Quantum entanglement in $H \to WW^*$ decays

Constructing qutrit Bell tests in the ATLAS detector | NNV meeting

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Outline

Quantum entanglement

Analysis at truth-level

Rest frame reconstruction and results

Conclusion

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Rest frame reconstruction and results

Conclusion

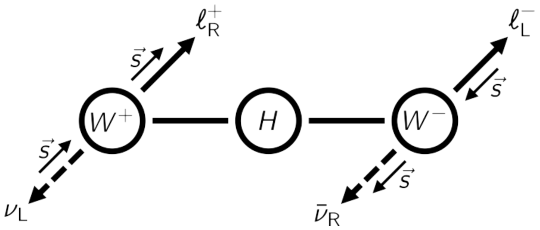
Quantum entanglement in $H \to WW^*$

Bell's theorem

"If (a hidden-variable theory) is local it will not agree with quantum mechanics, and if it agrees with quantum mechanics it will not be local."

- Bell, John S. (1987). *Speakable and Unspeakable in Quantum Mechanics*

- Study "spooky action" at TeV scale
- Construct a Bell test in $H \to WW^*$
- Exploit Higgs J^{CP} and the $V - A$ structure of weak interaction
- Verify entanglement in spin-1 WW pair from Higgs decay



From theory to experiment

$$\rho_{WW} = \frac{1}{9}\mathbb{I}_9 + \sum_{j=1}^{8} a_j \lambda_j \otimes \frac{1}{3}\mathbb{I}_3 + \sum_{k=1}^{8} \frac{1}{3}\mathbb{I}_3 \otimes b_k \lambda_k + \sum_{j=1}^{8}\sum_{k=1}^{8} c_{jk} \lambda_j \otimes \lambda_k$$

- **Weyl-Wigner transformation**: Quantum Tomography
  Invertible mapping between Hilbert space operators and phase space functions.
- [**Theory**] Forward mapping: *Operator* (ρ) $\rightarrow$ *Angle* ($\Phi_\rho^Q(\hat{\mathbf{n}})$) Wigner Q symbol
  Construct operators $\rightarrow$ obtain angular distribution
- [**Experiment**] Reverse mapping: *Angle* ($\Phi_\rho^P(\hat{\mathbf{n}})$) $\rightarrow$ *Operator* (ρ) Wigner P symbol
  Measure lepton angular distribution in W rest frame $\rightarrow$ obtain operators

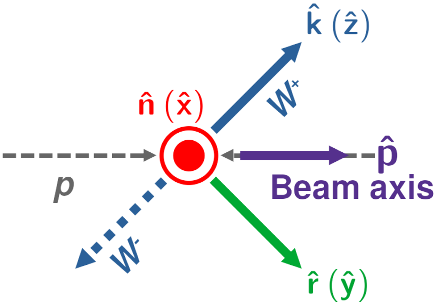
$$a_j, b_j \Leftarrow \hat{a}_j, \hat{b}_j = \frac{1}{2}\left\langle \Phi_j^P(\hat{\mathbf{n}}) \right\rangle \qquad c_{jk} \Leftarrow \hat{c}_{jk} = \frac{1}{4}\left\langle \Phi_j^P(\hat{\mathbf{n}}_1)\, \Phi_k^P(\hat{\mathbf{n}}_2) \right\rangle$$

Bell test in $H \to WW^*$

- Construct coordinate system in Higgs **center-of-momentum (CM) frame**
  Projective measurements in W CM frame
- **CGLMP Bell operator** ⏩ CGLMP inequality
  Optimal Bell test for qutrit pairs

$$\left\langle \mathcal{B}^{ij}_{\text{CGLMP}} \right\rangle = \frac{8}{\sqrt{3}} \left\langle \xi_i^+ \xi_i^- + \xi_j^+ \xi_j^- \right\rangle + 25 \left\langle [(\xi_i^+)^2 - (\xi_j^+)^2][(\xi_i^-)^2 - (\xi_j^-)^2] \right\rangle + 100 \left\langle \xi_i^+ \xi_j^+ \xi_i^- \xi_j^- \right\rangle \quad (i,j \in \{x,y,z\},\ i \neq j)$$

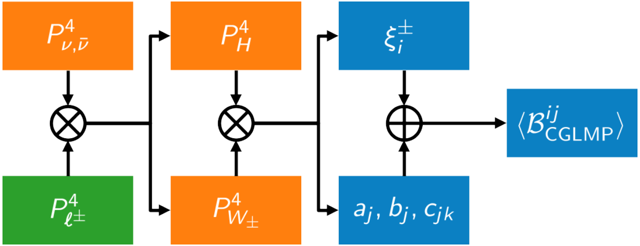
- $\max\left\{\langle \mathcal{B}^{ij}_{\text{CGLMP}} \rangle\right\} > 2$ "Entanglement"
  Exceeding classical bound



- $\xi_i^{\pm} = \hat{\mathbf{e}}_i \cdot \hat{\ell}^*_{\pm}$, $\hat{\mathbf{e}}_i \in \{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\}$
- Denote $\hat{\ell}^*_{\pm}$ as $\ell_{\pm}$ direction in parent $W^{\pm}$ boson CM frame

Approaches to quantum entanglement observables

- **Unknowns**: Neutrino, $W^{\pm}$, Higgs kinematics
- **Knowns**: Lepton kinematics, Missing Transverse Energy (MET)



- CM frame reconstruction is essential to extract W spin information from final-state leptons
- **Straightforward approach**: Regression on missing kinematic information. Calculate CM frame observables using estimated values.

