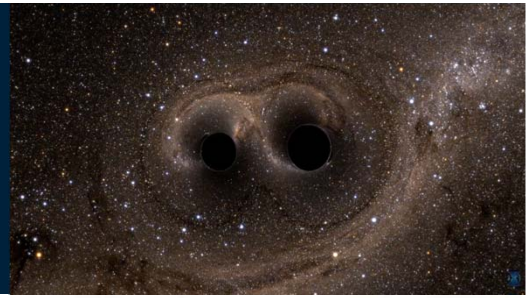

Thermal Noise in Gravitational Wave Detectors

Prof. Giles Hammond

Institute for Gravitational Research

University of Glasgow

Topical Lectures in Gravitational Waves, Nikhef, 20th June 2017

- It is not easy ....

Livingston

- Really not easy ….

OVERVIEW

- **Institute for Gravitational Research**
- **Thermal noise and units**
- **Brownian Motion**
- **Fluctuation-Dissipation theorem**
- **Sources of damping/Thermoelastic noise**
- **Thermal noise in simple mechanical systems**
- **Levin’s method for TN with inhomogeneous loss**
- **Thermal noise in a GW detector suspension**

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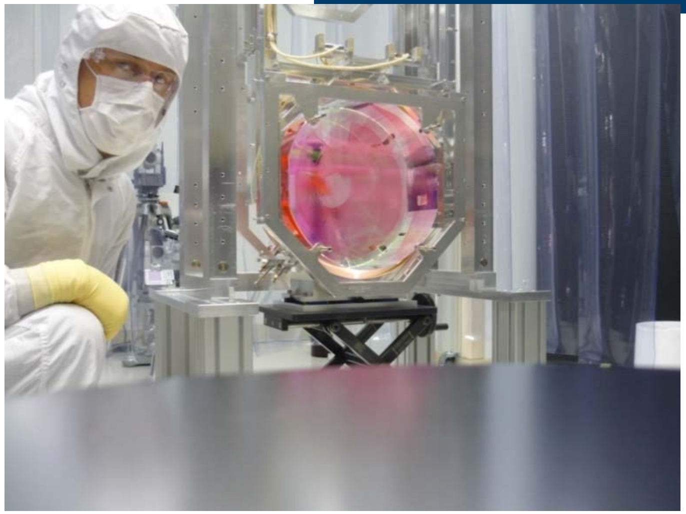
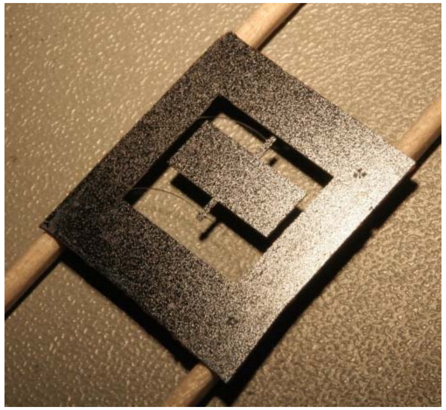
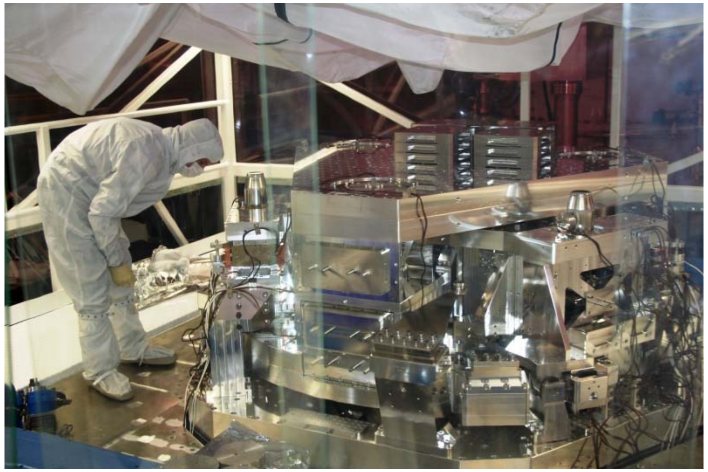
INSTITUTE FOR GRAVITATIONAL RESEARCH



- Suspension for Advanced gravitational wave detectors; silica, silicon, sapphire (now)

- Precision measurements of gravity; axion searches and Casimir force (Birmingham, PhD, postdoc)
- MEMS gravimeters for gravity imaging (now)





- aLIGO ISI (JILA, postdoc)

QUANTIC

Development of precision novel interferometric techniques

Development of systems of ultra low mechanical loss for the suspensions of mirror test masses

Astrophysical interpretation of collected data

Involved in the successful LISA Pathfinder mission and in developing space-based gravitational wave detector LISA

Working on a pan-European consortium carrying out investigations towards '3rd generation' gravitational wave detectors e.g. 'Einstein Telescope'

Group now comprised of approximately 70 members

~One third work in material science subgroup
(coatings, bonding, suspension R&D etc.)

THERMAL NOISE AND UNITS

Different noise sources occur in a gravitational wave detector

Excess Gas

Coating Thermo-optic noise

Substrate Brownian noise

Coating Brownian noise

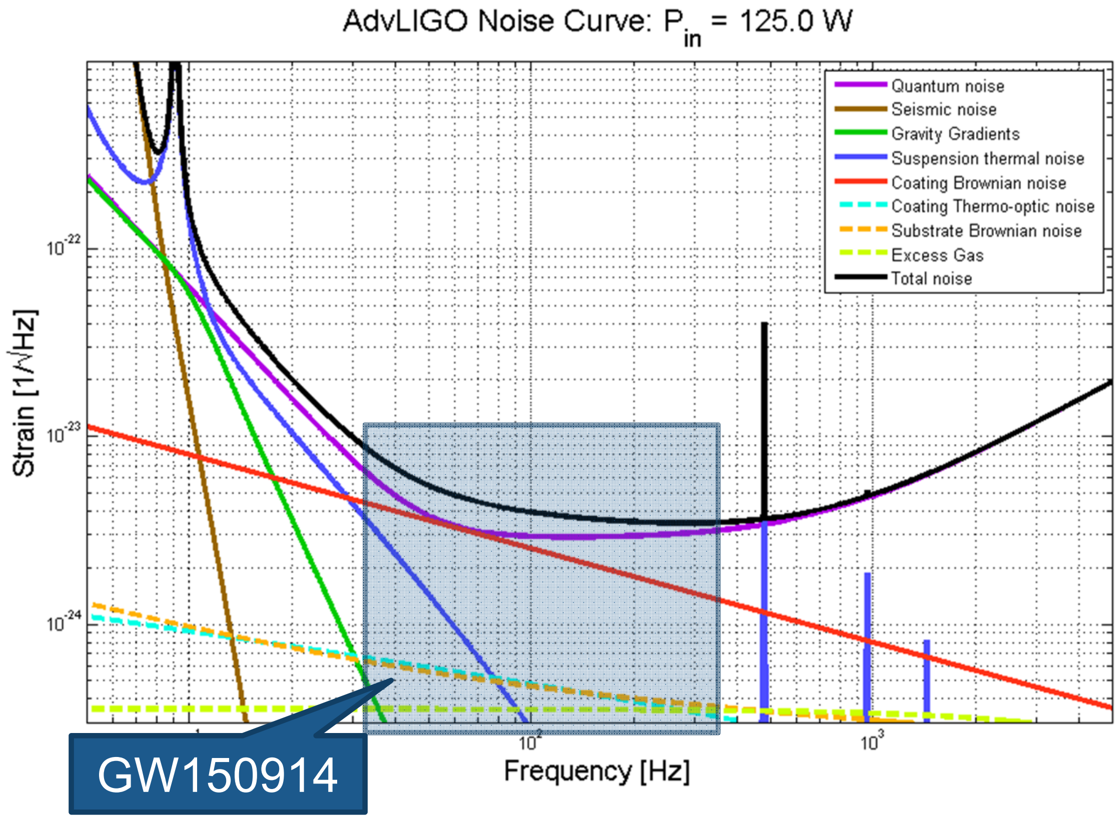
Gravity Gradients

Quantum Noise

Suspension thermal noise

Seismic Noise

Total Noise

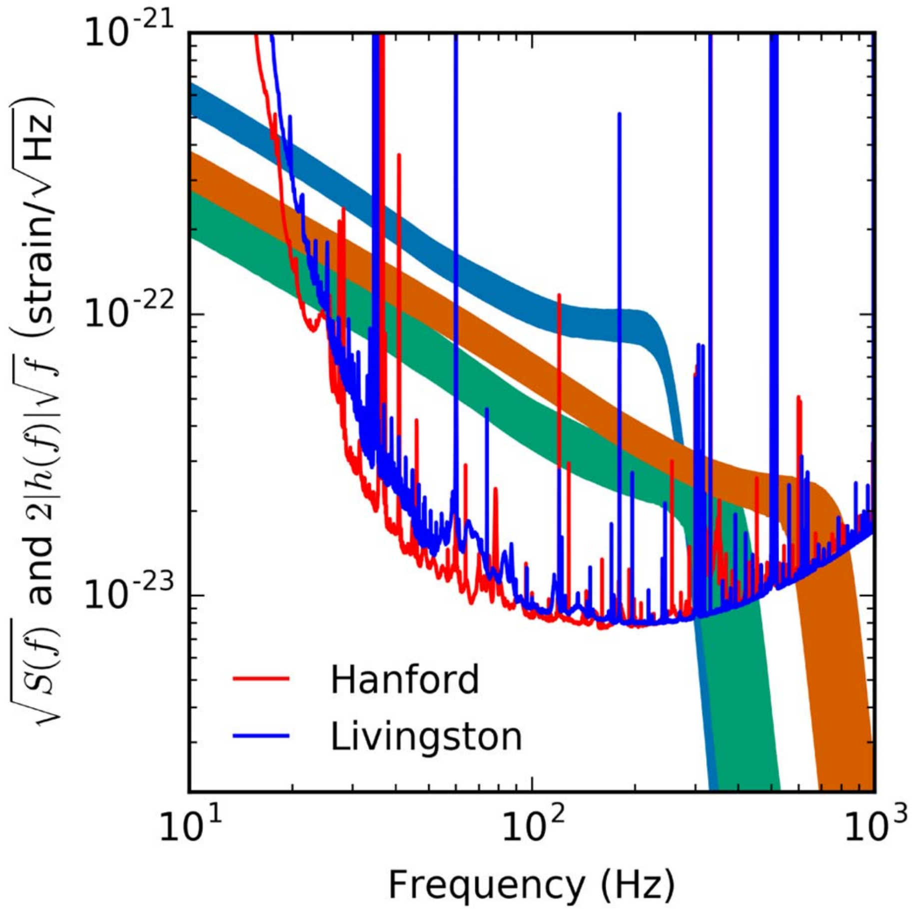


Thermal noise associated with suspension mirrors limits performance at the most sensitive frequencies

Aiming to detect signals with a $\Delta L/L < 10^{-21}$

Different noise sources occur in a gravitational wave detector

Excess Gas
Coating Thermo-optic noise
Substrate Brownian noise
Coating Brownian noise
Gravity Gradients
Quantum Noise
Suspension thermal noise
Seismic Noise
Total Noise



Abbott et al., Phys. Rev. Lett. 116, 241103, 2016
Abbott et al., Phys. Rev. X 6, 041015, 2016

Units of Noise

Noise is the statistical fluctuation of a parameter

All properties have fluctuations

e.g. pressure, voltage, light intensity, …

In high precision experiments noise limits the sensitivity

Usually noise power is normalised to unit bandwidth

Gives the power spectral density in units of 'power per Hz'

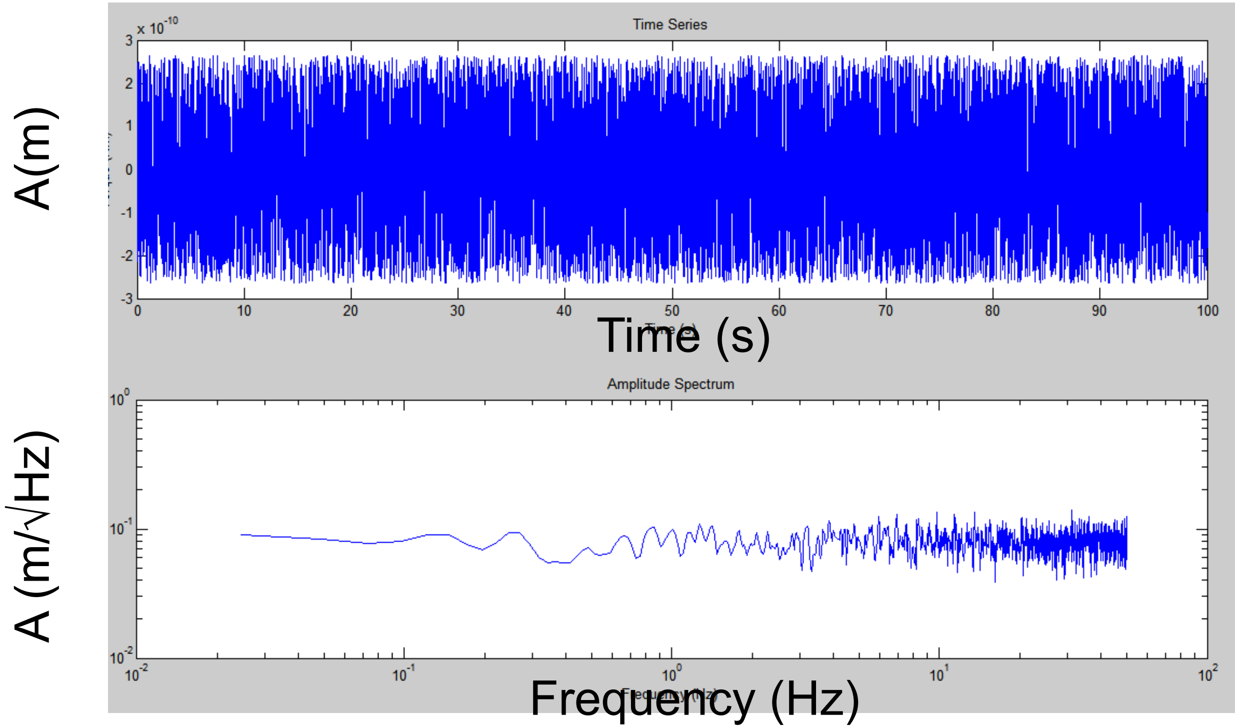
Note: $P \sim V^2$, so sometimes see it denoted V^2/Hz

Conversion to amplitude spectral density gives units of $\mathrm{amplitude}/\sqrt{\mathrm{Hz}}$

Use S_x to represent power spectral density m^2/Hz

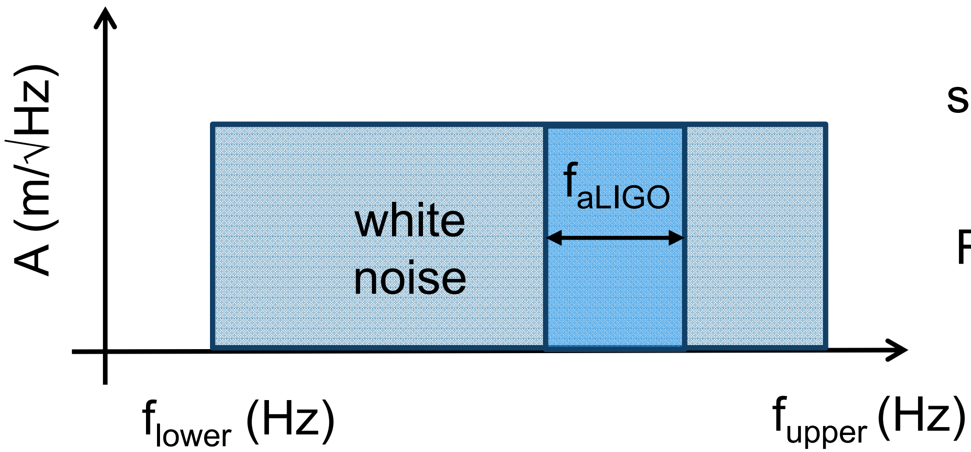
Noise in a detector is usually quoted as $x = \sqrt{S_x}$, $\left(\mathrm{m}/\sqrt{\mathrm{Hz}}\right)$

