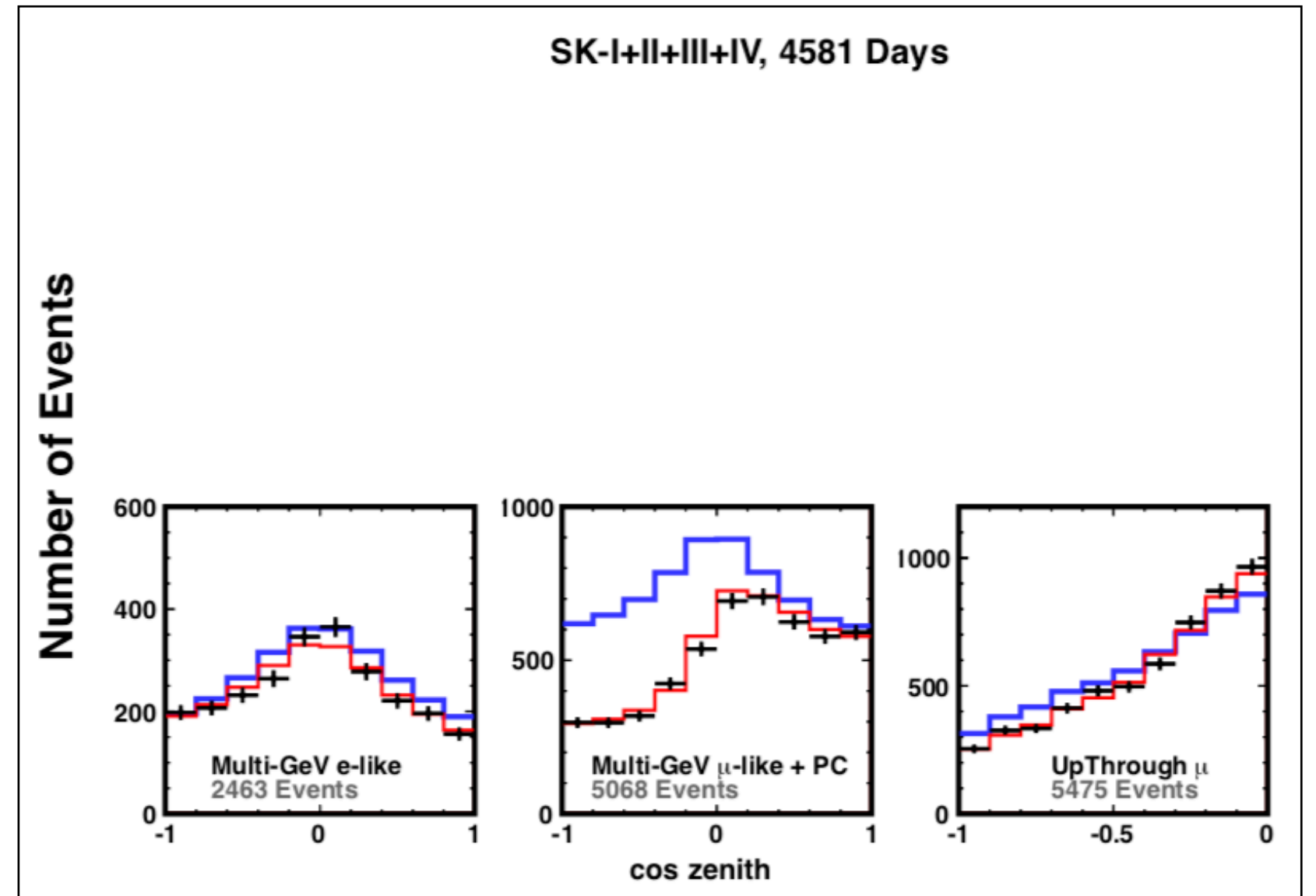
Borexino (2007-)

- Liquid scintillator
 - Very low threshold
 - Very good energy resolution
 - Isotropic (no direction information)
 - Can distinguish the various contributions to the solar neutrino spectrum
- The solar neutrino problem (electron neutrino flux deficiency) is energy-dependent!
- MSW effect, will come back to this later



SuperK Revisited: Atmospheric Neutrino Oscillations

- For higher energy atmospheric neutrinos, SuperK can distinguish between electron and muon neutrino interactions
- Muon neutrinos go missing
 - Atmospheric Neutrino Anomaly
- Note: No oscillation for ν_e
 - All ν_μ oscillate into ν_τ
 - Maximal mixing between ν_μ and ν_τ
- There is no escape: neutrinos REALLY oscillate



Blue: no oscillation

Red: oscillation included

Neutrino Oscillations (I)

$$\begin{array}{l} i = [1, 2, 3] \\ \alpha = [e, \mu, \tau] \end{array}$$

Neutrino:

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i}^* |\nu_{\alpha}\rangle \quad |\nu_{\alpha}\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

Antineutrino:

$$|\bar{\nu}_i\rangle = \sum_{\alpha} U_{\alpha i} |\bar{\nu}_{\alpha}\rangle \quad |\bar{\nu}_{\alpha}\rangle = \sum_i U_{\alpha i}^* |\bar{\nu}_i\rangle$$

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Time propagation:

$$|\nu_i(x, t)\rangle = e^{-iE_i t} |\nu_i(x, 0)\rangle$$
$$|\nu_i(x, 0)\rangle = e^{ip_i x} |\nu_i\rangle$$

Relativistic neutrinos:

$$E_i = \sqrt{m_i^2 + p_i^2} \simeq p_i + \frac{m_i^2}{2p_i} \simeq E + \frac{m_i^2}{2E}$$

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Jan Willem:

The point is, that different mass eigenstates propagate with different momentum $\mathbf{p}_k$ and therefore the relative phases of the different mass eigenstates vary with distance. This is

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Neutrino $|\nu_{\alpha}\rangle$ emitted by a source at $t=0$ develops with time into state: $|\nu(x, t)\rangle = \sum_i U_{\alpha i} e^{ip_i x} e^{-iE_i t} |\nu_i\rangle = \sum_{i, \beta} U_{\alpha i} U_{\beta i}^* e^{ip_i x} e^{-iE_i t} |\nu_{\beta}\rangle$

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Time-dependent transition amplitude for a flavour conversion $\nu_{\alpha} \rightarrow \nu_{\beta}$: $A(\alpha \rightarrow \beta)(t) = \langle \nu_{\beta} | \nu(x, t) \rangle = \sum_i U_{\beta i}^* U_{\alpha i} e^{ipx} e^{-iE_i t}$

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Natural units: $t=x=L$ and using the relativistic limit: $A(\alpha \rightarrow \beta)(t) = \langle \nu_{\beta} | \nu(x, t) \rangle = \sum_i U_{\beta i}^* U_{\alpha i} \exp(-i \frac{m_i^2}{2} \frac{L}{E}) = A(\alpha \rightarrow \beta)(L)$

Neutrino Oscillations (I)

$$\begin{array}{l} i = [1, 2, 3] \\ \alpha = [e, \mu, \tau] \end{array}$$

Neutrino: $|\nu_i\rangle = \sum_{\alpha} U_{\alpha i}^* |\nu_{\alpha}\rangle \quad |\nu_{\alpha}\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$

Antineutrino: $|\bar{\nu}_i\rangle = \sum_{\alpha} U_{\alpha i} |\bar{\nu}_{\alpha}\rangle \quad |\bar{\nu}_{\alpha}\rangle = \sum_i U_{\alpha i}^* |\bar{\nu}_i\rangle$

Time propagation: $|\nu_i(x, t)\rangle = e^{-iE_i t} |\nu_i(x, 0)\rangle$
 $|\nu_i(x, 0)\rangle = e^{ip_i x} |\nu_i\rangle$

Relativistic neutrinos: $E_i = \sqrt{m_i^2 + p_i^2} \simeq p_i + \frac{m_i^2}{2p_i} \simeq E + \frac{m_i^2}{2E}$

Neutrino $|\nu_{\alpha}\rangle$ emitted by a source at $t=0$ develops with time into state: $|\nu(x, t)\rangle = \sum_i U_{\alpha i} e^{ip_i x} e^{-iE_i t} |\nu_i\rangle = \sum_{i, \beta} U_{\alpha i} U_{\beta i}^* e^{ip_i x} e^{-iE_i t} |\nu_{\beta}\rangle$

Time-dependent transition amplitude for a flavour conversion $\nu_{\alpha} \rightarrow \nu_{\beta}$: $A(\alpha \rightarrow \beta)(t) = \langle \nu_{\beta} | \nu(x, t) \rangle = \sum_i U_{\beta i}^* U_{\alpha i} e^{ip_i x} e^{-iE_i t}$

Natural units: $t=x=L$ and using the relativistic limit: $A(\alpha \rightarrow \beta)(t) = \langle \nu_{\beta} | \nu(x, t) \rangle = \sum_i U_{\beta i}^* U_{\alpha i} \exp(-i \frac{m_i^2}{2} \frac{L}{E}) = A(\alpha \rightarrow \beta)(L)$

Similarly, for antineutrinos: $A(\bar{\alpha} \rightarrow \bar{\beta})(L) = \sum_i U_{\beta i} U_{\alpha i}^* \exp(-i \frac{m_i^2}{2} \frac{L}{E})$

The PMNS Matrix

- Pontecorvo, Maki, Nakagawa, Sakata

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \quad U_{\alpha i} = U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

The PMNS Matrix

- Pontecorvo, Maki, Nakagawa, Sakata
- Unitary $n \times n$
- $2n-1$ relative phases of the $2n$ neutrino states $\rightarrow (n-1)^2$ independent parameters
 - $1/2n(n-1)$ weak mixing angles
 - $1/2(n-1)(n-2)$ CP violating phases
- If $n=3$: 3 mixing angles and 1 CP violating phase

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$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{12} = \sin \theta_{12}, \text{ etc.}$$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

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The mixing angles can be defined via the elements of the neutrino mixing matrix (compare the first with the third representation):

$$\begin{aligned} s_{13}^2 &= \sin^2 \theta_{13} = |U_{e3}|^2 & c_{13}^2 &= 1 - s_{13}^2 = 1 - |U_{e3}|^2 \\ c_{12}^2 &= \cos^2 \theta_{12} = \frac{|U_{e1}|^2}{1 - |U_{e3}|^2} & s_{12}^2 &= \sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \\ c_{23}^2 &= \cos^2 \theta_{23} = \frac{|U_{\tau 3}|^2}{1 - |U_{e3}|^2} & s_{23}^2 &= \sin^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} \end{aligned}$$

Neutrino Oscillations (II)

Time-dependent transition amplitude for a flavour conversion $\nu_\alpha \rightarrow \nu_\beta$:
$$A(\alpha \rightarrow \beta)(t) = \sum_i U_{\beta i}^* U_{\alpha i} \exp\left(-i \frac{m_i^2}{2} \frac{L}{E}\right)$$

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$$\text{Use } \Delta m_{ij}^2 = m_i^2 - m_j^2 \quad = \sum_i \sum_j U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} \exp \left(-i \frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

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$$\text{Use } \sum_i \sum_j = \sum_{i=j} + \sum_{i>j} + \sum_{j>i} \quad \text{and} \quad A + A^* = (a + ib) + (a - ib) = 2a = 2 \operatorname{Re} A$$

$$= \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \operatorname{Re} \sum_{i>j} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} \exp \left(-i \frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

Neutrino Oscillations: the Master Equation

$$P(\alpha \rightarrow \beta)(t) = \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \operatorname{Re} \sum_{i>j} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} \exp \left(-i \frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

Experimental Physicist's View

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- Physics Parameters: $U_{\alpha i}$ Δm_{ij}^2
- Observables: L, E
- **Any neutrino oscillation experiment measures neutrino oscillations as a function of (at least) observables L and E, to determine PMNS matrix elements (or equivalently, 3 mixing angles and CP violating phase) and 3 mass splittings**

Neutrino Oscillations without CP Violation

$$P(\alpha \rightarrow \beta)(t) = \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \operatorname{Re} \sum_{i>j} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} \exp \left(-i \frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

IF no CP violation in the neutrino sector, PMNS elements are real. In that case, rewrite as

$$P_{CP \text{ holds}}(\alpha \rightarrow \beta)(t) = \sum_i U_{\alpha i}^2 U_{\beta i}^2 + 2 \sum_{i>j} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \cos \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

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Rewrite the cos using $\cos(2\theta) = 1 - 2\sin^2(2\theta)$

$$P_{CP \text{ holds}}(\alpha \rightarrow \beta)(t) = \sum_i U_{\alpha i}^2 U_{\beta i}^2 + 2 \sum_{i>j} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} - 4 \sum_{i>j} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

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For this part, use $\sum_i \sum_j U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} = \sum_i U_{\alpha i} U_{\beta i} \sum_j U_{\alpha j} U_{\beta j} = \delta_{\alpha\beta} \delta_{\alpha\beta} = \delta_{\alpha\beta}$

$$P_{CP \text{ holds}}(\alpha \rightarrow \beta)(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

Neutrino Oscillations Master Equation without CP Violation

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Neutrino Oscillations Master Equation including CP Violation

$$P(\overset{(-)}{\alpha} \rightarrow \overset{(-)}{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \overset{+}{(-)} 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

Some Remarks

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) + 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

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- One can detect flavour change in 2 ways:
 - Start with ν_{α} and look for ν_{β} ('appearance')
 - Start with ν_{α} and see if any disappears ('disappearance'/'survival')

Three Neutrino Probabilities are Messy...

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) + (-) 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

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- When substituting the PMNS matrix elements things get messy very quickly
 - Due to all the sums of products of four PMNS matrix elements
- So let's make some assumptions and simplify

Simplifications

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) + 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

- Assumption 1: no CP violation (U elements are real)

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- Assumption 1: no CP violation (U elements are real)
- Assumption 2: appearance

$$P(\alpha \rightarrow \beta)(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

$$P(\alpha \rightarrow \beta)(t) = -4 \sum_{i>j} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

Simplifications

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) + 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

- Assumption 1: no CP violation (U elements are real)
- Assumption 2: appearance

$$P(\alpha \rightarrow \beta)(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

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- Assumption 3: neglect (small) Δm_{12}^2

- Assumption 4: $\Delta m_{32}^2 \simeq \Delta m_{31}^2$

$$P(\nu_\alpha \rightarrow \nu_\beta) = -4 \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right) (U_{\alpha 1} U_{\beta 1} U_{\alpha 3} U_{\beta 3} + U_{\alpha 2} U_{\beta 2} U_{\alpha 3} U_{\beta 3})$$

Simplifications

$$P(\overset{(-)}{\alpha} \rightarrow \overset{(-)}{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \overset{+}{(-)} 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

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- Assumption 5: PMNS Unitarity

$$\sum_{i=1}^3 U_{\alpha i} U_{\beta i} = U_{\alpha 1} U_{\beta 1} + U_{\alpha 2} U_{\beta 2} + U_{\alpha 3} U_{\beta 3} = 0$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = 4 \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right) U_{\alpha 3}^2 U_{\beta 3}^2$$

$$(\alpha, \beta) = (\mu, \tau)$$

$$U_{\alpha i} = U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = 4 \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right) U_{\alpha 3}^2 U_{\beta 3}^2$$

$$(\alpha, \beta) = (\mu, \tau) \rightarrow P(\nu_{\mu} \rightarrow \nu_{\tau}) = 4c_{13}^4 s_{23}^2 c_{23}^2 \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right) = c_{13}^4 \sin^2(2\theta_{23}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$

$$(\alpha, \beta) = (e, \mu)$$

$$U_{\alpha i} = U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = 4 \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right) U_{\alpha 3}^2 U_{\beta 3}^2$$

$$(\alpha, \beta) = (e, \mu) \rightarrow P(\nu_e \rightarrow \nu_{\mu}) = 4s_{23}^2c_{13}^2s_{13}^2 \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right) = s_{23}^2 \sin^2(2\theta_{13}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$

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$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = 4 \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right) U_{\alpha 3}^2 U_{\beta 3}^2$$

$$(\alpha, \beta) = (e, \tau) \rightarrow P(\nu_e \rightarrow \nu_{\tau}) = 4c_{23}^2 c_{13}^2 s_{13}^2 \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right) = c_{23}^2 \sin^2(2\theta_{13}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$

Two-Neutrino Appearance Probabilities: Summary

$$U_{\alpha i} = U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$$P(\nu_{\mu} \rightarrow \nu_{\tau}) = c_{13}^4 \sin^2(2\theta_{23}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$

$$P(\nu_e \rightarrow \nu_{\mu}) = s_{23}^2 \sin^2(2\theta_{13}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$

$$P(\nu_e \rightarrow \nu_{\tau}) = c_{23}^2 \sin^2(2\theta_{13}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$

Two-Neutrino Appearance Probabilities: Summary

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$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$$P(\nu_{\mu} \rightarrow \nu_{\tau}) = c_{13}^4 \sin^2(2\theta_{23}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$

$$P(\nu_e \rightarrow \nu_{\mu}) = s_{23}^2 \sin^2(2\theta_{13}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$

$$P(\nu_e \rightarrow \nu_{\tau}) = c_{23}^2 \sin^2(2\theta_{13}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$

- What about numerical values?

Global Fit for Neutrino Oscillation Parameters: NuFIT 6.0 (2024)

NuFIT 6.0 (2024)				
	Normal Ordering ($\Delta\chi^2 = 0.6$)		Inverted Ordering (best fit)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2 \theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$
$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$
$\sin^2 \theta_{23}$	$0.561^{+0.012}_{-0.015}$	$0.430 \rightarrow 0.596$	$0.562^{+0.012}_{-0.015}$	$0.437 \rightarrow 0.597$
$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	$41.0 \rightarrow 50.5$	$48.6^{+0.7}_{-0.9}$	$41.4 \rightarrow 50.6$
$\sin^2 \theta_{13}$	$0.02195^{+0.00054}_{-0.00058}$	$0.02023 \rightarrow 0.02376$	$0.02224^{+0.00056}_{-0.00057}$	$0.02053 \rightarrow 0.02397$
$\theta_{13}/^\circ$	$8.52^{+0.11}_{-0.11}$	$8.18 \rightarrow 8.87$	$8.58^{+0.11}_{-0.11}$	$8.24 \rightarrow 8.91$

- Mnemonic:
- $\theta_{12} \approx 34^\circ$ (count up)
- $\theta_{23} \approx 45^\circ$ (count up)
- $\theta_{13} \approx 9^\circ$ (geometric series)

- PS: JUNO paper from last month, 59 days of data-taking (!):

$$\sin^2 \theta_{12} = 0.3092 \pm 0.0087$$

Substitute Mixing Angle Values to Further Simplify

$$\begin{array}{ll} P(\nu_\mu \rightarrow \nu_\tau) = c_{13}^4 \sin^2(2\theta_{23}) \sin^2\left(\Delta m_{32}^2 \frac{L}{4E}\right) & \theta_{13} = \text{small} \quad P(\nu_\mu \rightarrow \nu_\tau) = \sin^2(2\theta_{23}) \sin^2\left(\Delta m_{32}^2 \frac{L}{4E}\right) \\ P(\nu_e \rightarrow \nu_\mu) = s_{23}^2 \sin^2(2\theta_{13}) \sin^2\left(\Delta m_{32}^2 \frac{L}{4E}\right) & \theta_{23} = 45^\circ \quad P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2(2\theta_{13}) \sin^2\left(\Delta m_{32}^2 \frac{L}{4E}\right) \\ P(\nu_e \rightarrow \nu_\tau) = c_{23}^2 \sin^2(2\theta_{13}) \sin^2\left(\Delta m_{32}^2 \frac{L}{4E}\right) & \theta_{23} = 45^\circ \quad P(\nu_e \rightarrow \nu_\mu) = P(\nu_e \rightarrow \nu_\tau) \end{array}$$

PMNS Numerical Values

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$$\begin{aligned} s_{13}^2 &= \sin^2 \theta_{13} = |U_{e3}|^2 & c_{13}^2 &= 1 - s_{13}^2 = 1 - |U_{e3}|^2 \\ c_{12}^2 &= \cos^2 \theta_{12} = \frac{|U_{e1}|^2}{1 - |U_{e3}|^2} & s_{12}^2 &= \sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \\ c_{23}^2 &= \cos^2 \theta_{23} = \frac{|U_{\tau 3}|^2}{1 - |U_{e3}|^2} & s_{23}^2 &= \sin^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} \end{aligned}$$

$$U_{\alpha i} = U_{\text{PMNS}} = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} = \begin{pmatrix} 0.83 & 0.55 & 0.15 \\ 0.37 & 0.61 & 0.70 \\ 0.42 & 0.58 & 0.69 \end{pmatrix}$$

Flavour Content in Mass Eigenstates

$$U_{\alpha i} = U_{\text{PMNS}} = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} = \begin{pmatrix} 0.83 & 0.55 & 0.15 \\ 0.37 & 0.61 & 0.70 \\ 0.42 & 0.58 & 0.69 \end{pmatrix}$$

The diagram illustrates the flavor content of the neutrino mass eigenstates ν_1 , ν_2 , and ν_3 . Each state is represented by a vertical bar divided into three segments, corresponding to the electron (ν_e), muon (ν_μ), and tau (ν_τ) flavors. The segments are labeled with their respective values from the PMNS matrix.

Mass Eigenstate	ν_e (Red)	ν_μ (Green)	ν_τ (Blue)
ν_1	0.83	0.37	0.42
ν_2	0.55	0.61	0.58
ν_3	0.15	0.70	0.69

NuFIT 6.0 (2024) Mass Splittings

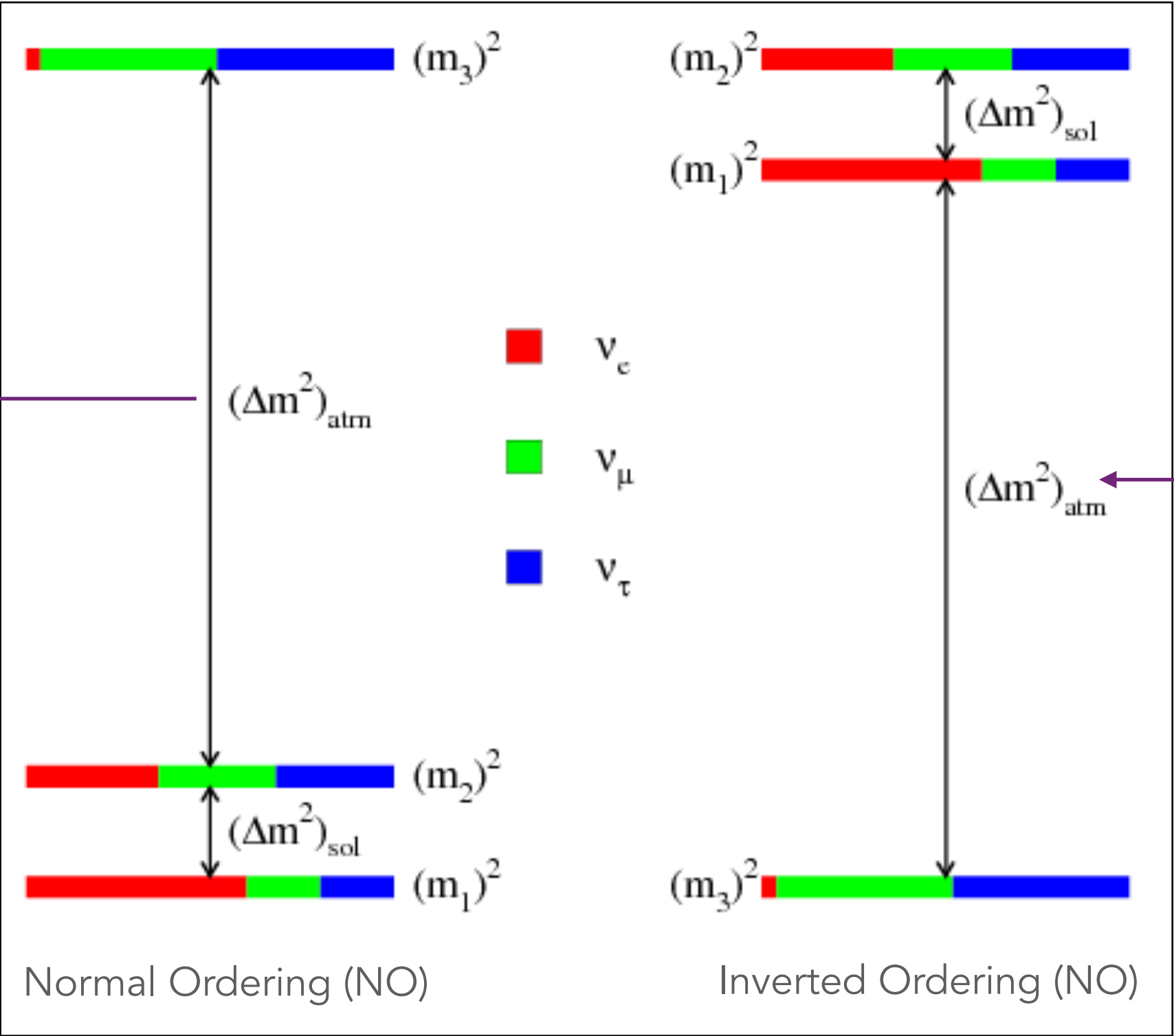
NuFIT 6.0 (2024)				
	Normal Ordering ($\Delta\chi^2 = 0.6$)		Inverted Ordering (best fit)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.534^{+0.025}_{-0.023}$	$+2.463 \rightarrow +2.606$	$-2.510^{+0.024}_{-0.025}$	$-2.584 \rightarrow -2.438$

- PS: JUNO paper from last month, 59 days of data-taking:

$$\Delta m_{21}^2 = (7.50 \pm 0.12) \times 10^{-5} \text{ eV}^2$$

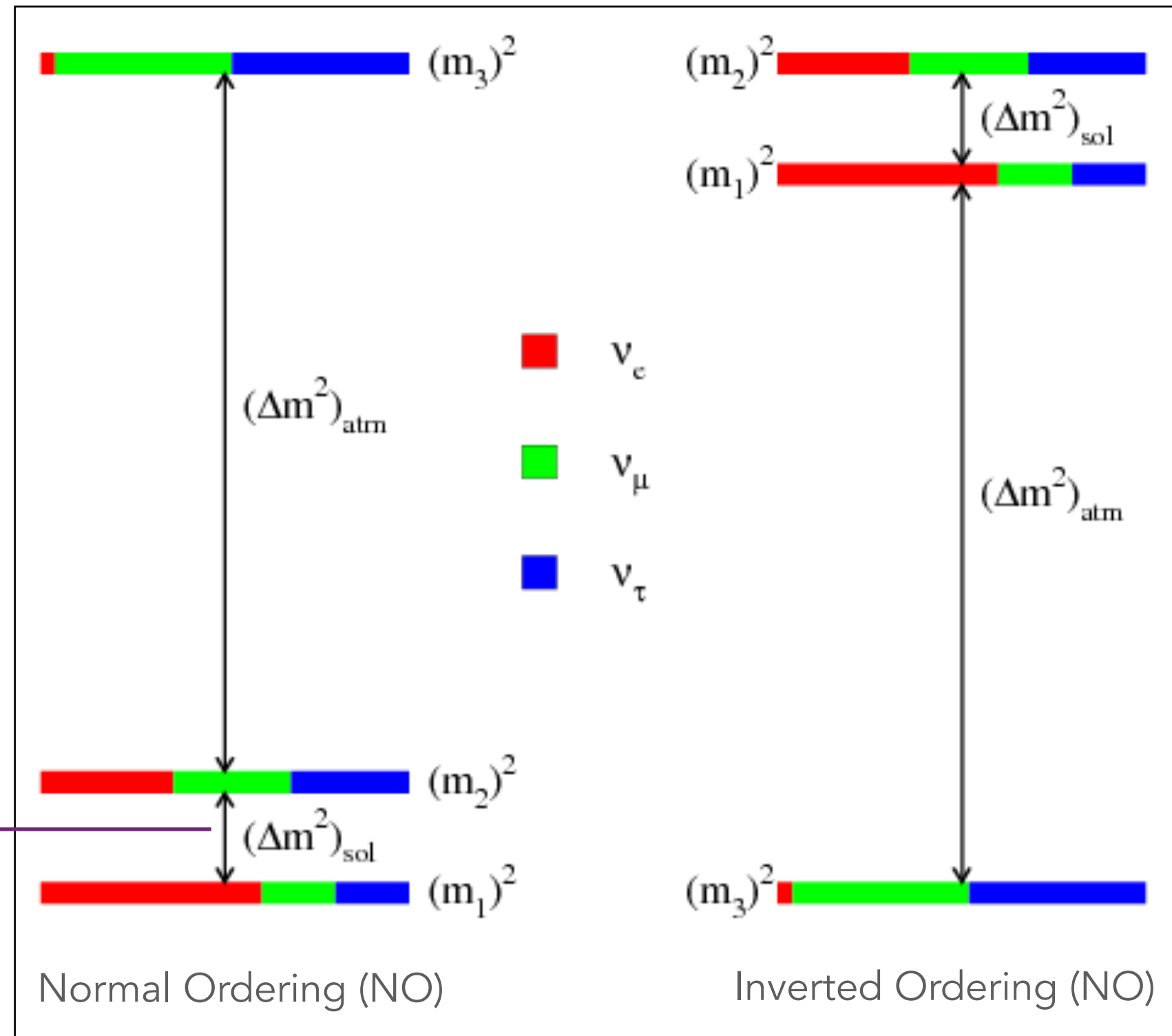
The Big Unknown: Neutrino Mass Ordering (NMO)

We do **NOT** know the sign of $\Delta m^2_{32} (= \Delta m^2_{31})$



We do **NOT** know the sign of $\Delta m^2_{32} (= \Delta m^2_{31})$

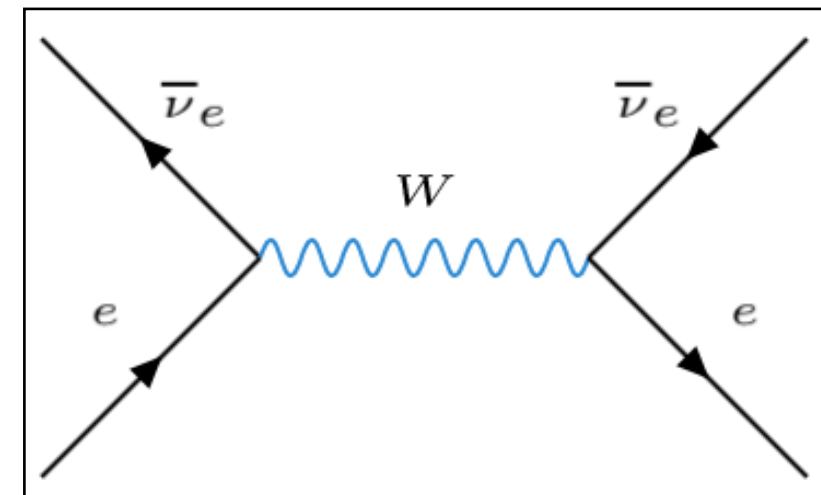
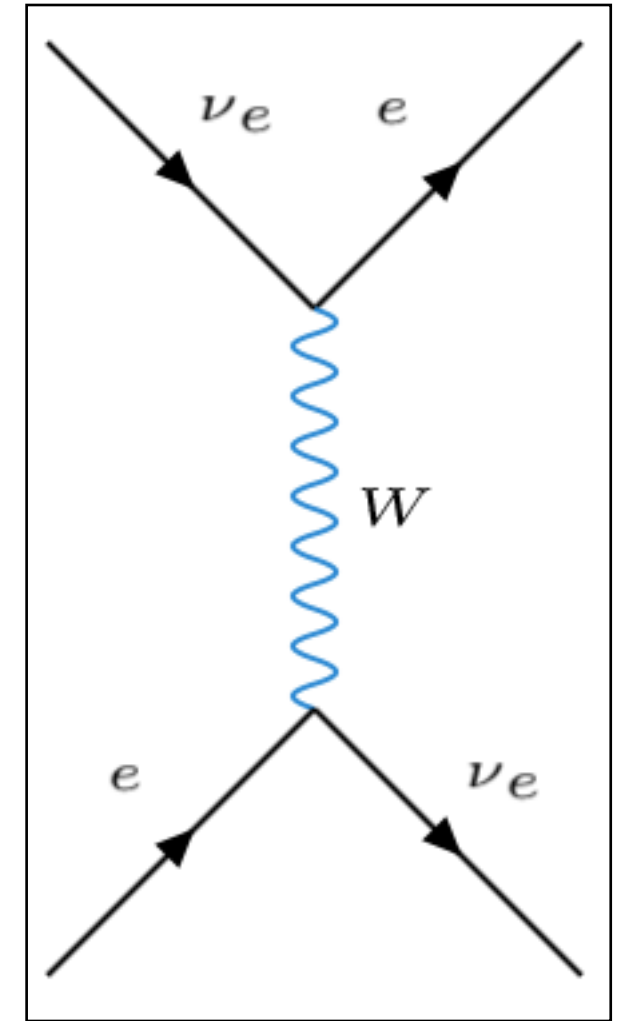
But: $m_2 > m_1$!



