# Subleading power corrections to the color-singlet transverse momentum spectra

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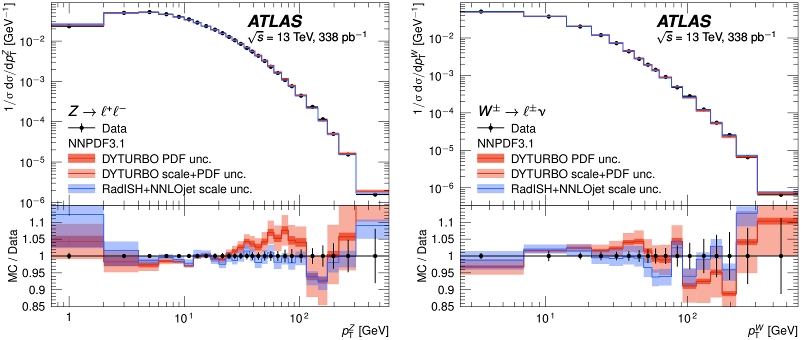
# The transverse momentum spectrum

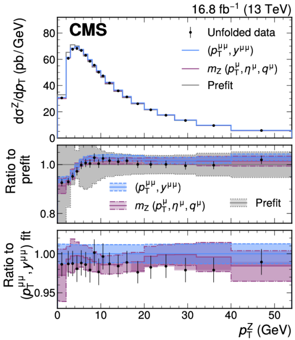
The boson **transverse momentum** $q_{\mathrm{T}}$ in Drell-Yan production processes is one of the **most precisely measured observables** at the LHC [ATLAS '24]

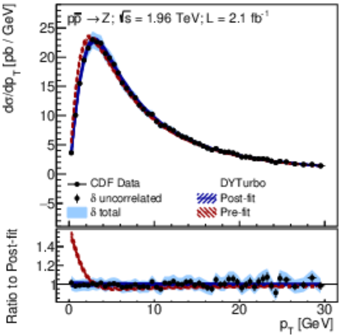


# The transverse momentum spectrum

In recent years, it has been used to obtain precise measurements of

- The $W$ **boson mass** [CMS '24]
- The **strong coupling** $\alpha_s(m_Z)$ [S. Camarda, G. Ferrera, M. Schott '22]



# Fighting the Pareto principle

- The theoretical predictions for the $q_\mathrm{T}$ spectrum are typically obtained by **matching resummation and fixed order** calculations

$$\frac{d\sigma}{dq_\mathrm{T}} = \frac{d\sigma^\mathrm{res}}{dq_\mathrm{T}} + \frac{d\sigma^\mathrm{NS}}{dq_\mathrm{T}} = \frac{d\sigma^\mathrm{res}}{dq_\mathrm{T}} + \frac{d\sigma^\mathrm{FO}}{dq_\mathrm{T}} - \left.\frac{d\sigma^\mathrm{res}}{dq_\mathrm{T}}\right|_\mathrm{FO}$$

## The Pareto principle

The Pareto principle states that for many outcomes, roughly **80% of consequences** come from **20% of causes**

- In our calculations, **80%** (or more) **of the computational time** is typically spent computing **20%** (or less) **of the result** (the **non-singular contributions**)

**Can we fix this imbalance?**

# Factorization and resummation beyond leading power

- **At leading power** (LP), in the limit where $q_{\mathrm{T}} \ll Q$, **the cross section** for color singlet production **factorizes** as

$$\frac{d\sigma}{d\Phi_0\, dq_{\mathrm{T}}} = \sum_{i,j} H_{ij}(\Phi_0;\mu) \int d^2\vec{k}_a\, d^2\vec{k}_b\, d^2\vec{k}_s\, \delta\Big(q_{\mathrm{T}} - \Big|\vec{k}_a - \vec{k}_b - \vec{k}_s\Big|\Big) \times B_i\left(x_a, \vec{k}_a; \mu, \frac{\nu}{\omega_a}\right) B_j\left(x_b, \vec{k}_b; \mu, \frac{\nu}{\omega_b}\right) S_{ij}\left(\vec{k}_s; \mu, \nu\right)$$

- The factorization allows for the resummation to be performed by solving **renormalization group equations** (RGEs)

**Can we write a factorization theorem at next-to-leading power?**

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# Subleading power corrections