- **Target**: Observables in CM frame, ensemble average

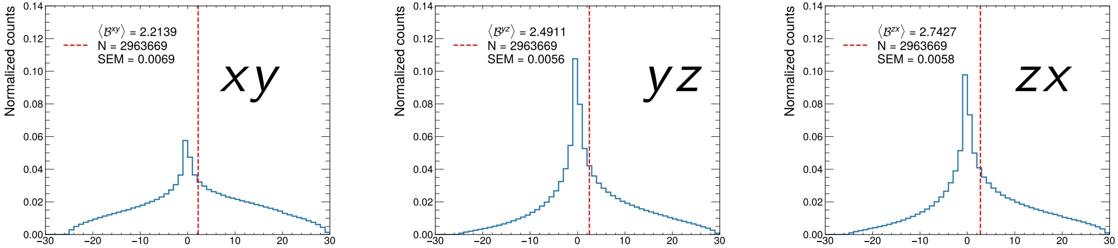
Outline

Overview of truth-level study

- **Motivation**
  - Verify theory using standard ATLAS simulation sample
  - Understand effects caused by experimental selection cuts
- **Truth-level study: Full knowledge of neutrinos, $W^{\pm}$, and Higgs four-momentum**
- **Generator**: `Powheg + Pythia8 (v8.212) + EvtGen (v1.2.0)`
- **Event Selection**: (ggF) $H \to WW^* \to \ell^+ \, \nu_\ell \, \ell^- \, \bar{\nu}_\ell$
  - **Different flavor channel**: $\ell \in \{e, \mu\}$
  - **Baseline selection**: $|\eta_\ell| < 5$, $p_{\mathrm{T}}^{\ell_{0(1)}} > 15\ (5)$ GeV ($\ell_{(1)0}$ denotes (sub)leading lepton)

Distribution of CGLMP Bell operator values ($\mathcal{B}^{xyz}_{\text{CGLMP}}$)

$$\left\langle \mathcal{B}^{ij}_{\text{CGLMP}} \right\rangle = \frac{8}{\sqrt{3}} \left\langle \xi_i^+ \xi_i^- + \xi_j^+ \xi_j^- \right\rangle + 25 \left\langle [(\xi_i^+)^2 - (\xi_j^+)^2][(\xi_i^-)^2 - (\xi_j^-)^2] \right\rangle + 100 \left\langle \xi_i^+ \xi_j^+ \xi_i^- \xi_j^- \right\rangle$$



- $\max\{\langle \mathcal{B}^{ij}_{\text{CGLMP}} \rangle\}$ observed in $ij = zx$, with expectation value of 2.74
- Referenced paper quotes 2.62 (no cuts) $\sim$ 2.75 (comparable cuts) arXiv:2106.01377

Selection cut definition

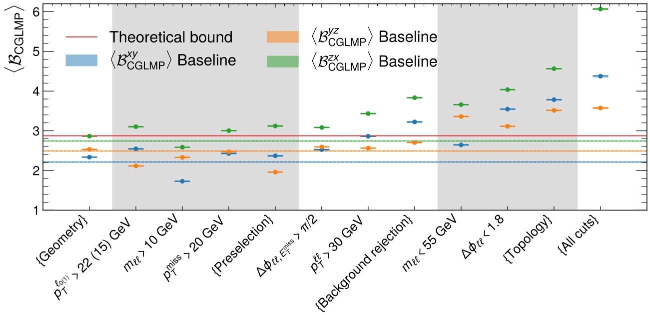
Referencing the Run 2 ggF $H \to WW^* \to e\nu\mu\nu$ signal region ▸ arXiv:2207.00338 (Table 2)

- **Geometry**:
  Detector coverage
- **Preselection**:
  Enhances hard scattering events
  Reduces multiple background sources
- **Background rejection**:
  Transverse plane kinematic requirements
- **Event topology**:
  Enhances resonant $H \to WW^*$ events

Category	Criteria
Geometry	$\lvert\eta_\ell\rvert < 2.5$ $\lvert\eta_e\rvert \notin [1.37, 1.52]$
Preselection	$p_T^{\ell_{0(1)}} > 22\ (15)$ GeV $m_{\ell\ell} > 10$ GeV $p_T^{\text{miss}} > 20$ GeV
Background rejection	$\Delta\phi_{\ell\ell, E_T^{\text{miss}}} > \pi/2$ $p_T^{\ell\ell} > 30$ GeV
Event Topology	$m_{\ell\ell} < 55$ GeV $\Delta\phi_{\ell\ell} < 1.8$

Overview on selection cut effects

- Dashed line: Sample mean at baseline (projection axis)
- Red line: Theoretical upper bound
  $\rightarrow 4/(6\sqrt{3}-9) \simeq 2.87$
- Negligible *Standard Error of the Mean* (SEM)
  $\rightarrow$ Sufficiently large sample