We DO know the sign of Δm^2_{12} from the MSW effect in the sun!

Solar Neutrino Puzzle and the MSW Effect

- If we assume 2 neutrino approximation for solar neutrinos:
 - $L \sim 10^8 \text{ km}$, $E \sim 1 \text{ MeV}$
 - Naively expect oscillations to be sizeable for $\Delta m^2 \sim E/L = 10^{-11} \text{ eV}^2$
 - Instead, we observe oscillations at $\Delta m_{12}^2 \sim 10^{-5} \text{ eV}^2$
- This is caused by the MSW effect (Mikheyev–Smirnov–Wolfenstein)
- Coherent forward scattering via W -boson exchange leads to an extra interaction potential energy in the Hamiltonian:
 - $\nu_e : +\sqrt{2}G_F N_e$
 - $\bar{\nu}_e : -\sqrt{2}G_F N_e$
 - G_F is the Fermi constant, N_e is the electron density in the medium
- This raises the effective mass of ν_e and lowers that of $\bar{\nu}_e$



MSW effect

- Fractional importance of matter effects on oscillations with vacuum mass splitting Δm^2 is $x \equiv \frac{\sqrt{2}G_F N_e}{\Delta m^2/2E}$ (this definition will start making sense later)
- The matter effect:
 - grows with neutrino energy E and electron density
 - is sensitive to the sign of Δm^2
 - Affects neutrinos different than antineutrinos
- Note: this last effect can 'fake' CP violation and therefore needs to be taken into account in searches for genuine CP violation

Experimental Physicist's View

- The matter effect:
 - Is sensitive to the sign of Δm^2
- The signs of the mass splittings are determined using the MSW effect!
- Therefore, the NMO can be determined using the MSW effect!

Intermezzo: Hamiltonian for 2-Neutrino Oscillations

In the **mass basis** (ν_1, ν_2, ν_3) , the Hamiltonian $\mathbf{H}_m$ is **diagonal**, as the mass eigenstates propagate independently:

$$\mathbf{H}_{\text{full}} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} = \begin{pmatrix} p + \frac{m_1^2}{2p} & 0 \\ 0 & p + \frac{m_2^2}{2p} \end{pmatrix} \quad E_j \approx p + \frac{m_j^2}{2p}$$

The oscillation probability $P(\nu_\alpha \rightarrow \nu_\beta)$ depends only on the **difference in phase** accumulated by the two mass eigenstates, $\Delta\phi = (E_2 - E_1)t$. A phase that is common to all components of the state vector will **cancel out** when computing the probability.

We can factor out and subtract the term p (since $E \approx p$) which is common to both eigenstates.

We define the **oscillation-relevant Hamiltonian** $\mathbf{H}_m$ by subtracting the term $p\mathbf{I}$:

$$\mathbf{H}_m = \mathbf{H}_{\text{full}} - p\mathbf{I} = \begin{pmatrix} \frac{m_1^2}{2p} & 0 \\ 0 & \frac{m_2^2}{2p} \end{pmatrix}$$

- Set energy level of first eigenstate to 0

$$\mathbf{H}'_m = \mathbf{H}_m - \frac{m_1^2}{2p}\mathbf{I} = \begin{pmatrix} \frac{m_1^2}{2p} - \frac{m_1^2}{2p} & 0 \\ 0 & \frac{m_2^2}{2p} - \frac{m_1^2}{2p} \end{pmatrix}$$

This matrix governs the flavor evolution according to the equation:

$$i \frac{d}{dx} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \mathbf{H}_{\text{vac}} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

$$\mathbf{H}'_m = \frac{1}{2E} \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix}$$

Hamiltonian for 2-Neutrino Oscillations in the flavour basis

The Hamiltonian in the flavor basis $\mathbf{H}_f$ is obtained by the similarity transformation:

$$\mathbf{H}_f = \mathbf{U}_{2 \times 2} \mathbf{H}_m \mathbf{U}_{2 \times 2}^\dagger$$

Let's write this out explicitly:

$$\mathbf{H}_f = \frac{1}{2E} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

To do this, we often subtract an overall multiple of the identity matrix, $\frac{\Delta m^2}{4E} \mathbf{I}$, which does not affect the oscillation probability.

$$\mathbf{H}_f = \frac{\Delta m^2}{4E} \left(\begin{pmatrix} 2 \sin^2 \theta & \sin(2\theta) \\ \sin(2\theta) & 2 \cos^2 \theta \end{pmatrix} - \mathbf{I} \right) + \frac{\Delta m^2}{4E} \mathbf{I}$$

$$\mathbf{H}_f - \frac{\Delta m^2}{4E} \mathbf{I} = \frac{\Delta m^2}{4E} \begin{pmatrix} 2 \sin^2 \theta - 1 & \sin(2\theta) \\ \sin(2\theta) & 2 \cos^2 \theta - 1 \end{pmatrix}$$

Applying $2 \sin^2 \theta - 1 = -(\cos^2 \theta - \sin^2 \theta) = -\cos(2\theta)$ and $2 \cos^2 \theta - 1 = \cos(2\theta)$, we get the standard, oscillation-relevant vacuum Hamiltonian:

$$\mathbf{H}_{vac} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}$$

$$\mathbf{H}'_m = \frac{1}{2E} \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix}$$

$$U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

This matrix governs the flavor evolution according to the equation:

$$i \frac{d}{dx} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \mathbf{H}_{vac} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

The MSW Effect for 2-Neutrino Oscillations

$$H_{\text{vac}} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

Now including matter effects in the Hamiltonian:

$$H_M = H_{\text{vac}} + \sqrt{2}G_F N_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$H_M = H_{\text{vac}} + \sqrt{2}G_F N_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \frac{\sqrt{2}}{2}G_F N_e \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Can subtract any contribution proportional to identity matrix as that doesn't change the eigenvalues.

$$H_M = \frac{\Delta m^2}{4E} \begin{pmatrix} -(\cos 2\theta - x) & \sin 2\theta \\ \sin 2\theta & (\cos 2\theta - x) \end{pmatrix}$$

$$\text{with } x \equiv \frac{\sqrt{2}G_F N_e}{\Delta m^2/2E} \text{ as before}$$

The MSW Effect for 2-Neutrino Oscillations

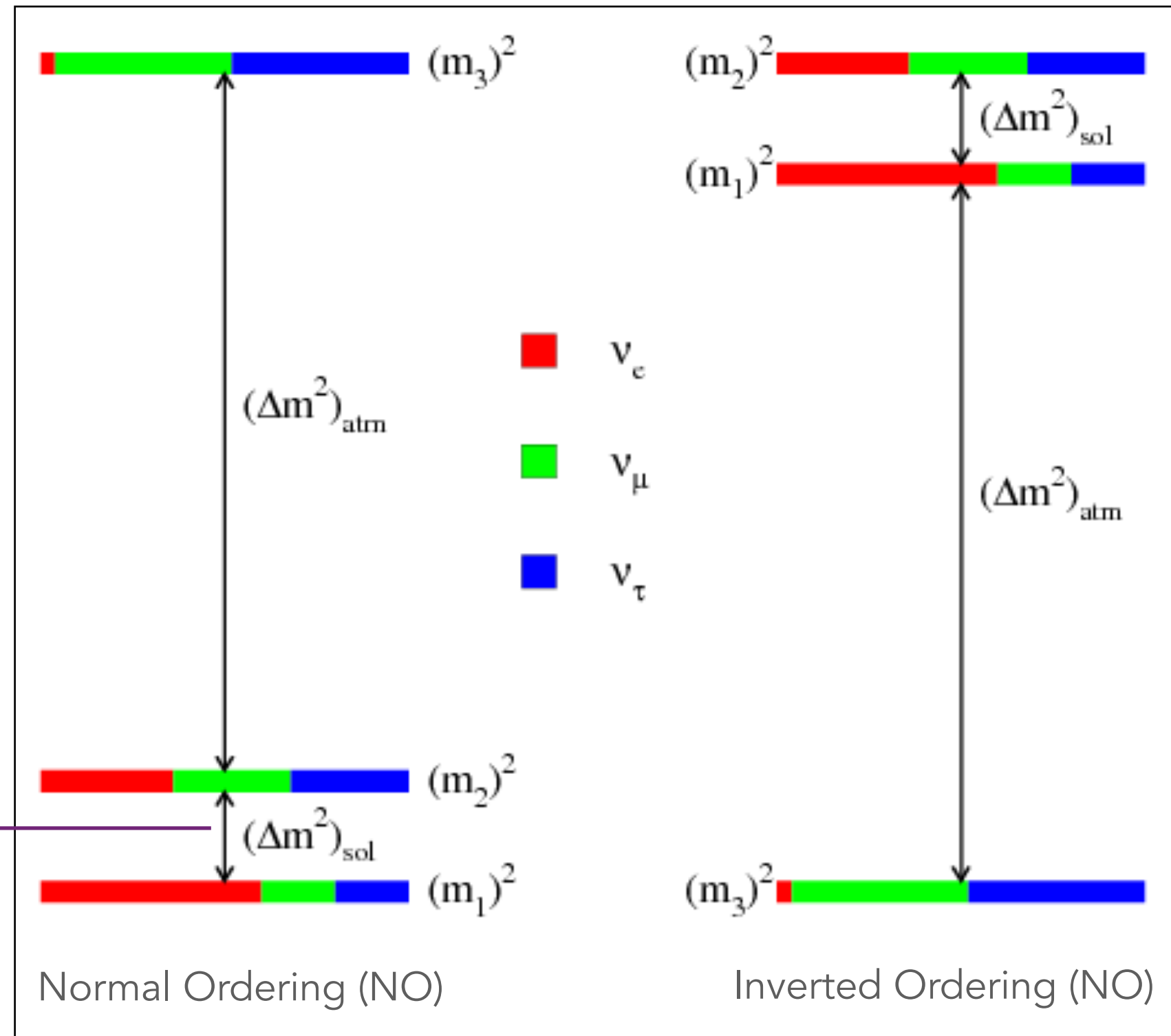
$$H_M = \frac{\Delta m^2}{4E} \begin{pmatrix} -(\cos 2\theta - x) & \sin 2\theta \\ \sin 2\theta & (\cos 2\theta - x) \end{pmatrix}$$

$$x \equiv \frac{\sqrt{2}G_F N_e}{\Delta m^2/2E}$$

- Then, defining $\Delta m_M^2 \equiv \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - x)^2}$ and $\sin^2 2\theta_M \equiv \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2}$
- $H_M = \frac{\Delta m_M^2}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$
- And all 2-neutrino approximations are adapted accordingly:
 - $P_M(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta_M \sin^2(\Delta m_M^2 \frac{L}{4E})$

Even if $\sin^2(2\theta)$ is small, $\sin^2(2\theta_M)$ can be maximal if $x \sim \cos(2\theta)$. This is called the MSW resonance

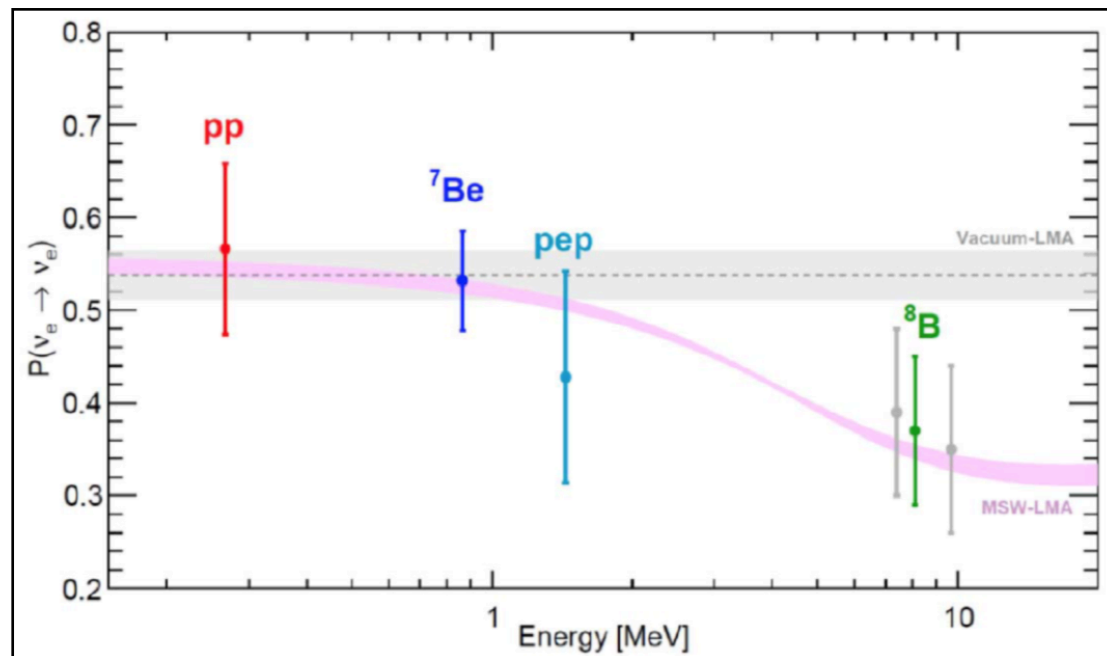
So how was the 'Solar Mass Splitting' measured?



We DO know the sign of Δm^2_{12} from the MSW effect in the sun!

MSW effect in Solar Neutrinos

- In the Sun things are even more subtle due to changing electron density as the neutrinos travel out from the Sun's core, leading to energy-dependent MSW-modified oscillation probabilities



PDG 2011 (<https://pdg.lbl.gov/2011/reviews/rpp2011-rev-neutrino-mixing.pdf>):

$$\bar{P}^{2\nu}(\nu_e \rightarrow \nu_e; t_s, t_0) \cong \frac{1}{2} - \frac{1}{2} \cos 2\theta.$$

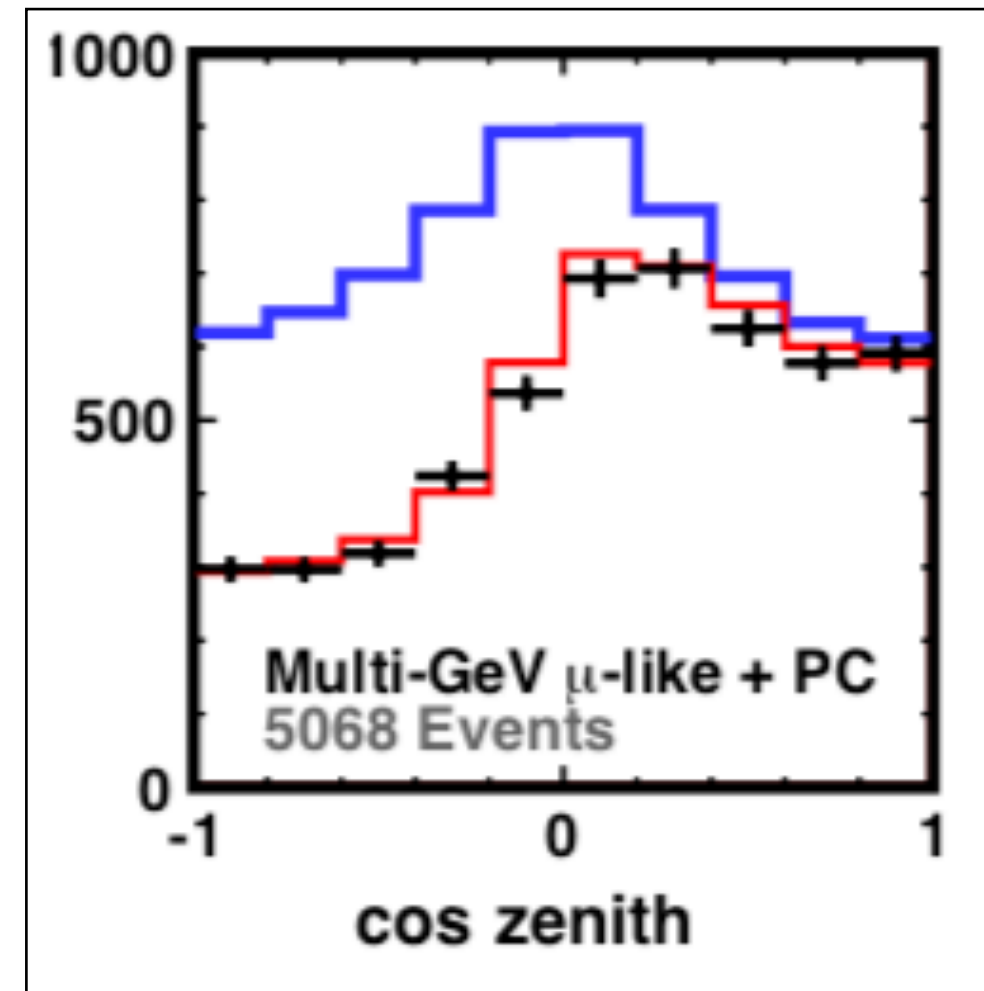
data. We see that depending on the sign of $\cos 2\theta \neq 0$, $\bar{P}^{2\nu}(\nu_e \rightarrow \nu_e)$ is either bigger or smaller than $1/2$. It follows from the solar neutrino data that in the range of validity (in $E/\Delta m^2$) of Eq. (14.59) we have $\bar{P}^{2\nu}(\nu_e \rightarrow \nu_e) \cong 0.3$. Thus, the possibility of $\cos 2\theta \leq 0$ is ruled out by the data. Given the choice $\Delta m^2 > 0$ we made, the data imply that $\Delta m^2 \cos 2\theta > 0$.

Takeaways Solar Neutrinos and Solar Mass Splitting

- Matter effects in the Sun become significant for higher energy solar neutrinos.
- They 'feel' a varying electron density as they travel outwards to vacuum
- Can redefine mixing angles and mass splittings to take matter effects into account
- Using these, the 2 neutrino approximation holds again, this time for oscillations including matter effects
- Benefit from matter effects: allows to determine the sign of Δm^2_{12}

Atmospheric Neutrinos, where the 2-Neutrino Approximation Works

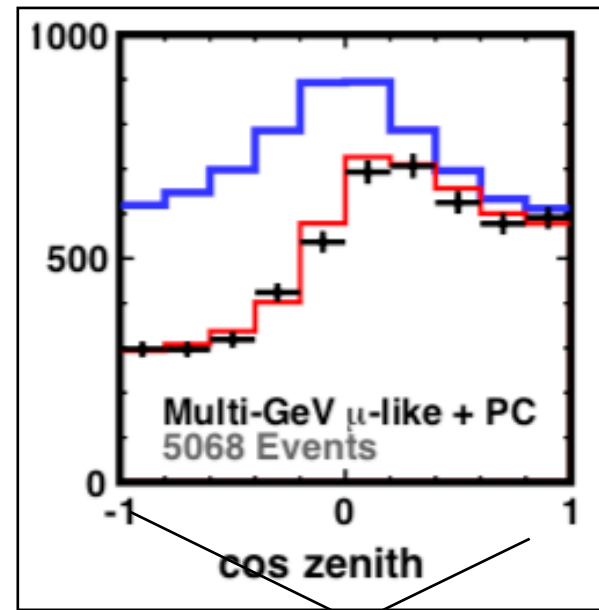
$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2(2\theta_{23}) \sin^2\left(\Delta m_{32}^2 \frac{L}{4E}\right)$$
$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta_{23}) \sin^2\left(\Delta m_{32}^2 \frac{L}{4E}\right)$$



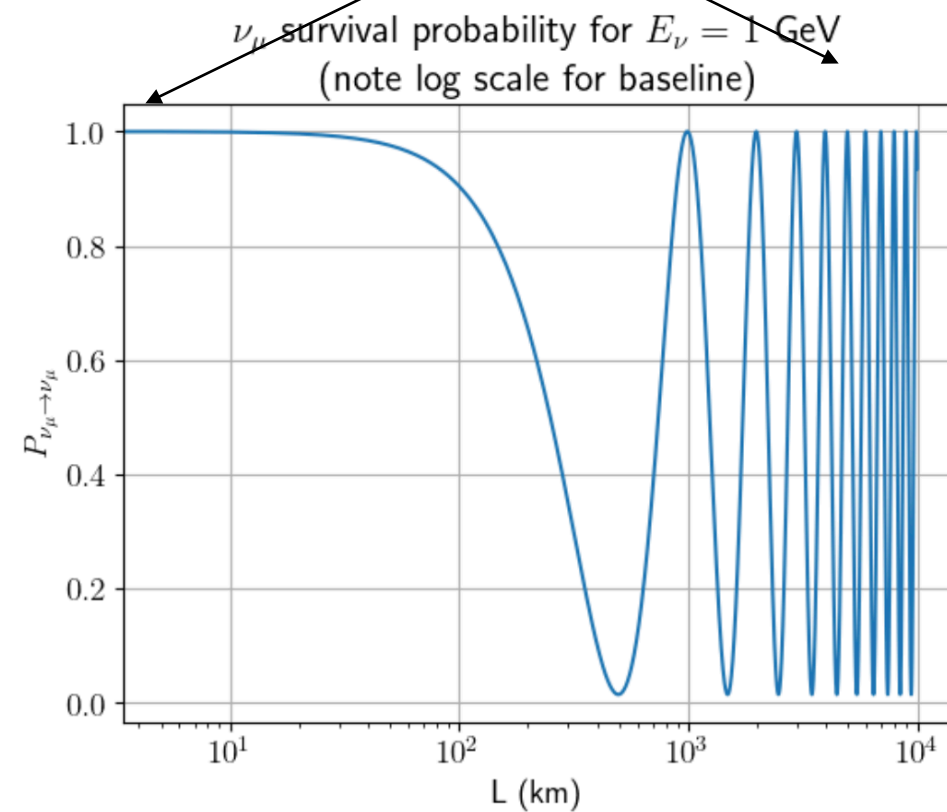
Up-going

Down-going

The Atmospheric Neutrino Deficit Explained with 2-Neutrino Oscillations



$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta_{23}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right)$$



$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta_{23}) \sin^2 \left(\Delta m_{32}^2 \frac{L}{4E} \right) \approx 1 - \frac{1}{2} \sin^2(2\theta_{23}) \approx 1 - \frac{1}{2} = \frac{1}{2}$$

Takeaways Atmospheric Neutrinos

- Confirmed neutrino oscillations through muon neutrino disappearance
- Maximal mixing between muon and tau neutrinos found for atmospheric neutrinos (the electron neutrinos do NOT disappear!)
- Easy to explain using two-neutrino approximation
- But if you use the full three neutrino picture including matter effects:
 - Day-night asymmetry shows electron neutrino regeneration confirming matter effects inside the Earth
- Future experiments: the matter effect is sensitive to the sign of Δm^2_{23} and solve the neutrino mass ordering question

Neutrino Physics isn't so Simple (Anymore)

- We've entered the phase of precision measurements
- 2-neutrino approximation not sufficient anymore
- Need 'higher-order' terms from full 3-neutrino oscillations
- Including matter effect
- Including CP violation

3-Neutrino Oscillations Including CP violation and Matter Effects

$$P(\nu_e \rightarrow \nu_\mu) \simeq P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3 \quad \text{Up to order } \alpha^2, \quad \Delta m_{21}^2 = \alpha \Delta m_{31}^2 \quad \Delta m_{32}^2 = (1 - \alpha) \Delta m_{31}^2$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(\hat{A} - 1)^2} \sin^2((\hat{A} - 1)\hat{\Delta}),$$

$$P_{\sin \delta} = \alpha \frac{\sin \delta \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}}{\hat{A}(1 - \hat{A})} \sin(\hat{\Delta}) \sin(\hat{A}\hat{\Delta}) \sin((1 - \hat{A})\hat{\Delta}),$$

$$P_{\cos \delta} = \alpha \frac{\cos \delta \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}}{\hat{A}(1 - \hat{A})} \cos(\hat{\Delta}) \sin(\hat{A}\hat{\Delta}) \sin((1 - \hat{A})\hat{\Delta}),$$

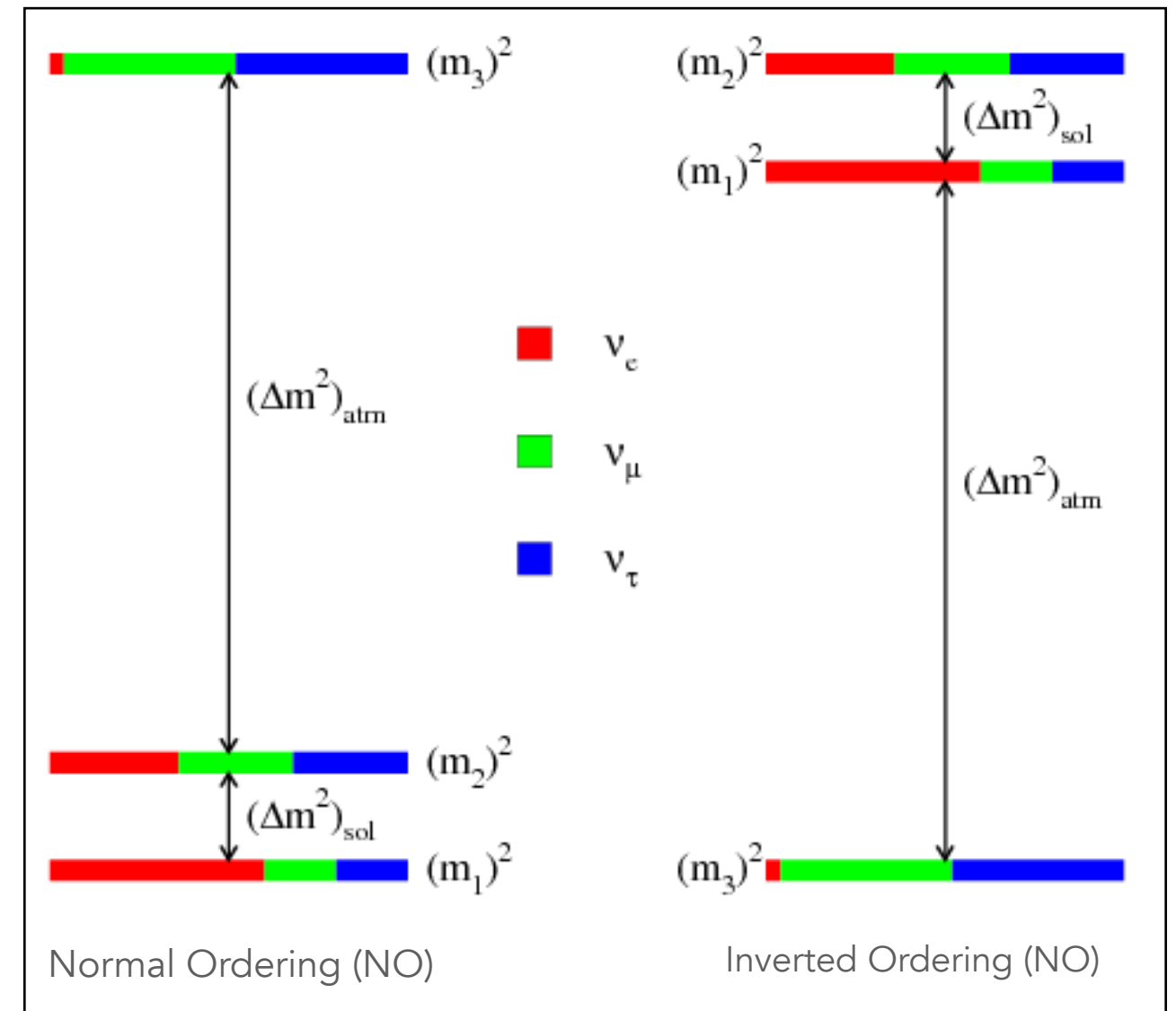
$$P_3 = \alpha^2 \frac{\cos^2 \theta_{23} \sin^2 2\theta_{12}}{\hat{A}^2} \sin^2(\hat{A}\hat{\Delta})$$

$$\hat{\Delta} = \Delta m_{31}^2 L / (4E_\nu) \quad \hat{A} = A / \Delta m_{31}^2 = 2VE_\nu / \Delta m_{31}^2 \quad V = \sqrt{2}G_F N_e$$

