Musical Representations: Violence, Purity and Mathematics

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Nikhef (NL)

Structure

1. Methodology

2. Violence and purity in music

3. Musical representations of Lie groups

4. Current project

1. Methodology

2. Violence and purity in music

3. Musical representations of Lie groups

4. Current project

Creation of musical **metaphors** of mathematical groups analysed in terms of violence and purity

Creation of musical **representations** of mathematical groups analysed in terms of violence and purity

Questions

- What is a musical representation?

- Why violence and purity?

- **What is a musical representation?**

- Why violence and purity?

Definition of a mathematical group

Let "$\circ$" be a binary operation on a set S. if:

i. $(x \circ y) \circ z = x \circ (y \circ z)$ is valid for all x, y, z in S,

ii. There exists in G an element e, called the identity, such that $x \circ e = e \circ x = x$ and for all x in G.

iii. To each element x of G there corresponds an element y of G, called the inverse of x, such that $x \circ y = y \circ x = e$.

then the pair (G, o) is called a group.

A set of transformations, such that any combination of these transformations gives us another which is contained in the set

The **elements** of the group (the transformations) can be considered as **actions**.

Musical representation - Key concepts

- **Metaphor** (general)
- **Homomorphism** (mathematical)
- **Representation** (mathematical)

Metaphor

“ **The fundamental form of metaphor is A is B**…where A is metaphorically B. A is the thing to be understood and is often relatively abstract; B offers the basis for understanding A and is normally relatively concrete. Individual metaphoric expressions do not necessarily take the form A is B directly…but if an expression is metaphoric we should be able to **recast** it in the normal form A is B."

Arnie Cox, *Music and Embodied Cognition: Listening, Moving, Feeling, and Thinking*, 58

Examples

Time is a thief → Time **= A**; thief **= B**

Life is a ride → Life**= A**; ride **= B**

Homomorphism

Let $(A,\circ)$ and $(B,*)$ be binary algebraic structures. A function $f:A\to B$ is a homomorphism if, for all $x,y \in A$,

$$f(x \circ y)= f(x) * f(y)$$

A **homomorphism** is a structure-preserving function or, equivalently, a relation-preserving map. Therefore, a homomorphism from a structure A to a structure B provides a way to reformulate A in terms of B.

Representation

A *representation* of a group G is a homomorphism $\varphi : G \rightarrow GL(V)$ for some (finite-dimensional) vector space V.

GL(V) is the general linear group of V, which consists of all automorphisms of V.

A **representation** is a specific type of homomorphism and, therefore, a specific act of reformulation in mathematics.

Key concepts

- **Metaphor:** ‘A is B', in spoken or written language

- **Homomorphism:** ‘A is B’, in mathematics

- **Representation:** ‘A is B’, in mathematics, but there are very precise rules for saying this; in particular, A and B are mathematical groups

Representations: metaphors of groups

Homomorphism	=	Mathematical metaphor
Representation	=	Mathematical metaphor of a group

Central Idea

Mapping:

group $\longrightarrow$ the representation of the group (its mathematical metaphor) $\longrightarrow$ the musical representation of the group (its musical metaphor)

Objective

Creation of a musical representation theory:

groups ⟶ $\boxed{\text{compositional methodology}}$ ⟶ musical representations of groups

- As the elements are actions, they are used to generate a musical form.
- The elements of the group are associated with sensations going from violence to purity.

- What is a musical representation?

- **Why violence and purity?**

The first perception

D3 Group → describes the symmetries of an equilateral triangle

- Rotational symmetry as a *pure* symmetry

- Reflection as a *violent* symmetry

Methodology

1. Choice of a mathematical group

(ex. D3)

2. Analysis in terms of violence and purity

(rotational symmetry– *pure* reflection – *violent*)

3. The musical form - The walk in the mathematical group

4. The musical representation

(D3 = <Gentleman, Jack> (2021))

CENTRAL IDEA

A mapping from a mathematical group into its musical metaphor

Collaboration with mathematicians to extend the scope of my ideas

D3 = <Gentleman, Jack>

Form:

$$\mathbf{A} \xrightarrow{r^3} \mathbf{A_1} \xrightarrow{f^2} \mathbf{A_2} \xrightarrow{rfr^2fr} \mathbf{A_3}$$

[0-10"] [10-44"] [45-54"] [54-57"] [58-1'05"] [1'06" – 1'27"] [1'27" – 2'58"]

r = rotation

f = reflection

D3= $\langle r,f \mid r^3=f^2=e,\ frf=r^{-1} \rangle$, where e is the identity and r,s are the generators of the group D3.

