Hough Transform and Bivariate Normal Distribution

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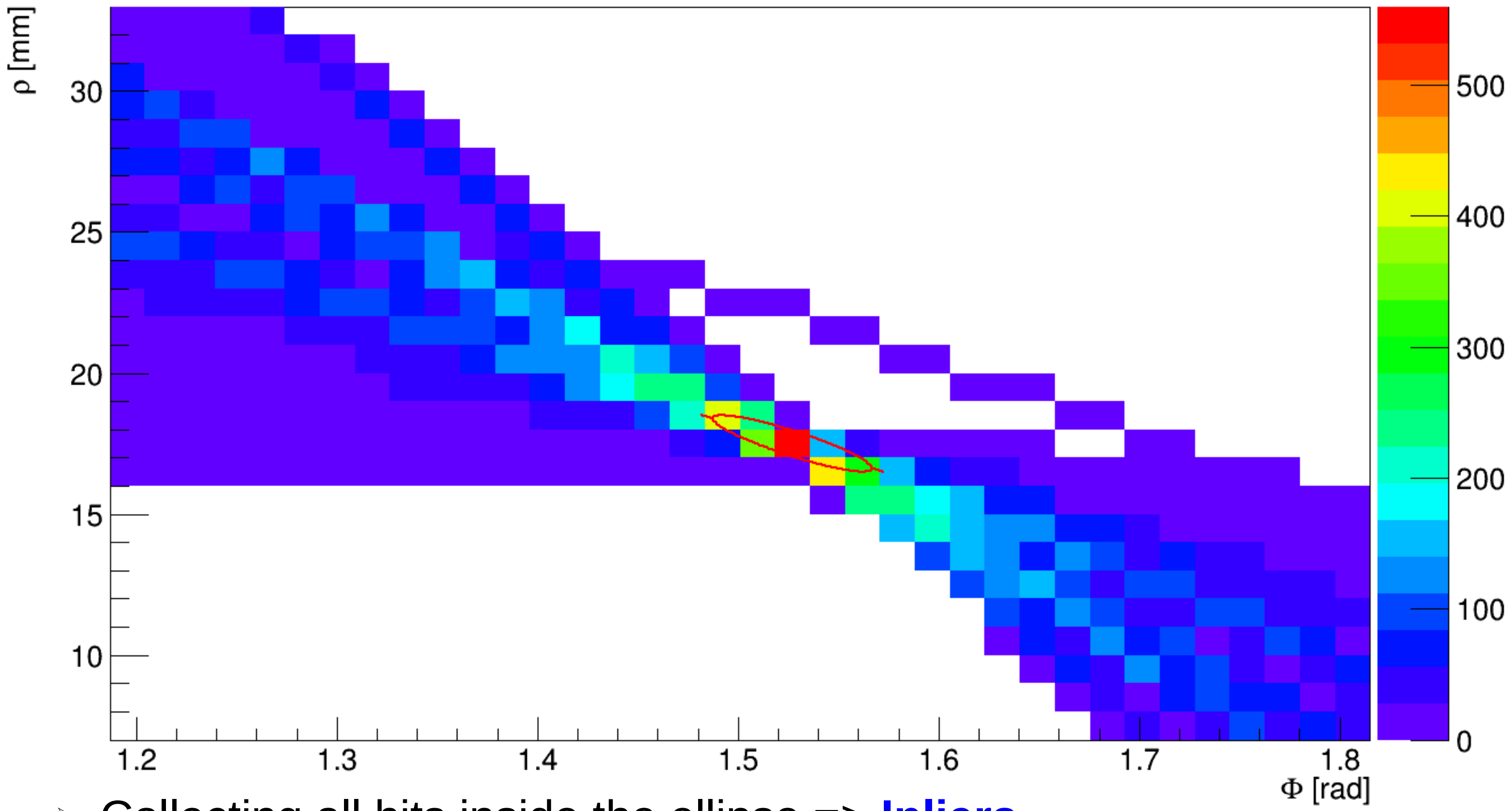
Inliers and 1/e of peak of BND:

$$t = \frac{1}{e}\left(\frac{1}{2\pi\sigma_\rho\sigma_\varphi\sqrt{1-r^2}}\right) \quad \text{and} \quad q = 2\left(1-r^2\right)$$



- Collecting all hits inside the ellipse => **Inliers**
- If number of inliers ≥ some % of all hits =>Next step
- Size of fitting is changeable => Number of inliers is changeable too

Diffusion and covariance matrix:

- Spacial Transverse resolution

$$\sigma_D^2 = ZD_T^2$$

- Covariance matrix: image space

$$\begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_D^2 \end{bmatrix}$$

- Covariance matrix: Hough space

$$\begin{bmatrix} \sigma_\rho^2 & \sigma_{\rho\varphi} \\ \sigma_{\varphi\rho} & \sigma_\varphi^2 \end{bmatrix}$$

First -order error propagation:

$$\rho = x\cos(\varphi) + y\sin(\varphi)$$

$$\begin{bmatrix} \sigma_\rho^2 & \sigma_{\rho\varphi} \\ \sigma_{\varphi\rho} & \sigma_\varphi^2 \end{bmatrix} = \nabla J \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_D^2 \end{bmatrix} \nabla J^T$$

$$\nabla J = \begin{bmatrix} \frac{\partial\rho}{\partial x} & \frac{\partial\rho}{\partial y} \\ \frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y} \end{bmatrix}$$

