Topical lectures

30-31 may 2017

https://www.nikhef.nl/~t58/BSM.pdf

Part 1

The strong CP problem and Axions

CP Conservation in the Presence of Pseudoparticles*

R. D. Peccei and Helen R. Quinn†
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305
(Received 31 March 1977)

We give an explanation of the *CP* conservation of strong interactions which includes the effects of pseudoparticles. We find it is a natural result for any theory where at least one flavor of fermion acquires its mass through a Yukawa coupling to a scalar field which has nonvanishing vacuum expectation value.

CP Conservation in the Presence of Instantons
R.D. Peccei, Helen R. Quinn (Stanford U., ITP). Mar 1977. 8 pp.
Published in **Phys.Rev.Lett. 38 (1977) 1440-1443**
ITP-568-STANFORD
DOI: 10.1103/PhysRevLett.38.1440

Symmetrie concepts involved:

- C, P and T
- Anomalies (QM breaking of classical symmetries)
- Peccei-Quinn Symmetry
- Axions: pseudo-Goldstone bosons of shift symmetries

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G^{I}_{\mu\nu}G^{\mu\nu,I}$$

Lorentz invariant
Gauge invariant

But there is another kind of term that is Lorentz and gauge invariant:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{4} G^I_{\mu\nu} \tilde{G}^{\mu\nu,I}$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \qquad \tilde{G}^I_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma,I}$$

Both are Lorentz invariant

$$\Lambda_{\mu}^{\ \mu'}\Lambda_{\nu}^{\ \nu'}\Lambda_{\rho}^{\ \rho}\Lambda_{\sigma}^{\ \sigma'}g_{\mu'\nu'}g_{\rho'\sigma'} = g_{\mu\nu}g_{\rho\sigma}$$

$$\Lambda_{\mu}^{\ \mu'}\Lambda_{\nu}^{\ \nu'}\Lambda_{\rho}^{\ \rho}\Lambda_{\sigma}^{\ \sigma'}\epsilon_{\mu'\nu'\rho'\sigma'} = \det(\Lambda)\epsilon_{\mu\nu\rho\sigma}$$

But the second is not invariant under P or T

$$\Lambda_P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \Lambda_T = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In quantum field theory, CPT is always a symmetry, so we may use CP instead of T

$$\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \to H = \int d^3x \left[\vec{E}^2 + \vec{B}^2\right]$$

$$\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} \to H = \int d^3x \left[\vec{E} \cdot \vec{B}\right]$$

$$P : \vec{E} \to -\vec{E}$$

$$\vec{B} \to \vec{B} \qquad \text{(like } \vec{r} \times \vec{p}\text{)}$$

$$T : \vec{E} \to \vec{E}$$

$$\vec{B} \to -\vec{B}$$

P, T are not symmetries of nature

In quantum field theory, CPT is always a symmetry, so we may use CP instead of T

$$i\mathcal{P}^\dagger\left[\bar\psi_{L,R}\gamma^\mu(\partial_\mu - ig_3 A^I_\mu T^I)\psi_{L,R}\right]\mathcal{P} = i\bar\psi_{R,L}\gamma^\mu(\partial_\mu - ig_3 A^I_\mu T^I)\psi_{R,L}$$

$$i\mathcal{C}^\dagger\left[\bar\psi_{L,R}\gamma^\mu(\partial_\mu - ig_3 A^I_\mu T^I)\psi_{L,R}\right]\mathcal{C} = i\bar\psi_{R,L}\gamma^\mu(\partial_\mu - ig_3 A^I_\mu T^I)\psi_{R,L}$$

$\mathcal{P}$ Parity operator

$\mathcal{C}$ Charge conjugation operator

P and C are symmetries of the strong and electromagnetic interactions;

P and C are not symmetries of the weak interactions, but the product CP is a symmetry of all gauge interactions

Yukawa interaction terms

$$
\begin{tikzcd}[column sep=tiny]
 & \mathcal{L}_Y = g\bar{\psi}_L\chi_R\phi + g^*\bar{\chi}_R\psi_L\phi^* \arrow[dl, "P"'] \arrow[dr, "C"] & \\
\mathcal{L}_Y = g\bar{\psi}_R\chi_L\phi + g^*\bar{\chi}_L\psi_R\phi^* \arrow[dr, "C"'] & & \mathcal{L}_Y = g\bar{\chi}_L\psi_R\phi^* + g^*\bar{\psi}_R\chi_L\phi \arrow[dl, "P"] \\
 & \mathcal{L}_Y = g\bar{\chi}_R\psi_L\phi^* + g^*\bar{\psi}_L\chi_R\phi &
\end{tikzcd}
$$

CP invariant only if $g = g^*$

In the Standard Model g and g^* are
3×3 matrices in family space that
determine all quark masses and the CKM matrix.

Cronin, Fitch (1964): CP is violated in $K_0 - \bar{K}_0$

Kobayashi-Maskawa (1973): This can be accommodated by means of 3-family CKM matrix.

Three new Standard Model parameters

$$\theta_3 \frac{g_3^2}{32\pi^2} \sum_{I=1}^{8} G^I_{\mu\nu} \tilde{G}^{\mu\nu,I}$$

Strong

$$+ \ \theta_2 \frac{g_2}{32\pi^2} \sum_{I=a}^{3} F^a_{\mu\nu} \tilde{F}^{\mu\nu,a}$$

Weak

$$+ \ \theta_1 \frac{g_1}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Y-charge

We cannot use P, or $T{=}CP$ to argue them away

New Feynman rules?
No! These terms are total derivatives

$$\frac{1}{4}\,\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu} = \partial_\mu K^\mu \ ,$$

$$K^\mu = \epsilon^{\mu\nu\rho\sigma}\ \mathrm{Tr}[A_\nu\partial_\rho A_\sigma + \tfrac{2}{3}g_3 A_\nu A_\rho A_\sigma]$$

