Heavy-ion physics, part 1

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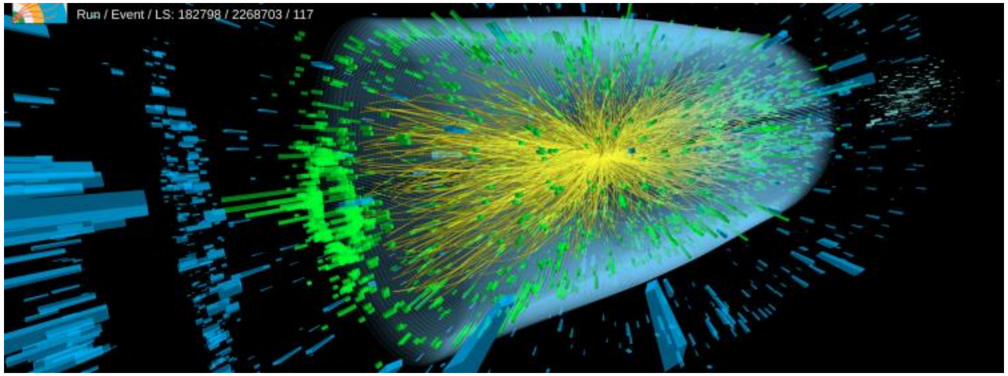
Who am I?

- Member of the CMS experiment at the LHC
- Trained in heavy-ion physics at the Relativistic Heavy-ion Collider (RHIC)
- Who am I am not:
  - A professor
  - A theorist

A heavy-ion collision in a HEP detector



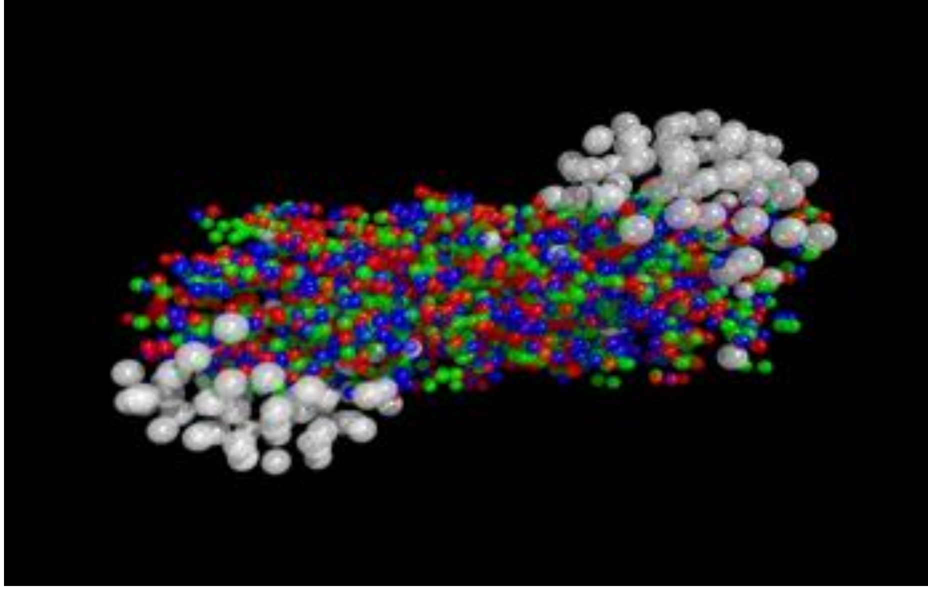
My colleagues and I during data taking at CERN

Part 1 contents

- Heavy ions and the quark-gluon plasma
- Some brief QCD reminders
- Thermodynamics of QCD
- Space-time evolution of heavy-ion collisions
  - Concept of collision centrality
  - Energy density in the Bjorken picture
- Particle production from the QGP
  - Statistical hadro-production
  - Strangeness enhancement

Focus of part 2 will be on a few types of measurements used to probe the quark gluon plasma created in heavy-ion collisions

Basic definition



Au: 79 protons & 118 neutrons

Pb: 82 protons & 126 neutrons

Note: Nuclear size of order 10 fm

Heavy

Ion

Collisions

Interested in colliding nuclei, we strip off the e's to accelerate them

The proton is also an ion (of hydrogen). proton-ion collisions are also of interest

Currently performed at RHIC and the LHC

Max energy per nucleon pair ($\sqrt{s_{NN}}$) of 5 TeV → 1PeV ($\sqrt{s}$) total energy !

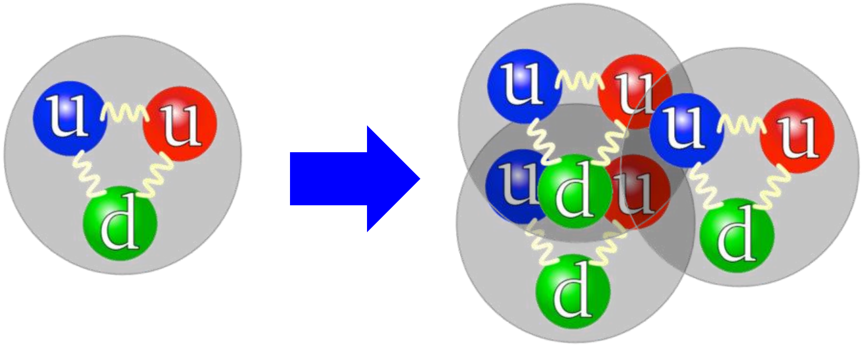
So one should expect large energy density from HIC's (we'll estimate it later)

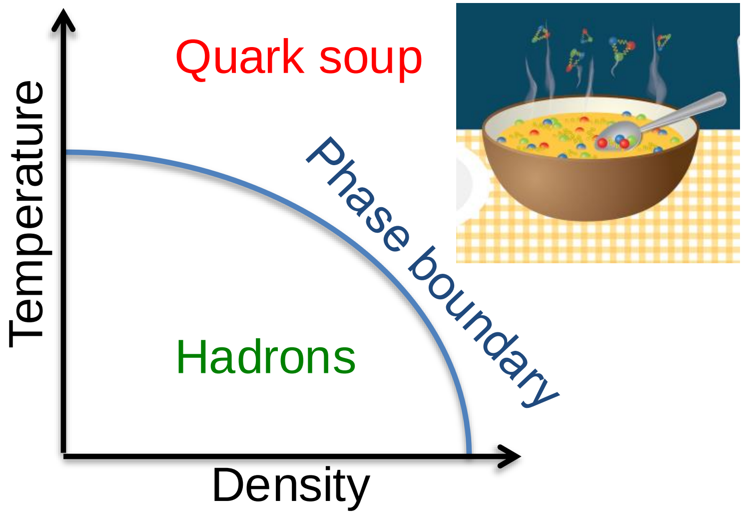
What happens to matter in these conditions?

Why collide heavy ions?

In normal matter, parton degrees of freedom confined to hadrons

Does this change at high temperature and/or large density?

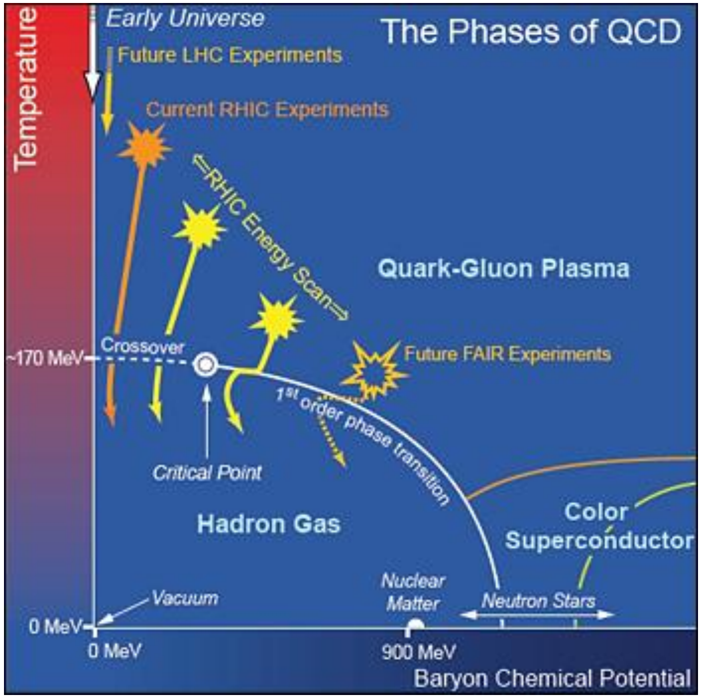




Based on the recent discovery of asymptotic freedom (1973), Collins & Perry hypothesized that it would in 1975:

> "A neutron has radius of about 0.5 – 1 fm and so has a density of about 8 x 10^{14} gm/cm^3, whereas the central density of a neutron star, can be as much as 10^{16} – 10^{17} gm / cm^3. In this case, one must expect the hadrons to overlap, and their individuality to be confused. Therefore we suggest that there is a phase change, and that the nuclear matter at such high densities is a quark soup"