- We consider the process of production of a **color singlet** (inclusive Drell-Yan for now) from a **hadronic scattering**

$$h_a\, h_b \to \mathrm{CS}(q) + X$$

- Two common parametrizations of the **color-singlet momentum** $q$ are

$$q = \left(\sqrt{Q^2 + q_{\mathrm{T}}^2}\cosh Y\,, q_{\mathrm{T}}\cos\phi, q_{\mathrm{T}}\sin\phi, \sqrt{Q^2 + q_{\mathrm{T}}^2}\sinh Y\right)$$

and

$$q = \left(\frac{q^- + q^+}{2}, q_{\mathrm{T}}\cos\phi, q_{\mathrm{T}}\sin\phi, \frac{q^- - q^+}{2}\right)$$

- In the following, we will use **light-cone coordinates** and study the differential cross section

$$\frac{d\sigma}{dq^-\, dq^+\, dq_{\mathrm{T}}^2} = 2\pi\,\frac{d\sigma}{dq^-\, dq^+\, dq_{\mathrm{T}}^2\, d\phi}$$

# Subleading power corrections

- The N$^k$LO QCD **perturbative expansion** of the differential cross section can be written as

$$\frac{d\sigma}{dq^- \, dq^+ \, dq_{\mathrm{T}}^2} = \sum_{n=0}^{k} \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^n \left[ D_n(q^-, q^+)\, \delta(q_{\mathrm{T}}^2) + \frac{C_n(q^-, q^+, q_{\mathrm{T}}^2)}{q_{\mathrm{T}}^2} \right]$$

- The **logarithmic structure** of the $C_n$ coefficients is well known and given by

$$C_n(q^-, q^+, q_{\mathrm{T}}^2) = \sum_{m=0}^{2n-1} \log^m \left(\frac{q^- q^+}{q_{\mathrm{T}}^2}\right) C_{nm}(q^-, q^+, q_{\mathrm{T}}^2)$$

- The **power structure** of the $C_{nm}$ coefficients is obtained by expanding them with respect to $q_{\mathrm{T}}^2$ and is given by

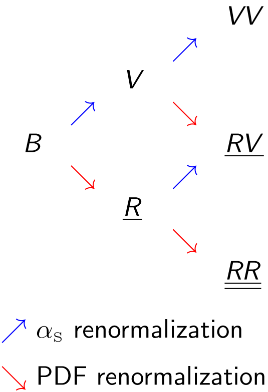
$$C_{nm}(q^-, q^+, q_{\mathrm{T}}^2) = \sum_{p=0}^{\infty} \left(\frac{q_{\mathrm{T}}^2}{q^- q^+}\right)^p C_{nmp}(q^-, q^+)$$

# State of the art

- At **NLO**, several results are available using both
  - $\mathcal{T}_0$ [Boughezal, Isgrò, Petriello '18] [Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18]
  - $q_\mathrm{T}$ [Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18] [Cieri, Oleari, Rocco '19] [Ferrera, Ju, Schönherr '24]

  as resolution variables
- At **NNLO**, only partial results are available including
  - The $\mathcal{T}_0$ LL contribution [Moult, Rothen, Stewart, Tackmann, Zhu '16] [Boughezal, Liu, Petriello '16]
  - A subclass of virtual diagrams for $q_\mathrm{T}$ [Oleari, Rocco '19]
- At $\mathbf{N^3LO}$, only the $\mathcal{T}_0$ LL contribution has been computed [Vita '24]
- In this talk, I will present a systematic way of computing the **subleading power corrections** at **NNLO**, i.e. the $C_{2mp}$ coefficients for $0 \le m \le 3$ and $p \ge 1$

# General picture




- $B$, $V$ and $VV$: No final-state partons, 2 degrees of freedom: $q^-$ and $q^+$
  - $\rightarrow$ Proportional to $\delta(q_{\rm T}^2)$: **do NOT contribute at subleading power**
- $R$, $RV$: One final-state parton, 4 degrees of freedom: $q^-$, $q^+$, $q_{\rm T}^2$ and $z$
  - $\rightarrow$ One PDF convolution, **no additional integrals to be done**
- $RR$: Two final-state partons, 7 degrees of freedom
  - $\rightarrow$ **Up to three integrals to be done**
  - $\rightarrow$ **Subject of the talk**

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# The method of regions

Given two parameters $a$ and $b$ with $a \gg b = \lambda\, a$, we study the integral

$$\int_0^a \frac{dx}{(x+a)\,(x+b)} = \frac{1}{a-b}\,\log\left(\frac{a+b}{2b}\right) = \frac{1}{a}\,\log\left(\frac{a}{2b}\right) + \mathcal{O}\left(\frac{b}{a}\right)$$