Taken from <https://arxiv.org/pdf/hep-ph/0103300.pdf>

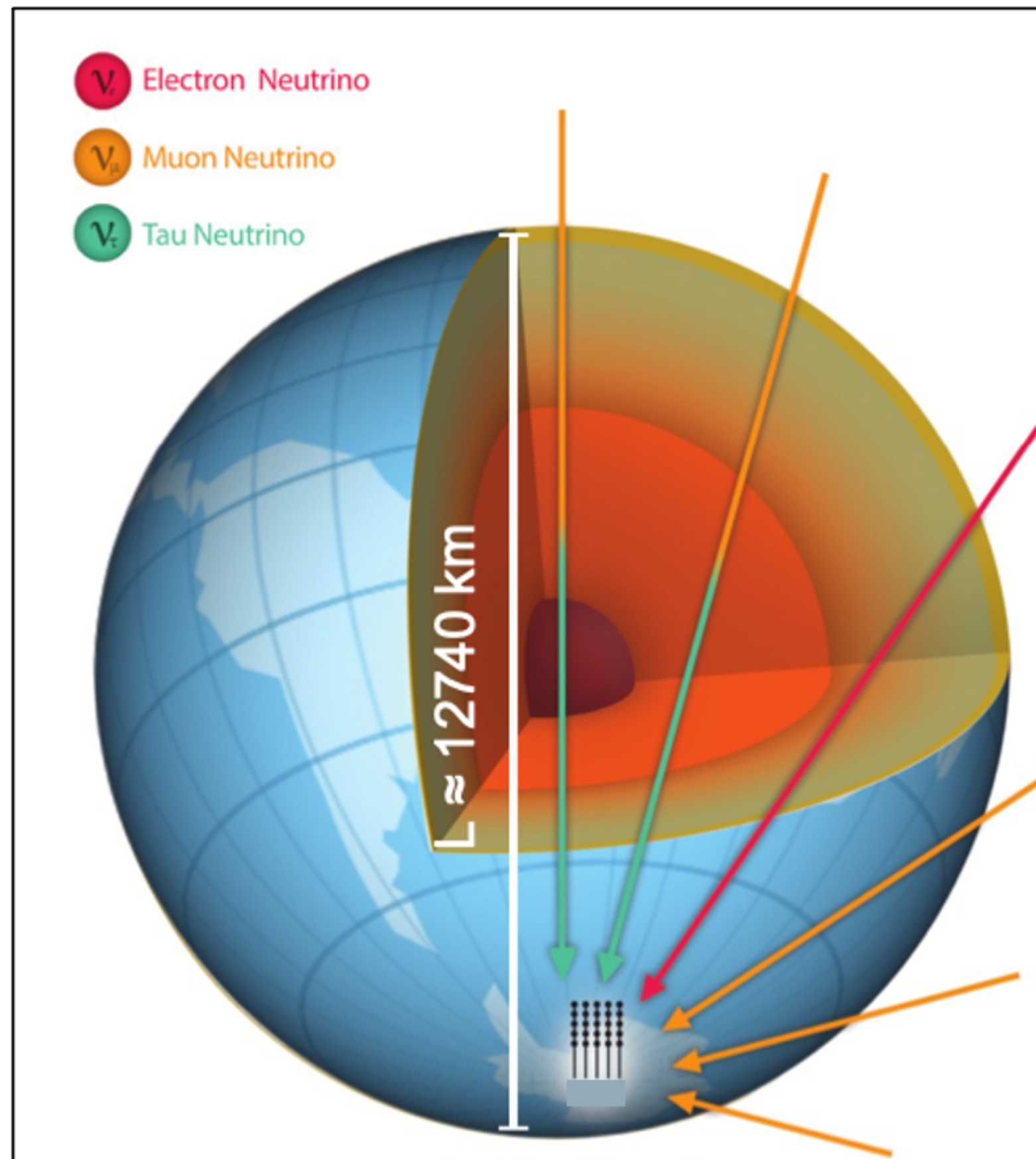
NMO Determination

- **MSW effect:**
 - Accelerator neutrinos
 - Currently: NOvA, T2K
 - Future: DUNE, HyperK
 - Atmospheric neutrinos
 - Currently: KM3NeT/ORCA, IceCube DeepCore, SuperK
- **Reactor neutrinos:**
 - Currently: JUNO
- **Non-oscillations:**
 - Neutrinoless double beta decay: If $0\nu\beta\beta$ is detected, the measured value of $\langle m_{ee} \rangle$ can constrain the mass ordering
 - Cosmological bounds: a strong upper limit on $\sum m_\nu$ from cosmology can put pressure on or potentially rule out the inverted ordering.

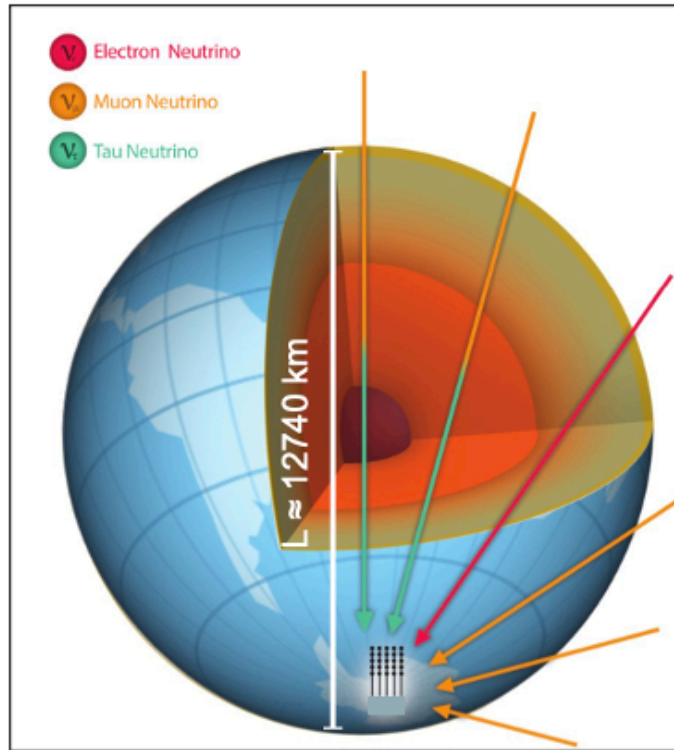


Focus on NMO Determination in KM3NeT/ORCA (Nikhef's Involvement)





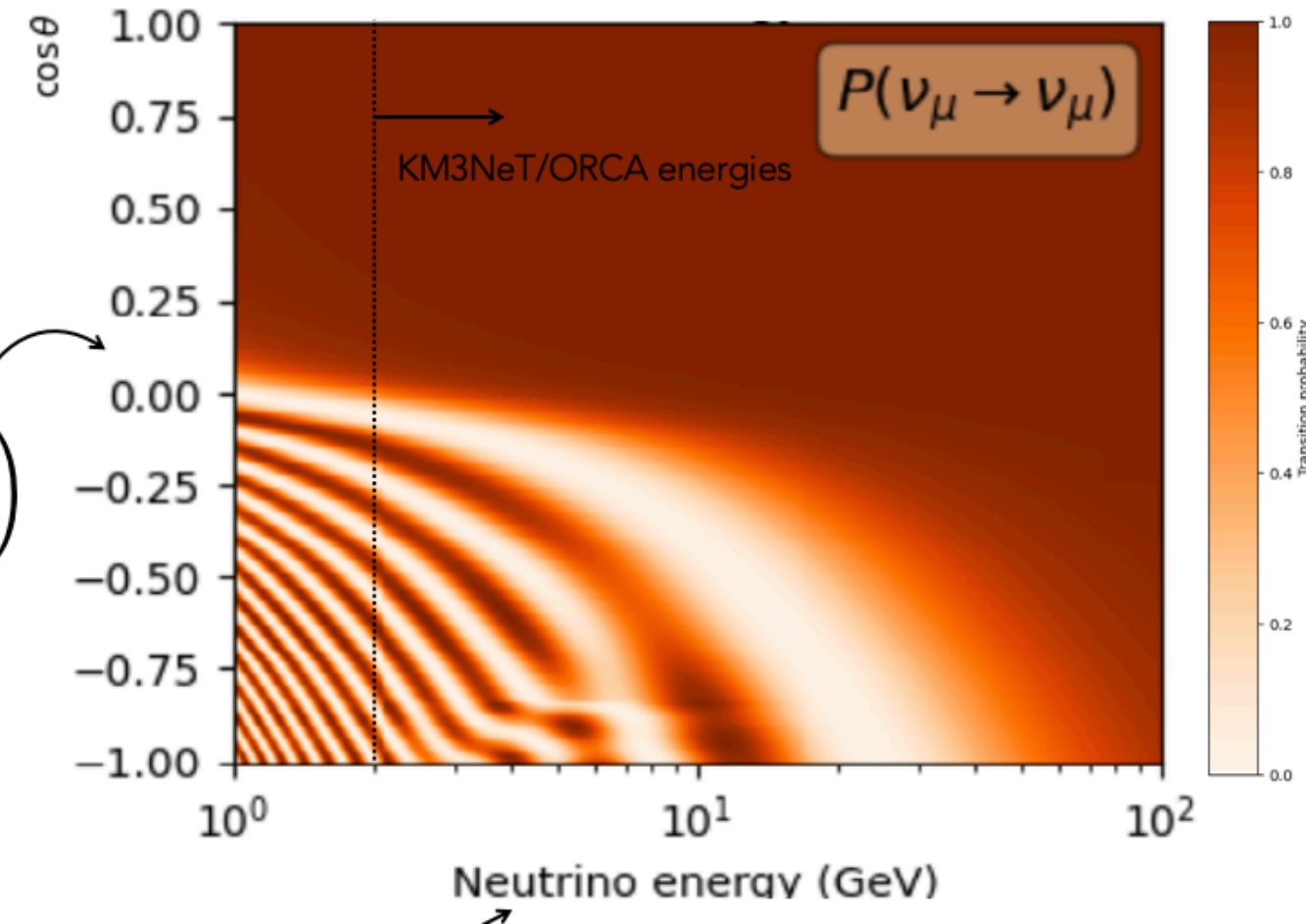
Atmospheric Neutrinos



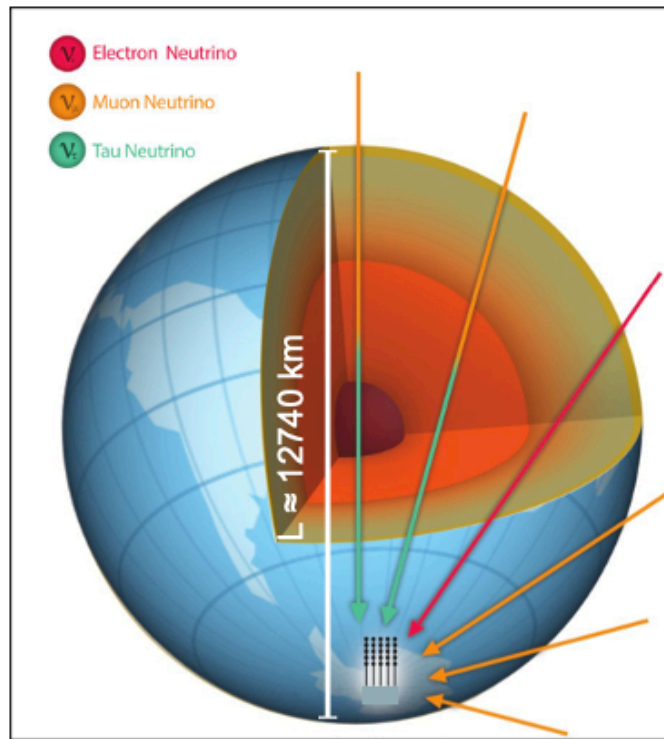
PMNS

$$P(\nu_\mu \rightarrow \nu_\tau) \sim \sin^2(2\theta_{23}) \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right)$$

Propagation



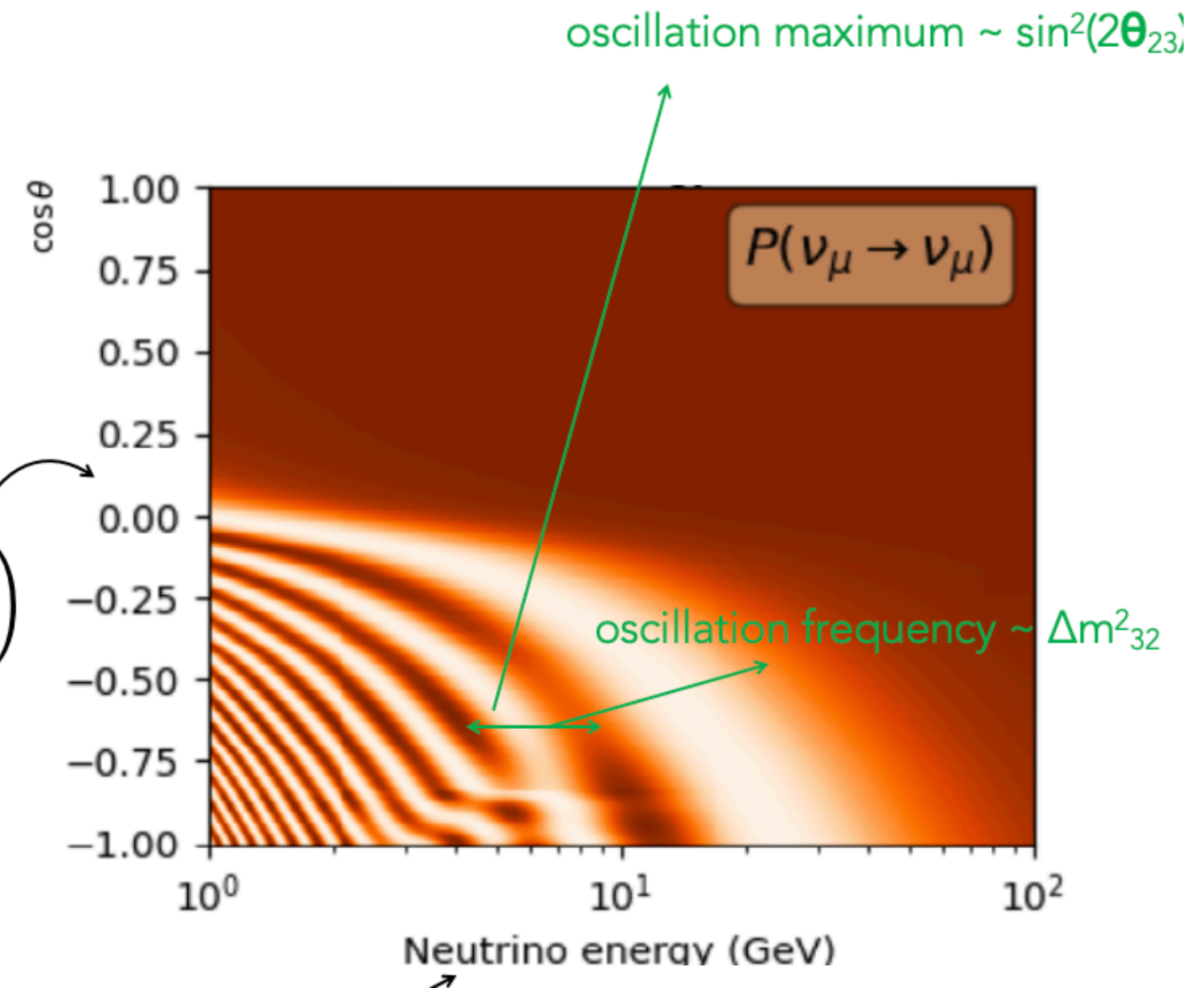
Atmospheric Neutrinos



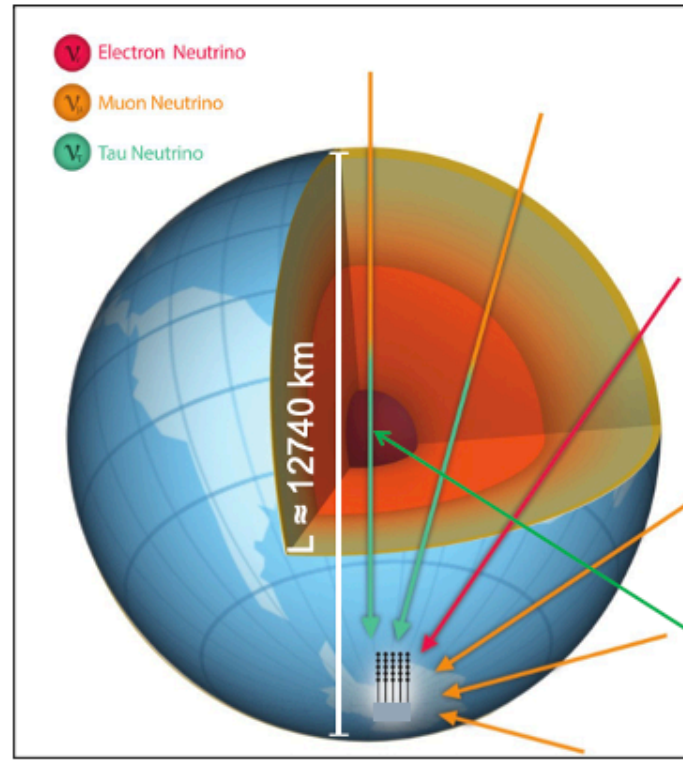
PMNS

$$P(\nu_\mu \rightarrow \nu_\tau) \sim \sin^2(2\theta_{23}) \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right)$$

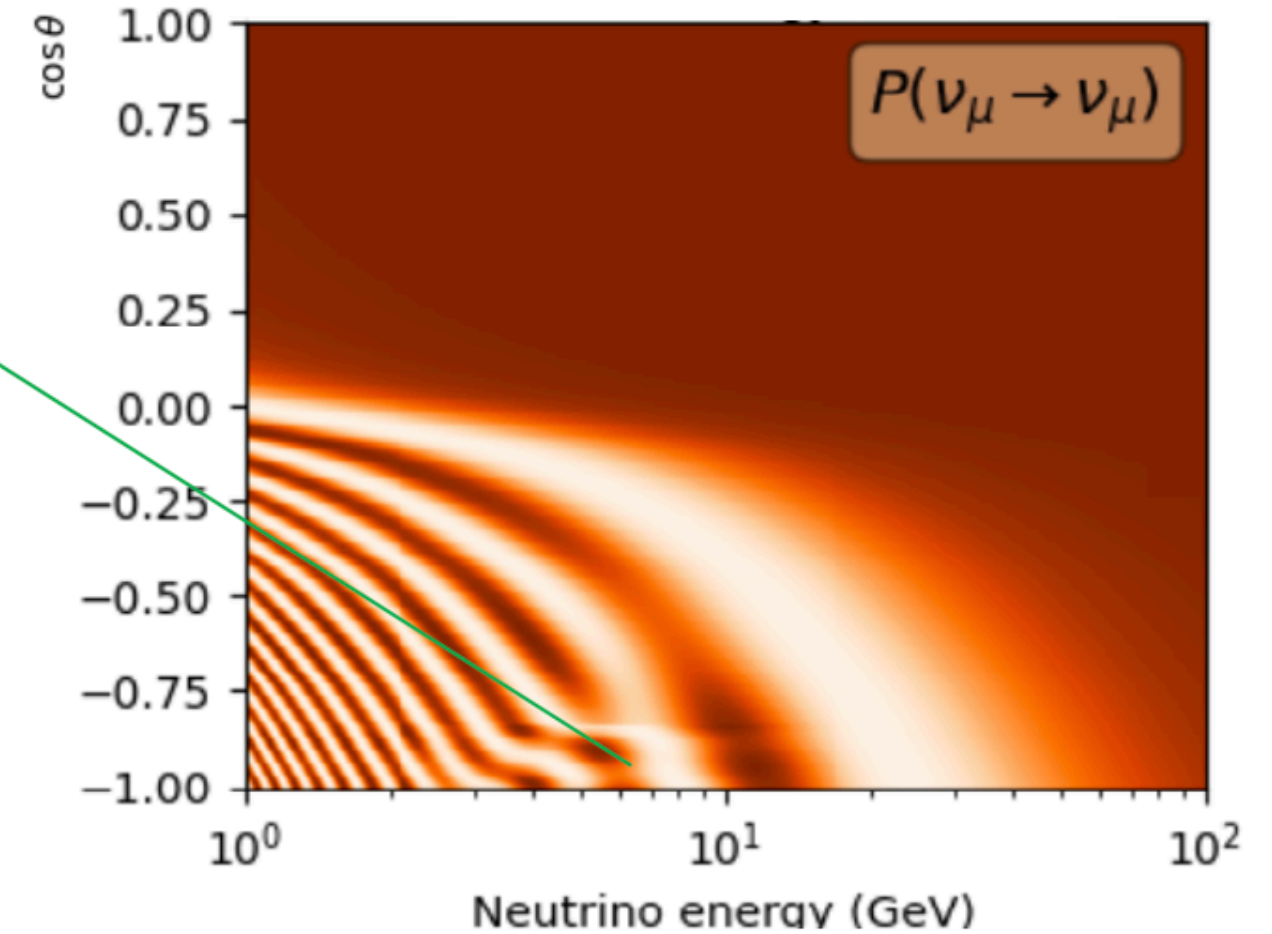
Propagation



Atmospheric Neutrinos: Matter Effect in Neutrino Oscillations



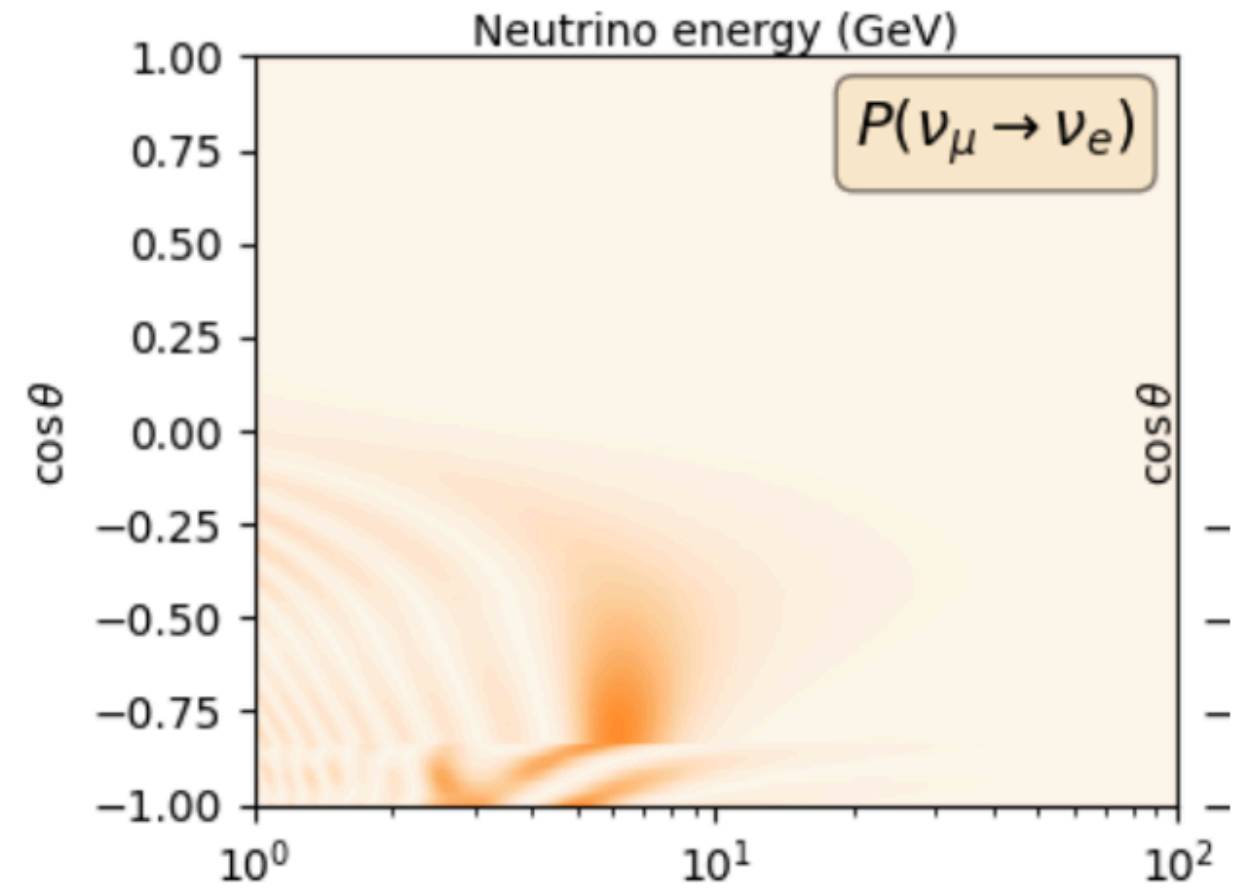
Distortions in regular neutrino oscillation patterns due to 'MSW' matter effect in dense materials



$$P_{\nu_\mu \rightarrow \nu_e} \approx \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \left(\frac{\Delta m_{31}^{2,m} L}{4E} \right)$$

At resonance: $\sin^2 2\theta_{13}^m \rightarrow 1$

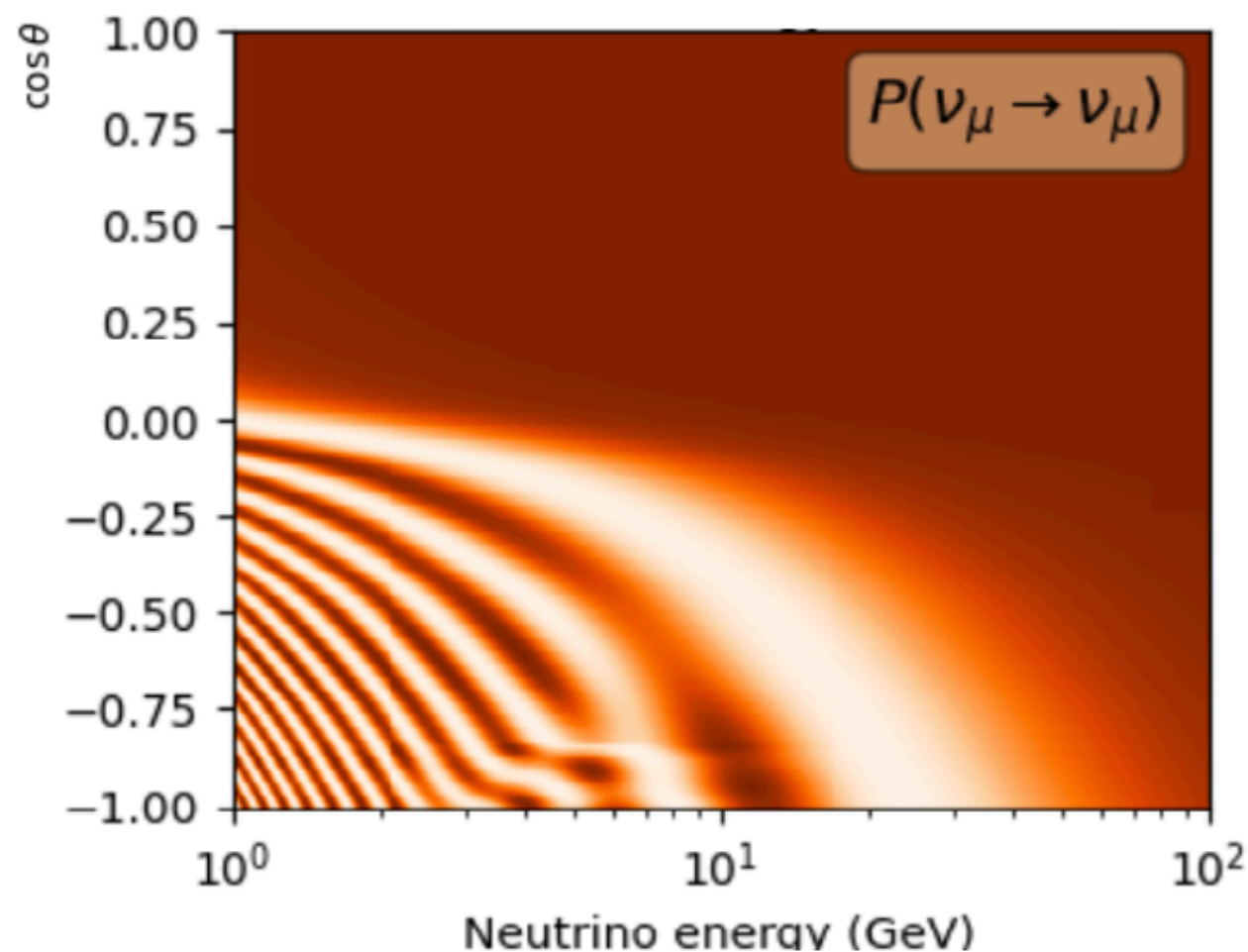
The **ν_e appearance probability** is strongly enhanced for NO.



$$P_{\nu_\mu \rightarrow \nu_\mu} \approx 1 - \cos^4 \theta_{13}^m \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m_{31}^{2,m} L}{4E} \right) - (\text{small solar term})$$

At resonance, $\theta_{13}^m \rightarrow \pi/4 \implies \cos^4 \theta_{13}^m \sim 0.25$