ASD and RMS



Time series : Random noise

FFT: Random noise



spectral density=A m/√Hz

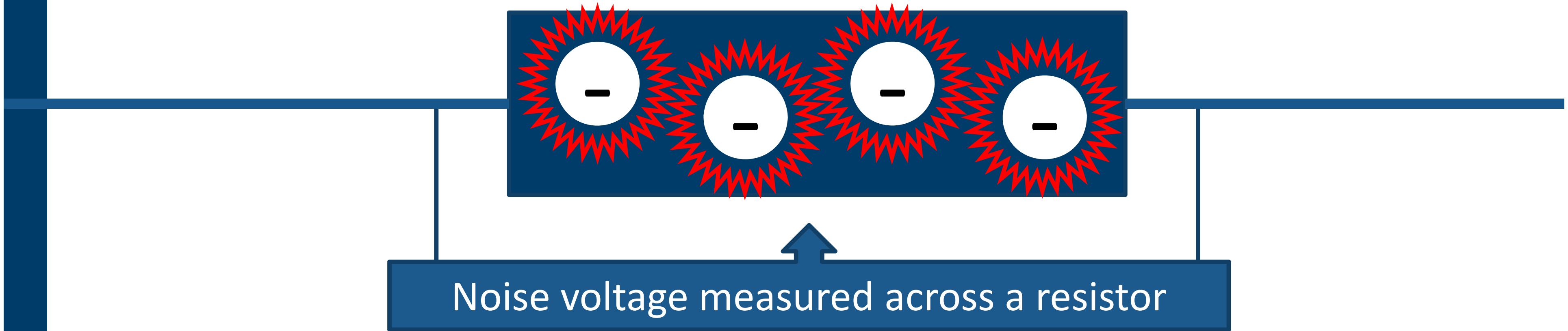
$$\text{RMS}=\sqrt{\int_{f_{lower}}^{f_{upper}} A^2 df} = \sqrt{A^2\left[f_{upper} - f_{lower}\right]} \text{ m}$$

BROWNIAN MOTION

Thermal Noise in Electric Circuits

Johnson–Nyquist is the electronic noise generated by the thermal agitation of the electrons inside an electrical conductor at equilibrium

This happens regardless of any applied voltage



The first independent description was given by Nyquist and Johnson in 1926 (at Bell labs)

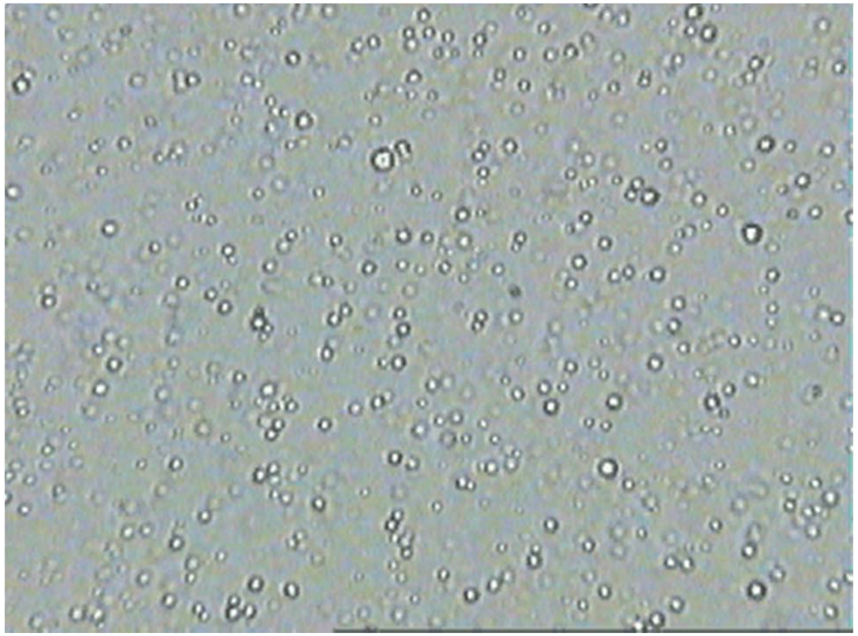
$$\overline{V_{th}^2} = 4k_B TRB$$

R is resistance and B bandwidth over which the noise is measured

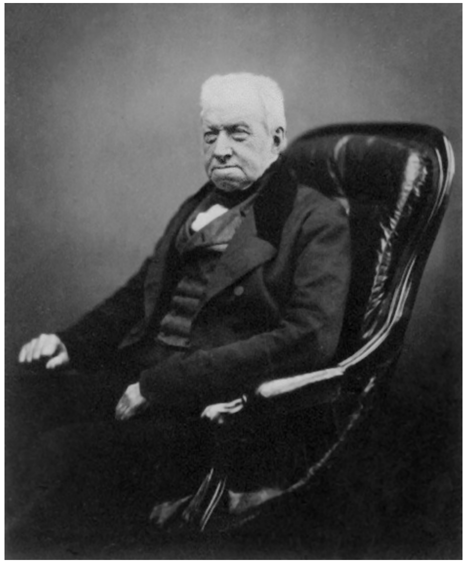
Thermal noise:

random fluctuations driven by thermal energy

Common example is Brownian Motion



Robert Brown
(botanist, born 1773)



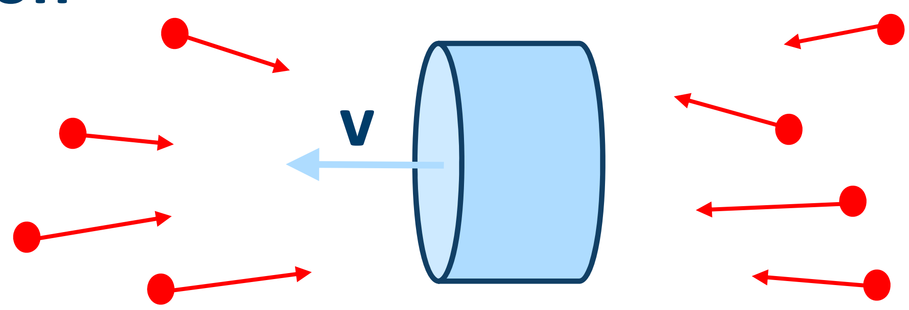
Brownian motion observed in fat droplets suspended in milk

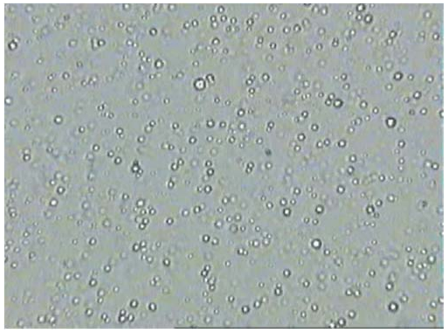
Brownian Motion

Brownian motion is the random motion of particles suspended in a fluid resulting from their collision with the fast-moving atoms or molecules in the gas or liquid

In 1827, Brown was observing grains of pollen suspended in water under a microscope

Although Brown did not provide a theory to explain the motion, phenomenon is now known as Brownian motion





In 1905 Einstein published the paper:

"Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen"

("On the movement of small particles suspended in a stationary liquid demanded by the molecular-kinetic theory of heat")

This explained in precise detail how the motion that Brown had observed was a result of the pollen being moved by individual water molecules

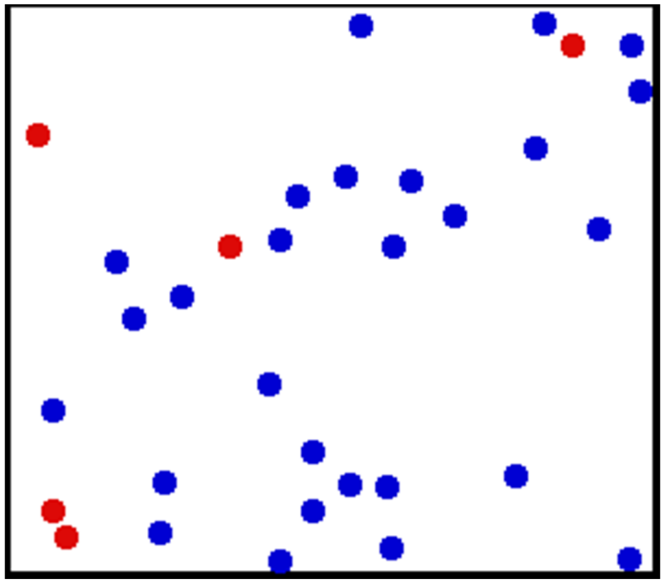
Brownian Motion

This explanation of Brownian motion served as convincing evidence that atoms and molecules exist

Experimentally verified by Jean Perrin in 1908

Perrin was awarded the Nobel Prize in Physics in 1926 "for his work on the discontinuous structure of matter"

The direction of the force of atomic bombardment is constantly changing, and at different times the particle is hit more on one side than another



Leads to the seemingly random nature of the motion

Einstein showed that this motion was a result of the stochastic collisions of the molecules of water with the pollen grains

i.e. the motion was due to the fluid surrounding the pollen and not the particles themselves

Einstein also realised that as a result of these impacts the pollen grains lost their initial kinetic energy while they moved through the water

This introduced a dissipation process

THE FLUCTUATION-DISSIPATION THEOREM

The Fluctuation-Dissipation Theorem

If a system has a dissipative process (turning energy into heat) there is a reverse process related to thermal fluctuations (i.e. thermal fluctuations can couple into the system)

Electrical system

Resistance dissipates energy in a resistor (Joule heating)

Corresponding fluctuation is Johnson noise

thermal fluctuations in a resistor create a fluctuating voltage in the resistor

Brownian motion

Drag when an object moves through fluid dissipates kinetic energy

Corresponding fluctuation is Brownian motion

An object in a fluid experiences forces from thermal fluctuations in the fluid

The Fluctuation-Dissipation Theorem

Strong correlation between the real (dissipative) part of the impedance (damping / loss) in a system and the thermal noise

Both thermal energy $k_B T$ and the dissipation in a system determine its thermal noise

Example

Johnson noise

$$S_U = 4 k_B T \, R \, \Delta f$$

thermal energy

dissipation

130nV/√Hz for 1MΩ

The Fluctuation-Dissipation Theorem

The relationship described by Einstein was an early example of the Fluctuation-Dissipation Theorem developed later by Callen et al.

This provided a general relationship between the excitation of a system (fluctuation) and the friction (dissipation) for any linear system in thermal equilibrium

PHYSICAL REVIEW VOLUME 83, NUMBER 1 JULY 1, 1951

Irreversibility and Generalized Noise*

HERBERT B. CALLEN AND THEODORE A. WELTON†
Randal Morgan Laboratory of Physics, University of Pennsylvania, Philadelphia, Pennsylvania
(Received January 11, 1951)

A relation is obtained between the generalized resistance and the fluctuations of the generalized forces in linear dissipative systems. This relation forms the extension of the Nyquist relation for the voltage fluctuations in electrical impedances. The general formalism is illustrated by applications to several particular types of systems, including Brownian motion, electric field fluctuations in the vacuum, and pressure fluctuations in a gas.

I. INTRODUCTION

THE parameters which characterize a thermodynamic system in equilibrium do not generally have precise values, but undergo spontaneous fluctuations. These thermodynamic parameters are of two classes: the "extensive" parameters,[1] such as the volume or the mole numbers, and the "intensive" parameters[2] or "generalized forces," such as the pressure or chemical potentials.

An equation relating particularly to the fluctuations in voltage (a "generalized force") in linear electrical systems was derived many years ago by Nyquist,[3] and such voltage fluctuations are generally referred to as Nyquist or Johnson "noise." The voltage fluctuations are related, not to the standard thermodynamic parameters of the system, but to the electrical resistance. The Nyquist relation is thus of a form unique in physics, correlating a property of a system in *equilibrium* (i.e., the voltage fluctuations) with a parameter which characterizes an irreversible process (i.e., the electrical resistance). The equation, furthermore, gives not only the mean square fluctuating voltage, but provides, in addition, the frequency spectrum of the fluctuations. The proof of the relation is based on an ingenious union of the second law of thermodynamics and a direct calculation of the fluctuations in a particular simple system (an ideal transmission line).

It has frequently been conjectured that the Nyquist relation can be extended to a general class of dissipative systems other than merely electrical systems. Yet, to our knowledge, no proof has been given of such a generalization, nor have any criteria been developed for the type of system or the character of the "forces" to which the generalized Nyquist relation may be applied. The development of such a proof and of such criteria is the purpose of this paper (Secs. II, III, and IV). The general theorem thus establishes a relation between the "impedance" in a general linear dissipative system and the fluctuations of appropriate generalized "forces."

Several illustrative applications are made of the general theorem. The viscous drag of a fluid on a moving body is shown to imply a fluctuating force, and application of the general theorem immediately yields the fundamental result of the theory of Brownian motion. The existence of a radiation impedance for the electromagnetic radiation from an oscillating charge is shown to imply a fluctuating electric field in the vacuum, and application of the general theorem yields the Planck radiation law. Finally, the existence of an acoustic radiation impedance of a gaseous medium is shown to imply pressure fluctuations, which may be related to the thermodynamic properties of the gas. The theorem thus correlates a number of known effects under one general principle and is able to predict a class of new relations.

In the final section of the paper, we discuss an intuitive interpretation of the principles underlying the theorem.

It is felt that the relationship between equilibrium fluctuations and irreversibility which is here developed provides a method for a general approach to a theory of irreversibility, using statistical ensemble methods. We are currently investigating such an approach.

II. THE DISSIPATION

A system may be said to be dissipative if it is capable of absorbing energy when subjected to a time-periodic perturbation (as an electrical resistor absorbs energy from an impressed periodic voltage). The system may be said to be linear if the power dissipation is quadratic in the magnitude of the perturbation. For a linear dissipative system, an impedance may be defined, and the proportionality constant between the power and the square of the perturbation amplitude is simply related to the impedance [in the electrical case, Power $=(\text{voltage})^2 \cdot R/|Z|^2$].

In the present section we treat the applied perturbation by the usual quantum mechanical perturbation

* This work was supported in part by the ONR.
† Now at Oak Ridge National Laboratory, Oak Ridge, Tennessee.
[1] For the theory of fluctuations of extensive parameters see Fowler, *Statistical Mechanics* (Cambridge University Press, London, 1936), second edition; or Tolman, *Principles of Statistical Mechanics* (Oxford University Press, London, 1938). A recent development of the theory is given by M. J. Klein and L. Tisza, Phys. Rev. **76**, 1861 (1949).
[2] A statistical mechanical theory of fluctuations of intensive parameters will be given in a subsequent paper by R. F. Greene and H. B. Callen.
[3] H. Nyquist, Phys. Rev. **32**, 110 (1928). A very neat derivation and an interesting discussion is given by J. C. Slater, Radiation Laboratory Report; "Report on Noise and the Reception of Pulses," February 3, 1941, unpublished.