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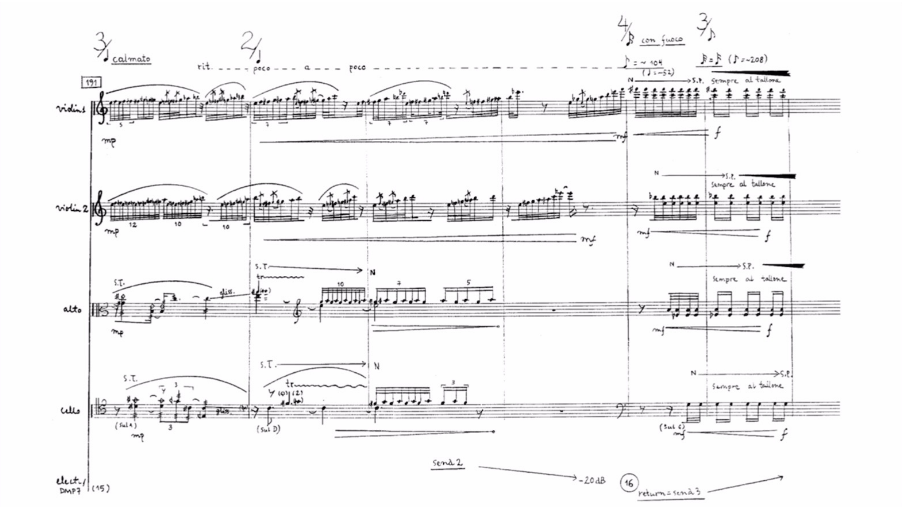
Violence and Purity

Conception → sentiments or sensations

Violence in music



Excerpt from *Nymphéa* (1987) by Kaija Saariaho

mm 195-204, the expression marking is initially “con fuoco” then “pesante, furioso,” leading to the instruction in mm 203-204, where the performers have to play with “tutta la forza.”

The overpressure generates distortion.

Excerpt from *SO(3) Etude No. 1* (2021-2023) by Petra Cini

Excerpt from *Etude No. 4* (2021-2022) by Petra Cini

less violence, a mix of violence and purity

Purity in music

Excerpt from *Extended Circular Music no. 3* by Jürg Frey

calm, regularity, simplicity and grace.

Excerpt from *Etude No.1* (2020-2021) by Petra Cini

First application of my methodology

Working with Lie groups

Application of my methodology to **continuous** groups - Lie groups

September 2021- March 2023 → monthly meetings with mathematicians Raf Bocklandt (University of Amsterdam (UVA)) and Eric Opdam (UVA) for the development of the meta-mathematical framework for the piece and later the preparation of the concert seminar I gave with Raf in late March of 2023

Collaboration

- Choice of Lie group to work with and collating of relevant knowledge relating to the group; the Lie group SO(3) was chosen.

- First stage of the creation of the meta-mathematical framework: analysis in terms of violence and purity of the Lie group SO(3).

In particular:

1. Asking the collaborating mathematicians what ideas would come to their mind when thinking of **violence and purity in the context of Lie groups.**

2. Asking the collaborators what characteristics and typology of elements of the Lie group SO(3) would be interesting to focus on to better convey what the **essence of the structure** is.

3. Asking questions about the SO(3) group aimed at showing through violence and purity, the **different characters, 'personalities,' of the elements** of the group.

1. How could we give different musical identities to the basis elements of the Lie algebra? How could we express the different 'characters' of the elements of the group starting from the Lie algebra?