The quark-gluon plasma



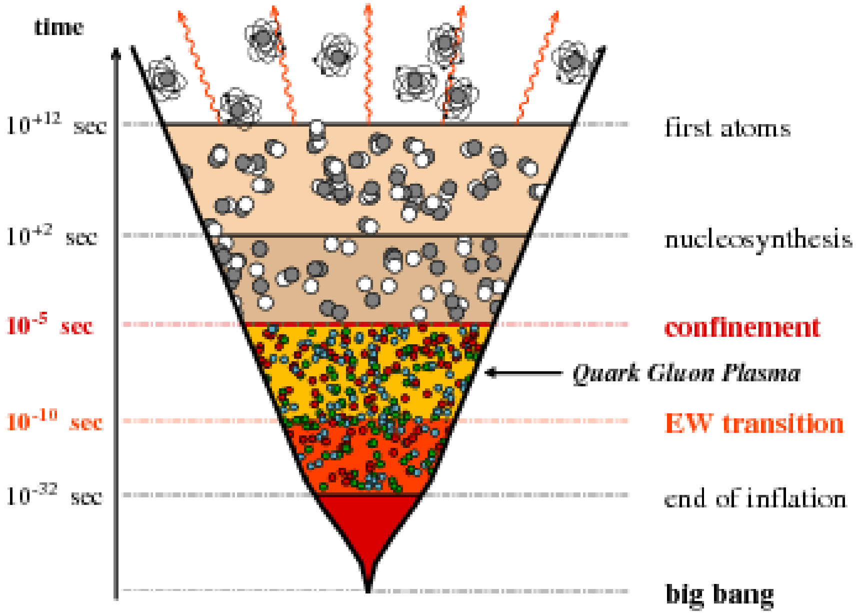
10^5 x hotter than core of the sun! ↘

- Critical temperature ($T_c \sim 170$ MeV) estimated pre-QCD by Hagedorn based on # of hadrons vs. energy
- Later confirmed with lattice QCD
- Heavy-ion collisions create high temperature, but low density → smooth crossover in this region
- Also phase transition at high density, but lattice calculations difficult there due to “sign problem”

Heavy-ion collisions allow us to probe QCD at non-zero temperature and density
→Requires different theoretical tools, e.g., high temperature QFT, thermodynamics, …
→QCD matter exhibits novel features in these conditions

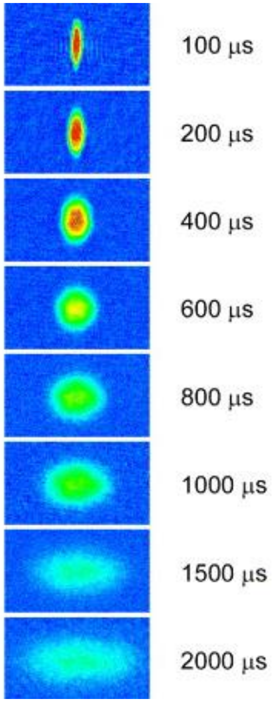
Far-reaching connections

Cosmology



The universe was composed of QGP from 10^{-10} to 10^{-5} sec after the Big Bang

Cold quantum gasses



Elliptic flow of a strongly interacting Fermi gas

Ideal fluid behavior observed in QGP & other strongly coupled systems

QCD reminder

Structure of QCD driven by SU(3) color gauge symmetry

Gluon self-interaction

In principle, all is calculable from the QCD Lagrangian:

$$\mathcal{L}_{QCD} = \sum_{\text{flavours}} \bar{\psi}_a \Big((i\gamma^\mu \partial_\mu - m)\delta_{ab} - g_s \gamma^\mu t^C_{ab} A^C_\mu \Big) \psi_b - \frac{1}{4} F^A_{\mu\nu} F^{\mu\nu,A}$$

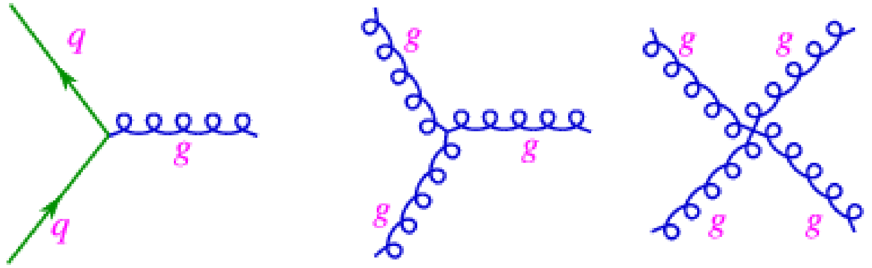
$$F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\nu - g_s\, f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu$$

$$[t^A, t^B] = i f_{ABC} t^C$$

Quark propagator

Quark-gluon vertex

Field strength tensor



$N_c^2 - 1 = 8$ gluons
$N_c * N_f$ quarks

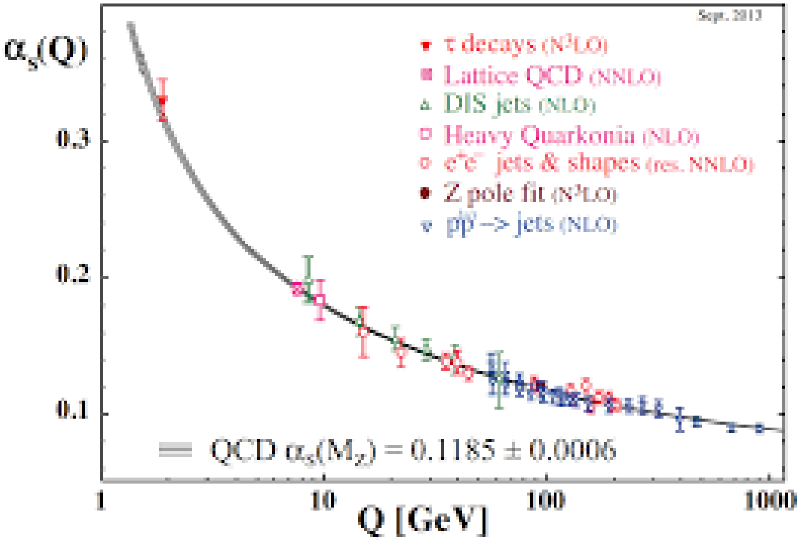
Quark masses

charge=+2/3	u (~5 MeV)	c (~1.5 GeV)	t (~175 GeV)
charge=-1/3	d (~10 MeV)	s (~100 MeV)	b (~5 GeV)

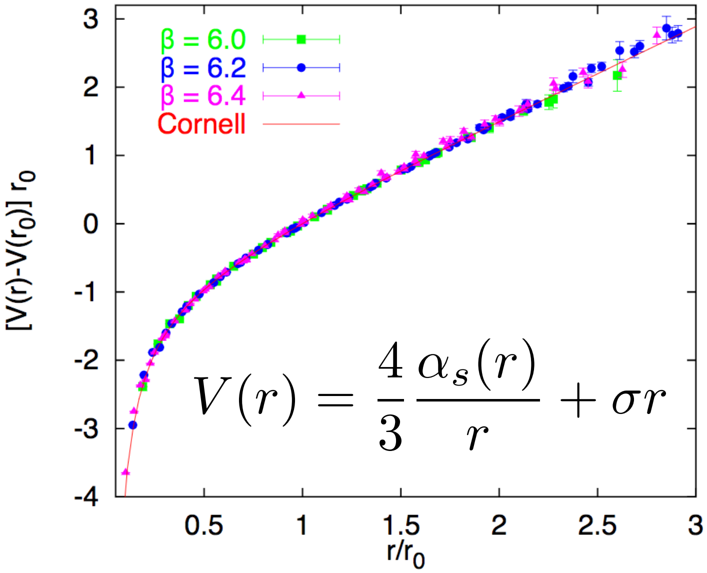
Compare to T_c ~ 170 MeV

Asymptotic freedom & confinement

Whereas in QED vacuum fluctuation screen charge,
in QCD they “anti-screen”



Quark-antiquark potential



$$V(r) = \frac{4}{3}\frac{\alpha_s(r)}{r} + \sigma r$$

- At short distance coupling is small
- Quarks can be probed inside the nucleon with high energy leptons (DIS)
- pQCD works well in this regime

- At “large” distance (1 fm), potential is linear
- Quarks confined inside hadrons
- Perturbation theory breaks at large coupling

QCD thermodynamics

In high T limit → QGP an ideal gas of weakly interacting quarks and gluons

Equation of state is given by pressure vs. temperature

or equivalently energy density vs. temperature

Can be calculated from the partition function

$$\varepsilon = \int \frac{d^3p}{(2\pi)^3} \sum_i \frac{E_i}{e^{\frac{E_i}{k_B T}} \pm 1}$$

+1, 0, 1 for fermions, classical or bosons

$$\Rightarrow \epsilon(T) \approx \frac{\pi^2}{30} N T^4$$

N = 3

Hadron phase can be approximated as a gas of pions, i.e., 3 states: □$^+$, □^{0}, □$^-$

For a QGP on the other hand, we expect

$$\epsilon(T) = \frac{\pi^2}{30}\left(N_B + \frac{7}{8}N_F\right)T^4$$

$$= \frac{\pi^2}{30}(47.5)T^4$$

$$N_B = \underset{\text{spin}}{2} \times \underset{\text{color}}{8}, \quad N_F = \underset{\text{spin}}{2} \times \underset{\text{particle/antiparticle}}{2} \times \underset{\text{color}}{3} \times \underset{\text{flavor: u/d/s}}{3}$$

