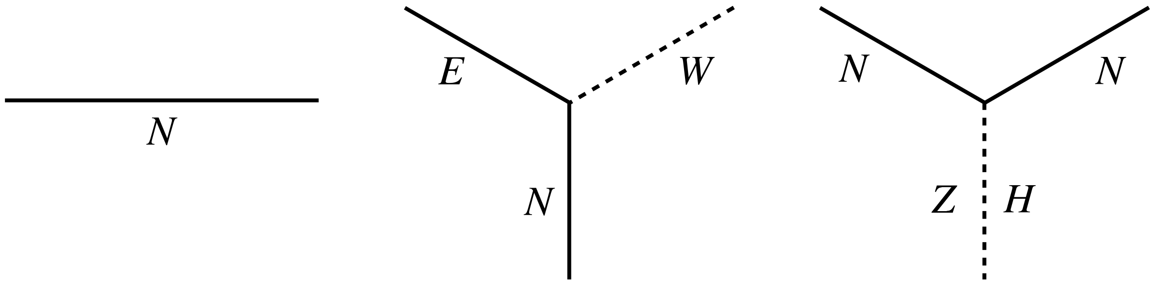
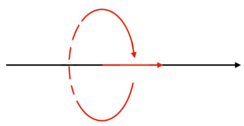
Neutrinos and their interactions



Jan-Willem van Holten, Dec. 9-10 2025

Chirality



Massless particles have their spin parallel or anti-parallel to the direction of propagation: positive/negative chirality

for massless fermions: chirality is eigenvalue of γ_5

properties: $\gamma_5^\dagger = \gamma_5 \qquad \gamma_5^2 = 1$

$$\gamma_5 \gamma_\mu = -\gamma_\mu \gamma_5 \qquad \mathrm{Tr}\, \gamma_5 = 0$$

diagonalized form:
$$\gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

left-handed fermions: $\gamma_5 \Psi = + \Psi$

$$\Psi_L = \frac{1}{2}\left(1 + \gamma_5\right) \Psi$$

right-handed fermions: $\gamma_5 \Psi = - \Psi$

$$\Psi_R = \frac{1}{2}\left(1 - \gamma_5\right) \Psi$$

Dirac equation and charge conjugation

Free fermion: $(-i\gamma \cdot p + m)\,\psi(p) = 0 \longleftrightarrow (\gamma \cdot \partial + m)\,\Psi(x) = 0$ (Dirac)

charge conjugation C: $C^{-1}\gamma_\mu C = -\gamma_\mu^T$ (T = transposition)

charge conjugate state: $\psi^c(p) = C\bar{\psi}^T(-p)$ (complex conjugation interchanges creation-annihilation operators)

$\longrightarrow$ $\boxed{(-i\gamma \cdot p + m)\,\psi^c(p) = 0}$ Charge conjugate also is a solution of the Dirac equation

$\downarrow$

anti-fermion

In position space and including electromagnetic interactions:

$$[\gamma \cdot (\partial - ieA) + m]\,\Psi(x) = 0 \longrightarrow [\gamma \cdot (\partial + ieA) + m]\,\Psi^c(x) = 0$$

with $\Psi^c(x) = C\bar{\Psi}^T(x)$

Majorana fermions

Majorana fermions are their own anti-particle: $\Psi(x) = \Psi^c(x)$

$\longrightarrow$ they cannot have electric or color charge

in the Standard Model only <u>neutrinos</u> can possibly be Majorana fermions

But: opposite sign for L- and R-chirality

$$(\gamma \cdot \partial - ig\,\gamma_5\gamma \cdot B + m)\,\Psi = 0 \longrightarrow (\gamma \cdot \partial - ig\,\gamma_5\gamma \cdot B + m)\,\Psi^c = 0$$

Majorana fermions can have *chiral* charges, coupling to axial vector fields!