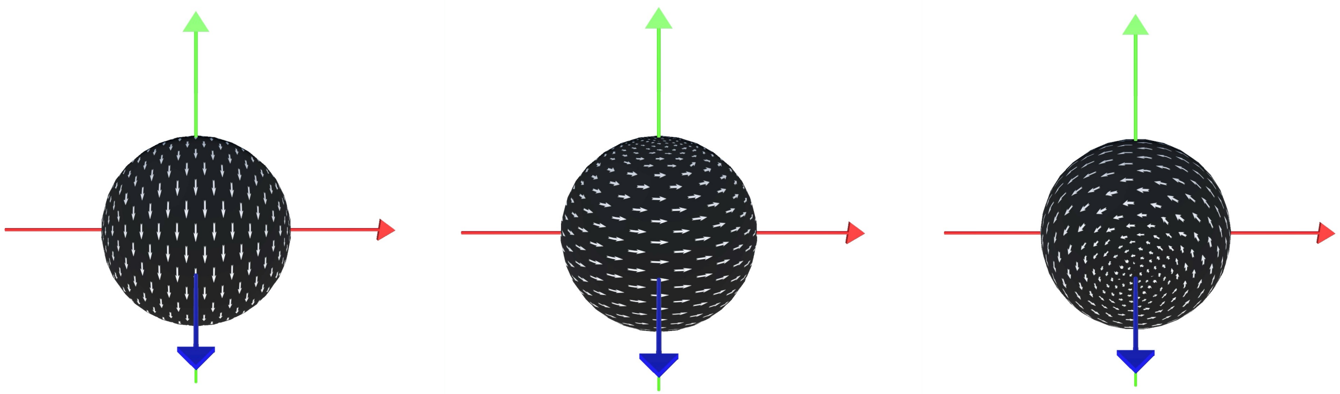
   Raf then explained to me that, in order to do that, we would need to obtain the complexification of the Lie algebra su(2), which is isomorphic to the Lie algebra so(3). Such complexification is sl(2,C), which is isomorphic to the Lie algebra so(3,C), the complexification of so(3). This would allow to get from three generators of rotations (the elements of the basis of the Lie algebra so(3)), one 'stretch' and two 'slants' (the elements of the basis of the Lie algebra sl(2,C)).

Key Facts

- Lie groups are simultaneously **groups and spaces**.
- Every Lie group has a **simpler structure** that is **associated** to it, namely a Lie algebra, from which a lot of information about the Lie group can be inferred.
- For every Lie group there is a way to express its elements by using the elements of its Lie algebra, namely the **exponential map,** which sends elements of the Lie algebra to elements of the Lie group.

The SO(3) group

The group of all rotations about the origin of three-dimensional Euclidean space, or, equivalently, the Lie group that represents **all the rotations of a 2-sphere.**



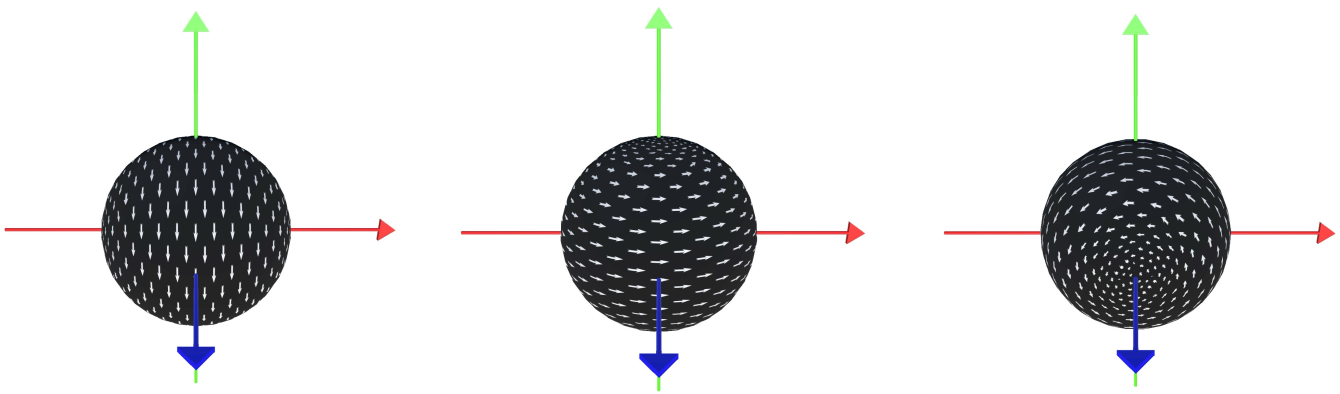
Infinite and continuous

$$SO(3) = \{X \in M_3(R) | X^T = X^{-1}, \mathbf{det}(X) = 1\}$$

The Lie algebra so(3)

The Lie algebra so(3) consists of generators of **infinitesimal** rotations.

It is a three-dimensional space.



$$so(3) = \{X \in M_3(R) | e^{tX} \in SO(3), \forall t \in R\}$$

SO(3) ETUDES (2021-2023)

- **SO(3) Etude No. 1:** focuses on the presentation of the Lie algebra so(3), with a focus on its elements as generators of micro rotations, and on the use of the exponential map to reach elements of the Lie group SO(3) starting from elements of the Lie algebra so(3).

- **SO(3) Etude No. 2 :** focuses on the musical representation of the Lie group SO(3), that is presented by generating a walk within the group leading through a selection of its elements.