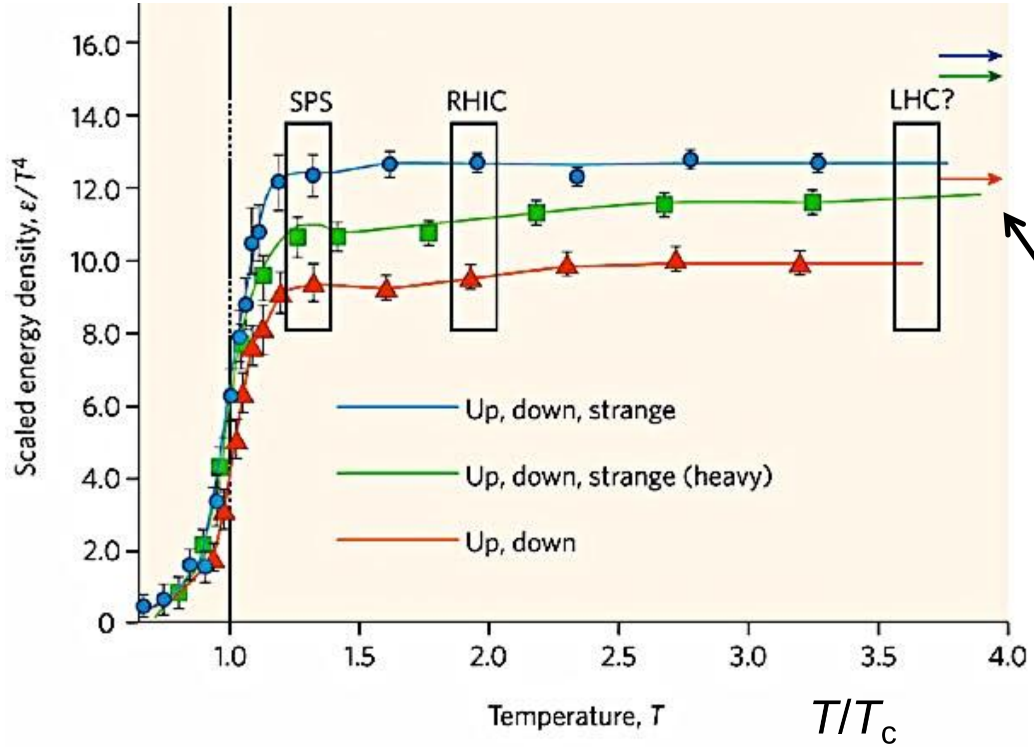
→ 15x jump in the energy density at T_c

E.O.S. from the lattice

Lattice calculations are reliable for zero net baryon density, which is a reasonable approximation for high energy HICs

They indeed show a spike in ratio of ε / T^4, Indicating a jump in the degrees of freedom

T_c ~ 170 MeV
→ ε_{SB} = 1.7 GeV/fm^3

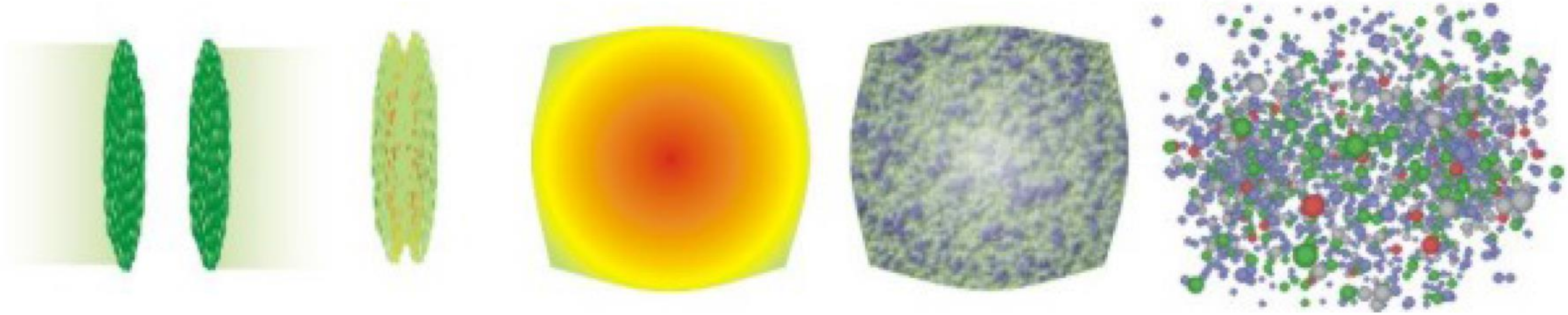


Does that mean the QGP behaves like an ideal gas of quarks and gluons?

No! Don't reach the SB limit

EOS of N=4 Super Yang-Mills
→ may be strongly coupled

Time evolution of a HIC

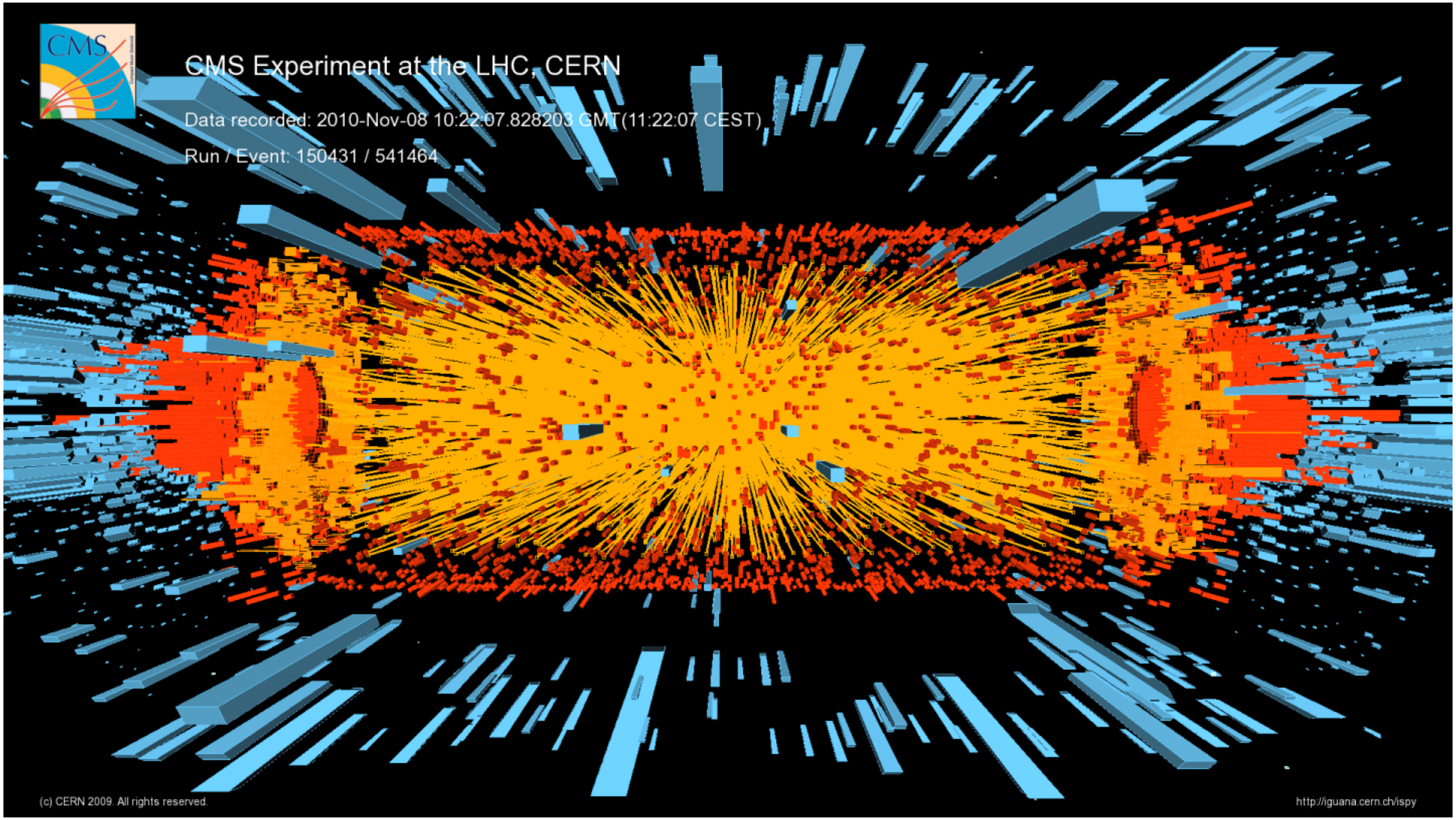


1) Prior to collision relativistic nuclei are Lorentz contracted
2) After crossing, particles start to scatter (| ~ 1/p)
3) After t ~ 1 fm, equilibrium is established, giving a thermalized QGP
4) QGP expands and cools until about 10 fm (3 x 10^{-23} sec !)
5) Hadronization occurs (chemical freeze-out), and shortly after particles stop interacting and free stream towards detectors (kinetic freeze-out)

How does one learn about the fleeting QGP from the mess of final state particles we detect?

Important to understand the *initial state*, as well as how to draw conclusions from the *final state* hadrons

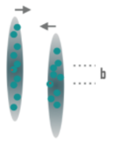
A real heavy-ion collision



The role of geometry

Not all heavy-ion collisions are created equal, but rather vary in *centrality*

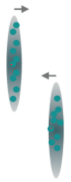
Some are head-on or “central”



Longitudinal view

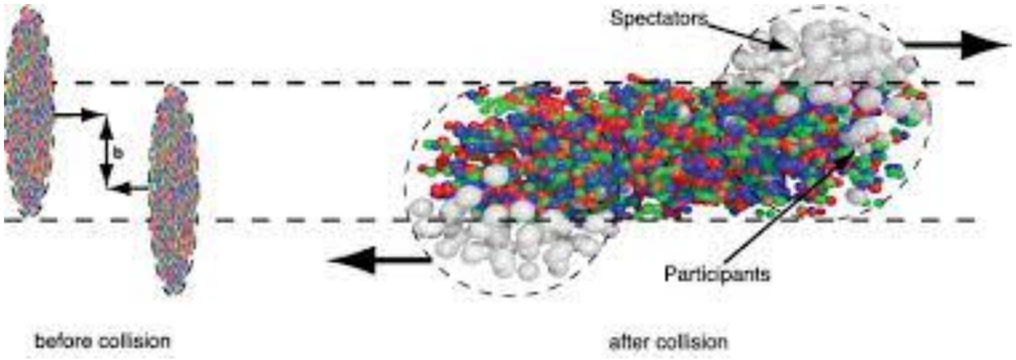
Transverse view

Some are glancing or “peripheral”



Longitudinal view

Transverse view



Struck nuclei are *participants*, others are *spectators*

Energy density achieved depends on # of participants

Not observable → how can we determine it a posteriori ?