Majorana fermions cannot be chiral eigenstates:

$$\gamma_5\,(\Psi_L)^c = -\,(\Psi_L)^c \qquad \gamma_5\,(\Psi_R)^c = +\,(\Psi_R)^c$$

$\uparrow$ right-handed $\qquad$ $\uparrow$ left-handed

Fermions in the Standard Model

Gauge interactions:

$$SU_c(3) \times SU_L(2) \times U_Y(1)$$

gluons W-bosons B-boson

electro-weak interactions

Dirac equation for massless L-fermions:

$$\gamma \cdot \left(\partial - \frac{ig_2}{2}\mathbf{W}\cdot\boldsymbol{\tau} - ig_1 Y B\right)\Psi_L = 0$$

Dirac equation for massless R-fermions:

$$\gamma \cdot (\partial - ig_1 Y B)\,\Psi_R = 0$$

		$SU_c(3)$	$SU_L(2)$	T_3	Y	Q
Q_L	U_L	$\underline{3}$	$\underline{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
	D_L	$\underline{3}$	$\underline{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
	U_R	$\underline{3}$	$\underline{1}$	0	$\frac{2}{3}$	$\frac{2}{3}$
	D_R	$\underline{3}$	$\underline{1}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
L_L	N_L	$\underline{1}$	$\underline{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
	E_L	$\underline{1}$	$\underline{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
	N_R	$\underline{1}$	$\underline{1}$	0	0	0
	E_R	$\underline{1}$	$\underline{1}$	0	-1	-1

$$Q = Y + T_3$$

Massive vector bosons

masses break gauge symmetry $\longrightarrow$ mixing of vector bosons

mass eigenstates: $W^{\pm}, Z, A$

charged gauge bosons: $$W^{\pm} = \frac{W^1 \mp iW^2}{\sqrt{2}}$$

neutral gauge bosons:

$$W^3 = \cos\theta_w\, Z + \sin\theta_w\, A \qquad B = -\sin\theta_w\, Z + \cos\theta_w\, A$$

redefine coupling constants: $$e = g_2 \sin\theta_w = g_1 \cos\theta_w$$

$\longrightarrow$ Dirac equations for massless leptons:

$$\gamma \cdot \left(\partial + \frac{ie}{\sin 2\theta_w} Z\right) N_L = \frac{ie}{\sqrt{2}\,\sin\theta_w}\gamma \cdot W^+ E_L$$

$$\gamma \cdot (\partial + ieA + ie\,\mathrm{cotan}\,2\theta_w\, Z)\, E_L = \frac{ie}{\sqrt{2}\,\sin\theta_w}\gamma \cdot W^- N_L$$

$$\gamma \cdot \partial\, N_R = 0,$$

$$\gamma \cdot (\partial + ieA - ie\tan\theta_w\, Z)\, E_R = 0$$

Massive fermions

masses break chirality $\longrightarrow$ mixing of L- and R-states

$$\gamma\cdot\left(\partial+\frac{ie}{2\sin 2\theta_w}(1+\gamma_5)Z+m_N\right)N=\frac{ie}{2\sqrt{2}\,\sin\theta_w}\gamma\cdot W^+\,\overbrace{(1+\gamma_5)\,E}^{E_L}$$

$$\gamma\cdot\left(\partial+ieA+\frac{ie}{2\sin 2\theta_w}\left(2\cos 2\theta_w-1+\gamma_5\right)Z+m_E\right]E=\frac{ie}{2\sqrt{2}\,\sin\theta_w}\gamma\cdot W^-\,\underbrace{(1+\gamma_5)\,N}_{N_L}$$

masses of vector bosons and charged leptons created dynamically
by Brout-Englert Higgs mechanism
neutrino masses more complicated (below)

3 fermion families:

$$\gamma\cdot\left[\partial+\frac{ie}{2\sin 2\theta_w}(1+\gamma_5)Z\right]N_i+m_{Nij}N_j=\frac{ie}{2\sqrt{2}\,\sin\theta_w}\gamma\cdot W^+\,(1+\gamma_5)\,E_i$$

$$\gamma\cdot\left[\partial+ieA+\frac{ie}{2\sin 2\theta_w}\left(2\cos 2\theta_w-1+\gamma_5\right)Z\right]E_i+\,m_{Eij}E_j=\frac{ie}{2\sqrt{2}\,\sin\theta_w}\gamma\cdot W^-\,(1+\gamma_5)\,N_i$$