The Fluctuation-Dissipation Theorem

In general, the power spectrum of the fluctuating thermal force acting on a system is

$$S_f(\omega) = 4k_B T\,\Re[Z(\omega)]$$

Where $Z(\omega)$ is the impedance of the system

$$Z(\omega) = \ F(\omega)\ /\ v(\omega)$$

For a velocity $v(\omega)$ resulting from a force $F(\omega)$

Alternatively, in terms of the power spectral density of the fluctuating motion of the system

$$S_x(\omega) = \frac{4k_B T}{\omega^2}\Re\left[Z^{-1}(\omega)\right]$$

Thermal displacement power spectral density

Real part of the system's inverse impedance

SOURCES OF DISSIPATION

Sources of Dissipation

Thermal vibrations will excite the mechanical resonances (e.g. violin modes and resonant modes of test mass)

Thermal noise power spectrum of a resonance calculated using FDT (if we know the damping)

For most real materials, the internal dissipation has been shown to follow a modified version of Hooke's Law

Two models of internal damping often used:

Structural damping – constant with frequency

Anelasticity – gives rise to broad peaks in the damping at some characteristic frequency

External Sources of Dissipation

There are a variety of external sources of dissipation

Gas Damping

External viscous damping is experienced by the mirrors and their suspensions due to the effects of residual gas molecules in the vacuum systems in which the suspensions are mounted

Recoil Damping

Energy may be lost from the pendulum suspension into a recoiling support structure

Frictional Damping

Friction at the suspension point and where suspension elements contact the test substrate may introduce stick-slip damping

Careful design of the technical aspects of the suspensions is essential to ensure these types of damping are minimised

Internal Sources of Dissipation

Once all the external sources of dissipation have been minimised, the dominant source of thermal noise is a result of the internal dissipation of the suspended optics

Internal dissipation can arise when a material responds anelastically to a force acted upon it

When an ideal elastic material is acted upon by a force, a stress σ is produced for which there is a resultant strain within the material

This strain, ε, is related to the Modulus of Elasticity M

$$\sigma = M\varepsilon$$

In an elastic response, the stress and strain are related to the compliance J of the material so that

$$\varepsilon = J\sigma$$

and J is related to the stored energy due to the induced deformation

Internal Sources of Dissipation

When a force is applied to anelastic materials the body returns to a new equilibrium state after a relaxation time

The stress and resulting strain can be modelled in complex form

Strain ε follows the stress σ after a phase lag ϕ

$$\sigma = \sigma_0 e^{i\omega t}$$
$$\varepsilon = \varepsilon_0 e^{i(\omega t - \phi)t}$$

ε_0 is the stress amplitude
$\omega = 2\pi f$, where f is the vibration frequency

ϕ is the angle the strain lags behind the stress
Commonly referred to as the loss angle

In an ideal elastic material $\phi = 0$

Internal Sources of Dissipation

$$\sigma = \sigma_0 e^{i\omega t}$$
$$\varepsilon = \varepsilon_0 e^{i(\omega t - \phi)t}$$

However, for the anelastic case ϕ is generally non-zero

Consequently, ratio $^{\sigma}/_{\varepsilon}$ is complex

Since $\sigma = M\varepsilon$ then the Modulus of Elasticity must also be complex

$$M^{+}(\omega) = M_1(\omega) + iM_2(\omega)$$

Note this is a function of ω

$M_1(\omega)$ and $M_2(\omega)$ are the real and imaginary parts and

$$\tan\phi = \frac{M_2}{M_1}$$

THERMOELASTIC DAMPING

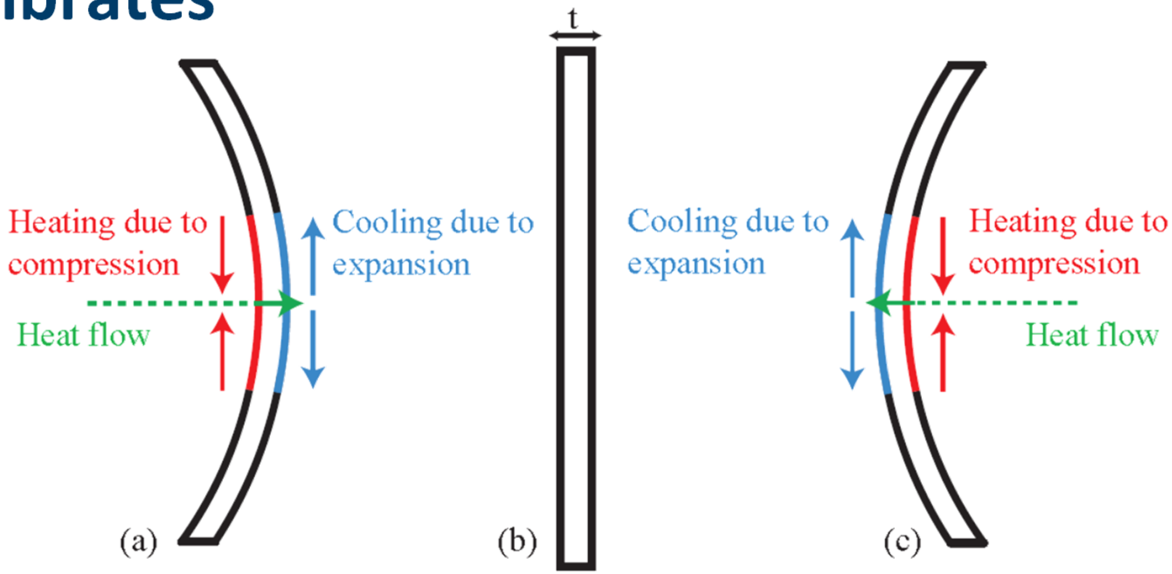
Thermoelastic Damping

Thermoelastic dissipation is an anelastic relaxation process related to the flow of heat in a material

Arises from the coupling of temperature and strain in a body via the thermal expansion coefficient

Thermoelastic dissipation in thin beams and fibres was studied by Zener, who calculated the differential heating across a fibre as it vibrates

$$\Delta L = \alpha L_0 \Delta T$$



Thermoelastic Damping

A thermal gradient is created

This leads to the flow of heat across the thickness of the beam

If the beam continues to vibrate there will be an oscillating heat flow

i.e. each side of the beam is compressed and stretched in turn, and the heat flow is reversed

This heat flow is a source of dissipation

Will have a peak at a frequency related to the time taken for heat to flow across the beam

Thermo-elastic and Thermo-refractive Noise

Temperature never constant throughout mirror

Local temperature fluctuations cause additional noise

Thermo-elastic noise

T fluctuations couple to the displacement of mirror face via the coefficient of thermal expansion α

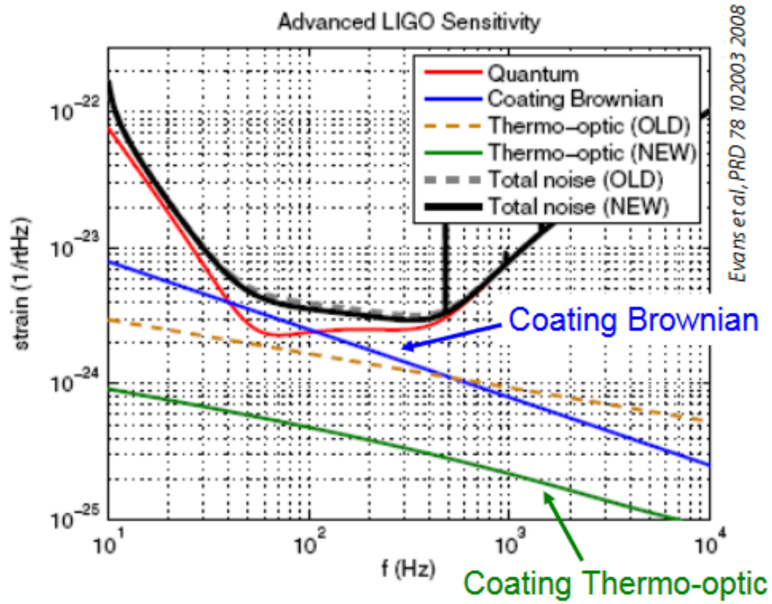
Thermo-refractive noise

T fluctuations can change the refractive index of mirror / coating, via dn/dT, changing phase of transmitted beam

Coating Thermal Noise

The Problem

- Initial LIGO mirror coatings are alternating layers of tantala/silica
- Tantala has high mechanical loss (internal friction).
  - 4 μm of tantala noisier than 15 cm silica.
- Dominant noise at most sensitive frequencies



Current Solution

Titania-doped tantala / Silica

- 40% lower loss
- Layer thicknesses optimized for low loss, equal reflectivity

Ongoing Research

- Amorphous coatings:
  - Annealing / Heated-substrate
  - Dopants
  - Structural measurements
- Crystalline Coatings:
  - AlGaAs
  - AlGaP

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Coating Thermal Noise

- Reductions in coating thermal noise required
- Upgrades to aLIGO detectors
  - A+: room temperature, 1064nm (could we change wavelength?)
  - Voyager - 120 K, Si optics, 1550nm / 2μm (20K possibility?)

  - For reference, aLIGO ETM coating loss (2 +/- 0.1)E-4
    - loss of Ti:Ta2O5 = 2.4E-4
    - loss of SiO2 = 5E-5

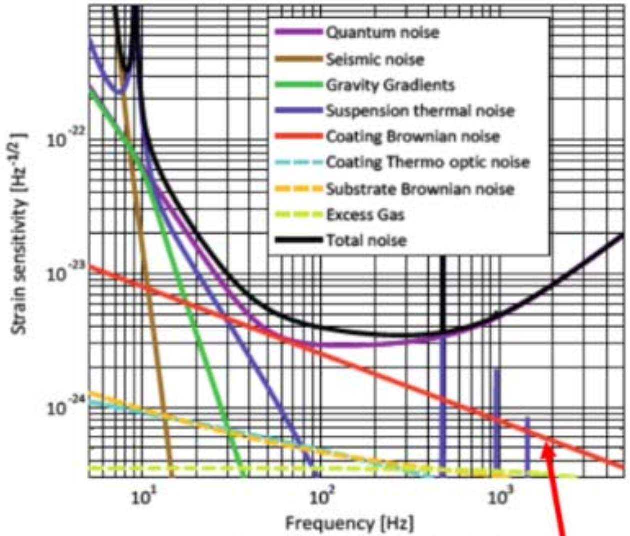
Temperature

Coating thickness

$$S_x(f,T) \approx \frac{2k_B T}{\pi^2 f}\frac{d}{w^2\,Y}\phi\left(\frac{Y'}{Y}+\frac{Y}{Y'}\right)$$

Laser beam radius

Coating mechanical loss



GWINC aLIGO noise budget

Coating thermal noise

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Example

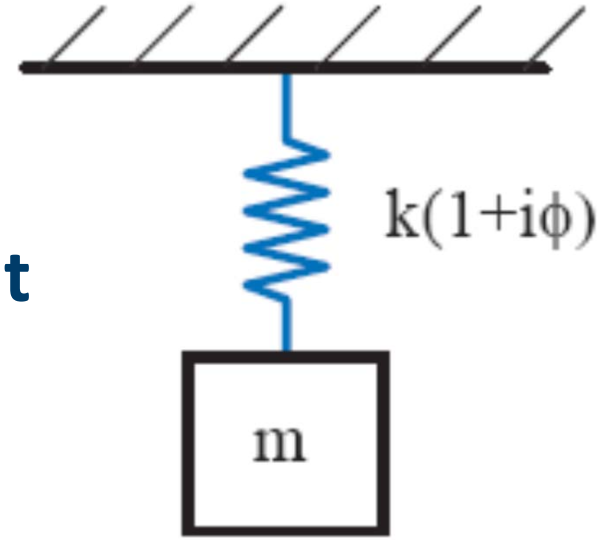
THERMAL NOISE IN A SIMPLE HARMONIC OSCILLATOR WITH INTERNAL FRICTION

The Fluctuation-Dissipation Theorem

Just shown internal friction in a material arises from the time delay between an applied stress and the development of the associated strain

In the frequency domain this appears as a phase lag ϕ between the applied force F and the response of the system x

This can be represented by a modified version of Hooke's law, involving a complex spring constant



$$F_{\text{spring}}(\omega) = -k\big(1 + i\phi(\omega)\big)x$$

The Fluctuation-Dissipation Theorem

$$F_{\text{spring}}(\omega) = -k\big(1 + i\phi(\omega)\big)x$$

If the resonant modes of the suspended mirror system in a gravitational wave detector are modelled as harmonic oscillators with internal damping, the equation of motion can be written as

$$F(\omega) = m\ddot{x} + k\big(1 + i\phi(\omega)\big)x$$

Using the standard expressions relating the acceleration and displacement to the velocity of a harmonic oscillator $x \propto e^{i\omega t}, v = i\omega x, \ddot{x} = i\omega v$

$$F(\omega) = i\omega m v - i\frac{k}{\omega}\big(1 + i\phi(\omega)\big)v$$

The Fluctuation-Dissipation Theorem

$$\boldsymbol{F(\omega) = i\omega m v - i\frac{k}{\omega}\big(1 + i\phi(\omega)\big)v}$$

The impedance Z(ω) can then be calulated

$$\boldsymbol{Z(\omega) = \frac{F(\omega)}{v}}$$

$$\boldsymbol{= i\omega m - i\frac{k}{\omega}\big(1 + i\phi(\omega)\big)}$$

$$\boldsymbol{= i\left(\omega m - \frac{k}{\omega}\right) + \phi(\omega)\frac{k}{\omega}}$$

The Fluctuation-Dissipation Theorem

This expression can be inverted to give the admittance

$$Y(\omega) = \frac{\omega}{k\phi(\omega) + i(\omega^2 m - k)}$$

Then rationalise the denominator by multiplying by numerator and denominator by the complex conjugate $k\phi(\omega) - i\left(\omega^2 m - k\right)$ gives

$$= \frac{\boxed{\phi(\omega)\frac{k}{\omega}} - i\left(\omega m - \frac{k}{\omega}\right)}{\boxed{\left[\phi(\omega)\frac{k}{\omega}\right]^2 + \left(\omega m - \frac{k}{\omega}\right)^2}}$$

The Fluctuation-Dissipation Theorem

$$S_x(\omega) = \frac{4k_BT}{\omega^2}\Re\left[Z^{-1}(\omega)\right] = \frac{4k_BT}{\omega^2}\Re[Y(\omega)]$$

Can now calculate the power spectral density of displacement thermal noise of an oscillator with mass m at temperature T associated with a resonant mode of frequency ω_0

$$S_x(\omega) = \frac{4k_BT}{\omega m}\frac{\phi(\omega)\omega_0^2}{\left[\left(\omega_0^2 - \omega^2\right)^2 + \omega_0^4\phi^2(\omega)\right]}$$

where $\omega_0^2 = {}^{k}/_{m}$ and $\phi(\omega)$ is the mechanical dissipation of the oscillator

The Fluctuation-Dissipation Theorem

Rather than putting in values, lets consider three cases

$$\boxed{\omega \ll \omega_0}, \omega \gg \omega_0 \text{ and } \omega = \omega_0$$

$$S_x(\omega) = \frac{4k_BT}{\omega m}\frac{\phi(\omega)\omega_0^2}{\left[\left(\omega_0^2 - \cancel{\omega^2}\right)^2 + \cancel{\omega_0^4\phi^2(\omega)}\right]}$$

If we have a low loss material (of the kind used in mirrors of a gravitational wave detector) then $\phi^2(\omega) \ll 1$

$$\omega \ll \omega_0 \qquad S_x(\omega) = \frac{4k_BT}{m\omega_0^2}\frac{\phi(\omega)}{\omega}$$

Far below resonance TN PSD due to resonance is directly proportional to the mechanical loss

The Fluctuation-Dissipation Theorem

Case Two $\omega \gg \omega_0$

$$S_x(\omega) = \frac{4k_BT}{\omega m}\frac{\phi(\omega)\omega_0^2}{\left[\left(\cancel{\omega_0^2} - \omega^2\right)^2 + \cancel{\omega_0^4\phi^2(\omega)}\right]}$$

Again, if we have a low loss material then $\phi^2(\omega) \ll 1$

$$\omega \gg \omega_0 \qquad S_x(\omega) = \frac{4k_BT\omega_0^2\phi(\omega)}{m\omega^5}$$

Far above resonance TN PSD due to resonance is directly proportional to the mechanical loss

Can minimize off-resonance TN using materials with low mechanical loss

The Fluctuation-Dissipation Theorem

Case Three $\omega = \omega_0$

$$S_x(\omega) = \frac{4k_BT}{\omega m}\frac{\phi(\omega)\omega_0^2}{\left[\cancel{(\omega_0^2-\omega^2)^2} + \omega_0^4\phi^2(\omega)\right]}$$

$$\omega = \omega_0 \qquad S_x(\omega) = \frac{4k_BT}{m\omega_0^3\phi(\omega)}$$

At resonance $S_x(\omega) \propto \dfrac{1}{\phi(\omega)}$

Results in a large noise spectral density at, and around, the resonant frequency

The Fluctuation-Dissipation Theorem

Reducing the mechanical loss effectively confines more of the thermal motion into a narrow frequency band centred on the resonance