$$\int d^4x\partial_\mu K^\mu = \text{Boundary Terms}$$

But in non-abelian theories K^μ is not gauge invariant

In abelian theories K^μ is gauge invariant, and
this argument eliminates θ_1

This leaves θ_2 and θ_3

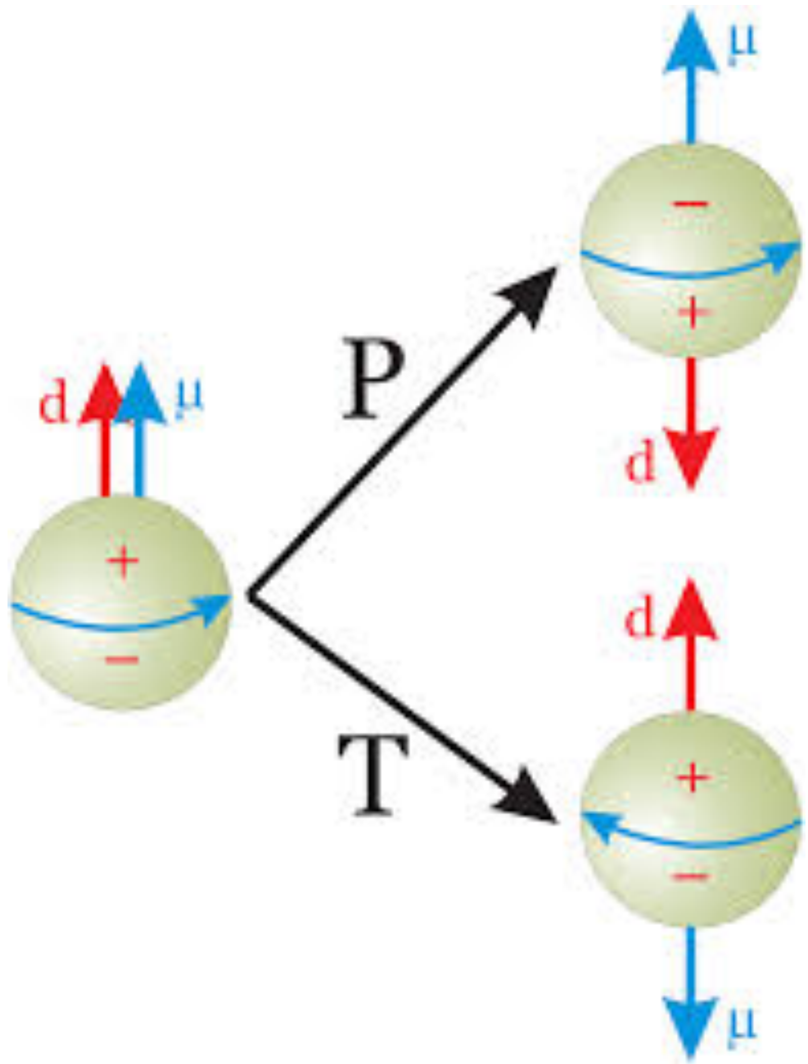
We will focus on θ_3, and just call it θ from now on

How can these terms matter if they do not contribute to Feynman diagrams?

Non-perturbative contributions!

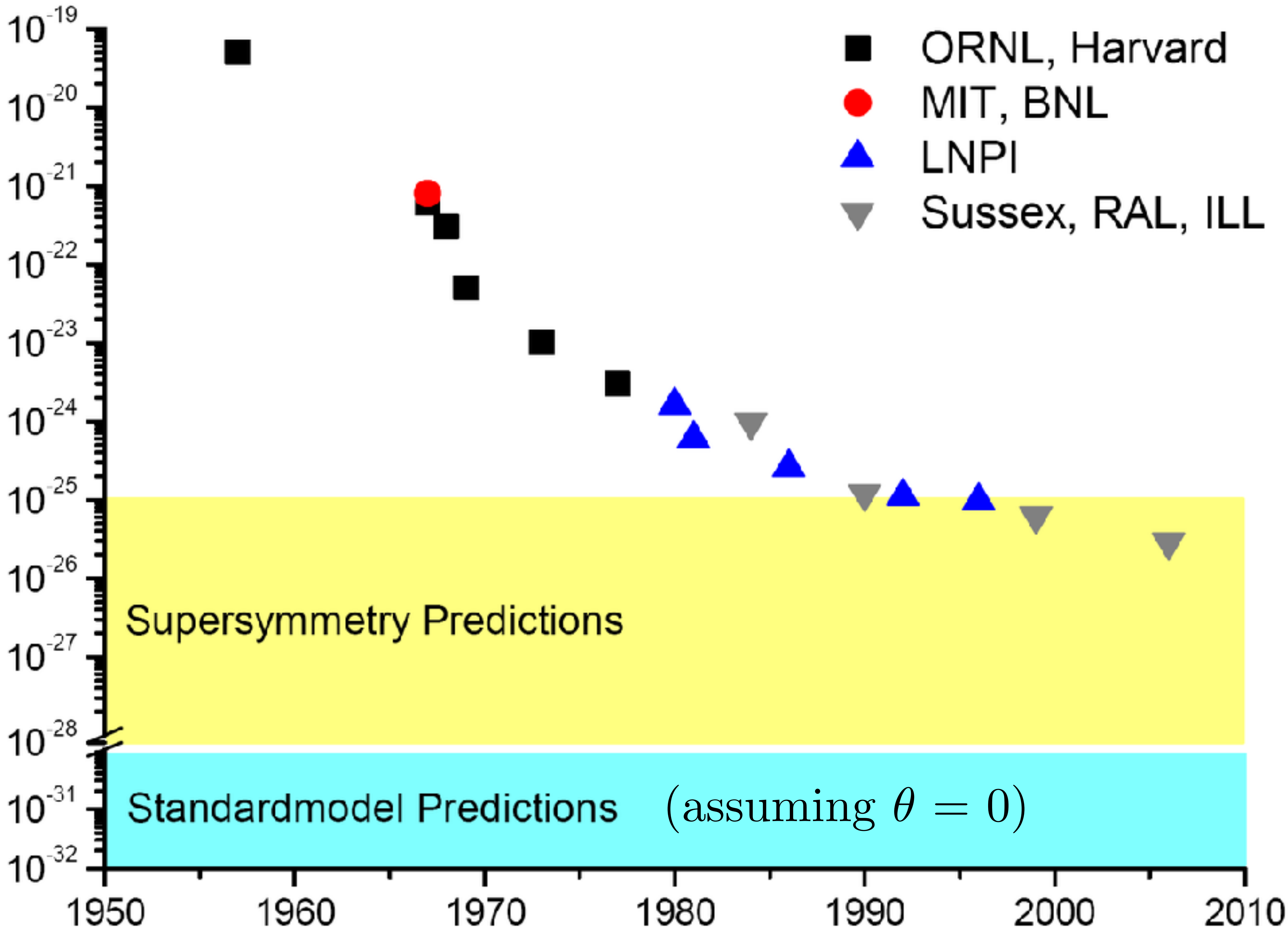
$$e^{-1/g^2} = 0 + 0g^2 + 0g^4 + 0g^6 + 0g^8 \ldots$$

Neutron electric dipole moment



$$\frac{e}{m_n}\theta\frac{m_u m_d}{m_u+m_d}\frac{1}{\Lambda_{\mathrm{QCD}}}$$

$$\theta < 10^{-10}$$



ORNL, Harvard
MIT, BNL
LNPI
Sussex, RAL, ILL
Supersymmetry Predictions
Standardmodel Predictions
(assuming $\theta = 0$)
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^{-28}
10^{-31}
10^{-32}
1950
1960
1970
1980
1990
2000
2010

Theoretically, θ is an angular variable:

$$0 \leq \theta < 2\pi$$

Hence there is an objective meaning to saying it is “small”
Furthermore, our existence does not depend on its smallness.
Hence there is no potential “anthropic” explanation.