Using the **method of regions**, we can write the integral as a **sum of** two **integrals computed in** two **different regions**

- **Hard** region: $x \sim a$

$$\int_0^a \frac{dx}{(x+a)\,x} = \frac{1}{a}\left(\log\frac{a}{2} - \log 0\right)$$

- **Soft** region: $x \sim \lambda\, a$ **NB:** $\theta(a-x) \to \theta(a) = 1$

$$\int_0^\infty \frac{dx}{a\,(x+b)} = \frac{1}{a}\left(\log\infty - \log b\right)$$

# The SCET power expansion

- From the **SCET**, we know that the contributions to the $q_{\mathrm{T}}$ spectrum will come from the phase space regions where the partons are either **soft** or **collinear** to the beam directions $n$ and $\bar{n}$
- Given the two **strongly separated scales**

$$Q \sim \sqrt{q^- q^+} \gg q_{\mathrm{T}} = \lambda Q$$

we expect non-zero contributions from the regions where the partons of momentum $k$ have a

→ **Soft** scaling: $k \sim (\lambda, \lambda, \lambda)\, Q$

→ ***n*-collinear** scaling: $k \sim \left(1, \lambda^2, \lambda\right) Q$

→ **$\bar{n}$-collinear** scaling: $k \sim \left(\lambda^2, 1, \lambda\right) Q$

# The SCET regions for double real radiation

The **9 relevant regions predicted by the SCET** are those with

→ **Double soft** scaling (1 possibility)

$$k_1^- \sim k_1^+ \sim k_2^- \sim k_2^+ \sim \lambda\, Q$$

→ Mixed **soft** and ($n$- or $\bar{n}$-) **collinear** scaling (4 possibilities)

$$k_1^- \sim k_1^+ \sim \lambda\, Q \qquad k_2^- \sim Q \qquad k_2^+ \sim \lambda^2\, Q$$

→ **Double** ($n$- or $\bar{n}$-) **collinear** scaling (2 possibilities)

$$k_1^- \sim k_2^- \sim Q \qquad k_1^+ \sim k_2^+ \sim \lambda^2\, Q$$

→ Mixed **$n$-collinear** and **$\bar{n}$-collinear** scaling (2 possibilities)

$$k_1^- \sim k_2^+ \sim Q \qquad k_1^+ \sim k_2^- \sim \lambda^2\, Q$$

# The analytic regulator

- The rapidity divergences are regulated through a **pure rapidity regulator** [Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18] that multiplies the phase space whenever there is at least one final state parton
- If $k$ is the **total momentum of the final-state partons**, the regulator is defined as

$$R_Y = \left( \upsilon^2 \, \frac{k^-}{k^+} \right)^{\alpha}$$

- This regulator has **two main advantages**
  - → It makes the **soft contributions zero** beyond leading order
  - → If there are two final state partons, it **does not depend separately on $k_1$ and $k_2$ but only on their sum** $k = k_1 + k_2$

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# The double collinear region

In the region where **the two partons are $n$-collinear**, it is convenient to introduce $k = k_1 + k_2$

$$z = \frac{q^-}{q^- + k^-} \qquad y = \frac{q_\mathrm{T}^2}{k^- k^+} \qquad d\Phi_{2j} = \left(\frac{\mu^2}{k^2}\right)^\epsilon \frac{d\Omega^{(3-2\epsilon)}}{2\,(4\pi)^{2-2\epsilon}}$$

and parametrize the phase space as

$$d\Phi_{\mathrm{CS}+2j} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{q_\mathrm{T}^2}\right)^\epsilon \frac{dq^-\, dq^+\, dq_\mathrm{T}^2\, q_\mathrm{T}^2}{(4\pi)^2\, S} \frac{dz}{z\,(1-z)} \frac{dy}{y^2} \frac{d\Phi_{2j}}{2\pi}\, d\Phi_{\mathrm{CS}}$$

## Integration variables

- Analytic integration over $\Phi_{2j}$ and $y$
- Numeric convolution over $z$, which enters the PDF $f_i\left(\frac{q^-}{z\sqrt{S}}\right)$

