

# Supersymmetry Phenomenology

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# Disclaimer

- First version of this script, likely contains a few bugs... (lets find them together and solve it)
- I am not a theorist, so quite often I am at the limits of my knowledge here... try to do my best, but we need theory people to go deeper than this
- SUSY is a wide field, 100 slides already and I think I could have made 200....

# Outline 1

- Supersymmetry: Motivation
- Generic SUSY
- Generic models: Minimal SUSY Standard Model
- MSSM Lagrangian
- Weak sector: Neutralinos and charginos
- Higgs sector
- Strong sector: Squarks , stops, gluinos
- R parity
- Specific GUT scale models: mSUGRA etc.

# Outline 2

- SUSY Dark Matter
- Dark Matter annihilation and relic density
- Fine tuning problem
- Running coupling constants
- Searches at the Large Hadron Collider
- Precision observables
- Worldwide data
- Outlook and Summary

# Material

- SUSY primer, S. Martin

# SUSY motivation and history

# Basics ☺

- Spin  $\frac{1}{2}$  fermions
  - Dirac equation can give 4 solutions (spinors) with fixed chirality (L,R) for massless (anti)particles
- Spin 1 bosons (W,B fields before symm. breaking):
  - three distinct spin projections (-1, 0 and 1) and only 2 for massless particles (0 would correspond to rest frame)
- Spin 0 bosons (higgs):
  - Klein Gordon equation: 2 solutions for particle and antiparticle

Remember that in the SM L-chiral fermions behave different in gauge interactions than right handed ones.

# SUSY transformations

A supersymmetry (SUSY) transformation turns a bosonic state into a fermionic state, and vice versa.

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle, \quad Q|\text{Fermion}\rangle = |\text{Boson}\rangle.$$

Operator  $Q$  that generates SUSY transformations must be a spinor (like a fermion): (*Why ?*)

$Q^\dagger$  (the hermitian conjugate of  $Q$ ) is also a symmetry generator. Because  $Q$  and  $Q^\dagger$  are fermionic operators, they carry spin angular momentum  $\frac{1}{2}$

→ supersymmetry must be a spacetime symmetry.



# SUSY transformations

Standard Model: chiral fermions (i.e., fermions whose left- and right-handed pieces transform differently under the gauge group) → parity-violating interactions

To make this work the so called Hagen Lopusanski theorem says that the generators  $Q$  and  $Q^\dagger$  must satisfy an algebra of anticommutation ( $Q$  are fermionic) and commutation relations:

$$\{Q, Q^\dagger\} = P_\mu$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$

$$[P_\mu, Q] = [P_\mu, Q^\dagger] = 0$$

where  $P_\mu$  is the four-momentum generator of spacetime translation  
(and  $Q$  has also an index).

$$\hat{P}_\mu = \left( \frac{1}{c} \hat{E}, -\hat{\mathbf{p}} \right) = i\hbar \left( \frac{1}{c} \frac{\partial}{\partial t}, \nabla \right) = i\hbar \partial_\mu$$

→ SUSY: Space-time Spin symmetry !

# SUSY particle states

The single-particle states of a supersymmetric theory fall into Supermultiples: They contains both fermion and boson states, which are commonly known as superpartners of each other.

Since two particle states in the supermultiplet are related by some  $Q$  and  $Q^\dagger$  and thus by  $P^2$

→ **The superpartners must have the same mass since  $P^2$  is the mass operator**

The supersymmetry generators  $Q$ ,  $Q^\dagger$  also commute with the generators of gauge transformations.

→ **particles in the same supermultiplet must also be in the same representation of the gauge group, and so must have the same electric charges, weak isospin, and color degrees of freedom.**

→ **SUSY particles couple as their SM partners !!!**

→ **Couplings are NO free parameters in SUSY !!!**

# SUSY supermultiplets

Each supermultiplet contains an equal number of fermion and boson degrees of freedom.

Important example:

→ Fermionic quark can be  $q_L$  and  $q_R$  ⇒ Two different scalar quarks  $q_L$  and  $q_R$

Simplest possibilities: (Weyl Fermion = solution of massless Dirac equation)

- Chiral supermultiplet
- Weyl fermion two spin  $\frac{1}{2}$  states with different helicity/chirality
  - two scalars (spin 0), often merged into a complex scalar field, one as partner for each chirality
- Gauge supermultiplet:
- one spin=1 field (must be massless gauge boson, i.e. two helicity states)
  - two spin=1/2 Weyl Fermions (two helicity states) with same gauge properties

# SUSY supermultiplets

Next possibility:

- spin-2 graviton (with 2 helicity states, => 2 degrees of freedom)
- spin-3/2 superpartner called the gravitino. The gravitino would be massless if supersymmetry were unbroken => again 2 degrees of freedom

These supermultiples are enough to describe a  $N = 1$  supersymmetry, with  $N$  referring to the number of supersymmetries (the number of distinct copies of  $Q, Q^\dagger$ )

$N > 1$  SUSY cannot describe parity violation or chiral fermions in 4d space time.

➔ Only interesting in high dim. Theories...

# SUSY supermultiplets

- All SM particles need to be grouped in either a chiral or gauge supermultiplet.
- Quarks and Leptons  $\rightarrow$  ?
- Massless bosons of the SM  $\rightarrow$  ? (which ones?)
- Higgs fields of the SM ?

# SUSY supermultiplets

- All SM particles need to be grouped in either a chiral or gauge supermultiplet.
- Quarks and Leptons  $\rightarrow$  Chiral
- Massless bosons of the SM  $\rightarrow$  Gauge
- Higgs fields of the SM  $\rightarrow$  Chiral

Which spin do their SUSY partners have ?

# SUSY supermultiplets → Spins

- All SM particles need to be grouped in either a chiral or gauge supermultiplet.
- Quarks and Leptons → Chiral → Spin 0 SUSY partners
- Massless bosons of the SM → Gauge → Spin 1/2
- Higgs fields of the SM → Chiral → Spin 1/2

Which spin do their SUSY partners have ?

# SUSY supermultiplets → Names

- All SM particles need to be grouped in either a chiral or gauge supermultiplet.
- → Spin 0 SUSY partners → sfermions (scalar fermions)
- → Spin  $\frac{1}{2}$  gauge partners → gauginos
- → Spin  $\frac{1}{2}$  higgs partners → Higgsinos

Which spin do their SUSY partners have ?



# Sfermions

SUSY partners of the left and right handed parts of electron field are called

left- and right-handed selectrons :

*(note that they have NOT a right-handed helicity since they are not fermions but have spin 0, but they have the **couplings** as their superpartners)*

Quarks → squarks

Bottom quark → sbottom

Stop → stop

# Higgs

- Sitting in chiral supermultiplet
- SM has 1 complex doublet higgs field ( $H_0, H_+$ ) giving mass to the  $W^{+-}$  and  $Z^0$
- However:  
In SUSY we need 2 complex doublet fields sitting in 2 chiral supermultiplets.

# Why 2 Higgs supermultiplets?

- Before electroweak symmetry breaking we have a complex isospin doublet in the SM Higgs sector:  $H^+$  and  $H^0$  with 2 degrees of freedom each and  $Y=1/2$
- > Q makes now the SUSY Higgs-fermions (they have 2 spins directions each) so everything seems to be OK

However so called triangular anomalies will appear!

What is this?

Higher order graphs

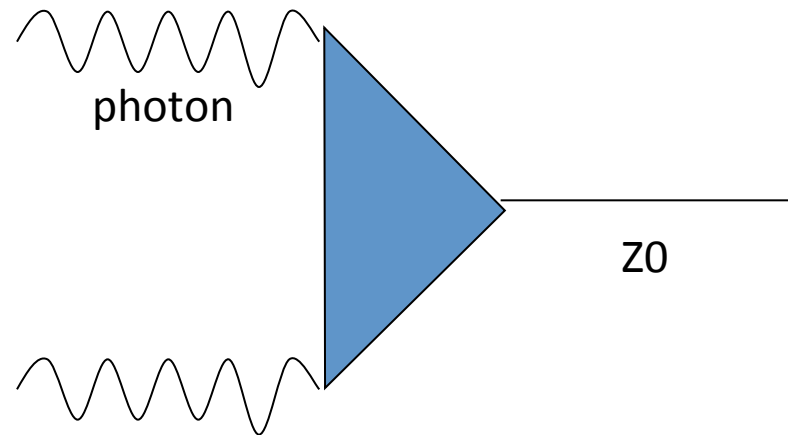
become divergent for left handed

fermions

if not  $\sum Y = 0$  ( $Y$  is the weak hypercharge)

(vanishes in the SM

for each generation -> Why?)



Solution:

Introduce at least two Higgs (Higgsino) doublets with opposite hypercharge  
This is called the 2HDM !

# Model building - supermultiplets

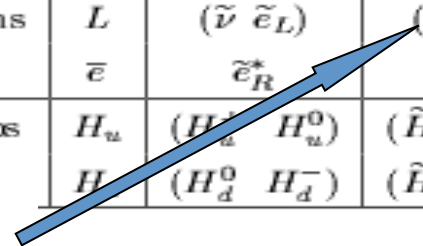
Organize fermions and bosons in spin multiplets

Table 1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model.

(Color, chirality, hypercharge =  $Q-I_{3L}$ )

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks (×3 families)	$Q$	$(\tilde{u}_L \tilde{d}_L)$	$(u_L d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons, leptons (×3 families)	$L$	$(\tilde{\nu} \tilde{e}_L)$	$(\nu e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, higgsinos	$H_u$	$(\tilde{H}_u^+ \tilde{H}_u^0)$	$(\tilde{H}_u^+ \tilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
	$H_d$	$(\tilde{H}_d^0 \tilde{H}_d^-)$	$(\tilde{H}_d^0 \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Always left handed  
In the SM → no index



(dimension of the multiplet)

Table 2: Boson supermultiplets in the Minimal Supersymmetric Standard Model.

Names	spin 1/2	spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
gluino, gluon	$\tilde{g}$	$g$	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\tilde{W}^\pm \tilde{W}^0$	$W^\pm W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\tilde{B}^0$	$B^0$	$(\mathbf{1}, \mathbf{1}, 0)$

# Degrees of freedom counting – auxiliary field

To make the numbers of bosonic and fermionic degrees of freedom match off-shell as well as on-shell, one has to introduce two more real scalar degrees of freedom into an auxiliary complex field  $F$ , which is eliminated when one goes on-shell.

The auxiliary field formulation is especially useful when discussing spontaneous supersymmetry

breaking...  $\mathcal{L} = \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{auxiliary}}$

# General SUSY Lagrangian

Very theoretical derivation of general SUSY Lagrangian: Skipped here:

Below is the most general set of renormalizable interactions for chiral fields that are consistent with supersymmetry:

See a “SUSY full-theory course” to derive them...

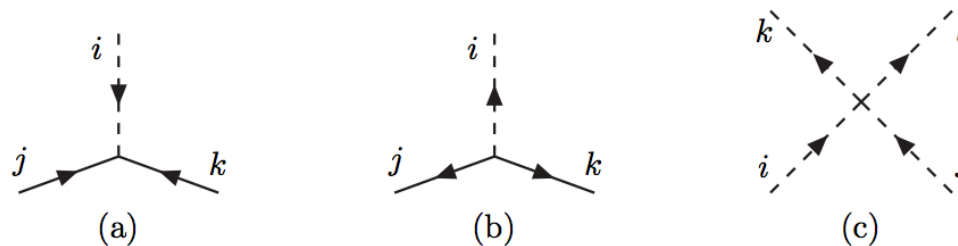


Figure 3.1: The dimensionless non-gauge interaction vertices in a supersymmetric theory: (a) scalar-fermion-fermion Yukawa interaction  $y^{ijk}$ , (b) the complex conjugate interaction  $y_{ijk}$ , and (c) quartic scalar interaction  $y^{ijn}y_{kln}^*$ .

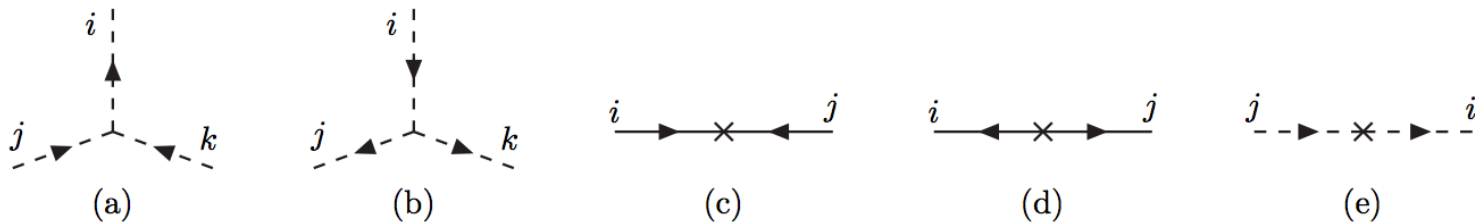


Figure 3.2: Supersymmetric dimensionful couplings: (a) (scalar)<sup>3</sup> interaction vertex  $M_{in}^*y^{jkn}$  and (b) the conjugate interaction  $M^{in}y_{jkn}^*$ , (c) fermion mass term  $M^{ij}$  and (d) conjugate fermion mass term  $M_{ij}^*$ , and (e) scalar squared-mass term  $M_{ik}^*M^{kj}$ .