The (in)famous gauge hierarchy problem is numerically much worse:

$$M^2_{\mathrm{Higgs}} \approx 10^{-34} M^2_{\mathrm{Planck}}$$

But:

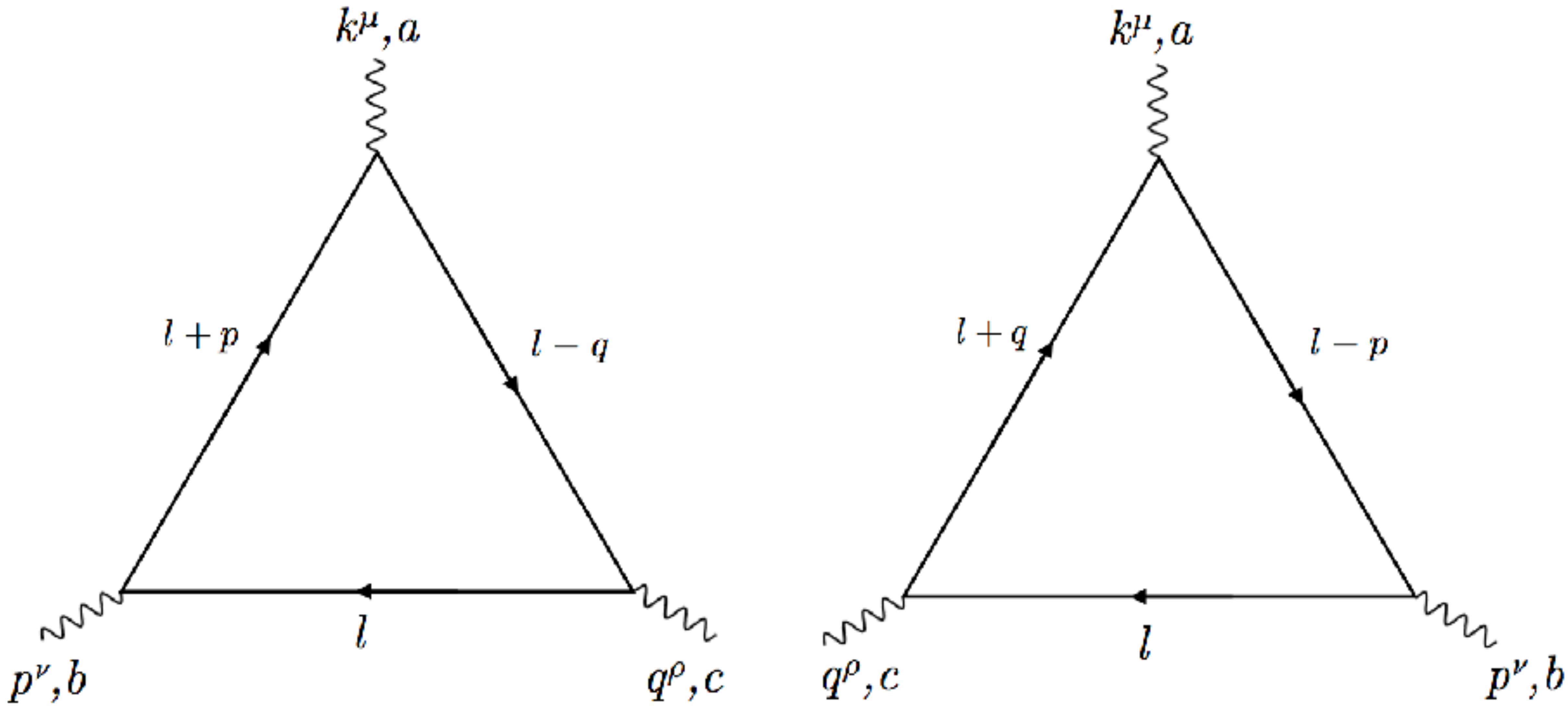
1. It requires a second scale to compare with
2. It has anthropic implications, hence the problem is less pure

The absurd smallness of θ is called the “strong CP problem”. It was first discussed around 1975

Without neutrino masses, the Standard Model has 18 parameters. All have been measured. Some have strange small values, like $m_{\mathrm{e}}/m_{\mathrm{top}}$. But it seems there is a 19th parameter that is so small we have not even measured it yet. There is no symmetry that can be invoked to set it to zero.

But in fact, the problem is much worse than that.

However, the reason it is worse also offers a clue to its solution.


k^μ, a
$l+p$
$l-q$
l
p^ν, b
q^ρ, c
k^μ, a
$l+q$
$l-p$
l
q^ρ, c
p^ν, b

$$iV^{\mu\nu\rho}_{abc}(p,q) = \; -\int \frac{d^4l}{(2\pi)^4}\, \mathrm{Tr}\left[(i\gamma^\mu\gamma_5 T_a)\left(\frac{i(\not{l}+\not{p})}{(l+p)^2}\right)(i\gamma^\nu T_b)\left(\frac{i\not{l}}{l^2}\right)(i\gamma^\rho T_c)\left(\frac{i(\not{l}-\not{q})}{(l-q)^2}\right)\right]$$

Linearly divergent; regulate it.

.....

The contraction with the external momentum is finite, and non-zero:

$$i(p+q)_\mu V^{\mu\nu\rho}_{abc} \quad = \quad \frac{1}{4\pi^2}\epsilon^{\nu\rho\alpha\beta}p_\alpha q_\beta \mathrm{Tr}\; T_a\left\{T_b, T_c\right\}$$