SO(3) Etude No. 1

5″ - 10″ Introduction of the organic complement	10″ - 16″ The presentation of the three dimensions of the Lie algebra so(3)	17″ - 50″ The presentation, one after the other, of the basis elements of so(3), i.e. the generators of micro rotations	51″ - 1′23″ The complexification of su(2) ≌ so(3), so from su(2) ≌ so(3) to sl(2,C)	1′23″ - 1′36″ The presentation of the three dimensions of the Lie algebra so(3,C)	1′37″ - 1′49″ And I will manage to fix you'; from so(3,C) back to so(3)
1′50″ - 1′56″ 'Mission completed'; back to so(3).	1′57″ - 2′57″ Exponential map applied to z to reach a point in SO(3)	2′58″ - 2′59″ Reaching of the corresponding point of SO(3)	3′00″ - 3′30″ Reaching the point corresponding to x in SO(3) through a combination of different exponentiated elements of the Lie algebra so(3)	3′31″ - 3′35″ Reaching of the corresponding point of SO(3)	3′36″ - 3′48″ The piece dies down, in the organicity of reality, we wait

17″ – 50″

The presentation, one after the other, of the basis elements of so(3), i.e. the generators of micro rotations.

The embellishments and the frantic quality of the musical realization of the basis elements x, y and z of so(3) evoke the elements of so(3)'s function of generators of micro rotations. The element x is presented first, then y and z.

The following three elements x, y and z constitute a basis of so(3)

$$x = \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; y = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}, \; z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}.$$

Every element $s \in so(3)$ can be written as $s = ax + by + cz$ for $a,b,c \in R$.

A micro rotation matrix generated by x is $dx = d\theta x + I$, one generated by y is $dy = d\beta y + I$, one generated by z is $dz = d\alpha z + I$, for $d\theta, d\beta$ and $d\alpha$ generated infinitesimally small. dx is a micro rotation about the z-axis, dy a micro rotation about the y-axis and dz a micro rotation about the x-axis.

1’37” – 1’49”
‘And I will manage to fix you’; from so(3,C) back to so(3)
By fixing through a determined Galois action the real part of so(3,C), we obtain so(3) again. A voice thus sings: “and I will manage to fix you”.
The simplest way to define so(3,C) is as the set of complex matrices X such that $X^T = -X$ (skew symmetric). In this case, if we take the simplest Galois action where $\alpha_\sigma(X)$ is the entry-wise complex conjugation of X, then the subalgebra fixed for this action by α is so(3).

3’31” – 3’35”
Reaching of the corresponding point of SO(3)
A voice whispers ‘sing for me ’and another one sings a melody that reminds of a jingle to signify that we have arrived. Then a loud, violent low-pitched sound erupts: we jump on the point we were walking to after the obsessive marching has come to an end. This point is the rotation by 30° around the x axis in SO(3), which will be the first element that will act on the musical theme in the musical space.
$R_x(30) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(30°) & -\sin(30°) \\ 0 & \sin(30°) & \cos(30°) \end{bmatrix}$

SO(3) Etude No. 2

The second stage of the creation of the meta-mathematical framework: the musical representation of the Lie group SO(3), taking into account the group's points of interest.

The action of the group SO(3) **on a musical object** contained within a “musical space.”

1. Choice of a group → **2.** Analysis in terms of violence and purity →

Lie group SO(3)

- The elements of the SO(3) group are identified and perceived as violent when seen as points, and as pure when seen as rotations acting on a musical vector space.
- Rotations with a larger angle of rotation are realised as transformations that take longer to be carried out and are therefore perceived as **purer.**

3. The musical form – The walk within the group

1. To the three rotations around the **x, y and z axes** — attribution of their **musical expressions** as actions on the musical space.

2. For each of the used elements of SO(3) — **axis/angle representation**

3. To the **values** of the axis/angle representations — attribution of expressions and musical functions

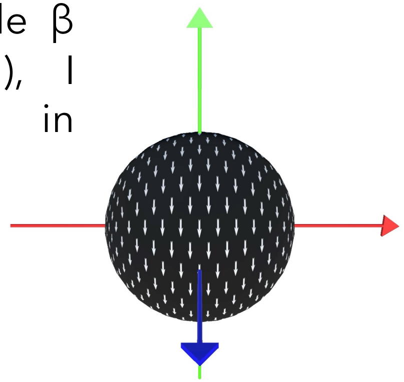
1. To the three rotations around the **x, y and z axes** → attribution of their **musical expressions** as actions on the musical space.

