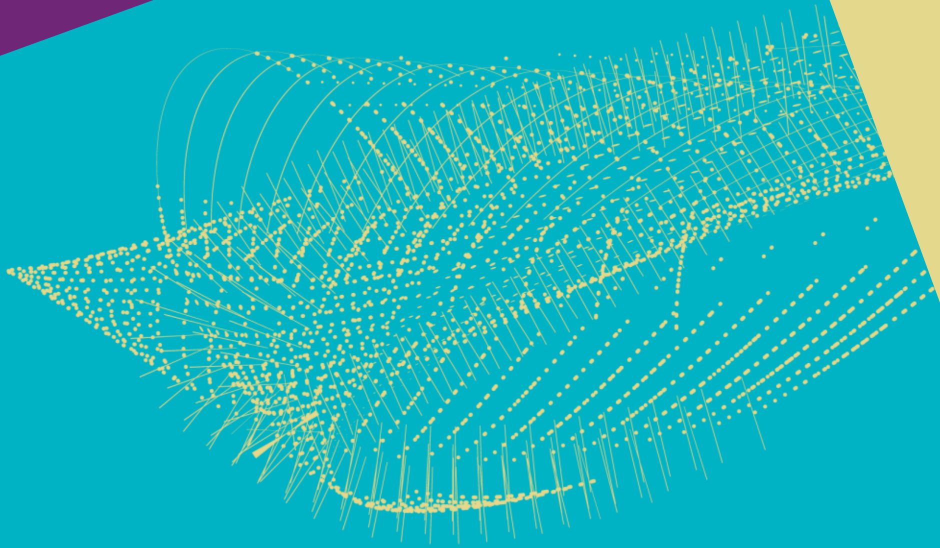
Nikhef

Maastricht University

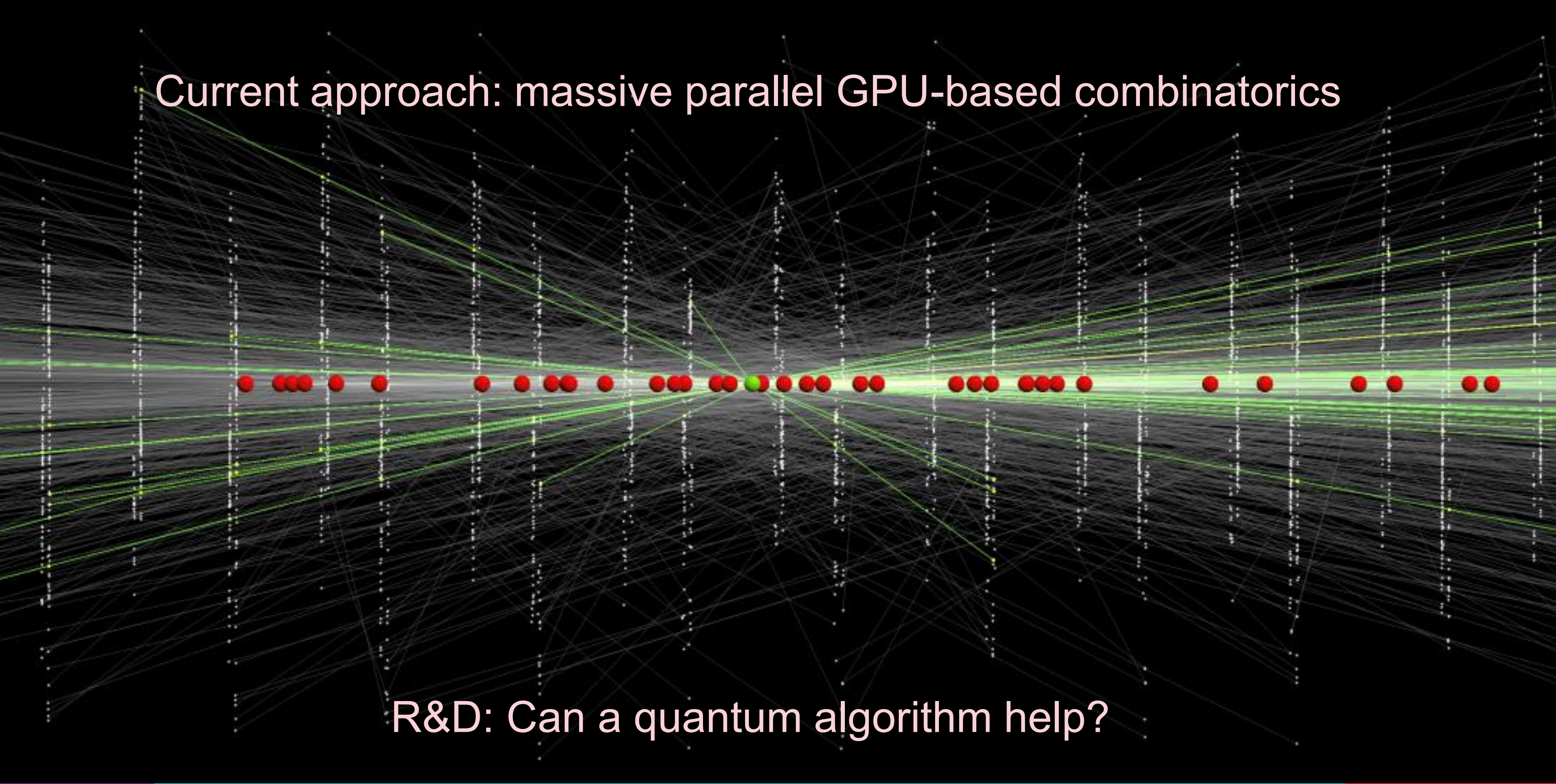
UHASSELT
KNOWLEDGE IN ACTION



Quantum Particle Tracking with TrackHHL

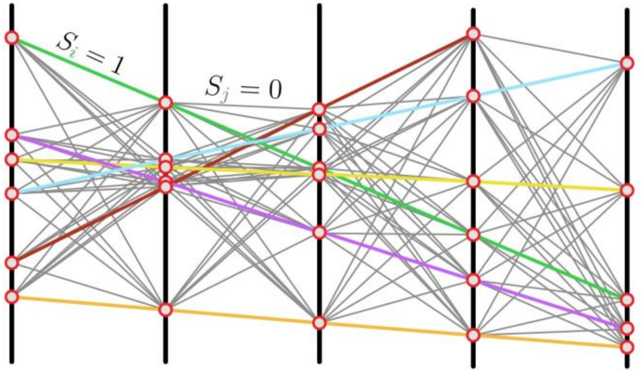
George William Scriven
Maastricht University, UHasselt, Nikhef
NNV
07/11/2025

Current approach: massive parallel GPU-based combinatorics



R&D: Can a quantum algorithm help?

Method: "Global" algorithm for trackfinding

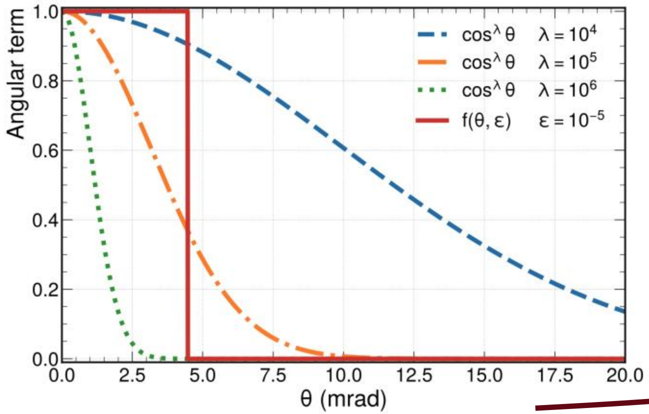


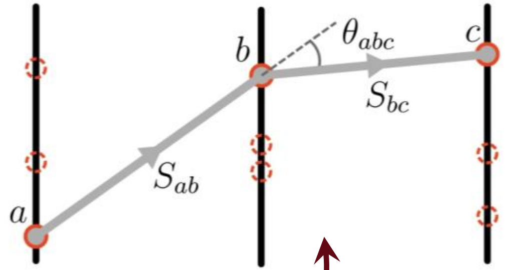
1) Build an Ising-like (quadratic) Hamiltonian

Define a Hamiltonian based on doublets of hits S_i being "on track" $S_i = 1$ or "off track" $S_j = 0$

$$\mathcal{H}(S) = \sum_{abc} f(\theta) S_{ab} S_{bc} + \gamma \sum_{ab} S_{ab}^2 + \delta \sum_{ab} (1 - 2S_{ab})^2$$

- Angular term: assigns value for scattering
- Spectral term: ($\gamma = 2.0$) makes the spectrum of A_{ij} positive
- Gap term: ($\delta = 1.0$) ensures gap in the solution spectrum





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1) Build an Ising-like (quadratic) Hamiltonian

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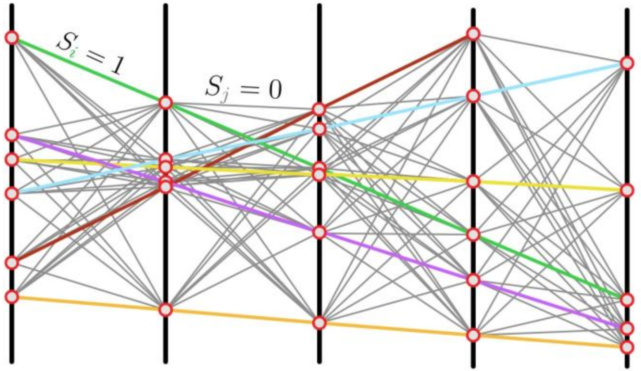


2) Find ground state of:

$$< S|\mathcal{H}|S >= \sum_{ij} A_{ij} S_i S_j + \sum_i b_i S_i$$

$$S_i \in \{0,1\}$$

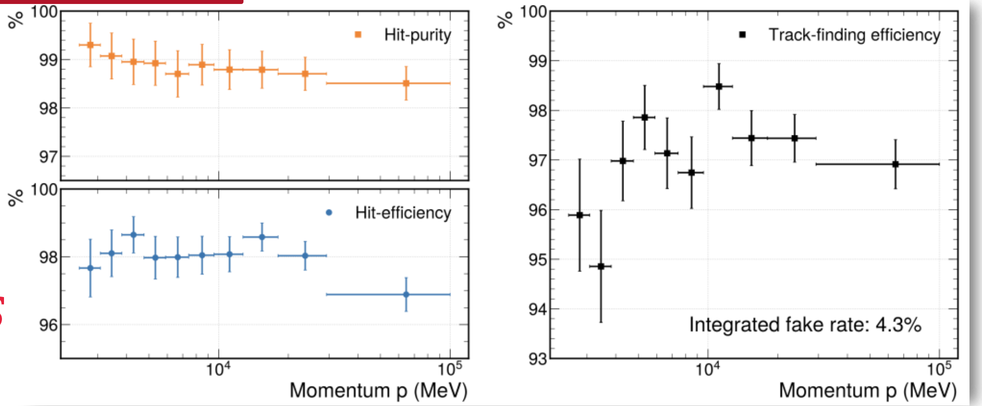
3) Minimisation:

- *Minimization*:
  - $\nabla_S < S|\mathcal{H}|S >= 0$
  - $\Rightarrow \quad A\boldsymbol{S} = b$
- Solve the large matrix inversion to find $\boldsymbol{S}$

Try by solving classically good performance! ↓

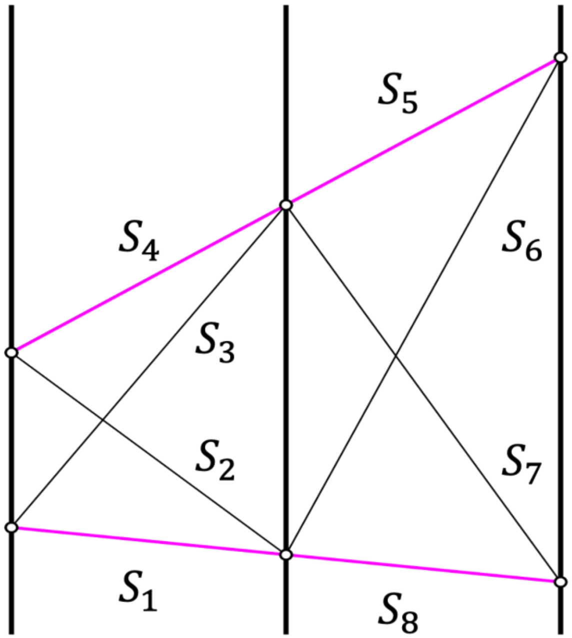
Tracking performance on LHCb simulated events

[Link]