- Using Bivariate normal distribution for XY_Plane for collecting inliers

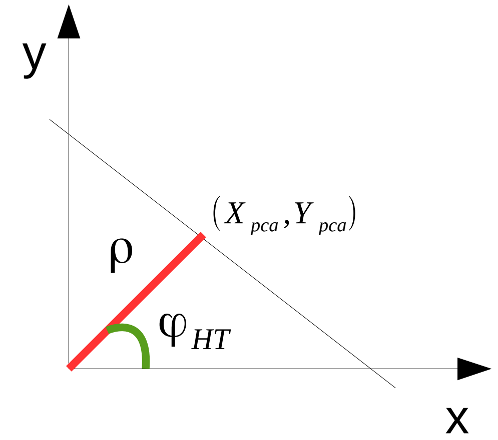
ZS_Plane

Diffusion and covariance matrix:

$$x_{pca} = -\rho \sin(\varphi)$$

$$y_{pca} = \rho \cos(\varphi)$$

$$S = \sqrt{(x_{hit} - x_{pca})^2 + (y_{hit} - y_{pca})^2}$$



$$\sigma_S^2 = \left(\frac{\partial s}{\partial x}\right)^2 \sigma_D^2 + \left(\frac{\partial s}{\partial y}\right)^2 \sigma_D^2$$

$$\sigma_L^2 = Z D_L^2$$

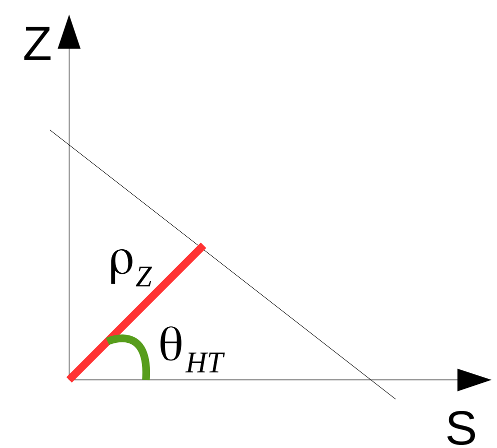
$$\Rightarrow \begin{bmatrix} \sigma_S^2 & 0 \\ 0 & \sigma_L^2 \end{bmatrix}$$

First -order error propagation for SZ_Plane:

$$\rho_Z = s \cos(\theta_{HT}) + z \sin(\theta_{HT})$$

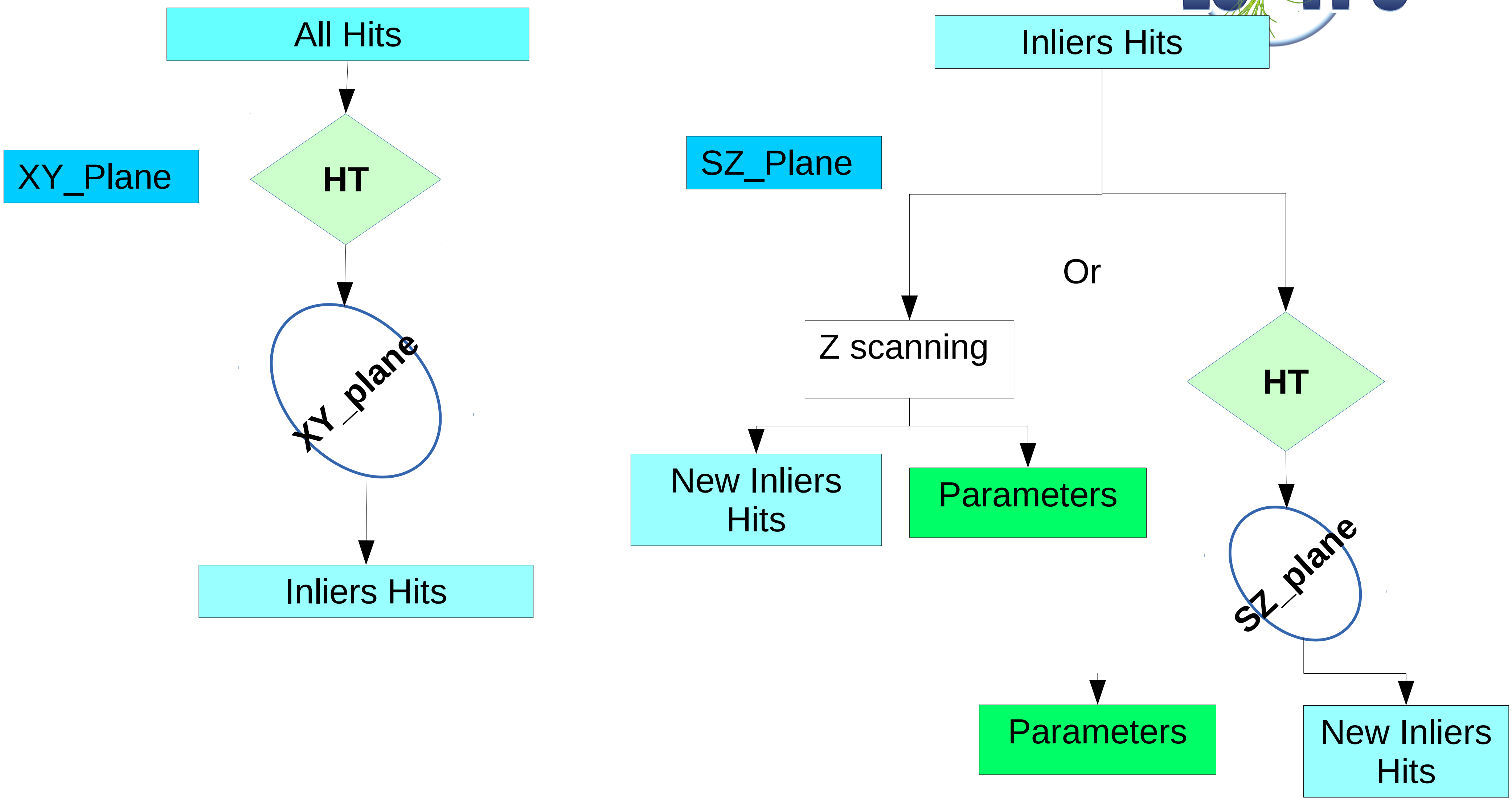
$$\begin{bmatrix} \sigma_{\rho_z}^2 & \sigma_{\rho_z \theta} \\ \sigma_{\theta \rho_z} & \sigma_\theta^2 \end{bmatrix} = \nabla J \begin{bmatrix} \sigma_S^2 & 0 \\ 0 & \sigma_L^2 \end{bmatrix} \nabla J^T$$

$$\nabla J = \begin{bmatrix} \frac{\partial \rho_Z}{\partial S} & \frac{\partial \rho_Z}{\partial z} \\ \frac{\partial \theta}{\partial S} & \frac{\partial \theta}{\partial z} \end{bmatrix}$$