The Glauber model

- Attributed to Roy Glauber (2005 Nobel for quantum optics)
- Main assumptions:
  - Probability of nucleon-nucleon scattering independent of previous s
  - Nucleons move in straight-line trajectories undeflected
- Woods-Saxon distribution of nuclear charge density

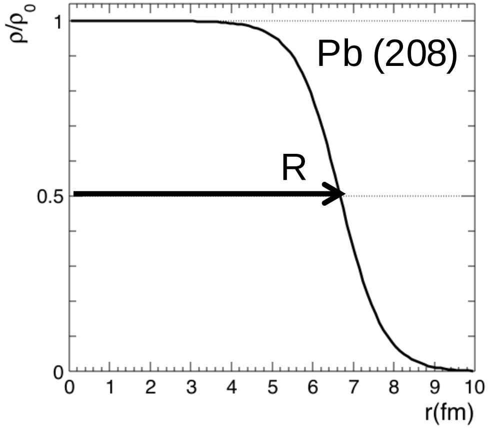
$$\rho(r) = \frac{\rho_0}{1 + exp[(r-R)/a]}$$

(spherical case)

nuclear size

surface thickness

e.g., for Pb(208)
R = 6.68 fm
a = 0.55 fm



- Nuclear thickness function in the optical limit

Define nuclear thickness

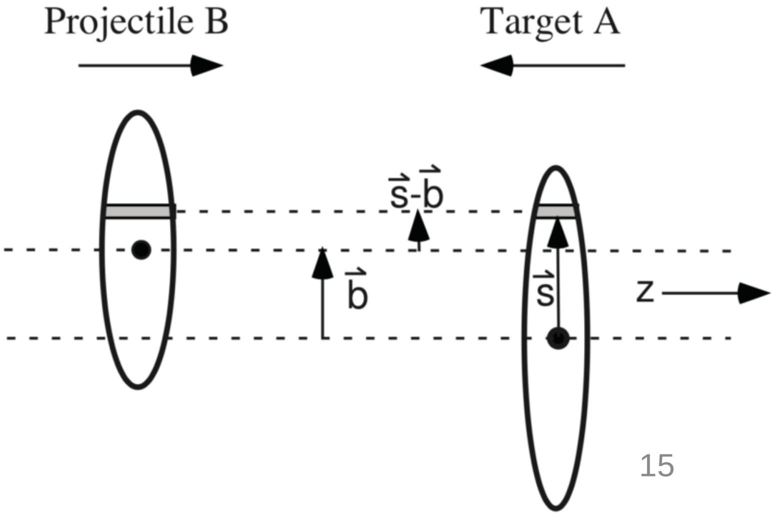
$$T_A(\vec{s}) = \int \rho_A(\vec{s}, z)\ \mathrm{d}z$$

where

$$\int T_A(\vec{s})\mathrm{d}^2 s = A$$

Take the product of nucleus A & B, and integrate over s:

$$T_{\mathrm{AB}}(b) = \int T_A(\vec{s}) \cdot T_B(\vec{s} - \vec{b})\mathrm{d}^2 s$$



Two key numbers: N_{coll} & N_{part}

- The total number of inelastic nucleon-nucleon collisions

$$N_{\text{coll}}(b) = T_{\text{AB}}(b) \cdot \sigma_{\text{inel}}^{\text{nn}}$$

- The total number of participants or "wounded nuclei", derivation:

Prob. for a nucleon in nucleus A to scatter with one from nucleus B:

$$p_{\text{int}} = T_B(\vec{s} - \vec{b}) \cdot \sigma_{\text{inel}}^{\text{nn}}/B$$

Prob. for a nucleon in A not to interact: $(1 - p_{\text{int}})^B$

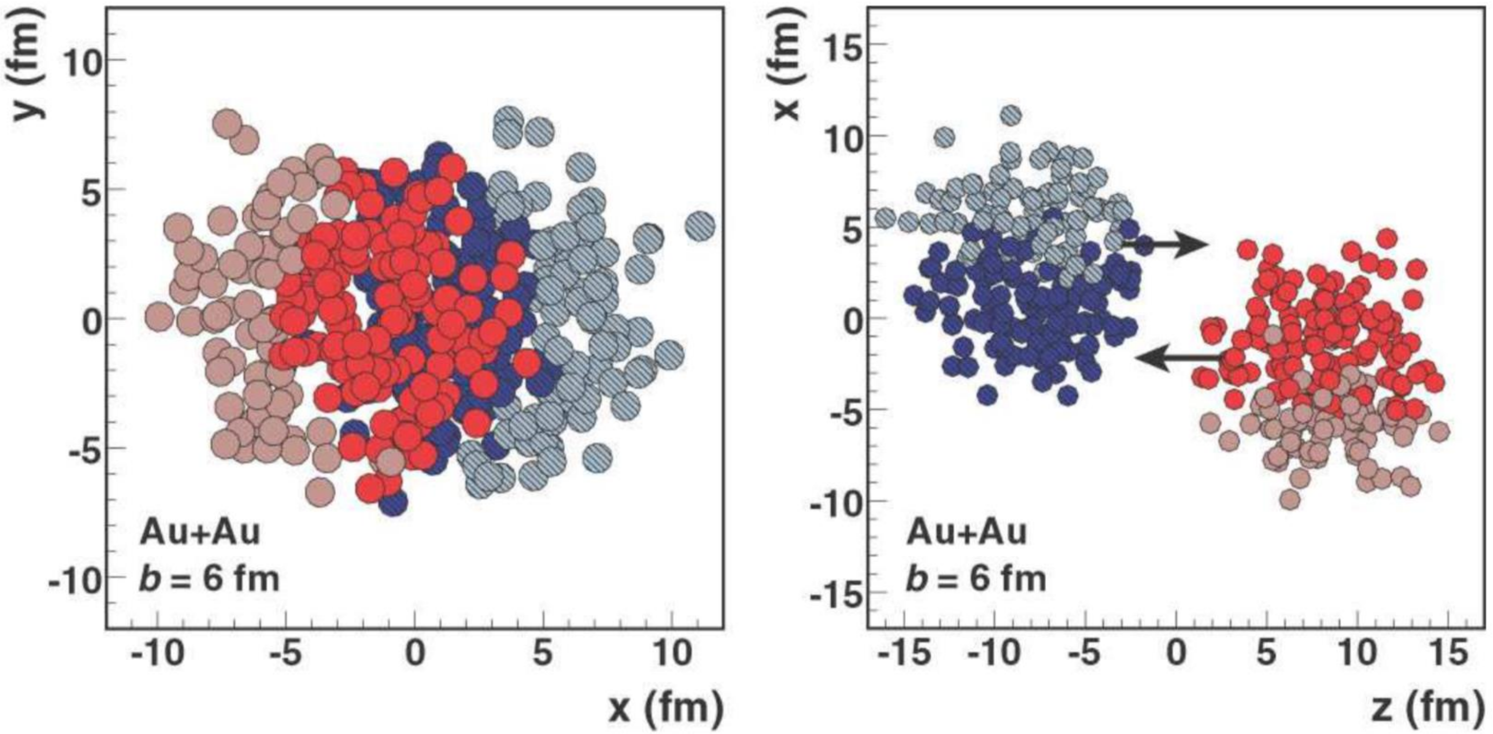
Prob. for a nucleon in A to interact at least once: $1 - (1 - p_{\text{int}})^B$

Number of participants from A:

$$N_{\text{part}}^{\text{A}} = \int T_{\text{A}}(\vec{s}) \cdot \left(1 - \left[1 - T_{\text{B}}(\vec{s} - \vec{b}) \cdot \sigma_{\text{inel}}^{\text{nn}}/B\right]^B\right) \mathrm{d}^2 s$$

Number of total participants: $N_{\text{part}}(b) = N_{\text{part}}^{\text{A}}(b) + N_{\text{part}}^{\text{B}}(b)$

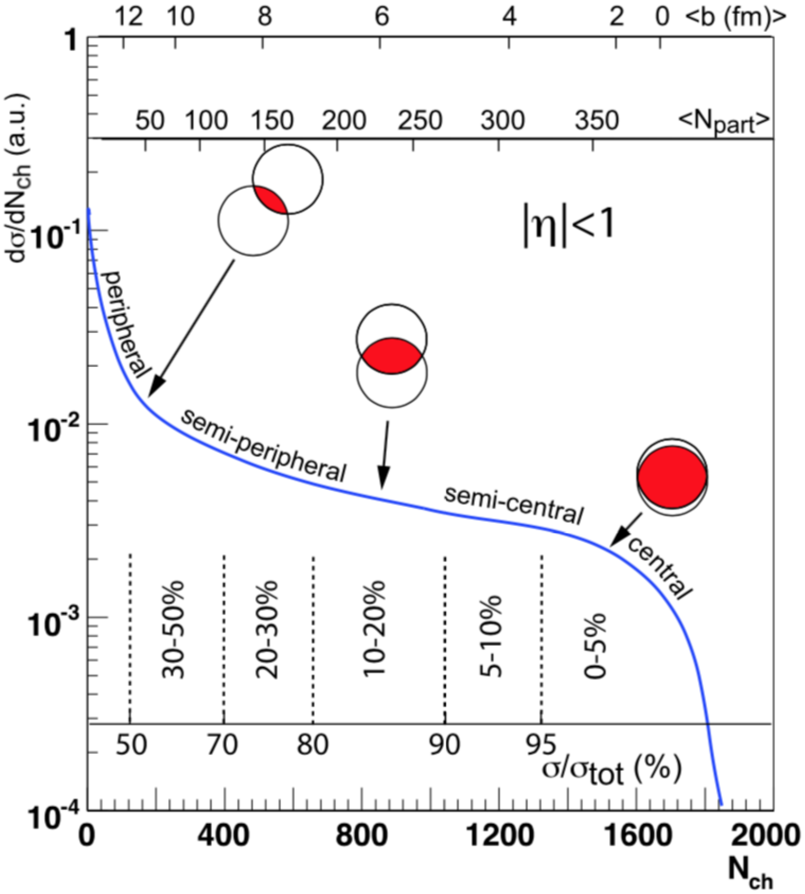
Glauber Monte Carlo



- In MC approach nucleon density discrete rather than continuous
- Sample nucleon positions according to nuclear density distribution
- Nucleons interact if their distance d < $\sqrt{(\sigma^{nn}/\pi)}$
- Fluctuations in overlap geometry will turn out to be important