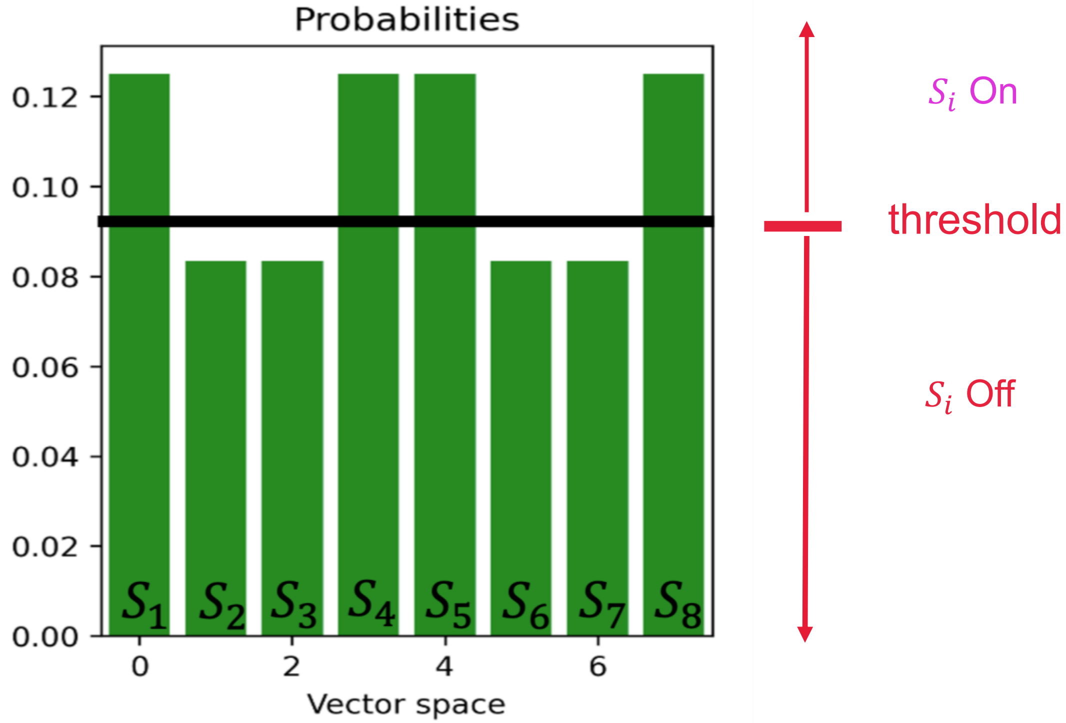


Simplest (trivial) case how it works classically

Two tracks in three layers



$\nabla_S \mathcal{H} = 0 \;\Rightarrow\; \boldsymbol{S} = A^{-1} b$



- Algorithm finds that:
- S_1, S_4, S_5, S_8 are good segments and S_2, S_3, S_6, S_7 are wrong cobinations

The Quantum Algorithm: Harrow-Hassadim-Lloyd

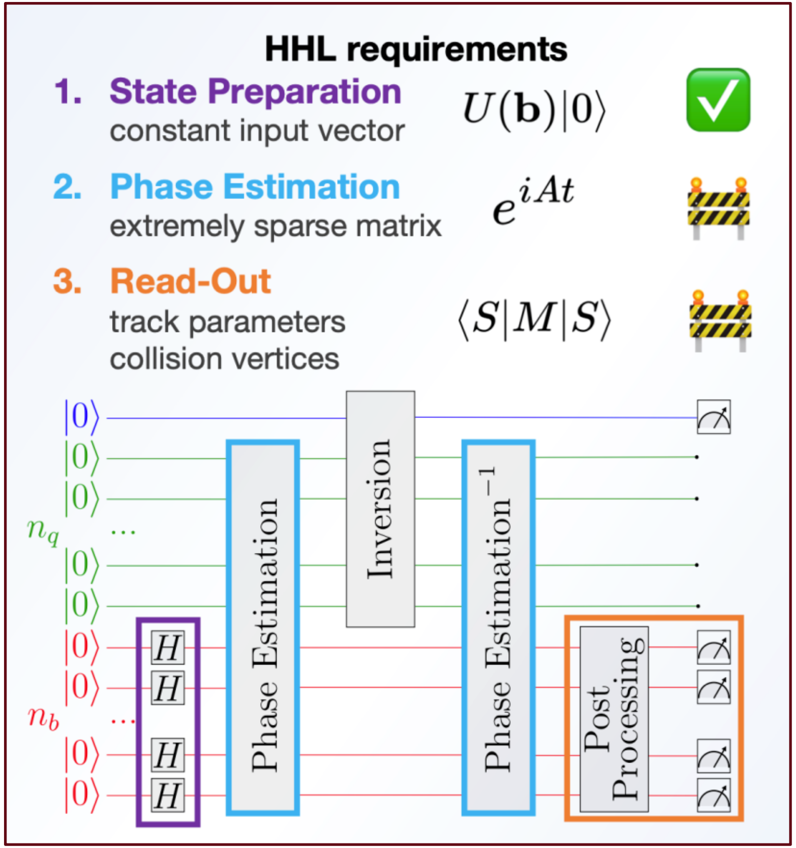
Solving the matrix equation to find solution $\boldsymbol{S}$

$\boldsymbol{A}\boldsymbol{S} = \boldsymbol{b} \Rightarrow \boldsymbol{S} = \boldsymbol{A}^{-1}\boldsymbol{b}$ Solve using HHL:

1. State preparation
2.
   - Quantum Phase estimation
   - Inversion
   - Quantum Phase Estimation†
3. Measurement

Quantum state: $|\Phi\rangle_i |\Phi\rangle_b |\Phi\rangle_q$

- $|\Phi\rangle_i$ ancilla qubit for controlled phase rotation
- $n_b = \log_2 N$ qubits to store vector $\boldsymbol{b}$
- Prepare $\boldsymbol{b} = (1,1,1,1,1,1)$ with Hadamards
- QPE: apply $U = e^{iA}$ with n bit precision
- n_q ancilla qubits for phase estimation
- Readout S



The Quantum Algorithm: Harrow-Hassadim-Lloyd

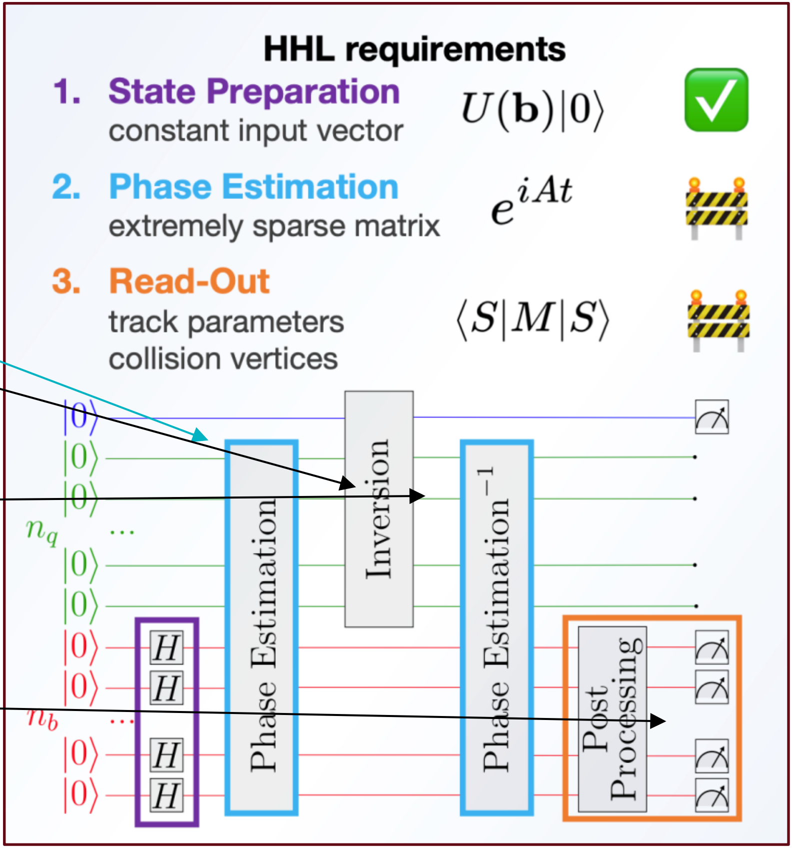
Full state space evolution:

$$\sum_{j=0}^{2^{n_b}-1} b_j |u_j\rangle |\tilde{\lambda}_j\rangle |0\rangle_a$$

$$\sum_{j=0}^{2^{n_b}-1} b_j |u_j\rangle |\tilde{\lambda}_j\rangle \left(\sqrt{1-\frac{C^2}{\tilde{\lambda}_j^{\,2}}}\,|0\rangle_a + \frac{C}{\tilde{\lambda}_j}|1\rangle_a\right)$$

$$\frac{1}{\sqrt{\sum_{j=0}^{2^{n_b}-1} \left|\frac{b_j C}{\tilde{\lambda}_j}\right|^2}} \sum_{j=0}^{2^{n_b}-1} b_j |u_j\rangle |\tilde{\lambda}_j\rangle \frac{C}{\tilde{\lambda}_j}|1\rangle_a$$

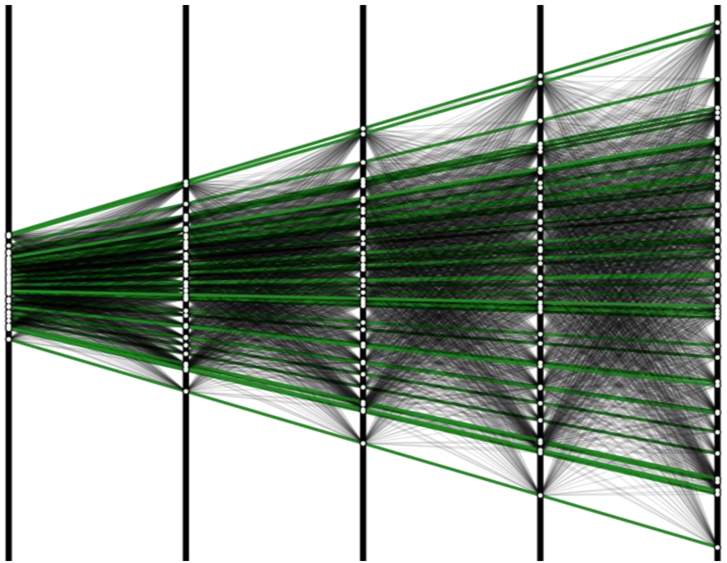
$$|x\rangle_b |0\rangle_q^{\otimes n} |1\rangle_a$$

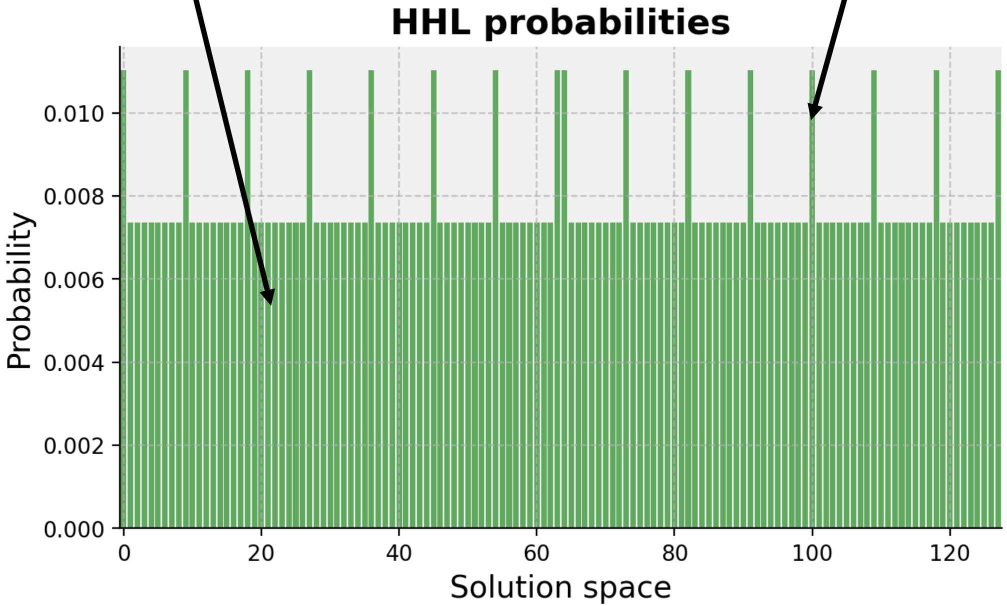


Consider the case:

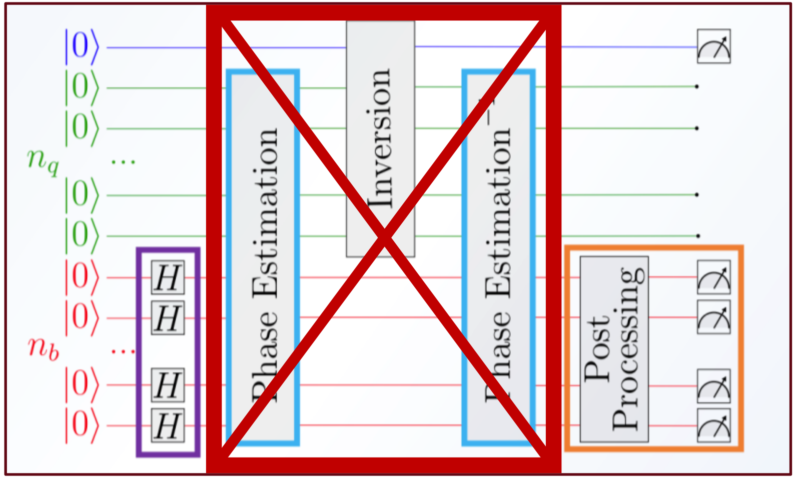
Solutions “off”
$S_i = 0$

Solutions “on”
$S_i = 1$

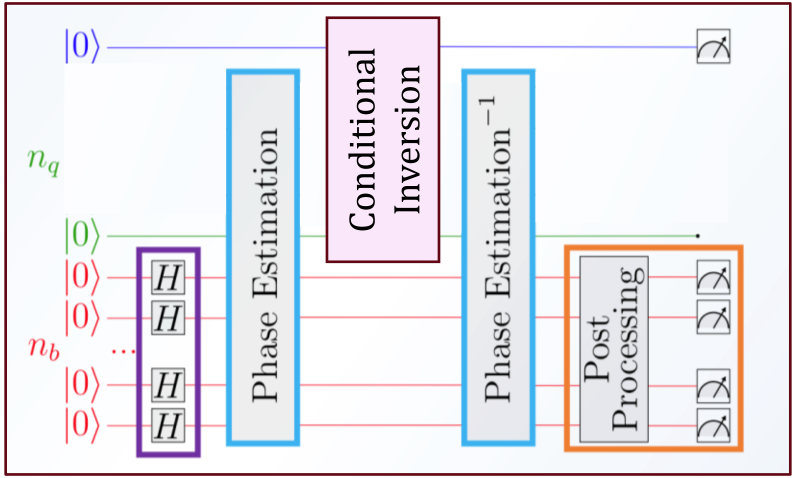




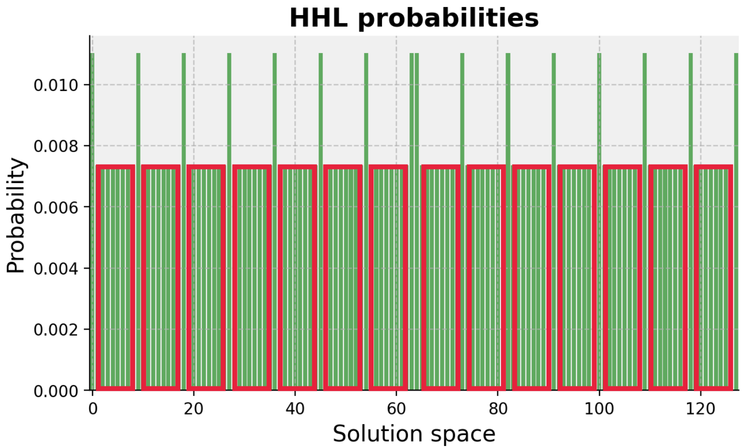
1-Bit HHL



$|0\rangle$
n_q
n_b
H
Phase Estimation
Inversion
Phase Estimation^{-1}
Post Processing



$|0\rangle$
n_q
n_b
H
Phase Estimation
Conditional Inversion
Phase Estimation^{-1}
Post Processing



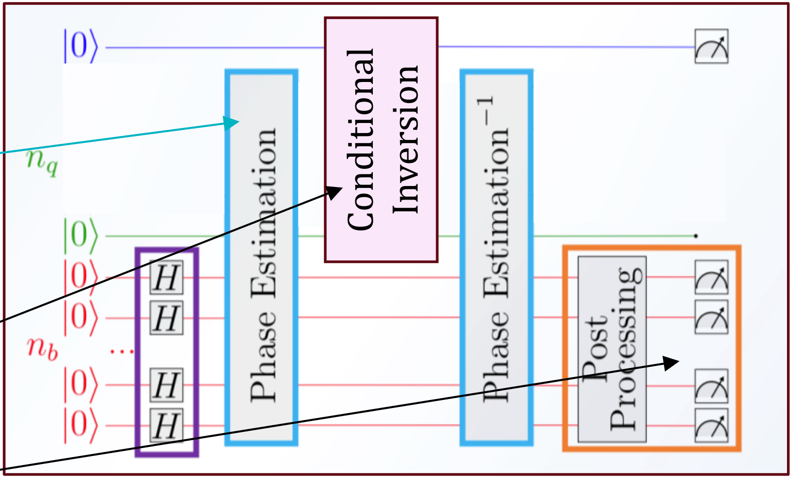
HHL probabilities
Probability
Solution space
0.000
0.002
0.004
0.006
0.008
0.010
0
20
40
60
80
100
120

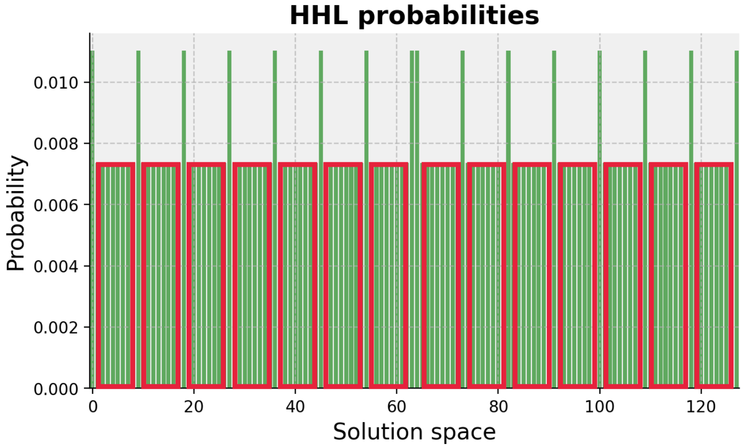
1-Bit HHL

$$\sum_{j=0}^{2^{n_b}-1} b_j |u_j\rangle |\tilde{\lambda}_j\rangle |0\rangle_a$$

$$\sum_{j=0}^{2^{n_b}-1} b_j |u_j\rangle |\tilde{\lambda}_j\rangle \left(\cancel{\sqrt{1-\frac{C^2}{\tilde{\lambda}_j^2}}}|0\rangle_a + \frac{C}{\tilde{\lambda}_j}|1\rangle_a\right)$$

$$|x\rangle_b |0\rangle_q^{\otimes n} |1\rangle_a$$





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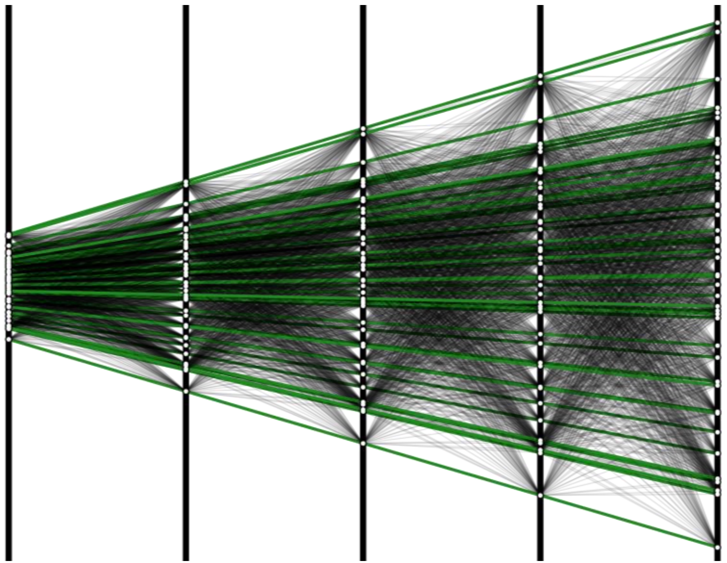
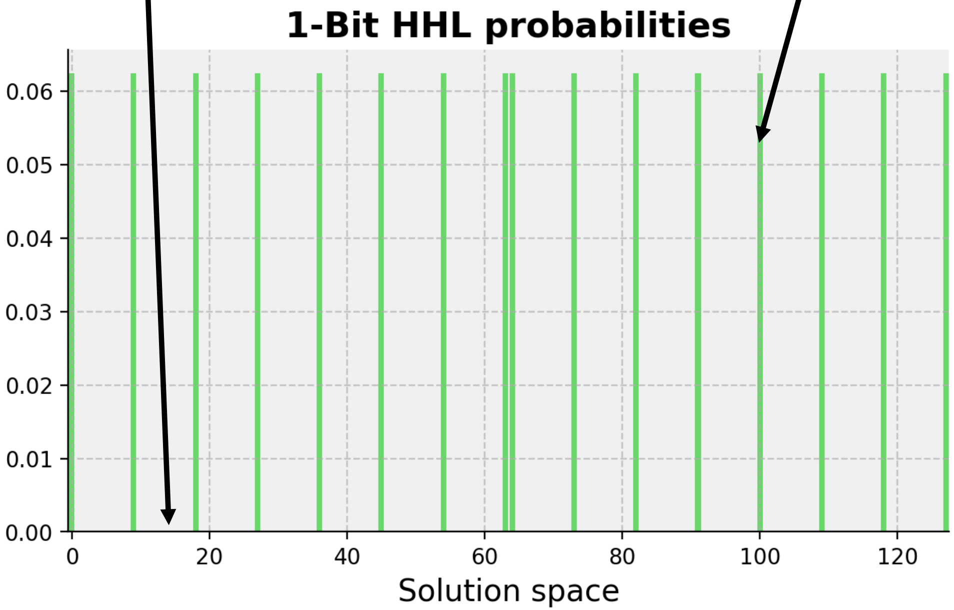
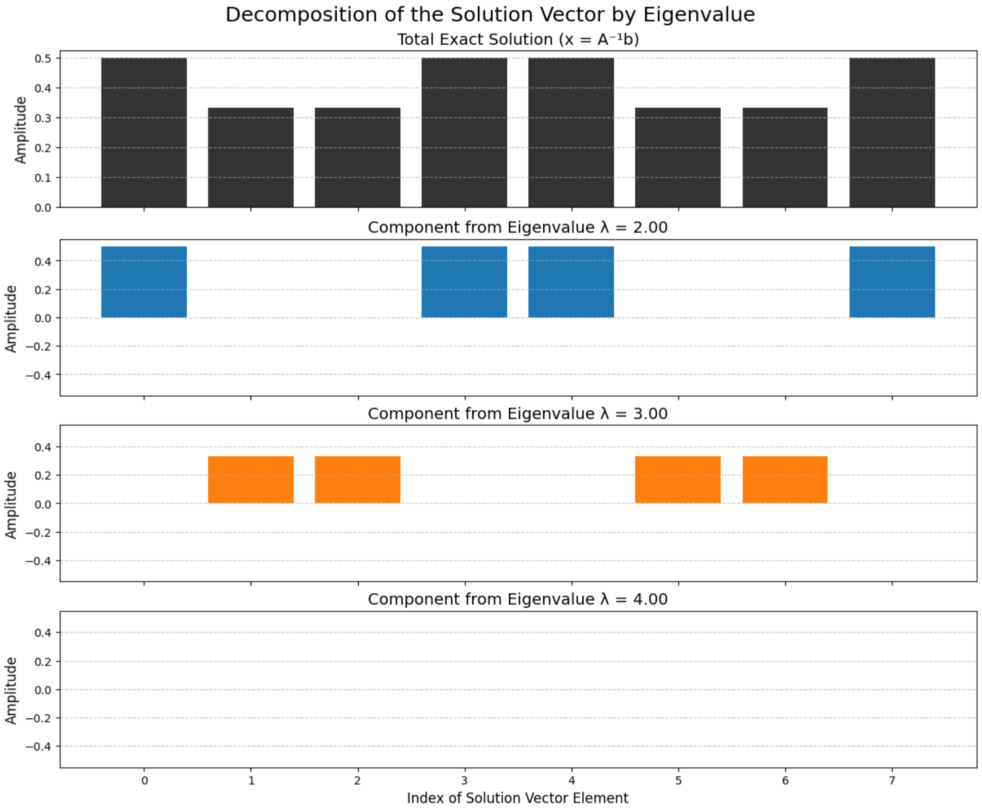
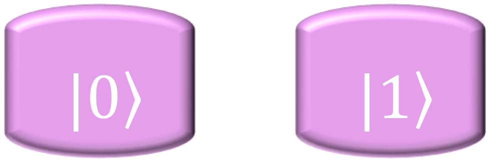




How does this work?