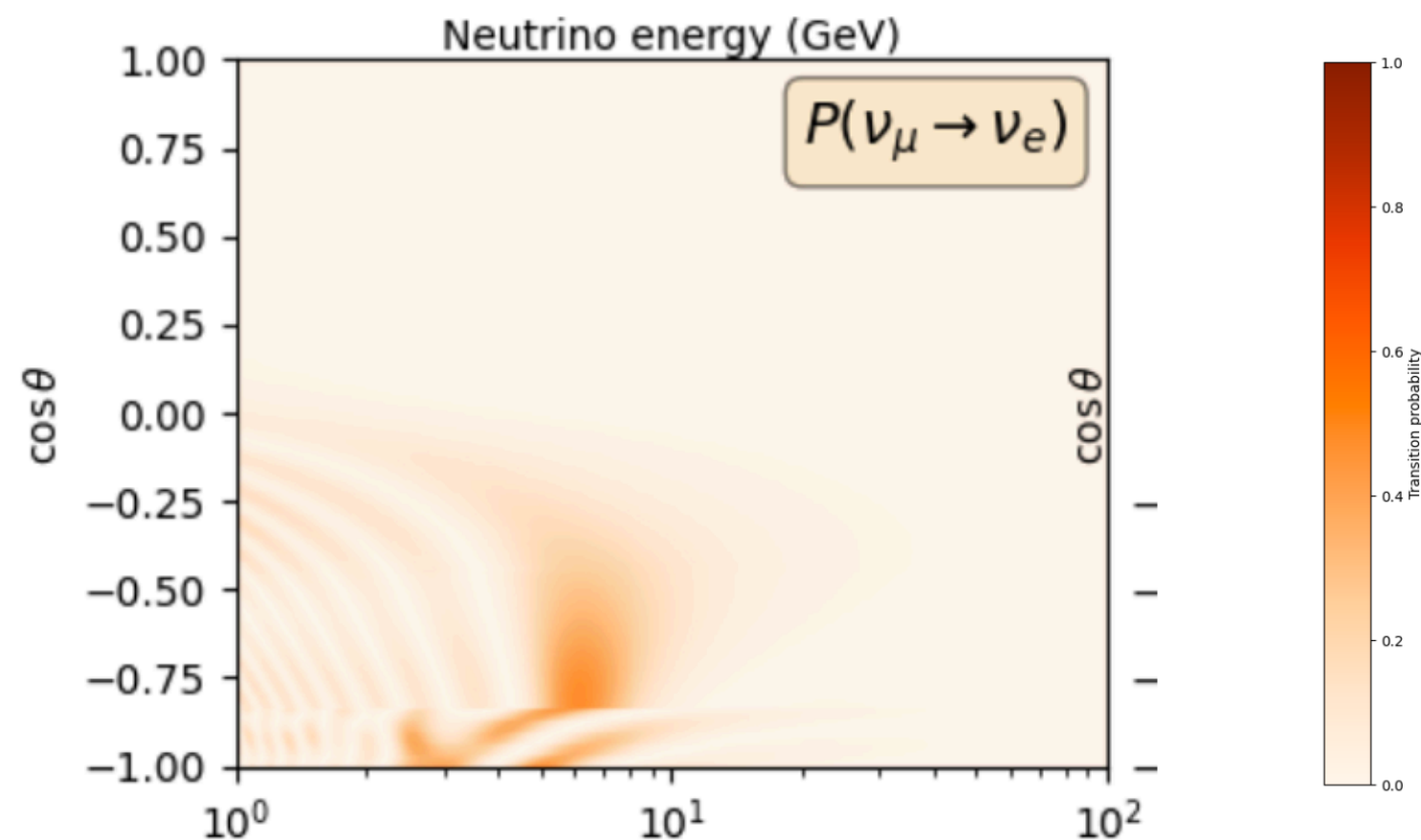
The **amplitude of ν_μ disappearance** changes from $\sin^2 2\theta_{23} \sim 1$ down to ~ 0.25 ,



$$P_{\nu_\mu \rightarrow \nu_e} \approx \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \left(\frac{\Delta m_{31}^{2,m} L}{4E} \right)$$

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$$P_{\nu_\mu \rightarrow \nu_\mu} \approx 1 - \cos^4 \theta_{13}^m \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m_{31}^{2,m} L}{4E} \right) - (\text{small solar term})$$

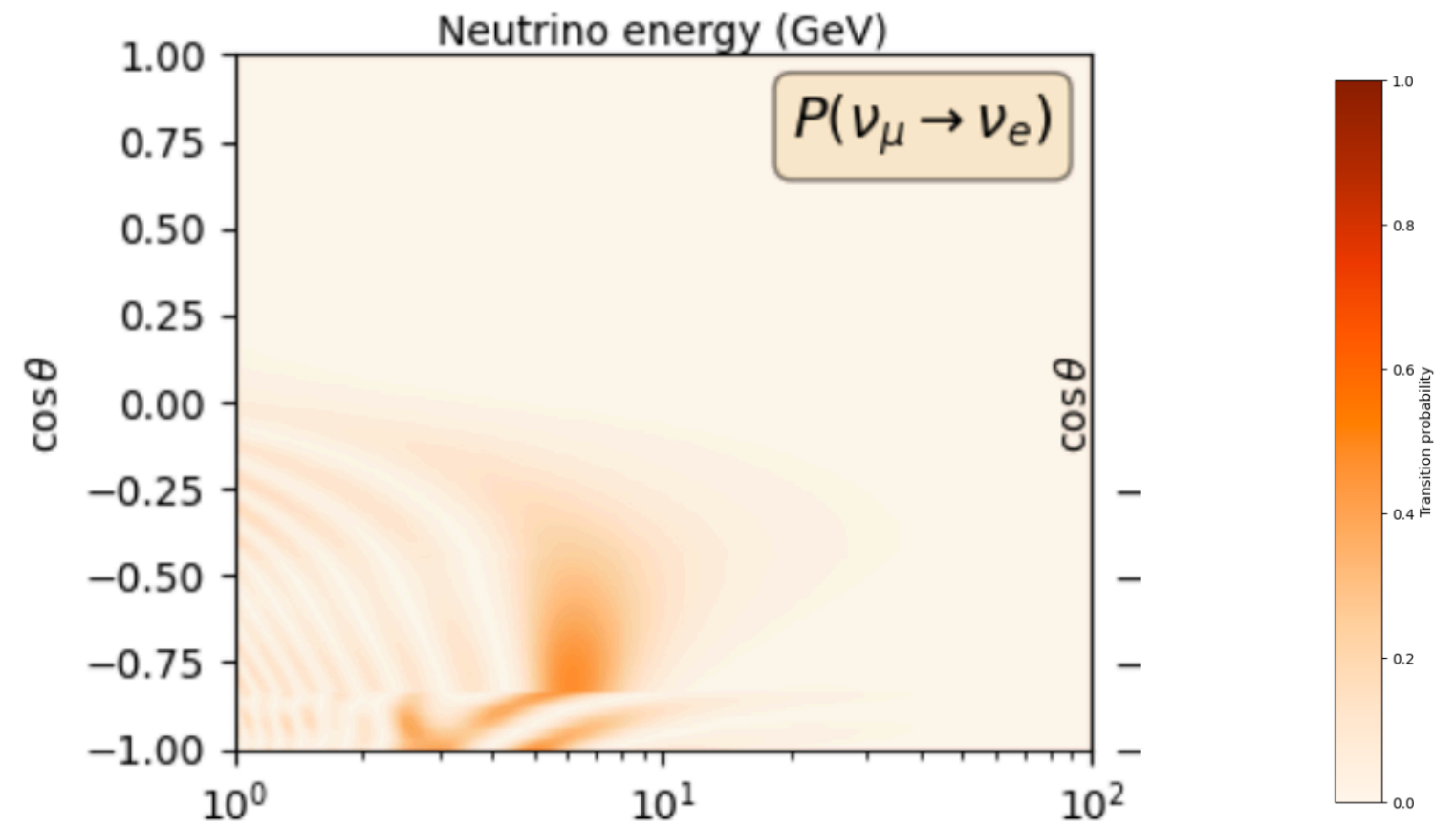
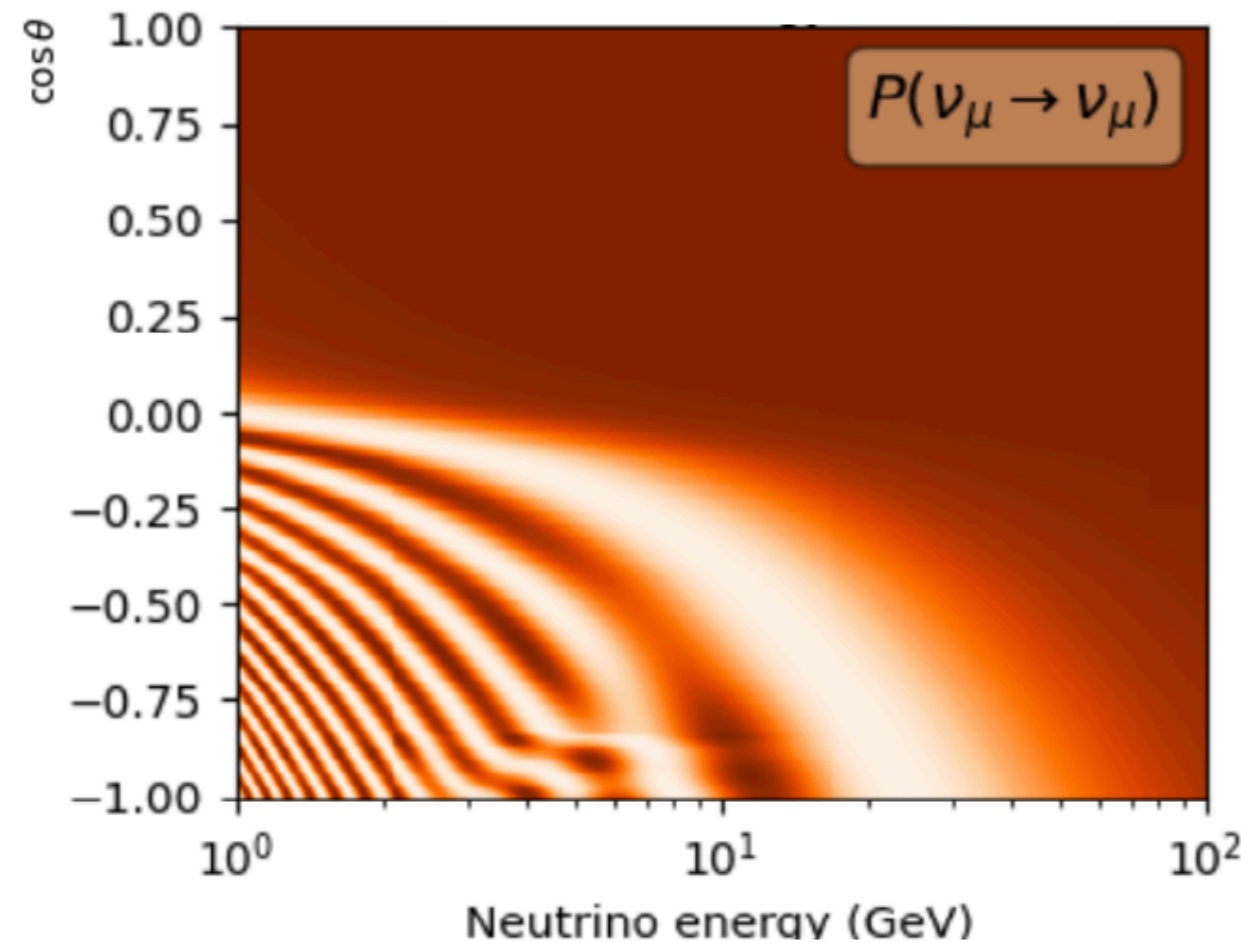
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$$P_{\nu_\mu \rightarrow \nu_e} \approx \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \left(\frac{\Delta m_{31}^{2,m} L}{4E} \right)$$

At resonance: $\sin^2 2\theta_{13}^m \rightarrow 1$

The **ν_e appearance probability** is strongly enhanced for NO.



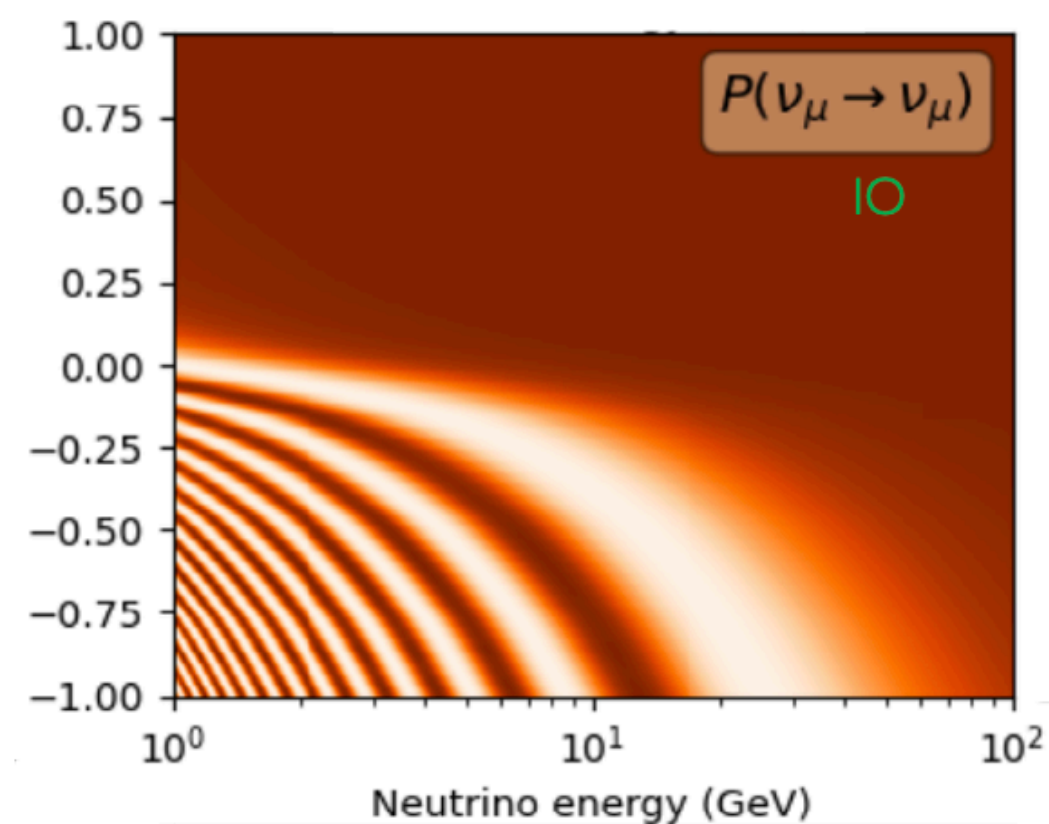
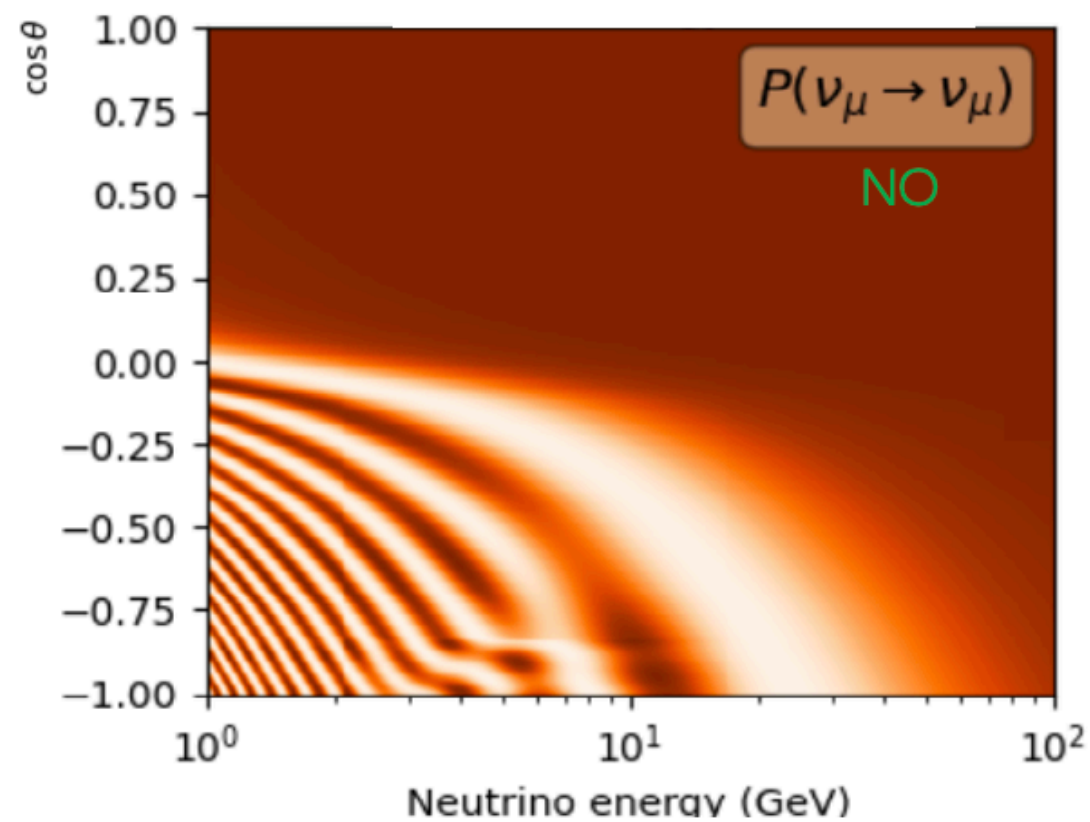
The **NMO sensitivity** comes mostly from **ν_e appearance**, because the **MSW matter resonance** selectively enhances ν_e appearance for **Normal Ordering** (and $\bar{\nu}_e$ for IO).

Therefore: NMO sensitivity mostly in showers

NMO Sensitivity: Neutrino Energies and Directions

$\Delta m_{32}^2 > 0$: normal ordering (NO)

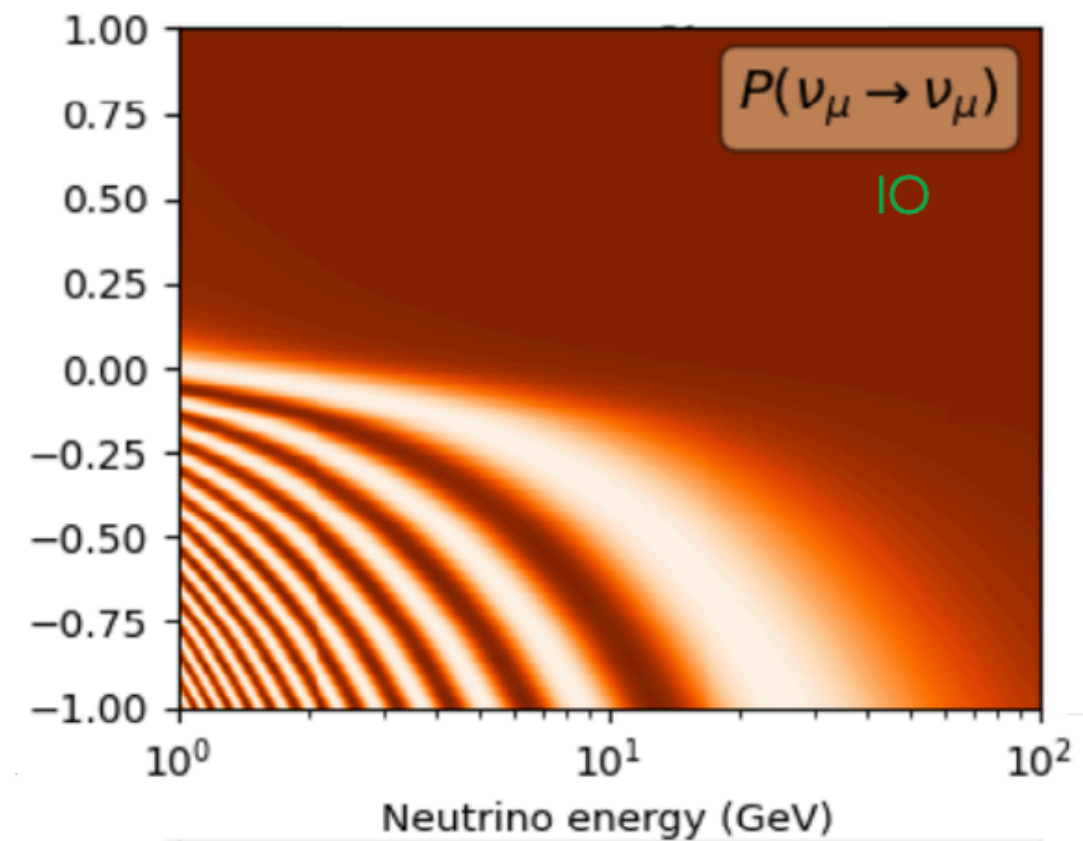
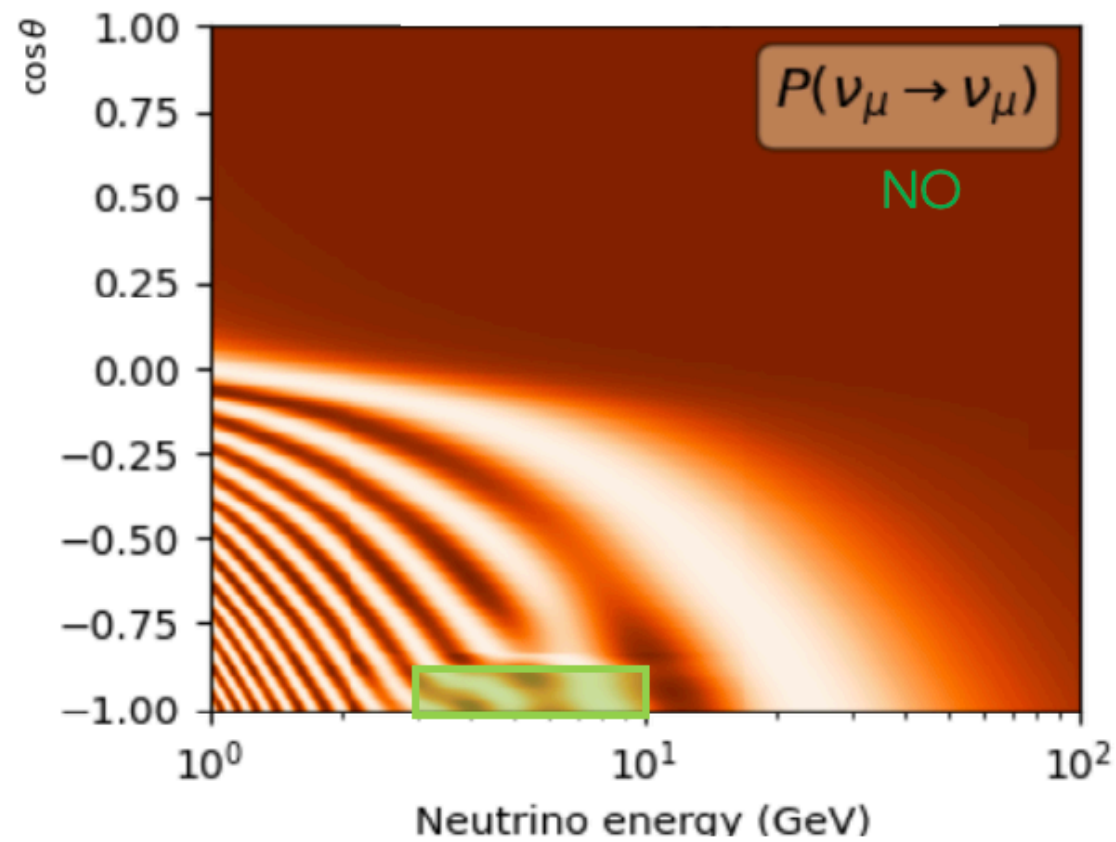
$\Delta m_{32}^2 < 0$: inverted ordering (IO)



NMO Sensitivity: Neutrino Energies and Directions

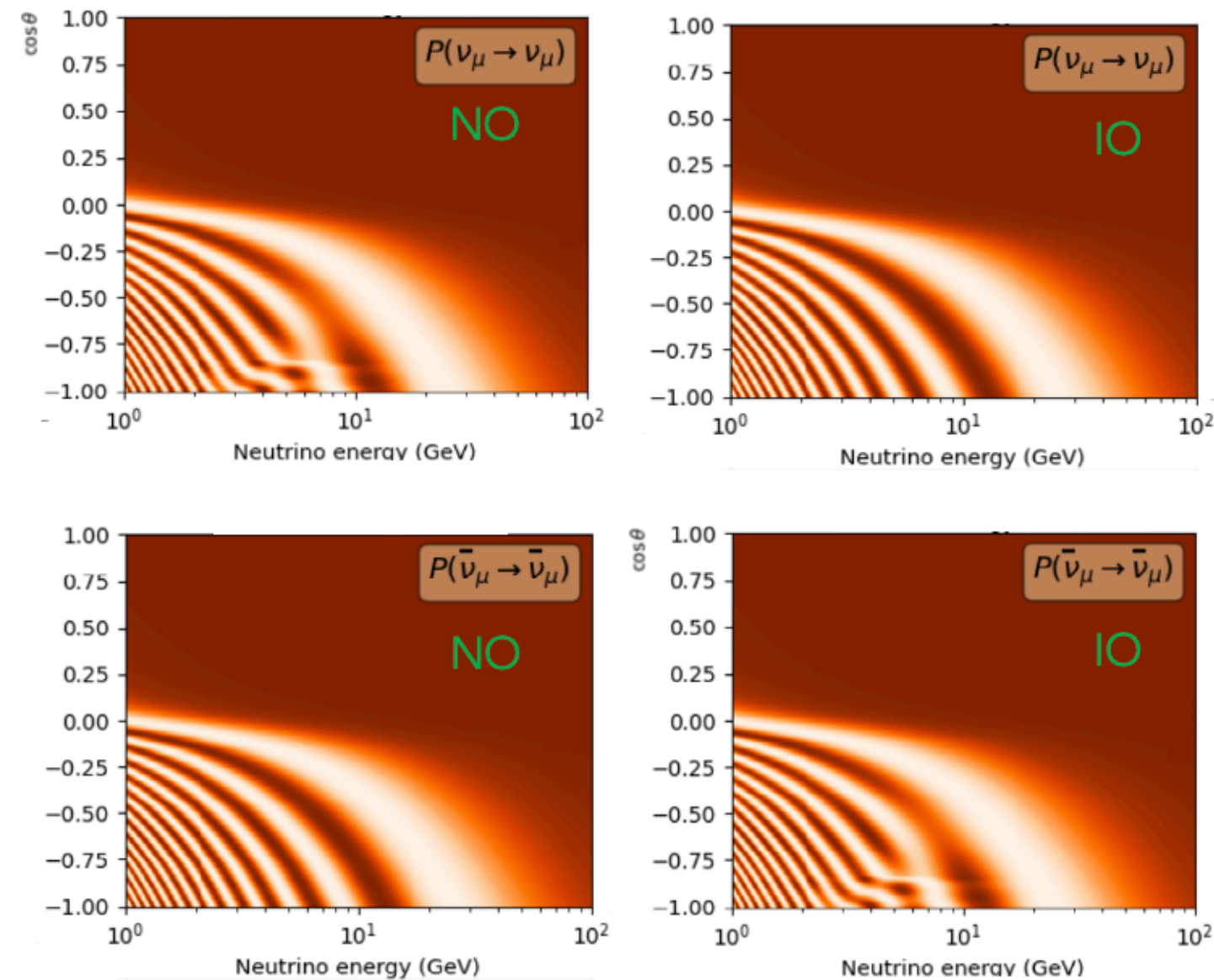
$\Delta m_{32}^2 > 0$: normal ordering (NO)

$\Delta m_{32}^2 < 0$: inverted ordering (IO)



NMO sensitivity in energy range 3-10 GeV and $-1 < \cos(\theta) < -0.8$

NMO Sensitivity: Neutrinos vs Anti-Neutrinos

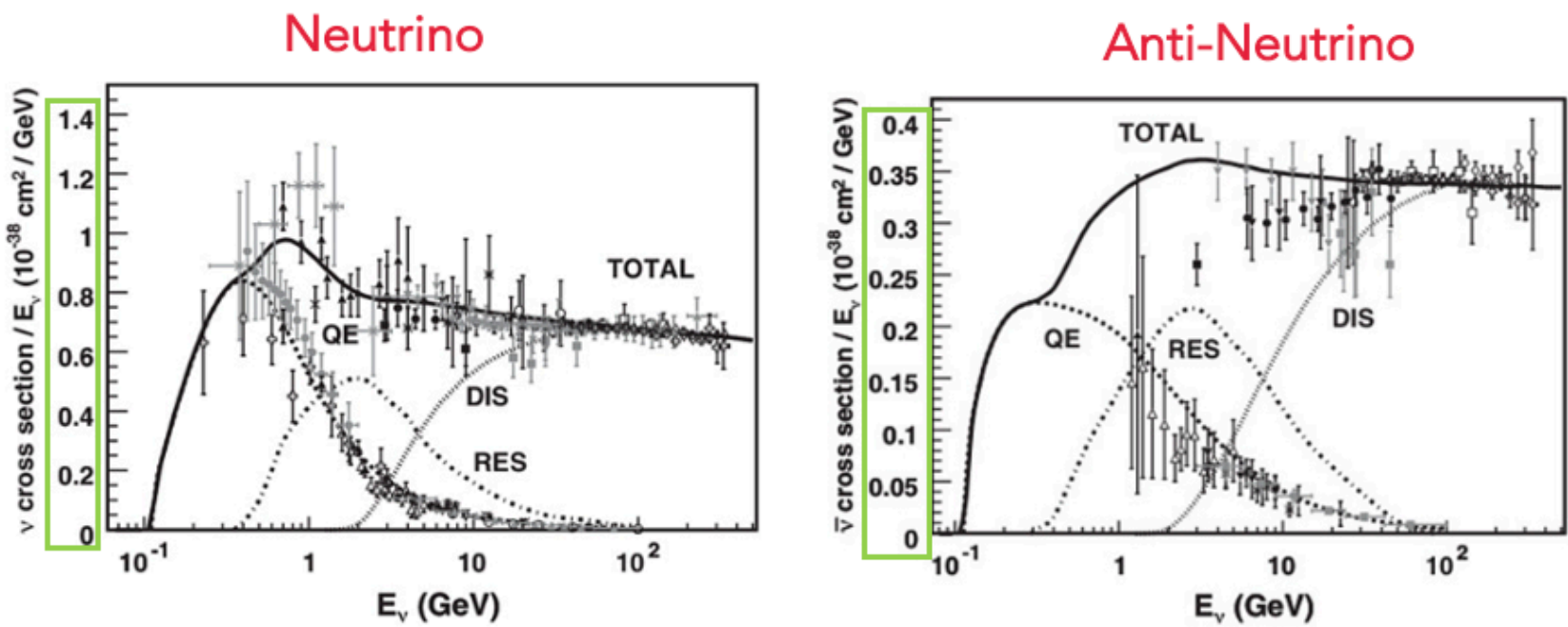


If this would all, there would be no sensitivity to NMO!