- $\langle \mathcal{B}^{zx}_{\text{CGLMP}} \rangle$ is consistently the greatest among the combinations
- Selection cuts bias expectation values to greater values [Exact values]

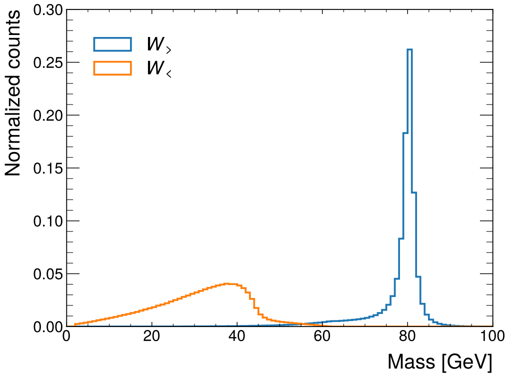
Outline

Overview of reconstruction-level study

- **Motivation**: To emulate practical setting
  - Implement neural network regressor in CM frame estimation
  - Account for detector effects
- **Objective**: Reconstruct four-momentum of intermediate particles H, W_0, W_1
- Reconstruction-level sample implemented
- Geometry and preselection cuts applied ($N = 590,902$)

Association with leading lepton

- SM $H \to WW^*$ produces off-shell W^*



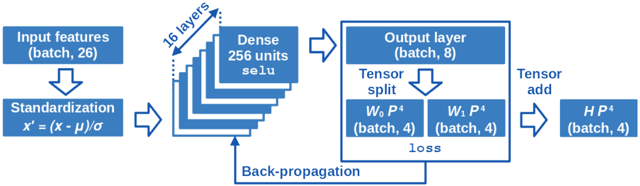
- (At baseline) 83% of ℓ_0 decayed from the *more massive* W boson
- Analytical Approximation (AA)
  - On-shell conditions for ℓ_0 constraint equations
- Neural Network (NN)
  - Possibility of on-shell W boson mass as implicit constraint
- Notation:
  - $\ell_{(1)0}$: lepton (sub)leading in p_T
  - Subscript (1)0 denotes association with the (sub)leading lepton $\ell_{(1)0}$

Neural Network setup

- **Target**: $W_0\ (E, p_x, p_y, p_z)$, $W_1\ (E, p_x, p_y, p_z)$
- **Features**:

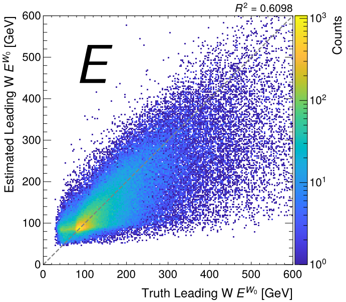
Object	**Component** (Total: 26)
ℓ_0, ℓ_1	$p_{\mathrm{T}}, p_x, p_y, p_z, E, \eta$
$\ell\ell$	$p_{\mathrm{T}}, p_x, p_y, p_z, E, \eta, \Delta\phi_{\ell_0,\ell_1}, m$
MET	$p_{\mathrm{T}}, p_x, p_y, \Delta\phi_{\ell_0,\mathrm{MET}}, \Delta\phi_{\ell_1,\mathrm{MET}}, \Delta\phi_{\ell\ell,\mathrm{MET}}$

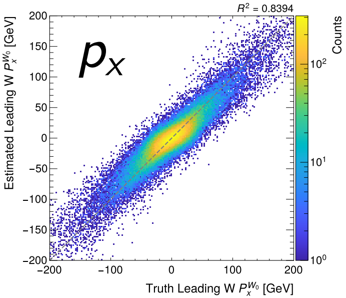
- **Model structure**: (`ADAM` optimizer, MAE loss, L2-regularization)

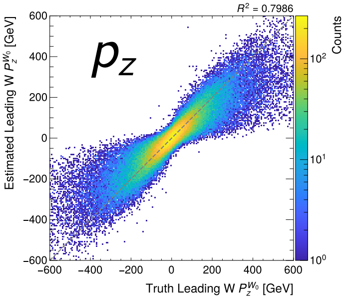


Reconstruction-level neural network regressor results

- Results in four-momentum estimation: ▸ W_0 results, ▸ W_1 results, ▸ H results
- Similar estimation performance with reconstruction-level input ▸ Reco-NN metrics
- Neural network regressor estimations $\longrightarrow$ entanglement observable calculation