# General SUSY Gauge Interactions

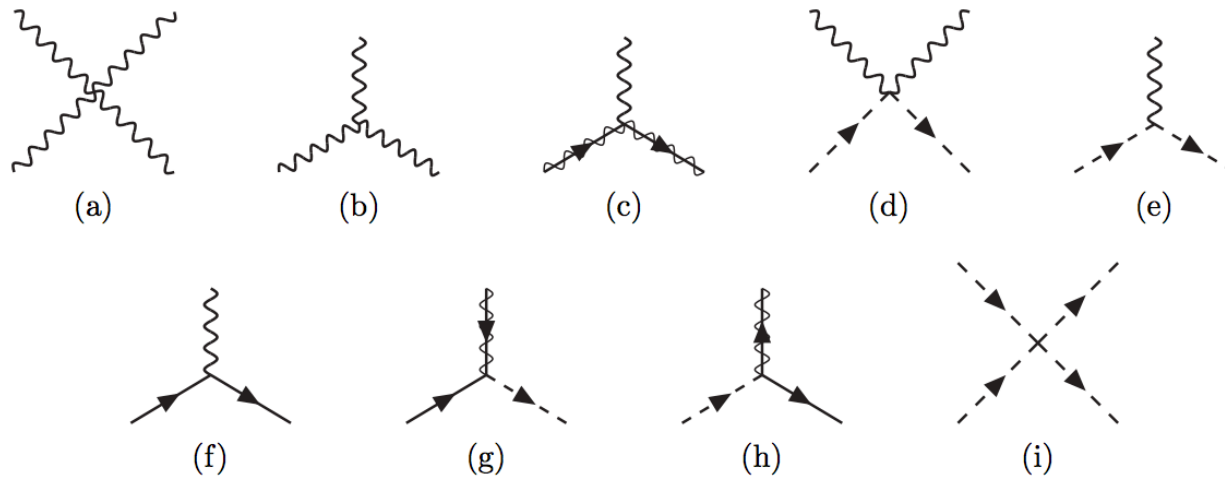


Figure 3.3: Supersymmetric gauge interaction vertices.

Figures 3.3a,b,c occur only when the gauge group is non-Abelian, for example for  $SU(3)_C$  color and  $SU(2)_L$  weak isospin in the MSSM.

Figure 3.3c shows the coupling of a gaugino to a gauge boson; the gaugino line in a Feynman diagram is traditionally drawn as a solid fermion line superimposed on a wavy line..

Figure 3.3g we have the coupling of a gaugino to a chiral fermion and a complex scalar (dashed line)

# MSSM lagrangian

The superpotential for the MSSM is

$$W_{\text{MSSM}} = \bar{u}y_u Q H_u - \bar{d}y_d Q H_d - \bar{e}y_e L H_d + \mu H_u H_d. \quad (6.1.1)$$

The objects  $H_u$ ,  $H_d$ ,  $Q$ ,  $L$ ,  $\bar{u}$ ,  $\bar{d}$ ,  $\bar{e}$  appearing here are chiral superfields corresponding to the chiral supermultiplets in Table 1.1. (Alternatively, they can be just thought of as the corresponding scalar fields, as was done in section 3, but we prefer not to put the tildes on  $Q$ ,  $L$ ,  $\bar{u}$ ,  $\bar{d}$ ,  $\bar{e}$  in order to

- The  $\mu$  term in eq. (6.1.1) is the supersymmetric version of the Higgs boson mass in the Standard Model.
- There are dimensionless Yukawa coupling parameters  $y_u$ ,  $y_d$ ,  $y_e$  as  $3 \times 3$  matrices in family space



# Yukawa couplings

In the limit that only 3<sup>rd</sup> generation masses are important we yield:

$$\mathbf{y}_u \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad \mathbf{y}_d \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad \mathbf{y}_e \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}.$$

$$W_{\text{MSSM}} \approx y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) - y_\tau(\bar{\nu}_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+ H_d^- - H_u^0 H_d^0).$$

*Minus signs due to SU(2)<sub>L</sub> structure and terms needed to get vacuum exp. value*

Terms like  $H_u^* H_u$  or  $H_d^* H_d$  are forbidden in the superpotential, which must be holomorphic (complex differentiable → no  $H H^*$  allowed)

In the Standard Model the down-type quarks couple to the Higgs field (which has  $Y=-1/2$ ) and the up-type quarks to its complex conjugate (which has  $Y=+1/2$ )

→ In SUSY this is not allowed  
 → 2<sup>nd</sup> reason why we need two separate Higgs doublets to give mass to the down and up type particles

# Yukawa coupling and new interactions

Examples of SM and SUSY interactions with strength  $y_t$

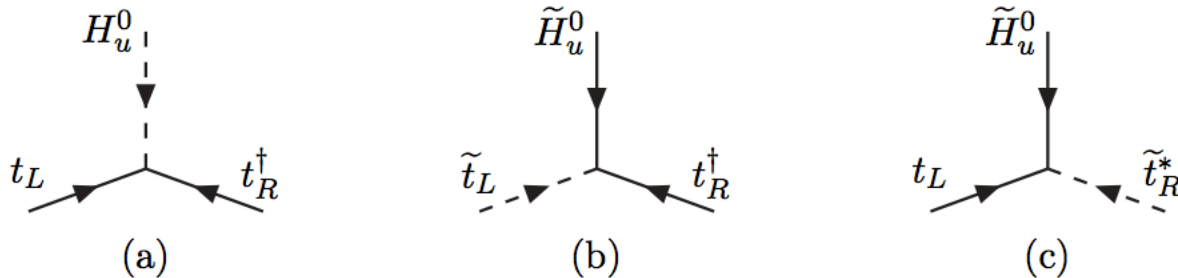


Figure 6.1: The top-quark Yukawa coupling (a) and its “supersymmetrizations” (b), (c), all of strength  $y_t$ .

For each of the three interactions, there is another with  $H_u^0 \rightarrow H_u^+$  and  $t_L \rightarrow -b_L$  (with tildes where appropriate),

# Further couplings

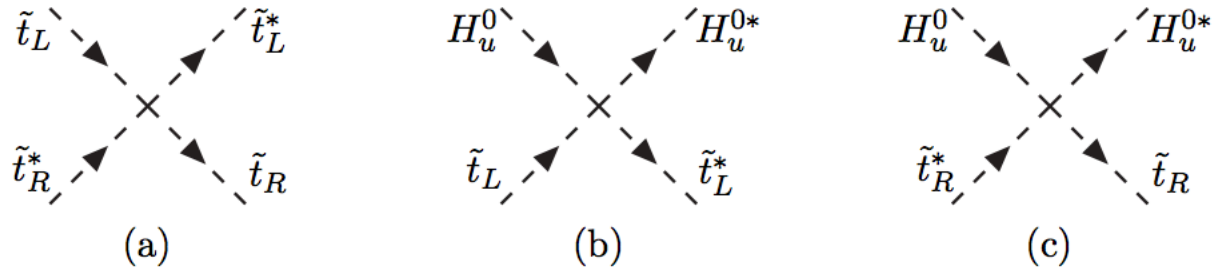


Figure 6.2: Some of the (scalar)<sup>4</sup> interactions with strength proportional to  $y_t^2$ .

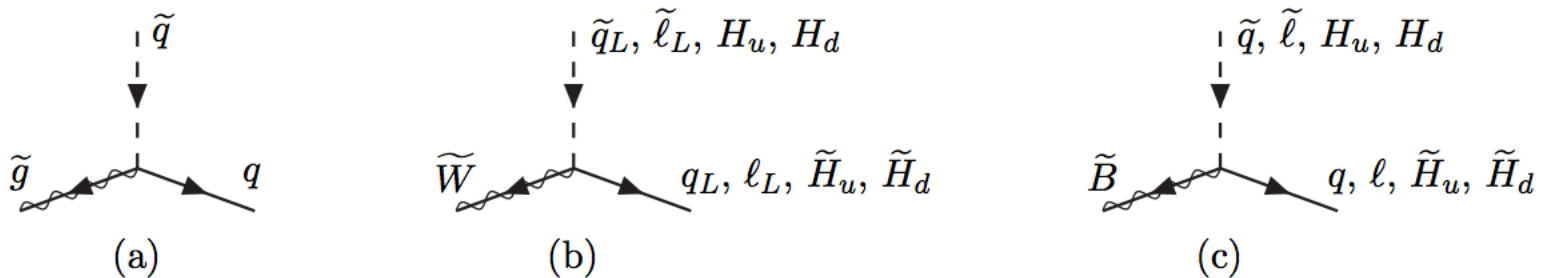


Figure 6.3: Couplings of the gluino, wino, and bino to MSSM (scalar, fermion) pairs.

Gaugino couplings highly important

→ Important: Wino couples only to left-handed particle

→ What is the Wino and Bino again ?

# Higgs and Higgsino mass terms

...Many terms in among them are (Higgs)<sup>4</sup> terms (see graph I on previous slides). Here we look at the dimensional terms

$$-\mathcal{L}_{\text{higgsino mass}} = \mu(\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0) + \text{c.c.},$$

as well as Higgs squared-mass terms in the scalar potential

$$-\mathcal{L}_{\text{supersymmetric Higgs mass}} = |\mu|^2(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2).$$

Potential is non-negative with a minimum at  $H_u^0 = H_d^0 = 0$

- No electroweak symmetry breaking without including a negative supersymmetry-breaking squared-mass soft term for the Higgs scalars
- Interesting is that electroweak scale (minimum= $v$ ) is coupled to the parameter  $\mu$  (**which is not SUSY breaking**)
- Not clear why this parameter should be around 100-1000 GeV (or introduce cancellation **with the soft-breaking terms**)
- Terms of different origin ?
- This is the so called “little- $\mu$ ” problem

# Solutions to little mu problem

$\mu$  term is absent before symmetry breaking, and then it arises from the VEV(s) of the symmetry breaking of some new field.



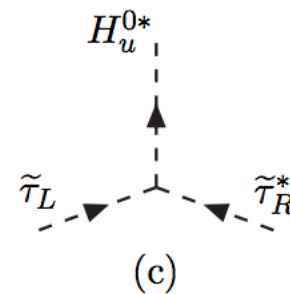
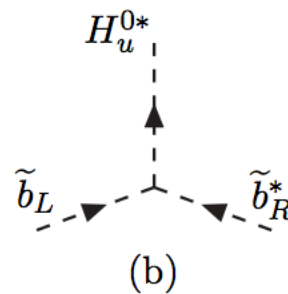
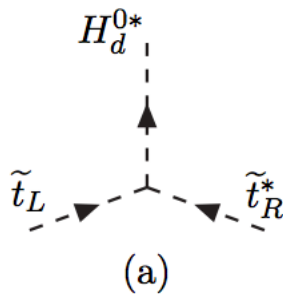
Then the term **is** related to SUSY breaking

Example is the NMSSM  
(where  $\mu$  is generated)

*But still need to explain why SUSY breaking masses are much lower than Planck scale...*

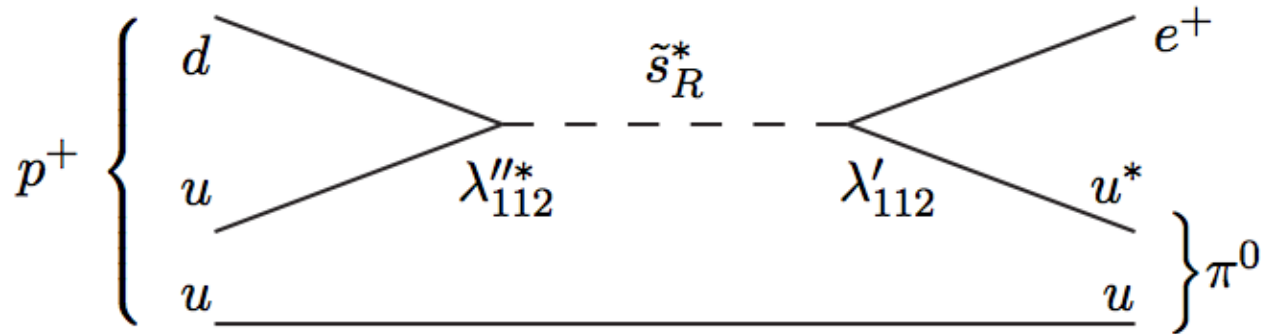
# Further Yukawa coupling terms

$$\mathcal{L}_{\text{supersymmetric (scalar)}^3} = \mu^* (\tilde{u}_L \tilde{u}_R H_d^{0*} + \tilde{d}_L \tilde{d}_R H_u^{0*} + \tilde{e}_L \tilde{e}_R H_u^{0*} + \tilde{u}_L \tilde{d}_R H_d^{-*} + \tilde{d}_L \tilde{u}_R H_u^{+*} + \tilde{e}_L \tilde{\nu}_R H_u^{+*}) + \text{c.c.}$$



➔ Mixing of left and right handed stops, sbottoms and staus !