QPE BIN



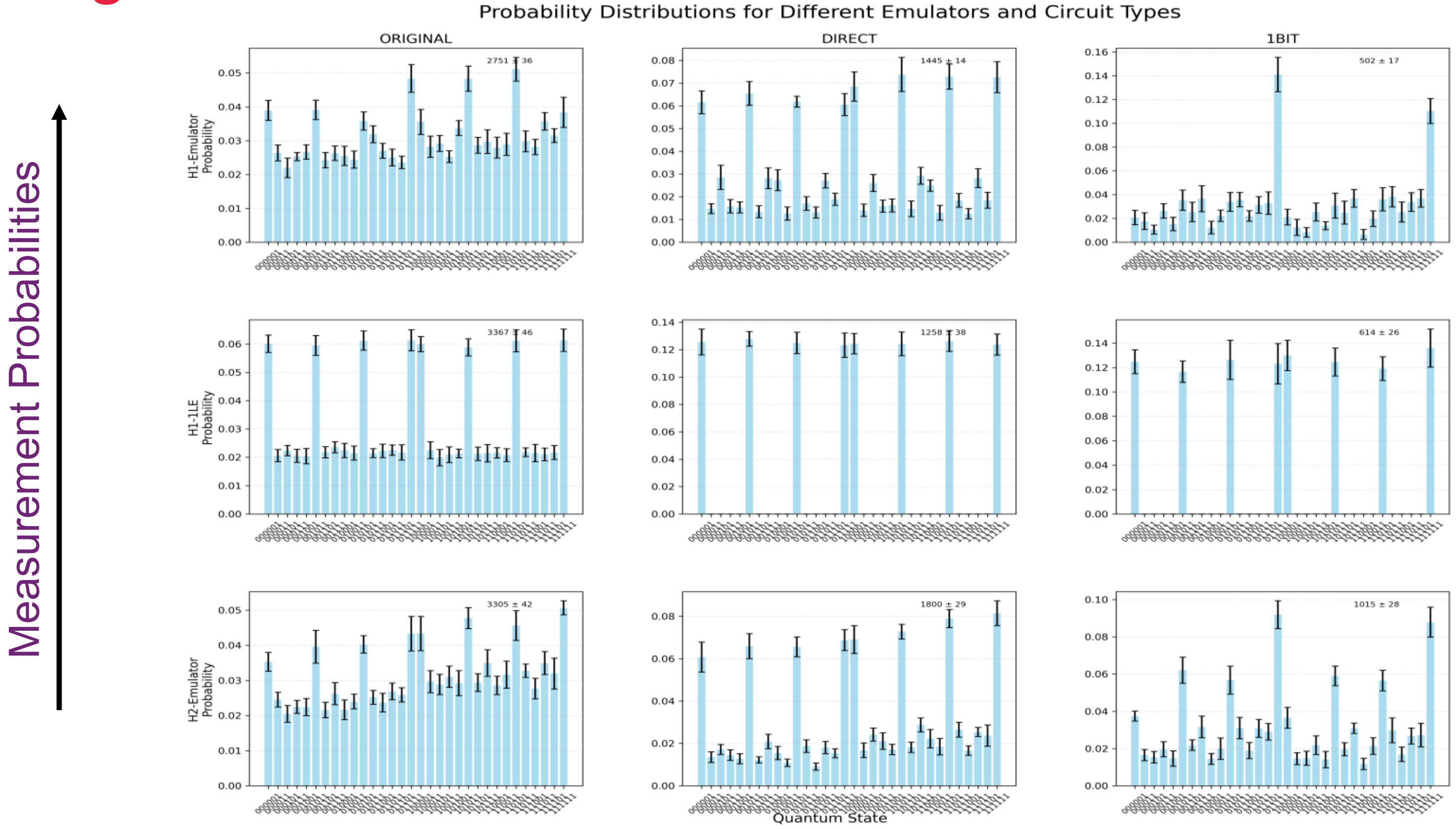
$$P(j=0|\lambda_k) = \cos^2\left(\frac{\lambda_k t}{2}\right)$$

$$\lambda_c = \frac{\lambda_{\min} + \lambda_{\max}}{2} \implies t = \frac{\pi}{\lambda_c}$$

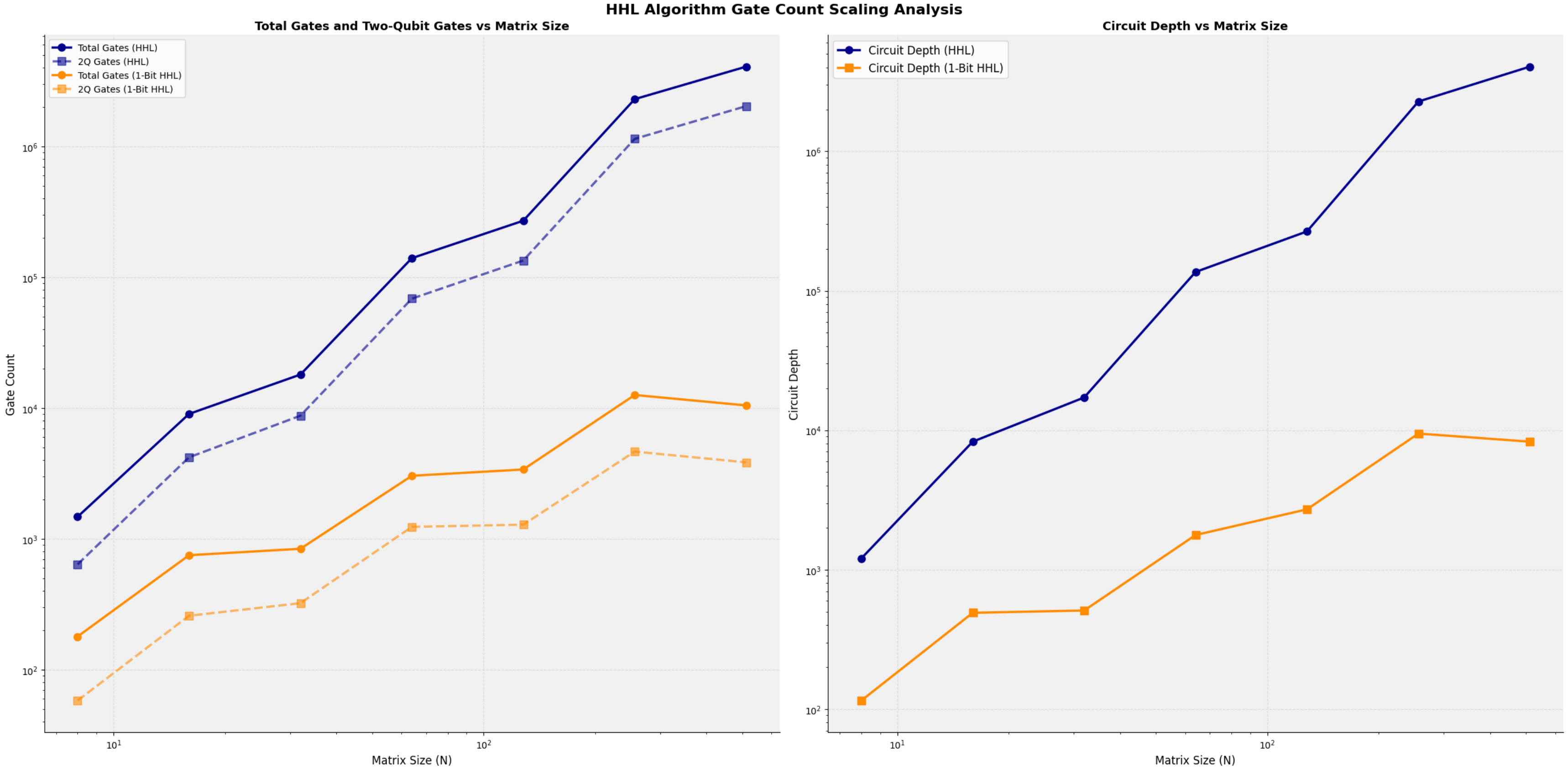
$$P(j=0|\lambda_c) = \cos^2\left(\frac{\lambda_c \pi}{2\lambda_c}\right) = \cos^2\left(\frac{\pi}{2}\right) = 0$$

$$P(j=1|\lambda_c) = \sin^2\left(\frac{\lambda_c \pi}{2\lambda_c}\right) = \sin^2\left(\frac{\pi}{2}\right) = 1$$

Testing Quantinuum Hardware Emulators

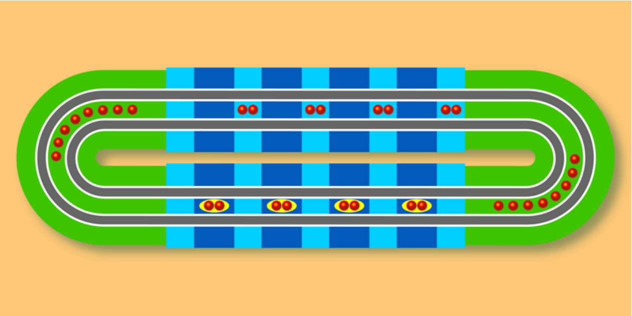


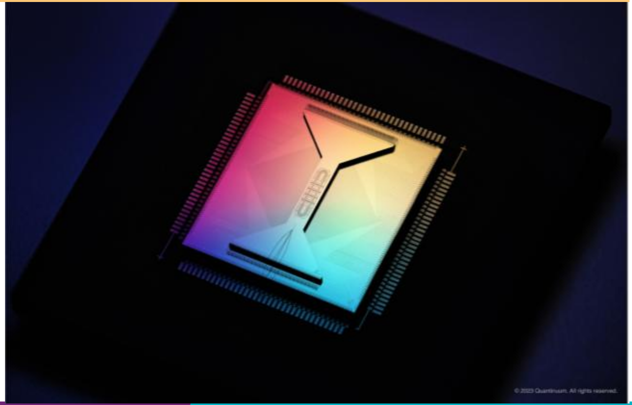
Circuit Depth (Hardware Agnostic)



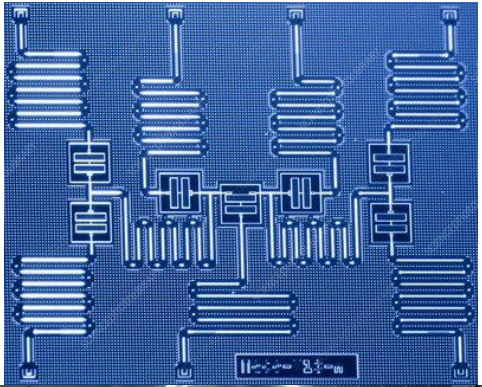
Hardware Choices

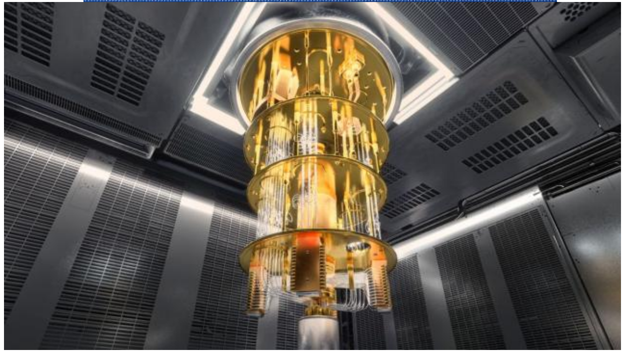
Trapped Ions (Quantinuum)



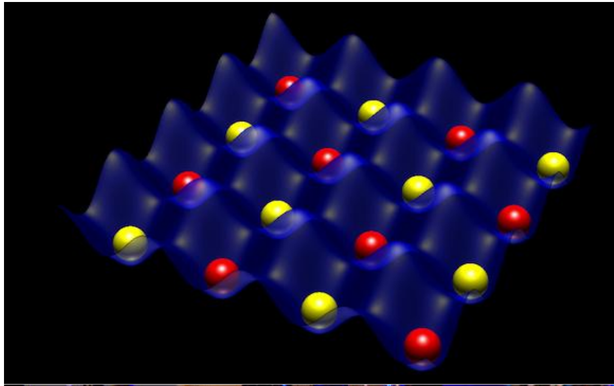


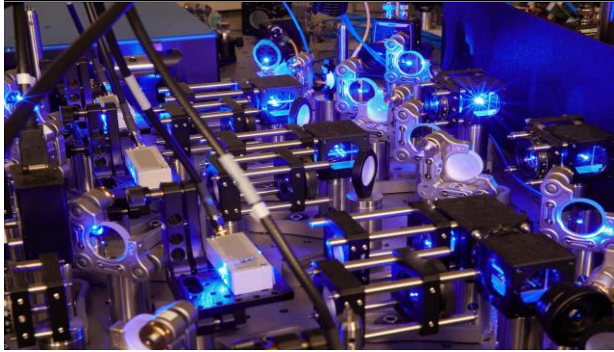
Superconducting (IBM)



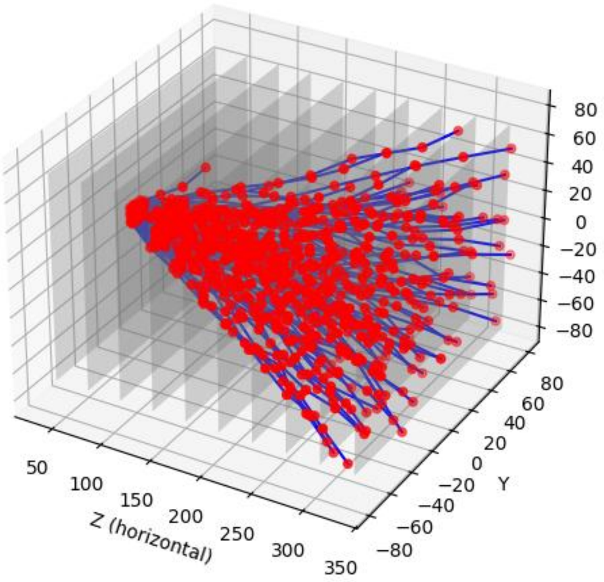


Neutral Atoms (TU Eindhoven)