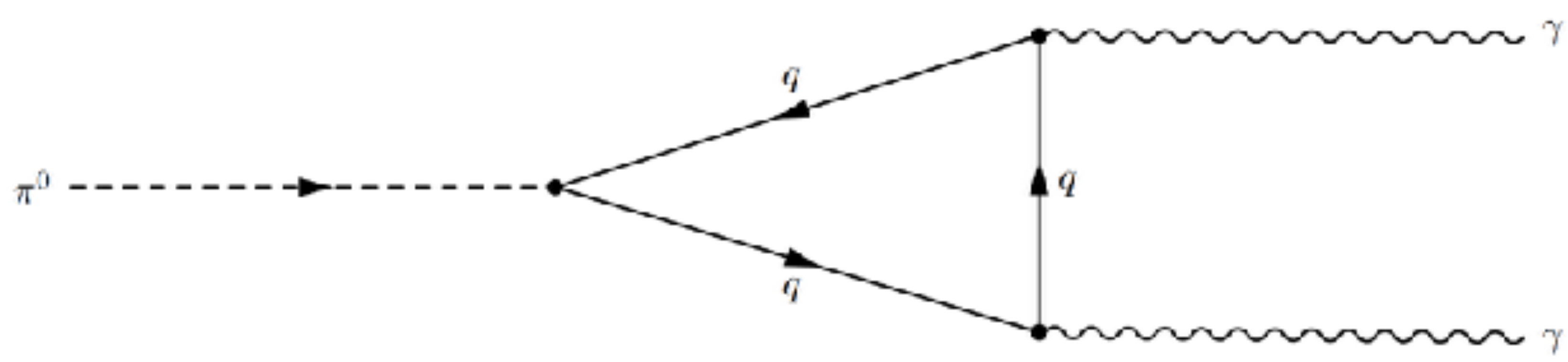
Experimentally verified example: pion decay

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2 m_\pi^3 N_c^2}{576\pi^3 f_\pi^2} = 7.73 \text{ eV}$$

7.8 ± 0.2 eV (Observed)

$f_\pi \approx 130$ MeV

$N_c = 3$



$$\partial_\mu J^\mu = \frac{g^2}{8\pi^2}\ \mathrm{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\mathcal{L} = -\tfrac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \theta \frac{g_3^2}{16\pi^2} \mathrm{Tr}\; G_{\mu\nu} \tilde{G}^{\mu\nu} + i\bar{\psi} D_\mu \gamma^\mu \psi + m \bar{\psi}_L \psi_R + m^* \bar{\psi}_R \psi_L$$

$$\mathcal{L} = -\tfrac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \theta \frac{g_3^2}{16\pi^2} \mathrm{Tr}\ G_{\mu\nu}\tilde{G}^{\mu\nu} + i\bar{\psi} D_\mu \gamma^\mu \psi + g\sigma\bar{\psi}_L\psi_R + g^*\sigma^*\bar{\psi}_R\psi_L$$

Axion effective action

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{g_3^2}{16\pi^2}\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$+ \ (\theta + \mathrm{arg}\ \mathrm{det}\ M)\frac{g_3^2}{16\pi^2}\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

A New Light Boson?

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 6 December 1977)

Axion mass in a two-Higgs model

$$m_A = \frac{N m_\pi f_\pi}{2\sqrt{m_u + m_d}} \left[\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\right]^{1/2} \frac{2^{1/4} G_F^{1/2}}{\sin 2\alpha}$$

$$m_A \approx \frac{(140\ \mathrm{keV})}{\sin 2\alpha}$$

Problem of Strong P and T Invariance in the Presence of Instantons

F. Wilczek[(a)]
Columbia University, New York, New York 10027, and The Institute for Advanced Studies, Princeton, New Jersey 08540[(b)]
(Received 29 November 1977)

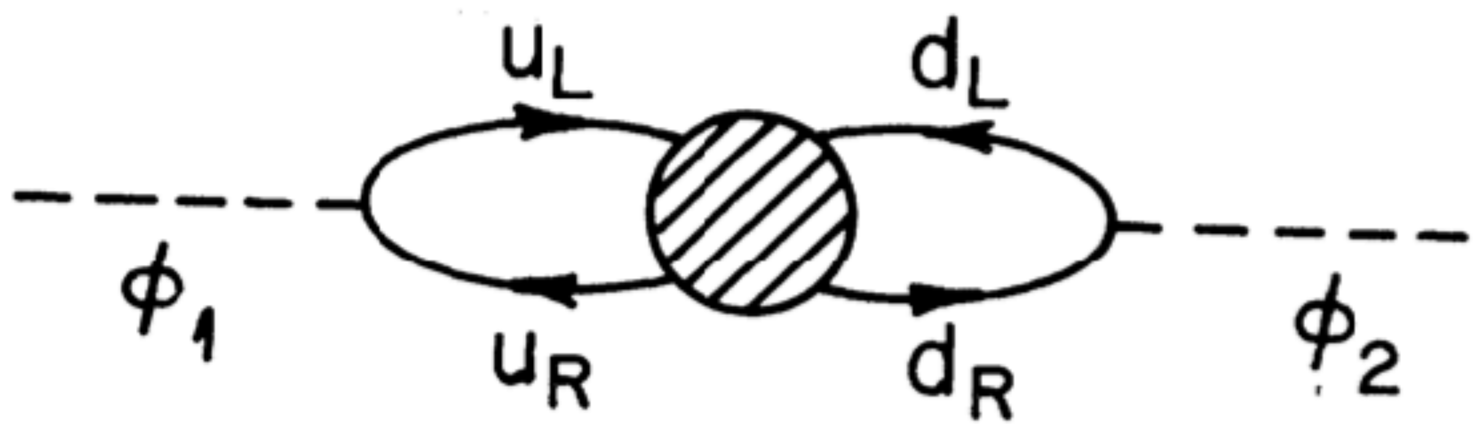


FIG. 1. Instanton interaction generating the axion mass.

Multiple axions

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a^i \partial^\mu a^i + \frac{a^i}{f_a^i}\frac{g_3^2}{16\pi^2}\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$\frac{a}{f_a} = \sum_i \frac{a^i}{f_a^i}$$

This combination is the QCD axion

The other linear combinations are “axion-like particles"

Axion dark matter

QCD axion

$$\Omega_a h^2 \approx 0.71 \times \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{7/6} \left(\frac{\Theta_a}{\pi}\right)^2$$

ALP (axion-like particle)

$$\Omega_{a_i} h^2 \approx 0.16 \times \left(\frac{m_i}{\mathrm{eV}}\right)^{1/2} \left(\frac{f_{a_i}}{10^{11}\ \mathrm{GeV}}\right)^2 \left(\frac{\Theta_i}{\pi}\right)^2$$