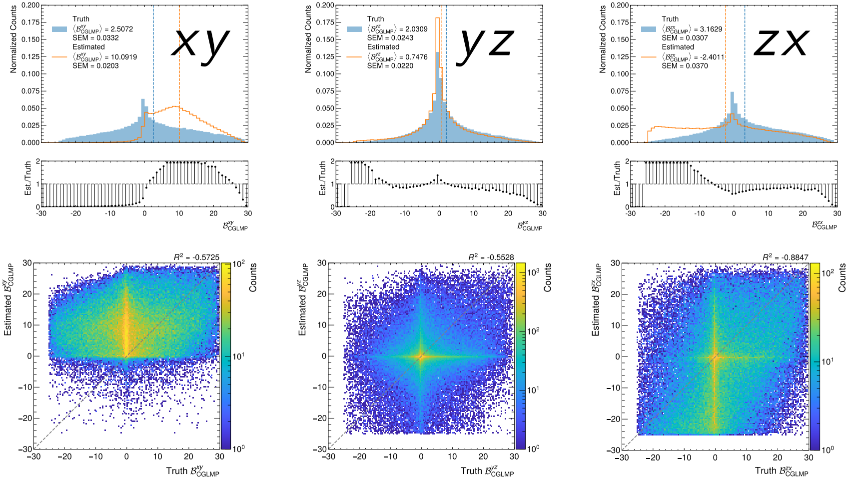






Reconstruction-level neural network regressor: $\mathcal{B}^{xyz}_{\mathrm{CGLMP}}$

- Observable of interest
- Significantly skewed distribtion
- Correlation observed in earlier stages does not carry over



Outline

Take home messages

- Truth-level results in agreement with phenomenology paper
- Selection cut shifts observable distribution and biases expectation value
- Proposed viable methods in intermediate particles' four-momentum estimation
  - Neural network regressor outperforms analytical approximation
  - Model viability verified at truth and reconstruction-level
- Entanglement observable calculation with neural network regressor estimations
  - Results highlight the sensitivity of entanglement observable
  - Significant impact from imperfect W_0, W_1, H four-momentum estimations

Outlook

- Optimization of experimental selection cuts p_T^ℓ cuts
- Quantitative measures to address experimental selections
- Utilization of physical constraints in neural network regressor
- Employ advanced machine learning models and strategies
- Thorough error analysis for every derived observable

Outline

Quantum Tomography

arXiv:2209.13990) back

- Lepton emission angle measurement in the $W^\pm$ boson rest frame
  $\rightarrow$ Projective measurement (von Neumann measurement)
- Probability density function for $W^\pm$ boson with density matrix ρ

$$\mathcal{P}(\ell^\pm_{\hat{\mathbf{n}}};\rho) = \frac{d}{4\pi}\,\mathrm{Tr}(\rho\,\Pi_{\pm,\hat{\mathbf{n}}})$$

- W^+ produces ℓ^+ in direction $\hat{\mathbf{n}}$
  $\rightarrow$ positive helicity selected by projection operator $\Pi_{+,\hat{\mathbf{n}}} \equiv |+\rangle_{\hat{\mathbf{n}}}\langle+|_{\hat{\mathbf{n}}}$
- Eigenstates of the S_z operator: $\{|+\rangle, |0\rangle, |-\rangle\}$
  Active rotaton of angle θ, ϕ about $\hat{\mathbf{y}}, \hat{\mathbf{z}}$ axis

$$\Pi_{\pm,\hat{\mathbf{n}}} = |\pm\hat{\mathbf{n}}\rangle\langle\pm\hat{\mathbf{n}}| = U_{\theta,\phi}\,|\pm\rangle_z\,\langle\pm|_z\,U^\dagger_{\theta,\phi}$$
$$U_{\theta,\phi} = \exp(-\mathrm{i}S_z\phi)\exp(-\mathrm{i}S_y\theta)$$

Spin and projection operators in $d = 3$

- Explicit form of the projection operator $\Pi_{+,\hat{\mathbf{n}}} =$

$$\begin{bmatrix} \cos^4\frac{\theta}{2} & \frac{1}{2\sqrt{2}}\sin\theta(1+\cos\theta) & \frac{1}{4}\exp(-2i\phi)\sin^2\theta \\ \frac{1}{2\sqrt{2}}\exp(i\phi)\sin\theta(1+\cos\theta) & \frac{1}{2}\sin^2\theta & \frac{1}{2\sqrt{2}}\exp(-i\phi)\sin\theta(1-\cos\theta) \\ \frac{1}{4}\exp(2i\phi)\sin^2\theta & \frac{1}{2\sqrt{2}}\sin\theta(1-\cos\theta) & \sin^4\frac{\theta}{2} \end{bmatrix}$$

- Recall: PDF for $W^{\pm}$ boson with density matrix ρ

$$\mathcal{P}(\ell^{\pm}_{\hat{\mathbf{n}}};\rho) = \frac{d}{4\pi}\mathrm{Tr}(\rho\,\Pi_{\pm,\hat{\mathbf{n}}})$$

- Wigner Q symbols:

$$\mathcal{P}(\ell^{\pm}_{\hat{\mathbf{n}}};\rho) = \frac{3}{4\pi}\left(\frac{1}{3} + \sum_{j=1}^{8}\Phi_j^{Q,\pm}a_j\right)$$

Wigner P, Q symbols for spin-1 ($j = 1$)

$$\Phi_j^{Q,\pm} = \langle \pm\hat{\mathbf{n}} | \lambda_j^{(3)} | \pm\hat{\mathbf{n}} \rangle = \mathrm{Tr}\left(\lambda_j^{(3)} \Pi_{\pm,\hat{\mathbf{n}}}\right)$$