# Leading $\epsilon$ poles at leading power

- The angular integration over $\Phi_{2j}$ exposes the **collinear singularity** of the 2nd emission
  $\rightarrow \frac{1}{\epsilon}$ pole
- The $y$ integration exposes the overlapping **soft singularity** of the 2nd emission
  $\rightarrow \frac{1}{\epsilon}$ pole
- The $q_T^2$ distribution exposes the **collinear singularity** of the 1st emission
  $\rightarrow \left(q_T^2\right)^{-1-\epsilon-\alpha} \sim \frac{1}{\epsilon+\alpha}\,\delta\!\left(q_T^2\right)$ pole
- The $z$ distribution exposes the **soft singularity** of the 1st emission
  $\rightarrow (1-z)^{-1+2\alpha} \sim \frac{1}{\alpha}\,\delta(1-z)$ pole

$$\frac{1}{\epsilon^2\,\alpha}\,\frac{1}{\epsilon+\alpha}\,\delta\!\left(q_T^2\right) = \frac{1}{\epsilon^3\,\alpha}\left(1-\frac{\alpha}{\epsilon}\right)\delta\!\left(q_T^2\right) = \left[\frac{1}{\epsilon^3\,\alpha} - \frac{1}{\epsilon^4} + \mathcal{O}(\alpha)\right]\delta\!\left(q_T^2\right)$$

# Leading $\epsilon$ poles at next-to-leading power

- The angular integration over $\Phi_{2j}$ exposes the **collinear singularity** of the 2nd emission
  $\to \frac{1}{\epsilon}$ pole
- The $y$ integration exposes the overlapping **soft singularity** of the 2nd emission
  $\to \frac{1}{\epsilon+\alpha}$ pole, besides $\frac{1}{\epsilon}$ and $\frac{1}{\alpha}$ poles
- The $q_\mathrm{T}^2$ distribution is now regular
- The $z$ distribution exposes the **soft singularity** of the 1st emission
  $\to (1-z)^{-1+2\alpha} \sim \frac{1}{\alpha}\,\delta(1-z)$ pole

$$\frac{1}{\epsilon\,\alpha}\,\frac{1}{\epsilon+\alpha} = \frac{1}{\epsilon^2\,\alpha}\left(1-\frac{\alpha}{\epsilon}\right) = \frac{1}{\epsilon^2\,\alpha} - \frac{1}{\epsilon^3} + \mathcal{O}(\alpha)$$

# The mixed collinear and anti-collinear region

In the region where **one parton is $n$-collinear and the other is $\bar{n}$-collinear**, it is convenient to introduce

$$z_a = \frac{q^-}{q^- + k_1^-} \qquad z_b = \frac{q^+}{q^+ + k_2^+}$$

and parametrize the phase space as

$$d\Phi_{\mathrm{CS}+2j} = \frac{dq^- \, dq^+ \, dq_{\mathrm{T}}^2}{S} \, d\Phi_{\mathrm{CS}} \, \mu^{2\epsilon} \frac{d^{2-2\epsilon} k_{1\mathrm{T}}}{2\,(2\pi)^{3-2\epsilon}} \, \frac{dz_a}{z_a\,(1-z_a)} \times \mu^{2\epsilon} \frac{d^{2-2\epsilon} k_{2\mathrm{T}}}{2\,(2\pi)^{3-2\epsilon}} \, \frac{dz_b}{z_b\,(1-z_b)} \, \delta\left(q_{\mathrm{T}}^2 - \left(\vec{k}_{1\mathrm{T}} + \vec{k}_{2\mathrm{T}}\right)^2\right)$$

## Integration variables

- Analytic integration over $k_{1\mathrm{T}}$ and $k_{2\mathrm{T}}$
- Numeric convolution over $z_a$ and $z_b$, which enter the PDFs

# The mixed collinear and anti-collinear region

The analytic regulator in the mixed collinear and anti-collinear region reads

$$R_Y = \left( \frac{q^- (1 - z_a)\, z_b}{q^+\, z_a\, (1 - z_b)} \right)^{\alpha} \left[ 1 + \alpha\, \frac{z_a z_b \left( {k_{2\mathrm{T}}}^2 - {k_{1\mathrm{T}}}^2 \right)}{q^- q^+ (1 - z_a)(1 - z_b)} + \mathcal{O}(\alpha^2) \right]$$

## NO $\epsilon^3$ POLES

The leading poles are proportional to

$$(1 - z_a)^{-1+\alpha} (1 - z_b)^{-1-\alpha} \frac{1}{\epsilon} \sim \frac{1}{\alpha^2\, \epsilon}$$