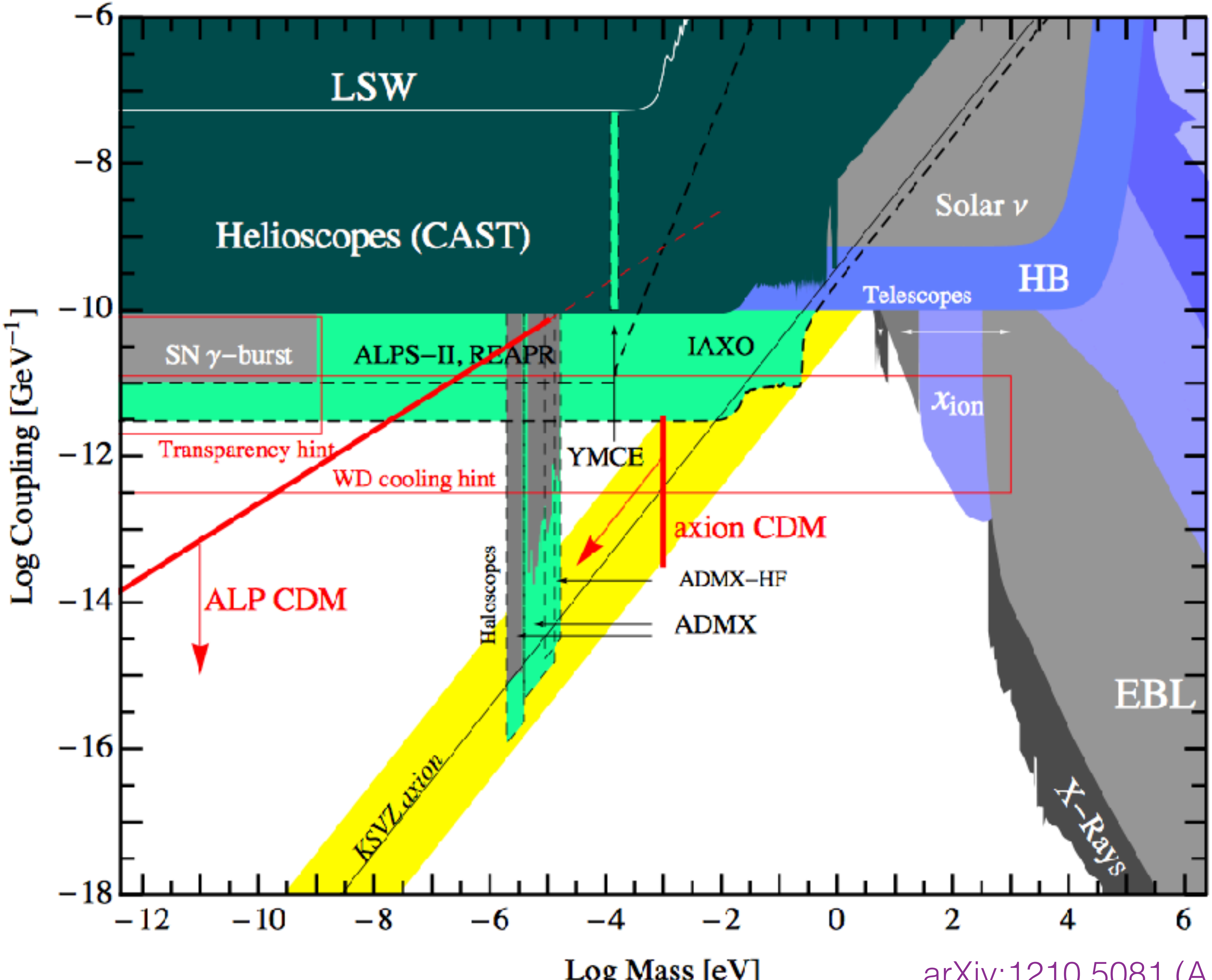
Weak theta angle:

$$+\,\theta_2 \frac{g_2}{32\pi^2} \sum_{I=a}^{3} F^a_{\mu\nu} \tilde{F}^{\mu\nu,a}$$

This angle changes under phase rotations

$$e^{i\alpha B}$$

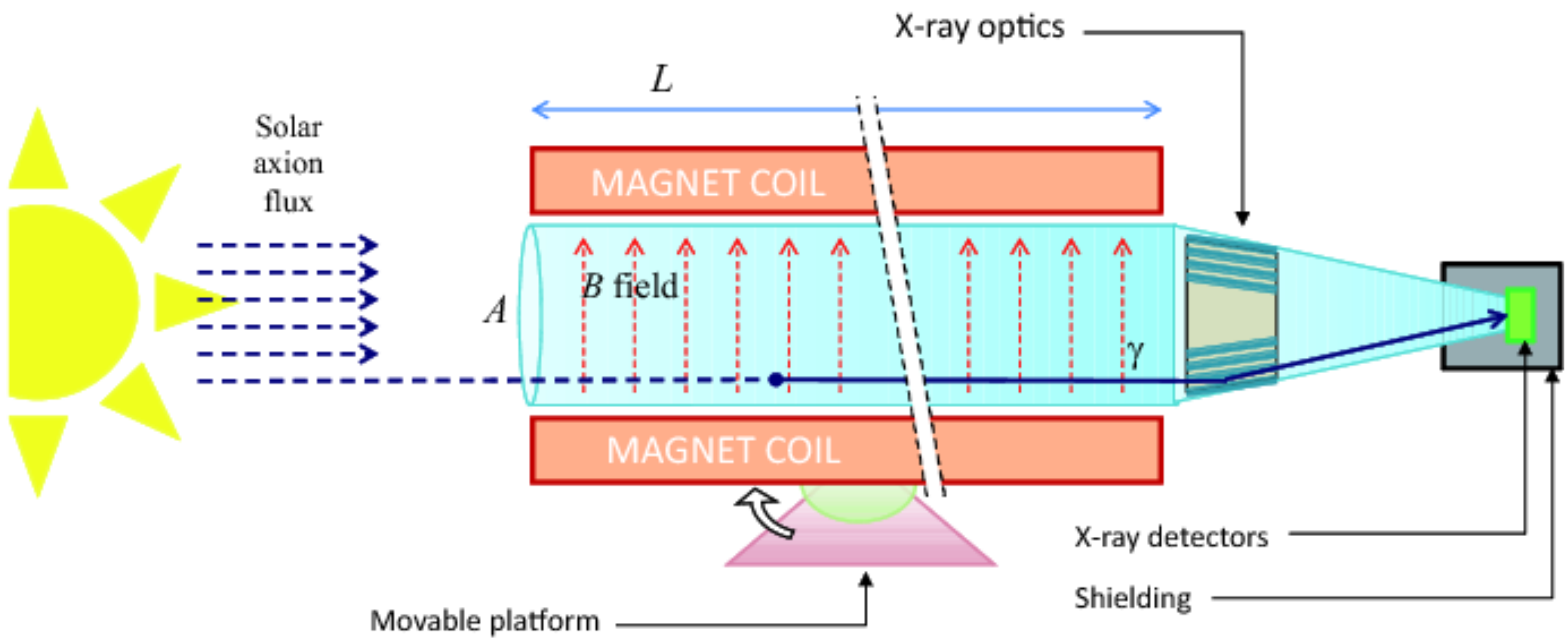
Where B is Baryon number. Hence we can rotate the angle to zero. There is no price to pay for this because baryon number is an exact symmetry apart from an anomaly with respect to $SU(2)_W$