# Proton decay



In general MSSM both couplings are allowed via Scalar-fermion-fermion interactions (page 21)

Proportional to yukawa coupling...

➔ These interactions must be tiny since we would otherwise observe proton decay

# Baryon and Lepton number violating terms

Need to forbid baryon or Lepton number violating terms (or both):

$$W_{\Delta L=1} = \frac{1}{2} \lambda^{ijk} L_i L_j \bar{e}_k + \lambda'^{ijk} L_i Q_j \bar{d}_k + \mu^i L_i H_u$$

$$W_{\Delta B=1} = \frac{1}{2} \lambda''^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

*L etc. are  
chiral supermultiplets*



# R-parity

Fast proton decay likely with very general SUSY Lagrangian

→ Solution: assume conservation of a new multiplicative quantum number called **R-parity**:

baryon and lepton numbers of particles are no longer assumed to be conserved. Instead R-parity may be conserved, where the R-parity is

$$R = (-1)^{2j+3B+L}.$$

- With spin  $j$ , baryons  $B$ , and leptons  $L$ .
- All Standard Model- like particles have R-parity of 1 while the new “supersymmetric” particles have R-parity -1.

# R-parity conservation consequences

1. Lightest SUSY particle stable  
a candidate for dark matter → Why?
2. Collider signals: SUSY particles are always produced in pairs

**The minimal SUSY model (MSSM) is defined to have r-parity conservation**

# Intermezzo

## EW symmetry breaking in the Standard Model:

- Reason: massive Z,W terms make theory non- renormalizable
- EW symmetry breaking:

SU(2)  $\times$  U(1) symmetry



U<sub>Q</sub>(1) symmetry

3 massless SU<sub>L</sub>(2) vector bosons:  
W<sub>1</sub>, W<sub>2</sub>, W<sub>3</sub>

1 massless U<sub>Y</sub>(1) vector boson : B

1 complex doublet self-interacting  
Higgs fields (=4 real scalar fields)

interaction between Higgs and fermions

3 massive vector bosons: W<sup>+</sup>, W<sup>-</sup>  
(W<sub>1</sub>, W<sub>2</sub>) , Z<sup>0</sup> (B, W<sub>3</sub>)

1 massless U<sub>Q</sub>(1) boson:  $\gamma$  (B, W<sub>3</sub>)

1 real scalar Higgs field

+3 *Goldstone Bosons*

*'eaten' by the massive vector bosons*

Mass terms for quarks and leptons

# SUSY breaking

## Supersymmetry is a broken symmetry

→ We expect a mechanism similar to electroweak symmetry breaking which yields a broken symmetry at low energies

Or: The underlying model should have a Lagrangian density that is invariant under supersymmetry, but a vacuum state that is not.

→ Mass terms for SUSY particles are introduced due to SUSY breaking

→ We do not know exactly how ?

→ Lets be ignorant on the exact mechanism and introduce all allowed Mass terms...

SUSY breaking should be soft (of positive mass dimension) in order to be able to naturally maintain a solution to the hierarchy problem

→ See later slides on hierarchy problem

# Soft breaking terms

$$\mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2} \left( M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + \text{c.c.} \right)$$

Remember:

M3 = Gluino mass

M2 = Wino mass

M1 = Bino mass

# Soft breaking terms

$$\mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2} \left( M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + \text{c.c.} \right) \\ - \left( \tilde{u} \mathbf{a}_u \tilde{Q} H_u - \tilde{d} \mathbf{a}_d \tilde{Q} H_d - \tilde{e} \mathbf{a}_e \tilde{L} H_d + \text{c.c.} \right)$$

- Later related to Yukawa couplings
- Again 3x3 matrices in family space (with mass dimension)

# Soft breaking terms

$$\begin{aligned}\mathcal{L}_{\text{soft}}^{\text{MSSM}} = & -\frac{1}{2} \left( M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + \text{c.c.} \right) \\ & - \left( \tilde{u} \mathbf{a}_u \tilde{Q} H_u - \tilde{d} \mathbf{a}_d \tilde{Q} H_d - \tilde{e} \mathbf{a}_e \tilde{L} H_d + \text{c.c.} \right) \\ & - \tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} - \tilde{L}^\dagger \mathbf{m}_L^2 \tilde{L} - \tilde{u} \mathbf{m}_u^2 \tilde{u}^\dagger - \tilde{d} \mathbf{m}_d^2 \tilde{d}^\dagger - \tilde{e} \mathbf{m}_e^2 \tilde{e}^\dagger\end{aligned}$$

- These are squared 3x3 mass matrices
- Different for left and right-handed
- Different for u and d-type
- Different for squarks and sleptons

# Soft breaking terms

$$\begin{aligned}\mathcal{L}_{\text{soft}}^{\text{MSSM}} = & -\frac{1}{2} \left( M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + \text{c.c.} \right) \\ & - \left( \tilde{u} \mathbf{a}_u \tilde{Q} H_u - \tilde{d} \mathbf{a}_d \tilde{Q} H_d - \tilde{e} \mathbf{a}_e \tilde{L} H_d + \text{c.c.} \right) \\ & - \tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{L}^\dagger m_L^2 \tilde{L} - \tilde{u} m_u^2 \tilde{u}^\dagger - \tilde{d} m_d^2 \tilde{d}^\dagger - \tilde{e} m_e^2 \tilde{e}^\dagger \\ & - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + \text{c.c.}) .\end{aligned}$$

➔ These are additional soft breaking terms for the Higgs

➔ Now with  $H_u^* H_u$  ➔ Why ?

➔ Is  $b$  and  $\mu$  related ?



# Soft breaking terms summary

Expect:

$$M_1, M_2, M_3, \mathbf{a}_u, \mathbf{a}_d, \mathbf{a}_e \sim m_{\text{soft}},$$
$$m_Q^2, m_L^2, m_u^2, m_d^2, m_e^2, m_{H_u}^2, m_{H_d}^2, b \sim m_{\text{soft}}^2,$$

All these terms together yield:

105 new parameters

(masses, phases and mixing angles in the MSSM Lagrangian that cannot be rotated away)

→ Is this a problem ?

# Many parameters?

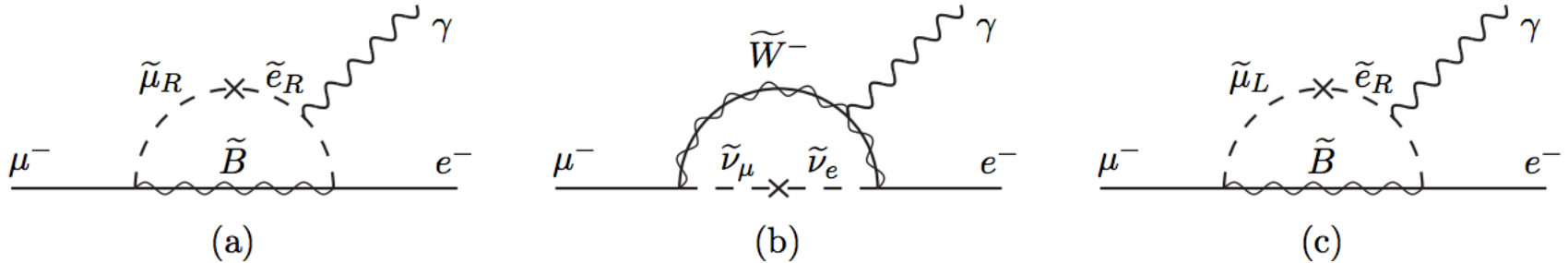
The true SUSY model (if existing) has likely much less parameters.

We see that random setting of some offdiagonal elements of the mass matrices yield again e.g. lepton number violation

→ Can reduce amount of “effective” parameters since we know that offdiagonal elements must be very small....

# Constraints of offdiagonal elements

Mu => e gamma



K0 mixing:

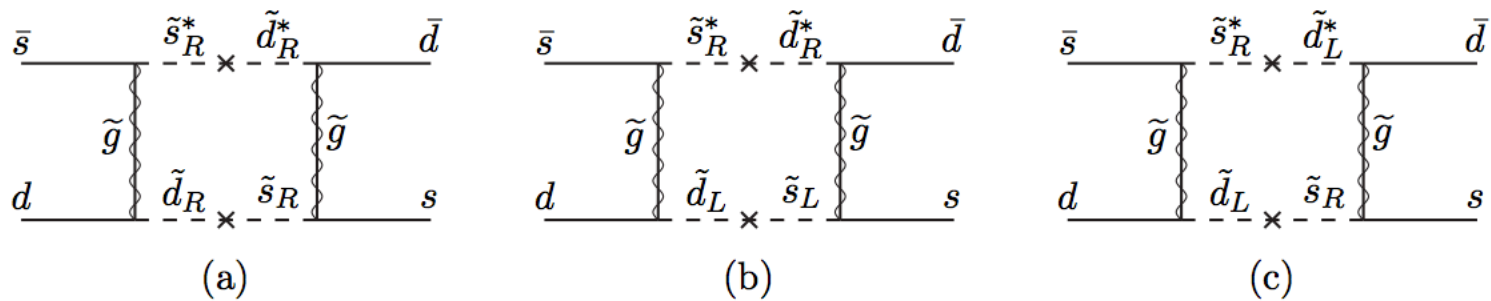


Figure 6.7: Some of the diagrams that contribute to  $K^0 \leftrightarrow \bar{K}^0$  mixing in models with strangeness-

# Phenomenological MSSM

$$m_{\mathbf{Q}}^2 = m_{\mathbf{Q}}^2 \mathbf{1}, \quad m_{\mathbf{u}}^2 = m_{\mathbf{u}}^2 \mathbf{1}, \quad m_{\mathbf{d}}^2 = m_{\mathbf{d}}^2 \mathbf{1}, \quad m_{\mathbf{L}}^2 = m_{\mathbf{L}}^2 \mathbf{1}, \quad m_{\mathbf{e}}^2 = m_{\mathbf{e}}^2 \mathbf{1}.$$

$$\mathbf{a}_{\mathbf{u}} = A_{u0} \mathbf{y}_{\mathbf{u}}, \quad \mathbf{a}_{\mathbf{d}} = A_{d0} \mathbf{y}_{\mathbf{d}}, \quad \mathbf{a}_{\mathbf{e}} = A_{e0} \mathbf{y}_{\mathbf{e}},$$

→ Only the squarks and sleptons of the third family can have large (scalar)<sup>3</sup> couplings.

Assume that CP violation only due to phase of CKM Matrix

- ⇒ Now typically about 15 – 25 parameters
- We call this phenomenologically relevant MSSM
- pMSSM is not a model, but a collection of possible SUSY models

# Phenomenological MSSM

Usually:  $\mathbf{m}_Q^2 \approx \begin{pmatrix} m_{Q_1}^2 & 0 & 0 \\ 0 & m_{Q_1}^2 & 0 \\ 0 & 0 & m_{Q_3}^2 \end{pmatrix}, \quad \mathbf{m}_{\bar{u}}^2 \approx \begin{pmatrix} m_{\bar{u}_1}^2 & 0 & 0 \\ 0 & m_{\bar{u}_1}^2 & 0 \\ 0 & 0 & m_{\bar{u}_3}^2 \end{pmatrix},$

$$\mathbf{a}_u \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_t \end{pmatrix}, \quad \mathbf{a}_d \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_b \end{pmatrix}, \quad \mathbf{a}_e \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_\tau \end{pmatrix}$$

→ Only the squarks and sleptons of the third family can have large (scalar)<sup>3</sup> couplings.

Assume that CP violation only due to phase of CKM Matrix

- ⇒ Now typically about 15 – 25 parameters
- We call this phenomenologically relevant MSSM
- pMSSM is not a model, but a collection of possible SUSY models

# This looks like a mess ?

The MSSM should be seen as our theoretical constraints of SUSY.

The “true” SUSY model is likely much simpler in structure and that is the reason why many of the 105 parameters are likely not relevant and should be set to specific values.

# The mass spectrum of the MSSM

# MSSM Higgs sector



# After the EW symmetry breaking

Gauge and Higgs fields are supersymmetrized before electroweak Symmetry breaking (hence they can be put into multiplets):

Higgs sector in 2HDM:

$(H^+_1, H^0_1)$  with  $Y=+1/2$  and  $(H^0_2, H^-_2)$  with  $Y=-1/2$

→ After the Higgs-Mechanism (eats 3 degrees of freedom from the  $8=2^*$  complex doublet)

These Higgs field mix to 5 observable Higgs bosons:

$h^0, H^0$  (neutral, CP even)

$A$  (neutral, CP odd)

$H^+, H^-$  (charged)

→ In addition we have the Higgsinos (8 degrees of susy higgs field transform to 4 Higgsinos with spin  $\frac{1}{2}$ )

$H^+_1, H^0_1, H^0_2, H^-_2$  (all with a tilde!!, I can't make the tilde in PowerPoint)

# In more detail

Scalar potential in the MSSM:

$$\begin{aligned}
 V = & (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\
 & + [b(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.}] \\
 & + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \frac{1}{2}g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2.
 \end{aligned}$$

Finding minimum  $\rightarrow$  Vacuum expectation values and prediction for Z mass

$$v_u = \langle H_u^0 \rangle,$$

$$v_d = \langle H_d^0 \rangle.$$

$$v_u^2 + v_d^2 = v^2 = 2m_Z^2 / (g^2 + g'^2) \approx (174 \text{ GeV})^2.$$

The ratio of the VEVs is traditionally written as

$$\tan \beta \equiv v_u / v_d.$$

$$m_Z^2 = \frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1 - \sin^2(2\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2.$$

$\rightarrow$  This is the SUSY version of the Hierarchy problem  
 $m_{H_u}$  and  $\mu$  need to cancel to yield  $M_Z$ !