- Every element (rotation) of SO(3) can be realised as the multiplication of three rotations with determined angles of rotation **around the x, y and z axes**

In particular:

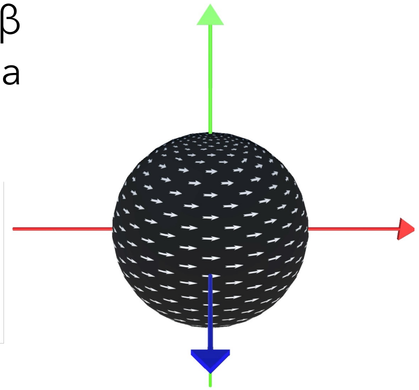
- To the rotation by an angle β around the x axis $R_x(\beta)$, I associated a change in **dynamics.**

$$R_x(\beta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix};$$



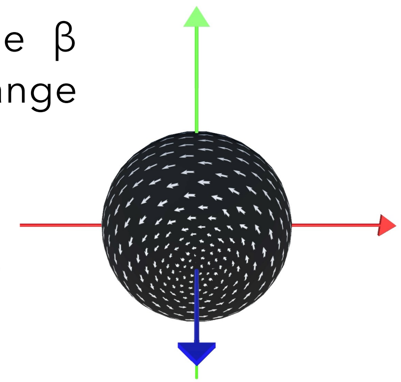
- To the rotation by an angle β around the y axis $R_y(\beta)$, a change in **velocity.**

$$R_y(\beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix};$$



- To the rotation by an angle β around the z axis $R_z(\beta)$, a change in **pitch.**

$$R_z(\beta) = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$



2. For each of the used elements of SO(3) $\longrightarrow$ **axis/angle representation**

$$R_y(45)R_x(60)= \begin{bmatrix} \cos(45°) & 0 & \sin(45°) \\ 0 & 1 & 0 \\ -\sin(45°) & 0 & \cos(45°) \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(60°) & -\sin(60°) \\ 0 & \sin(60°) & \cos(60°) \end{bmatrix}$$

Axis/angle representation of $R_y(45)R_x(60)$: { [0.7700762, 0.5524826, -0.318976], 73.7200938 }

3. To the **values** of the axis/angle representations → attribution of expressions and musical functions

- The magnitude of the angle of each axis/angle representation determines the temporal duration of the element's action.

$R_x(30)$: { [1, 0, 0], **30** } temporal duration = **8'' ca.**

$R_y(45)R_x(60)$: { [0.7700762, 0.5524826, -0.318976], **73.7200938** } temporal duration = **17'' ca.**

- The x, y and z components of the unit vectors of each axis/angle representation are realised by using, with **different intensities**, the **materials** that were used for the basis elements of the Lie algebra so(3)

$R_y(45)R_x(60)$: { [**0.7700762, 0.5524826, -0.318976**], 73.7200938 }

x=0.7700762, y=0.5524826 and z= -0.3189760,

the x component will be present with more intensity, followed by y and z.

4. The group representation

Musical object contained in the "musical space".

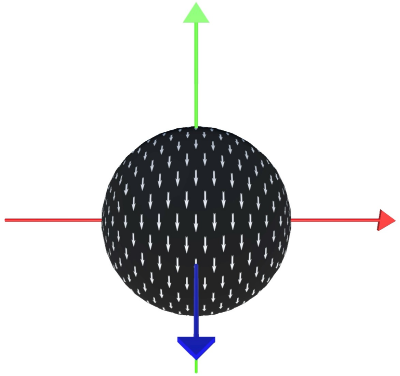
A

Musical material chosen as the theme of *SO(3) Etude No. 2 (2021/23)*; transformations of this material occur throughout the piece, following the musical actions of the group's elements.


arpeggiando, with increasing intensity
Piano
tr
intimate, molto espressivo, con Ped.
semplice, trattenuto