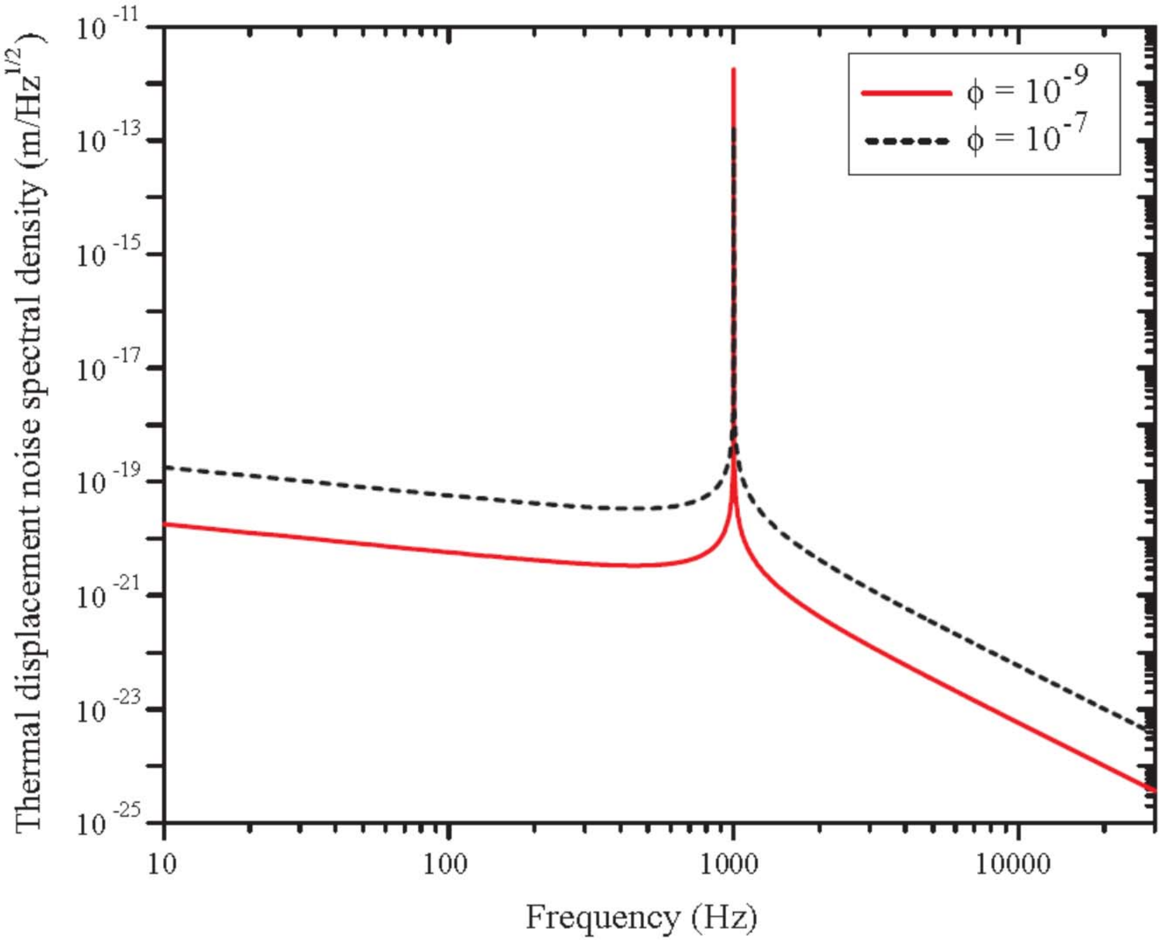
$$\omega = \omega_0 \qquad S_x(\omega) = \frac{4k_BT}{m\omega_0^3\phi(\omega)}$$

Results in a narrower and higher thermal noise peak at the resonant frequency and lower off-resonance thermal noise

$$\omega \ll \omega_0 \qquad S_x(\omega) = \frac{4k_BT}{m\omega_0^2}\frac{\phi(\omega)}{\omega}$$

$$\omega \gg \omega_0 \qquad S_x(\omega) = \frac{4k_BT\omega_0^2\phi(\omega)}{m\omega^5}$$

The Fluctuation-Dissipation Theorem



The thermal displacement noise spectrum for a mode of two otherwise identical mechanical oscillators with different $\phi(\omega)$

The Fluctuation-Dissipation Theorem

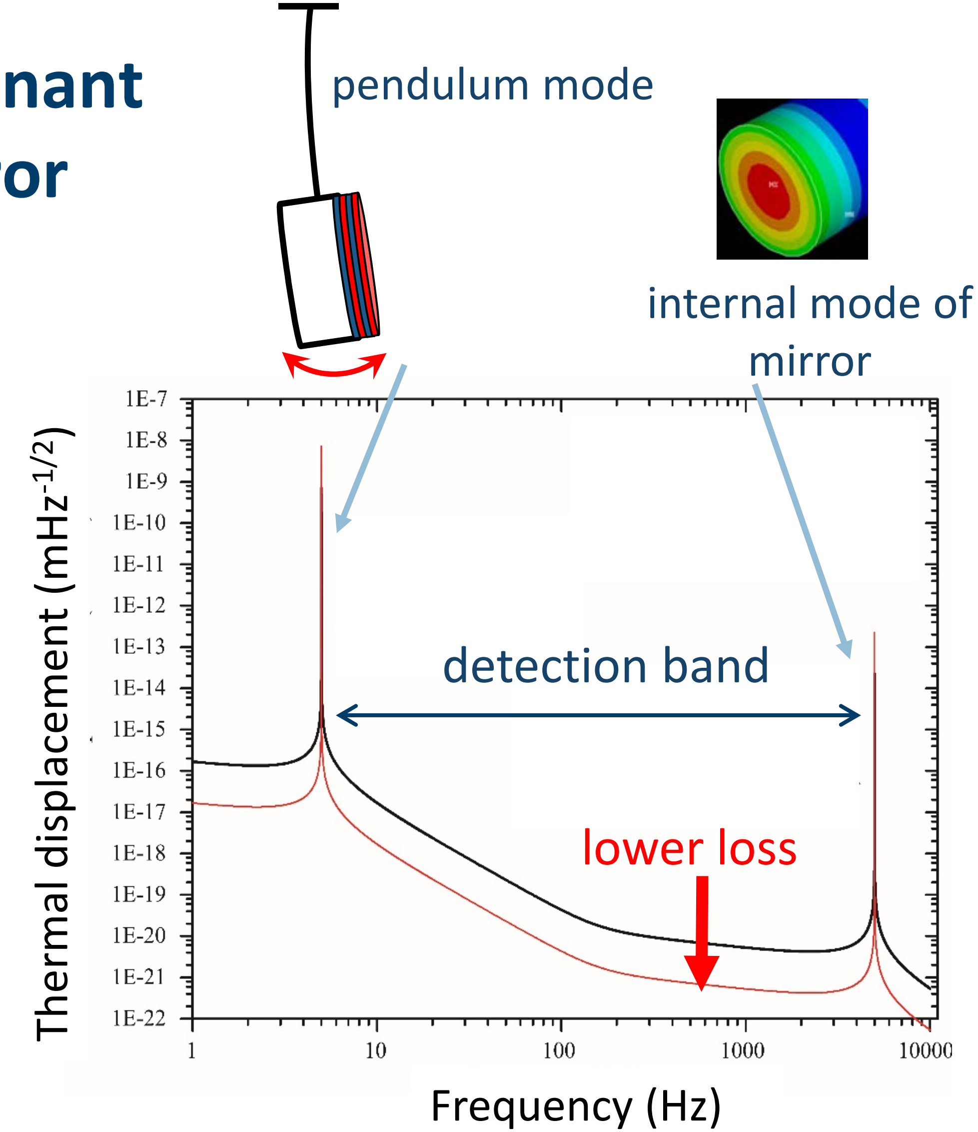
Apply this to a gravitational wave detector

Thermal energy drives resonant modes of a suspended mirror

Seen width of resonance related to mechanical loss of the material

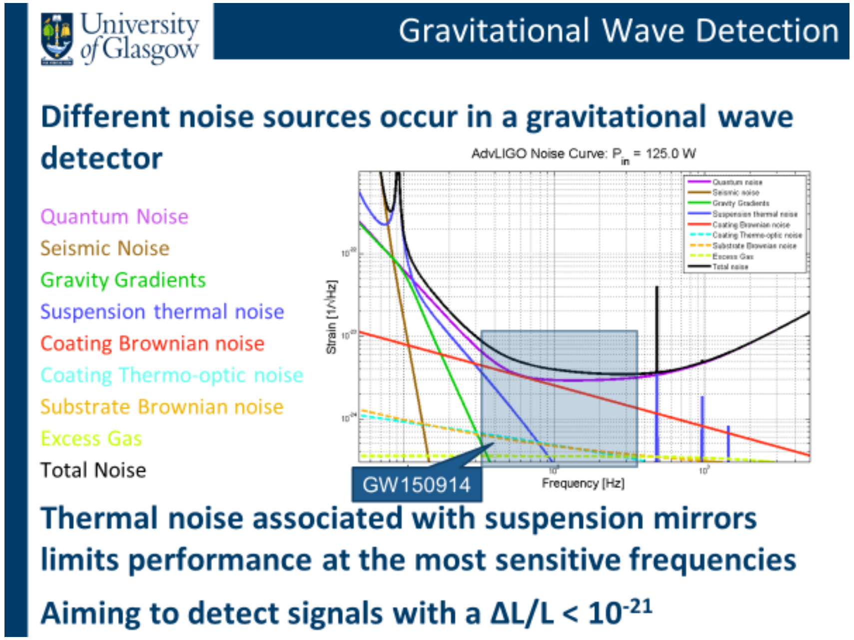
Lower loss material → lower off resonance TN

Room temperature GWDs use fuse silica mirror substrates and suspension fibres ($\varphi < 10^{-9}$)

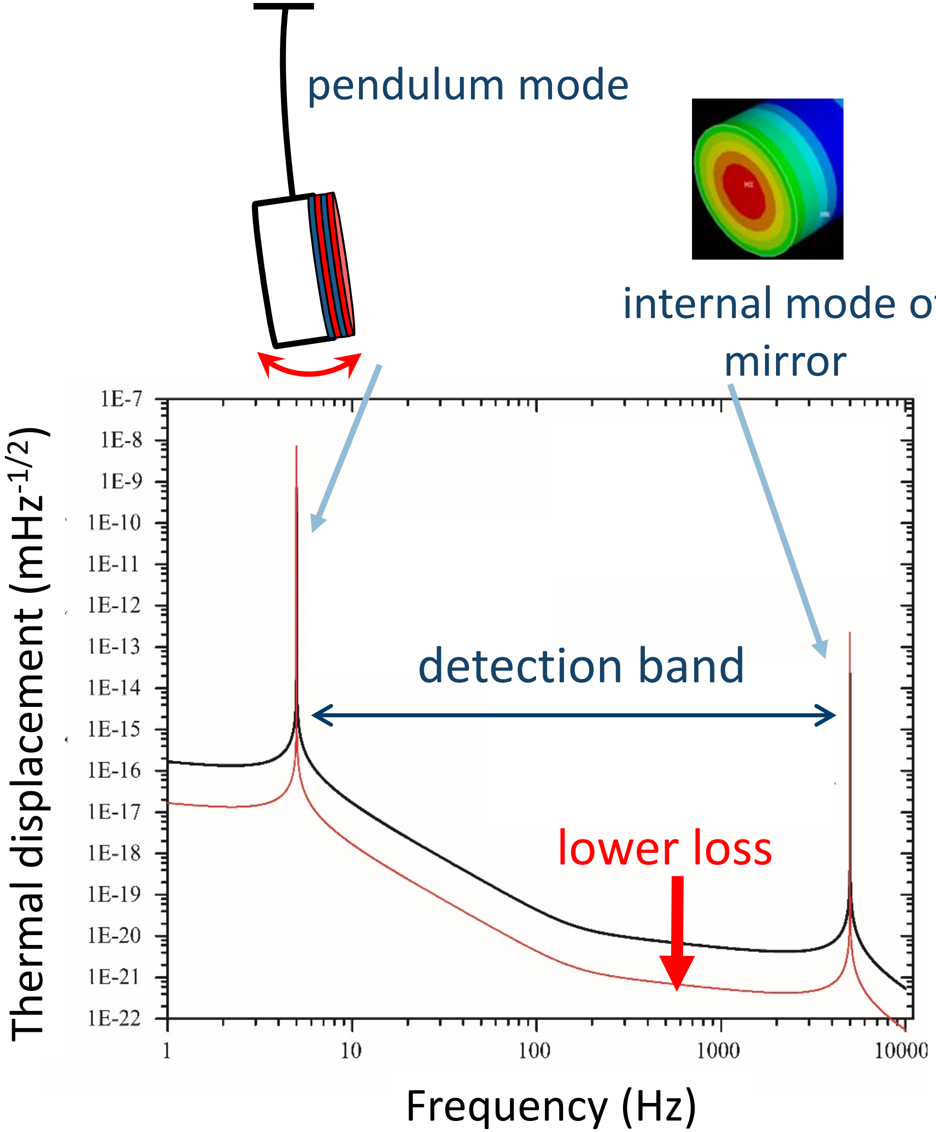


The Fluctuation-Dissipation Theorem

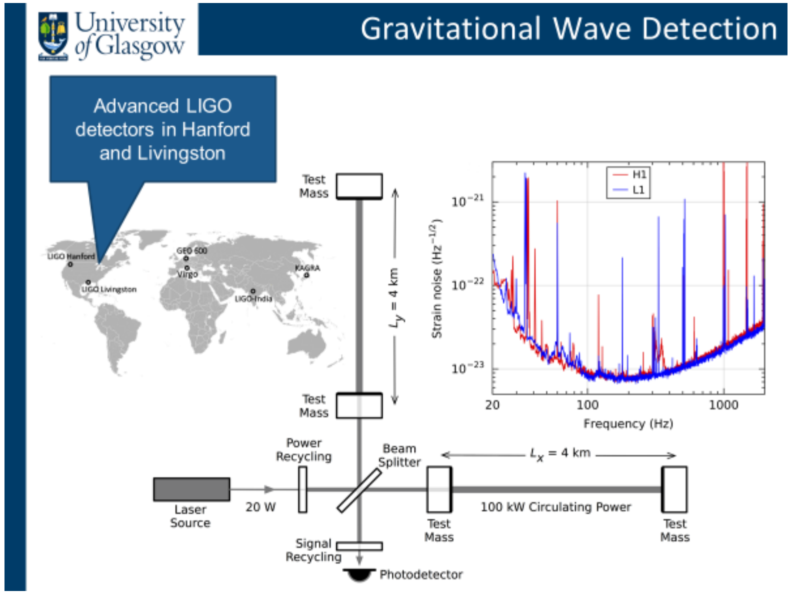
Slide from earlier:



Using low loss suspension materials ensures that thermal noise peaks from pendulum mode and first mirror resonance are suitably narrow