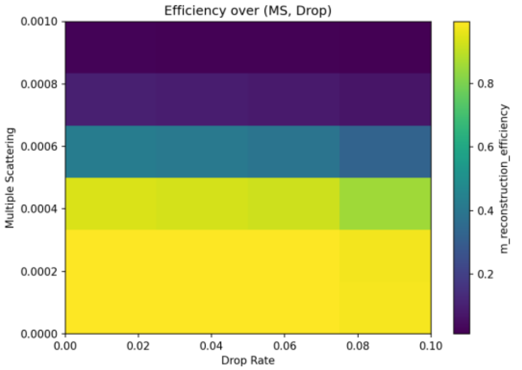
Determining Parameter Dependency and Numerical Stability

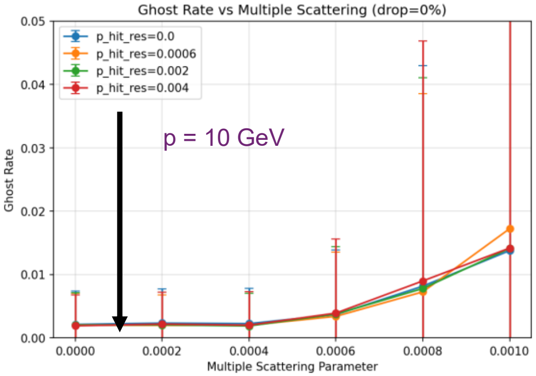


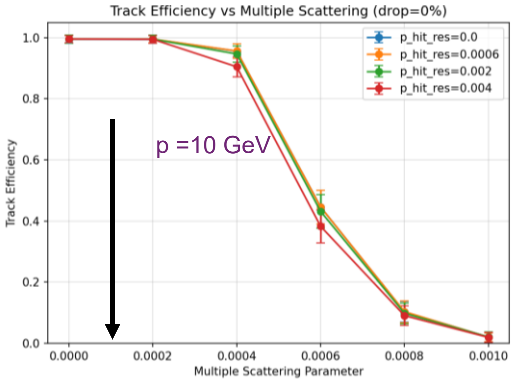
Toy Parameters:

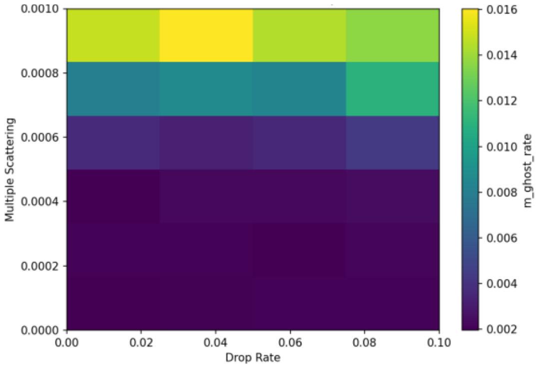
- Scattering within layers
- Pixel Resolution
- Detector Noise
- Missed Hits
- Acceptance window

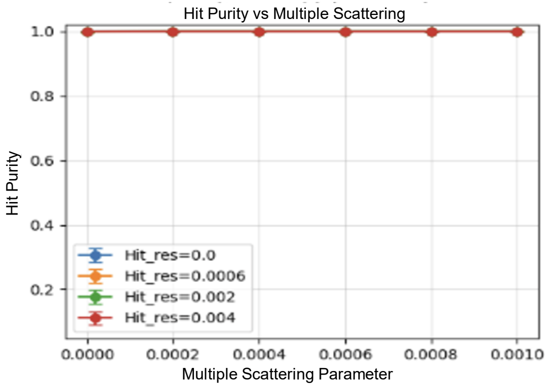
Toy Parameter Dependencies

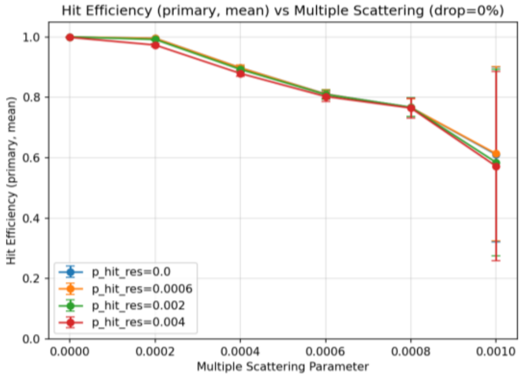












Remaining Tasks

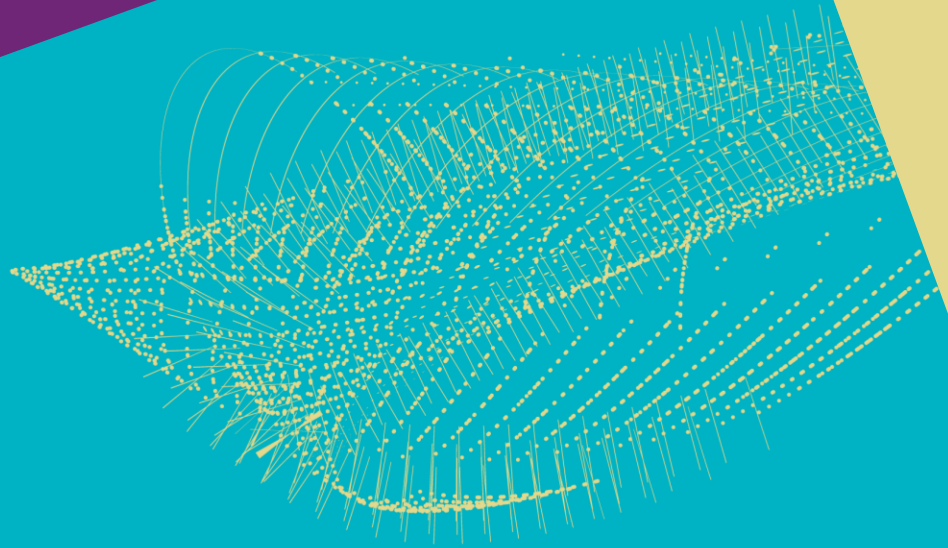
Full characterise model stability

Write the paper!!!

More efficient inversion algorithms QSVT

Quantum Method for PVs


That's all Folks!



Backup Slides

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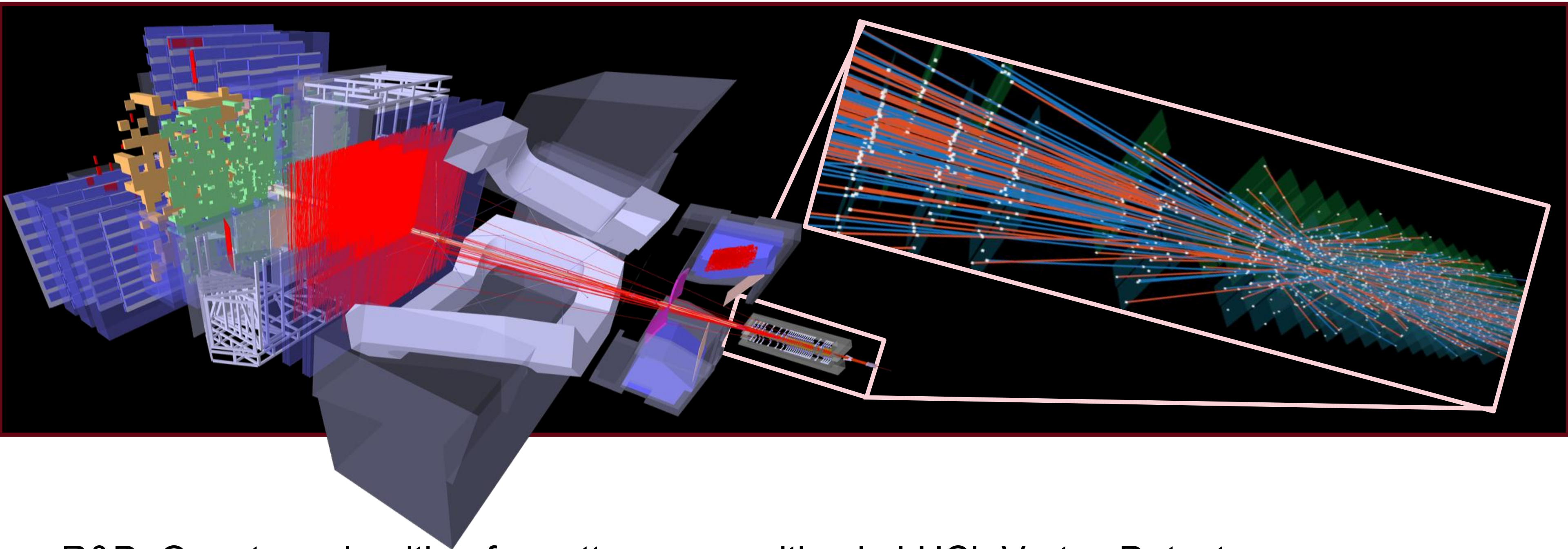
R&D: LHCb Velo detector



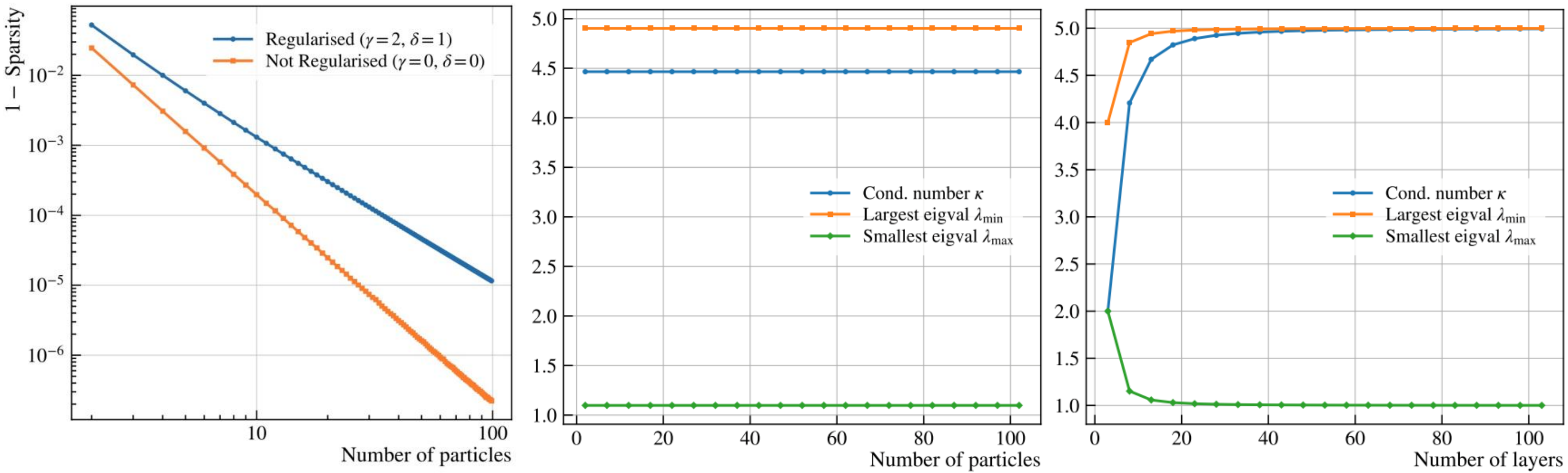
R&D: Quantum algorithm for pattern recognition in LHCb Vertex Detector:

- Algorithmic use case for a small scale quantum computer
  - Applications long term depending also on hardware developments
- R&D towards longer term with potential spin-offs
  - Investigated various approaches: HHL, QAOA, VQE, VQLS, annealing