- **Using Bivariate normal distribution for SZ_Plane for collecting inliers**

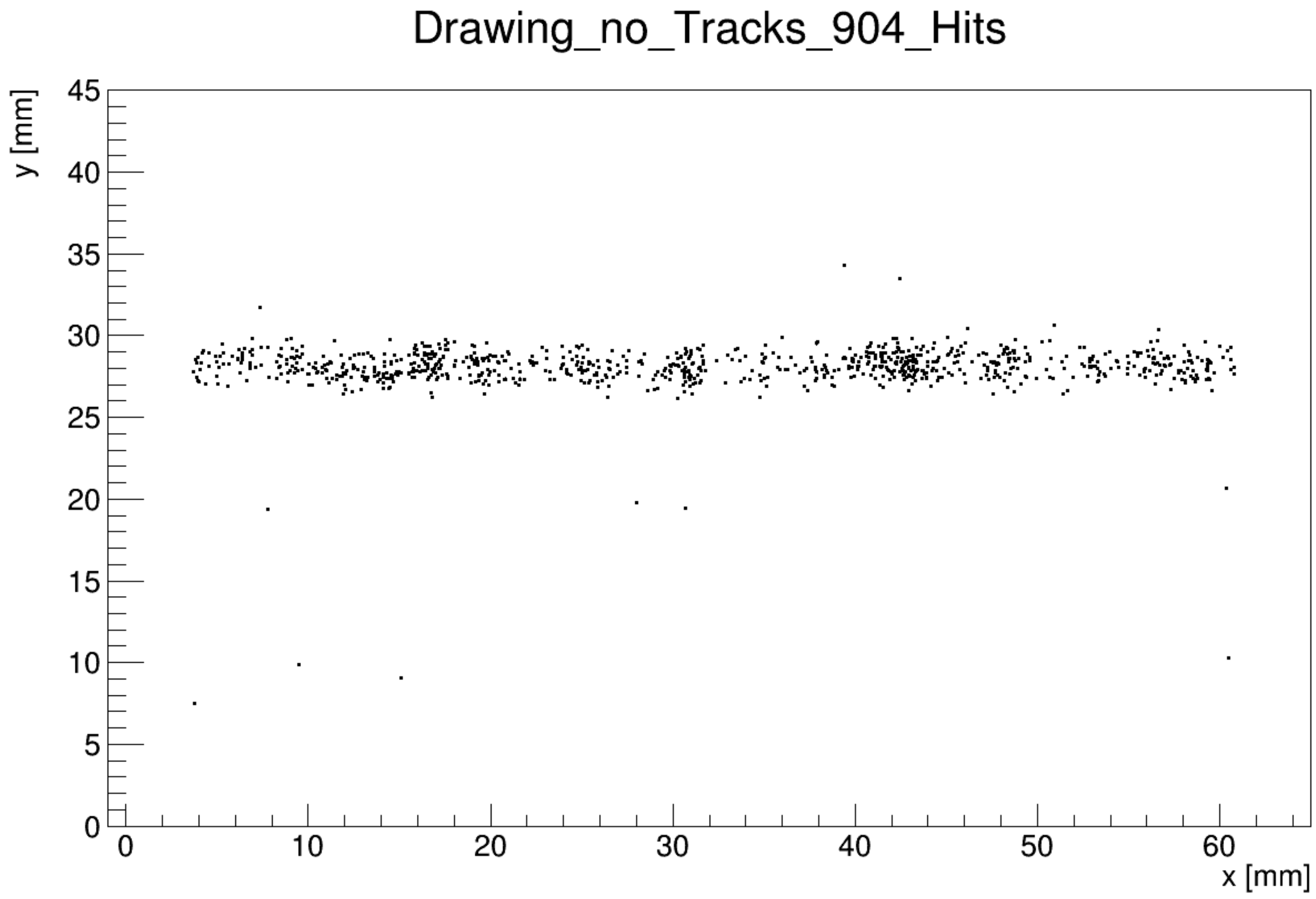
Strategy



All Hits
XY_Plane
HT
XY_plane
Inliers Hits
Inliers Hits
SZ_Plane
Or
Z scanning
HT
New Inliers Hits
Parameters
SZ_plane
Parameters
New Inliers Hits

Simulation

2 tracks:
Y1 = 29 mm , Z1 = 150 mm
Y2 = 27 mm , Z2 = 140 mm



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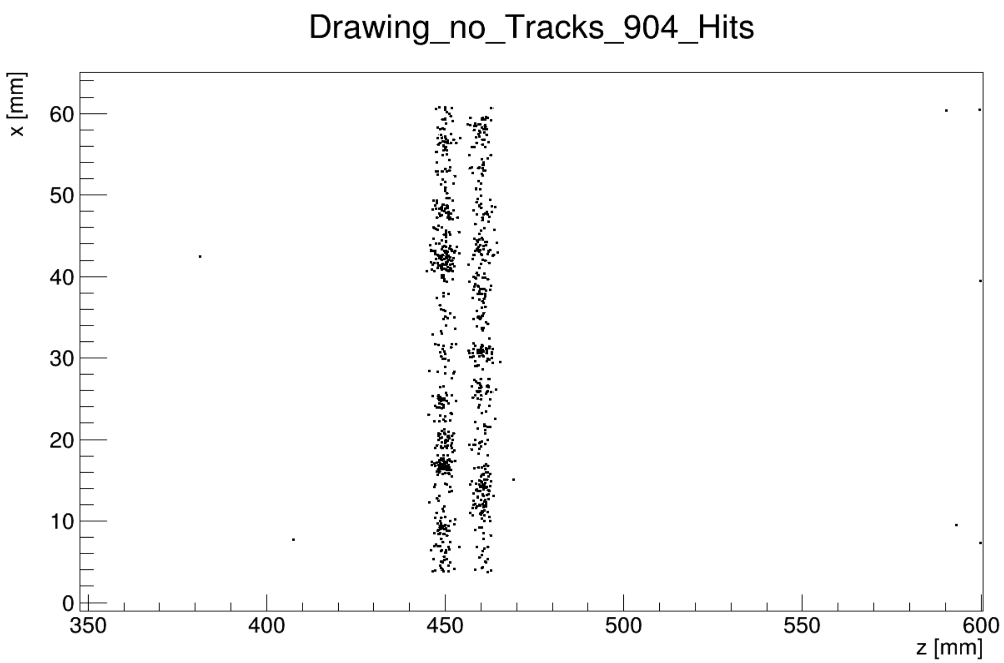