Diagonalize mass matrices simultaneously

$\longrightarrow$ requires relative rotation of N- and E-masses

when E-masses diagonal, still need to rotate N-masses

$$\nu_i=U_{ij}N_j\quad\longleftrightarrow\quad N_i=U^{-1}_{ij}\nu_j=U^{\dagger}_{ij}\,\nu_j$$

unitary PMNS-matrix

U is chosen to make m_N diagonal: $\quad\left(U\cdot m_N\cdot U^{-1}\right)_{ij}=m_{Ni}\delta_{ij}$

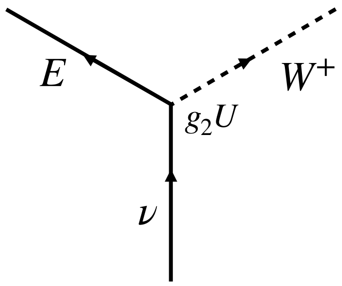
Dirac equations for mass eigenstates:

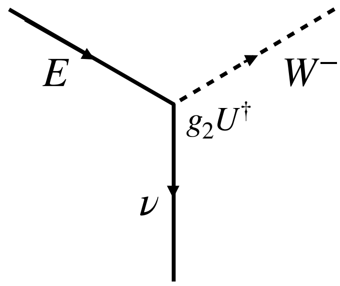
$$\gamma\cdot\left[\partial+\frac{ie}{2\sin 2\theta_w}(1+\gamma_5)Z+m_{Ni}\right]\nu_i=\frac{ie}{2\sqrt{2}\,\sin\theta_w}\gamma\cdot W^+\,(1+\gamma_5)\,U_{ij}E_j$$

PMNS-matrix

$$\gamma\cdot\left[\partial+ieA+\frac{ie}{2\sin 2\theta_w}\left(2\cos 2\theta_w-1+\gamma_5\right)Z+m_{Ei}\right]E_i\;=\frac{ie}{2\sqrt{2}\,\sin\theta_w}\gamma\cdot W^-\,(1+\gamma_5)\,U^{-1}_{ij}\nu_j$$

The PMNS-matrix appears only in the coupling to charged $W^{\pm}$-bosons





and $W^- \leftrightarrow W^+$, $U \leftrightarrow U^\dagger$ for the processes involving anti-particles

General parametrisation of PMNS matrix in terms of angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and complex phase δ (source of CP violation):

$$U[\theta_{ij}, \delta] = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{1} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

(for Dirac neutrinos)

Neutrino oscillations

At time $t = 0$ an neutrino in interaction state N_i is created with energy E:

$$|\Psi(0)\rangle = |N_i\rangle = U^\dagger_{ik}\,|\nu_k\rangle$$

this state is a superposition of mass-energy eigenstates, each of which develops in space and time as

$$|\nu_k(t, \mathbf{r})\rangle = e^{i(\mathbf{p}_k\cdot\mathbf{r} - Et)}\,|\nu_k\rangle \qquad \text{with} \qquad E = \sqrt{m_k^2 + \mathbf{p}_k^2}$$

$$\longrightarrow \qquad |\Psi(t)\rangle = e^{i(\mathbf{p}_k\cdot\mathbf{r} - Et)} U^\dagger_{ik}\,|\nu_k\rangle$$

The probability amplitude for the neutrino to be observed in the interaction state N_j at time t then is

$$\langle N_j|\Psi(t)\rangle = \sum_k e^{i(\mathbf{p}_k\cdot\mathbf{r} - Et)}\, U^\dagger_{ik} U_{kj}$$

the probability for the transition $N_i \to N_j$ in time t itself becomes

$$P_{i\to j}(t, \mathbf{r}) = \left|\sum_k e^{i(\mathbf{p}_k\cdot\mathbf{r} - Et)}\, U^\dagger_{ik} U_{kj}\right|^2 = \left|\sum_k e^{i\mathbf{p}_k\cdot\mathbf{r}}\, U^\dagger_{ik} U_{kj}\right|^2$$