Matrix Properties

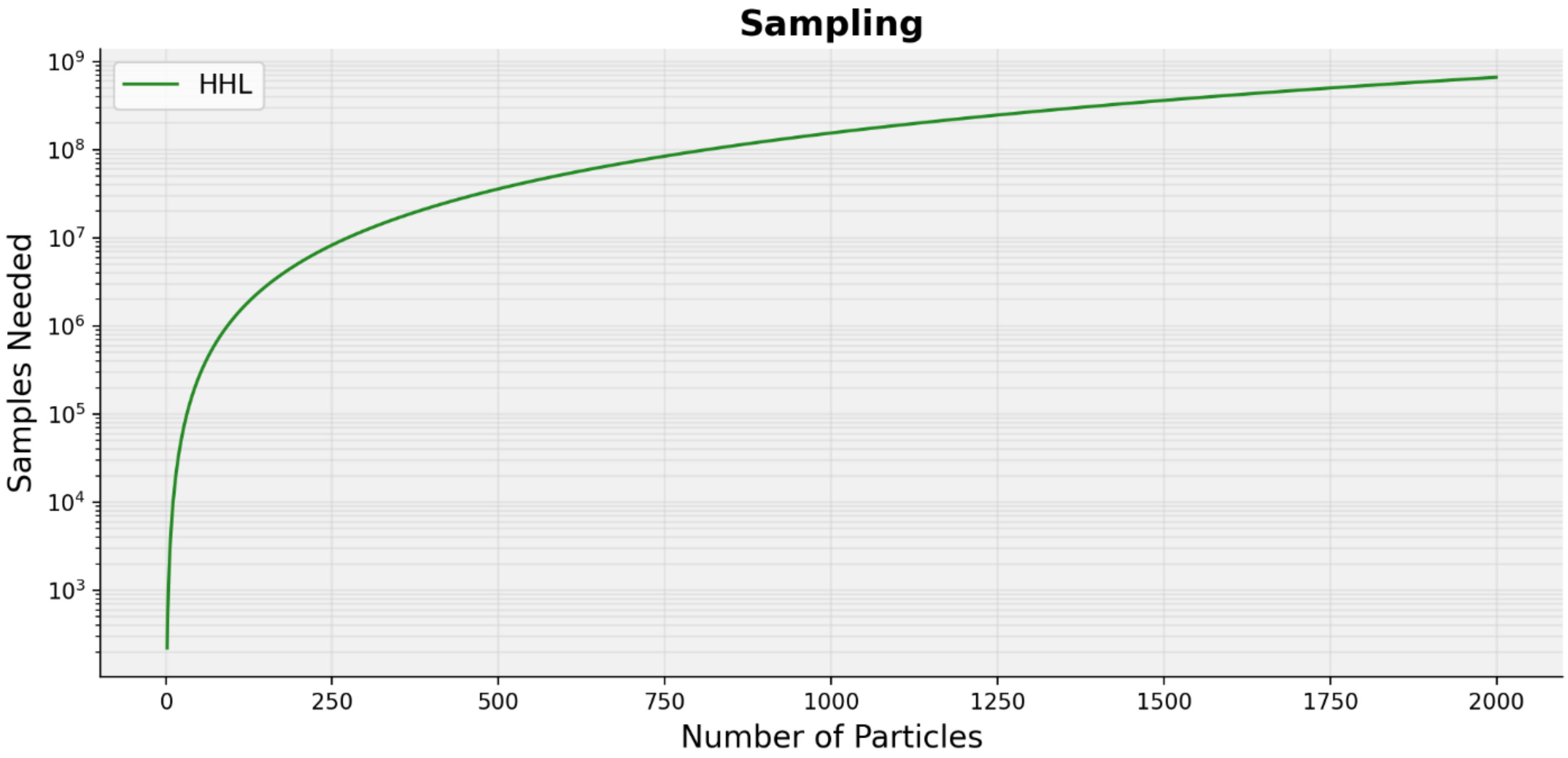
A is very well behaved: very Sparse

Low condition-number: $\kappa(A) = \frac{|\lambda_{\max}|}{|\lambda_{\min}|} \approx 5$

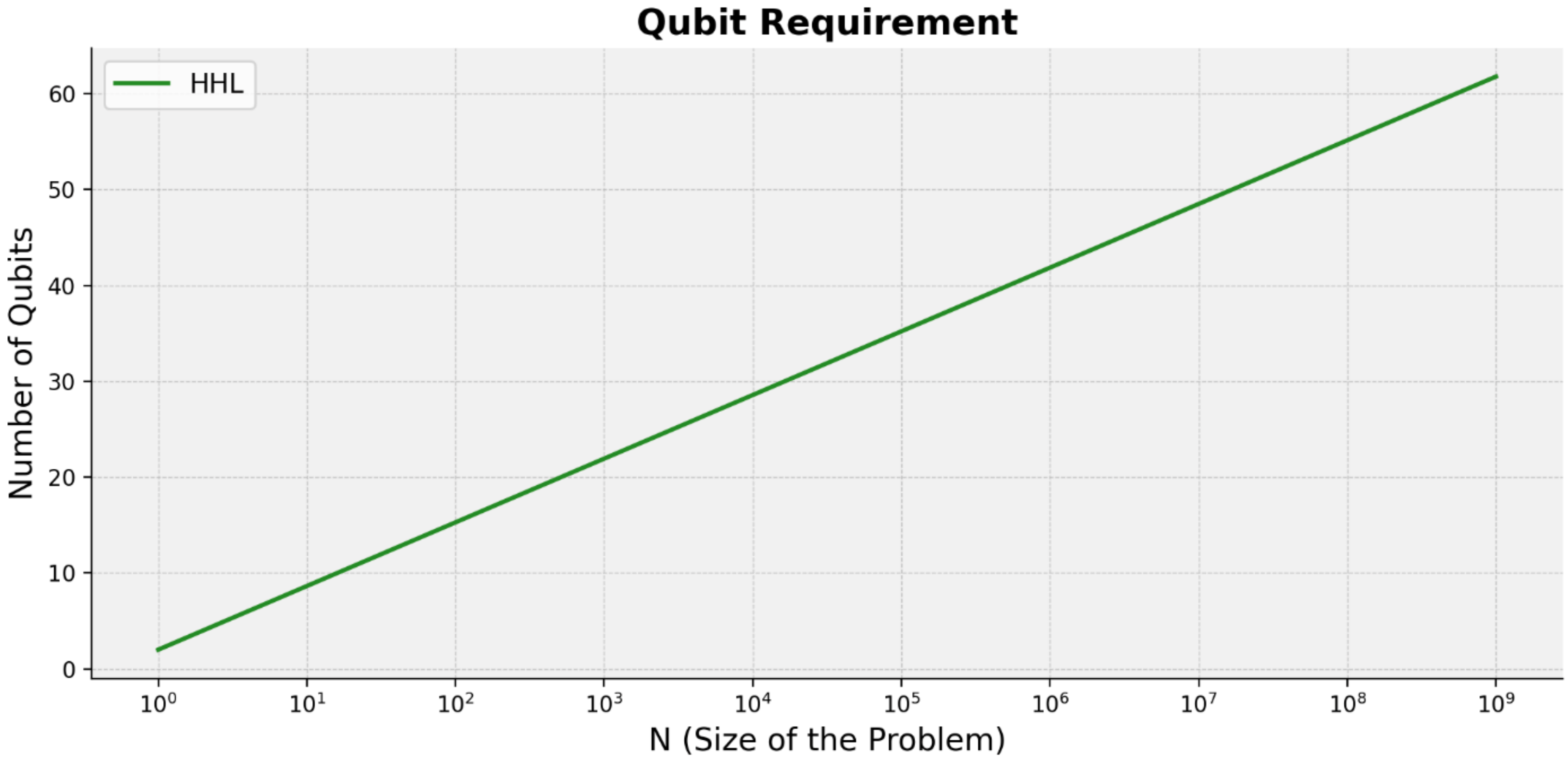


HHL Promises $O(\log(N)\kappa^2)$ runtime vs $O(N\sqrt{\kappa})$

Sampling Problem



Qubit Requirement



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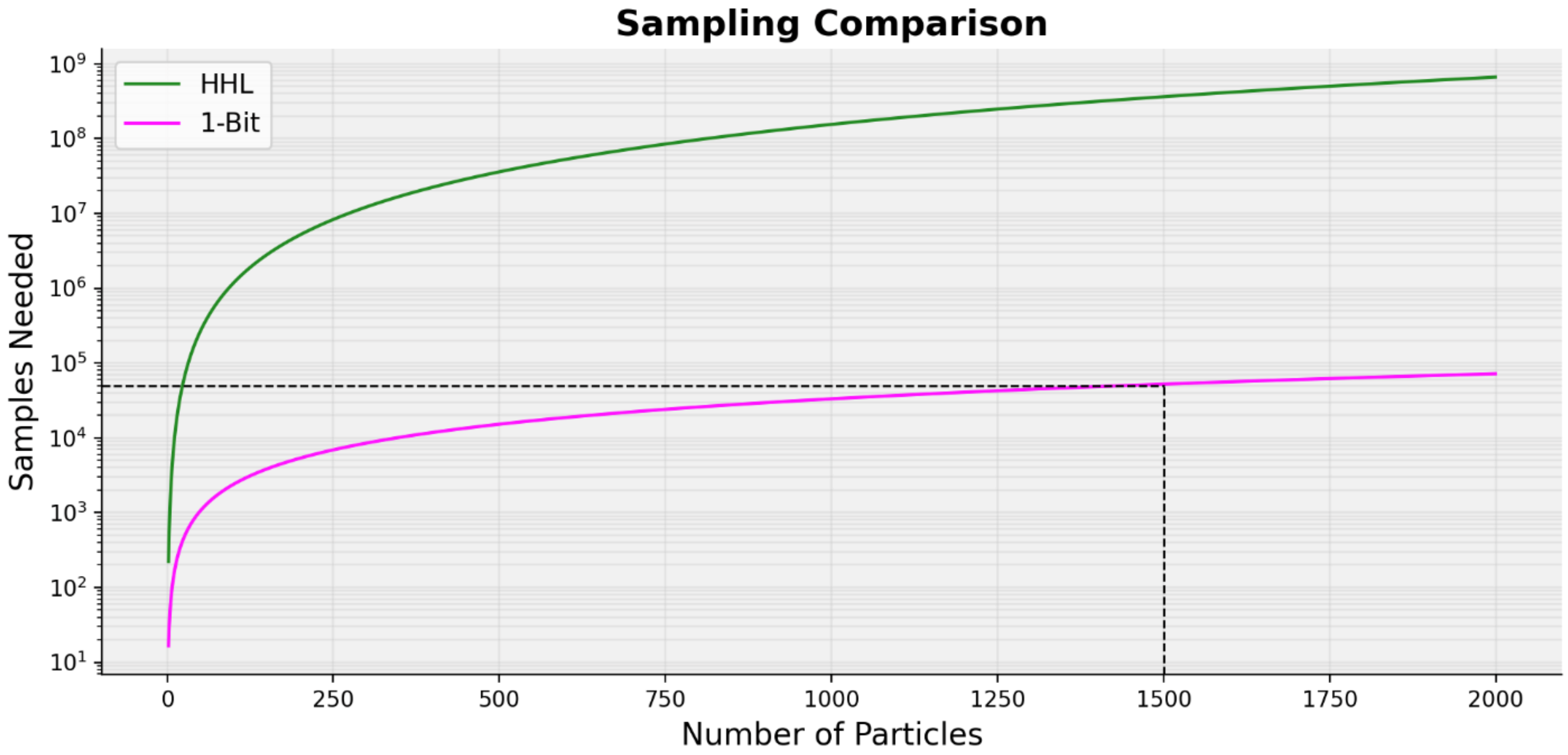
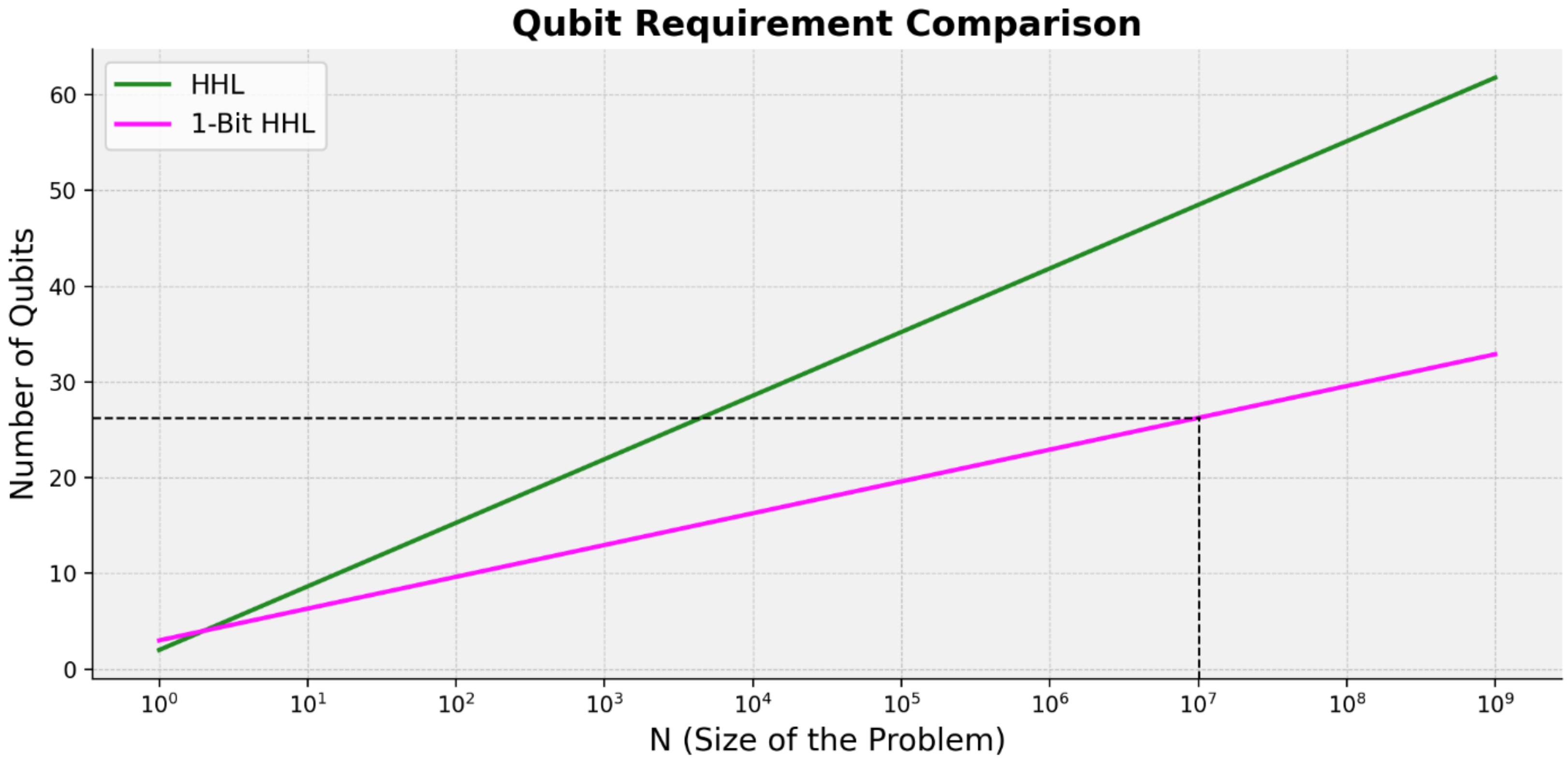
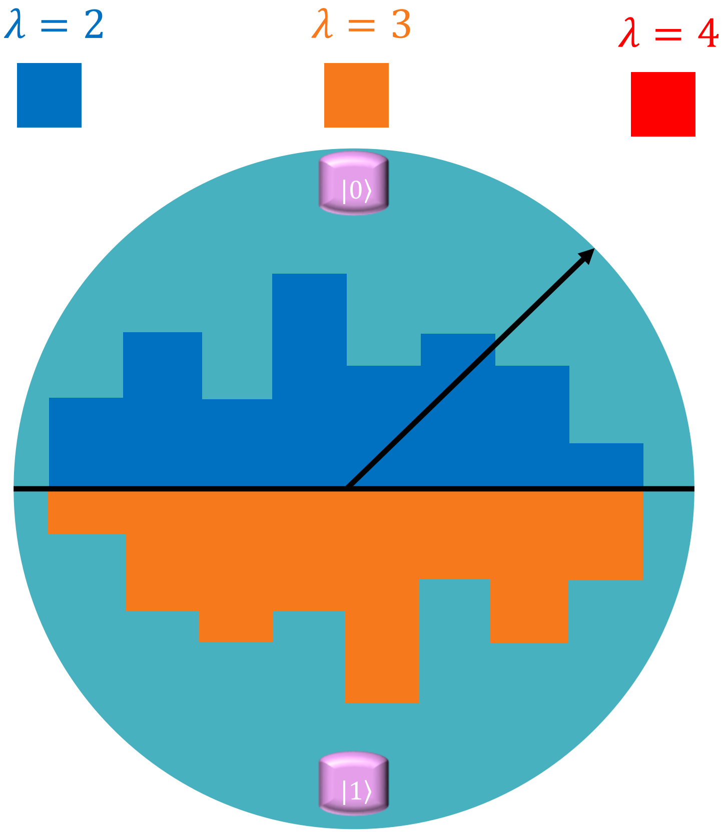