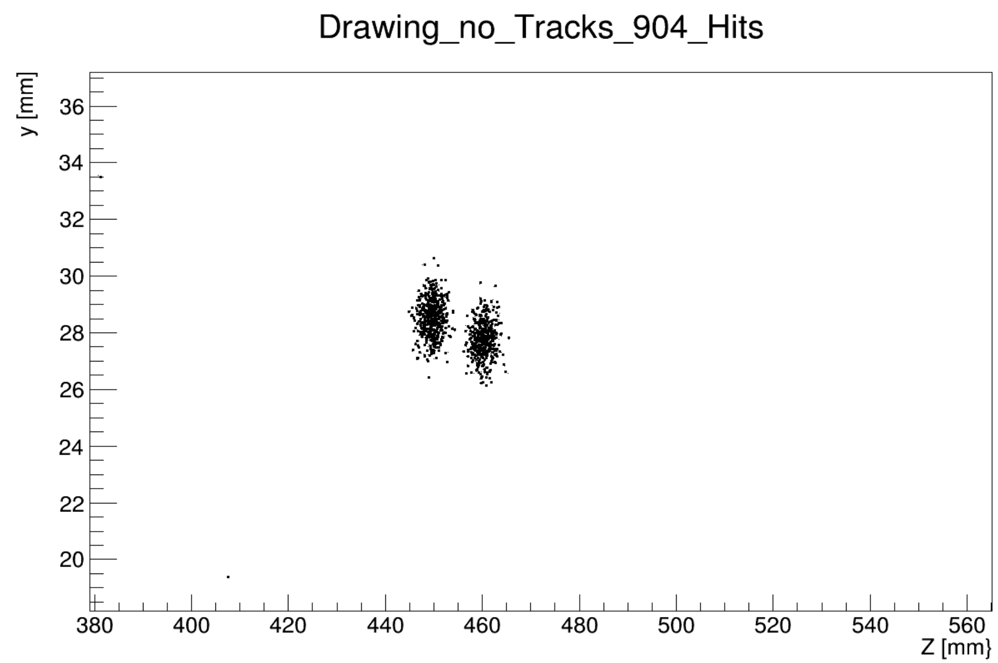

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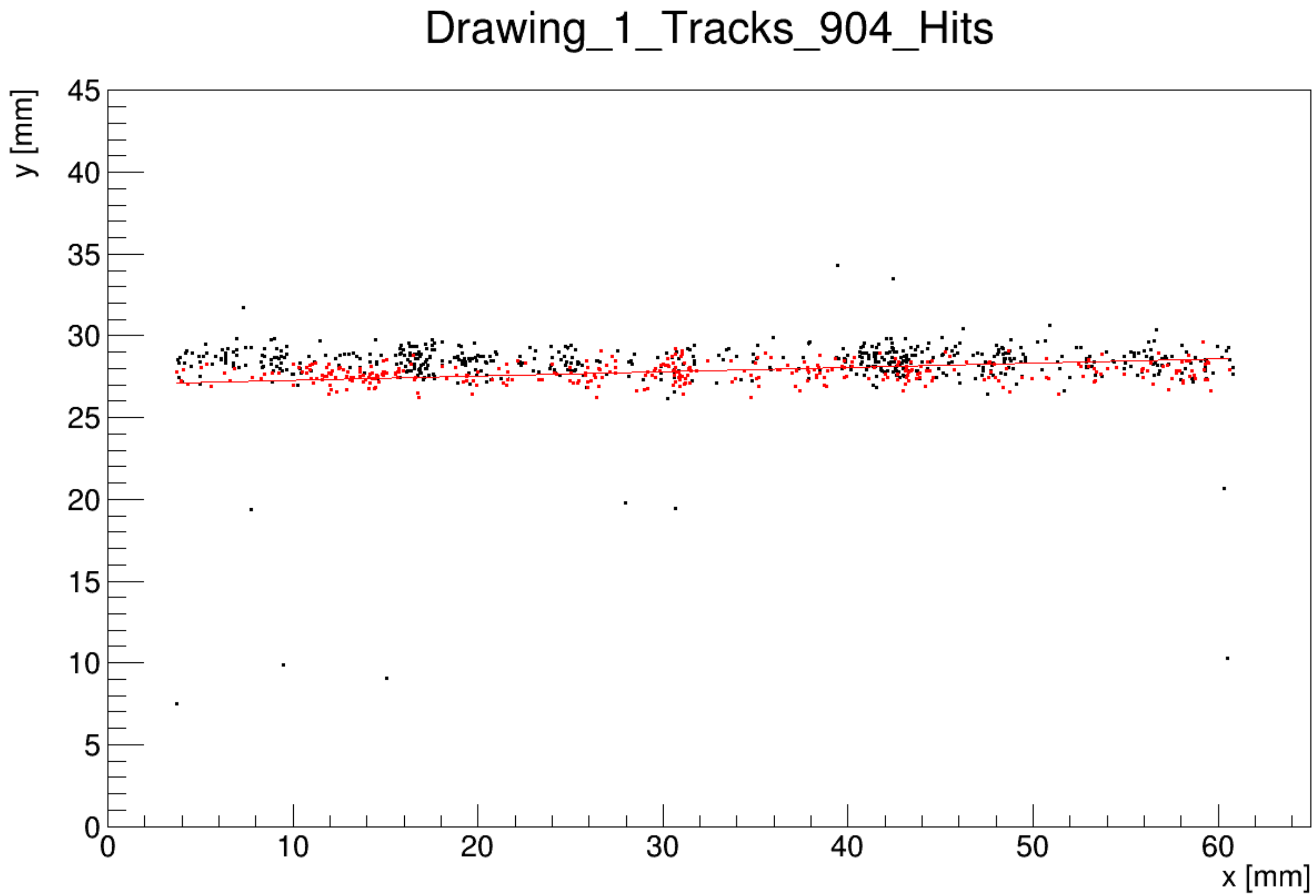