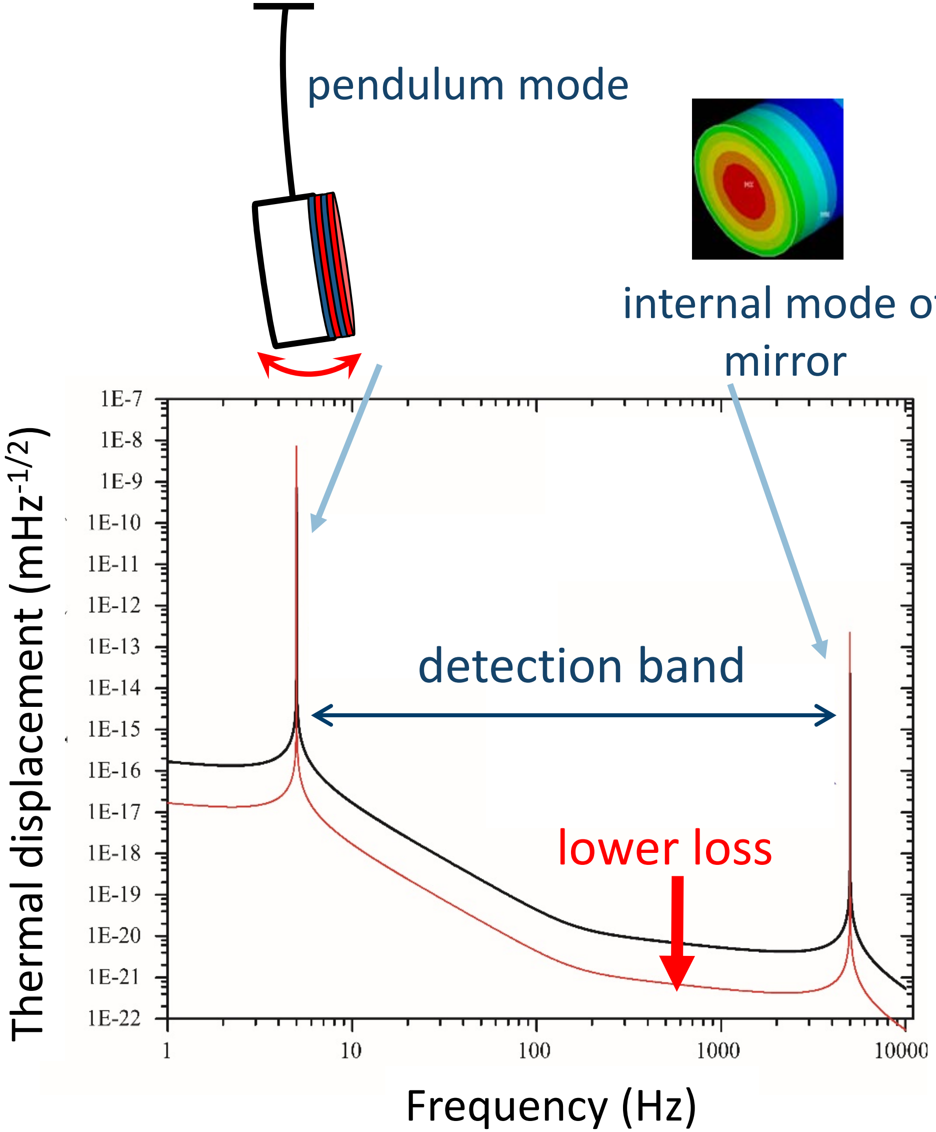


The Fluctuation-Dissipation Theorem



Reality is there are several violin and internal resonant modes

Ensuring the TN peaks are suitably narrow enables them to be removed from IFO signal by notch filtering



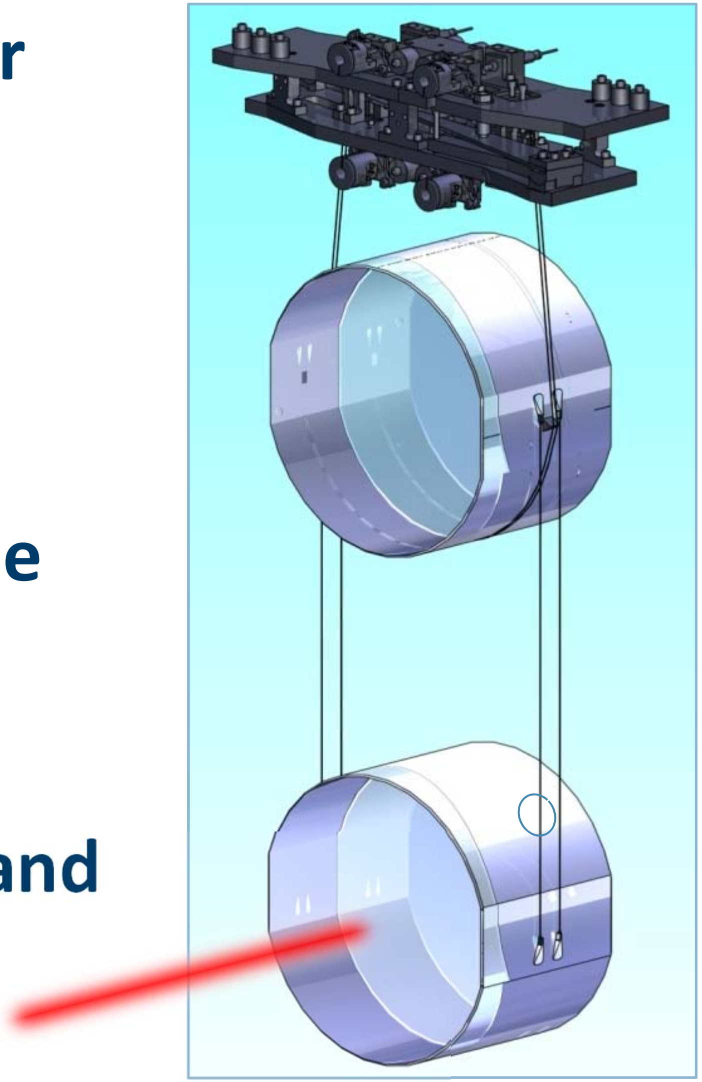
THERMAL NOISE IN REAL MIRRORS WITH INHOMOGENEOUS LOSS

Real Mirrors - Inhomogeneous loss

In reality, the presence of mirror coatings or at the attachment points of the suspension fibres means loss is spatially in-homogeneous

Instead, it is necessary to use the approach described by Levin

Allows the actual spatial distribution of mechanical loss and the detailed shape of the laser beam to be taken into account



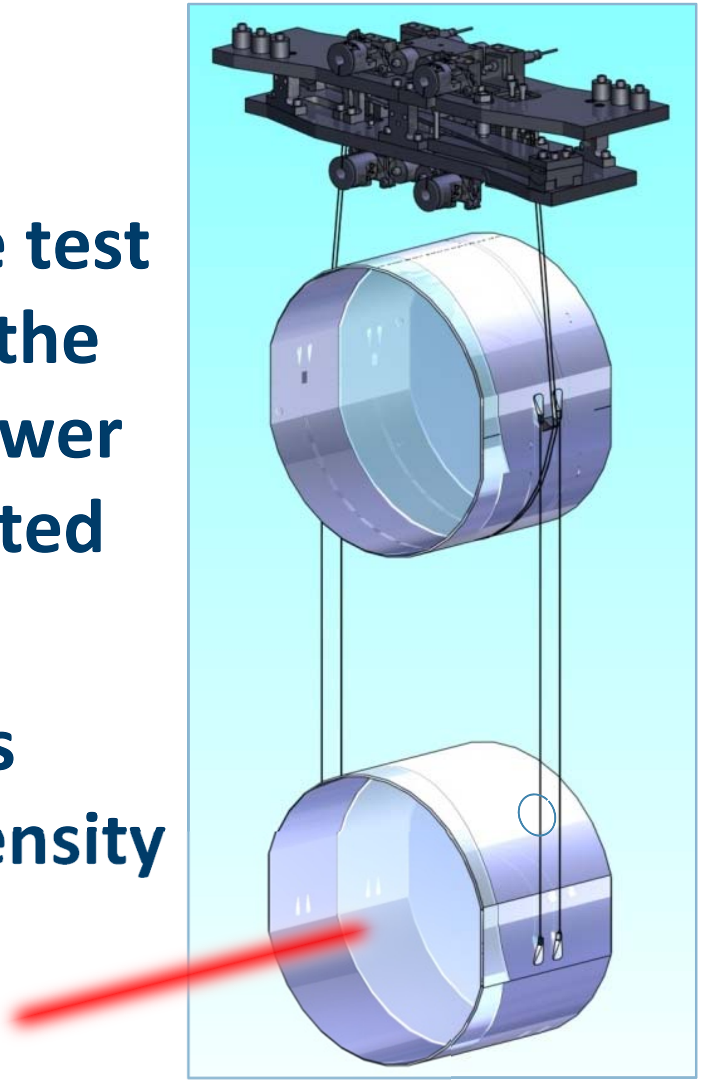
Laser incident on front surface of test mass mirror

Real Mirrors - Inhomogeneous loss

Levin considered the effect of applying a notional oscillating pressure to the front face of the test mass with the spatial profile of the laser beam and the resulting power dissipated in the mass is calculated

Application of the fluctuation dissipation theorem then allows calculation of power spectral density of thermal displacement

$$S_x(\omega) = \frac{8k_BT}{\omega^2}\frac{W_{\text{diss}}}{F_0^2}$$



Laser incident on front surface of test mass mirror

Real Mirrors - Inhomogeneous loss

$$S_x(\omega) = \frac{8k_BT}{\omega^2}\frac{W_{\mathrm{diss}}}{F_0^2}$$

F_0 is the peak amplitude of the notional oscillatory force

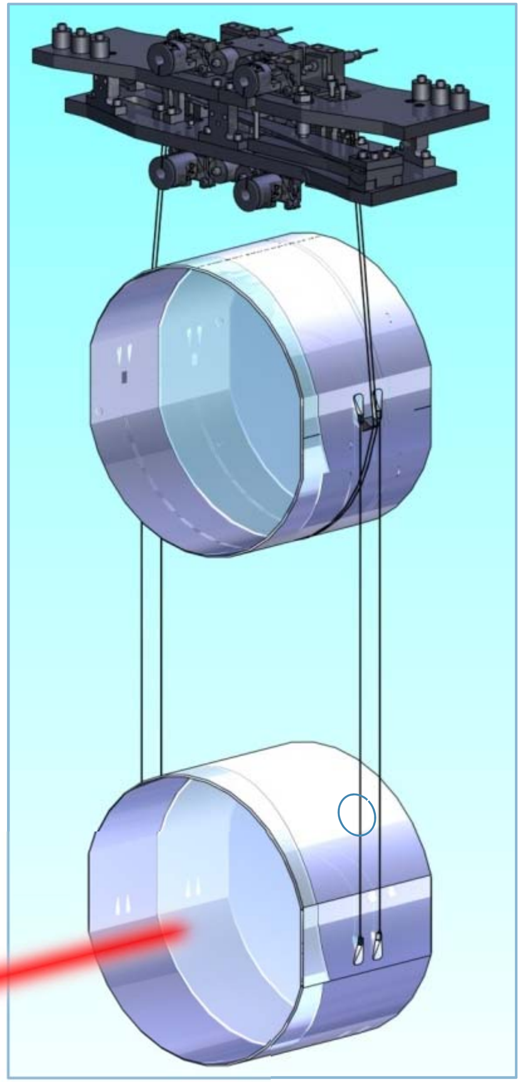
W_{diss} is the power dissipated in the mirror

The dissipated power is given by:

$$W_{\mathrm{diss}} = \omega \int_{\mathrm{vol}} \varepsilon(x,y,z)\phi(x,y,z,\omega)dV$$

ε is the energy density of the elastic deformation under the peak applied pressure

[In practise, more often calculated using Finite Element Analysis]



Laser incident on front surface of test mass mirror

Real Mirrors - Inhomogeneous loss

What does this mean for real mirrors?

$$W_{\text{diss}} = \omega \int_{\text{vol}} \varepsilon(x, y, z)\phi(x, y, z, \omega)dV$$

Location of loss is important

Deformation is biggest, closest to where the force is applied

Dissipation located close to the front surfaces of the mirrors contributes a greater level of thermal displacement noise than dissipation located far from the front face of the mirror

Therefore mirror coatings are very important!

Laser beam interacts with mirror coating

Loss of coating is ~1×10^{-4}

Compared to loss of mirror suspensions ~1×10^{-7}

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THERMAL NOISE IN A GW SUSPENSION

Suspension thermal noise

Thermally excited motion of the suspension fibres

Pendulum mode

Violin modes

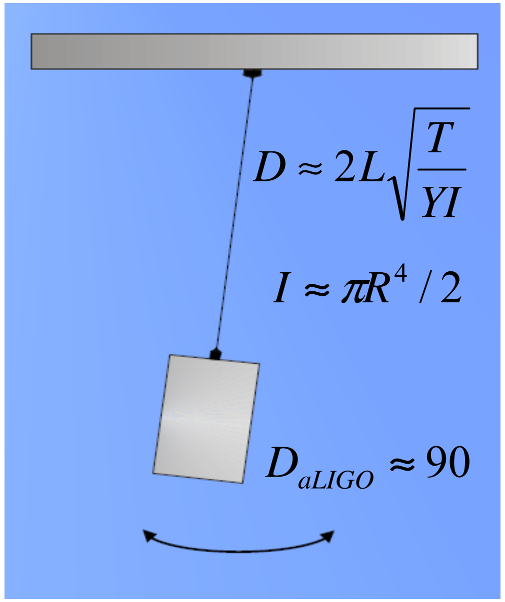
Vertical bounce mode

In pendulum, some of pendulum energy stored in gravity

Gravity is lossless, elasticity is lossy

These effects lead to dilution

$$\phi_{\text{observed}} = \frac{E_{\text{elastic}}}{E_{\text{total}}}\phi_{\text{material}} = \frac{1}{D}\phi_{\text{material}}$$

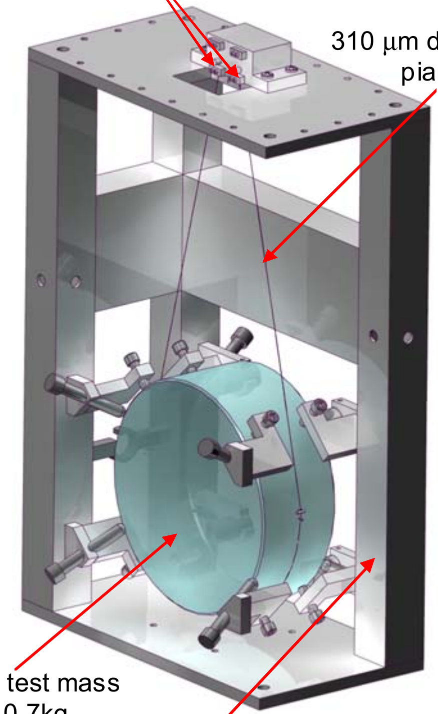


Storing more energy in gravity requires thin suspension fibres

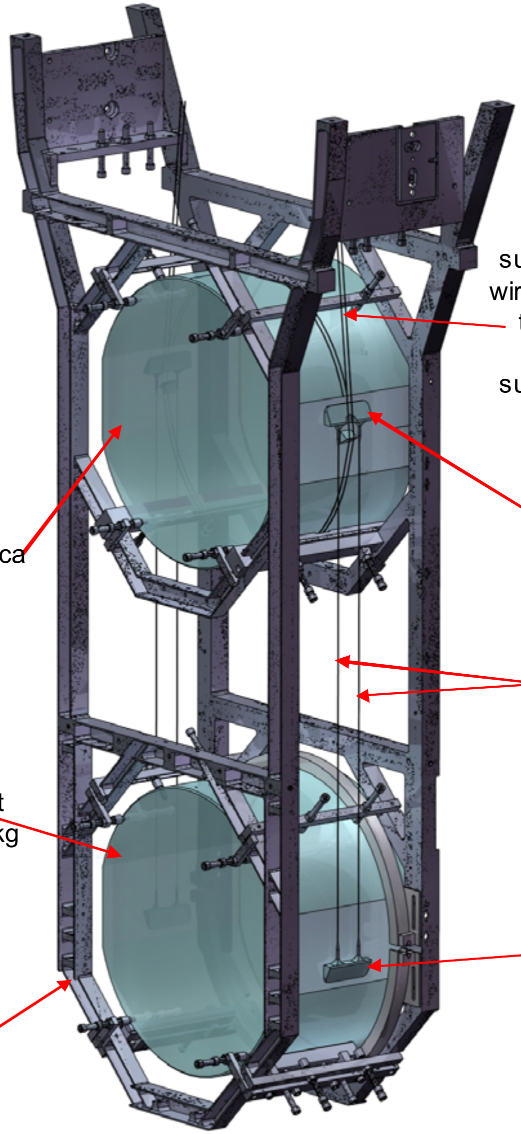
Bulk, surface, weld & thermoelastic losses are used in FEA to model fused silica suspension fibres