# Higgs mass predictions

$$m_{A^0}^2 = 2b / \sin(2\beta) = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2$$

$$m_{h^0, H^0}^2 = \frac{1}{2} \left( m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 - m_Z^2)^2 + 4m_Z^2 m_{A^0}^2 \sin^2(2\beta)} \right),$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2.$$

→ Prediction for all 5 Higgs masses

→ Can trade  $m_{H_u}$ ,  $m_{H_d}$  and  $b$  for

$m_A$ ,  $\mu$  and  $\tan(\beta)$  as pMSSM parameters

# Higgs mass prediction

This yields at tree level a prediction for the lightest Higgs mass:

$$m_{h^0} < m_Z |\cos(2\beta)|$$

$$\rightarrow M_{\text{higgs}} < 91 \text{ GeV}$$

# Higgs mass prediction

Beyond tree level → Loop contributions:

$$\Delta(m_{h^0}^2) = \text{---} h^0 \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} + \text{---} h^0 \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} + \text{---} h^0 \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$

Figure 8.2: Contributions to the MSSM lightest Higgs squared mass from top-quark and top-squark one-loop diagrams. Incomplete cancellation, due to soft supersymmetry breaking, leads to a large positive correction to  $m_{h^0}^2$  in the limit of heavy top squarks.

→  $M_{\text{higgs}} < 135 \text{ GeV}$

We know now:  $M_{\text{higgs}} = 125 \text{ GeV}$

→ SUSY scale usually  $> 1 \text{ TeV}$  (stops heavy or highly mixed)

# MSSM electroweak sector

# After the EW symmetry breaking

- Supersymmetrization happens “before” EW symmetry breaking

-> 2 Winos  $\tilde{W}^\pm$  have same quantum numbers as Higgsino fields  $H_{\pm 1}$ ,  $H_{\pm 2}$

-> They mix to 4 charginos  $\tilde{\chi}_{1,2}^\pm$

The neutral Wino and Bino and the Higgsinos  $H_{01}$ ,  $H_{02}$  mix to 4 neutralinos:

$$\tilde{\chi}_{1,2,3,4}^0$$

It may also be that Higgsinos and Winos+Bino stay separate (e.g. if susy would be unbroken)

→ We can get then two neutral Higgsinos + Photino + Zino

# Mixing matrix

$$\mathcal{L}_{\text{neutralino mass}} = -\frac{1}{2} (\psi^0)^T M_{\tilde{\chi}^0} \psi^0 + \text{h.c.},$$

where

$$\psi^0 = \begin{pmatrix} \tilde{B} \\ \tilde{W}^3 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix}$$

and

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -c_\beta s_{\theta_W} m_Z & s_\beta s_{\theta_W} m_Z \\ 0 & M_2 & c_\beta c_{\theta_W} m_Z & -s_\beta c_{\theta_W} m_Z \\ -c_\beta s_{\theta_W} m_Z & c_\beta c_{\theta_W} m_Z & 0 & -\mu \\ s_\beta s_{\theta_W} m_Z & -s_\beta c_{\theta_W} m_Z & -\mu & 0 \end{pmatrix}.$$

Here we have introduced abbreviations  $s_\beta = \sin \beta$ ,  $c_\beta = \cos \beta$ ,  $s_W = \sin \theta_W$ , and  $c_W = \cos \theta_W$ . The mass matrix  $\mathbf{M}_{\tilde{N}}$  can be diagonalized by a unitary matrix  $\mathbf{N}$  to obtain mass eigenstates:

$$\tilde{N}_i = \mathbf{N}_{ij} \psi_j^0, \quad (8.2.4)$$

so that

$$\mathbf{N}^* \mathbf{M}_{\tilde{N}} \mathbf{N}^{-1} = \begin{pmatrix} m_{\tilde{N}_1} & 0 & 0 & 0 \\ 0 & m_{\tilde{N}_2} & 0 & 0 \\ 0 & 0 & m_{\tilde{N}_3} & 0 \\ 0 & 0 & 0 & m_{\tilde{N}_4} \end{pmatrix} \quad (8.2.5)$$



# Mixing matrix simplified

Regime	Composition neutralinos	Composition charginos
$M_1 < M_2 <  \mu $	$(\widetilde{B}, \widetilde{W}, \widetilde{H}, \widetilde{H})$	$(\widetilde{W}, \widetilde{H})$
$M_1 <  \mu  < M_2$	$(\widetilde{B}, \widetilde{H}, \widetilde{H}, \widetilde{W})$	$(\widetilde{H}, \widetilde{W})$
$ \mu  < M_1 < M_2$	$(\widetilde{H}, \widetilde{H}, \widetilde{B}, \widetilde{W})$	$(\widetilde{H}, \widetilde{W})$
$ \mu  < M_2 < M_1$	$(\widetilde{H}, \widetilde{H}, \widetilde{W}, \widetilde{B})$	$(\widetilde{H}, \widetilde{W})$
$M_2 <  \mu  < M_1$	$(\widetilde{W}, \widetilde{H}, \widetilde{H}, \widetilde{B})$	$(\widetilde{W}, \widetilde{H})$
$M_2 < M_1 <  \mu $	$(\widetilde{W}, \widetilde{B}, \widetilde{H}, \widetilde{H})$	$(\widetilde{W}, \widetilde{H})$

**Table 1:** Composition of the neutralinos  $(\widetilde{\chi}_1^0, \widetilde{\chi}_2^0, \widetilde{\chi}_3^0, \widetilde{\chi}_4^0)$  and charginos  $(\widetilde{\chi}_1^\pm, \widetilde{\chi}_2^\pm)$ .

# Couplings matrix Chargino/Neutralino

	$\tilde{B}$	$\tilde{W}^3$	$\tilde{H}_S^0$	$\tilde{H}_A^0$	$\tilde{W}^\pm$	$\tilde{H}_{u/d}^\pm$
$\tilde{B}$			$h^0, H^0, A^0$	$h^0, H^0, A^0$		$H^\mp$
$\tilde{W}^3$			$h^0, H^0, A^0$	$h^0, H^0, A^0$	$W^\mp$	$H^\mp$
$\tilde{H}_S^0$	$h^0, H^0, A^0$	$h^0, H^0, A^0$		$Z$	$H^\mp$	$W^\mp$
$\tilde{H}_A^0$	$h^0, H^0, A^0$	$h^0, H^0, A^0$	$Z$		$H^\mp$	$W^\mp$
$\tilde{W}^\pm$		$W^\mp$	$H^\mp$	$H^\mp$	$Z$	$h^0, H^0, A^0$
$\tilde{H}_{u/d}^\pm$	$H^\mp$	$H^\mp$	$W^\mp$	$W^\mp$	$h^0, H^0, A^0$	$Z$

**Table 2:** Interactions between the Binos, Winos and Higgsinos. The entries indicate which fields are involved in the interaction.

# MSSM - Particle Content

Particle content of the MSSM:

Superpartners for Standard Model particles:

$$\left[ u, d, c, s, t, b \right]_{L,R} \quad \left[ e, \mu, \tau \right]_{L,R} \quad \left[ \nu_{e,\mu,\tau} \right]_L \quad \text{Spin } \frac{1}{2}$$

$$\left[ \tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b} \right]_{L,R} \quad \left[ \tilde{e}, \tilde{\mu}, \tilde{\tau} \right]_{L,R} \quad \left[ \tilde{\nu}_{e,\mu,\tau} \right]_L \quad \text{Spin } 0$$

$$g \quad \underbrace{W^\pm, H^\pm}_{\text{Spin } 1} \quad \underbrace{\gamma, Z, H_1^0, H_2^0}_{\text{Spin } 0}$$

$$\tilde{g} \quad \tilde{\chi}_{1,2}^\pm \quad \tilde{\chi}_{1,2,3,4}^0 \quad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector:

Two Higgs doublets, physical states:  $h^0, H^0, A^0, H^\pm$

# Generating SUSY breaking

No time to discuss this:

Examples:

Gravity mediated SUSY breaking: (Minimal Supergravity or MSUGRA)

Susy breaking through gravity at the Planck scale, gravitino is very heavy

Gauge mediated SUSY breaking: (GMSB)

Mediators are 'normal' gauge bosons, gravitino is lightest susy particle

Anomaly mediated SUSY breaking: (AMSB)

Breaking in higher dimensions

+ many others

**My conclusion: We do not really know the MSSM mass spectrum**

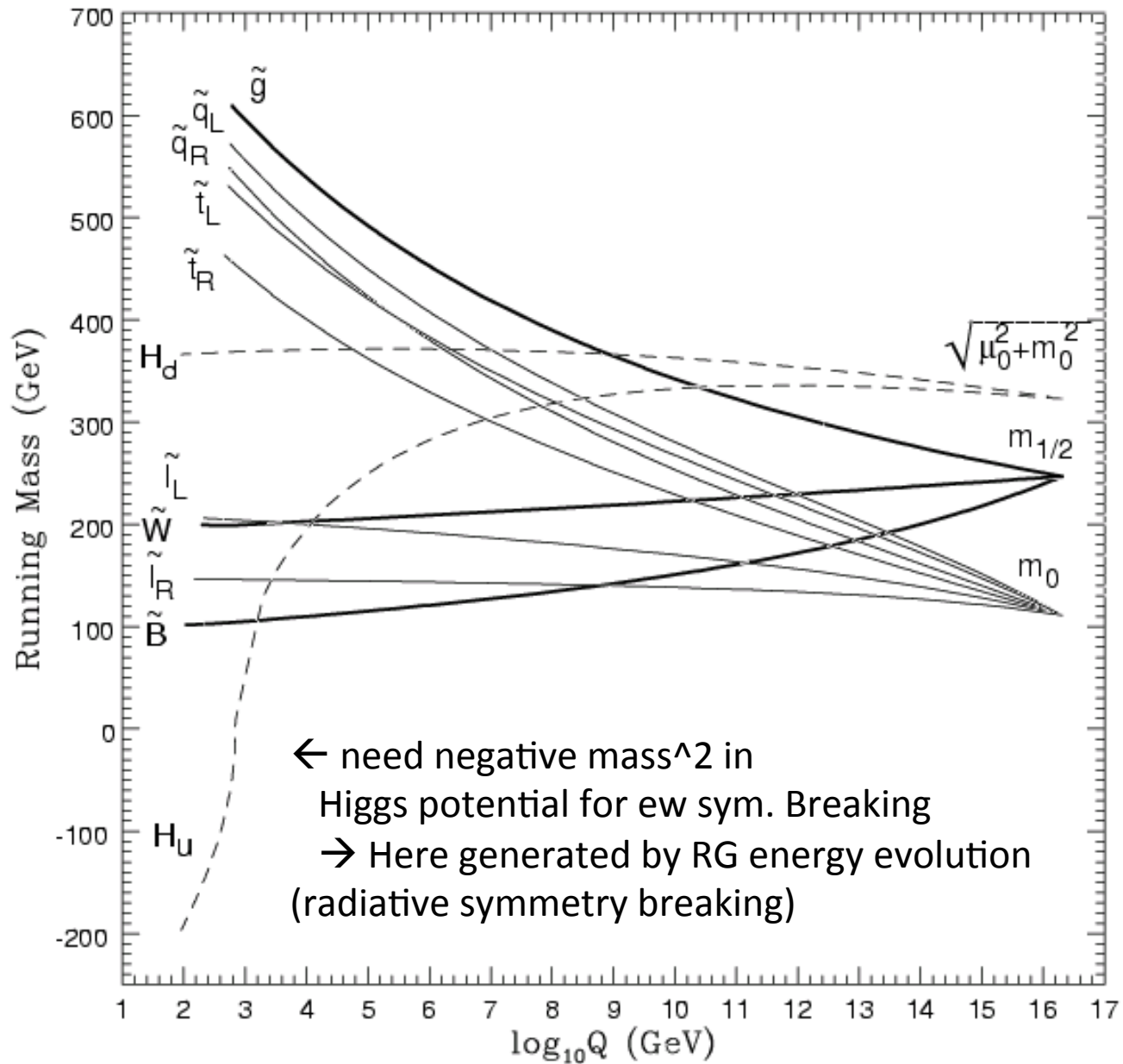
# What is the CMSSM?