Example: 2-neutrino case (N_e, N_μ)

$$\nu_i = U_{ij} N_j \qquad \text{with} \qquad U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

initial state: $|\Psi(0)\rangle = |N_e\rangle$

$$\longrightarrow \quad \langle N_\mu | \Psi(t) \rangle = e^{i(\mathbf{p}_e \cdot \mathbf{r} - Et)} U^\dagger_{ee} U_{e\mu} + e^{i(\mathbf{p}_\mu - Et)} U^\dagger_{e\mu} U_{\mu\mu}$$

$$= e^{i(\mathbf{p}_\mu - Et)} \left(-e^{i\Delta \mathbf{p} \cdot \mathbf{r}} + 1 \right) \cos\theta \sin\theta$$

here $\quad \Delta \mathbf{p} \cdot \mathbf{r} = \left(\sqrt{E^2 - m_e^2} - \sqrt{E^2 - m_\mu^2} \right) L \simeq \dfrac{(m_\mu^2 - m_e^2) L}{2E}$

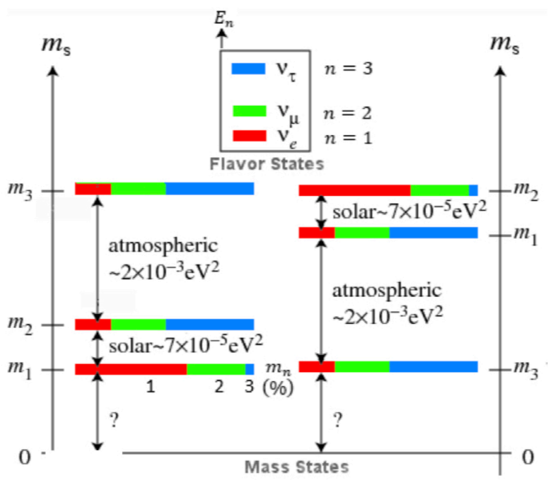
$$\longrightarrow \quad P_{e\to\mu}(t, \mathbf{r}) = \left| \left(1 - e^{i\Delta p L}\right) \sin\theta \cos\theta \right|^2$$

$$= \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E}$$

$$P_{e\to e}(t) = 1 - P_{e\to\mu}(t)$$

Neutrino oscillations provide information about Δm^2 of neutrinos

they have been observed for solar and atmospheric neutrinos



Neutrino masses

Rewrite Dirac equations for massive neutrinos in empty space in chiral form:

$$-\gamma \cdot \partial N_L = m_N N_R$$

$$-\gamma \cdot \partial N_R = m_N N_L$$

with $\quad m_N = f\langle\varphi_2\rangle = f_N v$

N_R has no electro-weak charges $\longrightarrow$ can have Majorana mass:

$$-\gamma \cdot \partial N_R = m_N N_L + M(N_R)^c$$

Majorana fermions:

$$\mathrm{N}_1 = \frac{1}{2}\left[N_L + (N_L)^c\right] = \mathrm{N}_1^c$$

$$\mathrm{N}_2 = \frac{1}{2}\left[N_R + (N_R)^c\right] = \mathrm{N}_2^c$$

violates lepton number!

$$\boxed{\longrightarrow \quad -\gamma \cdot \partial \begin{pmatrix} \mathrm{N}_1 \\ \mathrm{N}_2 \end{pmatrix} = \begin{pmatrix} 0 & m_N \\ m_N & M \end{pmatrix} \begin{pmatrix} \mathrm{N}_1 \\ \mathrm{N}_2 \end{pmatrix}}$$

diagonalize:

$$-\gamma \cdot \partial \mathrm{N}_\pm = \pm m_\pm \mathrm{N}_\pm \quad \longrightarrow \quad -\gamma \cdot \partial (\gamma_5 \mathrm{N}_-) = m_-(\gamma_5 \mathrm{N}_-)$$

with $\quad m_\pm = \frac{1}{2} M \left(\sqrt{1 + \frac{4m_N^2}{M^2}} \pm 1 \right)$

