Prediction of environmental vibration due to anthropogenic sources and computation of Newtonian Noise

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Objective

To develop a tool to predict Newtonian Noise at a test mass in an underground cavity as a post-processing step on a seismic displacement field resulting from anthropogenic sources of vibration.

Methodology

1. **Computation of the seismic wave field**. We developed several methods to compute the seismic wave field at a large number of output points (e.g. a grid surrounding an underground cavity) due to anthropogenic sources of vibration (road and railway traffic, wind turbines, industrial and construction activities). We list these in order of increasing complexity:
   - Analytical expressions of plane P-, S- or surface waves in homogeneous soil.
   - Fundamental solutions (Green's functions) due to point sources at the free surface or at depth in a horizontally layered soil (full space or half space). These fundamental solutions are computed with the direct stiffness method or the thin layer method [1], as efficiently implemented in the ElastoDynamics Toolbox (EDT) [2].
   - Two-and-a-half dimensional (2.5D) prediction models for road or railway induced vibration, as implemented in the TRAFFIC toolbox. The toolbox consists of semi-analytical or finite element (FE) models of the road and track that are coupled to boundary element (BE) models representing the horizontally layered halfspace.
   - Alternatively, 2.5D FE models with perfectly matched layers (PMLs) are used to model vibration in coupled track(road)-soil models, as implemented in the WAVEFEM toolbox. We use the open source mesh generator GMSH [4] to generate a FE mesh.
   - 3D FE-PML models [3] subjected to arbitrary sources. We use GMSH to mesh heterogeneous soils and arbitrary cavity shapes. The FE computations are performed in MATLAB using the software Stabil [5] developed by the Structural Mechanics Section of KU Leuven.
   - As an alternative to PMLs, boundary element formulations are available through the BEMFUN toolbox, also developed by the Structural Mechanics Section of KU Leuven.
   - If required, wave scattering effects by the cavity are taken into account using a subdomain approach developed by Bielak et al. [8], as demonstrated in Reumers et al. [6].

   For large models, we have access to the computational resources provided by the Flemish Supercomputer Center (VSC).
   These tools have been successfully cross-validated in the past (e.g. with commercial software and with tools developed at Ecole Centrale Paris (presently Centrale Supélec), University of Cambridge and University of Southampton. These models are used to design vibration mitigation measures and to perform environmental impact studies.
   The tools were licensed to many universities and consultancy companies worldwide (Vibratec, Systra, RATP, Tyrens, Esteyco, Ineco, Nikhef, ATS Consulting, Aecom, Mott MacDonald, Géodynamique et Structure, Sirve); in-house training was organized at several occasions. Our software is used to conduct vibration studies in flagship projects as Crossrail (Elizabeth line) in London, HS2 London-Birmingham, California High-Speed Rail and Grand Paris Express.

2. **Newtonian Noise modeling**: the computation of Newtonian Noise due to a seismic wave field on a grid of output points is tackled in a two-step approach:
    - First, a 3D FE model of the cavity and surrounding soil domain (extending over several wavelengths) is modeled with GMSH [4]. Such model can incorporate cavities with arbitrary shape and size as well as soil heterogeneity. The mesh size is refined in the vicinity of the cavity to accurately compute the volume and surface integrals that contain terms of the order $1/r^2$ and/or $1/r^3$.
    - This FE mesh is subsequently used to compute three matrices A_t, A_b and A_s (size 3 x 3N) which, when multiplied individually with the seismic wave field (3N x 1 vector for every frequency) interpolated at the N nodes of the FE mesh yield the total Newtonian Noise (3 x 1 vector for every frequency), as well as the bulk and surface contribution [6,7]. The matrices are computed using Gaussian quadrature on the element level, as is customary in FE analysis. Their computation is independent of the seismic wave field; therefore, the matrices A_t, A_b and A_s only have to computed once for a particular soil layering and geometry of the cavity.

    In summary, the developed subroutines require as input the cavity geometry and soil layering and give as output the matrices A_t, A_b and A_s.
    The computation of Newtonian Noise for a particular seismic wave field then reduces to a post-processing step involving only a matrix-vector multiplication [6].

Current status

All tools have been developed and are ready to use [7]. Current applications include a homogeneous full space with spherical cavity subjected to plane P- and S-waves (represented analytically). These results have been validated with analytical expressions provided by Harms [9]. More complex cases will be cross-validated in collaboration with teams at ULiège, RWTH, and Nikhef.

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