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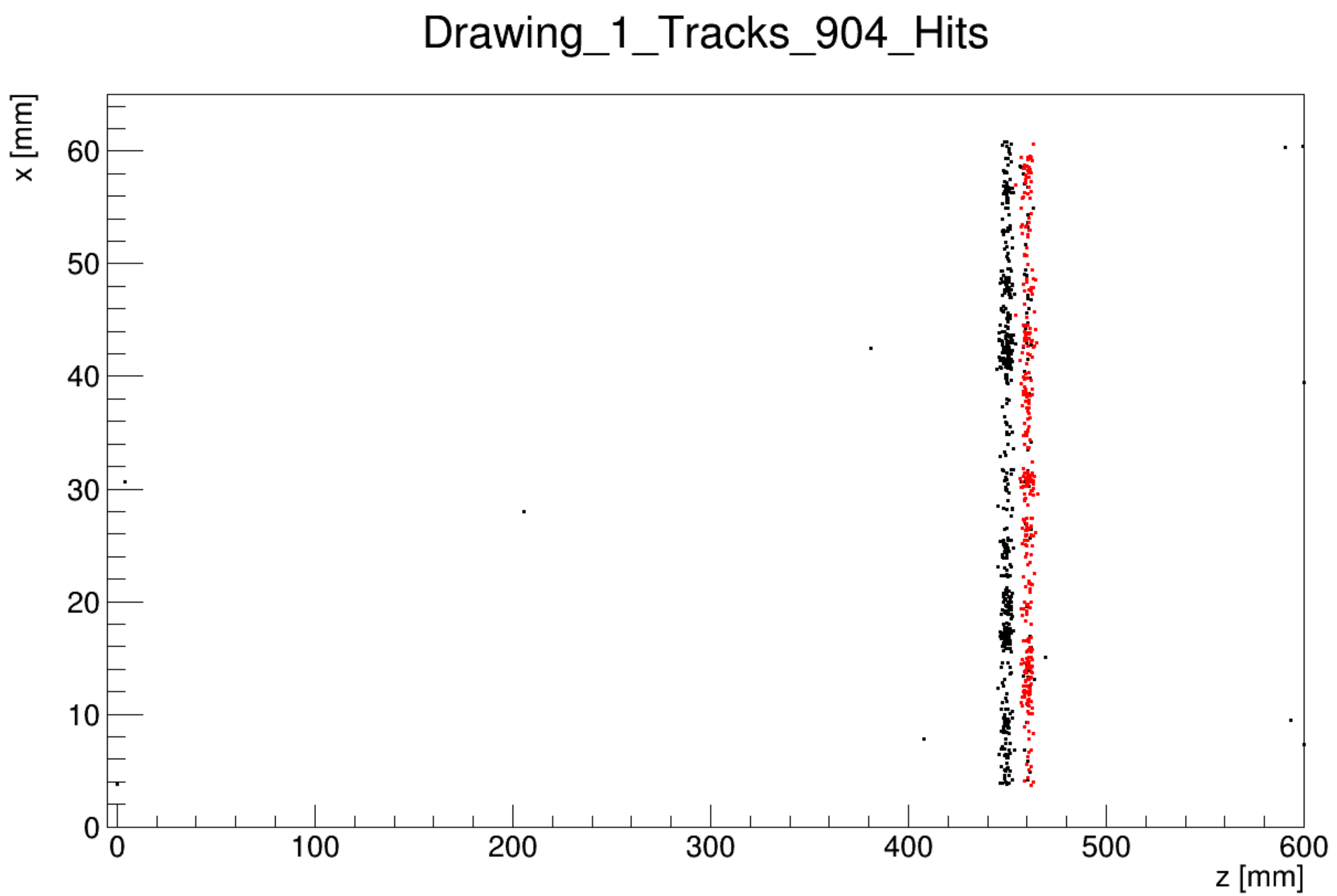
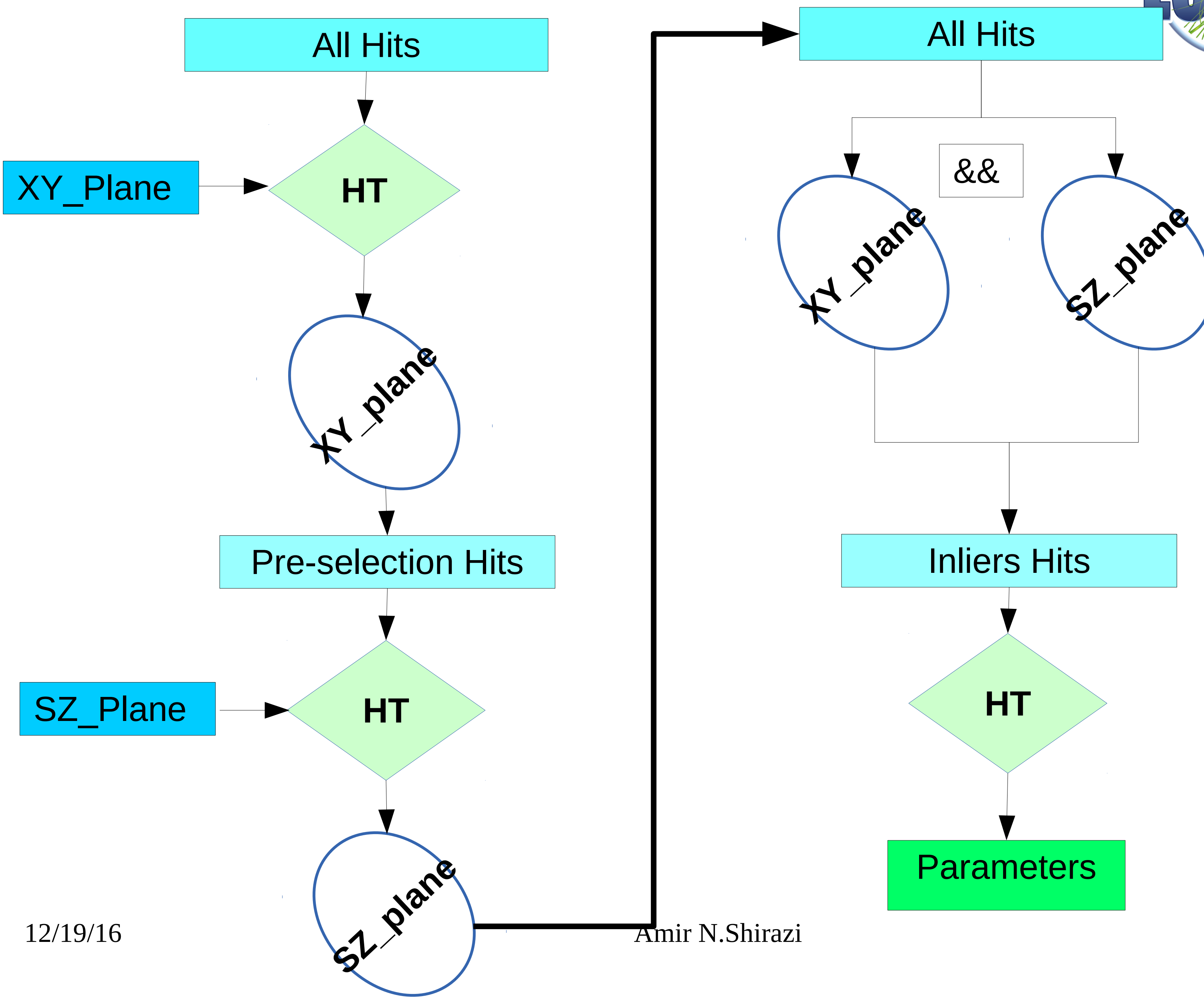