- Lepton angular distributions are also Wigner transformations

$$\mathrm{Tr}(\rho \Pi_{\pm,\hat{\mathbf{n}}}) = \langle \hat{\mathbf{n}} | \rho | \hat{\mathbf{n}} \rangle = \frac{3}{4\pi} \Phi_\rho^{Q,\pm}$$

$$2a_k = \int \mathrm{d}\Omega_{\hat{\mathbf{n}}} \mathcal{P}(\ell_{\hat{\mathbf{n}}}^{\pm}; \rho) \Phi_k^{P,\pm} \Rightarrow \hat{a}_k = \frac{1}{2} \left\langle \Phi_k^{P,\pm} \right\rangle$$

- Linear terms a_j, b_k : $\hat{a}_j = \hat{b}_j = \frac{1}{2} \left\langle \Phi_j^P \right\rangle$
- Quadratic terms c_{jk} : $\hat{c}_{jk} = \frac{1}{4} \left\langle \Phi_j^P(\hat{\mathbf{n}}_1)\, \Phi_k^P(\hat{\mathbf{n}}_2) \right\rangle$

Explicit expression of Wigner P, Q symbols

back

Wigner Q symbols

- $\Phi_1^{Q,\pm} = \frac{1}{\sqrt{2}} \sin\theta(\cos\theta \pm 1)\cos\phi$
- $\Phi_2^{Q,\pm} = \frac{1}{\sqrt{2}} \sin\theta(\cos\theta \pm 1)\sin\phi$
- $\Phi_3^{Q,\pm} = \frac{1}{8}(\pm 4\cos\theta + 3\cos 2\theta + 1)$
- $\Phi_4^{Q,\pm} = \frac{1}{2}\sin^2\theta\cos 2\phi$
- $\Phi_5^{Q,\pm} = \frac{1}{2}\sin^2\theta\sin 2\phi$
- $\Phi_6^{Q,\pm} = \frac{1}{\sqrt{2}} \sin\theta(-\cos\theta \pm 1)\cos\phi$
- $\Phi_7^{Q,\pm} = \frac{1}{\sqrt{2}} \sin\theta(-\cos\theta \pm 1)\sin\phi$
- $\Phi_8^{Q,\pm} = \frac{1}{8\sqrt{3}}(\pm 12\cos\theta - 3\cos 2\theta - 1)$

Wigner P symbols

- $\Phi_1^{P,\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\cos\phi$
- $\Phi_2^{P,\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\sin\phi$
- $\Phi_3^{P,\pm} = \frac{1}{4}(\pm 4\cos\theta + 15\cos 2\theta + 5)$
- $\Phi_4^{P,\pm} = 5\sin^2\theta\cos 2\phi$
- $\Phi_4^{P,\pm} = 5\sin^2\theta\sin 2\phi$
- $\Phi_6^{P,\pm} = \sqrt{2}(\pm 1 - 5\cos\theta)\sin\theta\cos\phi$
- $\Phi_7^{P,\pm} = \sqrt{2}(\pm 1 - 5\cos\theta)\sin\theta\sin\phi$
- $\Phi_8^{P,\pm} = \frac{1}{4\sqrt{3}}(\pm 12\cos\theta - 15\cos 2\theta - 5)$

CGLMP inequality

arXiv:2106.01377 back

- CGLMP inequality: (computational basis: $|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$)

$$2 \geq \mathcal{I}_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ - P(A_1 = B_1 - 1) - P(B_1 = A_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)$$

- $\mathcal{B}^{xy}_{\text{CGLMP}} = \frac{-2}{\sqrt{3}}(S_x \otimes S_y + S_y \otimes S_x) + \lambda_4 \otimes \lambda_4 + \lambda_5 \otimes \lambda_5$
  spin basis: $|\psi_s\rangle = \frac{1}{\sqrt{3}}(|+-\rangle - |00\rangle + |+-\rangle)$
- $\mathcal{I}_3 = \text{Tr}(\rho_{WW}\mathcal{B}_{\text{CGLMP}}) \to \max\left\{\langle \mathcal{B}^{ij}_{\text{CGLMP}}\rangle\right\}$
- $S_x = \frac{1}{\sqrt{2}}(\lambda_1 + \lambda_6) \quad S_y = \frac{1}{\sqrt{2}}(\lambda_2 + \lambda_7) \quad S_z = \frac{1}{2}(\lambda_3 + \sqrt{3}\lambda_8)$
- $\text{Tr}(\rho_{WW} S_i \otimes S_j) = -4\left\langle \xi_i^+ \xi_j^- \right\rangle$
- $\left\langle (\xi_x^+)^2 - (\xi_y^+)^2 \right\rangle = \frac{2}{5}a_4 \quad \left\langle \xi_x^+ \xi_y^+ \right\rangle = \frac{2}{5}a_5$