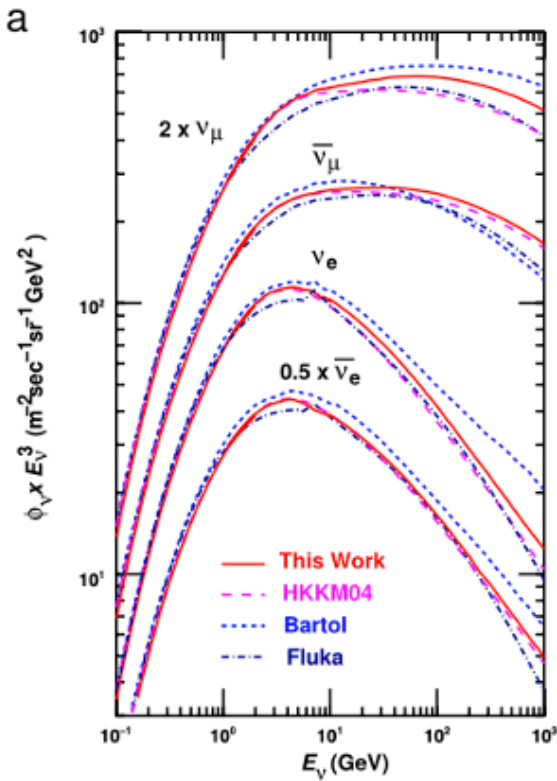
(Because KM3NeT can't distinguish (well) between neutrinos and anti-neutrinos)

NMO Sensitivity: Fluxes and Cross Sections

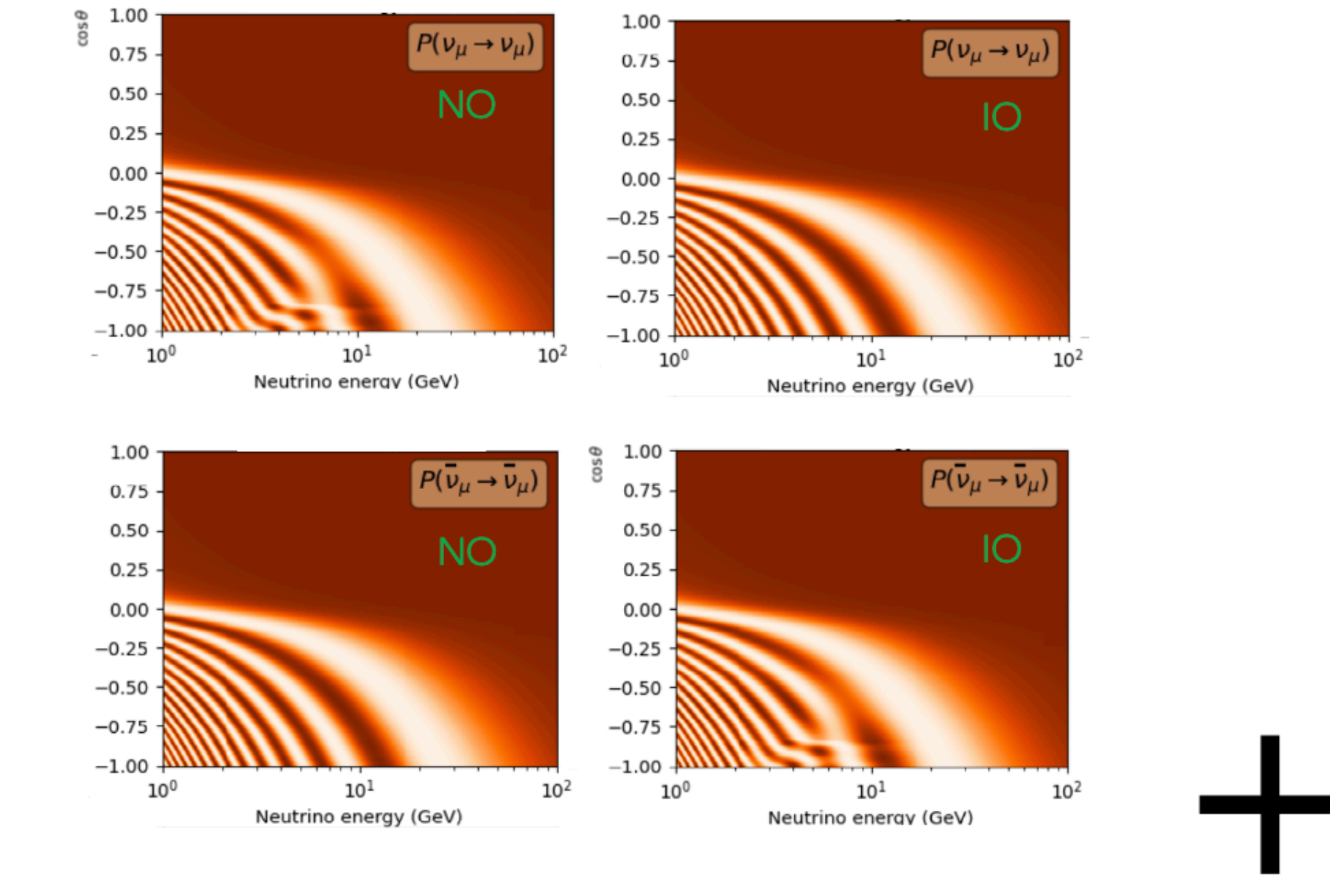
Cross section higher for neutrinos



Flux higher for atmospheric neutrinos



NMO Sensitivity: Neutrinos vs Anti-Neutrinos

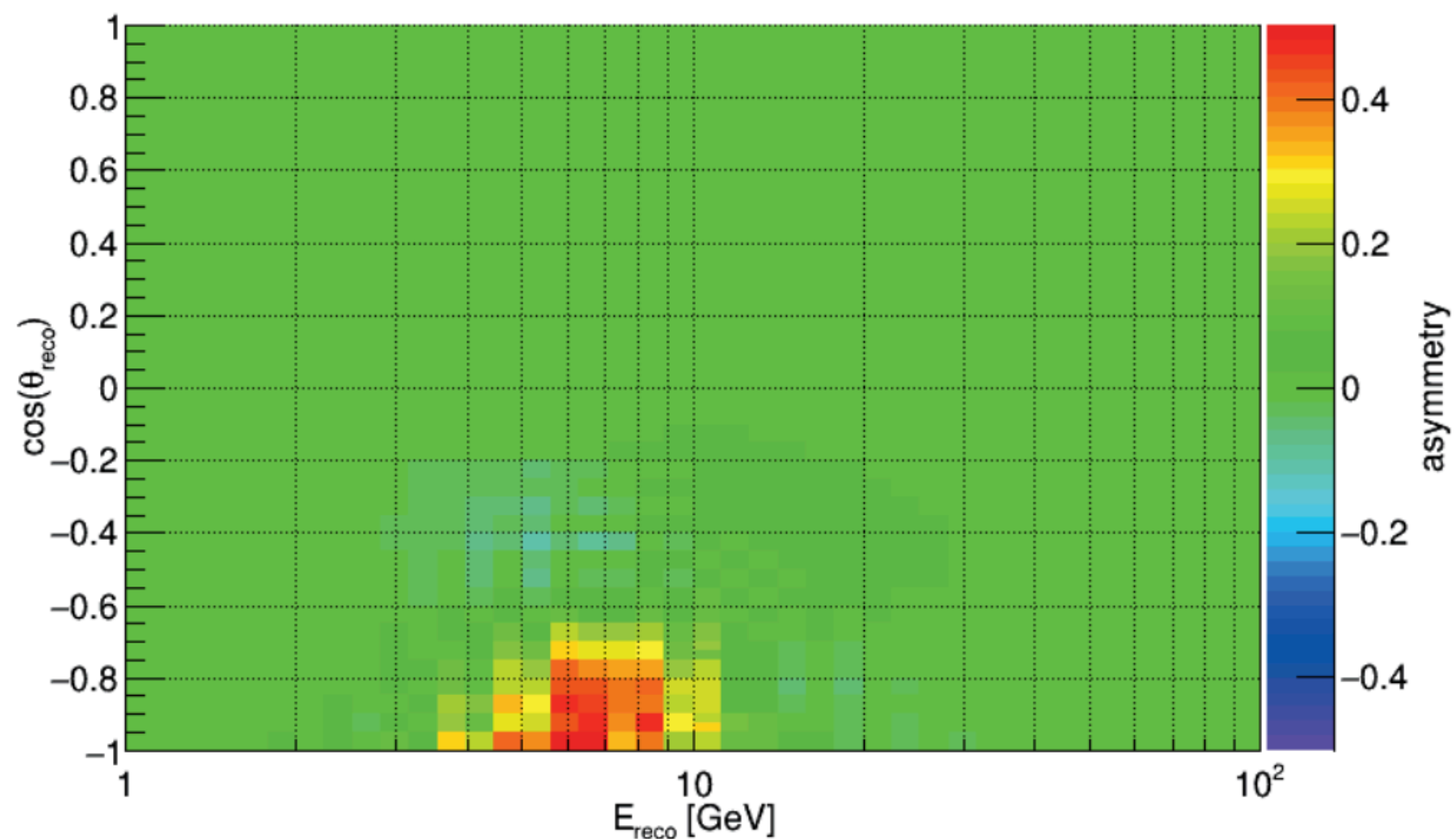


Asymmetric fluxes and cross sections for atmospheric neutrinos and antineutrinos saves the NMO sensitivity

Summary NMO Sensitivity (for atmospheric neutrinos)

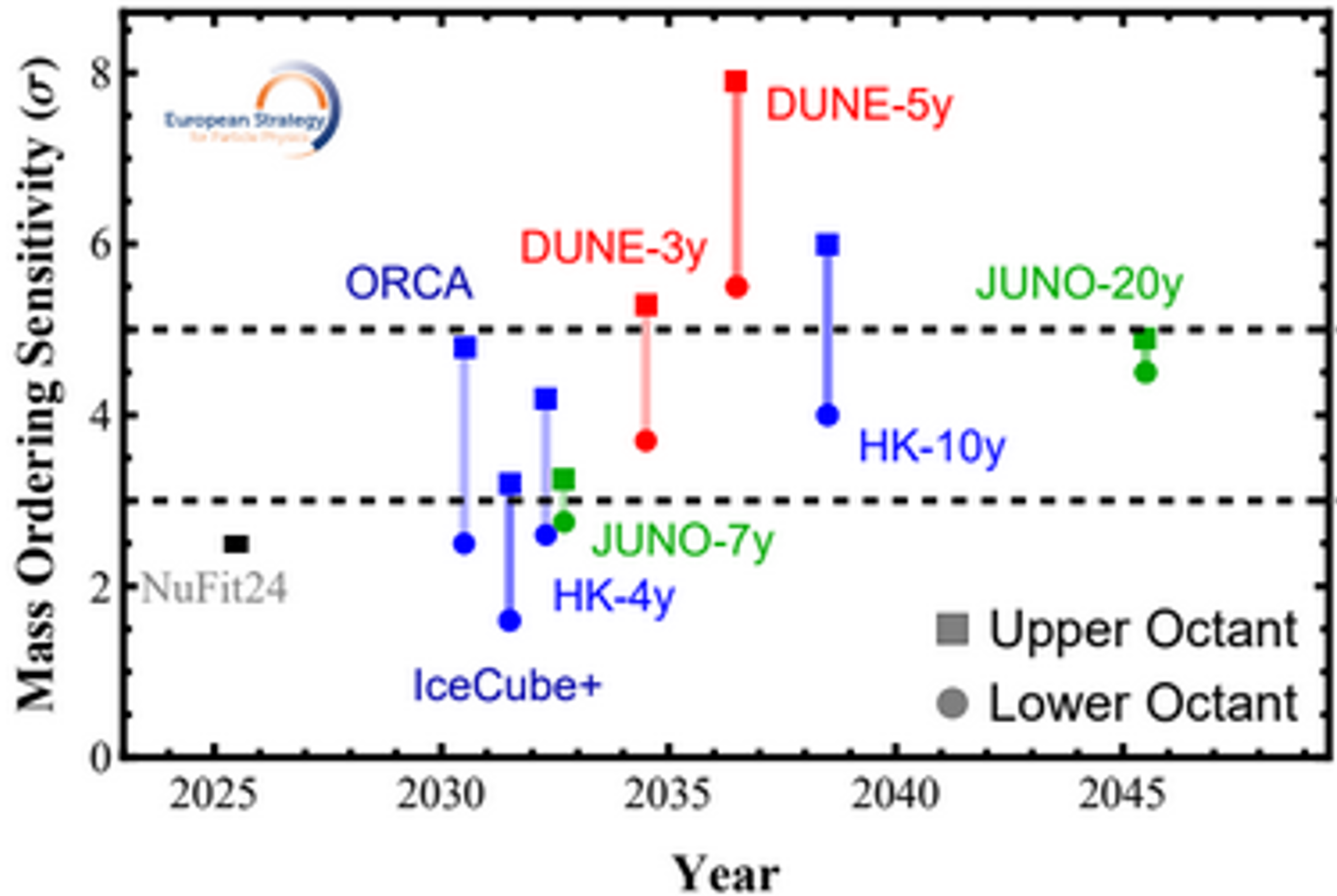
- NMO sensitivity in energy range 3-10 GeV
- NMO sensitivity for $-1 < \cos(\theta) < -0.8$
- NMO sensitivity due to MSW effect (asymmetric between NO and IO)
- NMO sensitivity mostly from shower channel
- NMO sensitivity due to unequal neutrino/antineutrino cross sections
- NMO sensitivity due to unequal neutrino/antineutrino fluxes

(NO-IO)/IO



(b) cascade channel

Figure 4.1: The asymmetry as defined in Equation 4.2 for an operation time of 3 years. The **simulation chain** has been set up as described in chapter 3. The two hypotheses being compared are the **NH** and **IH reference parameter values** as given in Appendix A. Note that the colour axis range is not the same in the two figures. The χ^2 -**sensitivity** (see Equation 4.3) is 2.16 (track channel), 3.58 (cascade channel), 4.18 (combined).



Backup

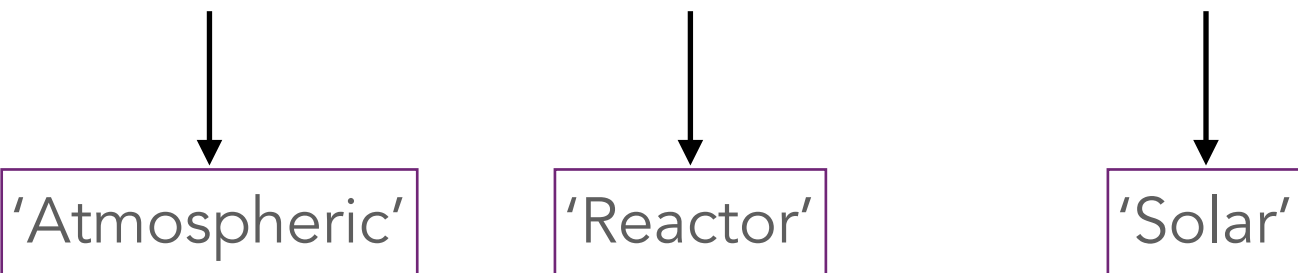
Summary from Frank Close's Neutrino

- Muon discovered in 1937 (p72) ('found, but not predicted' (p74): discovered first, anticipated as the pion)
- Pion discovered in 1947 (p72) ('predicted, not yet found' (p74))
- Wu: P violated in beta decay
- Lederman: P violated in muon decay. See Niels's lectures (<https://www.nikhef.nl/~h71/Lectures/2015/ppII-cpviolation-29012015.pdf>, p6): As an aside the curves also show an asymmetry in the height of the oscillation caused by the violation of parity in the muon decay.
- Lederman at Brookhaven (see below): P violated in pion decay.
- Brookhaven accelerator (1957) (p80):
 - protons on target gives intense beams of pions, 20% decays (the others semileptonically) into muon and neutrino (of which at that time, it's still thought that it's 'the' neutrino), both in the forward direction of the pion, **but** with opposite spin. The 'gift of nature' (Frank Close) is that the muon then decays into an electron and two neutrinos (*). So: stop the muons and see which direction the electrons emerge. If parity is violated, you expect more electrons in the direction the muon was originally moving. This measurement of $H(\mu^+) = +1$ (p8 of Zuber) establishes maximal parity violation.
 - (*) This was proven by Steinberger in 1948 (p75) (before the neutrino discovery!), who showed that the muon did NOT decay into an electron and a photon (then the electron energy would have to be fixed), but instead, energy was carried away by two additional particles! The $\mu^- \rightarrow e^- + \gamma$ was expected if the muon was 'just' a heavier version of the electron.
 - Also, it's all an indirect measurement of the neutrino helicity (it was the chirality of the electrons in Wu and the chirality of the muons in Lederman). Goldhaber proposes a direct measurement of the neutrino helicity.
- Steinberger 1958 (p76): $\pi^+ \rightarrow \mu^+ + \nu$ > 50000 times, $\pi^+ \rightarrow e^+ + \nu$ six times $\rightarrow$ consistent with parity violation (as already found by Lederman, see above).
- All this leads Pontecorvo to ask: are these neutrinos somehow different from each other (p76)? That's his 1959 paper that only appears in English in 1960. It contains two important points: 1) higher energy neutrinos have a higher cross section (in fact, in Fermi's effective theory it rises continuously), so use accelerator neutrinos. 2) there might be a difference in the type of neutrinos!
- Schwartz, Steinberger and Lederman (1962) (p81) take up the challenge (sort of independent from Pontecorvo). Use the high energy neutrinos themselves: intense proton beams on beryllium gives large numbers of pions, which then decay to muons and neutrinos. Filter out the muons with thick steel plates and see how every now and then a neutrino creates either a muon or an electron. After ten days and 10^{12} neutrinos, 51 muons were found and no electrons. The conclusion: there is a muon-neutrino and it's completely different from the electron-neutrino.
- Schwartz was onto the neutral current interaction, but his follow-up experiments never got funded and he left physics! (p82)

Remarks On A Confusing Naming Convention

Maury Goodman: "Despite the large ν_e component of the atmospheric neutrino flux, it is difficult to measure Δm^2_{21} effects. This is because of a cancellation between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ together with the fact that the ratio of ν_μ and ν_e atmospheric fluxes, which arise from sequential π and μ decay, is near 2. "

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



'Atmospheric' 'Reactor' 'Solar'

Boris Kayser: "The 'solar' matrix dominates the mixing of solar neutrinos. The mixing angle in this matrix, θ_{12} , is approximately the solar mixing angle θ_{sol} determined by treating the solar neutrino problem as a two-neutrino problem. This is a very good approximation, since the solar neutrinos are born as ν_e , and $|U_{e3}|^2$, the ν_3 fraction of ν_e , is quite small. Thus, ν_e is approximately a mixture of just ν_1 and ν_2 , and it will remain so as it propagates."

- Only makes sense under certain assumptions
 - 2-neutrino approximations
 - Mostly due to (numerical) coincidences
 - Small $\Delta m^2_{12} \sim 10^{-5} \text{eV}^2$
 - Small θ_{13}
 - Near-maximal 'atmospheric' mixing

The third, Solar, matrix on the right-hand side of Eq. (40) dominates the mixing of solar neutrinos. The mixing angle in this matrix, θ_{12} , is approximately the solar mixing angle θ_{sol} determined by approximating the solar neutrino problem as a two-neutrino problem. This is a very good approximation, since the solar neutrinos are born as ν_e , and $|U_{e3}|^2$, the ν_3 fraction of ν_e , is quite small. Thus, ν_e is approximately a mixture of just ν_1 and ν_2 , and it will remain so as it propagates.

The Significance of $P(\nu_e \rightarrow \nu_e)$

For SNO-energy-range solar neutrinos,
there is a very pronounced solar matter effect.
(Mikheyev and Smirnov)

At these energies —

A solar neutrino is born in the core of the sun as a ν_e .

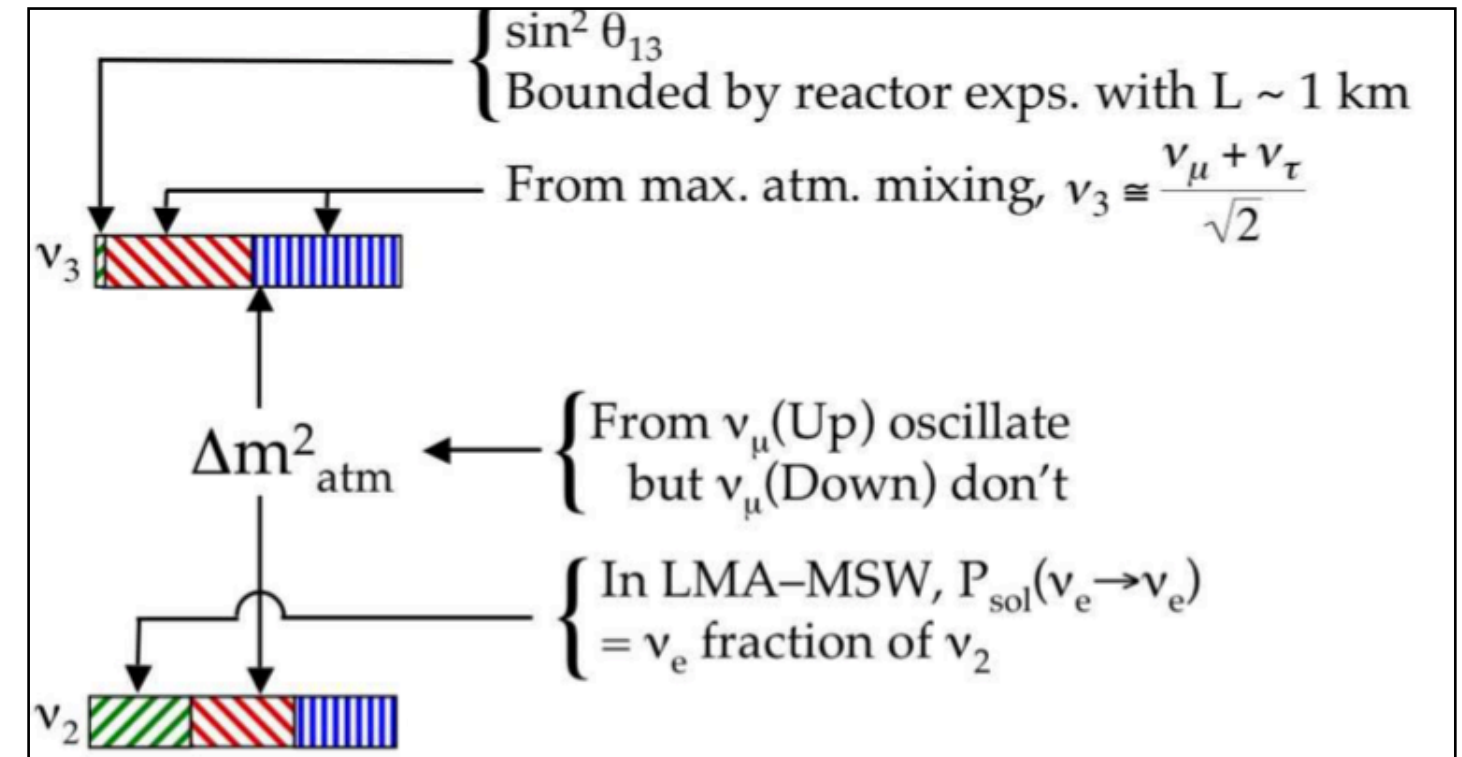
But by the time it emerges from the outer edge
of the sun, with 91% probability it is a ν_2 .
(Nunokawa, Parke, Zukanovich-Funchal)

Then $P(\nu_e \rightarrow \nu_e)$ at earth $= \left| \langle \nu_e | \nu_2 \rangle \right|^2 = |U_{e2}|^2$.

ν_2 
↑ $|U_{e2}|^2 = 0.3$.

So What Does U_{PMNS} look like?

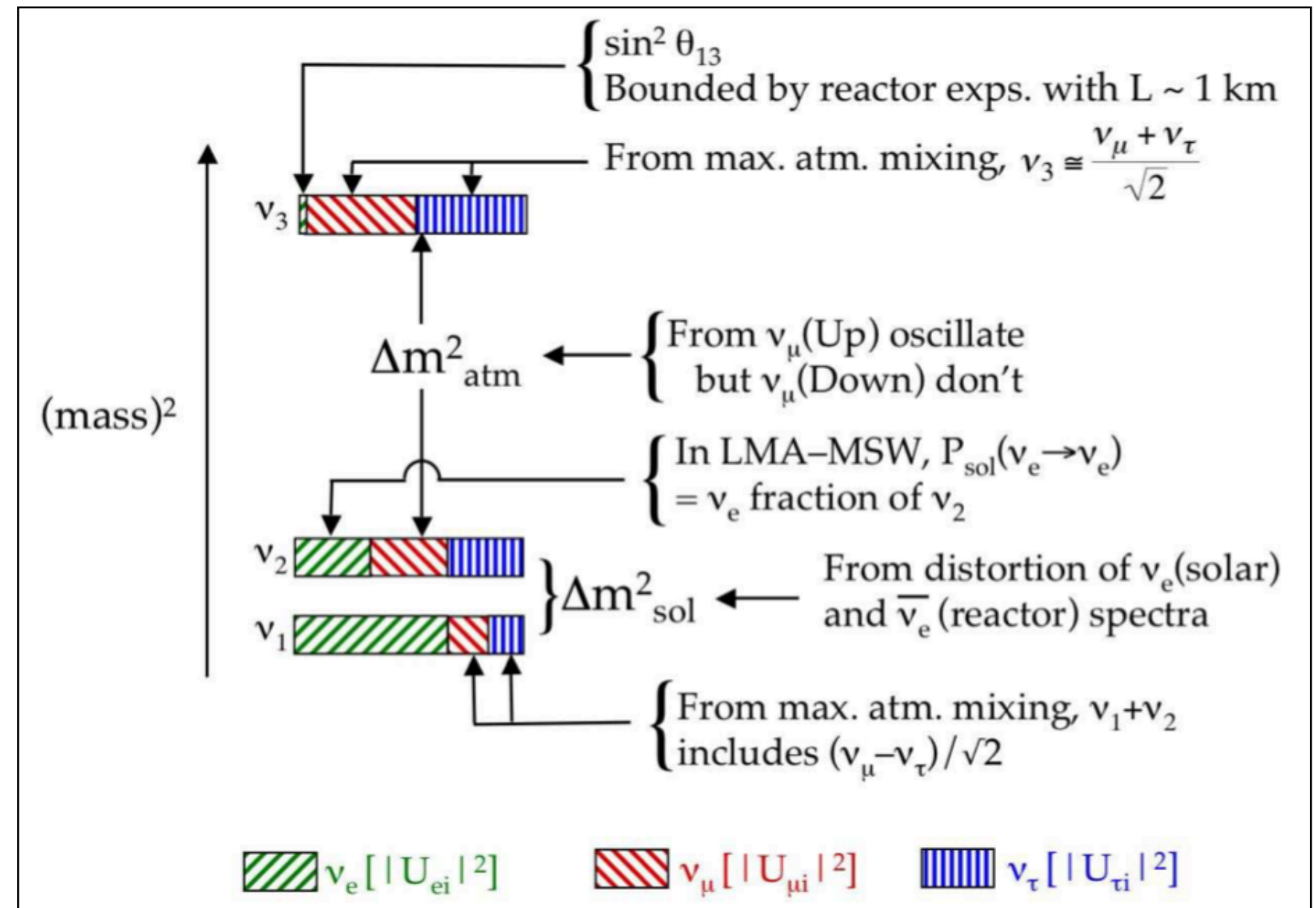
- $P_{\text{sol}}(\nu_e \rightarrow \nu_e)$, i.e. the measured solar electron neutrino flux is roughly 1/3
- Neutrinos in the sun start out as the higher-energy eigenstate ν_e of $H_M(r=0)$ where electron density is high
- The neutrino propagates outward as the slowly moving eigenstate of the slowly changing Hamiltonian. When it leaves the Sun it's still the higher-energy eigenstate. But now the electron density is zero and the Hamiltonian is just the vacuum Hamiltonian. We called that higher-energy eigenstate ν_2
- Thus, solar neutrinos leave the Sun as ν_2
- So electron content of ν_2 is 1/3
- Solar neutrino flavour change is ν_e to ν_x , where in ν_x , the flavour content $\nu_\mu - \nu_\tau$ is governed by the maximal atmospheric mixing again



So What Does U_{PMNS} look like?