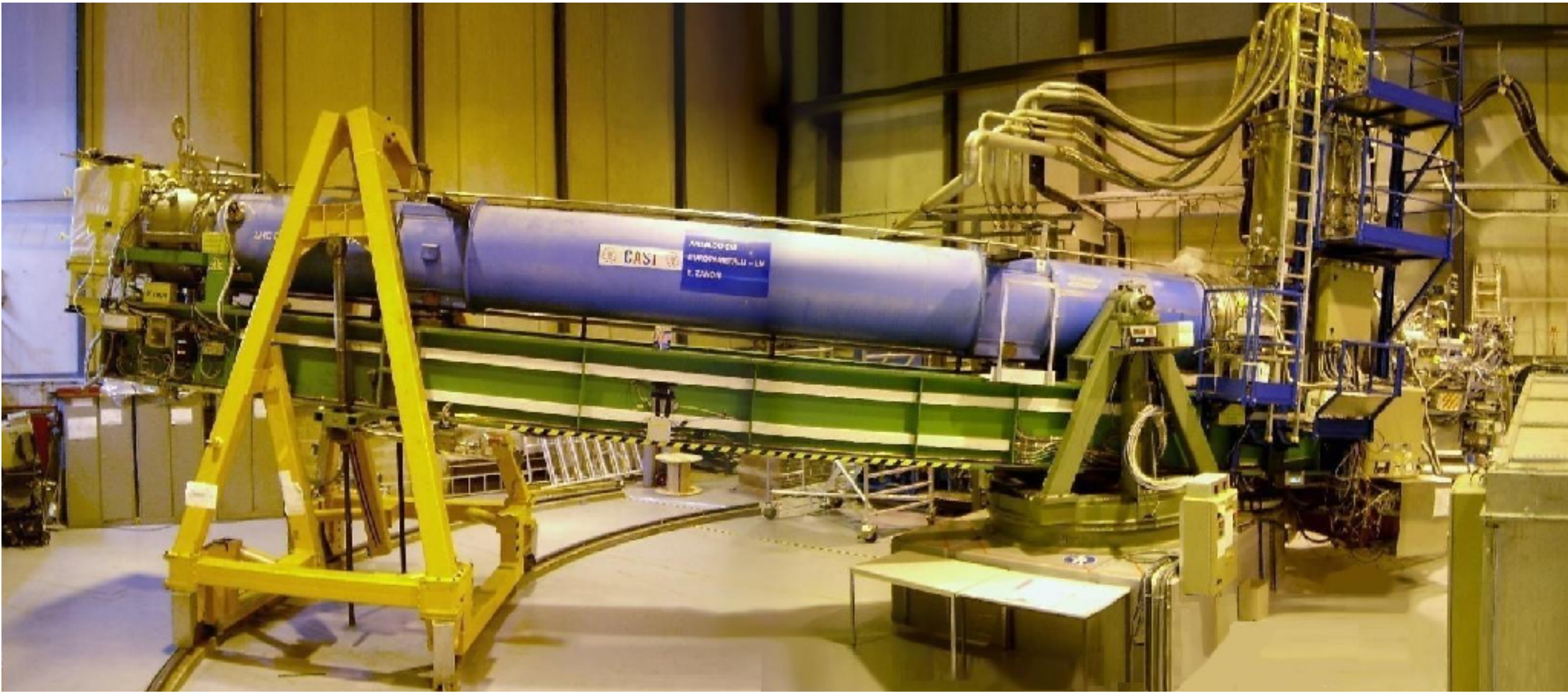


arXiv:1210.5081 (A. Ringwald)

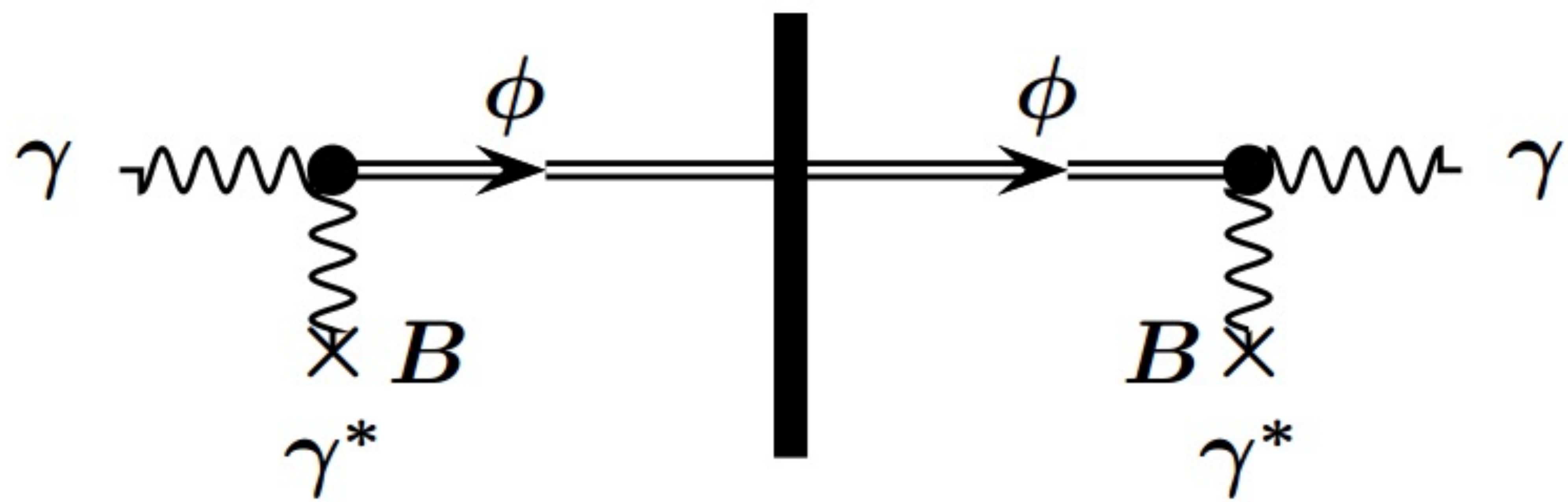
Axion helioscope: assume the sun produces axions

Axion haloscope: assume axions exist in galactic halo




CAST

Light shining through a wall experiments (e.g. ALPS)



Part 2

Grand Unification

Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

Unity of All Elementary Particle Forces

H. Georgi, S.L. Glashow (Harvard U.). Feb 1974. 4 pp.
Published in **Phys.Rev.Lett. 32 (1974) 438-441**
DOI: 10.1103/PhysRevLett.32.438


GROUP
THEORY
AHEAD

- Different groups can have the same Lie-algebra
  (e.g. $SU(2)$ and $SO(3)$)

- Some Lie algebra representations may not be representations of the corresponding group
  (e.g. spinor representations are not representations of $SO(3)$)

List of irreducible representations

- SU(2): 1,2,3,4,5,6,7,…..
- SO(3): 1,3,5,7,9,11,…..
- SU(3): 1,3,3*,6,6*,8,10,10*,15,15*…..
- SU(4): 1,4,4*,6,10,10*,15,20,20*,20’,……
- SU(5): 1,5,5*,10,10*,15,15*,24,…..