CGLMP Truth values at individual cut stages

back

Category	Criteria	Individual cut	$\langle \mathcal{B}^{xy}_{\mathrm{CGLMP}} \rangle$	$\langle \mathcal{B}^{yz}_{\mathrm{CGLMP}} \rangle$	$\langle \mathcal{B}^{zx}_{\mathrm{CGLMP}} \rangle$
Geometry	$\lvert\eta_\ell\rvert < 2.5$ $\lvert\eta_e\rvert \notin [1.37, 1.52]$	2238610	2.3363 ± 0.0079	2.5316 ± 0.0064	2.8648 ± 0.0068
Preselection	$p_{\mathrm{T}}^{\ell_{0(1)}} > 22\ (15)$ GeV	1628280	2.5468 ± 0.0094	2.1147 ± 0.0071	3.1033 ± 0.0082
	$m_{\ell\ell} > 10$ GeV	2806878	1.7284 ± 0.0071	2.3305 ± 0.0057	2.5851 ± 0.0059
	$p_{\mathrm{T}}^{\mathrm{miss}} > 20$ GeV	2675839	2.4276 ± 0.0073	2.4702 ± 0.0058	3.0046 ± 0.0063
Background rejection	$\Delta\phi_{\ell\ell, E_{\mathrm{T}}^{\mathrm{miss}}} > \pi/2$	2436549	2.5259 ± 0.0076	2.5929 ± 0.0061	3.0832 ± 0.0066
	$p_{\mathrm{T}}^{\ell\ell} > 30$ GeV	2326116	2.8625 ± 0.0077	2.5644 ± 0.0061	3.4332 ± 0.0069
Event Topology	$m_{\ell\ell} < 55$ GeV	2579470	2.6443 ± 0.0075	3.3608 ± 0.0060	3.6579 ± 0.0062
	$\Delta\phi_{\ell\ell} < 1.8$	2162319	3.5454 ± 0.0079	3.1135 ± 0.0065	4.0372 ± 0.0071

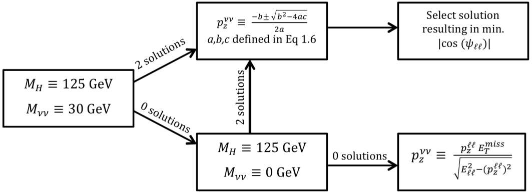
Analytical approximation method

Method proposed in the dissertation *Spinning the Higgs* by R. Aben ▸ CDS ▸ Details

1. Di-lepton di-neutrino energy-momentum relation, under $m_{H_{\mathrm{SM}}}$ constraint

$$m_H{}^2 = 2E_{\ell\ell}E_{\nu\nu} - 2\vec{p}_{\ell\ell}\cdot\vec{p}_{\nu\nu} + m_{\ell\ell}{}^2 + m_{\nu\nu}{}^2$$

2. Solve quadratic equation in terms of $p_Z^{\nu\nu}$, assuming $m_{\nu\nu} = \langle m_{\nu\nu}^{\mathrm{truth}}\rangle$



3. Collinear $\nu\nu$ approximation. Solve for a scaling factor to separate estimated $\vec{p}_{\nu\nu}$
4. Correct the neutrino energy in the "off-shell" $\ell\nu$ pair, such that $m_{\nu\nu} = \langle m_{\nu\nu}\rangle$

Spinning the Higgs Method - 1

1. Start with the di-neutrino di-lepton system energy momentum relation

$$m_H{}^2 = 2E_{\ell\ell}E_{\nu\nu} - 2\vec{p}_{\ell\ell}\cdot\vec{p}_{\nu\nu} + m_{\ell\ell}{}^2 + m_{\nu\nu}{}^2$$

2. Solve the quadratic system:

$$0 = \underbrace{\left[p_z^{\ell\ell 2} - E_{\ell\ell}{}^2\right]}_{a} p_z^{\nu\nu 2} + \underbrace{m_{\mathrm{fix}}{}^2 p_z^{\ell\ell}}_{b} p_z^{\nu\nu} + \underbrace{\left\{\frac{1}{4}m_{\mathrm{fix}}{}^2 - E_{\ell\ell}{}^2\left[E_{\mathrm{T}}^{\mathrm{miss}\,2} + m_{\nu\nu}{}^2\right]\right\}}_{c}$$

Where

$$m_{\mathrm{fix}}{}^2 = m_H{}^2 - m_{\ell\ell}{}^2 - m_{\nu\nu}{}^2 + 2p_x^{\ell\ell}E_x^{\mathrm{miss}} + 2p_y^{\ell\ell}E_y^{\mathrm{miss}}$$

Spinning the Higgs Method - 2

3. If $b^2 - 4ac < 0$ retry with $m_{\nu\nu} = 0$
4. If $m_{\nu\nu} = 0$ still causes negative determinant, extremize m_H^2 with respect to $p_z^{\nu\nu}$

$$p_z^{\nu\nu} = p_z^{\ell\ell} \frac{\sqrt{{E_{\mathrm{T}}^{\mathrm{miss}}}^2 + m_{\nu\nu}^2}}{\sqrt{{E_{\mathrm{T}}^{\ell\ell}}^2 - {p_z^{\ell\ell}}^2}}$$

5. Choose solution where $\ell\ell$ is system closer to the $\vec{p}_{\mathrm{T}}$ plane in reconstructed Higgs CM frame
6. Designate the lepton where $m_{\nu\nu\ell}$ is greater as the "on-shell" pair, denote with subscript 0