- The ν_e content of ν_1 follows from unitarity: 2/3
- The ν_μ and ν_τ content of ν_1 follows again from maximal mixing
 - ν_1 and ν_2 contain the neutrino for $(\nu_\mu - \nu_\tau)/\sqrt{2}$, the 50% mixture that is orthogonal to the one in ν_3

$$U \approx \begin{array}{ccc} & \nu_1 & \nu_2 & \nu_3 \\ \left[\begin{array}{ccc} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{array} \right] & \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \end{array}$$



CP violation

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \begin{matrix} + \\ - \end{matrix} 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

- If CP is conserved: $P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$
- CP violation if: $P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq 0 = 4 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$
- If $\alpha=\beta$, the imaginary part is always zero due to $U_{\alpha i} U_{\alpha i}^* = |U_{\alpha i}|^2$
- Can only find CP violation in appearance experiments!
- Defining the so-called Jarlskog invariant $J_{\alpha\beta} \equiv \text{Im}(U_{\alpha 1} U_{\alpha 2}^* U_{\beta 1}^* U_{\beta 2}) = \pm c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta$, with the $\pm$ denoting the cyclic (anticyclic) permutations of $(\alpha, \beta) = (e, \mu), (\mu, \tau), (\tau, e)$
- $P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = \Delta P_{\alpha\beta}^{CP} = -16 J_{\alpha\beta} \sin \left(\Delta m_{12}^2 \frac{L}{4E} \right) \sin \left(\Delta m_{23}^2 \frac{L}{4E} \right) \sin \left(\Delta m_{13}^2 \frac{L}{4E} \right)$
- Note the signs of the Δm_{ij}^2 and how the sum of sines became a product using trigonometric identities

Neutrino Oscillations including CP Violation

Both imaginary!

$$P(\alpha \rightarrow \beta)(t) = \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \operatorname{Re} \sum_{i>j} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} \exp \left(-i \frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

$$2\operatorname{Re}(A \exp(-ix)) = 2\operatorname{Re}((a + ib) \exp(-ix)) = 2\operatorname{Re}((a + ib)(\cos(x) - i \sin(x)))$$

$$= 2\operatorname{Re}(a \cos(x) + b \sin(x) + i(b \cos(x) - a \sin(x)))$$

$$= 2(a \cos(x) + b \sin(x)) = 2(\operatorname{Re} A \cos(x) + \operatorname{Im} A \sin(x))$$

Neutrino Oscillations including CP Violation

Both imaginary!

$$P(\alpha \rightarrow \beta)(t) = \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \operatorname{Re} \sum_{i>j} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} \exp \left(-i \frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

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$$= 2\operatorname{Re}(a \cos(x) + b \sin(x) + i(b \cos(x) - a \sin(x)))$$

$$= 2(a \cos(x) + b \sin(x)) = 2(\operatorname{Re} A \cos(x) + \operatorname{Im} A \sin(x))$$

$$P(\alpha \rightarrow \beta)(t) = \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \sum_{i>j} \operatorname{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \cos \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right) + 2 \sum_{i>j} \operatorname{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

Neutrino Oscillations including CP Violation

$$P(\alpha \rightarrow \beta)(t) = \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \cos\left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E}\right) + 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin\left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E}\right)$$

Neutrino Oscillations including CP Violation

$$P(\alpha \rightarrow \beta)(t) = \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \cos\left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E}\right) + 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin\left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E}\right)$$

For Re term: rewrite the cos again using $\cos(2\theta) = 1 - 2\sin^2(\theta)$.

For Im term: use $2\sin(2\theta) = 4\sin(\theta)\cos(\theta)$.

$$P(\alpha \rightarrow \beta)(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2\left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E}\right) + 4 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin\left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E}\right) \cos\left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E}\right)$$

Neutrino Oscillations including CP Violation

$$P(\alpha \rightarrow \beta)(t) = \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \cos \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right) + 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

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Antineutrino: all $U_{\alpha i} \rightarrow U_{\alpha i}^*$

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) + 4 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \cos \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

Neutrino Oscillations including CP Violation

$$P(\alpha \rightarrow \beta)(t) = \sum_i |U_{\alpha i} U_{\beta i}^*|^2 + 2 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \cos \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right) + 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

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$$P(\alpha \rightarrow \beta)(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) + 4 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \cos \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

Antineutrino: all $U_{\alpha i} \rightarrow U_{\alpha i}^*$

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) + 4 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \cos \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

Equivalently

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) - 4 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \cos \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

Neutrino Oscillations Master Equation including CP Violation

Neutrinos

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) + 4 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \cos \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

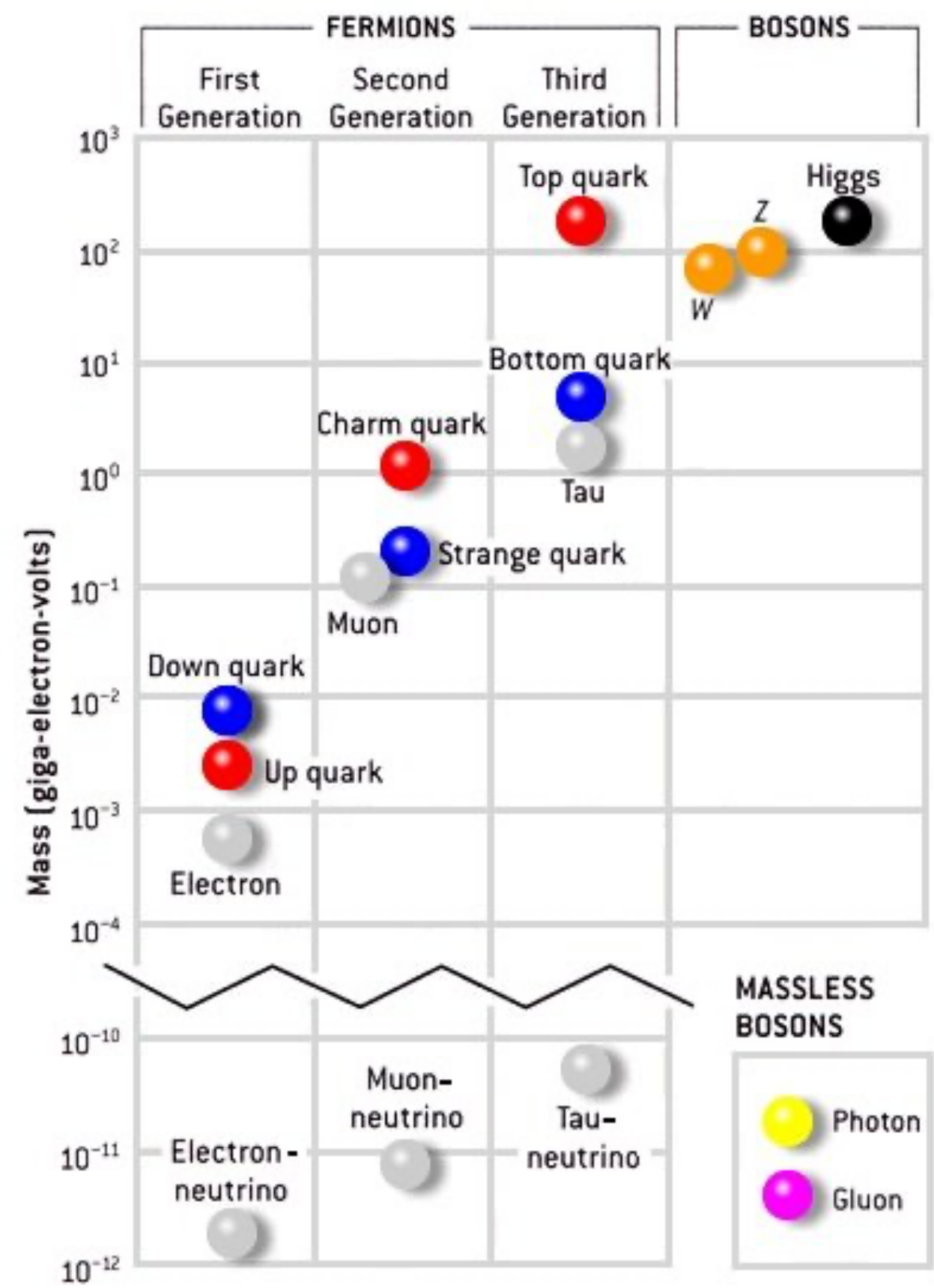
Anti-Neutrinos

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) - 4 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \cos \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right)$$

Combining and using $2\sin(x)\cos(x)=\sin(2x)$

$$P(\bar{\alpha} \rightarrow \bar{\beta})(t) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin^2 \left(\frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right) \pm 2 \sum_{i>j} \text{Im}(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}) \sin \left(\frac{\Delta m_{ij}^2}{2} \frac{L}{E} \right)$$

SM Particle Masses



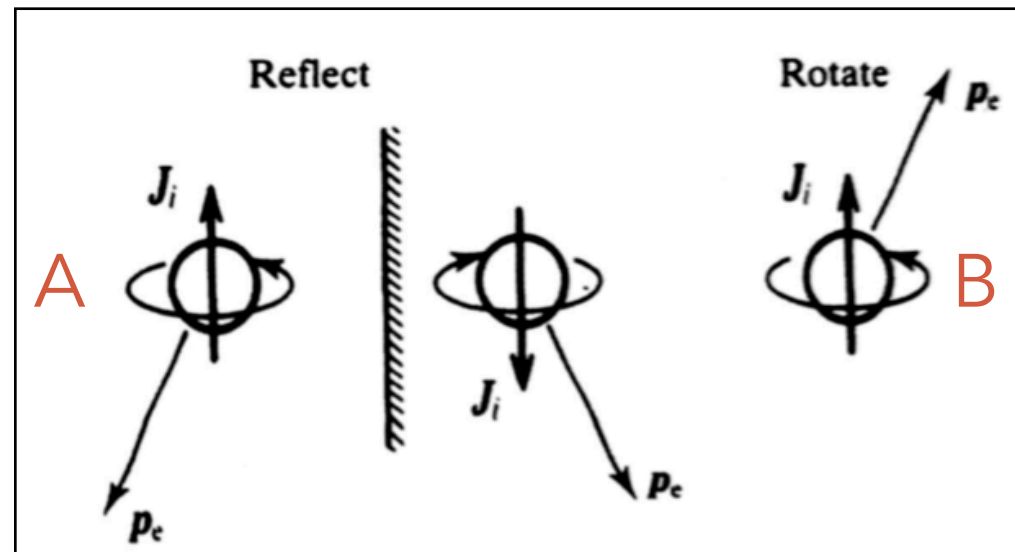
Properties of the Weak Interaction

- 1956: Question from Lee and Yang following the θ - τ puzzle: everyone assumes parity is conserved in weak interaction, but did we actually check???
- Parity (P) transformation:

$$\mathbf{P} : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$

- Similarly, P is 1) reflection in a plane, 2) 180 degrees rotation around the axis perpendicular to that plane

- $P(A) = B$



Reflections on Reflections

- Feynman: “Never mind [...mere conventions]. Let us talk about *phenomena*.”

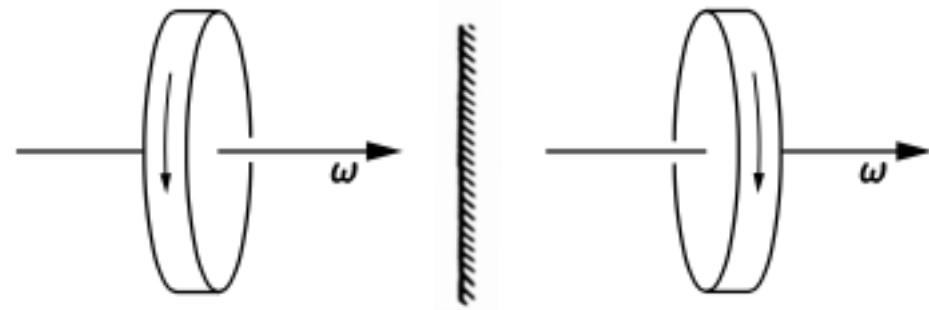


Fig. 52-3. A rotating wheel and its mirror image. Note that the angular velocity “vector” is not reversed in direction.

Never mind (the convention of) the vector. Would you be shocked to find the wheel rotating in the other direction???

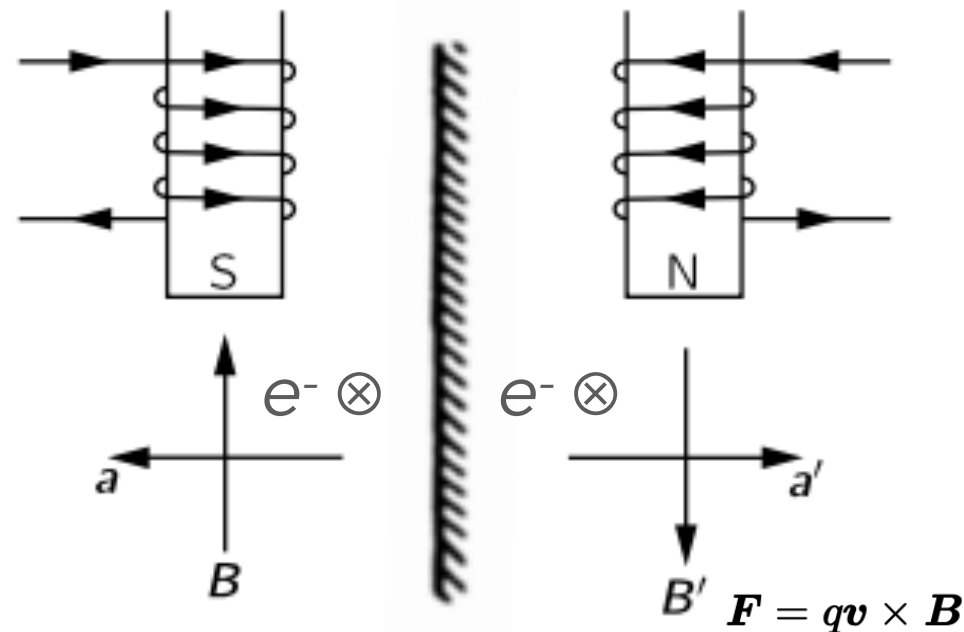


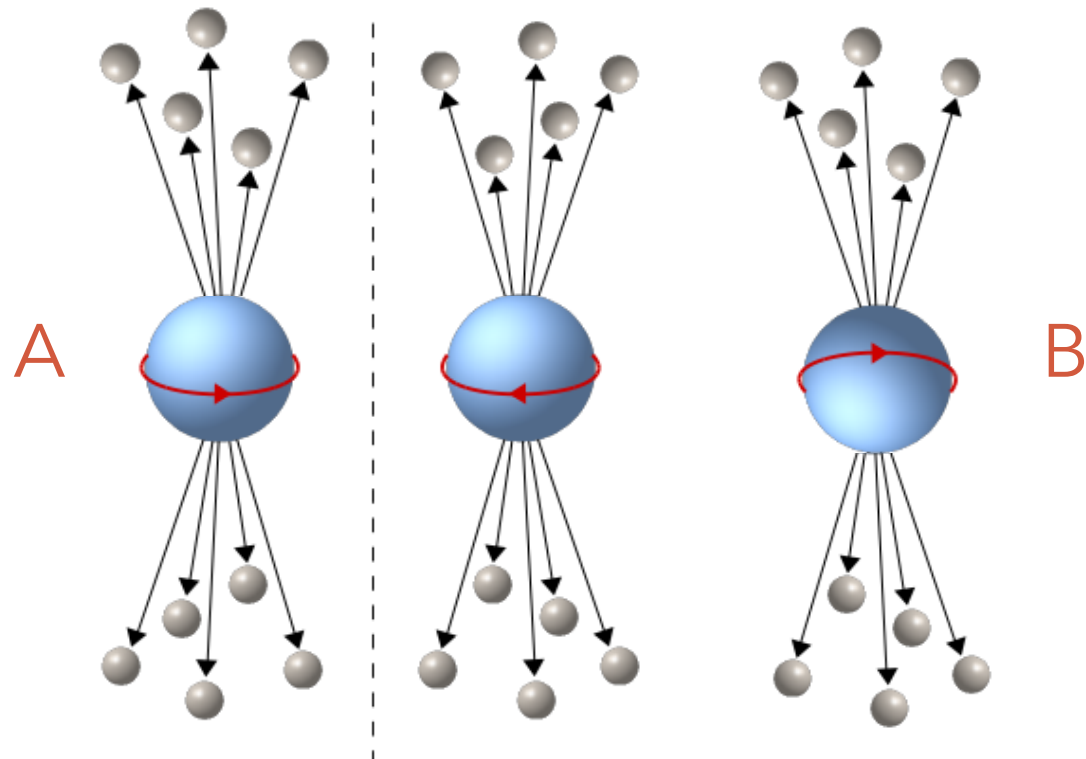
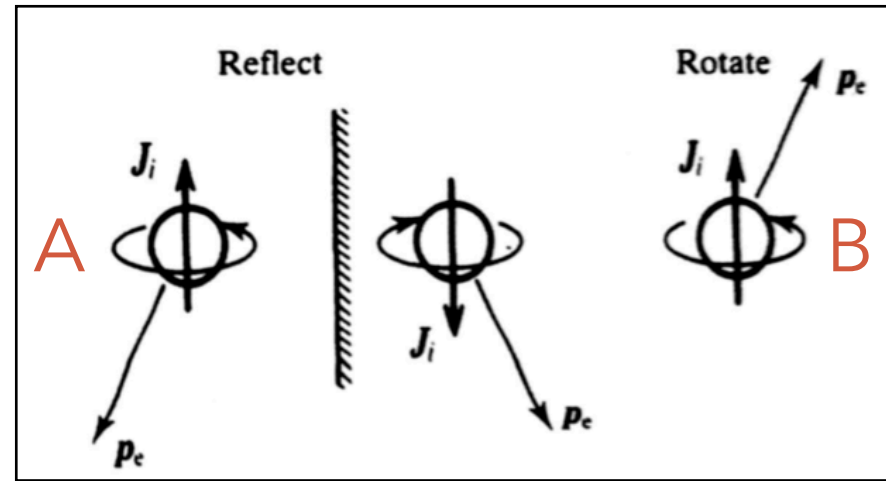
Fig. 52-4. A magnet and its mirror image.

Never mind the conventions (like north and south). Would you be shocked to find the electron deviation to be in the same direction???

What About This?

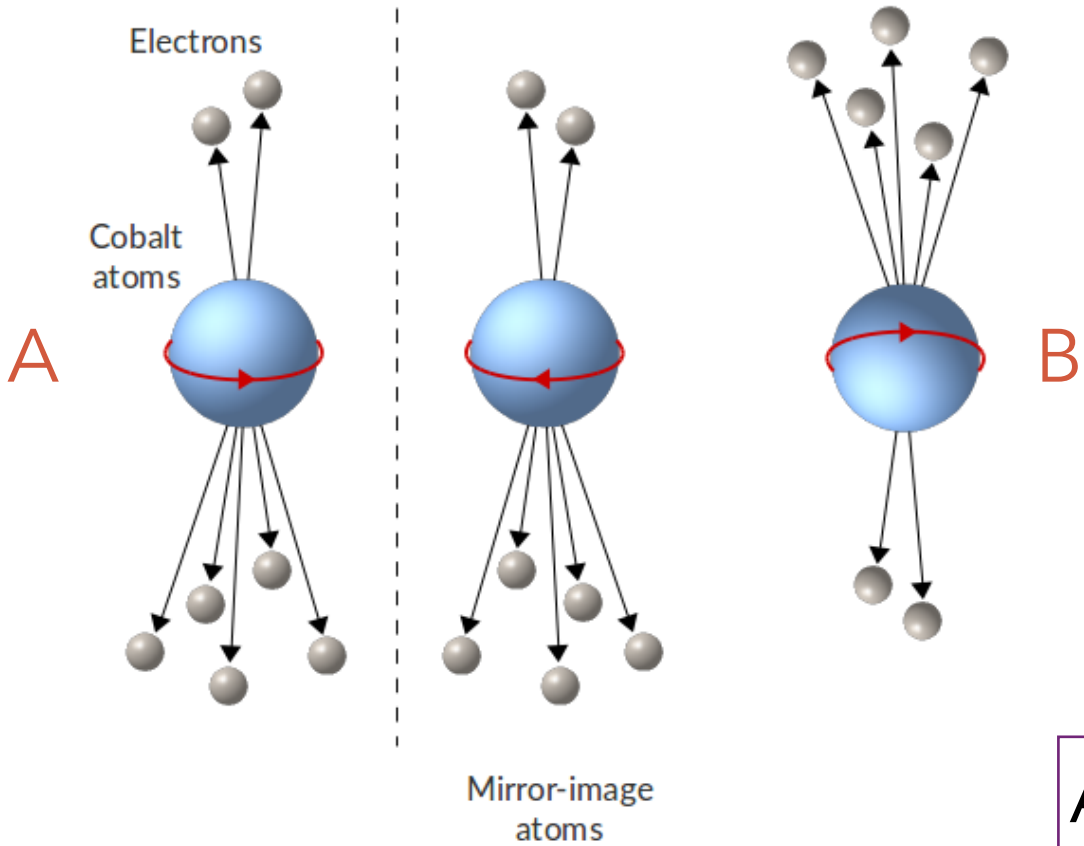
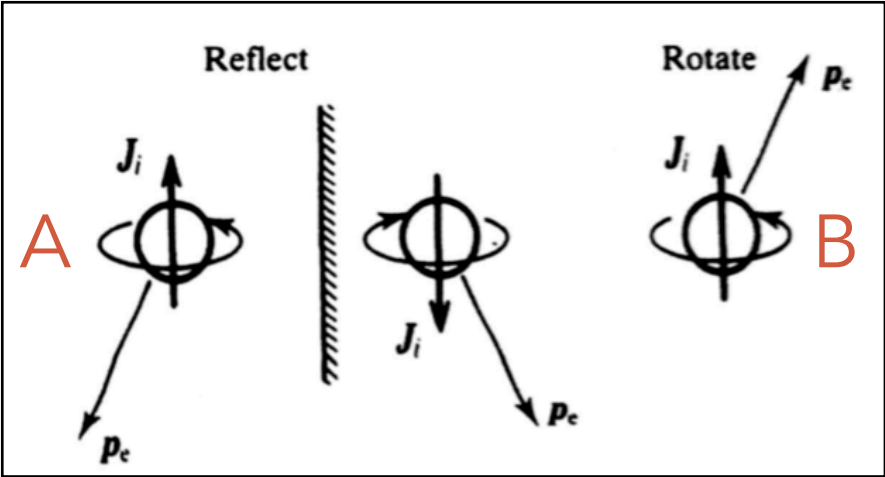


Weak Force Parity Experiment if P is Conserved



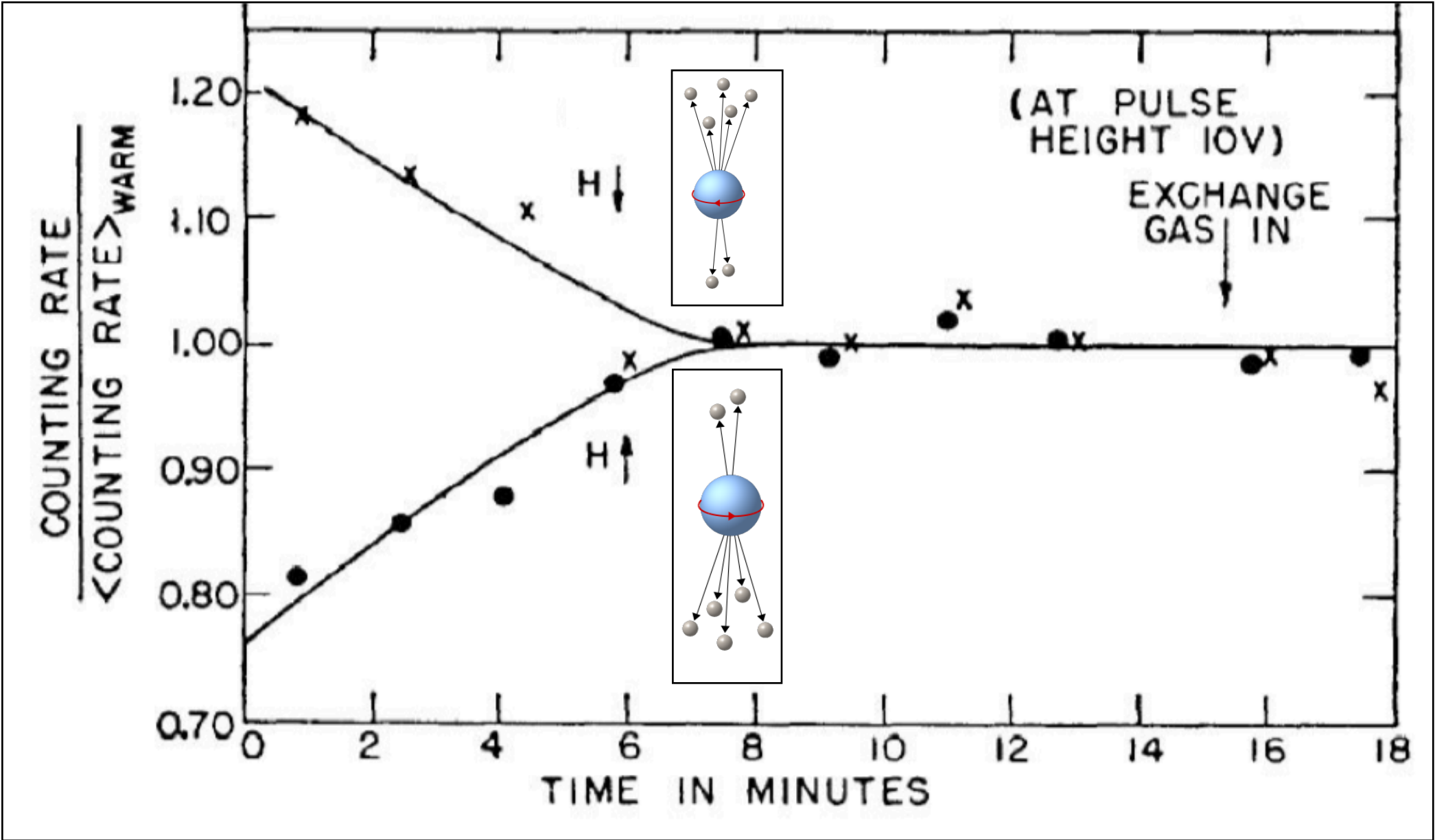
Assuming P-symmetry holds, would you be shocked to find more electrons being emitted downward or upward?

Weak Force Parity Experiment if P is Violated

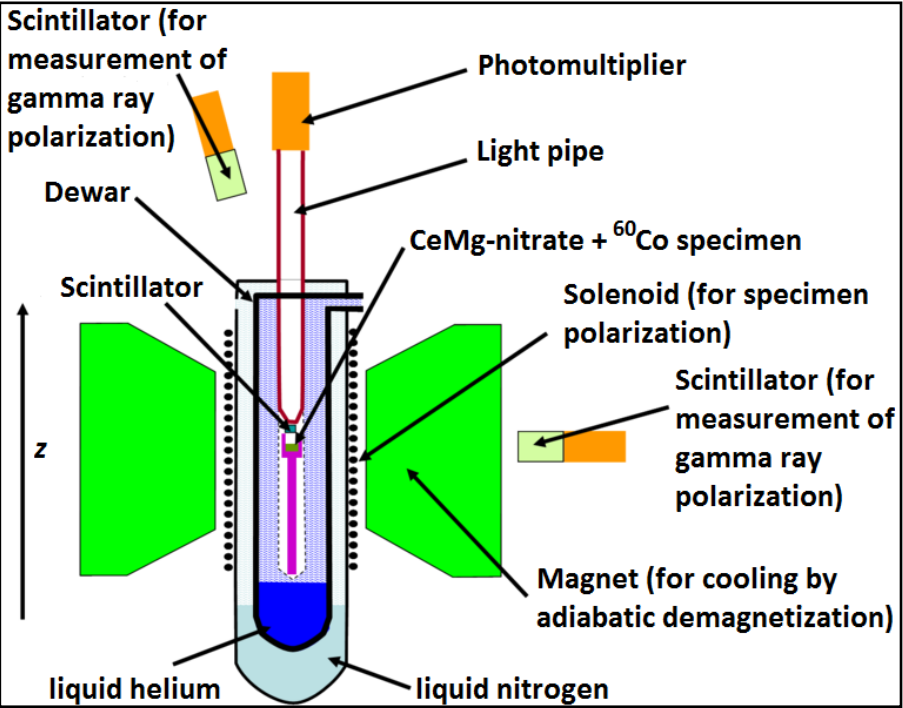


A and B do NOT look the same -> P symmetry is violated!!!

Wu Experiment (1956)



Parity is (maximally) violated in beta decays!