Strategy


All Hits
XY_Plane
HT
XY_plane
Pre-selection Hits
SZ_Plane
HT
SZ_plane
All Hits
&&
XY_plane
SZ_plane
Inliers Hits
HT
Parameters

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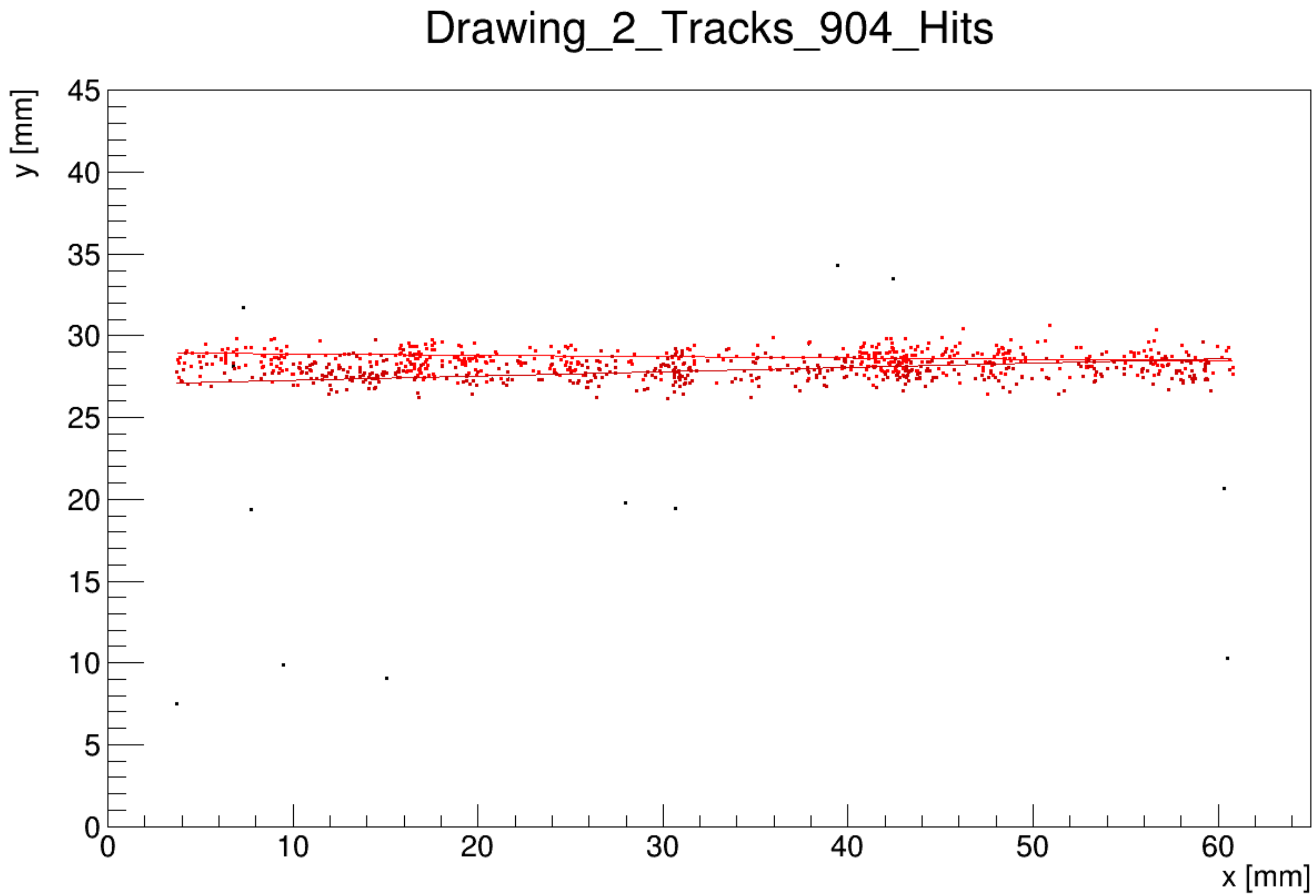