Spinning the Higgs Method - 3

7. Assume $\vec{p}_{\nu\nu} = \alpha^2 \vec{p}_{\nu_0} + (1+\alpha^2)\vec{p}_{\nu_1}$
8. The collinear approximation leads to the assumption that $M_{\nu\nu} = 0$, since the massless constraint on individual neutrinos require the total energy of this newly approximated di-neutrino system to:

$$E'_{\nu\nu} = \alpha^2|\vec{p}_{\nu\nu}| + (1-\alpha^2)|\vec{p}_{\nu\nu}| = |\alpha^2\vec{p}_{\nu\nu} + (1-\alpha^2)\vec{p}_{\nu\nu}| = |\vec{p}_{\nu\nu}|$$

$$\begin{aligned} m_W^2 = E_W^2 - |\vec{p}_W|^2 &= (E_{\nu_0} + E_{\ell_0})^2 - |\vec{p}_{\nu_0} + \vec{p_{\ell_0}}|^2 \\ &= (\alpha^2 E'_{\nu\nu} + E_{\ell_0})^2 - |\alpha^2\vec{p}_{\nu\nu} + \vec{p}_{\ell_0}|^2 \end{aligned}$$

$$\alpha^2 = \frac{m_W^2 - m_{\ell_0}^2}{2(E'_{\nu\nu}E_{\ell_0} - \vec{p}_{\nu\nu}\cdot\vec{p}_{\ell_0})}$$

Spinning the Higgs Method - 4

9. Correct the di-neutrino mass to be equal to the mean value of calculated from the simulation sample $\langle M_{\nu\nu}\rangle$, and add the correction term m_f to the sub-leading neutrino energy

$$E_{\nu\nu}^2 - |\vec{p}_{\nu\nu}|^2 = \langle m_{\nu\nu}\rangle^2 = \left[\alpha^2|\vec{p}_{\nu\nu}| + \sqrt{(1-\alpha^2)^2|\vec{p}_{\nu\nu}|^2 + {m_f}^2}\right]^2 - |\vec{p}_{\nu\nu}|^2$$

$$\Rightarrow \sqrt{-(1-\alpha^2)^2|\vec{p}_{\nu\nu}|^2 + \left[-\alpha^2|\vec{p}_{\nu\nu}| + \sqrt{|\vec{p}_{\nu\nu}|^2 + \langle m_{\nu\nu}\rangle^2}\right]^2} = m_f$$

10. The final neutrino four-momentum in terms of (m, p_x, p_y, p_z)

$$p_{\nu_0} = (0, \alpha^2 p_x^{\nu\nu}, \alpha^2 p^{\nu\nu}, \alpha^2 p_z^{\nu\nu})$$
$$p_{\nu_1} = (m_f, (1-\alpha^2)p_x^{\nu\nu}, (1-\alpha^2)p_y^{\nu\nu}, (1-\alpha^2)p_z^{\nu\nu})$$

Reconstruction-level neural network regressor metrics

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Particle	Component	NN (Reconstruction-level)	
		MAE	R^2
W_0	E	38.49	0.61
	p_x	12.20	0.84
	p_y	12.22	0.84
	p_z	44.69	0.79
W_1	E	29.46	0.47
	p_x	10.90	0.73
	p_y	10.93	0.73
	p_z	51.39	0.71
H	E	85.01	0.66
	p_x	11.83	0.93
	p_y	11.84	0.93
	p_z	70.01	0.83

Reconstruction-level NN: W_0 CM frame reconstruction

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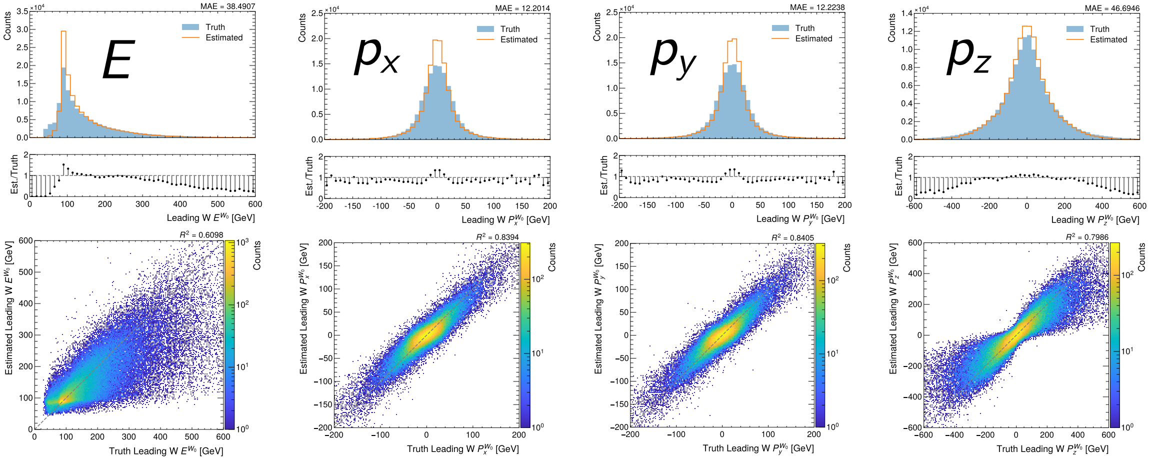