Quark doublet

$$\mathcal{L}_{\mathcal{Q}} = i \sum_{l=1}^{3} \sum_{\alpha=1}^{4} \sum_{\mu=1}^{4} \sum_{i=1}^{3} \sum_{p=1}^{2} \bar{\psi}_L^{\alpha,l,i,p} \gamma^{\mu}_{\alpha\beta} \left(\delta_{ij}\delta_{pq}\partial_\mu - ig_3 \delta_{pq} \sum_{I=1}^{8} A_\mu^I T_{ij}^I - ig_2 \delta_{ij} \sum_{a=1}^{3} A_\mu^a T_{pq}^a - ig_1 \tfrac{1}{6} \delta_{ij}\delta_{pq} B_\mu \right) \psi_L^{\beta,l,j,q}$$

$$\mathcal{L}_{\mathcal{Q}} = i\bar{\psi}_L^l \gamma^\mu \left(\partial_\mu - ig_3 A_\mu^I T^I - ig_2 A_\mu^a T^a - ig_1 B_\mu Y \right) \psi_L^l$$

$$(3, 2, \tfrac{1}{6})_L$$

Charge Quantization

		$SU(3)^3$	$SU(2)^3$	$SU(3)^2 \times U(1)$	$SU(2)^2 \times U(1)$	$U(1)^3$	(Grav) x $U(1)$
Q	$(3,2,1/6)$	2	0	1/3	1/2	1/36	1
U*	$(3^*,1,-2/3)$	-1	0	-2/3	0	-8/9	-2
D*	$(3^*,1,1/3)$	-1	0	1/3	0	1/9	1
L	$(1,2,-1/2)$	0	0	0	-1/2	-1/4	-1
E*	$(1,1,1)$	0	0	0	0	1	1
	Sum	0	0	0	0	0	0

GUTs, anomalies and Charge Quantization

One can try to impose one-family charge quantization on all three families by requiring that they all couple to the same Higgs.

But even that does not work:
One can have chiral fermions with irrational charges (in SM units) that get their mass from the SM Higgs

$$(3,2,\frac{1}{6}-\frac{x}{3})+(\bar{3},1,-\frac{2}{3}+\frac{x}{3})+(\bar{3},1,\frac{1}{3}+\frac{x}{3})$$
$$+(1,2,-\frac{1}{2}+x)+(1,1,1-x)+(1,1,-x)$$

$$\mathbf{5} \rightarrow (3,1,-\tfrac{1}{3}q) + (1,2,\tfrac{1}{2}q)$$

$$\mathbf{24} \rightarrow (8,1,0) + (1,3,0) + (1,1,0) + (3,2,-\tfrac{5}{6}q) + (3^*,2,\tfrac{5}{6}q)$$

$$\mathbf{10} \rightarrow (3^*,1,-\frac{2}{3}q) + (1,1,q) + (3,2,\frac{1}{6}q) \ ,$$
$$\mathbf{15} \rightarrow (6,1,-\frac{2}{3}q) + (1,3,q) + (3,2,\frac{1}{6}q) \ ,$$

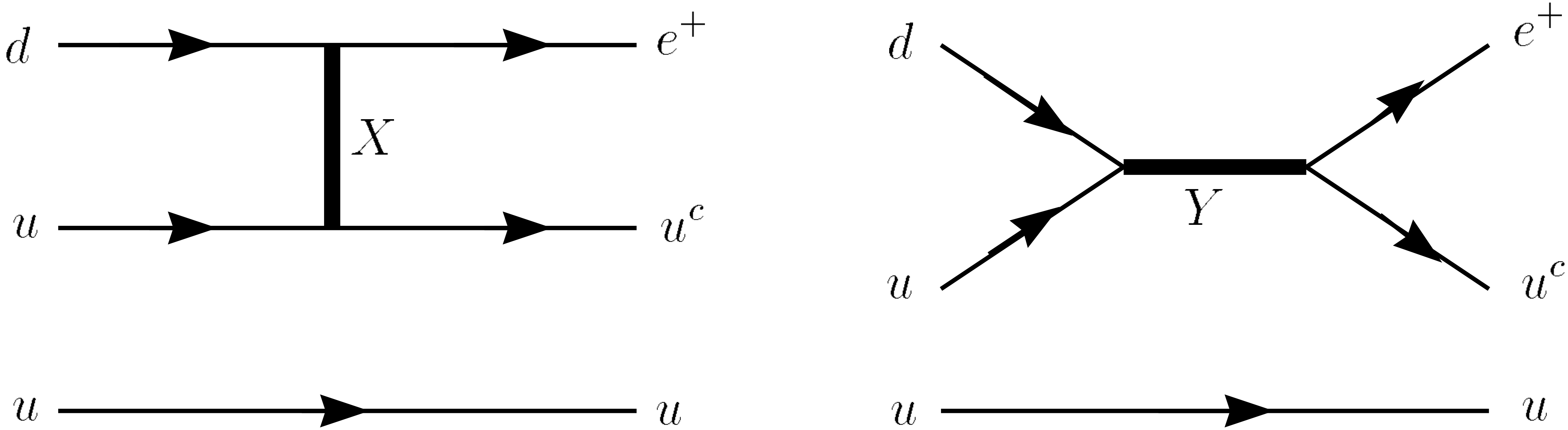
The first family

$$\Psi = (d_1^c, d_2^c, d_3^c, e^-, \nu)$$

$$\Delta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}$$

$$
\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ e^+ \\ \nu \end{bmatrix}_R , \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \bar{p}_3 & -\bar{p}_2 & -p_1(\theta) & -n_1 \\ -\bar{p}_3 & 0 & \bar{p}_1 & -p_2(\theta) & -n_2 \\ \bar{p}_2 & -p_1 & 0 & -p_3(\theta) & -n_3 \\ p_1(\theta) & p_2(\theta) & p_3(\theta) & 0 & -e^+ \\ n_1 & n_2 & n_3 & e^+ & 0 \end{bmatrix}_L
$$

$$\mathcal{L}_X = \frac{g_5}{\sqrt{2}} X_\mu^- [\bar{e}^- \gamma_\mu d^c + \bar{d} \gamma_\mu e^+ - \bar{u}^c \gamma_\mu u] + \text{c.c}$$
$$\mathcal{L}_Y = \frac{g_5}{\sqrt{2}} Y_\mu^- [\ \bar{\nu} \gamma_\mu d^c - \bar{u} \gamma_\mu e^+ - \bar{u}^c \gamma_\mu d] + \text{c.c}$$



From simple beginnings we have constructed the unique simple theory. It makes just one easily testable prediction, $\sin^2\theta_w = \frac{3}{8}$. It also predicts that the proton decays—but with an unknown and adjustable rate. More theoretical work is needed to determine whether the idea of infrared slavery, necessary for our unification, actually makes sense.