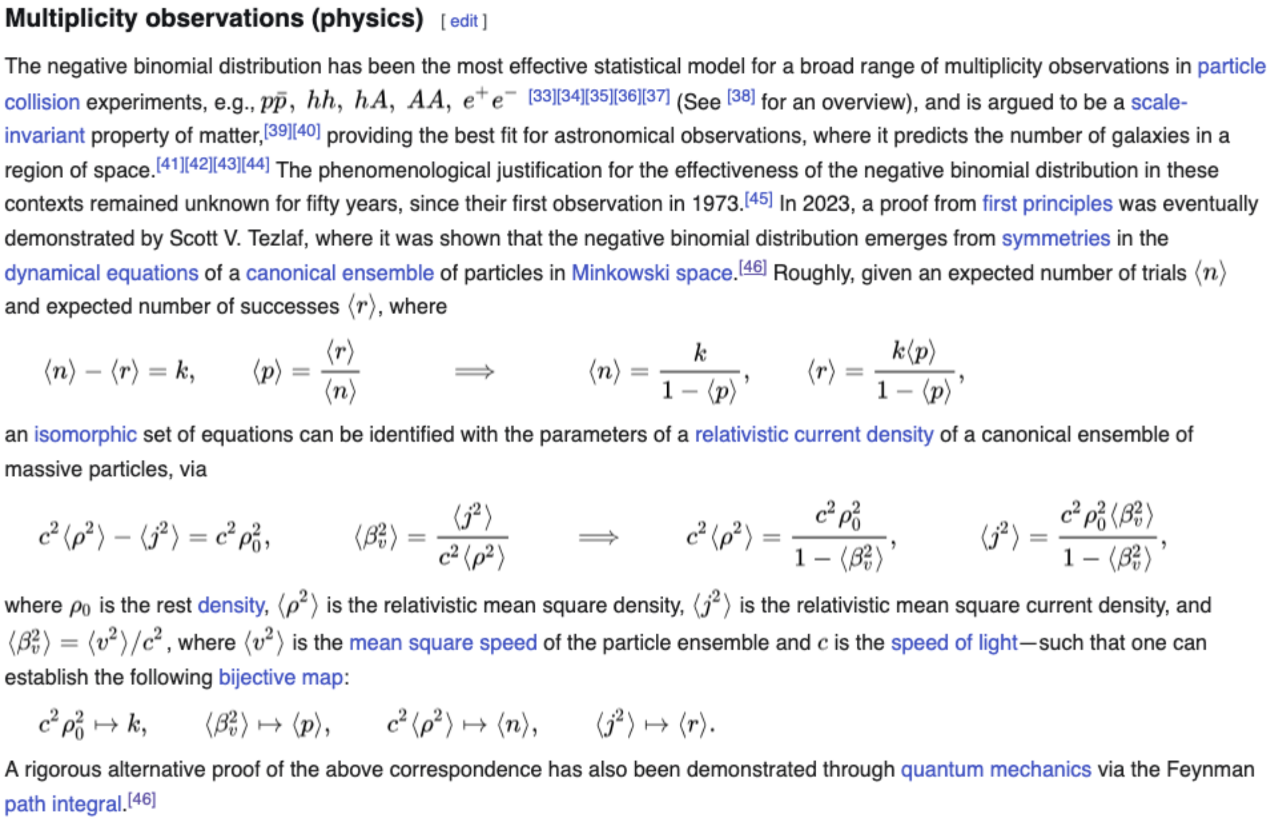
Centrality

- Centrality is based on an observable, typically event multiplicity, with a monotonic dependence on Glauber quantities, e.g., N_{part}
- Multiplicity then calculated from Glauber
  - Nucleon-nucleon multiplicity parameterized by a Negative Binomial Distribution, verified in pp
  - Total AA multiplicity assumed to scale w/ N_{part} → randomly sample the N.B.D. N_{part} times
- The multiplicity distributions in data and Glauber then divided into centrality classes, i.e., percentiles of the total cross section
- While centrality is detector dependent, corresponding Glauber values, e.g., $\langle N_{part} \rangle$, are approximately experiment independent



Aside: Negative Binomial Distribution

https://en.wikipedia.org/wiki/Negative_binomial_distribution#Overdispersed_Poisson



Multiplicity observations (physics) [edit]

The negative binomial distribution has been the most effective statistical model for a broad range of multiplicity observations in particle collision experiments, e.g., $p\bar{p},\ hh,\ hA,\ AA,\ e^+e^-$ [33][34][35][36][37] (See [38] for an overview), and is argued to be a scale-invariant property of matter,[39][40] providing the best fit for astronomical observations, where it predicts the number of galaxies in a region of space.[41][42][43][44] The phenomenological justification for the effectiveness of the negative binomial distribution in these contexts remained unknown for fifty years, since their first observation in 1973.[45] In 2023, a proof from first principles was eventually demonstrated by Scott V. Tezlaf, where it was shown that the negative binomial distribution emerges from symmetries in the dynamical equations of a canonical ensemble of particles in Minkowski space.[46] Roughly, given an expected number of trials $\langle n\rangle$ and expected number of successes $\langle r\rangle$, where

$$\langle n\rangle - \langle r\rangle = k, \qquad \langle p\rangle = \frac{\langle r\rangle}{\langle n\rangle} \qquad \implies \qquad \langle n\rangle = \frac{k}{1-\langle p\rangle}, \qquad \langle r\rangle = \frac{k\langle p\rangle}{1-\langle p\rangle},$$

an isomorphic set of equations can be identified with the parameters of a relativistic current density of a canonical ensemble of massive particles, via

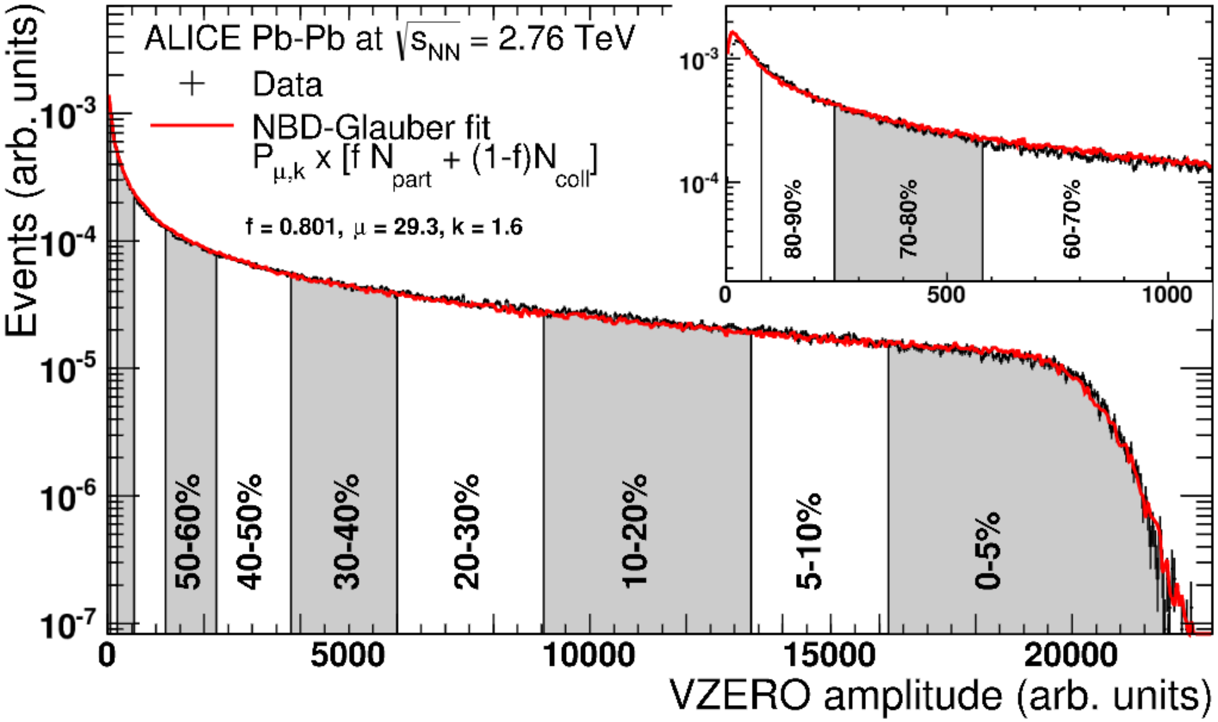
$$c^2\langle\rho^2\rangle - \langle j^2\rangle = c^2\rho_0^2, \qquad \langle\beta_v^2\rangle = \frac{\langle j^2\rangle}{c^2\langle\rho^2\rangle} \qquad \implies \qquad c^2\langle\rho^2\rangle = \frac{c^2\rho_0^2}{1-\langle\beta_v^2\rangle}, \qquad \langle j^2\rangle = \frac{c^2\rho_0^2\langle\beta_v^2\rangle}{1-\langle\beta_v^2\rangle},$$

where ρ_0 is the rest density, $\langle\rho^2\rangle$ is the relativistic mean square density, $\langle j^2\rangle$ is the relativistic mean square current density, and $\langle\beta_v^2\rangle = \langle v^2\rangle/c^2$, where $\langle v^2\rangle$ is the mean square speed of the particle ensemble and c is the speed of light—such that one can establish the following bijective map:

$$c^2\rho_0^2 \mapsto k, \qquad \langle\beta_v^2\rangle \mapsto \langle p\rangle, \qquad c^2\langle\rho^2\rangle \mapsto \langle n\rangle, \qquad \langle j^2\rangle \mapsto \langle r\rangle.$$

