Tutorial heavy ion theory

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1 Heavy ion observables

First, download the file containing the observables. There are APIs allowing you to open it in Mathematica, Python, C++, . . . , so you can pick your preferred environment. There is also an online viewing tool from which you can convert datasets in the file to CSV. In the cases of Mathematica and python, we have created helper functions which are distributed inside the HDF5 files themselves. For Mathematica, you can load these functions as follows (where you should replace "filename.h5" by the correct file name on your computer):

```
ToExpression[Import["filename.h5","/wolframfunctions/LoadDataSingleObservable"]];
```

Example usage for observable "multiplicitycharged":

```
LoadDataSingleObservable["filename.h5","multiplicitycharged/ALICETPC/centralitybinned",
{0,5,10,20,30,40,50,60},{0,100}]
```

Here the third argument contains the centrality bin boundaries, and the fourth argument the ESE bins.

For Python, you can similarly load the function by importing h5py and running (again, replace "filename.h5" to what it is on your computer):

```
exec(h5py.File("filename.h5", "r")["/pythonfunctions/get_trajectum_obs"][()])
```

In the root directory of the H5 file, there are datasets called "centrality", "dcentrality", "ESE" and "dESE". These contain the centers of the centrality bins, widths of centrality bins, centers of the event shape engineering bins, and their widths, respectively. All other datasets you will encounter in this tutorial are 2D datasets, with the first dimension corresponding to centrality, and the second to event shape engineering. For each observable, there is a folder in the H5 file. Inside each such folder, there is an "ALICETPC" folder.[1] Inside that, there is a "centralitybinned" folder.[2] Finally, inside that there are datasets called "values" (containing the data points), "lowererrors" (containing the lower stat error), "uppererrors" (containing the upper stat error), as well as two labels datasets (which include the axis labels you should use as the 0th and 4th entries) called "labelsLaTeX" and "labelsWolfram".

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[1] These names correspond to different centrality definitions, you will only need this one in this tutorial.

[2] It is also possible to bin in, for example, offline tracks, but you will not need that in this tutorial.

a) Plot the observables “multiplicitychargedpion” and “multiplicitychargedkaon” as a function of centrality (use only the 0–100% event shape engineering bin). These particles are produced thermally as the QGP freezes out at a temperature T_c. From Bose-Einstein statistics, we know that the number of produced mesons equals

$$N_i = g_i \int d^3p \frac{1}{\exp(\frac{\sqrt{p^2+m_i^2}}{T_c}) - 1}.$$

Looking at the ratio of the multiplicities for $\pi^\pm$ and $K^\pm$ as a function of centrality, do you think T_c depends on centrality? Can you extract the value of T_c?[3]

b) Download the file containing observables in the ultracentral regime. Extract the data for “multiplicitycharged” and “meanptcharged”, and make a parametric plot of $\langle p_T \rangle$ vs. $dN/d\eta$. It is claimed in [1] that the speed of sound squared of the QGP at $T_{\text{eff}} = \langle p_T \rangle/3$ is given by

$$c_s^2 = \frac{d \log\langle p_T \rangle}{d \log(dN/d\eta)},$$

where the derivative is to be taken along the rising slope for the highest multiplicity collisions. Can you extract c_s^2? Does your value agree with the lattice QCD computation in [2]? Does your extraction depend on whether you use the “ALICETPC” or “ATLASMAC” centrality definitions?

c) Plot the observable “v2chargedcumulantsbrak2” as a function of centrality. The multiplicity distribution in the azimuthal angle goes like

$$\frac{dN/d\eta}{\propto} 1 + v_2 \cos(2(\phi - \Psi_2)) + \dots$$

Knowing that the centrality is a proxy for the impact parameter of the collision, and knowing that hydrodynamics converts initial state spatial anisotropy into final state momentum anisotropy, can you explain the shape of the curve?

d) Plot now the same observable, but for 3rd order (“v3chargedcumulantsbrak2”). From symmetry considerations, do you expect this observable to be nonzero? Argue that this must mean that there are event-by-event fluctuations.