and $\quad \mathrm{N}_\pm = \frac{1}{\sqrt{1 + \frac{m_\pm^2}{m_N^2}}} \left(\mathrm{N}_1 + \frac{m_\pm}{m_N} \mathrm{N}_2 \right)$

note: for $M \gg m_N$

$$m_+ \simeq M \qquad m_- \simeq \frac{m_N^2}{M} \ll m_N$$

$$\mathrm{N}_+ \simeq \mathrm{N}_2 \qquad \mathrm{N}_- \simeq \mathrm{N}_1$$

seesaw mechanism to generate small neutrino masses

- predicts heavy sterile neutrinos
- violates lepton number

PMNS-matrix for Majorana neutrinos

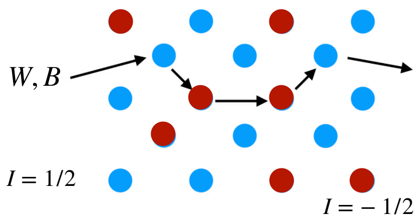
Majorana fermions cannot change their phase as $\mathsf{N}^c = C\,\bar{\mathsf{N}}^T = \mathsf{N}$

$\longrightarrow$ fewer phase factors can be eliminated from the PMNS-matrix by redefining phases of leptons (3 in stead of 5)

$\longrightarrow$ 2 additional phase factors in PMNS-matrix:

$$U \to U_M[\theta_{ij}, \gamma_{ij}] = U[\theta_{ij}, \delta = \gamma_{23}] \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma_{12}} & 0 \\ 0 & 0 & e^{i\gamma_{13}} \end{pmatrix}$$

The origin of masses (BEH)

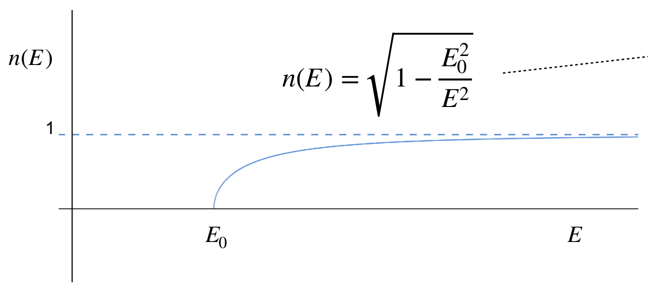


in a medium of weak charges massless (W, B)-fields are scattered

$\longrightarrow$ these charges create an index of refraction $n(E)$

dispersion: $E = \dfrac{pc}{n(E)} \longrightarrow n^2(E)\,E^2 = p^2c^2$

$$E^2 = E_0^2 + p^2c^2$$



$$n(E) = \sqrt{1 - \frac{E_0^2}{E^2}}$$

$$\boxed{\text{effective mass:} \quad E_0 = mc^2}$$

Rem.: - medium is transparent for combination $A = \cos\theta_w\,B + \sin\theta_w\,W^3$ (photon)

- medium is created by weak charges of scalar fields (Brout-Englert-Higgs)

Yukawa interactions

Fermions interact with scalar fields φ through Yukawa interactions:

$$(\gamma \cdot \partial + f\varphi)\,\Psi = 0$$

Condensation of scalar field: $|\varphi|^2 = v^2 > 0 \longrightarrow \varphi(x) = v + h(x)$

Higgs field

in vacuum: $(\gamma \cdot \partial + m)\,\Psi = 0$

with $m = fv$

In Standard Model doublet $\Phi = (\varphi_1, \varphi_2)$ of scalar fields

	$SU_c(3)$	$SU_L(2)$	T_3	Y	Q
φ_1	$\underline{0}$	$\underline{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
φ_2	$\underline{0}$	$\underline{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0

$$\langle \Phi \rangle = (0, v)$$

Rem.: $(\gamma \cdot \partial + m)\,\Psi = 0 \quad \longleftrightarrow \quad \left[\begin{array}{l} \gamma \cdot \partial \Psi_L = -\,m\Psi_R \\ \gamma \cdot \partial \Psi_R = -\,m\Psi_L \end{array}\right.$

Gauge field of $U_X(1)$: Z'

if $X = (B - L)/2$: gauge field Z' does not mix with (W, Z)