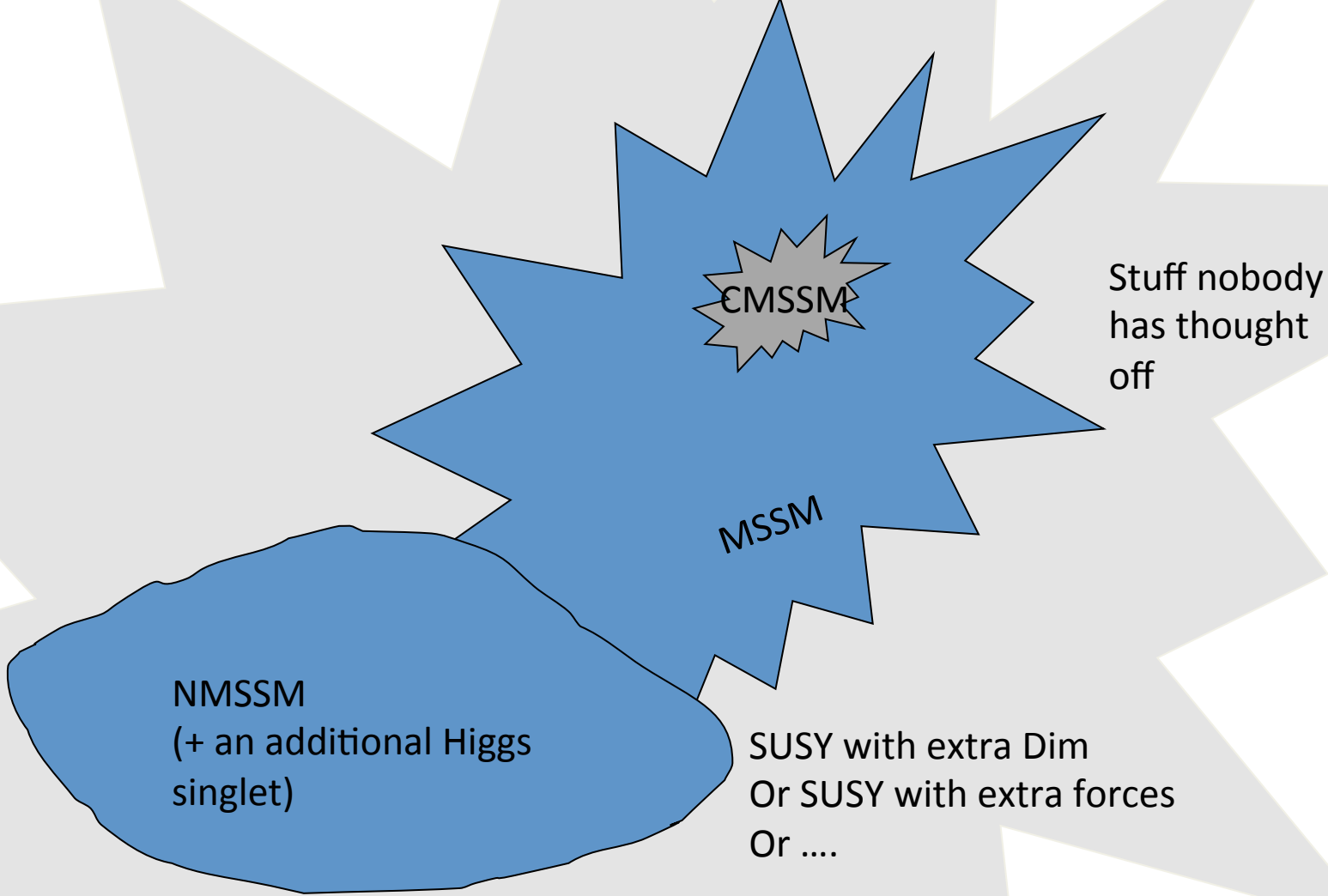
- Constrained Minimal Supersymmetric SM  
(also called minimal supergravity = MSugra)

Assume at  $M_X$  :      all scalar masses are the same =  $m_0$   
                                 all gaugino masses are the same =  $m_{1/2}$

- universal trilinear coupling  $A_0$
- Tan beta
- Sign of susy higgs parameter  $\mu$  ( $|\mu|$  constrained by  $M_z$ )

→ 4 1/2 parameters :  $m_0^2$  ,  $m_{1/2}$  ,  $A_0$  , tan beta , sign( $\mu$ )





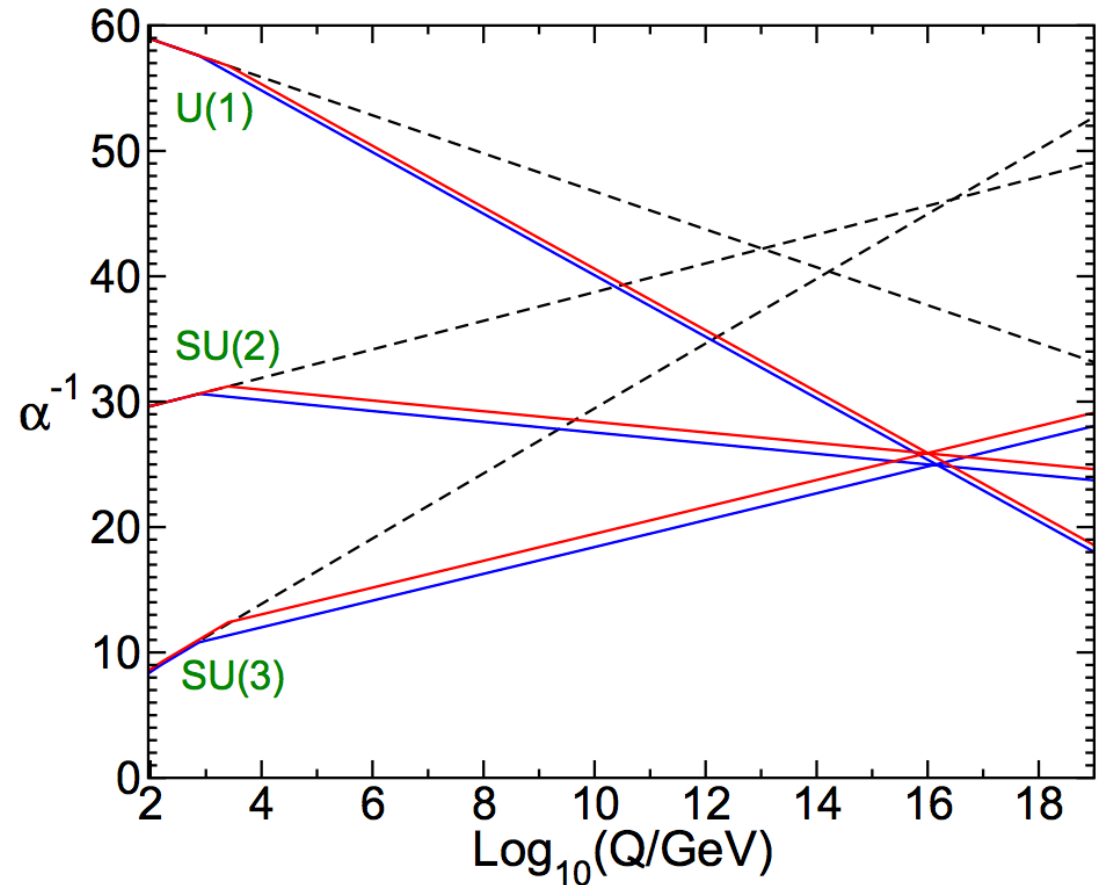
SUSY extensions  
OF THE SM

# Why Supersymmetry ?



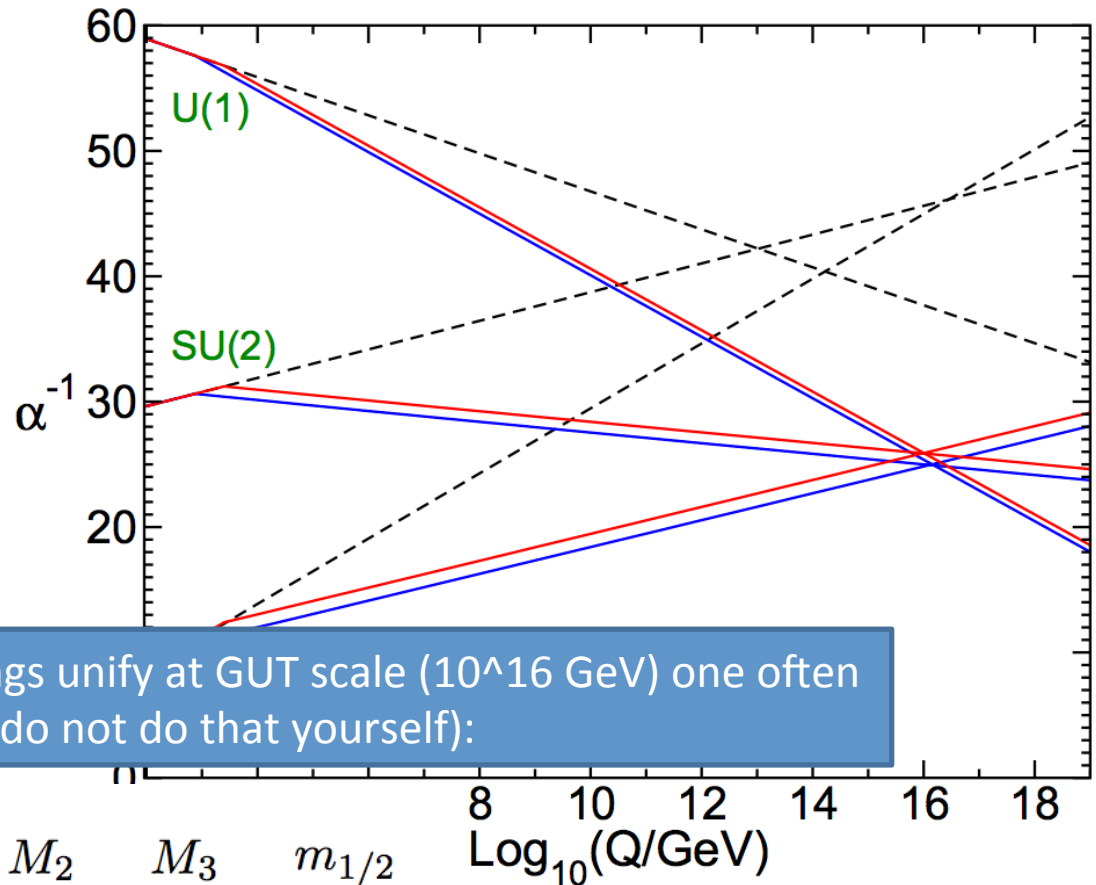
# Gauge couplings

Figure 6.8: Two-loop renormalization group evolution of the inverse gauge couplings  $\alpha_a^{-1}(Q)$  in the Standard Model (dashed lines) and the MSSM (solid lines). In the MSSM case, the sparticle masses are treated as a common threshold varied between 750 GeV and 2.5 TeV, and  $\alpha_3(m_Z)$  is varied between 0.117 and 0.120.



# Gauge couplings

Figure 6.8: Two-loop renormalization group evolution of the inverse gauge couplings  $\alpha_a^{-1}(Q)$  in the Standard Model (dashed lines) and the MSSM (solid lines). In the MSSM case, the sparticle masses are treated as a common threshold varied between 750 GeV and 2.5 TeV, and  $\alpha_3(m_Z)$  is varied between 0.117 and 0.120.



Since the couplings unify at GUT scale ( $10^{16}$  GeV) one often assumes (Please do not do that yourself):

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} = \frac{m_{1/2}}{g_U^2}$$

# Dark Matter

Dark Matter candidates in the MSSM

Which ones ?

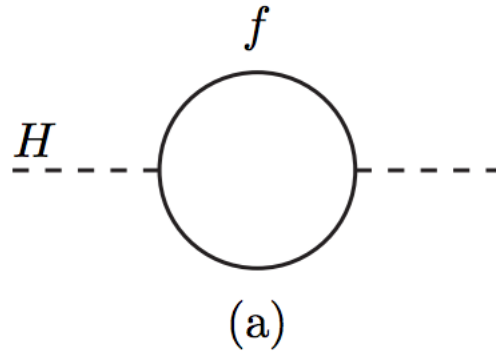
# Dark Matter

Dark Matter candidates in the MSSM

**Neutralino<sub>1</sub>**: Perfect candidate ? How perfect?

**Sneutrinos** : Not possible in MSSM, if light seen in Z decays, if heavy excluded by direct detection (only possible beyond MSSM)

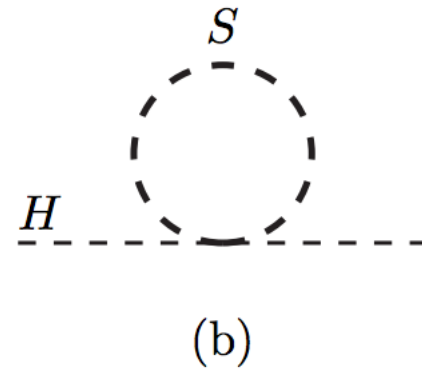
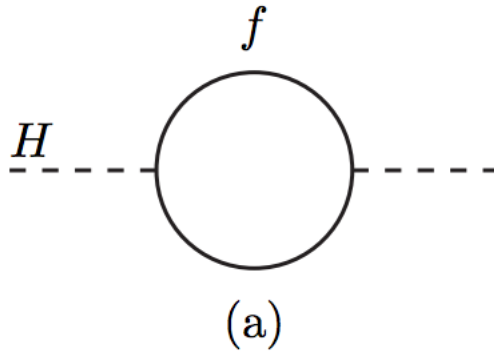
# Hierarchy problem



Yields quadratic divergence to the higgs mass:

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda_{UV}^2 + \dots$$

# Hierarchy problem



Unbroken SUSY  $\Delta m_H^2 = \frac{1}{8\pi^2}(\lambda_S - |\lambda_f|^2)\Lambda_{UV}^2 + \dots$

And:  $\lambda_S = |\lambda_f|^2$

# Fine tuning problem

Every beyond the SM theory “coupling” to any of the SM particles and defined at the scale  $\Lambda$  will contribute to the Higgs mass:

Higgs Mass =  $X$  + Quantum Corrections ( $\Lambda$ )

# Fine tuning problem

$$\text{Higgs Mass} = X + \text{Quantum Corrections } (\Lambda)$$

**Solution 1: New physics at Planck scale coupling to SM**

$$\Lambda = 10^{18} \text{ GeV}$$

$$125 \text{ GeV} = X + 123456789123456789 \text{ GeV}$$

Conclusion X needs to be highly **“fine tuned”** to get the right Higgs mass !