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# The need for an additional region

After combining the contributions from the SCET regions

→ All the $\alpha$ poles cancel

→ The $\epsilon$ poles **DO NOT** cancel

**SOMETHING IS MISSING**

We need to study the phase space more carefully

The phase space constraint

$$\theta\left(\left[q_{\mathrm{T}}^2 - (k_{1\mathrm{T}} - k_{2\mathrm{T}})^2\right]\left[(k_{1\mathrm{T}} + k_{2\mathrm{T}})^2 - q_{\mathrm{T}}^2\right]\right)$$

also allows for a new region (sometimes referred to as the central jet region in the literature [Chiu, Jain, Neill, Rothstein '12]) where the two partons are **back-to-back** in the transverse plane

$$k_{\mathrm{T}}^{+} = k_{1\mathrm{T}} + k_{2\mathrm{T}} \sim Q \qquad k_{\mathrm{T}}^{-} = k_{1\mathrm{T}} - k_{2\mathrm{T}} \sim \lambda\, Q$$

# The back-to-back region

In the **back-to-back region**, it is convenient to write the phase space as

$$d\Phi_{\mathrm{CS}+2j} = \frac{(4\pi)^{2\epsilon}}{\Gamma(1-2\epsilon)} \frac{dq^- \, dq^+ \, dq_{\mathrm{T}}^2}{\pi S} \left(\frac{\mu^2}{q^- q^+}\right)^{\epsilon} \left(\frac{\mu^2}{q_{\mathrm{T}}^2}\right)^{\epsilon} \frac{q^- q^+}{(4\pi)^4} \, d\Phi_{\mathrm{CS}}$$
$$\times \frac{dz_a}{{z_a}^2} \frac{dz_b}{{z_b}^2} \left(\frac{z_a}{1-z_a}\right)^{\epsilon} \left(\frac{z_b}{1-z_b}\right)^{\epsilon}$$
$$\times \frac{dv}{2} \left(1-v^2\right)^{-\epsilon} dw \left(1-w^2\right)^{-\frac{1}{2}-\epsilon} + \mathcal{O}(\lambda^4)$$

where we defined

$$z_a = \frac{q^-}{q^- + \frac{k_{\mathrm{T}}^+}{2}\left(e^{y_1}+e^{y_2}\right)} \qquad z_b = \frac{q^+}{q^+ + \frac{k_{\mathrm{T}}^+}{2}\left(e^{-y_1}+e^{-y_2}\right)}$$

$$v = \sqrt{1 - \frac{\left(k_{\mathrm{T}}^+\right)^2}{q^- q^+} \frac{z_a}{1-z_a} \frac{z_b}{1-z_b}} \qquad w = \frac{k_{\mathrm{T}}^-}{\sqrt{q_{\mathrm{T}}^2}}$$

# Properties of the back-to-back region

- The region requires at least two final state partons
  $\rightarrow$ It does **NOT contribute at NLO**
- The region is naturally power-suppressed since no propagator can go on-shell
  $\rightarrow$ It does **NOT contribute at leading power**
- The region does **NOT produce any $\alpha$ pole**
- The terms of the power expansion proportional to an odd power of $q_\mathrm{T}$ vanish after the integration over $w$
  $\rightarrow$ **NO odd powers** of $q_\mathrm{T}$ in the power expansion
- After combining the results from this region with those from the other (collinear) regions **all the $\epsilon$ poles cancel**

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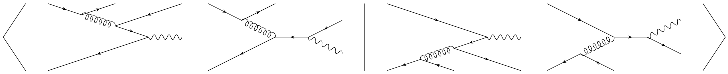
# Testing ground

We begin by studying the process

$$p\,p \to \gamma^* + X$$

and focus on the contributions to the total cross section proportional to

$$Q_{q_a} Q_{q_b} C_{\mathrm{F}} T_{\mathrm{F}} f_{q_a}\big(\zeta_a,\, \mu_{\mathrm{F}}^2\big)\, f_{\bar{q}_b}\big(\zeta_b,\, \mu_{\mathrm{F}}^2\big)$$



+ h.c.