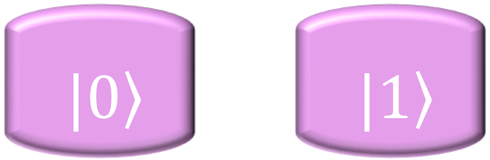
Qubit Requirement



A Quantum Clock



QPE BIN



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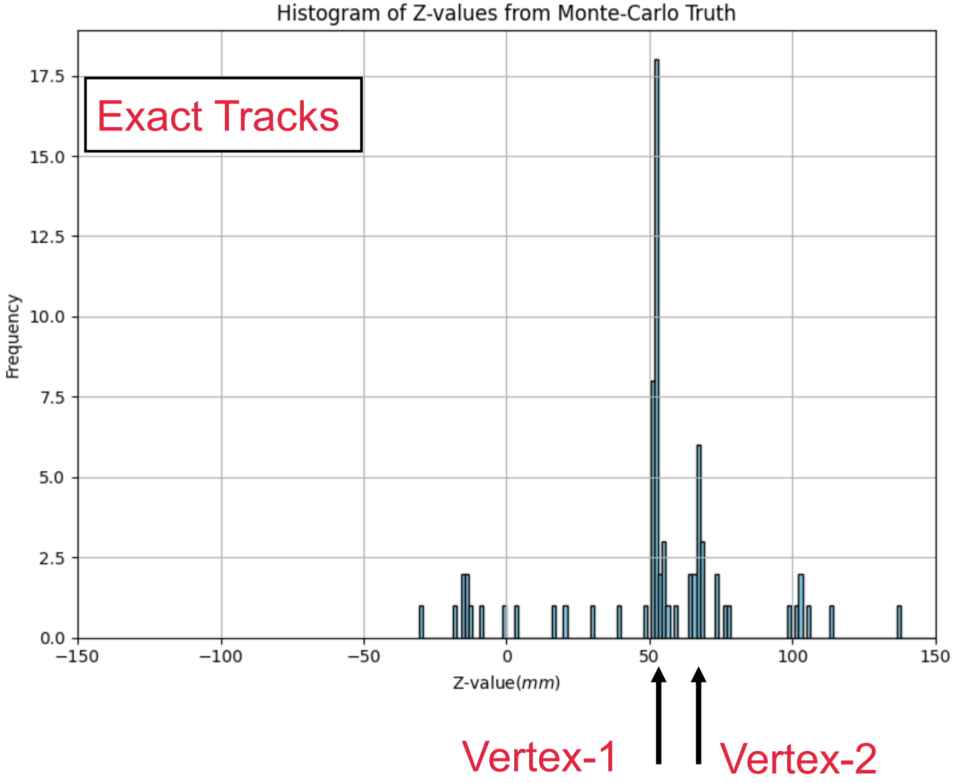
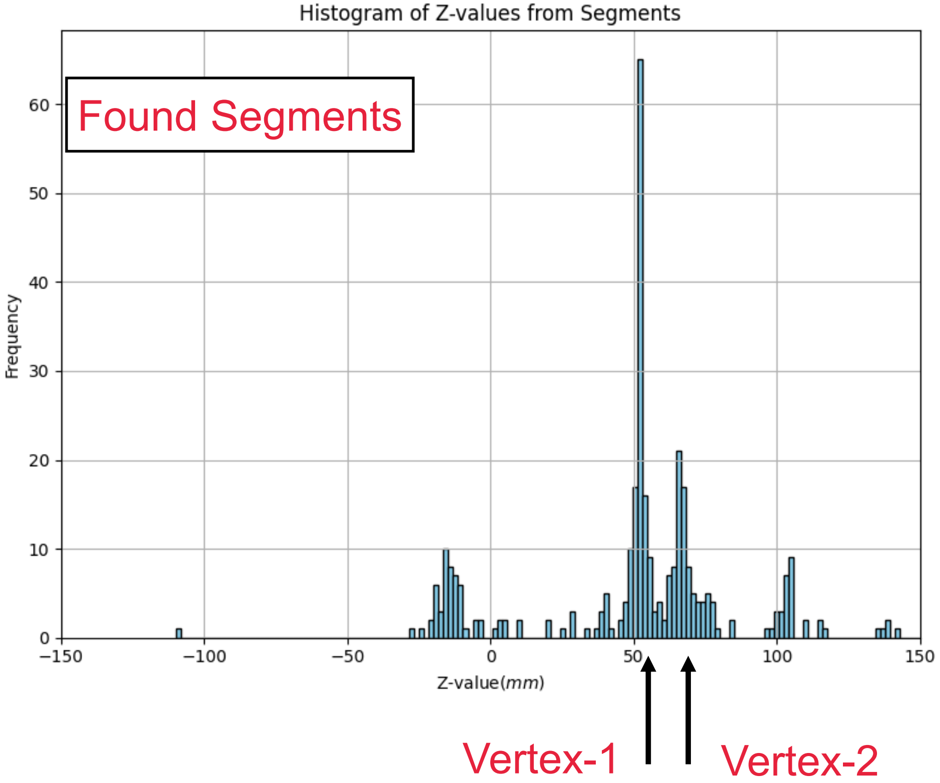
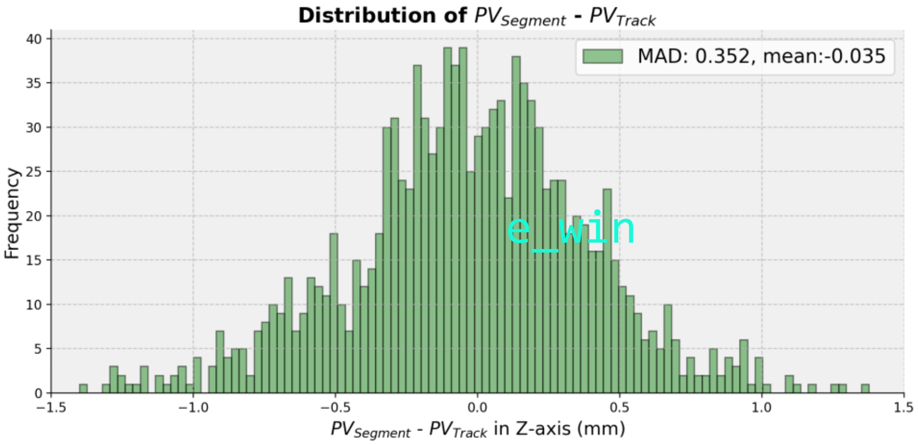
Vertex finding precision



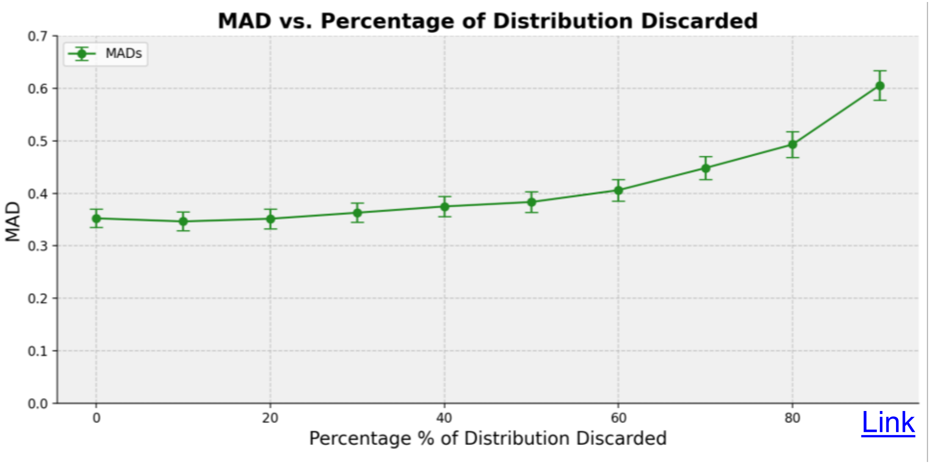


Vertex precision with fraction of segments

Distribution of precision of the extrapolated vertex point



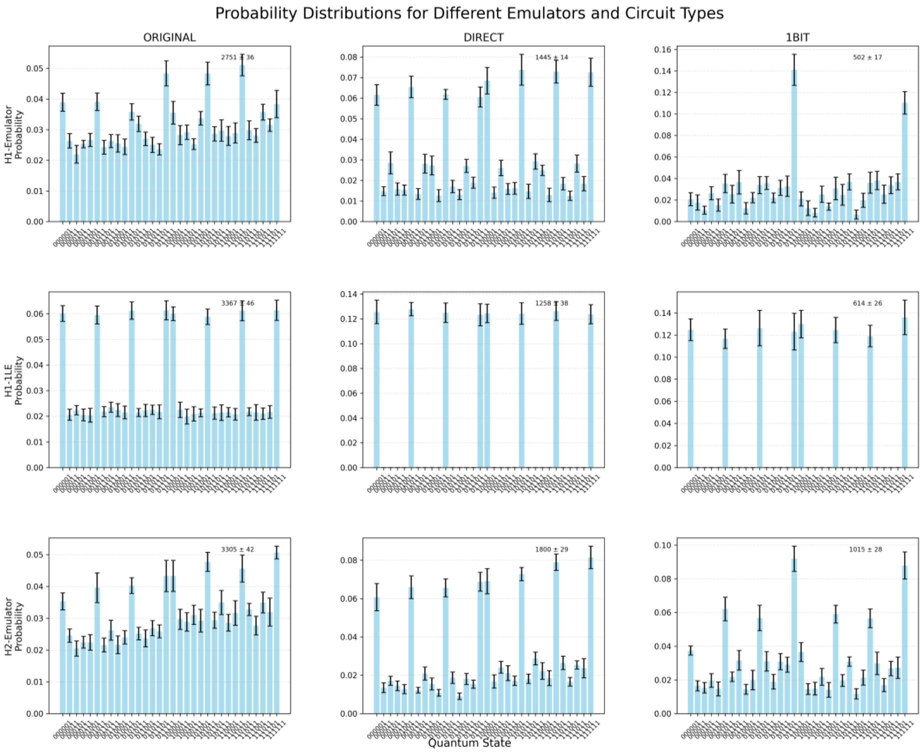
Mean Average Distance (MAD) of recontructed vertex versus the fraction of segments obtained in readout