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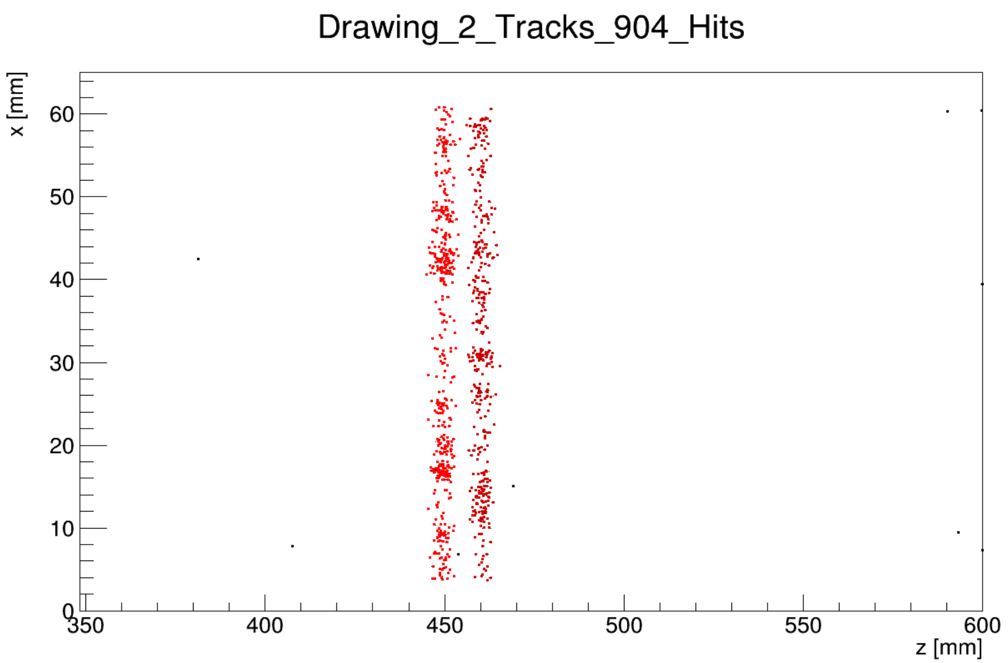
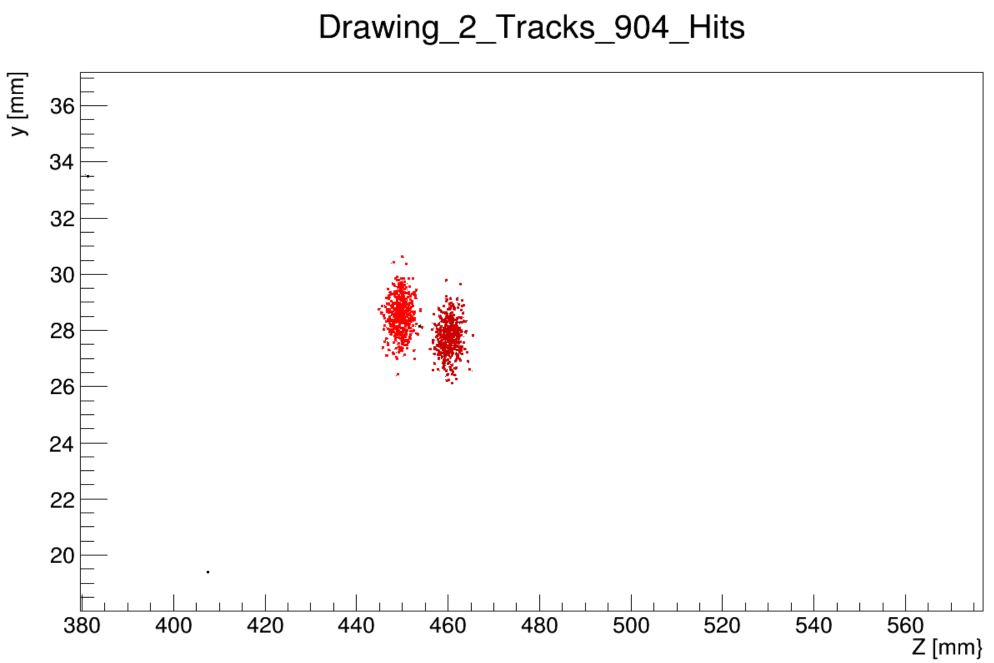




Summary and Outlook:

- Processor for finding Ghost and Clone tracks
- Analysis of BND Hough transform for one track for different Z position is ongoing.
- Analysis of BND Hough transform for two tracks :
  - With 2mm distance from each other in XY_plane
  - With less than 10mm distance from each other in Z direction
- Connecting tracklet in order to have full track (Kalman filter)