2 Bayesian analysis results

First, download the HDF5 file containing the results of the Bayesian analysis from [3]. In this file, the “chainsamples” dataset is an $N_{\text{walkers}} \times N_{\text{steps}} \times N_{\text{parameters}}$ array of samples drawn from the posterior, and the “keys” dataset contains the names of the parameters (in the same order).

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[3] In reality, some of the particles are produced by resonance decays, so this purely thermal estimate is not quantitatively accurate.

a) Find the index of the “trento_p” parameter in the “keys” dataset. Using this index, select this parameter from the “chainsamples” dataset, discarding the first 50% of steps from each walker. The reason for this is that the Monte Carlo needs some time to converge to the right distribution, so the first steps are not reliable. Make a histogram of the remaining datapoints, and compute the mean and standard deviation.

b) The neutron skin of ^{208}Pb is an interesting quantity from the perspective of neutron stars, so an extraction of it from heavy ion data is highly sought after. We modelled ^{208}Pb by a Woods-Saxon distribution, where the distribution of protons inside a nucleus is given by:

$$\rho_{\mathrm{WS}}(r) \propto \frac{1}{1+\exp(\frac{r-R_p}{a_p})},$$

with R_p and a_p parameters, and with a similar equation for neutrons (with R_n and a_n as parameters). The neutron skin is defined as the difference between the neutron and proton RMS values:

$$\Delta r_{np} = \langle r^2\rangle_n^{1/2} - \langle r^2\rangle_p^{1/2}.$$

Using the fact that for a Woods-Saxon distribution, the RMS is given by

$$\langle r^2\rangle_{\mathrm{WS}} = \frac{12a^3\mathrm{Li}_5(-e^{R/a})}{\mathrm{Li}_3(-e^{R/a})},$$

we can fill in the formula for Δr_{np} in terms of R_p, a_p, R_n and a_n. Using the fixed values (all in fm) $R_p = 6.68$, $a_p = 0.447$, $R_n = 6.69$, take a_n from the Bayesian analysis (the parameter is called “neutronskina”), and convert the values to Δr_{np}. Then create a histogram, making sure you again discard the first half of the steps. What is the value for the neutron skin that you extract?

c) By creating two-dimensional histograms (or by computing the Pearson correlation coefficient), find out which parameters are (anti)correlated with each other.

3 Running *Trajectum* yourself

The above results were generated using the *Trajectum* heavy ion code. If you are interested in running this code yourself, you can find it here. In the source code, you can also find an extensive readme on installation, as well as several worked examples. Suggestions (note that these take more time to run than this tutorial is long if you want to have enough statistics to see the effects):

a) In the “PbPb2Dnarrowrapidity” worked example, in the “collision.par” parameter file, find the “shear0”, “shear150” and “shear300” parameters. Double the values for these parameters, which encode the shear viscosity. Do you see a change in any of the observables? You should see a change in for example “v2chargedcumulantsbrak2”. Can you explain the behavior?

b) In the same example, raise the value of “freezeouttemp” (this is the T_c referred to in exercise 1a) to 200 MeV. Then plot the identified particle multiplicities again. Is the change consistent with your expectations?

c) In the same example, double the value of “sigmafluct”. This increases the amount of fluctuations in the initial state of the collisions. Then plot the “v3chargedcumulantsbrak2” observable again (see exercise 1d). Is the change consistent with your expectations?

References

[1] C. A. Bernardes, “Extracting the speed of sound in the strongly interacting matter created in relativistic nuclear collisions with the CMS experiment,” *EPJ Web Conf.*, vol. 296, p. 06006, 2024.

[2] A. Bazavov *et al.*, “Equation of state in (2+1)-flavor QCD,” *Phys. Rev. D*, vol. 90, p. 094503, 2014.

[3] G. Giacalone, G. Nijs, and W. van der Schee, “Determination of the Neutron Skin of Pb208 from Ultrarelativistic Nuclear Collisions,” *Phys. Rev. Lett.*, vol. 131, no. 20, p. 202302, 2023.