**→ Unnatural**



# Fine tuning problem

$$\text{Higgs Mass} = X + \text{Quantum Corrections} (\Lambda)$$

**Solution 2: New physics at TeV scale coupling to SM**

$$\Lambda = 10^3 \text{ GeV}$$

$$125 \text{ GeV} = X + 1000 \text{ GeV}$$

Conclusion X needs to be very softly **tuned** to get the right Higgs mass.

**→ Natural ?**

**... but how natural precisely given no new particles at LHC ?**

# Fine tuning in SUSY

$$\begin{aligned} \text{Higgs mass} &= Z \text{ mass} + \text{Quantum Corrections (M\_SUSY)} \\ 125 &= 91 + \text{Quantum Corrections (M\_SUSY)} \end{aligned}$$

Fine tuning of Higgs mass can be rewritten in fine-tuning of Z mass

$$Z \text{ mass} = \text{Higgs mass} - \text{Quantum Corrections (M\_SUSY)}$$

How large is the fine-tuning of the MSSM?

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

# Fine tuning in SUSY

$$\begin{aligned} \text{Higgs mass} &= Z \text{ mass} + \text{Quantum Corrections (M\_SUSY)} \\ 125 &= 91 + \text{Quantum Corrections (M\_SUSY)} \end{aligned}$$

Fine tuning of Higgs mass can be rewritten in fine-tuning of Z mass

$$Z \text{ mass} = \text{Higgs mass} - \text{Quantum Corrections (M\_SUSY)}$$

Ruud Peters  
Master thesis !

$$\text{FT} = \Delta_{\text{EW}} = \max_i \left| \frac{C_i}{m_Z^2/2} \right|, \quad (2)$$

where the  $C_i$  are defined as:

$$\begin{aligned} C_{m_{H_d}} &= \frac{m_{H_d}^2}{\tan^2 \beta - 1}, & C_{m_{H_u}} &= \frac{-m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1}, & C_\mu &= -\mu^2 \\ C_{\Sigma_d^d} &= \frac{\max(\Sigma_d^d)}{\tan^2 \beta - 1}, & C_{\Sigma_u^u} &= \frac{-\max(\Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1}. \end{aligned}$$

# How can we determine if SUSY is fine-tuned already ?

We determine how much a **parameter set** of the MSSM is fine tuned via:

$$\text{FT} = \text{max. Quantum-Corrections}^2 / M_Z^2$$

FT = 1-10 → Natural, **perfect** !

FT = 10-100 → a bit of tuning, **so la la**

FT = 100-1000 → **not so good.**

FT > 10<sup>(10)</sup> → highly FT models, **bad...**

# What is the minimum ?

Idea:

- We apply all constraints from all experimental data to our points (with 2 sigma)
- We apply constraints from direct detection experiments (Xenon, Lux) with 3 sigma
- We apply LHC bounds ... How do we do this?

# What is the minimum

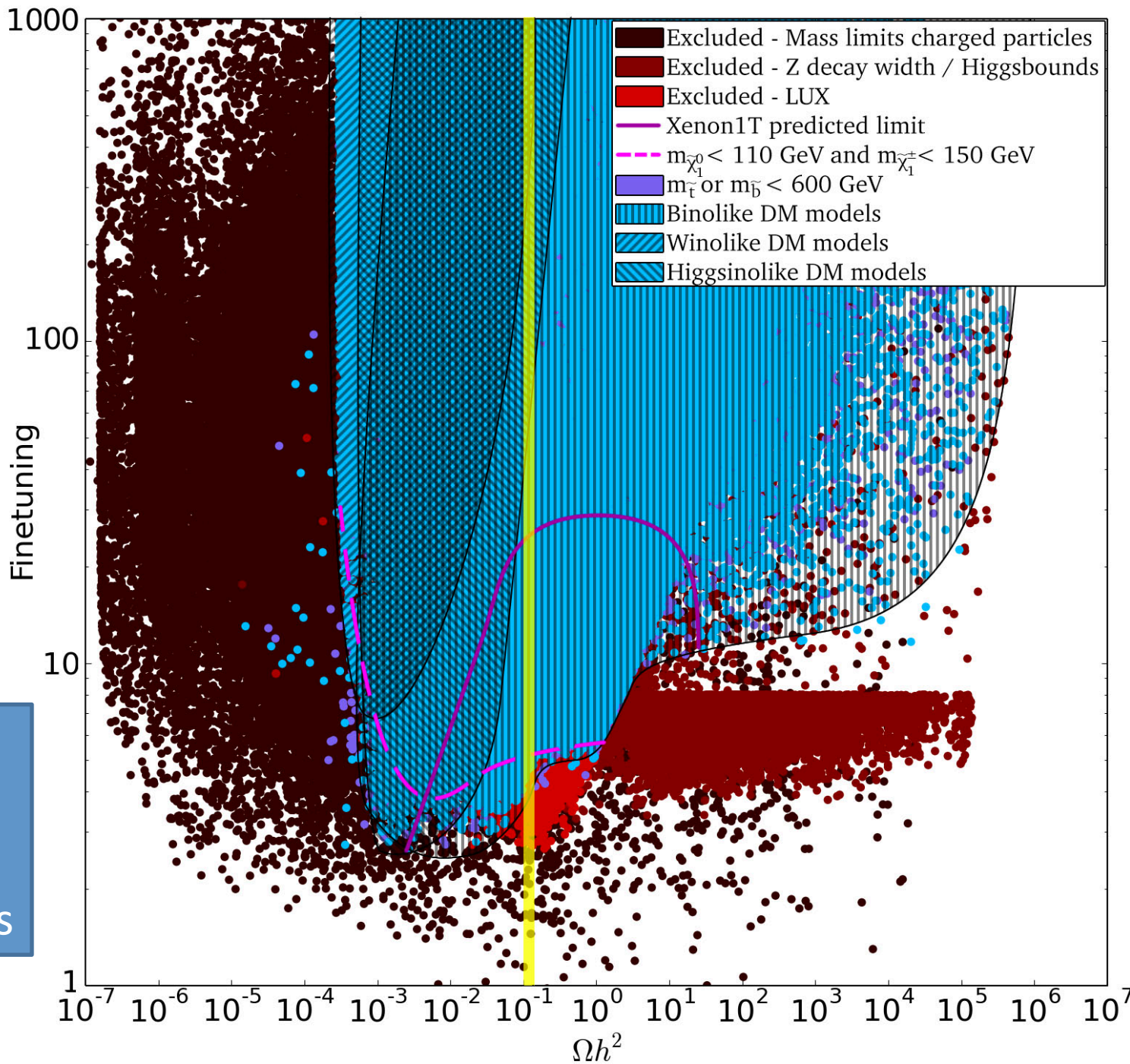
**Found solutions with FT around 3,5 ...**

**Let us look at the Dark Matter experiments ...**

*$\Omega_{DM}h^2 = 0.12$  as required by observations*

*Here  $\Omega_{DM}$  is the dark matter density in units of the critical density and  $h = H_0/(100 \text{ km/s per Mpc}) = 0.68$  with  $H_0$  the Hubble constant*