Initial LIGO and aLIGO

LIGO

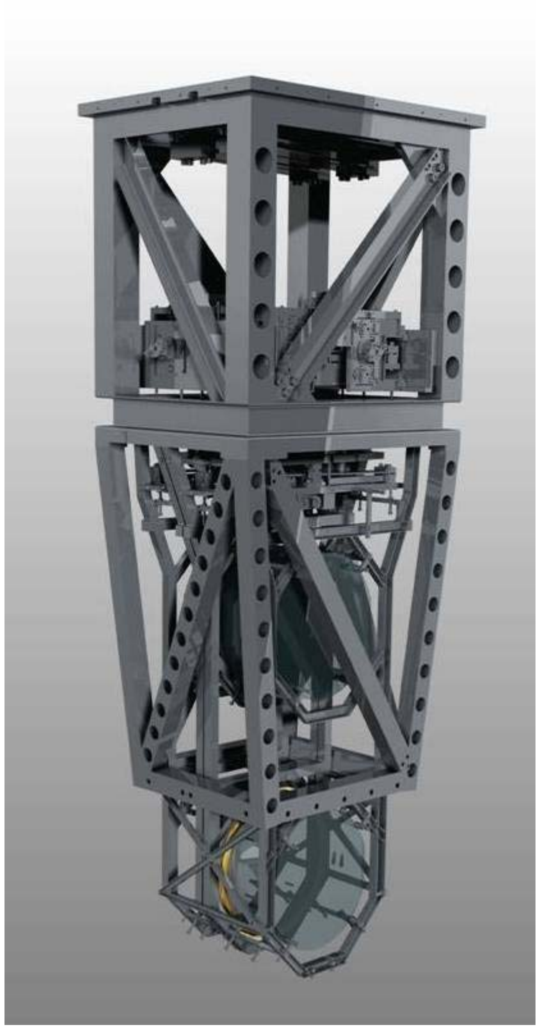


Advanced LIGO



aLIGO Quadruple Suspension

- The input test masses (ITM) and end test masses (ETM) of Advanced LIGO will be suspended via a quadruple pendulum system
- **Seismic isolation:** use quadruple pendulum with 3 stages of maraging steel blades for horizontal/vertical isolation
- **Thermal noise reduction:** monolithic fused silica suspension as final stage
- **Control noise minimisation:** use quiet reaction pendulum for global control of test mass position
- **Actuation:** Coil/magnet actuation at top 3 stages, electrostatic drive at test mass



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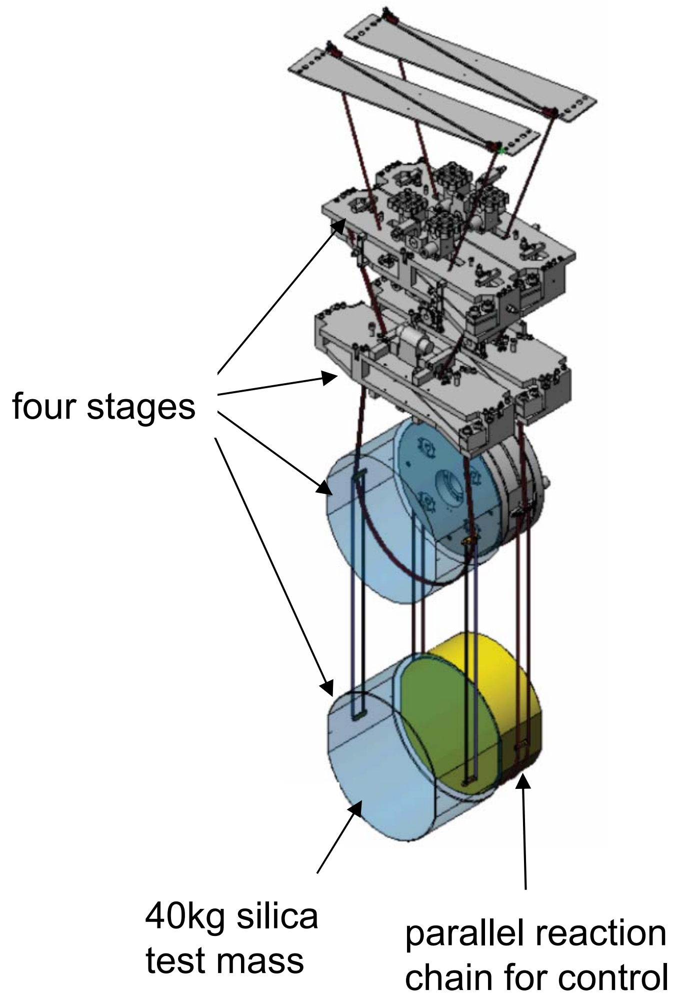
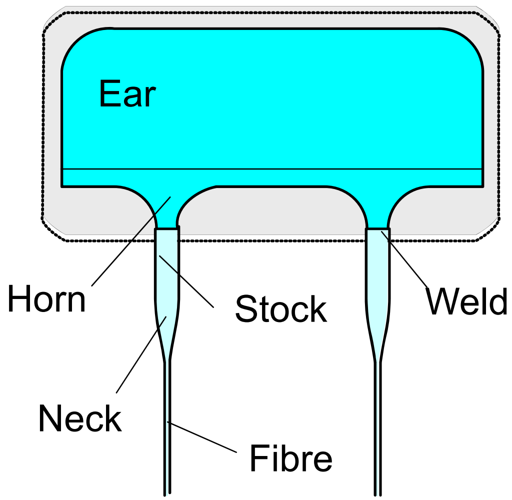
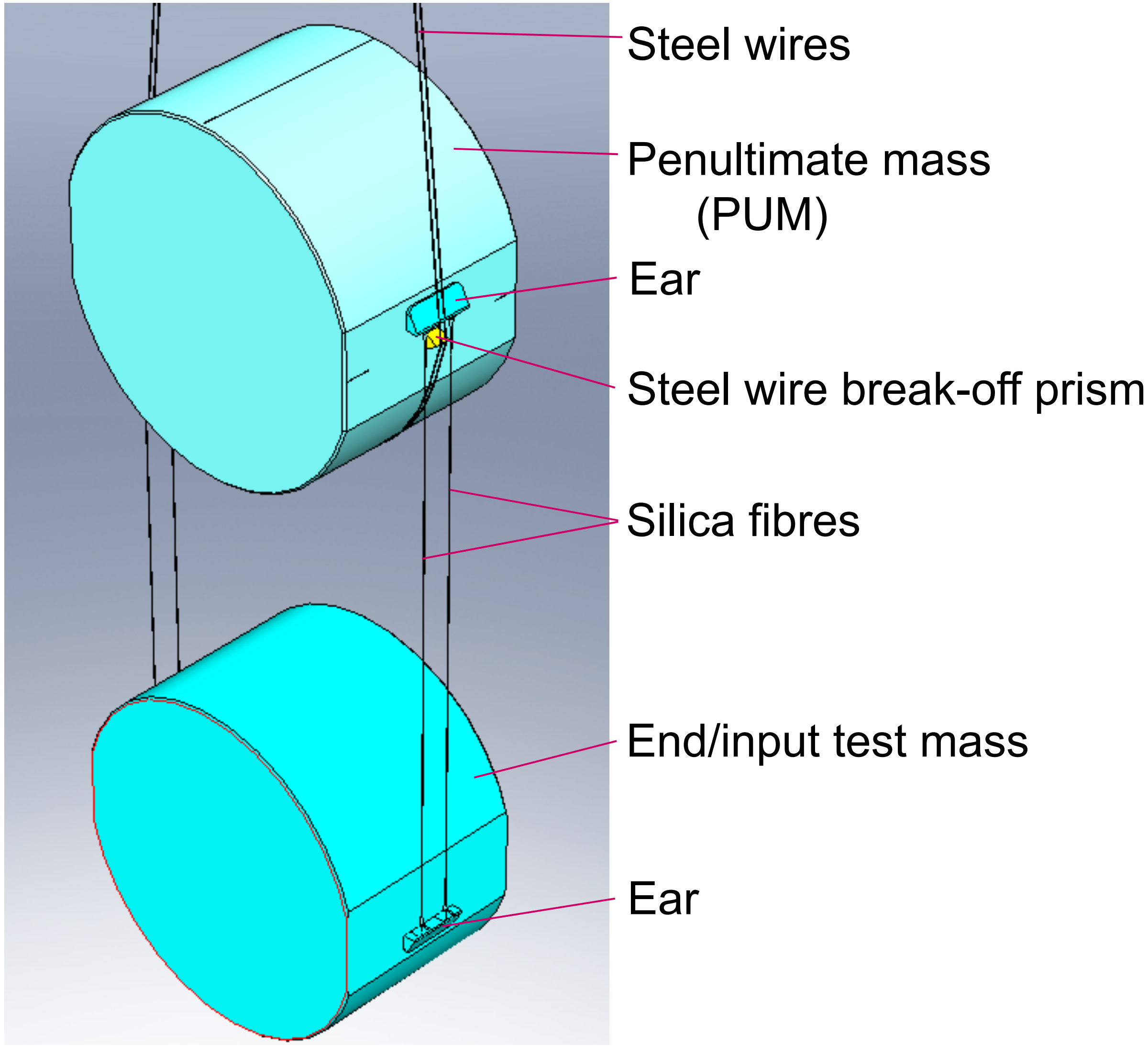
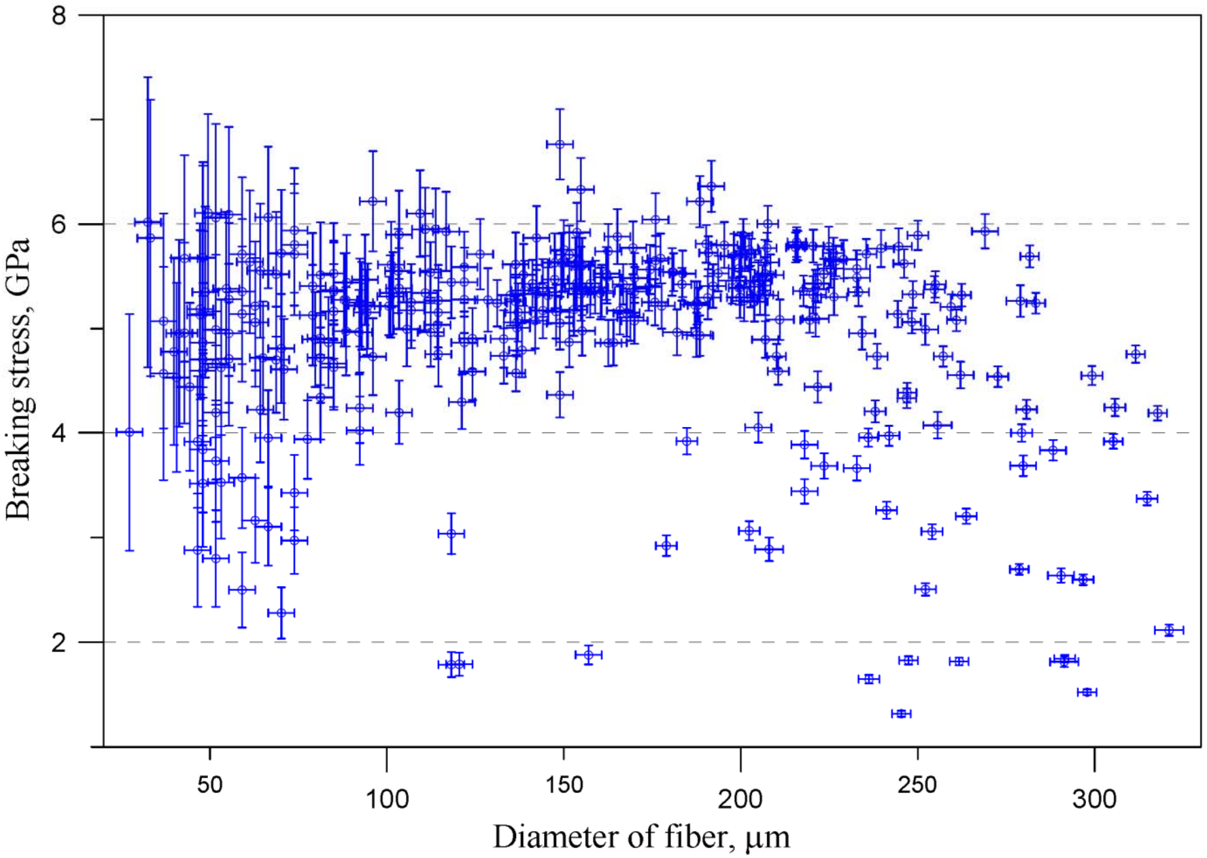


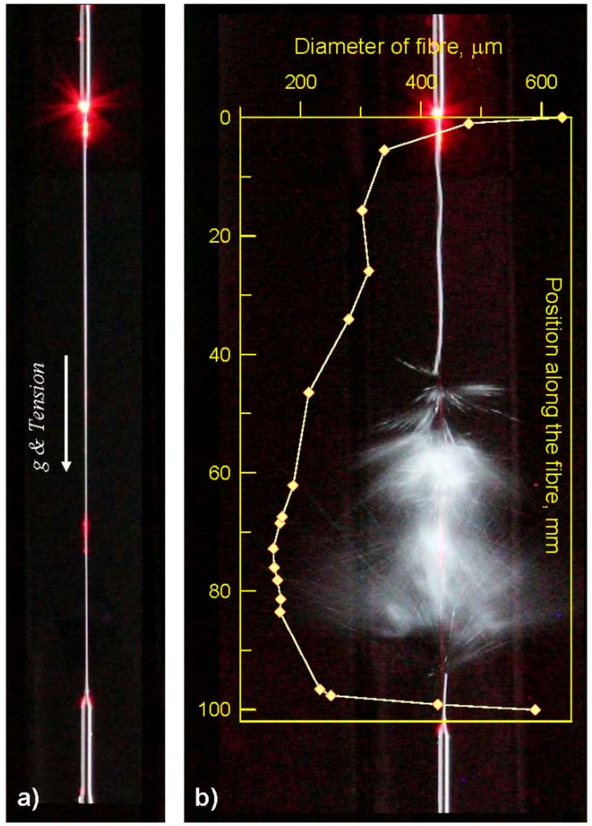
aLIGO Monolithic Stage





Fibre Strength





Fused Silica

Fused silica is a remarkable material: high strength, low mechanical loss, can be welded and drawn into fibres

$$Y = \frac{\sigma}{\varepsilon}$$

With 10kg on a 400 μm diameter, 60 cm long fibre:

$$\sigma = 10 \times 9.81 / \pi \times \left(400 \times 10^{-6}\right)^2 = 780 MPa$$

and the extension is

$$\Delta L = \varepsilon \times L = \left[\frac{\sigma}{Y}\right] \times L = \left[\frac{780 \times 10^6}{72 \times 10^9}\right] \times 0.6 = 6 mm$$

- The strain is 1% at this load !!!