Z' must have a very large mass $m_{Z'}$

$\longrightarrow$ BEH-scalar $\phi(x)$ with $\langle \phi \rangle = v_2$:

$$\phi(x) = v_2 + \frac{a(x)}{\sqrt{2}}$$

$$|D_\mu \phi|^2 = |(\partial_\mu - g_x Z'_\mu)\,\phi|^2 \quad \longrightarrow \quad g_x^2 v_2^2\, Z'^2 = \frac{1}{2} m_{Z'}^2\, Z'^2$$

requires $m_{Z'}^2 = 2 g_x^2 v_2^2 \gg m_W^2$

Most general renormalizable scalar potential for $\Phi = (\varphi_1, \varphi_2)$ and ϕ:

$$V[\Phi, \phi] = \frac{\lambda_1}{4}\left(|\Phi|^2 - v_1^2\right)^2 + \frac{\lambda_2}{4}\left(|\phi|^2 - v_2^2\right)^2 + \frac{\lambda_m}{4}\left(|\phi|^2 - v_2^2\right)\left(|\Phi|^2 - v_1^2\right)$$

with minima $\langle |\Phi|^2 \rangle = v_1^2 \qquad \langle |\phi|^2 \rangle = v_2^2$

scalar dynamics and masses

$$\Phi = \left(0, v_1 + \frac{h}{\sqrt{2}}\right) \qquad\qquad \phi(x) = v_2 + \frac{a(x)}{\sqrt{2}}$$

$$\longrightarrow \quad V[h,a] = \frac{\lambda_1 v_1^2}{2} h^2 + \frac{\lambda_2 v_2^2}{2} a^2 + \frac{\lambda_m v_1 v_2}{2} ha + \mathcal{O}[(h,a)^3]$$

$$= \frac{1}{2} m_-^2 h_-^2 + \frac{1}{2} m_+^2 h_+^2 + \mathcal{O}[(h_-, h_+)^3]$$

with $\quad m_\pm^2 = \frac{1}{2}\left(\lambda_1 v_1^2 + \lambda_2 v_2^2 \pm \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + \lambda_m^2 v_1^2 v_2^2}\right)$

and $\quad h = h_- \cos\theta_s + h_+ \sin\theta_s, \qquad a = -h_- \sin\theta_s + h_+ \cos\theta_s,$

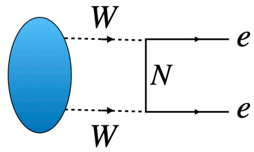
in which $\quad \tan 2\theta_s = \dfrac{\lambda_m v_1 v_2}{\lambda_1 v_1^2 - \lambda_2 v_2^2}$

in the limit $v_1^2/v_2^2 \to 0$:

$$\boxed{\begin{array}{ll} m_-^2 = \dfrac{\lambda_m^2 v_1^2}{4\lambda_2} & m_+^2 = \lambda_2 v_2^2 \\ h_- = h & h_+ = a \end{array}}$$

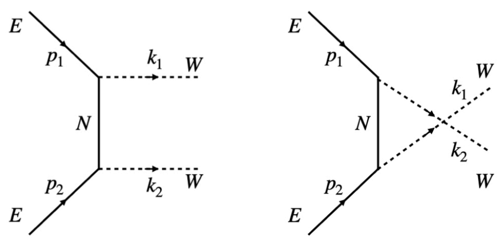
A test for Majorana neutrinos

a) neutrinoless double-β decay



b) same-sign charged lepton scattering $EE \to WW$

In both cases a Majorana neutrino is exchanged and lepton number is violated ($\Delta L = 2$)



Total cross section for b):

$$\sigma(E_i E_j \to WW) = \frac{G_F^2 |m_{Nij}|^2}{2\pi} \sqrt{1 - \frac{4M_W^2}{s}} \left(3 + \frac{s(s - 4M_W^2)}{4M_W^4}\right) \quad \sim \frac{s^2}{M_W^4}$$

$$m_{Nij} = \sum_k m_k U_{ik} U_{jk}$$

neutrino mass eigenvalues