To include hadrons in the theory, we must use the Glashow-Iliopoulos-Maiani (GIM) mechanism and introduce a fourth quark p' carrying charm.[2] Still, decisions must be made: Should the quarks have fractional or integer charges? Should there be one quartet of quarks or several? Bouchiat, Iliopoulos, and Meyer suggested what seems the most attractive alternative: *three quartets of fractionally charged quarks.*[3] This combination

of the GIM mechanism with the notion of colored quarks[4] keeps the successes of the quark model and gives an important bonus: Lepton and hadron anomalies cancel so that the theory of weak and electromagnetic interactions is renormalizable.[5]

Hierarchy of Interactions in Unified Gauge Theories*

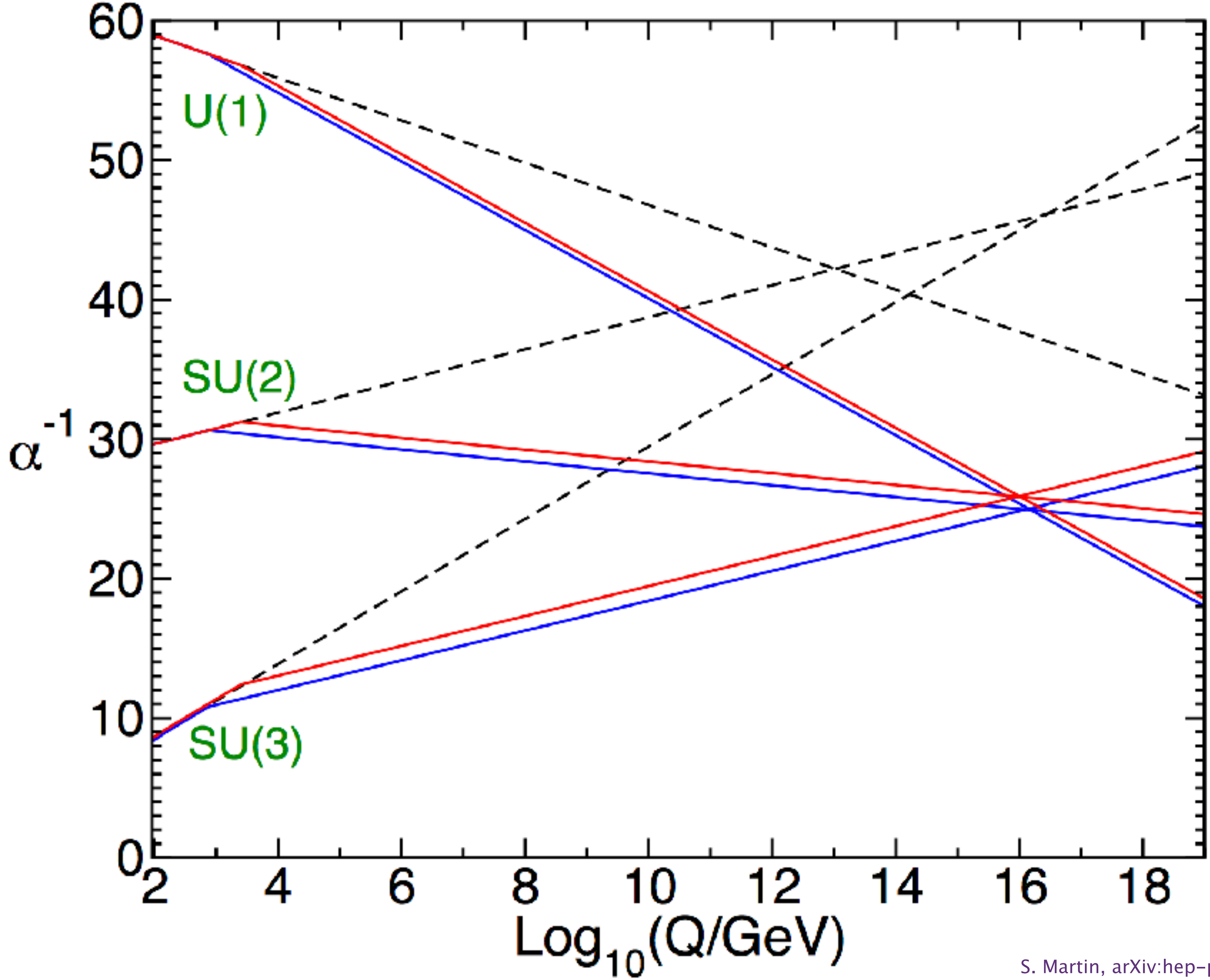
H. Georgi,† H. R. Quinn, and S. Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 15 May 1974)

We present a general formalism for calculating the renormalization effects which make strong interactions strong in simple gauge theories of strong, electromagnetic, and weak interactions. In an SU(5) model the superheavy gauge bosons arising in the spontaneous breakdown to observed interactions have mass perhaps as large as 10^{17} GeV, almost the Planck mass. Mixing-angle predictions are substantially modified.

Hierarchy of Interactions in Unified Gauge Theories

H. Georgi, Helen R. Quinn, Steven Weinberg (Harvard U.). Aug 1974. 12 pp.

Published in **Phys.Rev.Lett. 33 (1974) 451-454**
Print-74-1122 Rev. (HARVARD), PRINT-74-1122 (HARVARD)
DOI: 10.1103/PhysRevLett.33.451



S. Martin, arXiv:hep-ph/9709356

Proton stability (PDG 2016)

Mean life $\tau > 2.1 \times 10^{29}$ years, CL $= 90\%$ [f] ($p \to$ invisible mode)
Mean life $\tau > 10^{31}$ to 10^{33} years [f] (mode dependent)

Susy GUTS:

$$10^{34} \text{ to } 10^{36} \text{ years (dimension 6 operators)}$$