Useful Equations

$$x^2_{thermal} = \frac{4k_B T k\phi}{\omega m^2\left(\left(\omega_0^2-\omega^2\right)^2+\left(\phi\omega_0^2\right)^2\right)}$$

Thermal displacement noise

$$S_x(\omega) = \frac{4k_\mathrm{B}T}{\omega^2}\Re\left[\frac{1}{Z(\omega)}\right]$$

Fluctuation Dissipation theorem

$$\frac{\Delta\omega}{\omega_0} = \phi = \frac{1}{Q}$$

Quality factor

$$D = \frac{E_\mathrm{total}}{E_\mathrm{elastic}} \approx \frac{k_\mathrm{gravity}}{k_\mathrm{fibre}} \approx 2L\sqrt{\frac{T}{YI}}$$

Dissipation dilution (in horizontal direction)

$$\phi_\mathrm{surface} \approx \frac{8h\phi_s}{d}$$

$$\phi_\mathrm{TE}(\omega) = \frac{YT}{\rho C}\left(\alpha-\sigma_o\frac{\beta}{Y}\right)^2\left(\frac{\omega\tau}{1+(\omega\tau)^2}\right)$$

Dominant fibre loss term (thermoelastic is minimised with stress cancellation)

Other useful equations

$$f_{pendulum} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$f_{bounce} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

m=mass on 1 fibre

$$f_{free\_test\_mass} = \sqrt{2}f_{bounce}$$

$$I = \pi R^4/2$$

$$Y = \frac{\sigma}{\varepsilon}$$

$$\Delta L = \left[\frac{\sigma}{Y}\right]L$$

Suspension Thermal Noise

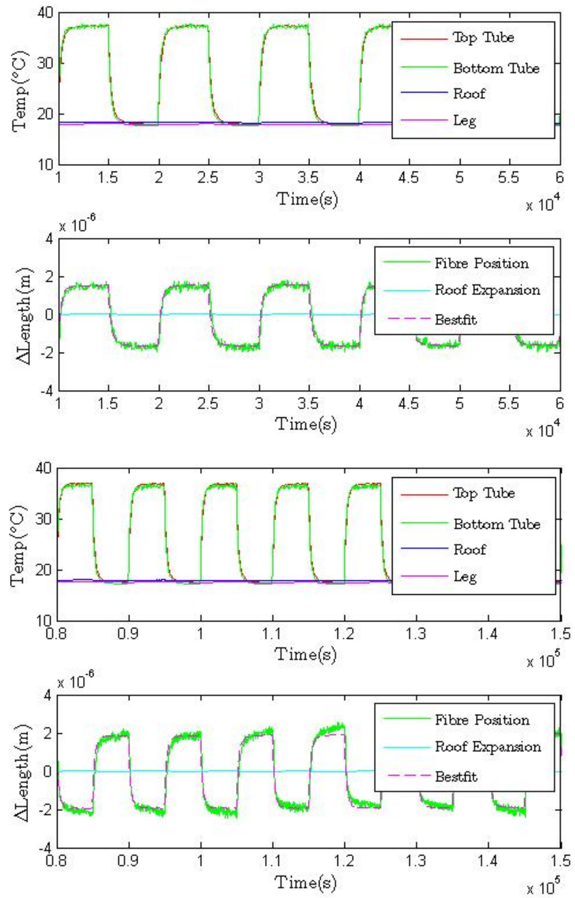
- aLIGO utilises thermoelastic cancellation to meet the noise requirement of 10^{-19} m/√Hz at 10Hz.
- A fused silica fibre has zero thermal expansion coefficient at a stress of 200MPa => no thermoelastic contribution

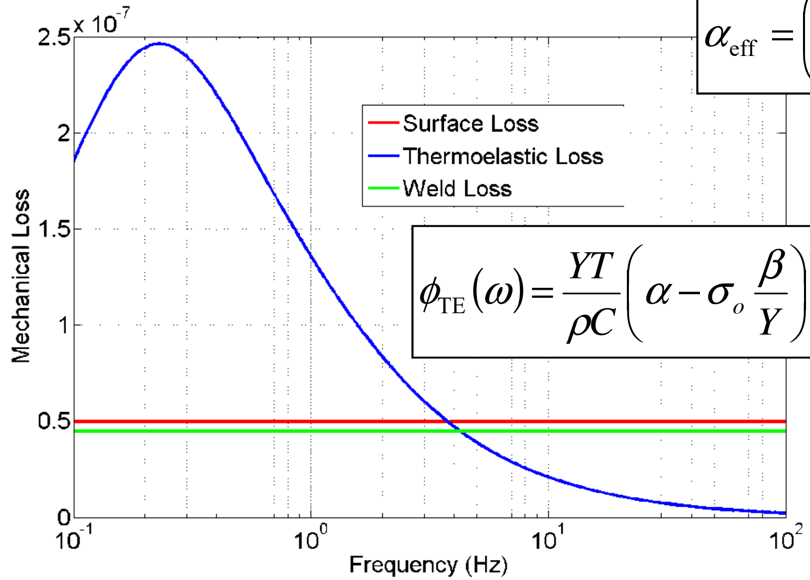
$$\alpha_{\text{eff}} = \left(\alpha - \sigma_o \frac{\beta}{Y}\right)$$

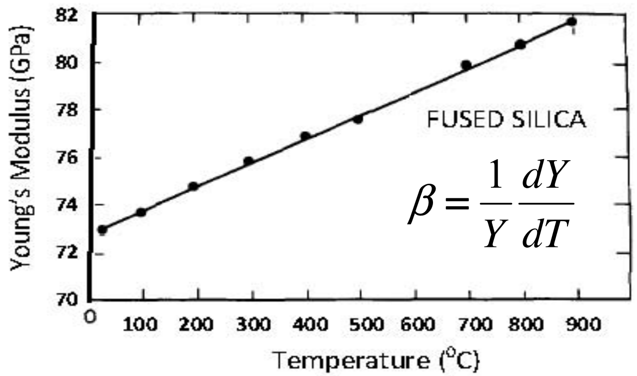
$$\phi_{\text{TE}}(\omega) = \frac{YT}{\rho C}\left(\alpha - \sigma_o \frac{\beta}{Y}\right)^2 \left(\frac{\omega\tau}{1+(\omega\tau)^2}\right)$$

Expansion at low stress

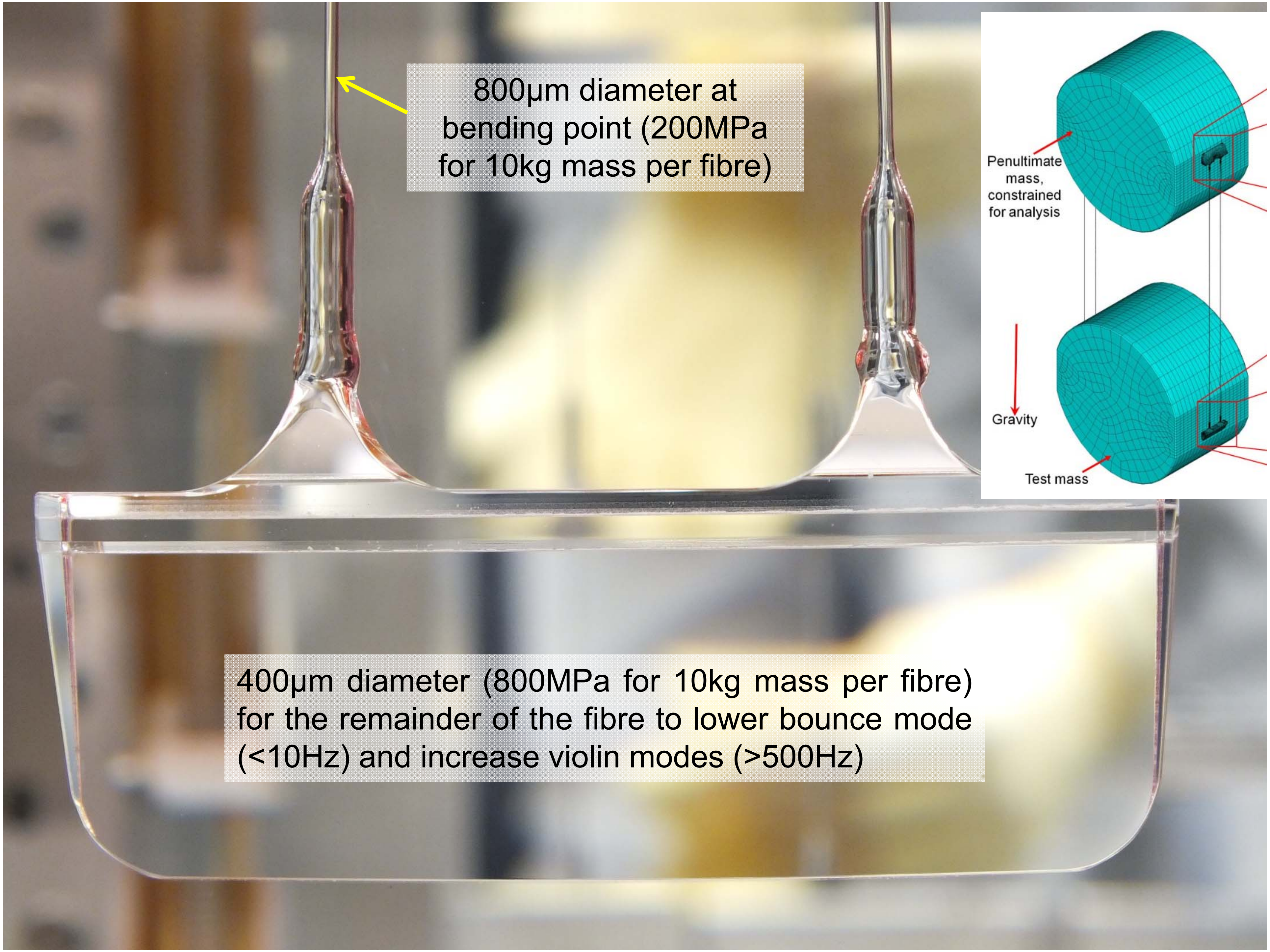
Contraction at high stress



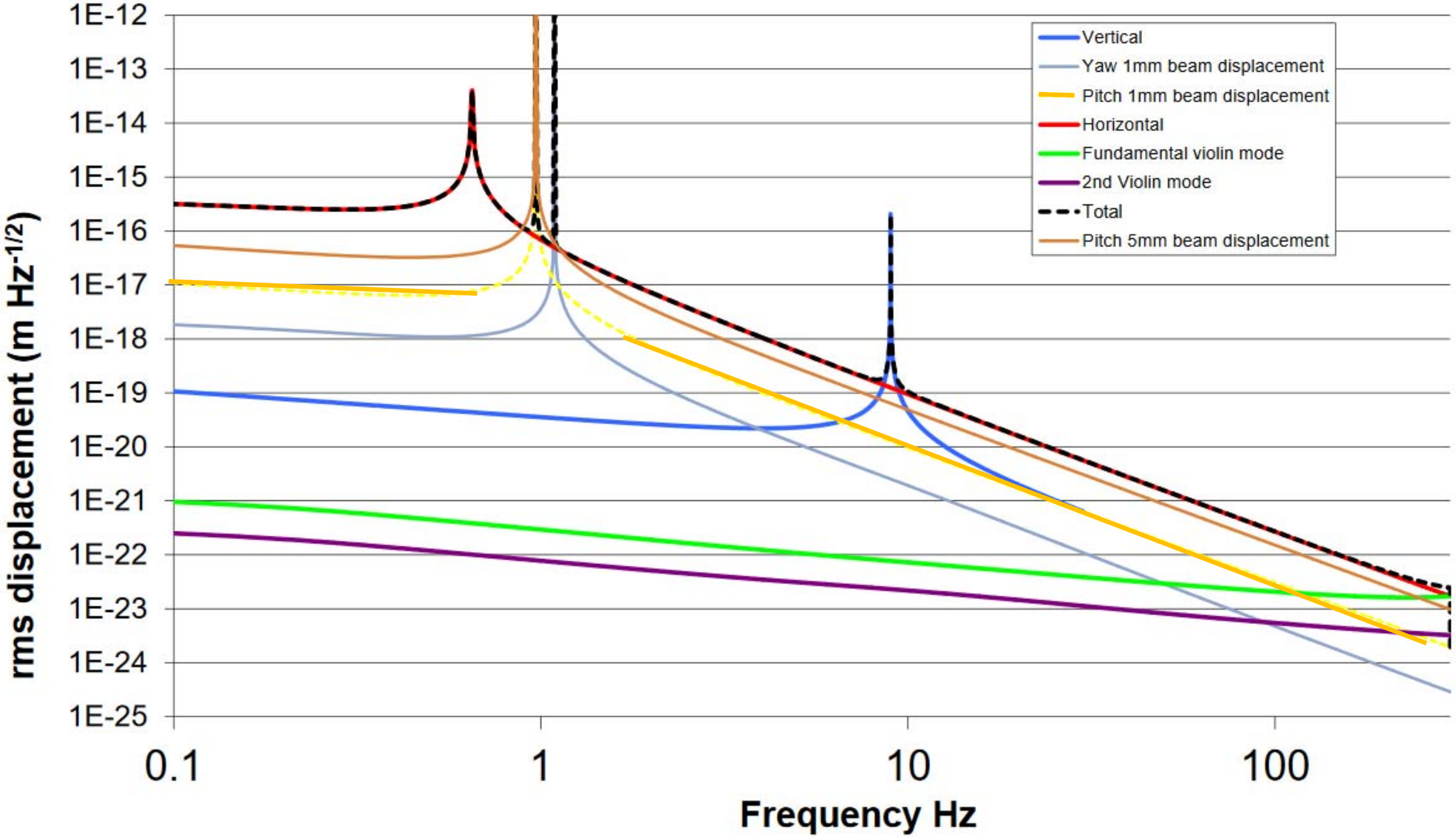




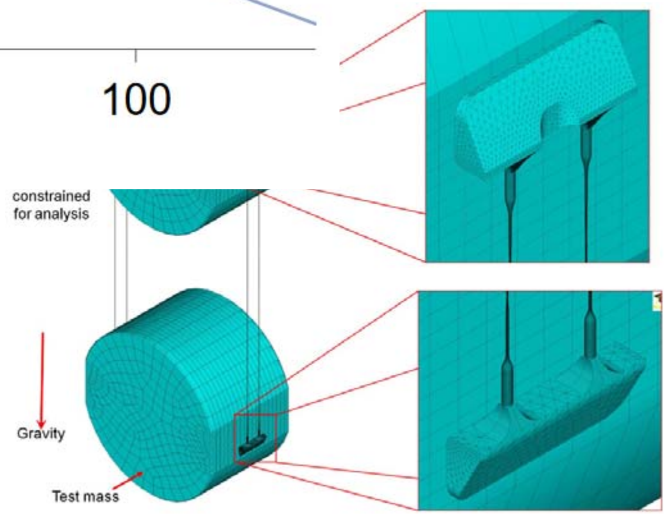
$$\beta = \frac{1}{Y}\frac{dY}{dT}$$


800µm diameter at bending point (200MPa for 10kg mass per fibre)
Penultimate mass, constrained for analysis
Gravity
Test mass
400µm diameter (800MPa for 10kg mass per fibre) for the remainder of the fibre to lower bounce mode (<10Hz) and increase violin modes (>500Hz)

aLIGO Suspension Thermal Noise



- FEA models to estimate final stage thermal noise
- Mathematica models to predict the transmissibility of all stages/transfer functions