Intermezzo: (Neutrino) Helicity

- Helicity is the projection of the spin along the direction of the momentum:

$$h = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|} = \pm 1$$

- Particle with $h=+1$ is called right-handed (spin and momentum parallel)
- Particle with $h=-1$ is called left-handed (spin and momentum anti-parallel)
- h is a pseudoscalar (dot product between ordinary polar vector and an axial or pseudovector)
 - As a result, h changes sign under the parity operation
- h is invariant only for massless particles
- Explanation that fits all observations of parity violation: the weak interaction only couples to left-handed fermions or right-handed antifermions.
- Note that none of the mentioned experiments measured the neutrino helicity directly
 - Goldhaber did just that in 1958 in an experiment that is far too difficult to explain here...
- Neutrinos are left-handed and antineutrinos are right-handed

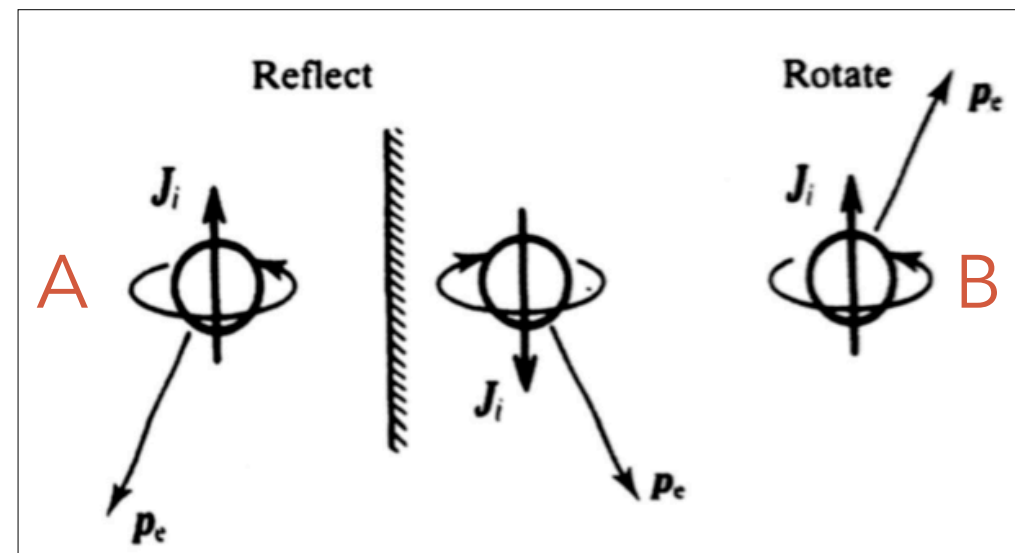
Pseudovectors and Parity

The cross product of two ('normal' or 'polar') vectors is a pseudovector

So orbital momentum $L = \vec{r} \times \vec{p}$ is a pseudovector, just like spin $\vec{J}$

Parity of a pseudovector (like J_i) is even; see picture below

Parity of a normal vector (like p_i) is odd; see picture below

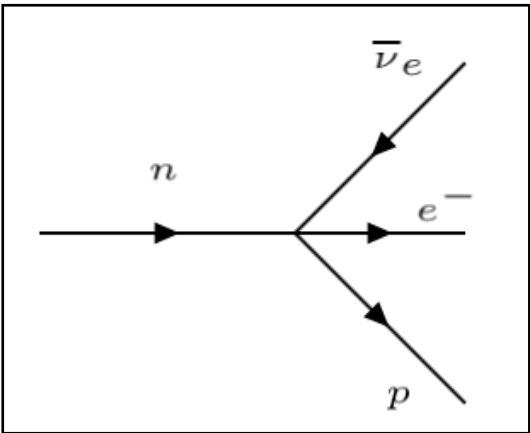


Any scalar (=dot) product between a pseudovector and an ordinary vector is a pseudoscalar. Pseudoscalar is odd under parity. Example: helicity, which is $J_i \cdot p_i$

V - A Structure of the Weak Interaction

- Fermi's effective theory for beta decays

$$M \propto G_F [\overline{\psi}_u \hat{O} \psi_d] [\overline{\psi}_e \hat{O} \psi_\nu]$$



- Only left-handed fermions contribute
- Left-hand operator: $P_L = \frac{1}{2}(1 - \gamma^5)$
- So operator O must be proportional to P_L to form a combination with other γ matrices to create a parity-violating interaction
- After a huge experimental effort it was found that the correct combination is

Name	Symbol	Current	Number of components	Effect under Parity
Scalar	S	$\overline{\psi}\psi$	1	+
Vector	V	$\overline{\psi}\gamma^\mu\psi$	4	(+, -, -, -)
Tensor	T	$\overline{\psi}\sigma^{\mu\nu}\psi$	6	
Axial Vector	A	$\overline{\psi}\gamma^\mu\gamma^5\psi$	4	(+, +, +, +)
Pseudo-Scalar	P	$\overline{\psi}\gamma^5\psi$	1	-

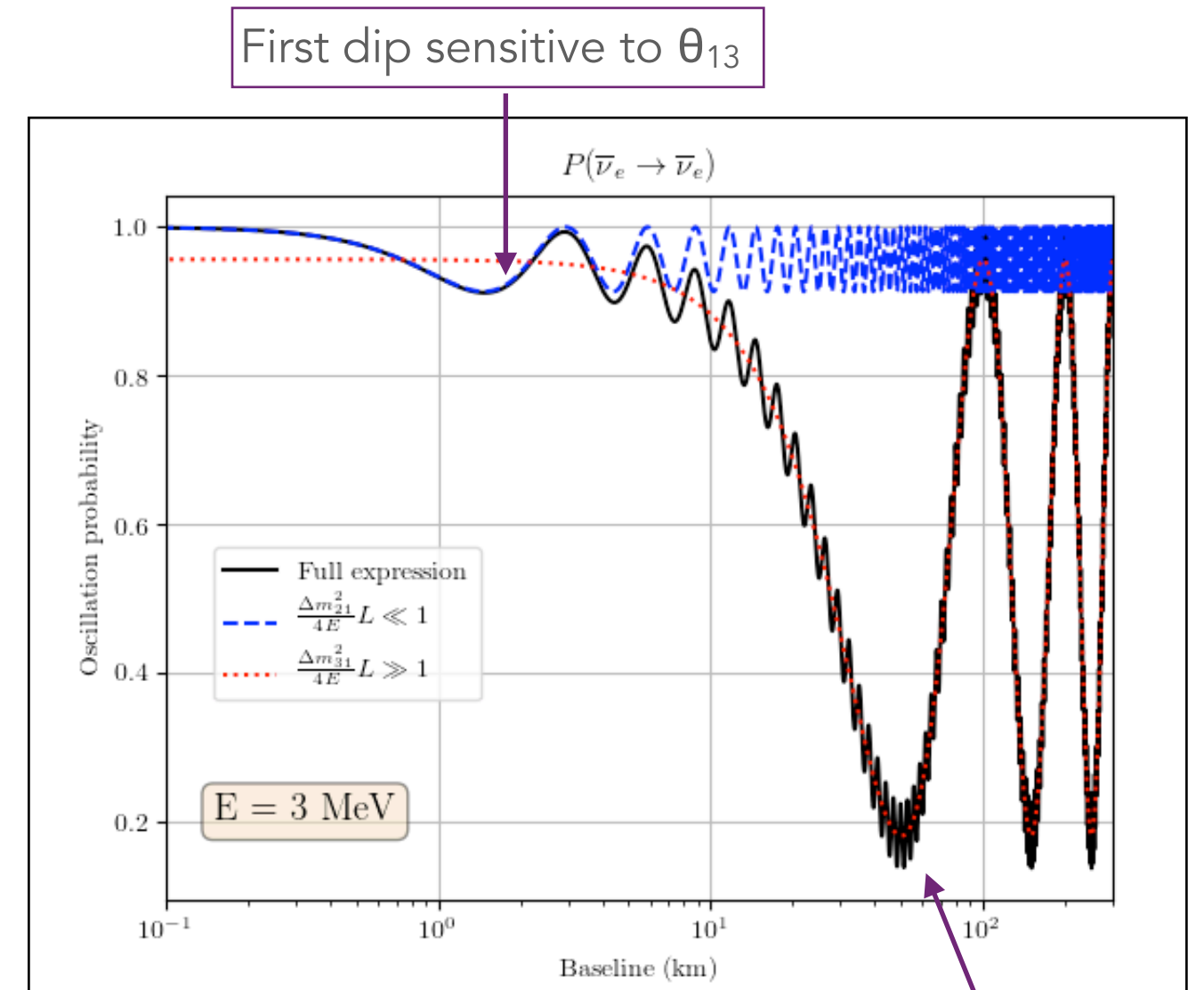
Table 1: All possible bilinear covariant combinations of γmatrices

$$\gamma^\mu \frac{1}{2}(1 - \gamma^5) \propto \gamma^\mu - \gamma^\mu \gamma^5$$

V (vector) - A (axial vector)

Reactor Neutrinos

- Were used to discover the (anti)neutrino (1950)
- Are used to measure reactor flux and spectrum
- Were used as a cross-check for solar neutrino oscillations parameters
- Are sensitive to neutrino mass ordering



$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \simeq 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right)$$

Second dip sensitive to θ_{12}

Takeaways Reactor Neutrinos

- Sensitive to both 'solar' section of PMNS matrix ('second dip') and the 'reactor' section ('first dip')
- Were first to measure non-zero θ_{13}
- That opens the door to CP-violation

$$P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = \Delta P_{\alpha\beta}^{CP} = -16J_{\alpha\beta} \sin\left(\Delta m_{12}^2 \frac{L}{4E}\right) \sin\left(\Delta m_{23}^2 \frac{L}{4E}\right) \sin\left(\Delta m_{13}^2 \frac{L}{4E}\right)$$

$$J_{\alpha\beta} \equiv \text{Im}(U_{\alpha 1} U_{\alpha 2}^* U_{\beta 1}^* U_{\beta 2}) = \pm c_{12} s_{12} c_{23} s_{23} c_{13}^2 \sin \delta$$

Long Baseline Neutrino Experiments

- Accelerator complexes create intense muon neutrino beams
- Were used to discover the muon neutrino (1962)
- Were used as a cross-check for atmospheric neutrino oscillations parameters
- Measuring neutrino oscillation parameters
- Are sensitive to neutrino mass ordering (through MSW effect) and CP violation effects
- Main players:
 - Historic: K2K, MINOS, OPERA
 - Current: T2K, NOvA
 - Future: DUNE

Detection Mode 1: ν_μ Disappearance

$$P_{\mu\mu} = P_{\bar{\mu}\bar{\mu}} \approx 1 - \sin^2 2\theta_{\mu\mu} \sin^2 \frac{\Delta m_{\mu\mu}^2 L}{4E}$$

$$\sin^2 \theta_{\mu\mu} = \cos^2 \theta_{13} \sin^2 \theta_{23}$$

$$\Delta m_{\mu\mu}^2 = \sin^2 \theta_{12} \Delta m_{31}^2 + \cos^2 \theta_{12} \Delta m_{32}^2 + \cos \delta \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23} \Delta m_{21}^2$$

$$P_{\mu\mu} \approx 1 - \sin^2 2\theta_{\mu\mu} \sin^2 \frac{\Delta m_{\mu\mu}^2 L}{4E} \simeq 1 - c_{13}^2 \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2 L}{4E} + \mathcal{O}(s_{13}^2, \Delta m_{21}^2)$$

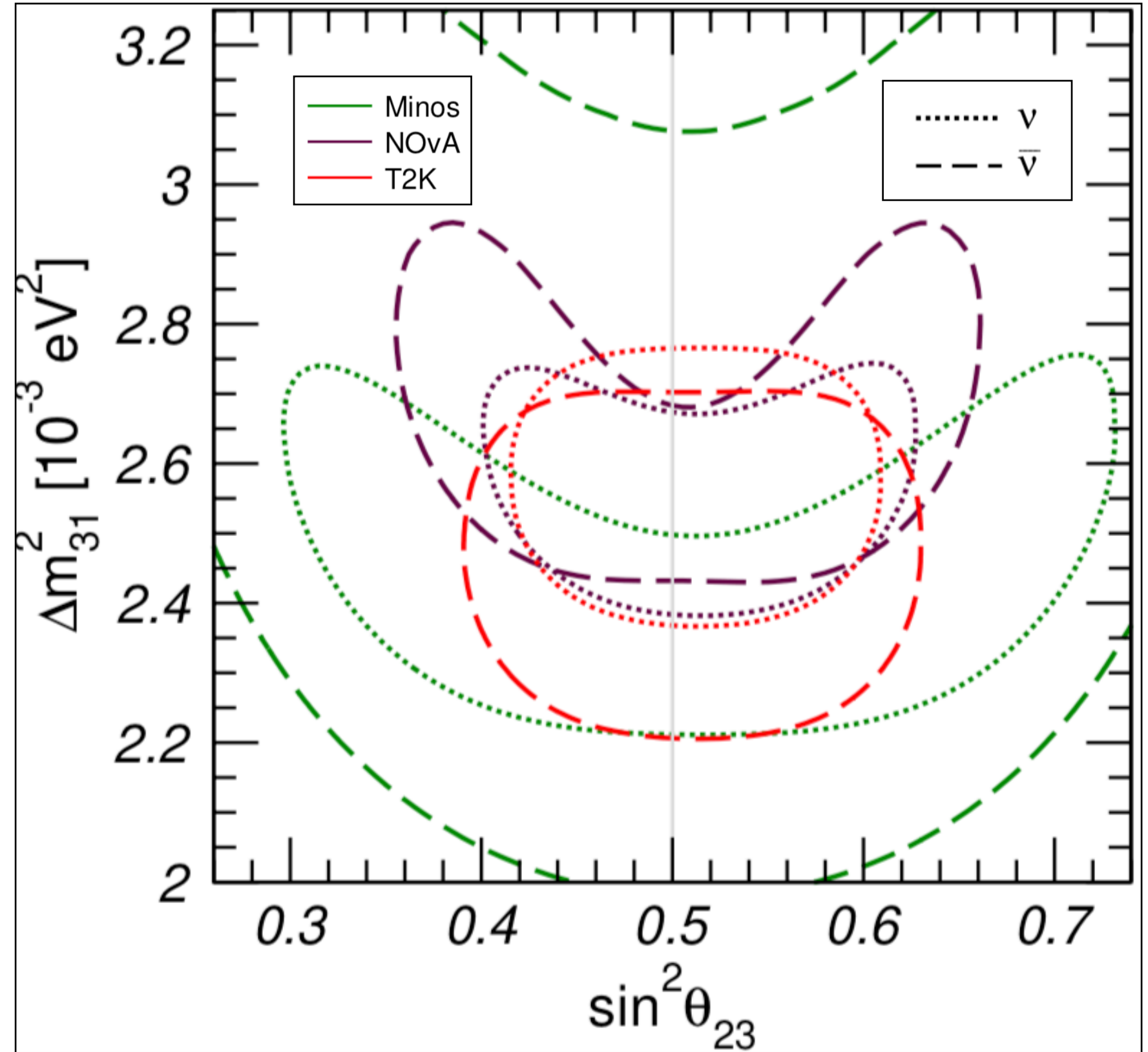
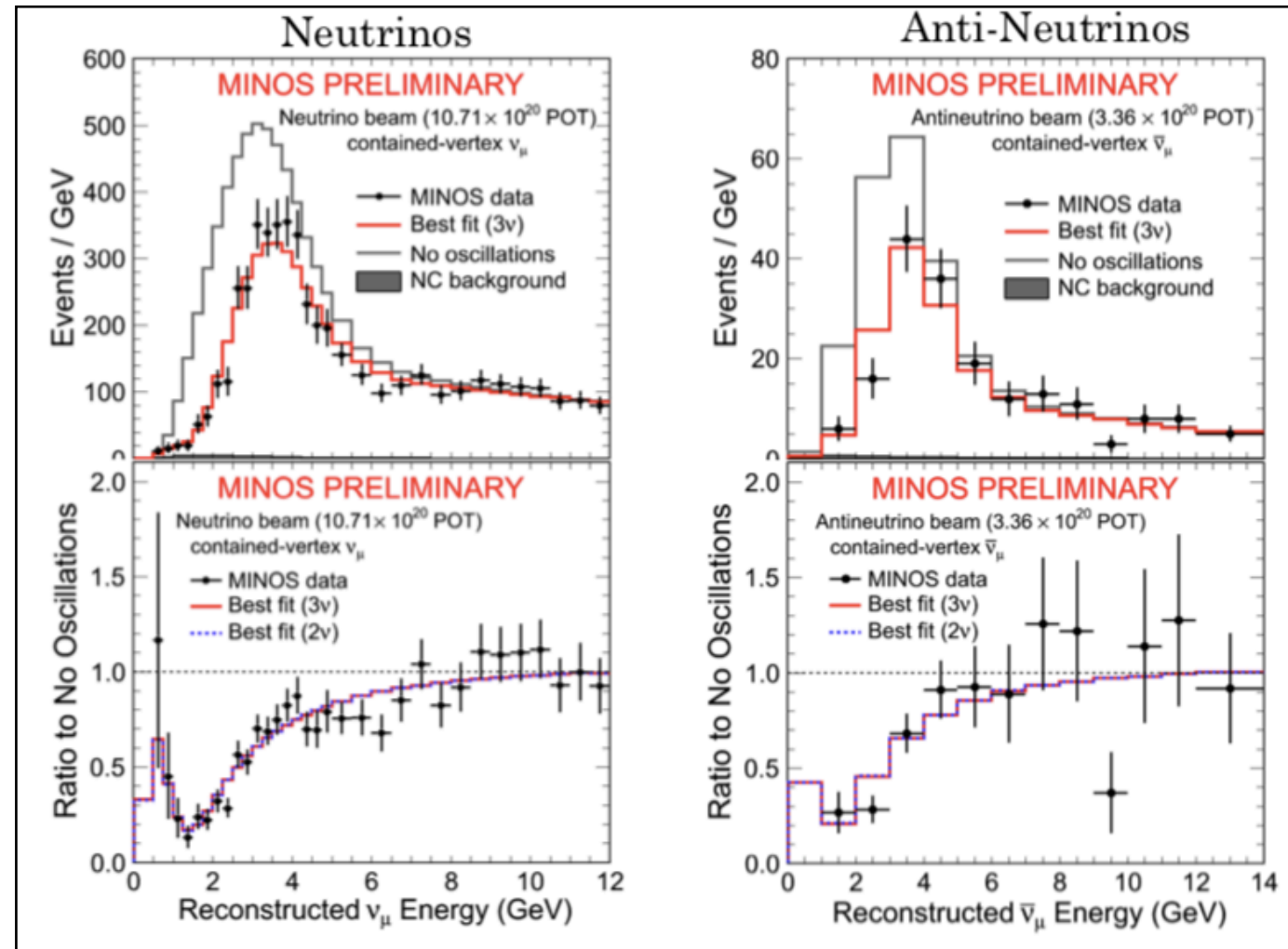
- Location (in L/E) of maximal disappearance gives sensitivity to $|\Delta m_{31}^2| \simeq |\Delta m_{32}^2|$
- Survival probability at minimum gives sensitivity to θ_{23} up to the octant due to

$$P_{\mu\mu}(\theta_{23}) \simeq P_{\mu\mu}\left(\frac{\pi}{2} - \theta_{23}\right)$$

- In particular, in case of maximal mixing, at oscillation minimum

$$P_{\mu\mu}\left(\theta_{23} \simeq \frac{\pi}{4}\right) \sim 0$$

Results ν_μ Disappearance



Detection Mode 2: ν_e Appearance

$$P(\nu_\mu \rightarrow \nu_e) \simeq P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3$$

$$\Delta m_{21}^2 = \alpha \Delta m_{31}^2 \quad \Delta m_{32}^2 = (1 - \alpha) \Delta m_{31}^2$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(\hat{A} - 1)^2} \sin^2((\hat{A} - 1)\hat{\Delta}),$$

$$P_{\sin \delta} = \alpha \frac{\sin \delta \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}}{\hat{A}(1 - \hat{A})} \sin(\hat{\Delta}) \sin(\hat{A}\hat{\Delta}) \sin((1 - \hat{A})\hat{\Delta}),$$

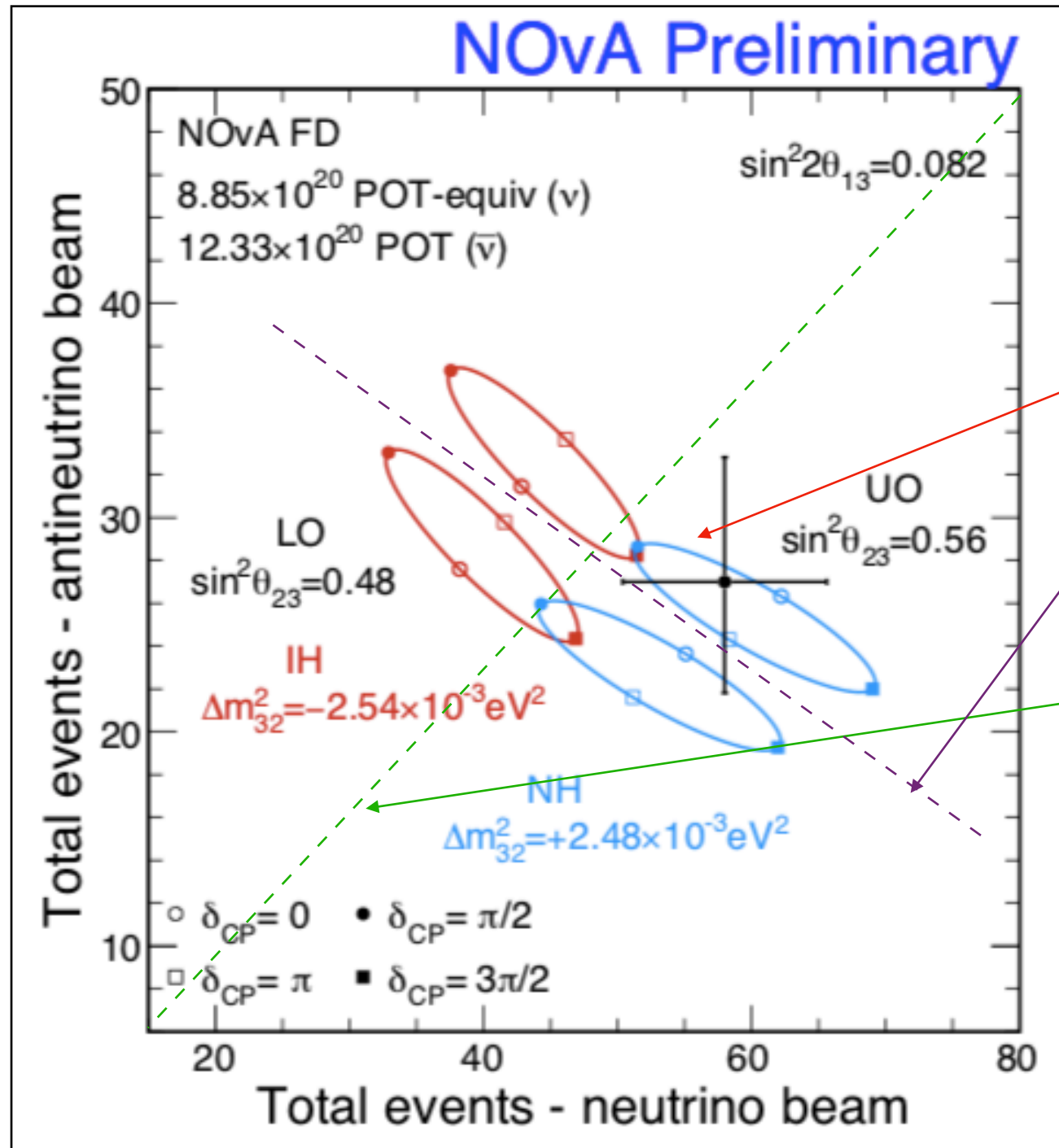
$$P_{\cos \delta} = \alpha \frac{\cos \delta \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}}{\hat{A}(1 - \hat{A})} \cos(\hat{\Delta}) \sin(\hat{A}\hat{\Delta}) \sin((1 - \hat{A})\hat{\Delta}),$$

$$\cancel{P_3} = \alpha^2 \frac{\cos^2 \theta_{23} \sin^2 2\theta_{12}}{\hat{A}^2} \sin^2(\hat{A}\hat{\Delta})$$

$$\hat{\Delta} = \Delta m_{31}^2 L / (4E_\nu) \quad \hat{A} = A / \Delta m_{31}^2 = 2VE_\nu / \Delta m_{31}^2 \quad V = \sqrt{2}G_F N_e$$

- CP odd phase can result in asymmetry $P(\nu_\mu \rightarrow \nu_e) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$
- $\sin^2 \theta_{23}$ as opposed to $\sin 2\theta_{23}$ allows to disentangle the θ_{23} octant
- Sensitivity to mass ordering is in A through the matter effect: enhancement of ν_e (anti- ν_e) in normal (inverted) ordering

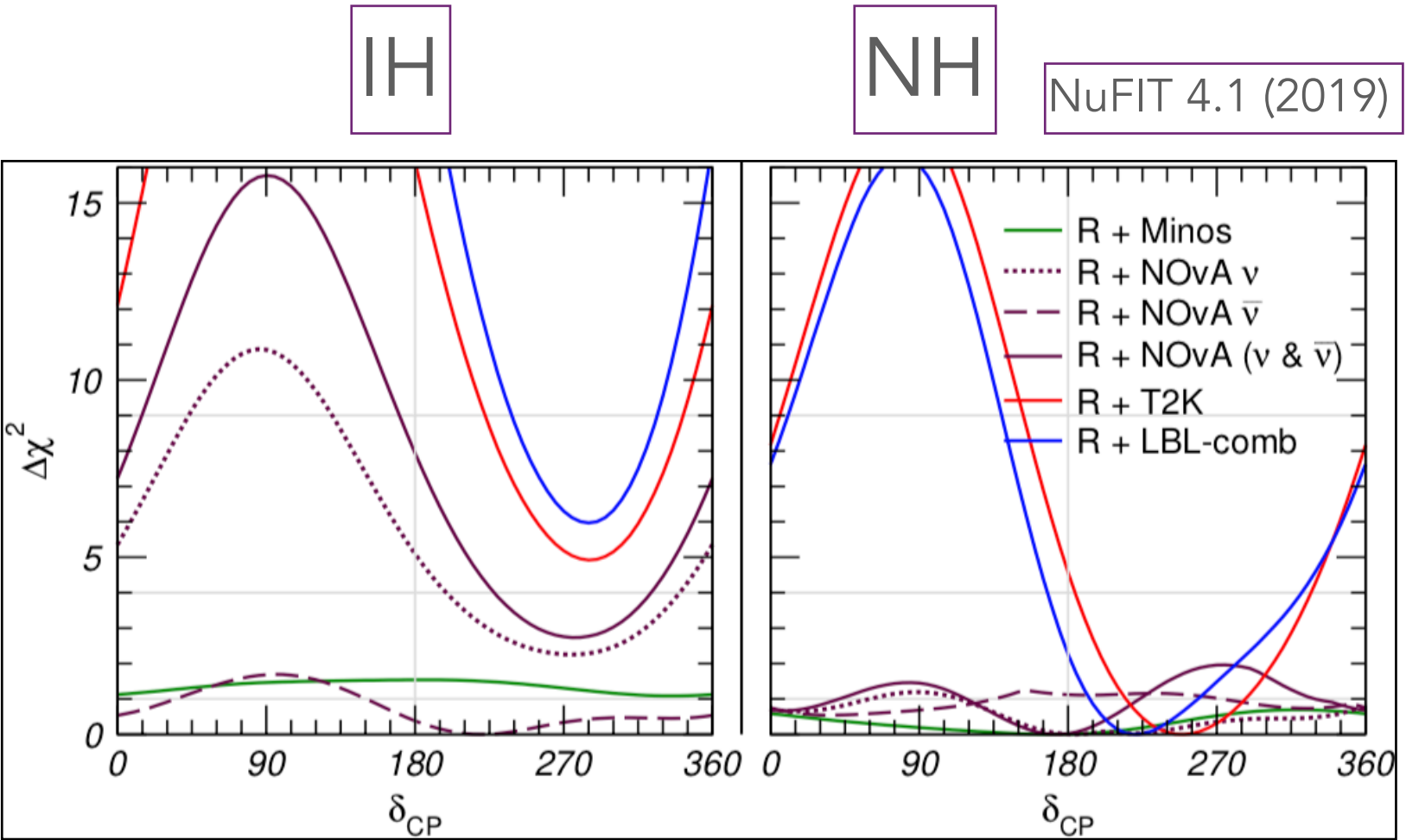
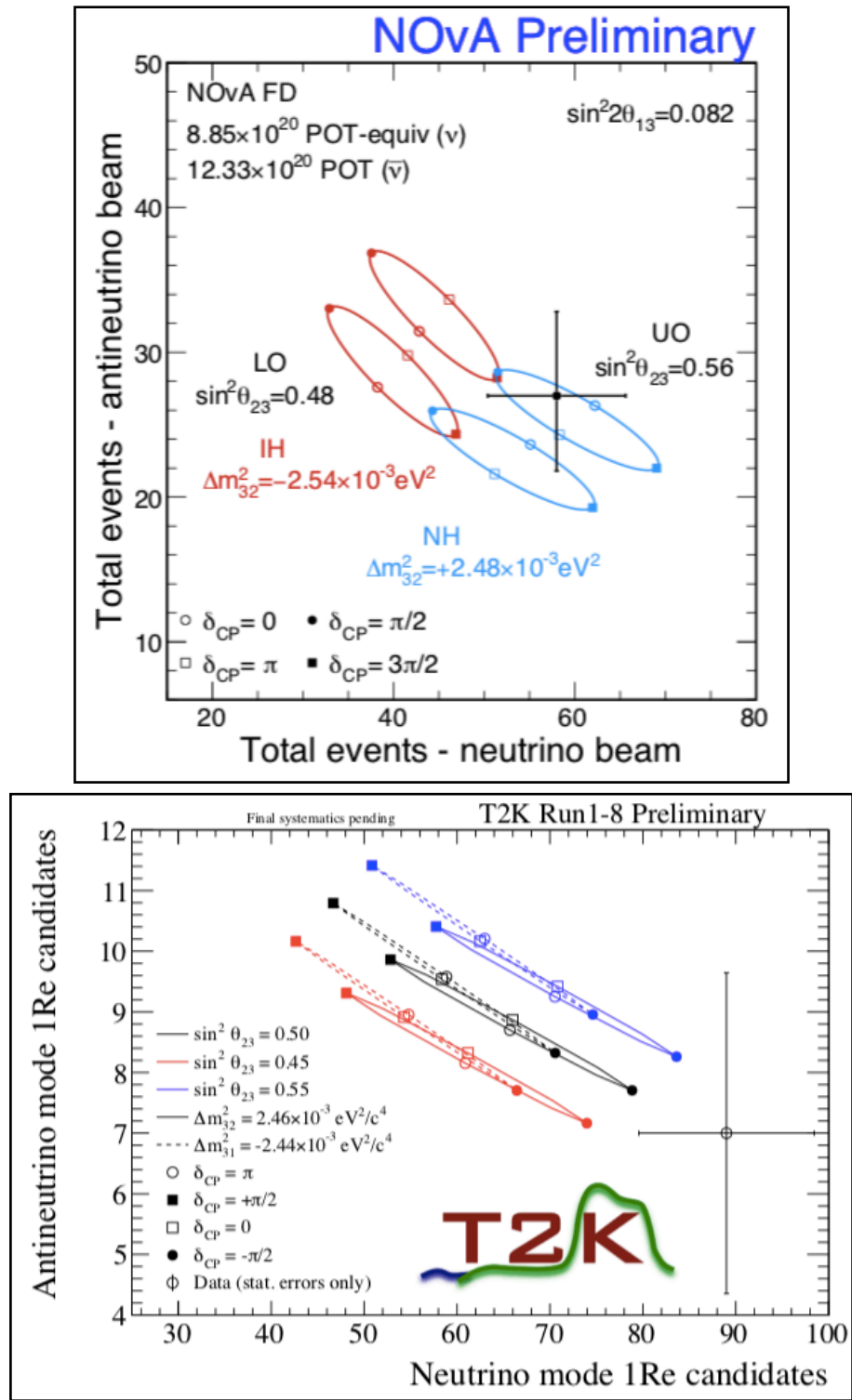
Neutrino Parameter Degeneracies



- CP odd phase can result in asymmetry $P(\nu_\mu \rightarrow \nu_e) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$
- $\sin^2 \theta_{23}$ as opposed to $\sin 2\theta_{23}$ allows to disentangle the θ_{23} octant
- Sensitivity to mass ordering is in A through the matter effect: enhancement of ν_e (anti- ν_e) in normal (inverted) ordering

Long Baseline Neutrino Results

- It looks like CP violation and matter effects are constructively contributing
 - Both pulling to 'lower-right'
- But very large errors still