A rigorous alternative proof of the above correspondence has also been demonstrated through quantum mechanics via the Feynman path integral.[46]

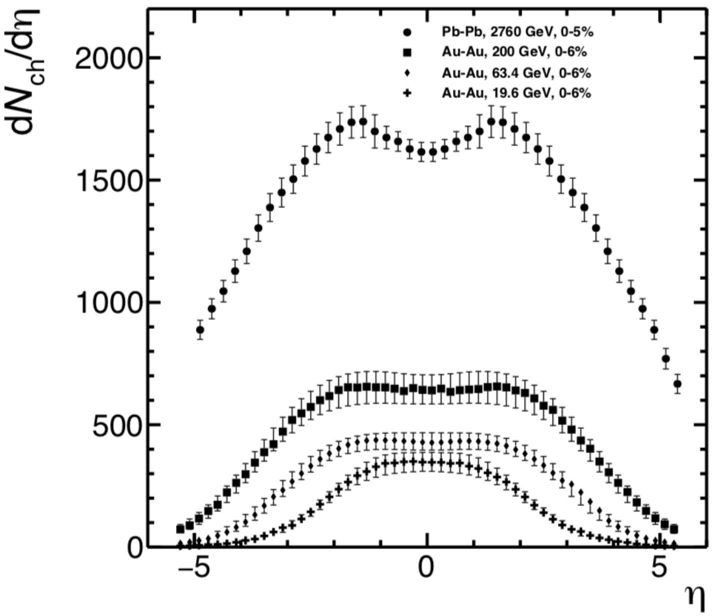
Centrality example: ALICE



- VZERO are scintillators at forward η, collected charged proportional to multiplicity
- Centrality typically measured at forward η to be independent from mid-rapidity measurements
- Glauber-based fit provides an excellent description of the VZERO charge distribution

Multiplicity

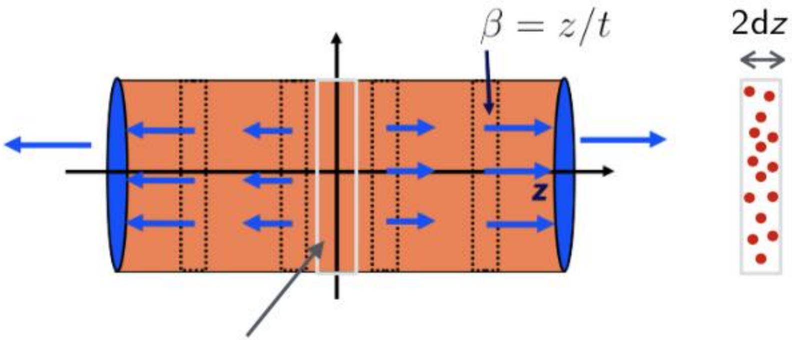
Multiplicity can also be used to constrain the energy density created in HICs



1.7k charged particles
per unit η at mid-rapidity

Notice the flat (pseudo)rapidity dependence near mid-rapidity
This *longitudinal boost invariance* emerges naturally in a
commonly used space-time picture of heavy-ion collisions

The Bjorken picture



Consider total energy in slice at $z = 0$ at time τ_0

Consider nuclei as relativistic pancakes, which cross at the speed of light

- Beam remnants continue at forward y
- Mid-y dominated by produced particles

Radius given by nuclear size

Main assumptions of the model:

- Homogenous expansion of cylinder in z direction, i.e., 1D ideal hydrodynamics
- Particles materialize at formation time τ_0 in their rest frame

Velocity in this system: $\beta = z/t \qquad \tau = t/\gamma$

$$\epsilon = \frac{E}{V} = \frac{1}{A}\frac{dE}{dz}\bigg|_{z=0} = \frac{1}{A}\frac{dy}{dz}\bigg|_{z=0}\frac{dE}{dy}\bigg|_{y=0} \underset{\uparrow}{=} \frac{1}{A\cdot\tau_0}\frac{dE}{dy}\bigg|_{y=0}$$

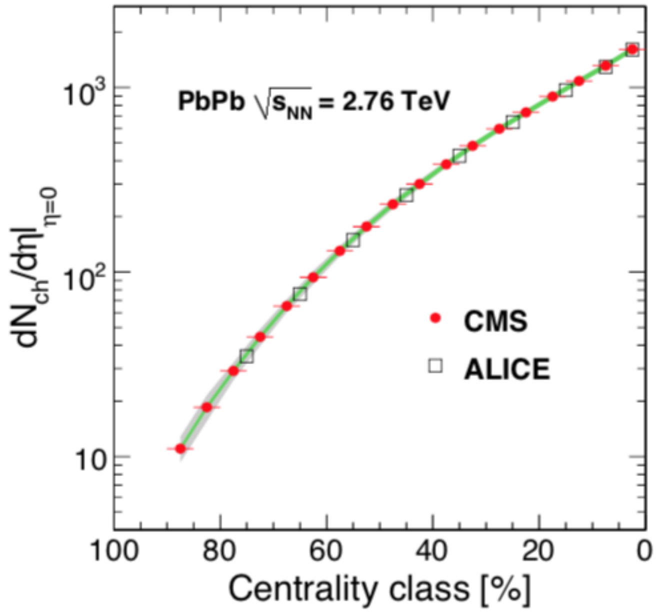
Standard relation for rapidity $\sinh y = \beta\gamma = z/\tau$

From multiplicity to E_T

Although E_T is measurable with calorimeters, heavy-ion experiments typically measure charged particle multiplicity

$$\frac{dE_T}{d\eta} \approx \langle m_T \rangle \, \frac{3}{2} \frac{dN_{ch}}{d\eta}$$

$m_T^2 = m^2 + p_T^2$
About 0.65 GeV for pions at the LHC



Mass isn't known unless particles are identified → need a correction

$$\frac{dE_\mathrm{T}}{dy} = J(y,\eta)\frac{dE_\mathrm{T}}{d\eta}$$

J(y,η) ≈10%

Estimating energy density

$$\epsilon = \frac{1}{A \cdot \tau_0} \frac{dE_T}{dy}\bigg|_{y=0}$$

A = πR^2 = π(7 fm)^2 for lead

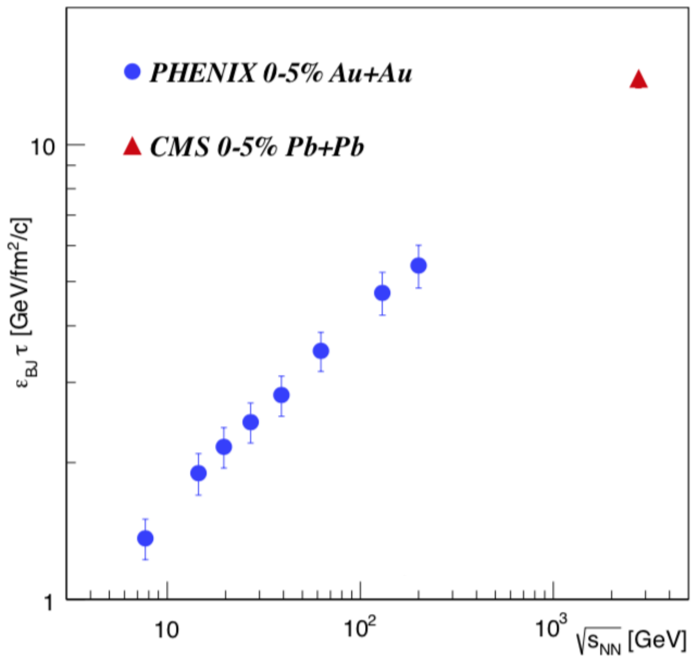
Traditionally assumed that τ_0 ~ 1 fm
We'll see later that short formation time is justified

dE$_T$ / dy|$_{y=0}$ ~ 2 TeV in central
collisions at the LHC

→ ε = 13 GeV/fm^3

Well in excess of ε = O(1) GeV/fm^3
for phase transition from lattice QCD

PHENIX, arXiv:1509.06727



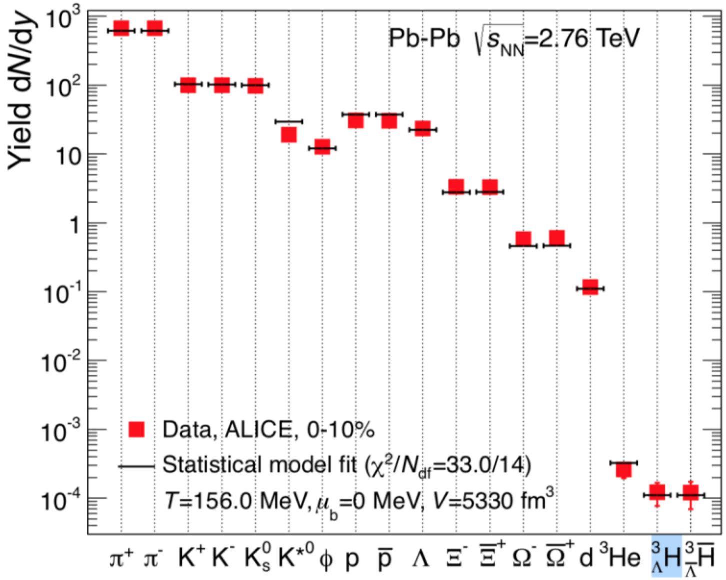
Statistical hadroproduction

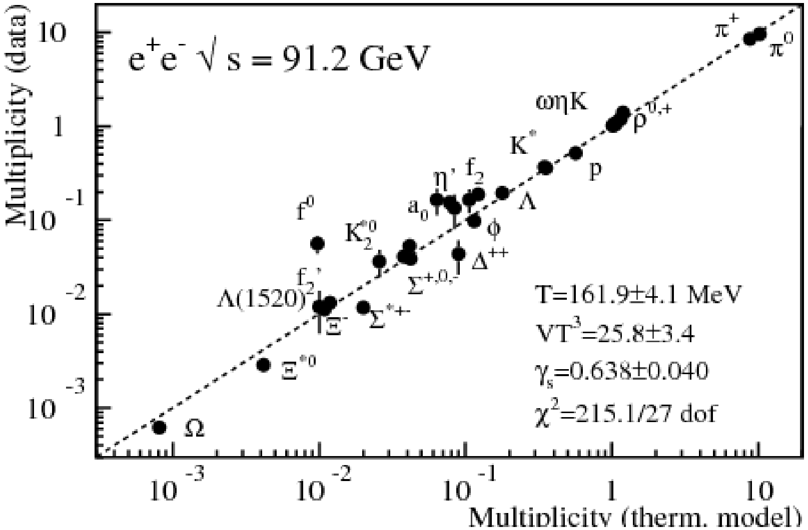
- Idea dates back to Fermi in the 1950’s
- Revived to consider production of hadron states in heavy ions
- Postulate that QGP hadronizes to equilibrated hadron gas. Particle number conserved only over entire volume, not locally → grand canonical ensemble
- Particle density given by derivative of the partition function

$$n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp((E_i-\mu_i)/T)\pm 1}$$

- Each conserved quantum number gives a chemical potential $\mu_{i,}$ e.g., charge, baryon number, etc.
- Temperature can be extracted by a fit to the abundance of states