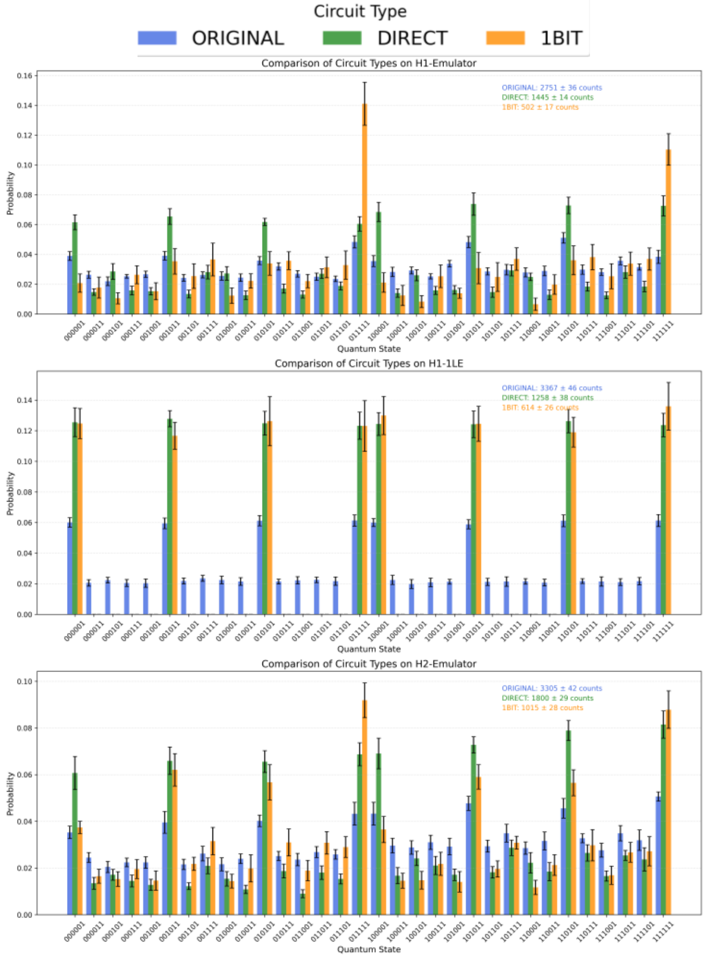


Link

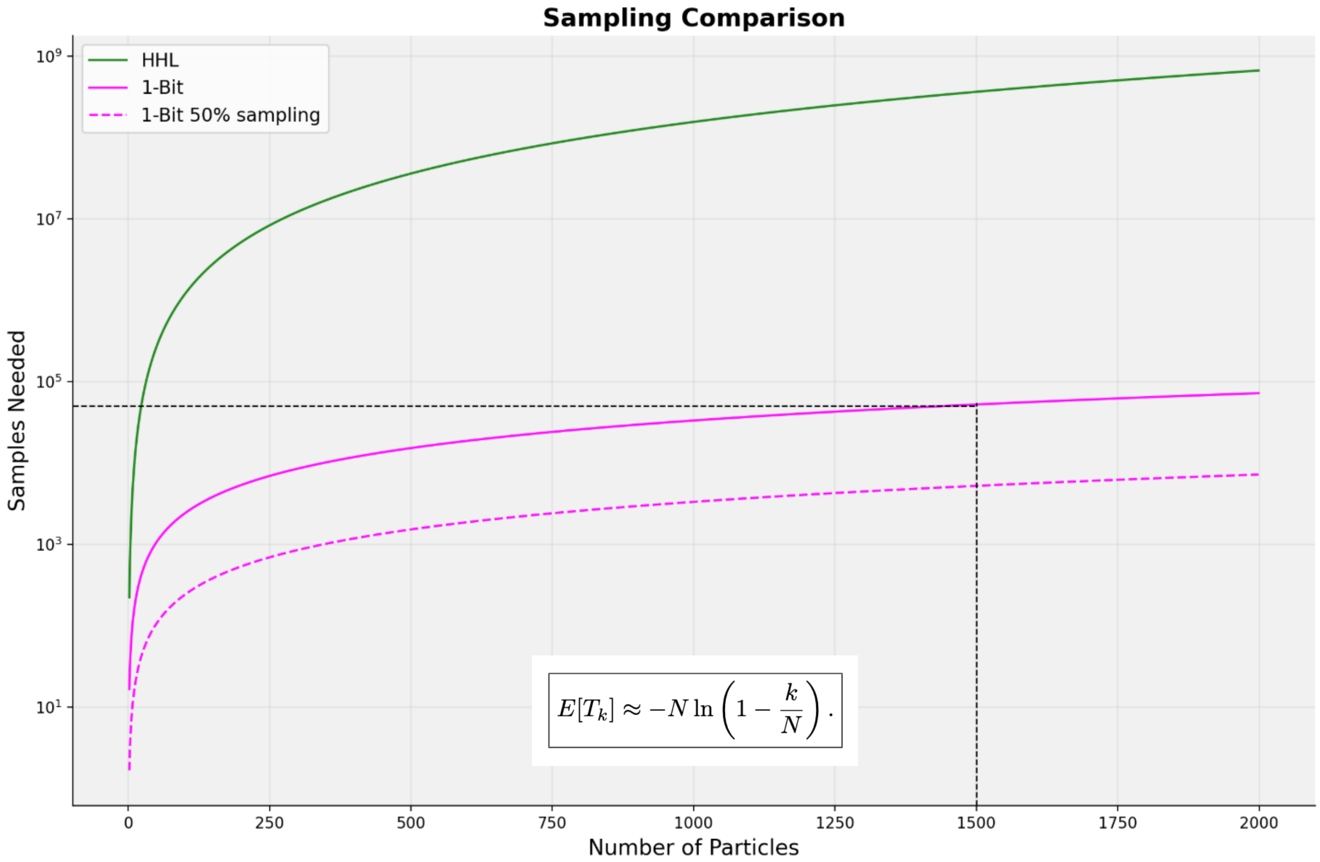
Requiring **only fraction (~50%) of all states** → strong reduction of number of readouts

Testing Quantinuum Hardware Emulators

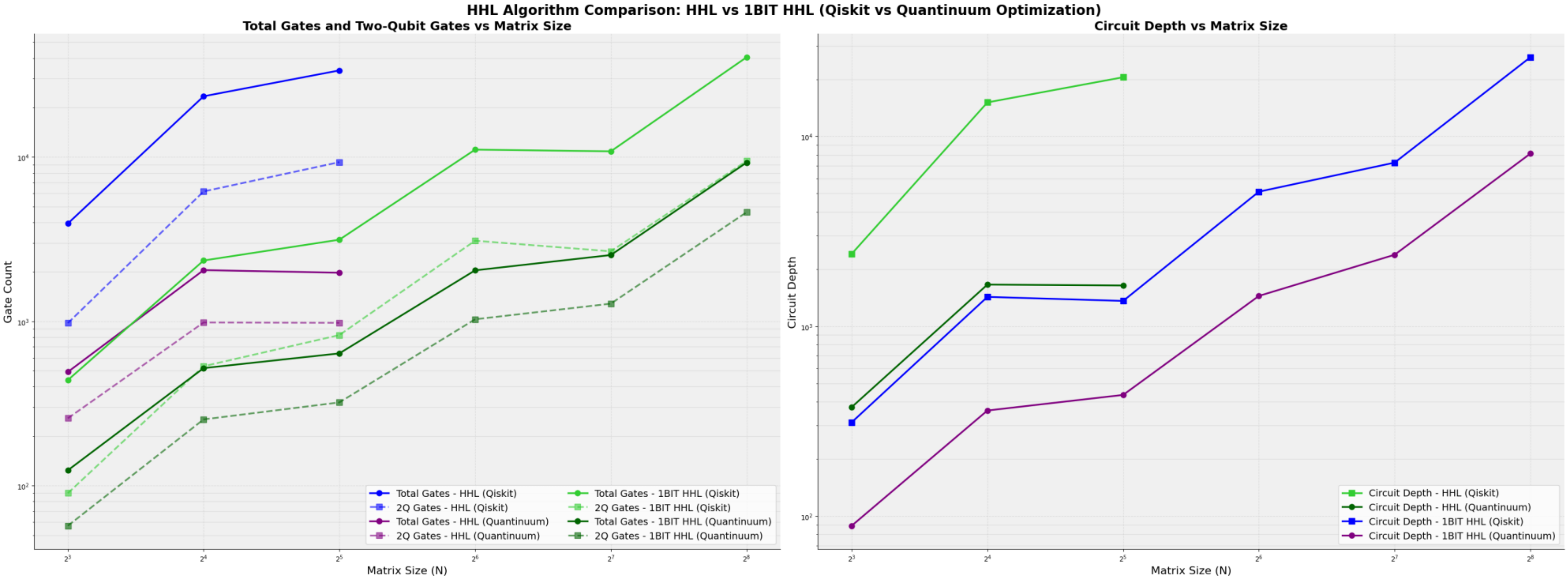




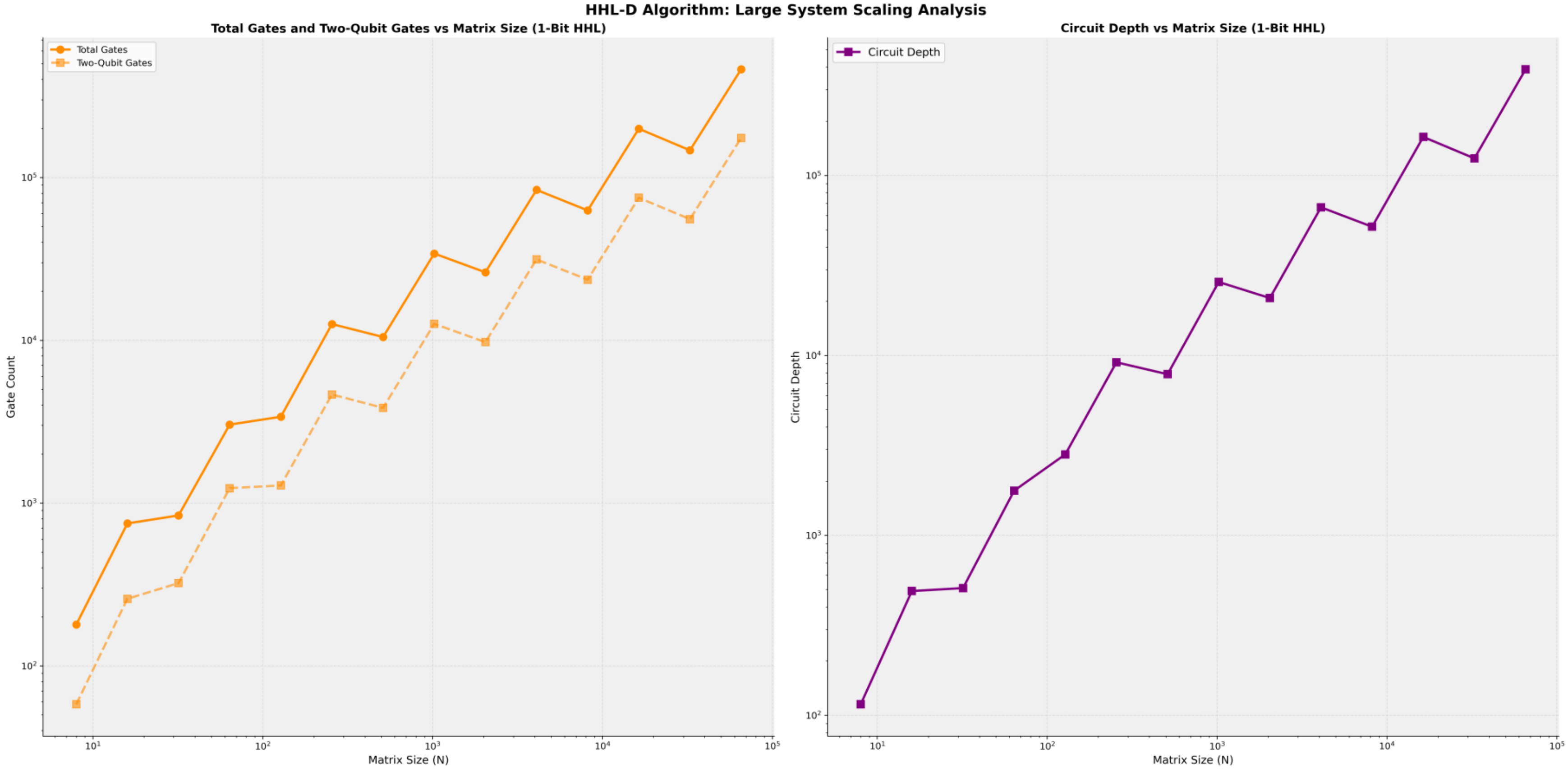
Readout issue: 1-bit QPE



Circuit Transpiled for different hardware



Circuit Depth 1-Bit



Nikhef