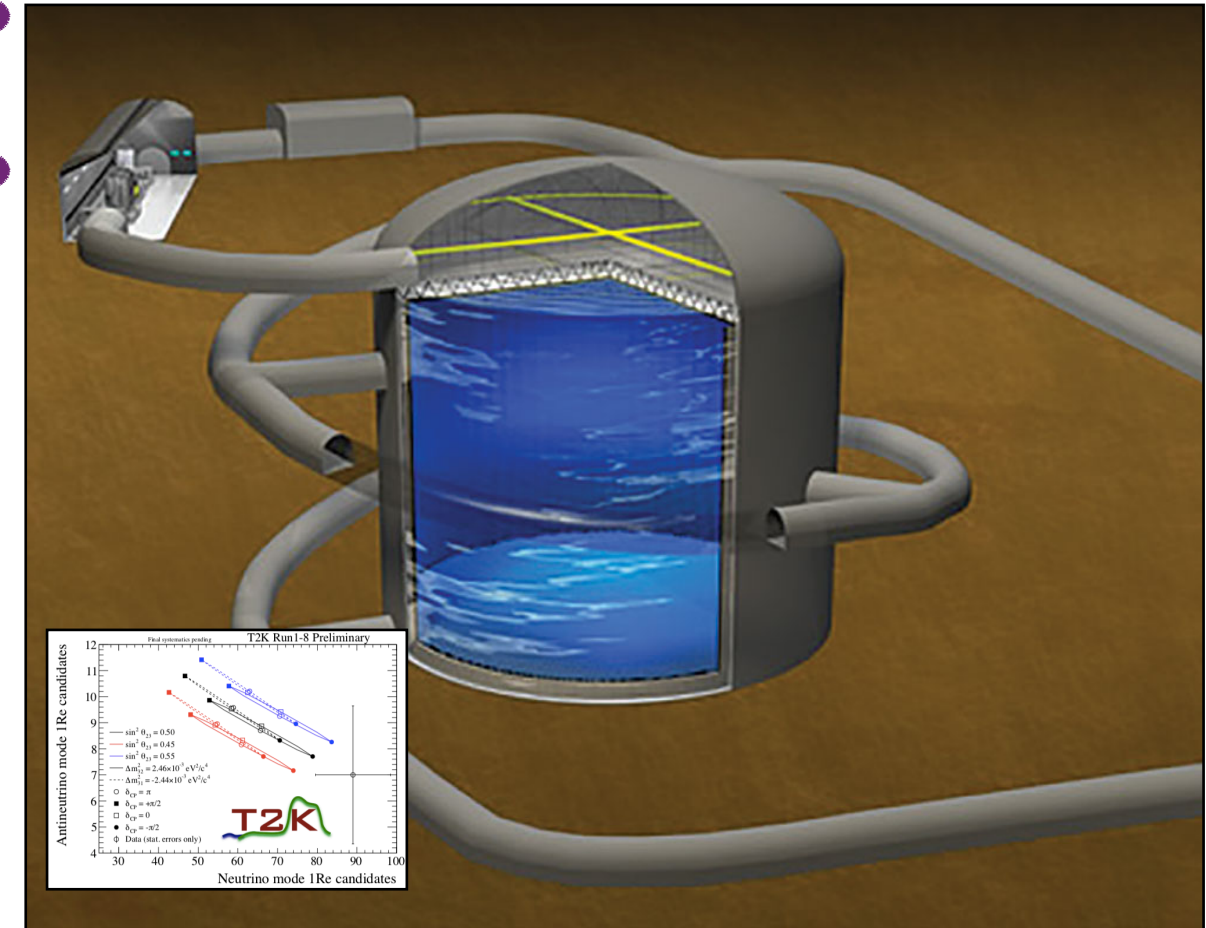
- NOvA: normal hierarchy preferred at 1.9 sigma
- T2K: normal hierarchy preferred at 89%

Future Long Baseline Detectors

- SuperK -> HyperK:
 - Large volume, 258kt total volume: two 74 x 60 m tanks
 - Construction 2019, beam 2026
- DUNE/LBNF: new high-energy beam line from Fermilab to Lead, South Dakota and LAr TPC detector
 - Higher energy means stronger matter effects, hence good capability to resolve mass ordering
 - Construction 2019, beam 2026

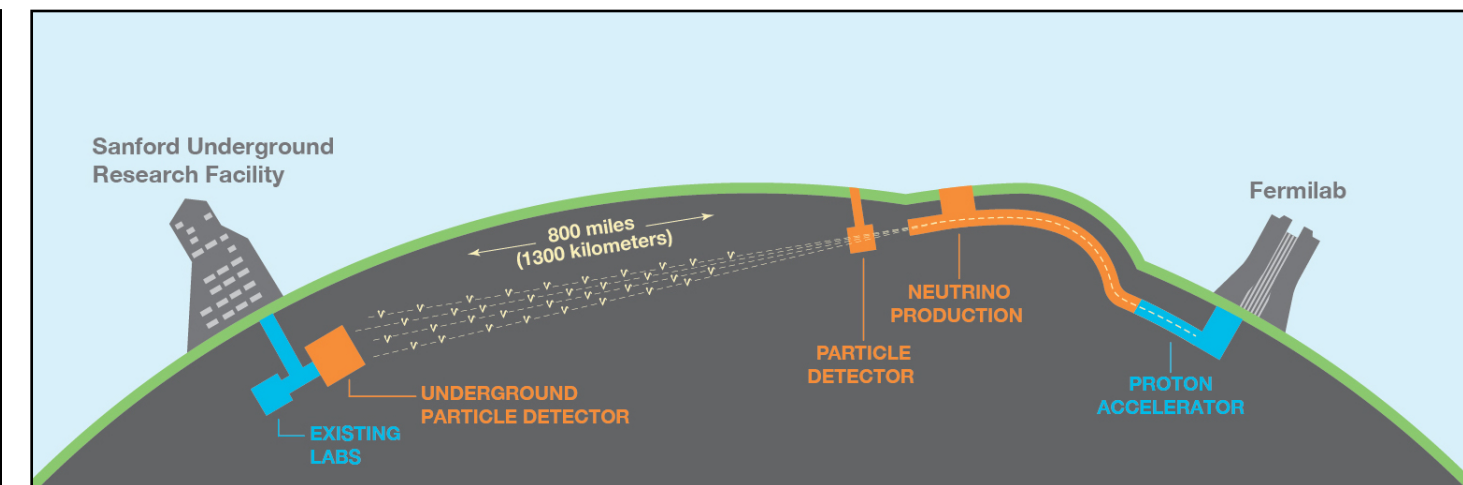
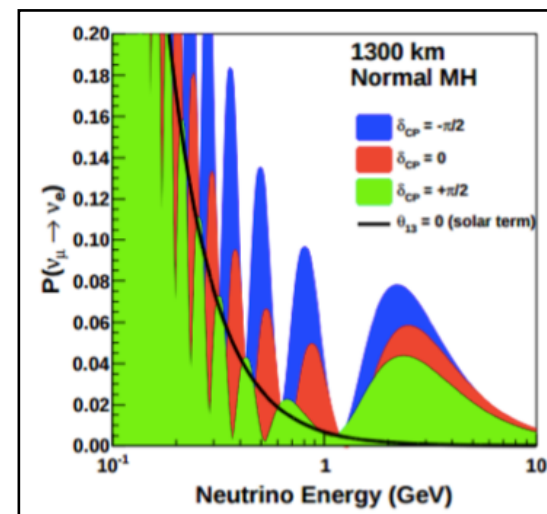
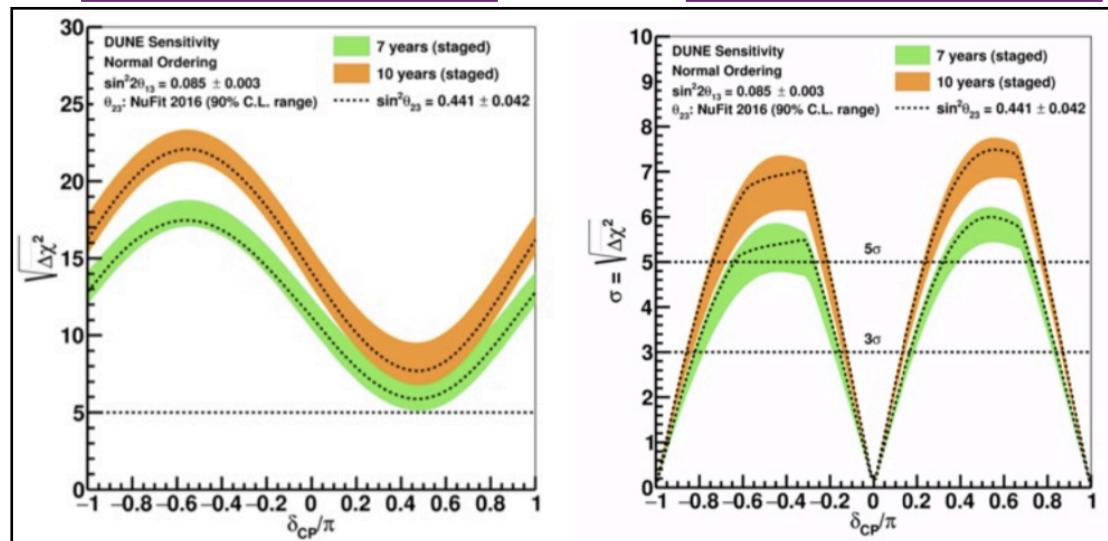
Do Neutrinos Violate CP?

What Is The Neutrino Mass Ordering?



Mass Ordering

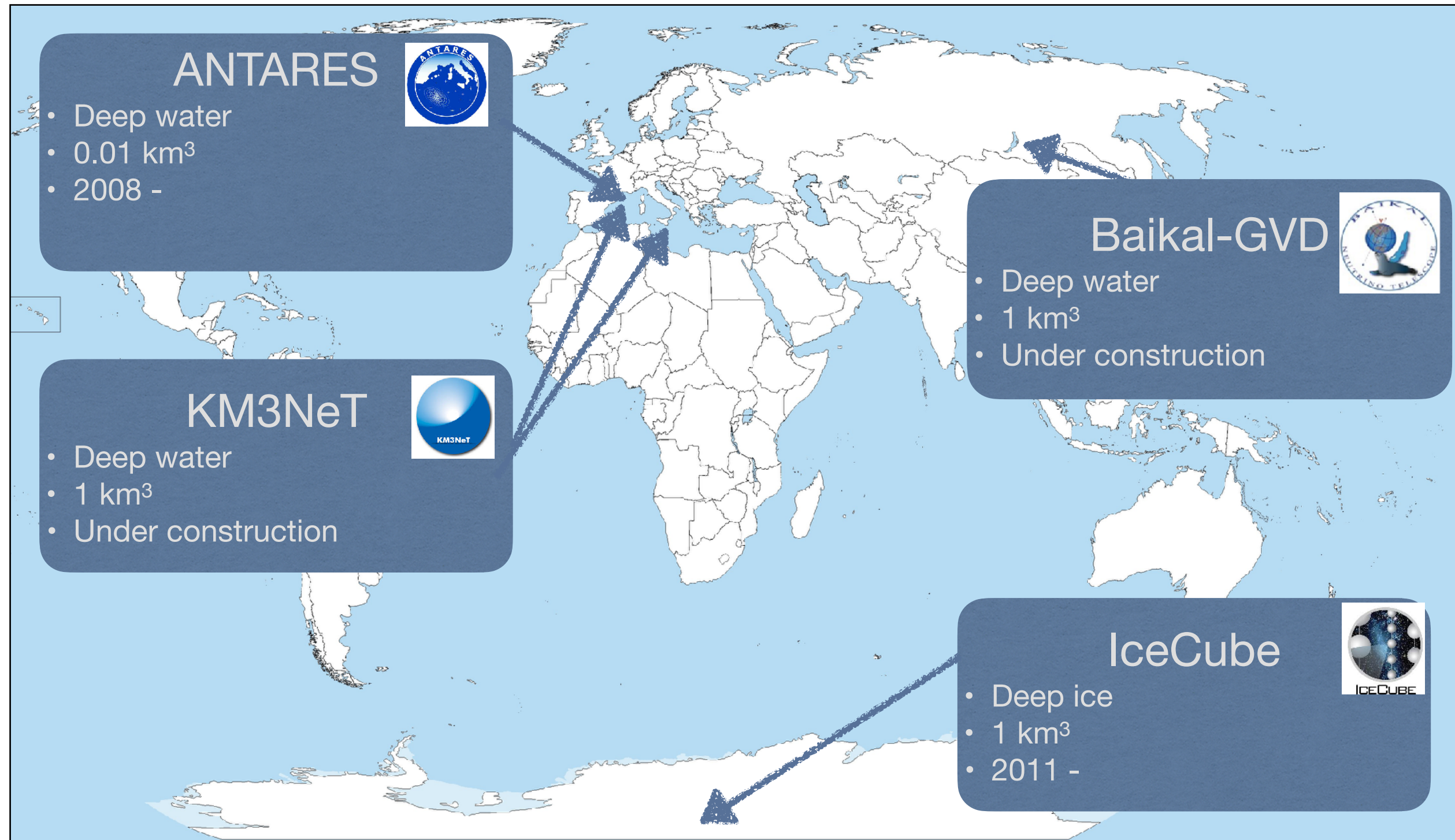
CP violation



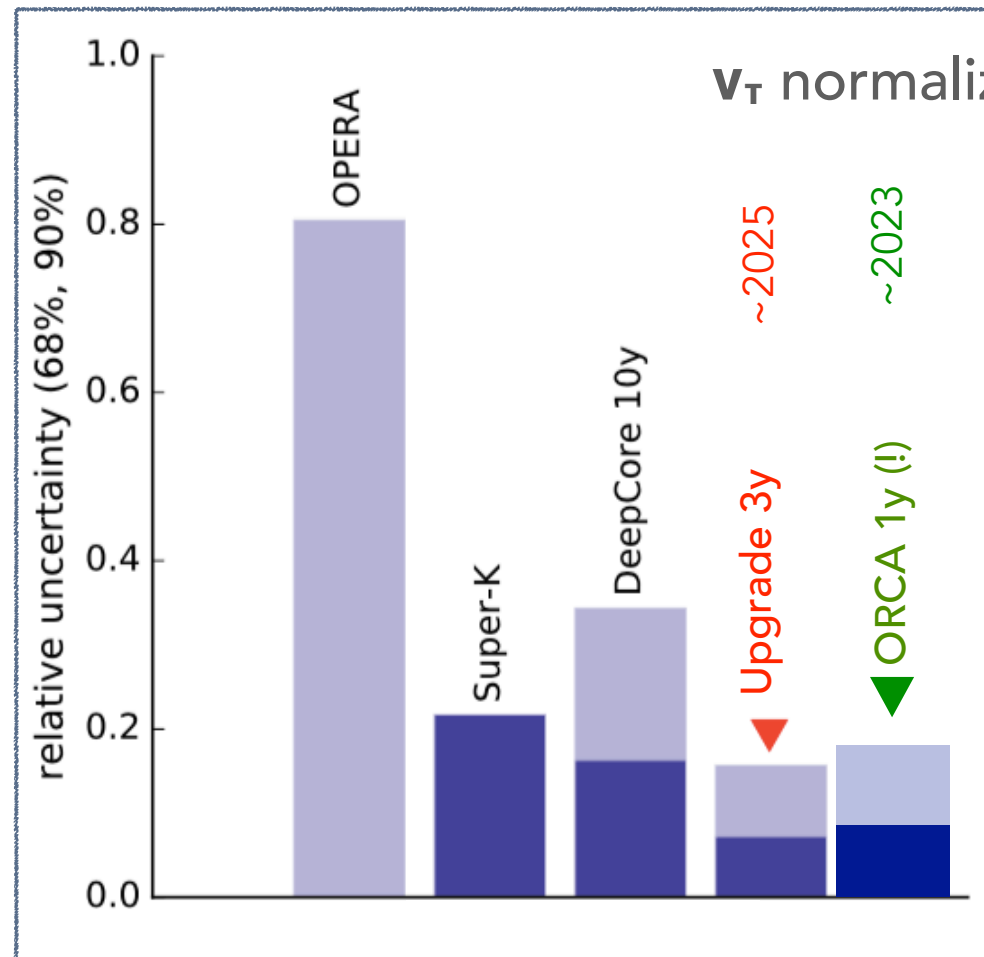
Takeaways Long Baseline Neutrinos

- Two detection modes: electron neutrino appearance and muon neutrino disappearance
- These are all precision measurements: higher order expansion of three neutrino oscillations gives sensitivity (and degeneracy) to octant of θ_{23} , mass ordering and CP violation

Ice/Water Cherenkov Neutrino Detectors Around the Globe



KM3NeT ORCA Physics Projections (2)



Neutrino Mass Ordering (NMO),
depending on the true ordering and true
value of $\sin^2(\theta_{23})$:

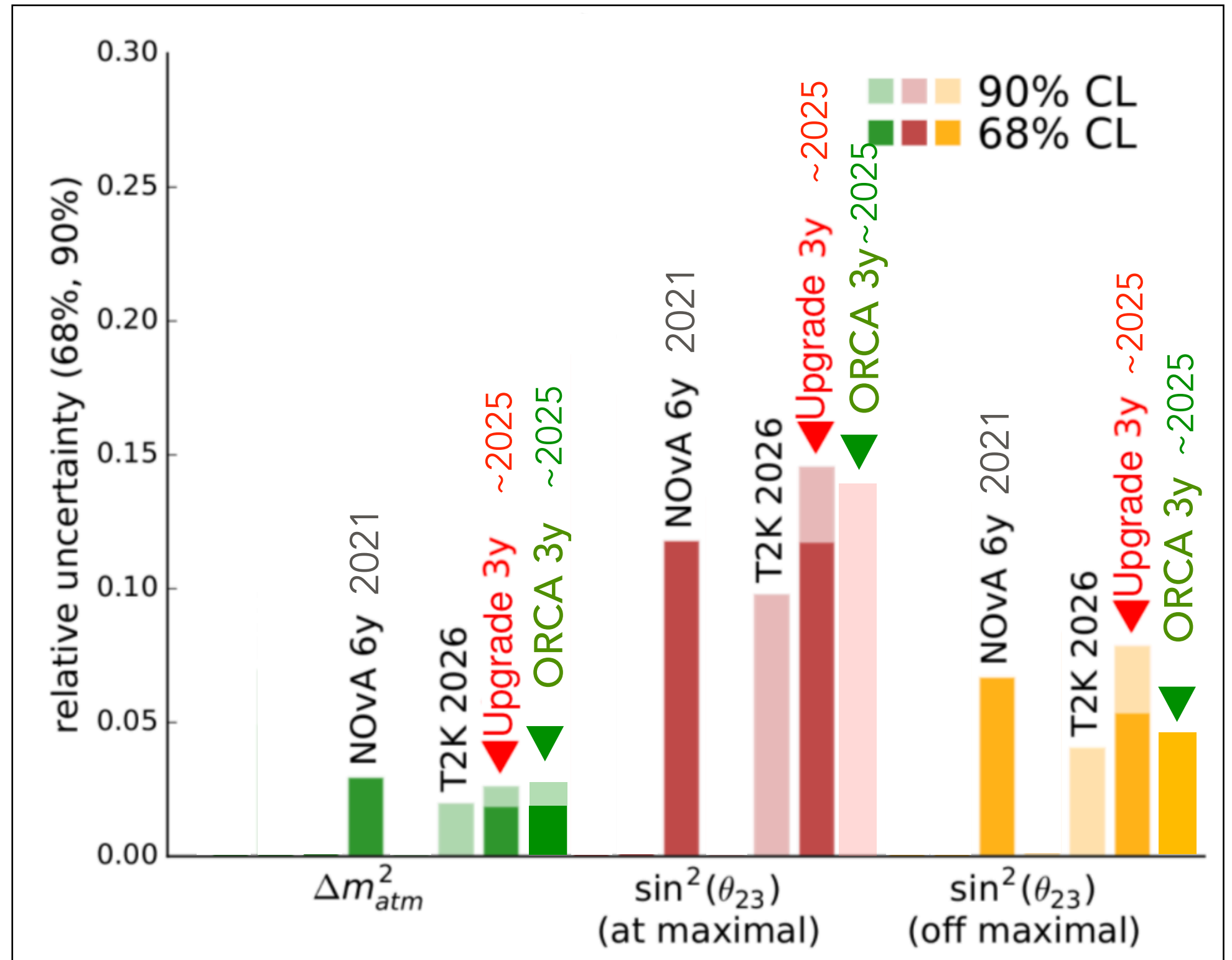
- IceCube Upgrade: 3 sigma in 3-8 years
- KM3NeT ORCA: 3-6 sigma in 3 years

- KM3NeT ORCA: under construction, scheduled completion by 2022
- IceCube Upgrade: funded, deployment in 2022-2023

KM3NeT ORCA has a very competitive physics case!

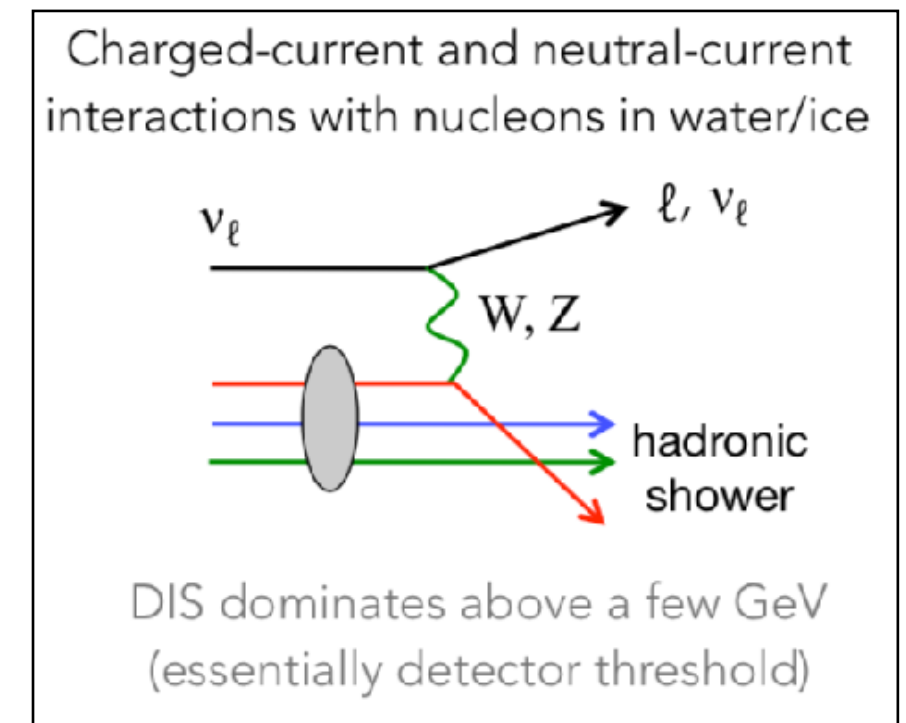
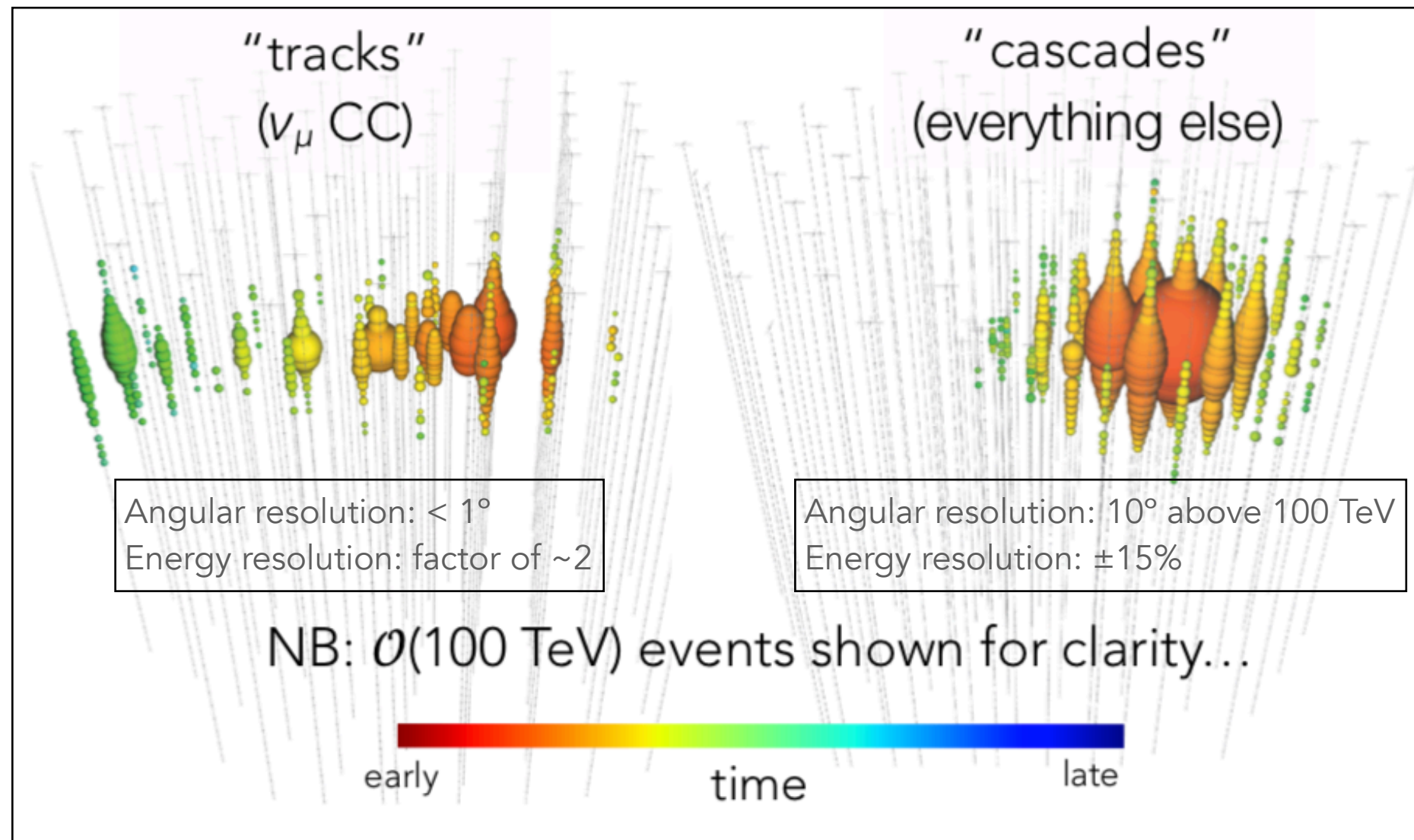
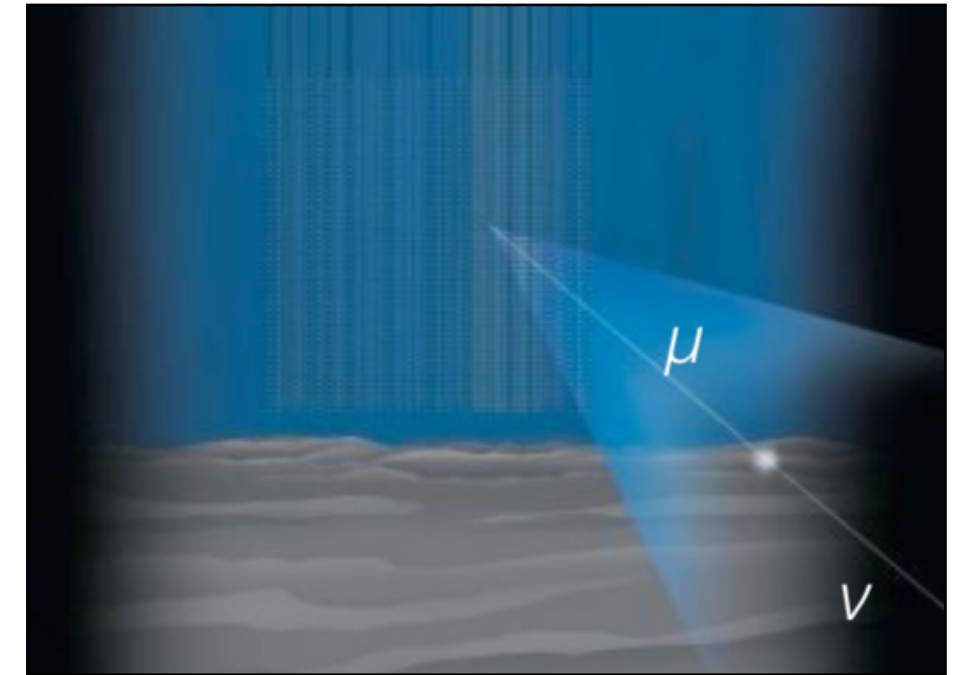
Neutrino Oscillations in KM3NeT and IceCube

- Both KM3NeT and IceCube have an aggressive (timeline-wise) and competitive low-energy (few GeV) neutrino oscillation physics program
 - KM3NeT ORCA (under construction)
 - IceCube DeepCore (running) and IceCube Upgrade (under construction)
- Determination of θ_{23} octant, mass ordering, ν_τ appearance, BSM physics



Neutrino Detection

- Neutrino-induced charged particles emit Cherenkov radiation
- Backgrounds from atmospheric muons reduced via timing, direction, energy and vetoing techniques



Resources

Web:

- History (and much more) of the neutrino:
 - <https://neutrino-history.in2p3.fr>
- International Neutrino Summer School:
 - <https://indico.fnal.gov/event/19346/timetable/#20190805> (2019 edition)
- Slac Summer Institute 2015 'The Universe of Neutrinos':
 - <https://www-conf.slac.stanford.edu/ssi/2015/>
- A wikipedia list of (many, not all) neutrino experiments:
 - https://en.wikipedia.org/wiki/List_of_neutrino_experiments

Lectures

- Joachim Kopp, Theoretical Neutrino Physics:
 - <https://indico.fnal.gov/event/19346/contribution/55/material/slides/1.pdf>
- Steve Boyd:
 - https://warwick.ac.uk/fac/sci/physics/staff/academic/boyd/warwick_week/neutrino_physics/

Books

- Neutrino, Frank Close
- Neutrino Physics, Kai Zuber
- Nuclear and Particle Physics, Burcham and Jobes
- Multimessenger Astronomy (free ebook), Imre Bartos and Marek Kowalski: <https://iopscience.iop.org/book/978-0-7503-1369-8.pdf>