# Dark Matter relic density

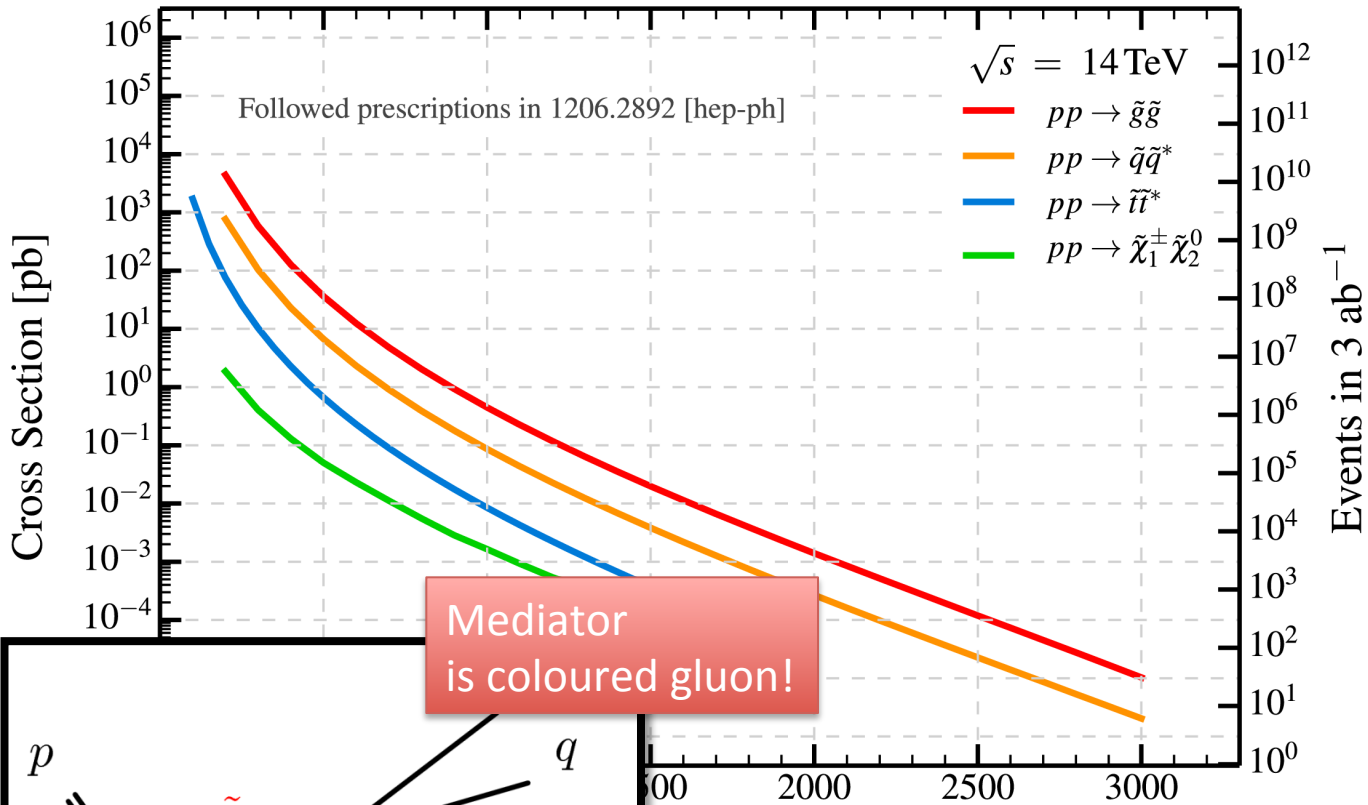


Plots by  
Melissa van  
Beekveld  
and Ruud Peters

# LHC SUSY searches

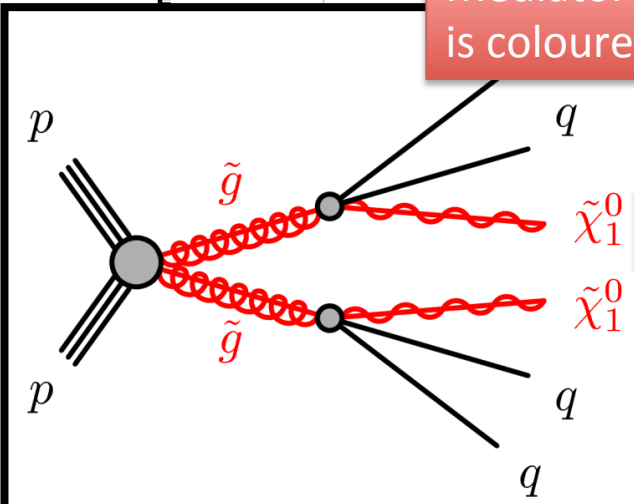


# Production rate: Supersymmetry

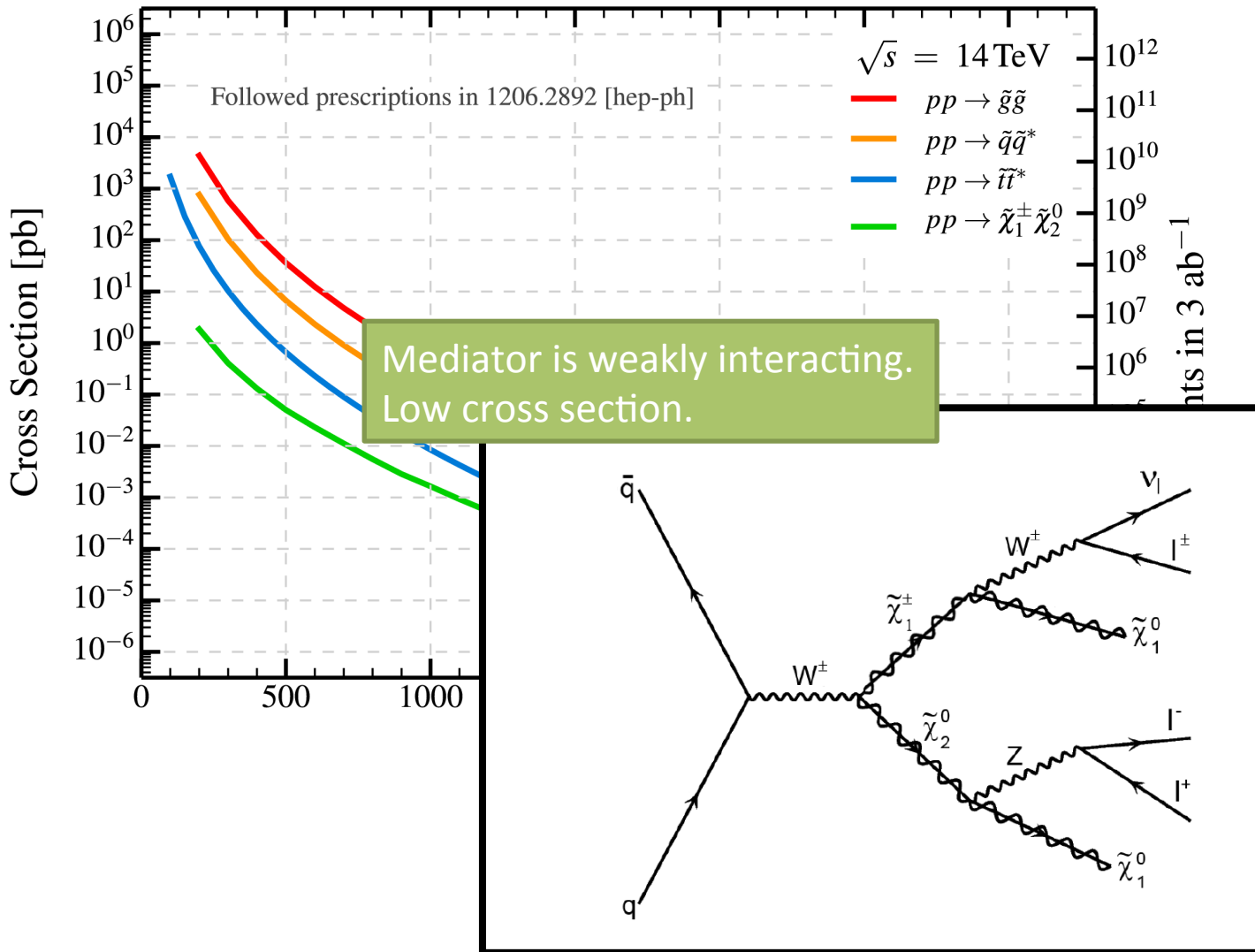


Mediator is coloured gluon!

Large Cross section



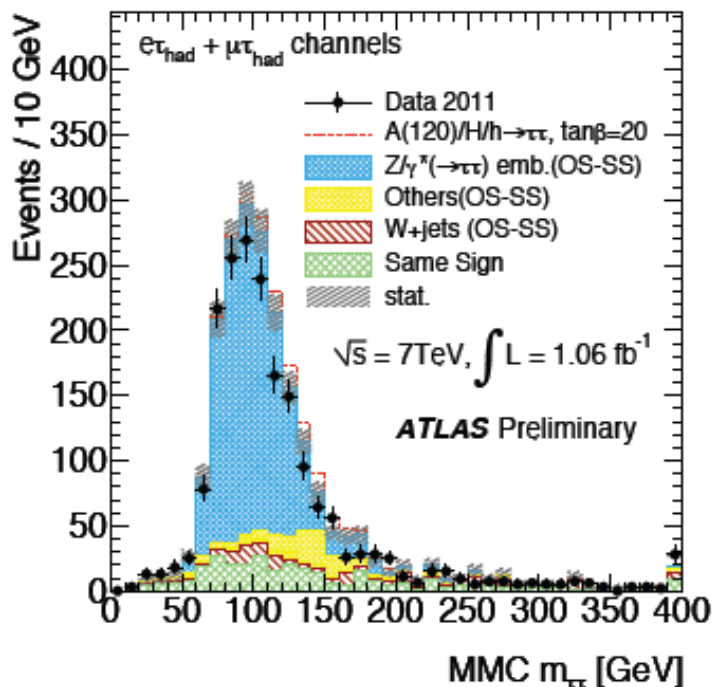
# Production rate: Supersymmetry



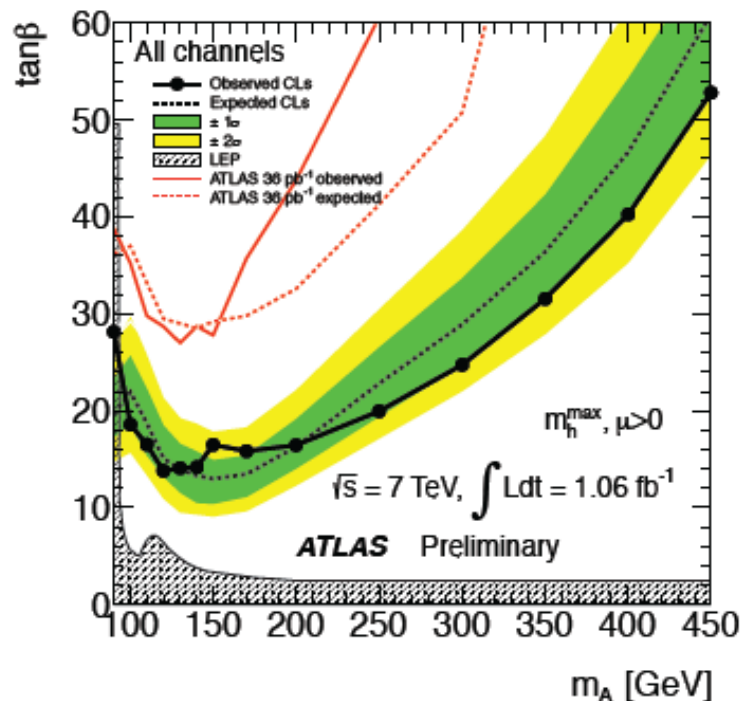
# MSSM H/A $\rightarrow \tau\tau$



83



Effective mass distribution for  $\tau_{had}$ . The data are compared with the background expectation and an added hypothetical signal. “OS-SS” denotes the difference between the opposite-sign and same-sign event yields.



Expected and observed exclusion limits based on CLs in the  $m_A - \tan \beta$  plane of the MSSM derived from the combination of the analyses for the  $e\mu$ ,  $\tau_{had}$  and  $\tau_{had}\tau_{had}$  final states. The dark green and yellow) bands correspond to the  $\pm 1\sigma$  and  $\pm 2\sigma$  error bands, respectively.

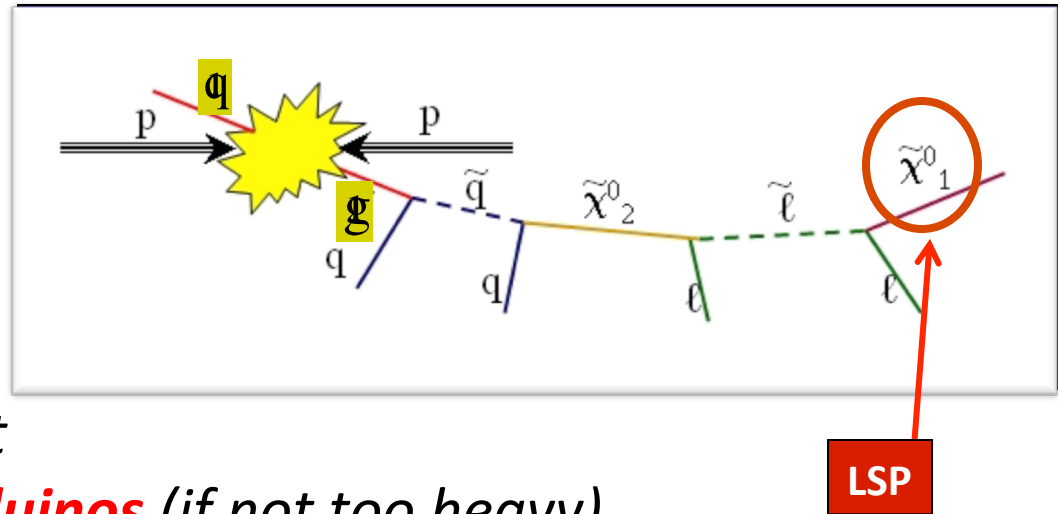
- Most sensitive at early LHC:  
SUSY search for squarks and gluinos

# SUSY and the LHC : Signal

If R-Parity is conserved then SUSY particles are pair produced

LHC:

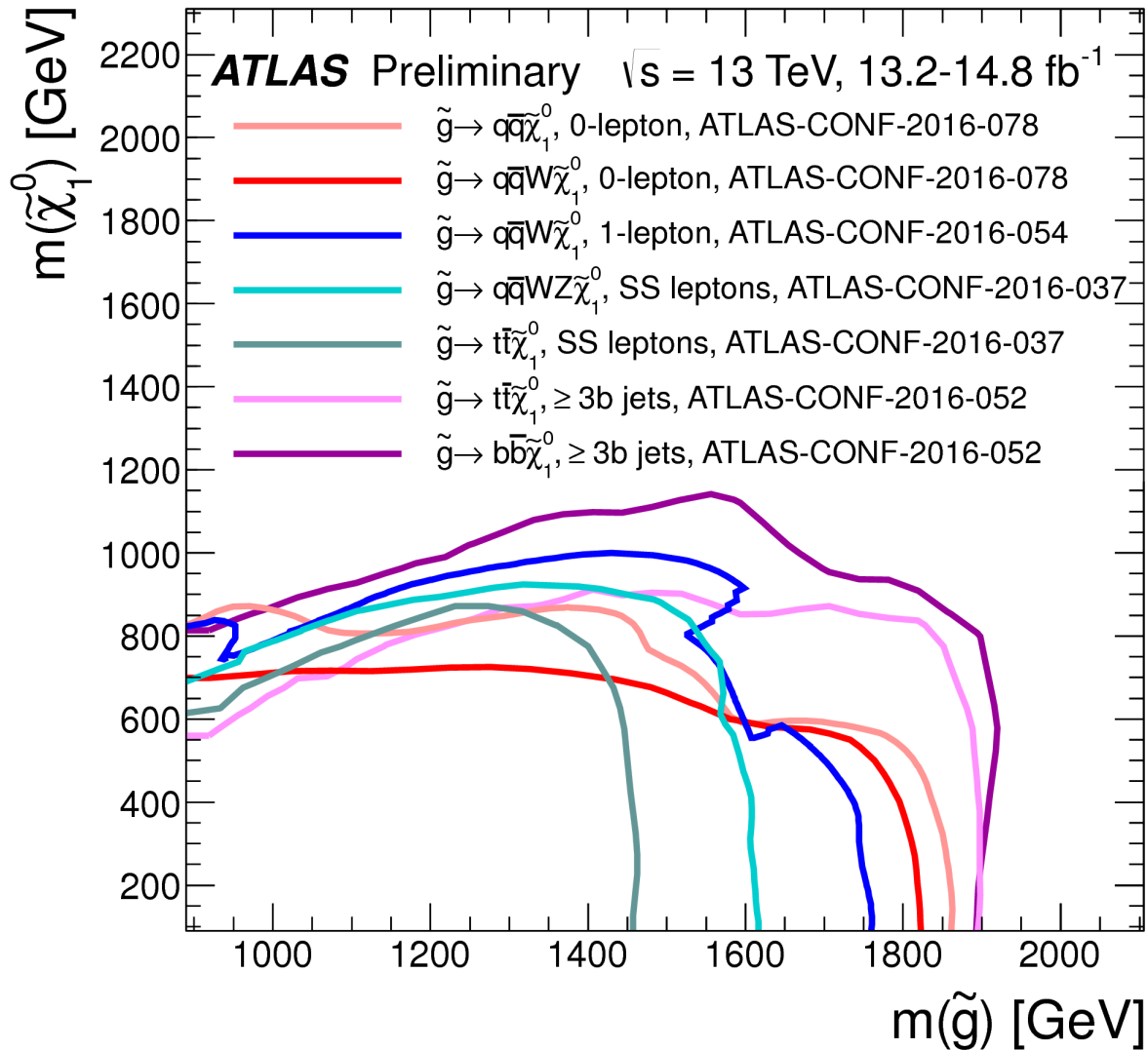
*Due to strong force dominant production of **squarks** and **gluinos** (if not too heavy) Cascade decay to lighter SUSY particles and finally the lightest SUSY particle (LSP)*