Reconstruction-level NN: W_1 CM frame reconstruction

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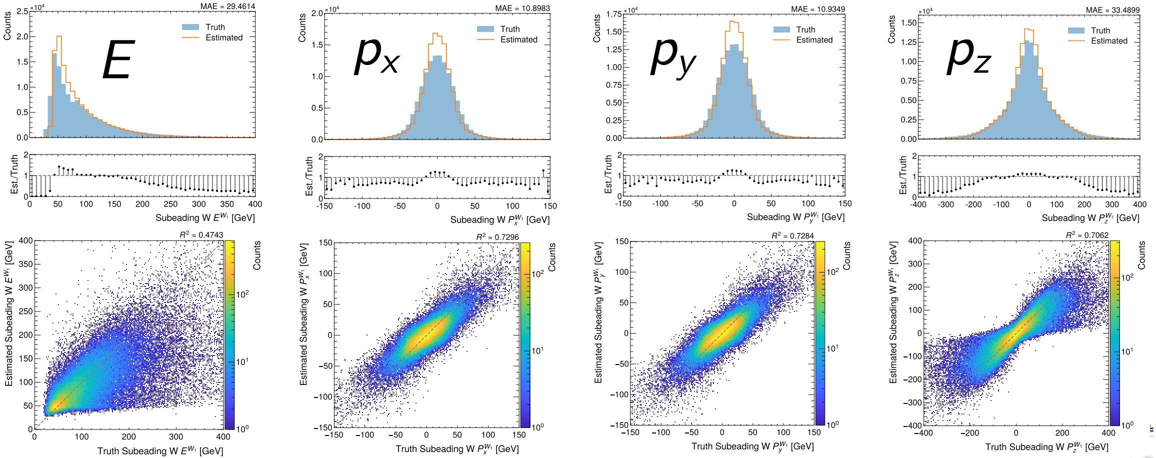

Reconstruction-level NN: Higgs CM frame reconstruction

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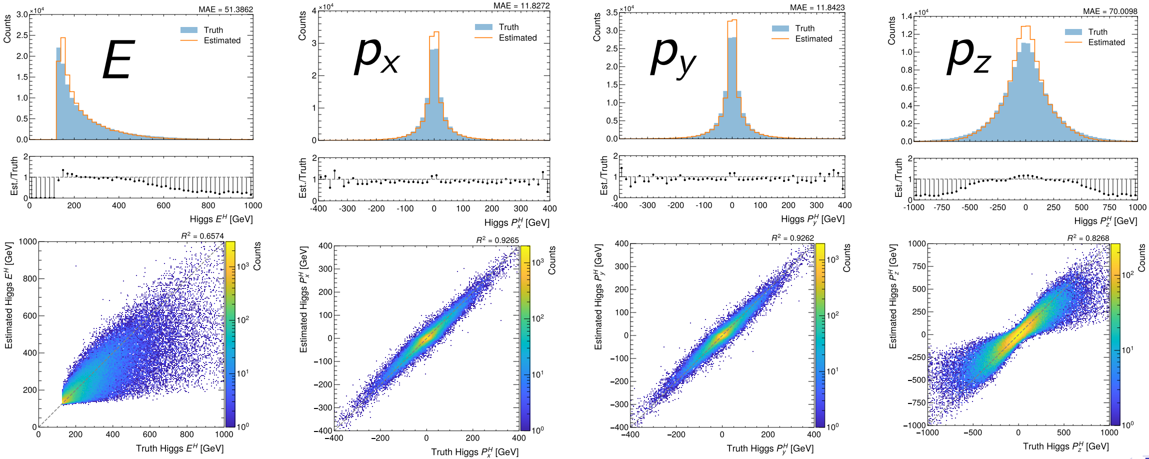
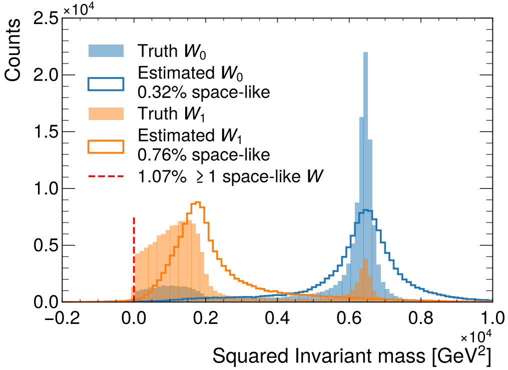


Estimated $W_{0,1}$ invariant mass

$$m^2 = E^2 - |\vec{p}|^2$$

- Model captures difference in invariant mass between $W_{0,1}$
- Smooth distribution of the *squared* mass extends below 0 $\rightarrow$ space-like W
- Rectify space-like four-momentum by correcting energy component
  - Impose $m_{H_{SM}}$ or $m_{W_{SM}}$ conditions
  - Keep $\vec{p}$ fixed



Preliminary ideas in lepton p_T cut optimization

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