Statistical description of data





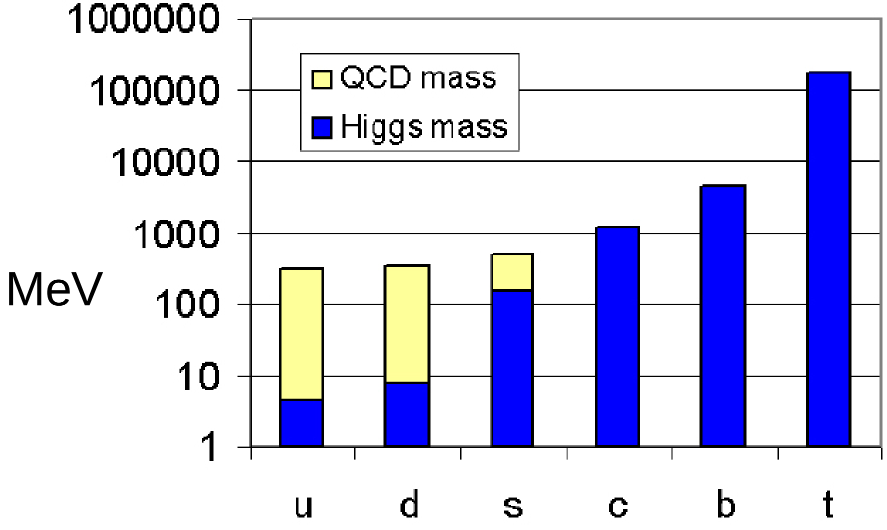
Fit to the heavy-ion data gives a T of 156 MeV close to the predicted critical temp.

Also work for e^+e^-, but with canonical ensemble (local, not global equilibrium)

→ Statistical production appears to be a generic feature of hadronization

However, an extra chemical potential γ_s is needed in e^+e^- for strange particles

The strange quark



- Light quark (“dressed”) mass dynamically generated by QCD
- Heavy quark mass directly from the Higgs mechanism
- Strange quark is special: both mechanisms contribute
  - Bare mass ~ 100 MeV
  - Kaon mass ~ 500 MeV

Because T_c in excess of the strange quark mass, expect thermal production

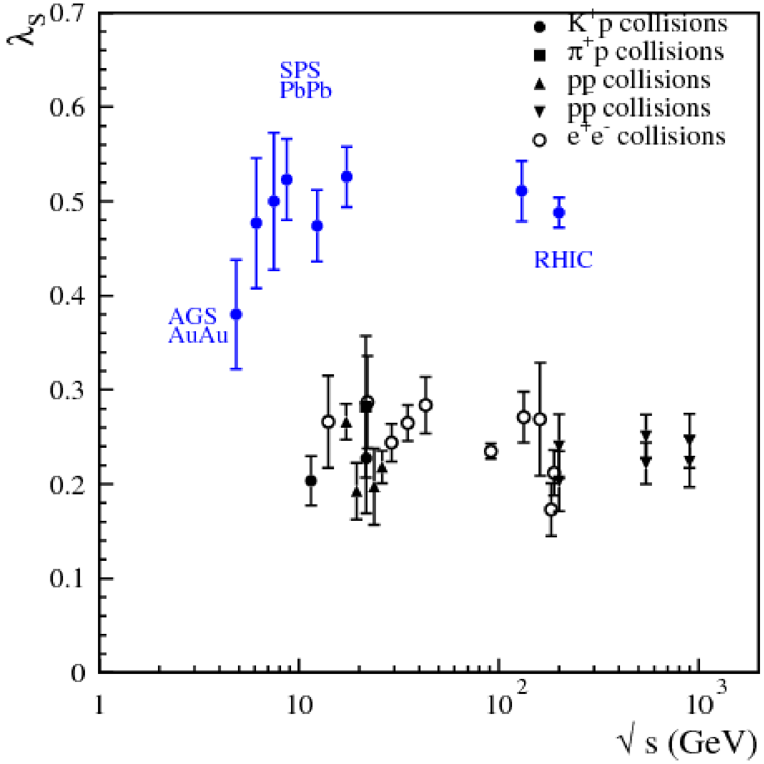
In elementary collisions strangeness is locally conserved →

γ_s reflects this “canonical suppression”

In an equilibrated QGP, expect conservation only over global volume ($\gamma_s = 1$)

Strangeness enhancement

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u}\rangle + \langle d\bar{d}\rangle}$$

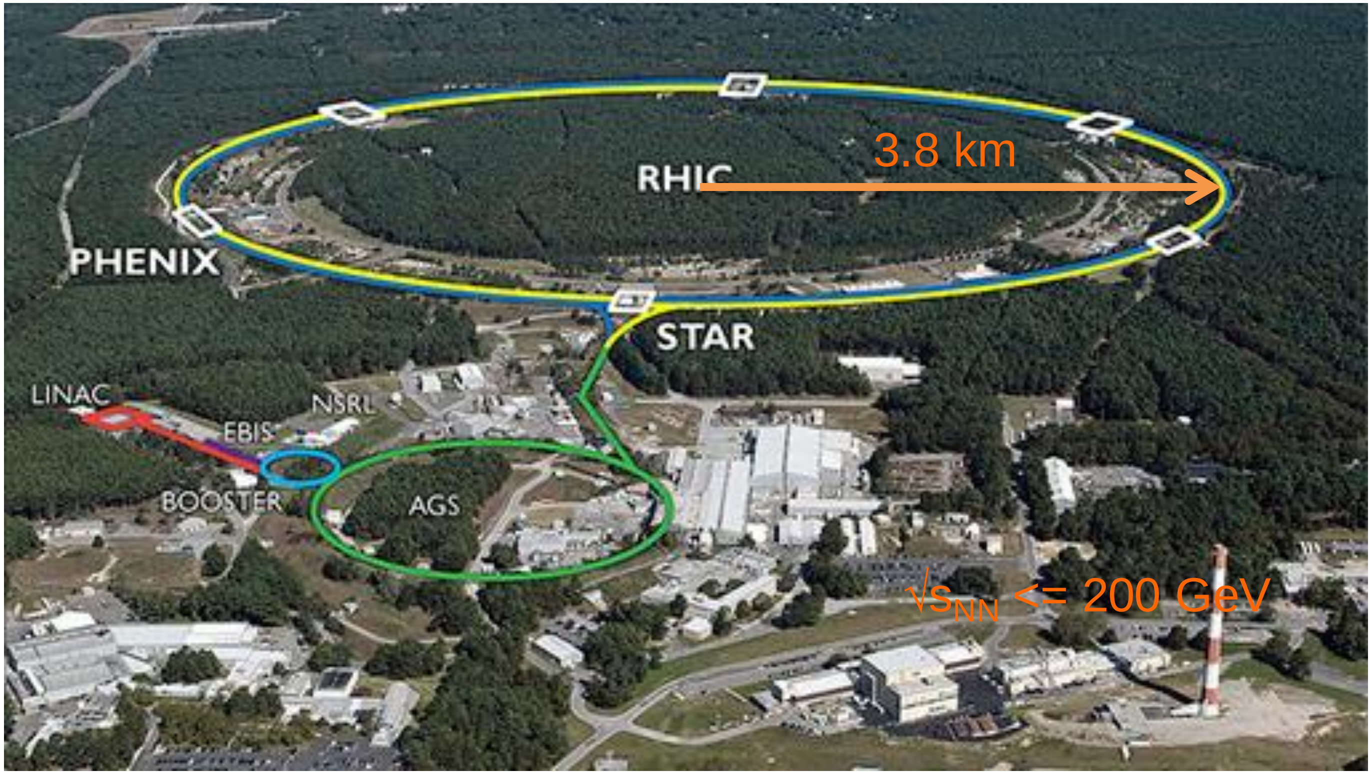


Strangeness indeed enhanced by 2x in AA, compared to elementary collisions

Supports picture of QGP as an equilibrated state over a large volume

Heavy-ion experiments

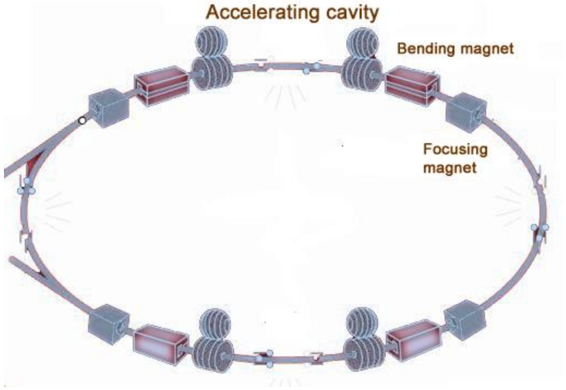
RHIC @ Brookhaven



- The first HI collider, after many years of fixed target expt's
- First collisions in 2000, continues to operate today
- STAR searching for the critical point with a beam energy scan
- PHENIX upgraded to sPHENIX, which is taking data now