- Separately **Gauge-invariant** structure
- **NO contributions** from loop (real-virtual) diagrams
- **NO contributions** from renormalization counterterms

# Cancellation of the leading poles

$$K = \frac{Q_{q_a} Q_{q_b} e^2 C_{\mathrm{F}} T_{\mathrm{F}} d\Phi_{\mathrm{CS}}}{2N_{\mathrm{C}} S\, q^- q^+} \left( \frac{\alpha_{\mathrm{S}}(\mu_{\mathrm{R}}^2)}{2\pi} \right)^2 \int_{\frac{q^-}{\sqrt{S}}}^1 \frac{dz_a}{z_a} f_{q_a}\!\left( \frac{q^-}{z_a \sqrt{S}}, \mu_{\mathrm{F}}^2 \right) \int_{\frac{q^+}{\sqrt{S}}}^1 \frac{dz_b}{z_b} f_{\bar{q}_b}\!\left( \frac{q^+}{z_b \sqrt{S}}, \mu_{\mathrm{F}}^2 \right)$$

$$\frac{d\sigma_{nn}}{dq^-\, dq^+\, dq_{\mathrm{T}}^2} = K \left( \frac{\mu_{\mathrm{R}}^2}{q_{\mathrm{T}}^2} \right)^{2\epsilon} \left( \frac{\upsilon q^-}{q_{\mathrm{T}}} \right)^{2\alpha} \left\{ \left[ \frac{1}{\alpha^2 \epsilon} - \frac{1}{\alpha \epsilon^2} + \frac{1}{\epsilon^3} + \ldots \right] \delta(1-z_a)\, \delta(1-z_b) + \ldots \right\}$$

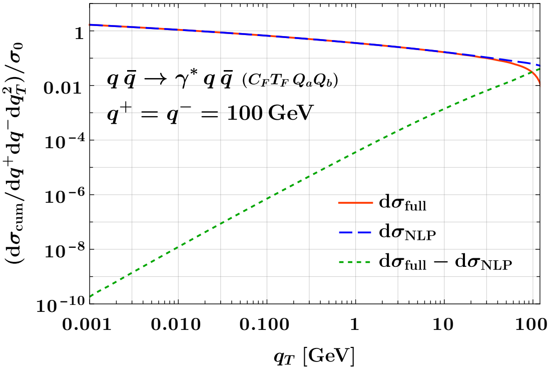
$$\frac{d\sigma_{\bar{n}\bar{n}}}{dq^-\, dq^+\, dq_{\mathrm{T}}^2} = K \left( \frac{\mu_{\mathrm{R}}^2}{q_{\mathrm{T}}^2} \right)^{2\epsilon} \left( \frac{\upsilon q_{\mathrm{T}}}{q^+} \right)^{2\alpha} \left\{ \left[ \frac{1}{\alpha^2 \epsilon} + \frac{1}{\alpha \epsilon^2} + \frac{1}{\epsilon^3} + \ldots \right] \delta(1-z_a)\, \delta(1-z_b) + \ldots \right\}$$

$$\frac{d\sigma_{n\bar{n}}}{dq^-\, dq^+\, dq_{\mathrm{T}}^2} = K \left( \frac{\mu_{\mathrm{R}}^2}{q_{\mathrm{T}}^2} \right)^{2\epsilon} \left( \frac{\upsilon^2 q^-}{q^+} \right)^{\alpha} \left\{ \left[ -\frac{2}{\alpha^2 \epsilon} + \ldots \right] \delta(1-z_a)\, \delta(1-z_b) + \ldots \right\}$$

$$\frac{d\sigma_{bb}}{dq^-\, dq^+\, dq_{\mathrm{T}}^2} = K \left( \frac{\mu_{\mathrm{R}}^4}{q^- q^+ q_{\mathrm{T}}^2} \right)^{\epsilon} \left\{ \left[ -\frac{2}{\epsilon^3} + \ldots \right] \delta(1-z_a)\, \delta(1-z_b) + \ldots \right\}$$

Each region contributes to the logarithmic structure of the final result and has its **own canonical scales**

# Numeric check




First numeric check with flat PDFs (**NLP result** vs. **full QCD result**)

# Conclusions

- We presented a framework for computing the NNLO $q_{\mathrm{T}}$ **subleading power corrections** to the differential cross section for the production of an electroweak boson
- We derived the soft and collinear contributions by expanding the cross section in the regions predicted by the leading power SCET
- In the case where there are two final state partons, the leading-power **SCET regions are NOT enough** to correctly cancel all the $\epsilon$ poles, but we need an **additional region**, where the two partons are hard and **back-to-back in the transverse plane**
- After adding the extra region **all the poles properly cancel**

**Thanks for your attention!**

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