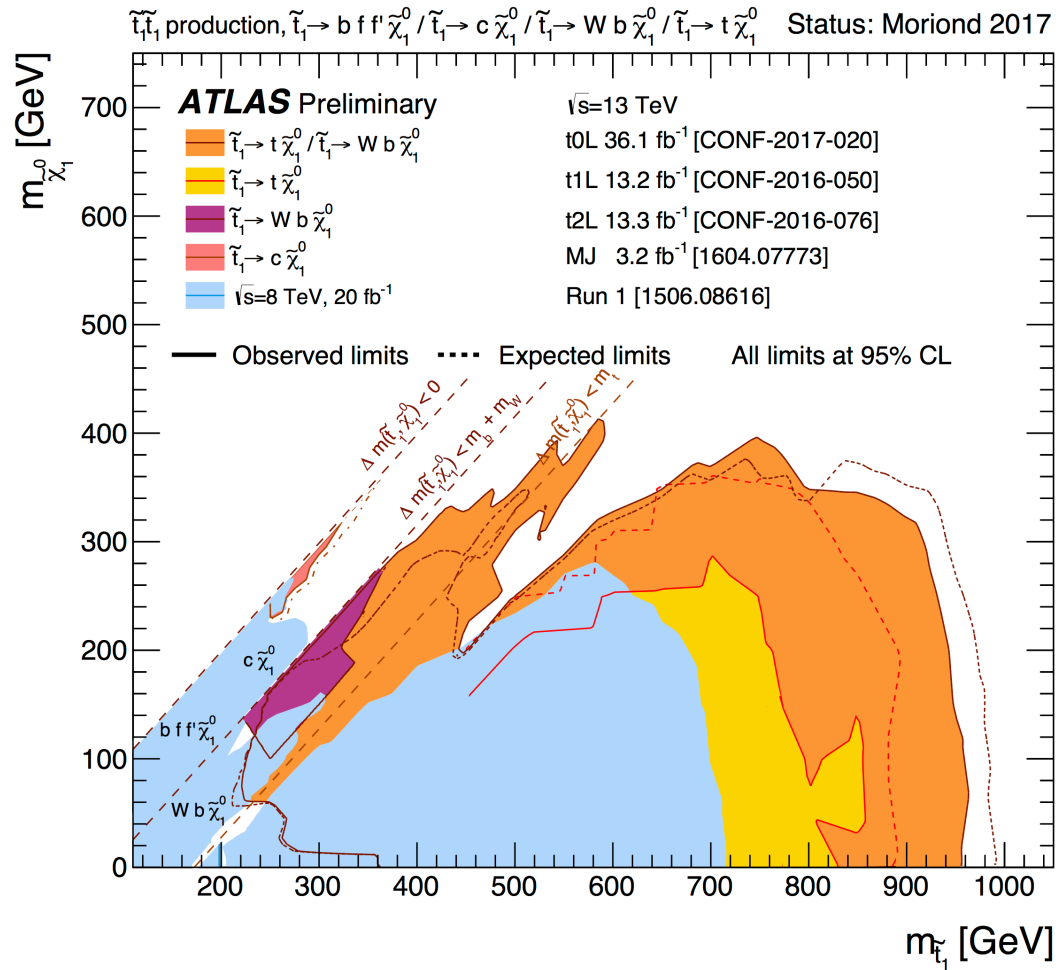
*Similar conclusions /channels  
For many other models  
( Universal Extra Dimension,  
ADD, Little Higgs, ....)*

*Mass pattern in general SUSY  
unknown ! Searches need to be quite  
general and  
model-parameter-independent*

# Gluginos



# Stops



# Heavy Higgs



# Wikipedia

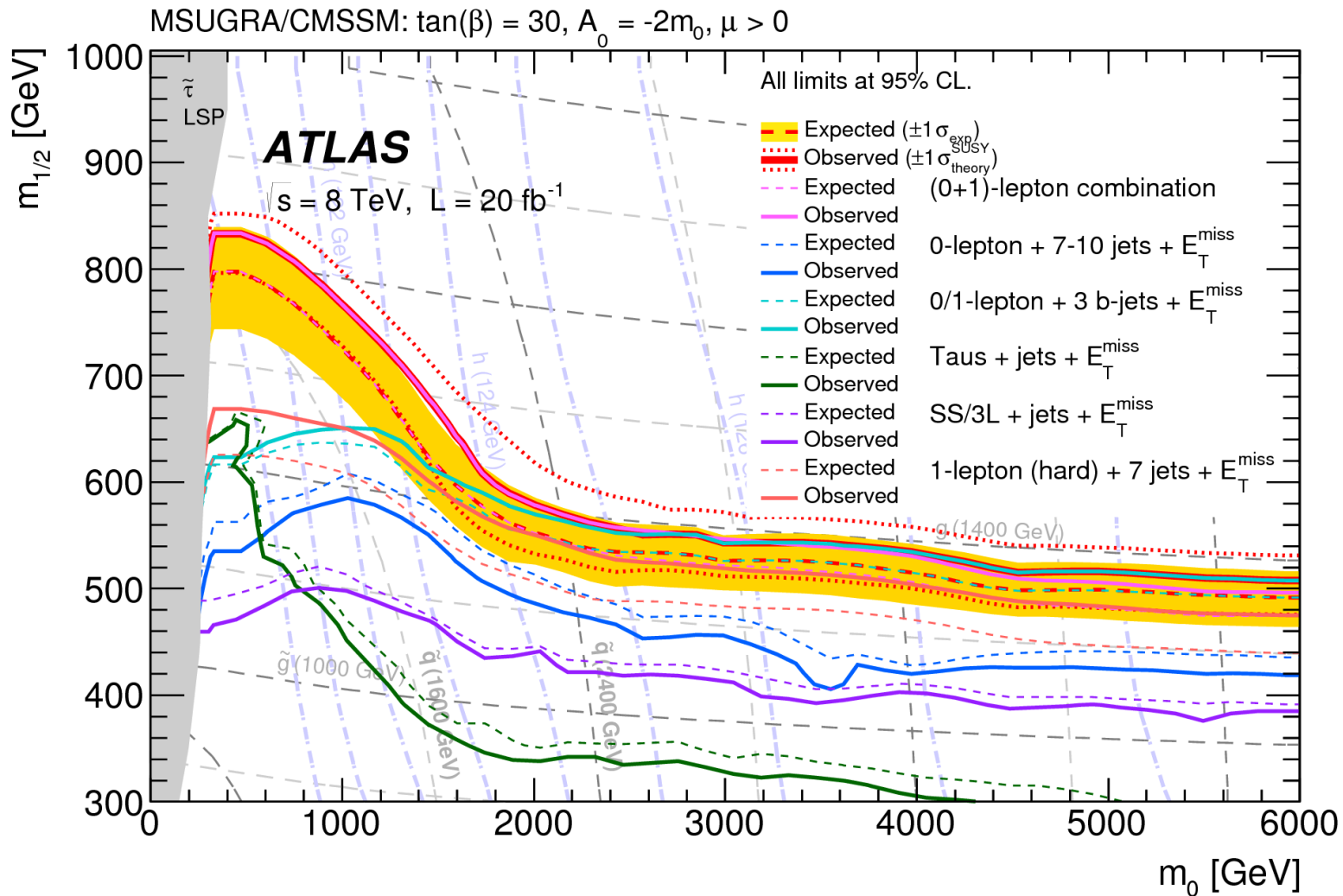
“ ... the failure to produce evidence of supersymmetry in the LHC experiment has cast doubt on the simplest WIMP hypothesis...”

Yes, that might be true, but SUSY still solves:

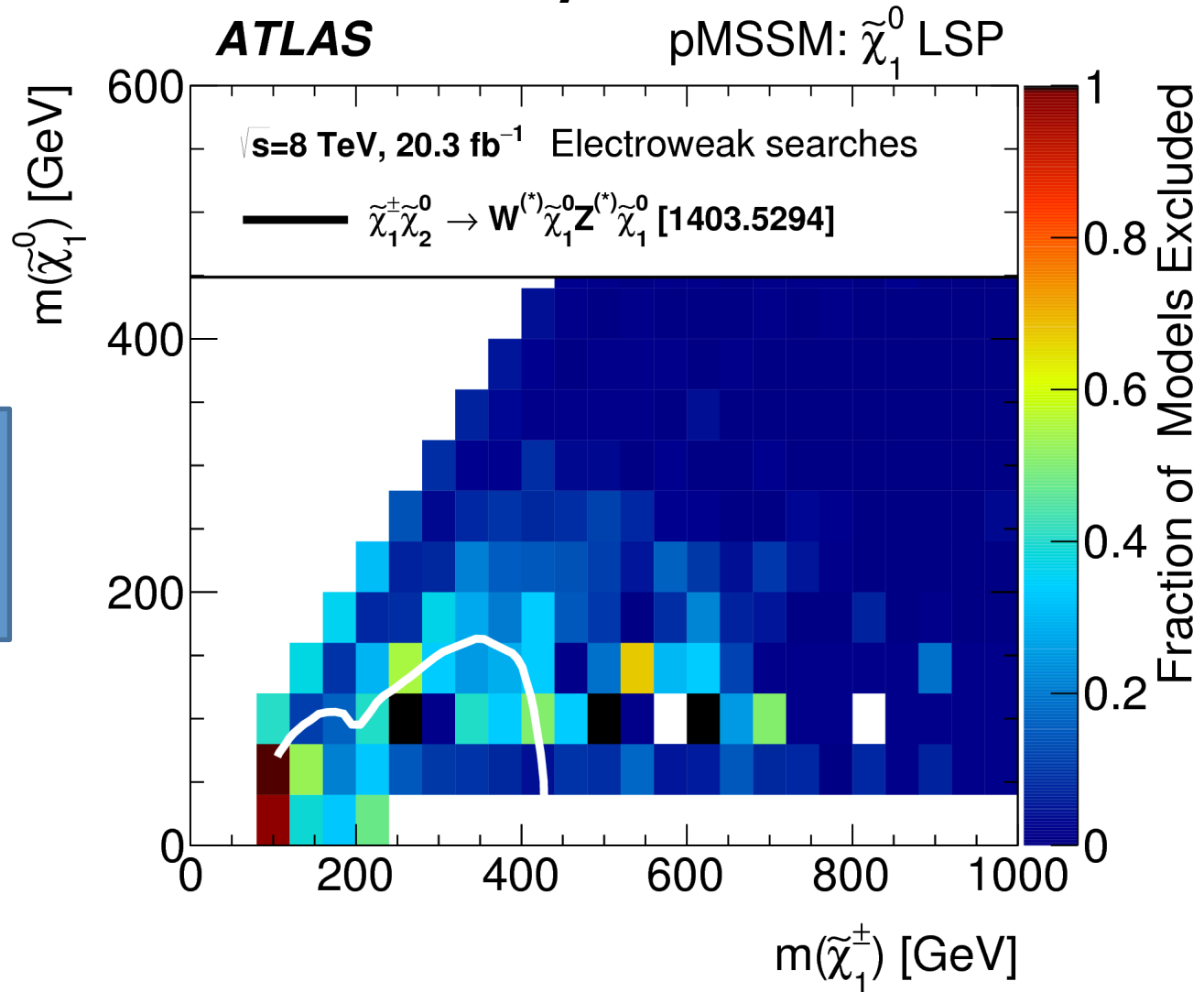
- DM
- Finetuning problem
- etc.

# Extra slides

# Run-1 results “constrained” MSSM



# OK, what if the gluinos and squarks are heavy ?



No real constrain  
on electroweak  
SUSY 100 GeV  
DM particles...

# Run-1/early run-2 results

- No sign of new physics
- Constraints on colored SUSY particles, e.g. Squarks and Gluinos are likely heavy ( $>1-1.5$  TeV)
- Constraints on **most simple** models (e.g. cMSSM)  $\rightarrow$  Need to work in less simple/more-parameter models
- No real strong constraints on weakly interacting Dark sector particles !

Run-2

13 TeV

# Extra material

Chiral Superfield charges under:

(SM) group  $G = SU(3)_C \times SU(2)_L \times U(1)_Y$ ,

$$\begin{aligned} L &: (1, 2, -1/2), & E &: (1, 1, 1), & Q &: (3, 2, 1/6), & U &: (3, 1, 2/3), \\ D &: (3, 1, -1/3), & H_1 &: (1, 2, -1/2), & H_2 &: (1, 2, 1/2), \end{aligned}$$

while the vector multiplets have the following charges under  $G$ ,

$$g : (8, 1, 0), \quad W : (1, 3, 0), \quad B : (1, 1, 0).$$