The LHC @ CERN

Collides heavy ions for ~ 1 month per year (PbPb, pPb, XeXe so far)
In operation, since 2010
ALICE dedicated to HI program, but interesting things can also be done with HEP detectors (ATLAS, CMS, and more recently LHCb)



Ion beams

Max energy determined by bending power of magnets

$$E \propto \frac{Z}{A}$$

charge

mass

4-vectors

Using the Mandelstam variable s = $(p_1 + p_2)^2$,

can derive the equivalent COM beam energy for heavy-ion beams

LHC

Early Run 1, max pp √s = 7 TeV → PbPb $\sqrt{s_{NN}}$ = 2.76 TeV (7*Z/A)

Later Run 1, max pp √s = 8 TeV → pPb $\sqrt{s_{NN}}$ = 5.02 TeV (8*√(Z/A))

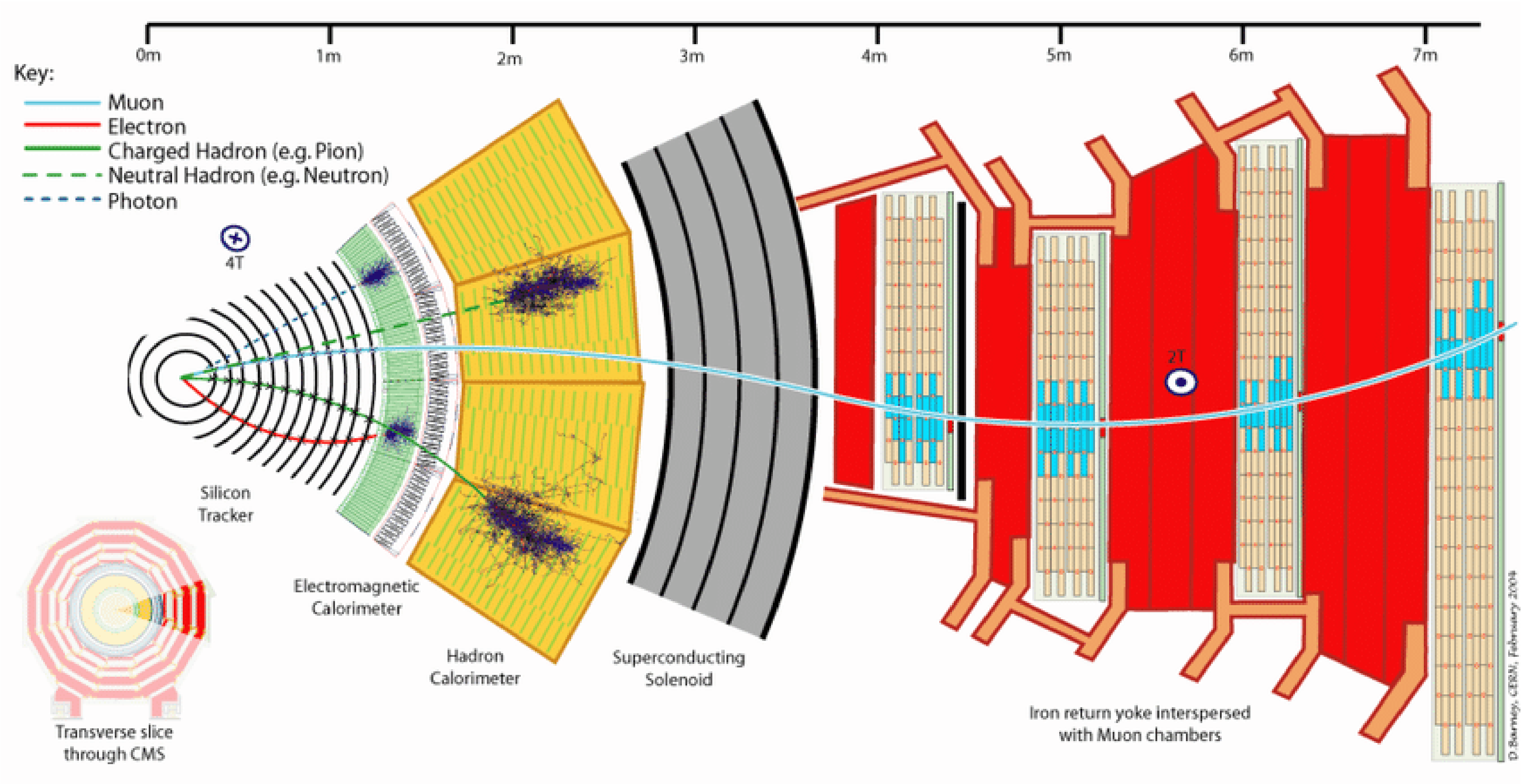
Run 2, max pp √s = 13 TeV → PbPb $\sqrt{s_{NN}}$ = 5.12 TeV (13*Z/A)

→ pPb $\sqrt{s_{NN}}$ = 8.16 TeV (13*√Z/A))

Z = 82
A = 208

At RHIC, AuAu $\sqrt{s_{NN}}$ = 200 GeV → pp √s = 500 GeV (polarized!)

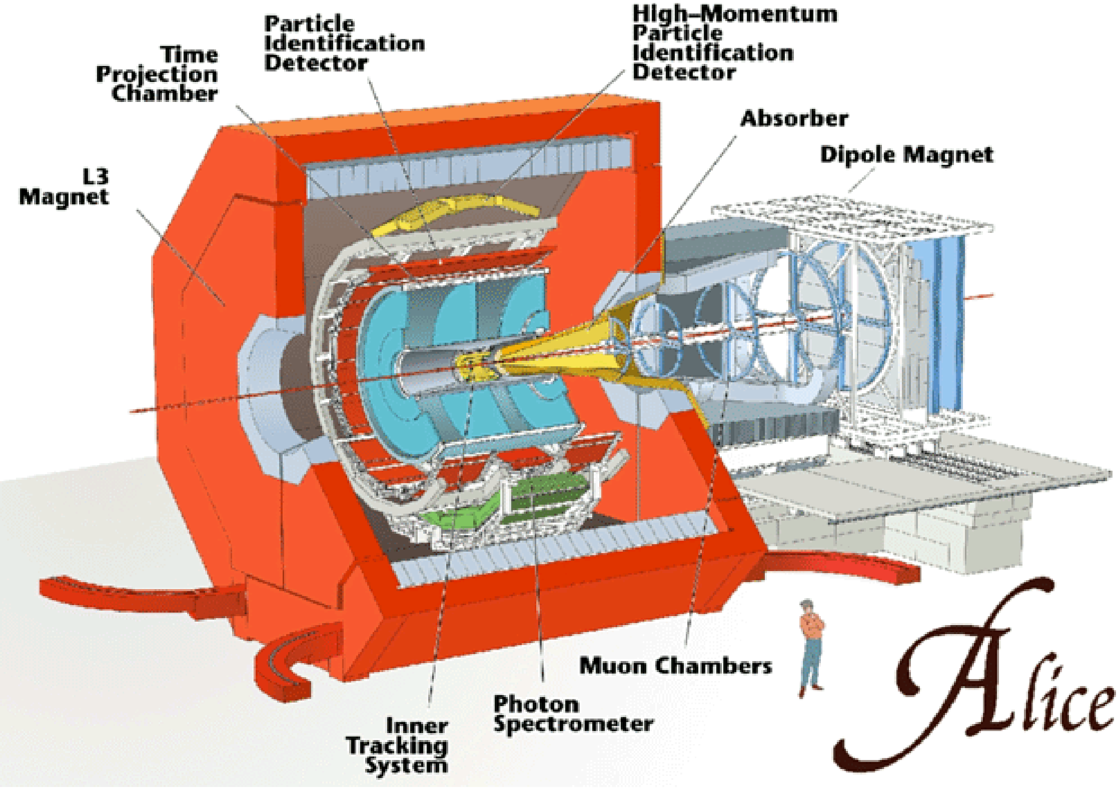
HEP detectors



Fast & triggerable for huge luminosity
Precision silicon tracking + high B field, e.g, for b-tagging
Hermetic calorimeters for jets and missing E_T
Isolated photons and leptons for EW bosons, Higgs, top

Ex: ATLAS, CMS
Also sPHENIX
follows HEP model

Heavy-ion detectors



Tracking w/ gaseous detectors for high occupancy, e.g., TPCs (STAR, ALICE)
Low p_T reach is essential for “bulk” observables
Emphasis on particle ID, e.g., TOF, dE/dx, RICH
Low p_T photons and leptons important, often forward η muon detectors independent from barrel system (ALICE, PHENIX)

Probes of the QGP

Observable	Property of the QGP
Strangeness enhancement	Chemical equilibrium
Jet quenching	Density
Quarkonia melting	Temperature
Particle correlations	Collectivity, Viscosity
Thermal photons	Temperature
Hanbury Brown Twiss correlations	Size

And many more…

Summary of this lecture

- Heavy-ion collisions used to explore QCD at high energy density
- Matter crosses over to a deconfined state, the quark-gluon plasma
- Collision centrality related to observables via a Glauber model
- Energy density estimated from mid-rapidity multiplicity in Bjorken's picture
- Extended QGP alters chemical composition of produced matter, increasing strangeness
- Currently colliding heavy ions at RHIC & LHC w/ various detectors

Next part

Select phenomena / observables for heavy ions: collective flow, jet quenching and quarkonia melting