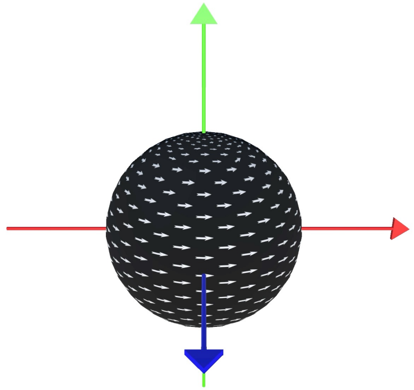
Change in **dynamics**

$$R_x(\beta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix};$$



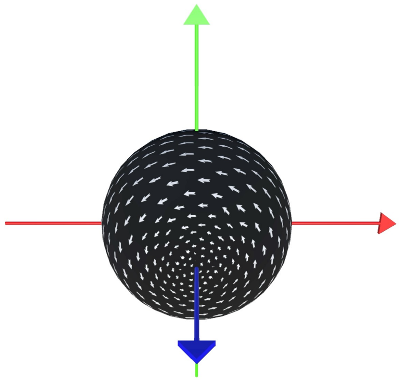
Change in **velocity**

$$R_y(\beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix};$$



Change in **pitch**

$$R_z(\beta) = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$



SO(3) ETUDE 2

m. 1 Introduction of the organic complement	**mm. 2-8** The theme is presented	**m. 9** $R_x(30)$	**mm. 10-16** The theme is presented again in its transformed form	**m. 17** $R_y(30)$	**mm.18-24** The theme is presented again in its transformed form	**m. 25** $R_z(60)$
mm. 26-32 The theme is presented again in its transformed form	**m. 33** $[R_z(60)R_y(30)R_x(30)]^{-1}$	**mm. 34-41** The theme is presented	**m. 42** $R_y(30)R_z(45)$	**mm. 43-49** The theme is presented again in its transformed form	**m.50** $R_x(90)$	**mm. 51-57** The theme is presented again in its transformed form
m. 58 $[R_x(90)R_y(30)R_z(45)]^{-1}$	**mm. 59-65** The theme is presented	**m. 66** $R_y(45)R_x(60)$	**mm. 67-73** The theme is presented	**m. 74** $R_x(30)R_z(30)$	**mm. 75-81** The theme is presented again in its transformed form	**m. 82** $R_z(180)$
mm. 83-89 The theme is presented again in its transformed form	**m. 90** $[R_z(180)R_x(30)R_z(30)R_y(45)R_x(60)]^{-1}$	**mm. 91-97** The theme is presented	**m. 98** $R_x(180)R_y(180)R_z(180)$	**mm. 99-106** The theme is presented		

m. 33

$[R_z(60)R_y(30)R_x(30)]^{-1}$

Since the angle of rotation of $R_z(60)R_y(30)R_x(30)$ is (approximately) 65°, we have that its inverse $[R_z(60)R_y(30)R_x(30)]^{-1}$ will have angle of rotation of (approximately) 295°. Hence, in accordance with the temporal length of associated to the previous elements, it takes 1'13" ca. to be made.

The material used in the first etude for the loud, violent low-pitched sound used to represent that we have reached the point of SO(3) is stretched for the duration of the 1'13'' ca. and reversed, to signify that the we are taking the inverse of the composition of the previous elements. We are walking back to our initial position.

The materials that had been used in the exponentiation processes, derived from the musical realizations of x,y, and z, are presented to signify that the rotation $[R_z(60)R_y(30)R_x(30)]^{-1}$ has been constructed using the Lie algebra elements x,y,z. These materials are present with different intensities in accordance with the values of the entries x, y, z of the axis of rotation of the element $[R_z(60)R_y(30)R_x(30)]^{-1}$, following the methodology mentioned at the beginning of the section. Such materials are also reversed to signify that we are taking the inverse of the composition of the previous elements. The piano materials have a coloring function.

$$[R_z(60)R_y(30)R_x(30)]^{-1} = \left(\begin{bmatrix} \cos(60°) & -\sin(60°) & 0 \\ \sin(60°) & \cos(60°) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(30°) & 0 & \sin(30°) \\ 0 & 1 & 0 \\ -\sin(30°) & 0 & \cos(30°) \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(30°) & -\sin(30°) \\ 0 & \sin(30°) & \cos(30°) \end{bmatrix}\right)^{-1}$$

Axis-angle representation of $R_z(60)R_y(30)R_x(30)$: { [0.1693787, 0.6321298, 0.7561236], 65.40094}

The axis of rotation of $[R_z(60)R_y(30)R_x(30)]^{-1}$ is the same of $R_z(60)R_y(30)R_x(30)$, its angle of rotation is approximately 360° - 65° = 295°.

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Current project

About the SL(2,C) group

- With Eric Opdam and Raf Bocklandt
- The Dutch collective **Nieuw Amsterdams Peil**
- 26 April 2026 Muziekgebouw aan 't IJ (Amsterdam, NL)

Muziekgebouw NL

SUN 26 APR 2026 16:00 - 17:00

Violence and Purity

Nieuw Amsterdams Peil

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