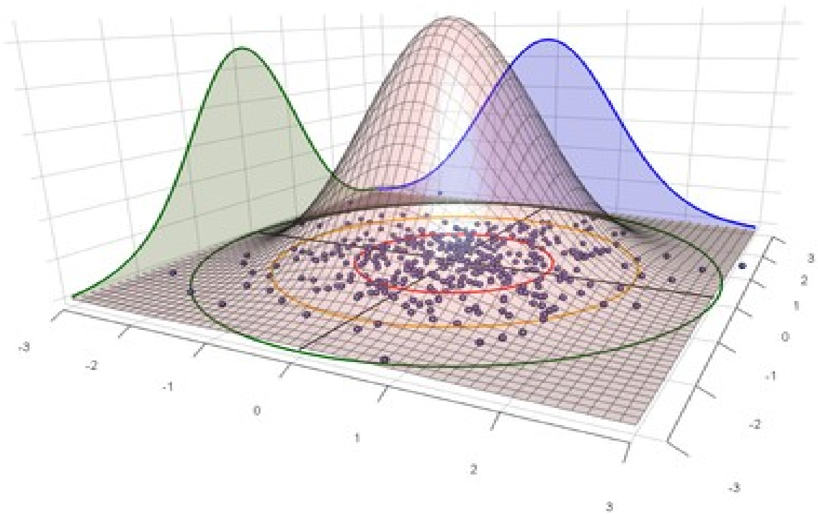
Backup

XY_Plane

Bivariate Normal Distribution (BND):

$$G(\varphi,\rho)=\frac{1}{2\pi\sigma_\rho\sigma_\varphi\sqrt{1-r^2}}\exp\left[\frac{-c}{2(1-r^2)}\right]$$

$$c\equiv\frac{(\varphi-\mu_\varphi)^2}{\sigma_\varphi^2}+\frac{(\rho-\mu_\rho)^2}{\sigma_\rho^2}-\frac{2r(\varphi-\mu_\varphi)(\rho-\mu_\rho)}{\sigma_\varphi\sigma_\rho}$$



http://ballistipedia.com/index.php?title=Closed_Form_Precision

- Correlation : $r\equiv\frac{\sigma_{\rho\varphi}}{\sigma_\rho\sigma_\varphi}$

Ellipse equation:

- General ellipse equation: $$1 = Ax^2 + Bxy + Cy^2$$
- **a** and **b** are semi_minor and semi_major axis $$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$
- Finding ellipse equation from BND:

$$t = \frac{1}{2\pi\sigma_\rho\sigma_\varphi\sqrt{1-r^2}}\exp\left[\frac{-c}{2(1-r^2)}\right]$$

$$k = 2\pi\sigma_\rho\sigma_\varphi\sqrt{1-r^2}$$ and $$q = 2(1-r^2)\ln\left(\frac{1}{kt}\right)$$

$$1 = \frac{(\varphi-\mu_\varphi)^2}{q\sigma_\varphi^2} + \frac{(\rho-\mu_\rho)^2}{q\sigma_\rho^2} - \frac{2r(\varphi-\mu_\varphi)(\rho-\mu_\rho)}{q\sigma_\varphi\sigma_\rho} \qquad (1)$$

XY_Plane

Ellipse equation:

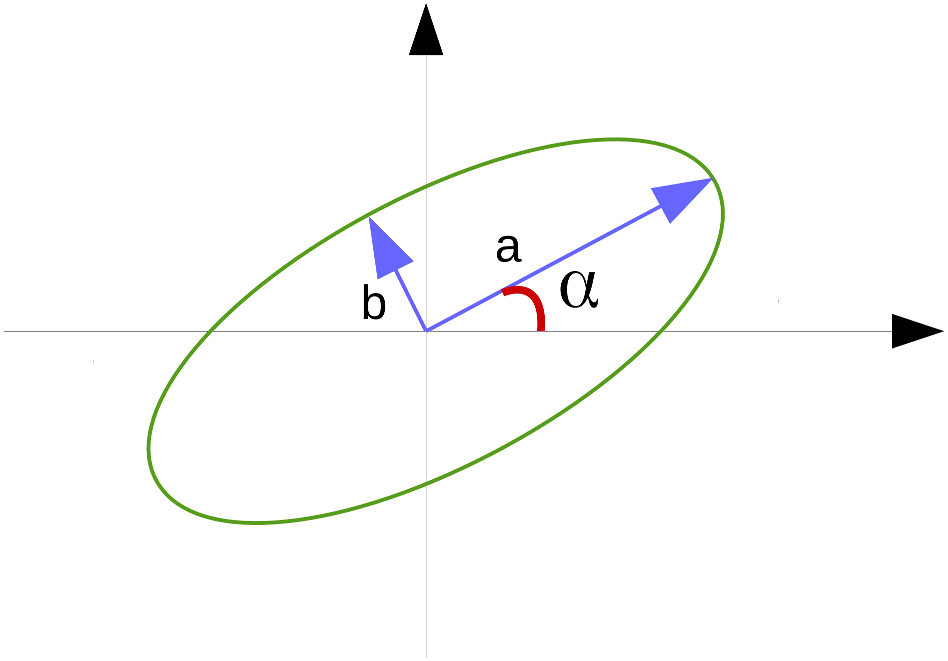
- Rotated ellipse:

$$1=\left(\frac{\cos^2(\alpha)}{a^2}+\frac{\sin^2(\alpha)}{b^2}\right)x^2-2\cos(\alpha)\sin(\alpha)\left(\frac{1}{a^2}-\frac{1}{b^2}\right)xy+\left(\frac{\sin^2(\alpha)}{a^2}+\frac{\cos^2(\alpha)}{b^2}\right)y^2 \qquad (2)$$

- From (1) and (2):

$$a^2=\frac{q\,\sigma_\varphi^2\sigma_\rho^2\cos(2\alpha)}{\sigma_\rho^2\cos^2(\alpha)-\sigma_\varphi^2\sin^2(\alpha)}$$

$$b^2=\frac{-q\,\sigma_\varphi^2\sigma_\rho^2\cos(2\alpha)}{\sigma_\rho^2\sin^2(\alpha)-\sigma_\varphi^2\cos^2(\alpha)}$$



ZS_Plane

ZS Hough Transform:

- **p**oint of **c**losest **a**pproach from XY -plane

$$x_{pca} = -\rho \sin(\varphi) \qquad y_{pca} = \rho \cos(\varphi)$$

- Arc length (S): The shortest distance between the hit and PCA

$$s = \sqrt{(x_{hit} - x_{pca})^2 + (y_{hit} - y_{pca})^2}$$

- In the ZS_plane a track is a straight line.