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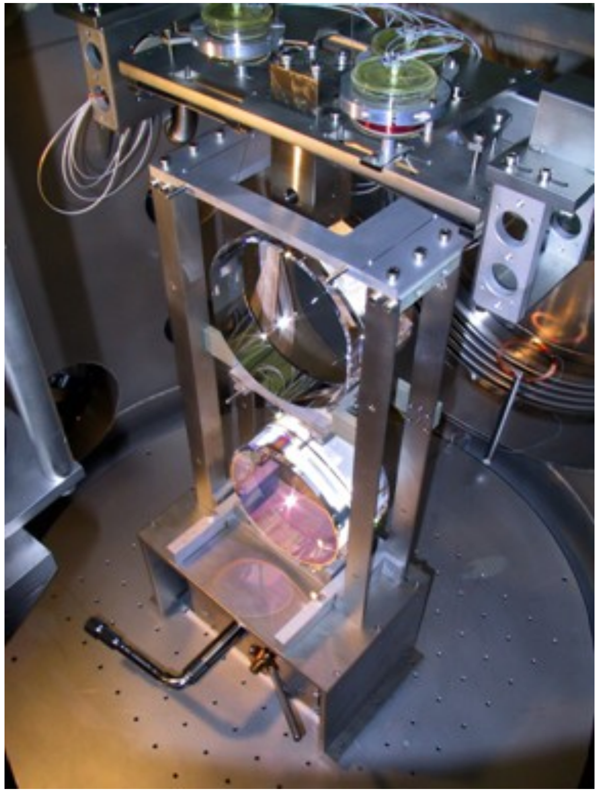
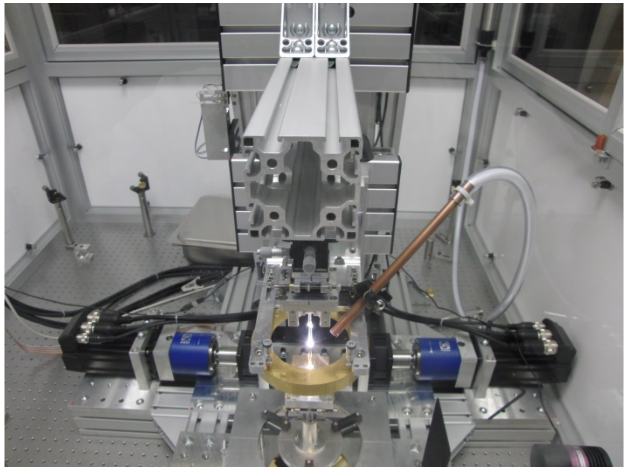
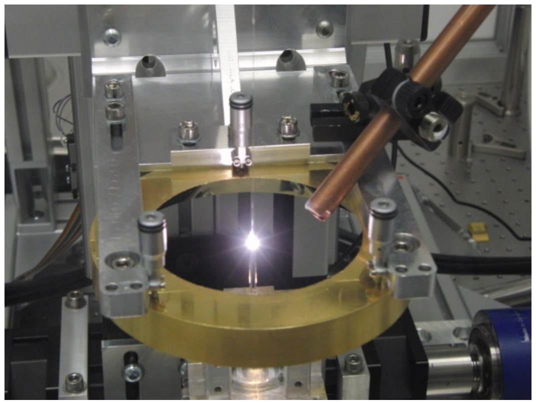
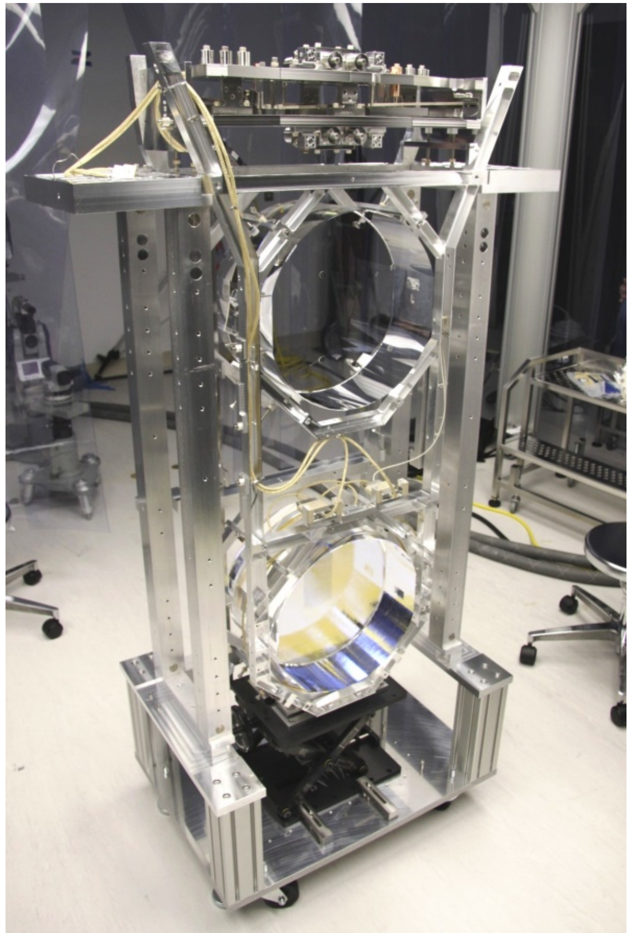
Monolithic Suspensions

- Monolithic suspensions & signal recycling pioneered in GEO-600 → upscaled to aLIGO









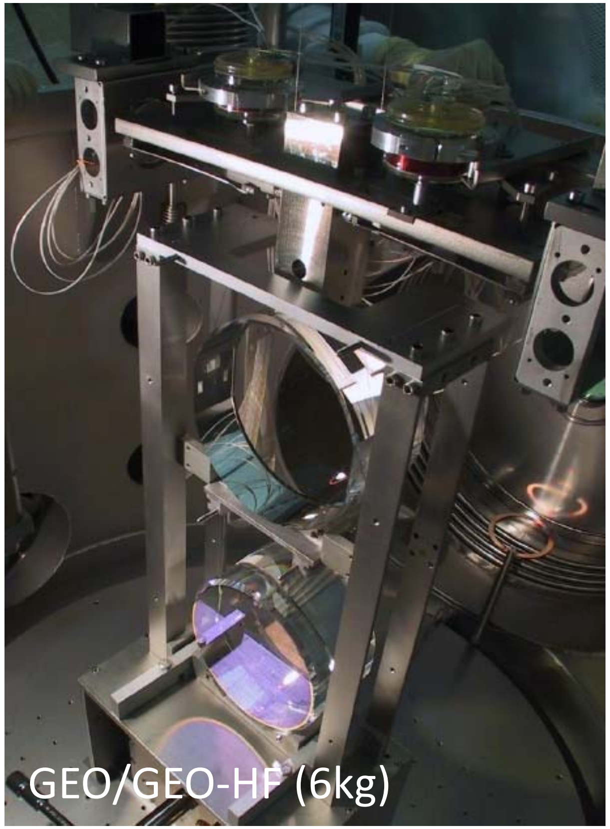
IGR advancedligo

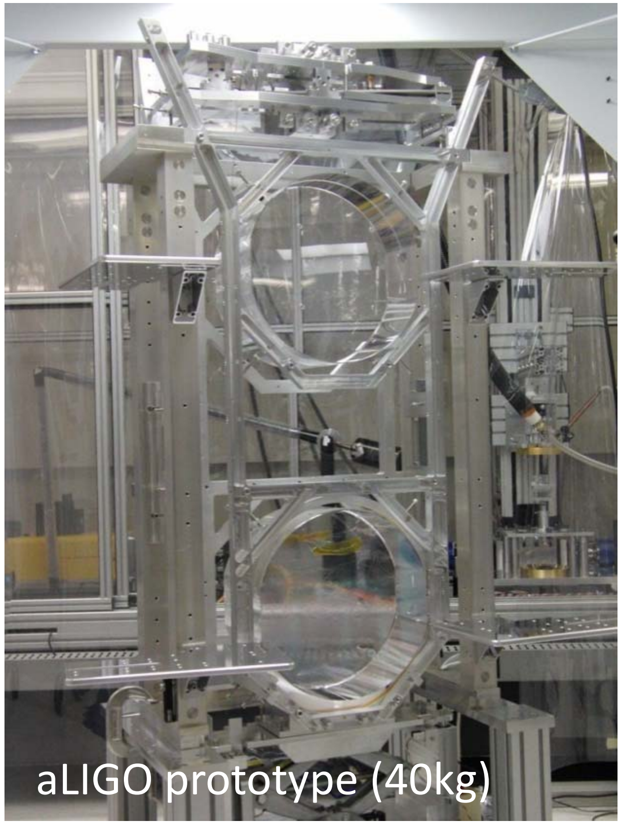
Monolithic Suspension

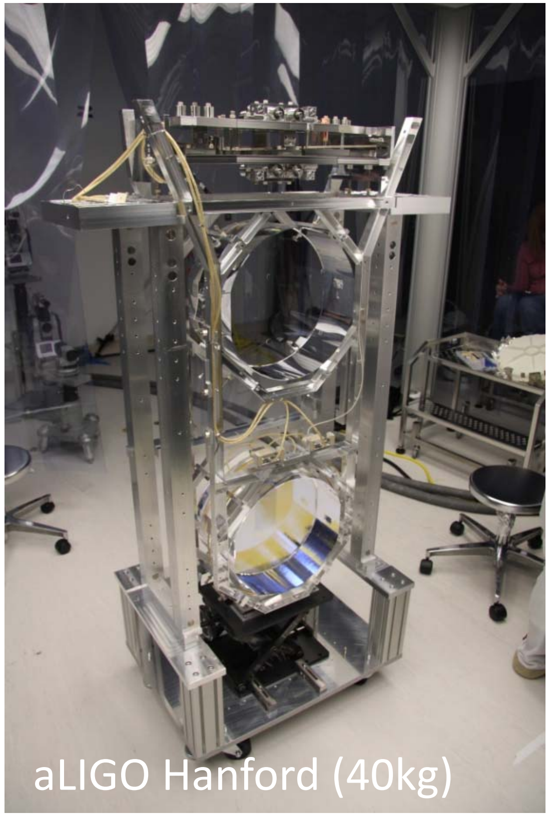
Require TN performance of $10^{-19}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 10 Hz

Chosen to be equal to the seismic noise floor at 10Hz

This is achieved with monolithic fused silica suspensions







TOTAL THERMAL NOISE IN A DETECTOR MIRROR

Total Thermal Noise in a Detector Mirror

Substrate thermal noise

Due to intrinsic mechanical loss of the test mass material

Due to thermoelastic effects

Here arising from temperature fluctuations in the test mass

Coating thermal noise

Due to the intrinsic mechanical loss of the coating materials

Due to thermoelastic effects

Here arising from temperature fluctuations and the differing thermodynamic properties of the coating and substrate

Suspension thermal noise

Due to the pendulum mode

Due to violin modes of the suspension fibres

SUMMARY – THERMAL NOISE IN GRAVITATIONAL WAVE DETECTORS

Summary – Thermal Noise in Gravitational Waves detectors

Fluctuating thermal energy

Brownian thermal noise

Fluctuating temperature

Thermo-elastic, thermo-refractive noise

Summary – Thermal Noise in Gravitational Waves detectors

Use low loss materials for mirrors and suspensions

This will minimise Brownian TN in detection band

E.g. fused silica mirror substrates, fused silica suspension fibres

Coating Brownian TN expected to limit performance of 2nd generation detectors at their most sensitive frequencies (~50 – 100 Hz)

Due to relatively high loss of coatings

Due to proximity of coatings to reflected laser beam

Suspension thermal noise is dominant thermal noise at lower frequencies (<30 Hz)

Reduce thermal noise by

Reducing loss

Reducing temperature

Increasing beam radius (for coatings)

EXTRA SLIDES

Thermoelastic Cancellation

- Consider a fibre under a static stress σ_0
- For a temperature change, ΔT, the strain due to the coefficient of linear thermal expansion, α and the change of the Young's modulus with temperature is

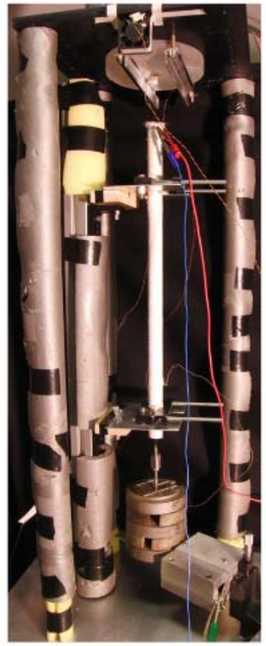
$$\frac{\Delta L}{L} = \alpha \Delta T + \frac{\sigma_o}{Y} = \alpha \Delta T + \frac{\sigma_o}{Y_0 + \frac{dY}{dT}\Delta T}$$

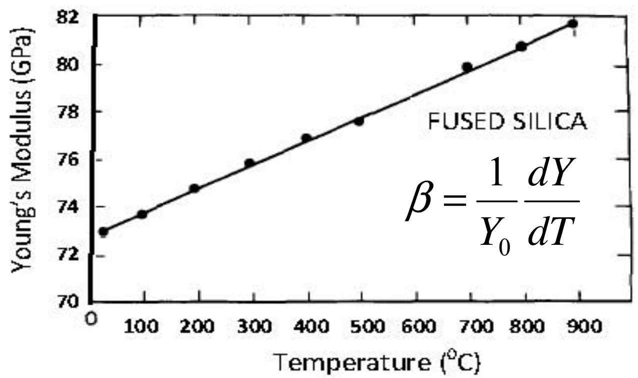
- The term dY/dT allows for a temperature dependent Young's modulus. Rearranging to 1st order gives

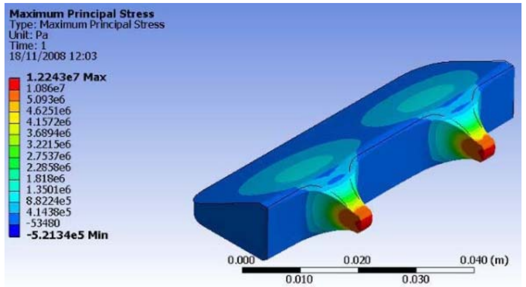
$$\frac{\Delta L}{L} = \alpha \Delta T + \frac{\sigma_o}{Y_0}\left(1 - \frac{1}{Y_0}\frac{dY}{dT}\right)\Delta T = \alpha \Delta T + \frac{\sigma_o}{Y_0}(1-\beta)\Delta T$$

$$\alpha_{eff} = \left(\alpha - \sigma_o \frac{\beta}{Y_0}\right)$$

$$\boxed{\phi_{\mathrm{TE}}(\omega) = \frac{YT}{\rho C}\left(\alpha - \sigma_o \frac{\beta}{Y}\right)^2 \left(\frac{\omega\tau}{1+(\omega\tau)^2}\right)}$$





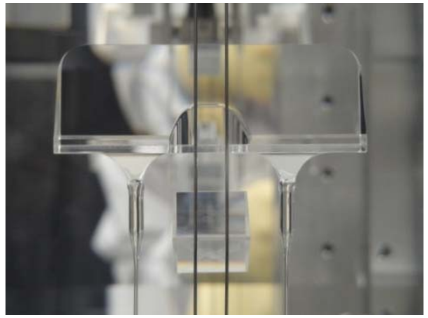


Suspension Thermal N

- Use the following loss terms to model the welds, ear horns and fibres

$\phi_{\text{bulk}} = 1.2\times10^{-11} f^{\,0.77}$	$\phi_{\text{TE}}(\omega) = \dfrac{YT}{\rho C}\left(\alpha - \sigma_o \dfrac{\beta}{Y}\right)^2 \left(\dfrac{\omega\tau}{1+(\omega\tau)^2}\right)$
$\phi_{\text{surface}} \approx \dfrac{8h\phi_s}{d}$	$\phi_{\text{weld}}(\omega) = 5.8\times10^{-7}$

$$\phi_i(\omega) = \left(\phi_{\text{bulk,i}}(\omega) + \phi_{\text{TE,i}}(\omega) + \phi_{\text{surface,i}}(\omega) + \phi_{\text{weld,i}}(\omega)\right)$$



$$\phi_{total}(\omega) = \frac{1}{D}\left[\frac{E_1}{E_{\text{elastic}}}\phi_1(\omega) + \frac{E_2}{E_{\text{elastic}}}\phi_2(\omega) + \ldots + \frac{E_{\text{n}}}{E_{\text{elastic}}}\phi_{\text{n}}(\omega)\right]$$

$$S_x(\omega) = \frac{4k_BT}{m\omega}\left(\frac{\omega_o^{\,2}\phi_{total}(\omega)}{\omega_o^{\,4}\phi_{total}^{\,2}(\omega) + \left(\omega_o^{\,2} - \omega^2\right)^2}\right)$$

- Surface loss: dislocations, un-terminated dangling bonds and micro-cracks on the pristine silica surface
- Thermoelastic loss: heat flow across the fibre due to expansion/contraction leads to dissipation.
- Bulk loss: strained Si-O-Si bonds have two stable minima which can redistribute under thermal fluctuations.

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