(QCD) Resummation 101 [cont'ed] 2025-03-11

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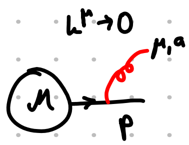
IV. (Im)proving the LL result

i) Factorization

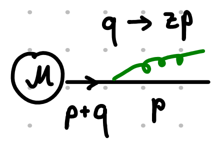
$$f(a,b) = g(a)\,h(b)$$

Think: "axiom" of indep. emissions in LL calc.



$\approx \frac{g p^\mu}{p\cdot k} T^a \mathcal{M}(p)$

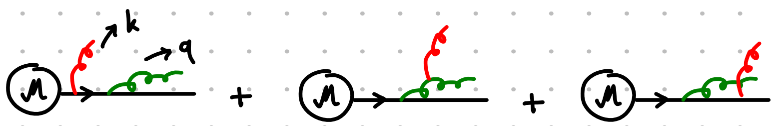
soft



$\approx \frac{g}{p\cdot q} P(z)\, \mathcal{M}(p+q)$

collinear

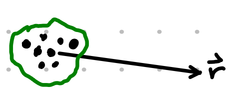
$\Rightarrow$ hard process (Z vs. W) factorizes



$$\approx \frac{g p^\mu}{p\cdot k} T^a \, \frac{g}{p\cdot q} P(z)\, \mathcal{M}(p+q)$$

$\Rightarrow$ soft & collinear emissions factorize from each other

Cf. multipole expansion in electrostatics:

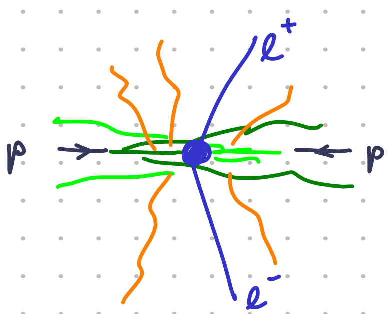


$$V(\vec{r}) = \underbrace{\frac{q}{r}}_{\text{keep this}} + \frac{\vec{e}_r\cdot\vec{d}}{r^2} + \frac{\vec{e}_r\cdot\overline{Q}\cdot\vec{r}}{r^3} + \dots$$

square $\Rightarrow$

$$\boxed{\frac{d\sigma}{dp_T^2} = H(Q)\, B_a(\vec{k}_T) \otimes B_b(\vec{k}_T) \otimes S(\vec{k}_T)}$$

Factorization theorem

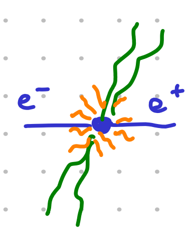


$\otimes \equiv$ convolution adding momenta from each sector

(see slides for picture)

cf. $\sigma(p_T^{cut}) = H(Q)\, B_a(Q, p_T^{cut})\, B_b(Q, p_T^{cut})\, S(p_T^{cut})$

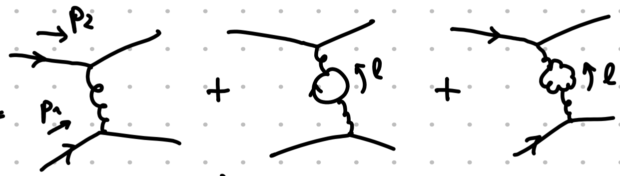
$[P(\text{no collinear jet})]^2 \times P(\text{no soft jet})$



$$\frac{d\sigma}{d\tau} = H(Q)\, J_a(Q\tau) \otimes J_b(Q\tau) \otimes S(\tau)$$

ii) Resummation from the RGE

$$\mathcal{L} \supset \bar{\Psi}(i\partial\!\!\!/ + g A\!\!\!/)\Psi$$

$\mathcal{A} =$ 

$$= g^2(\Lambda) + g^4 \beta_0 \int_Q^{\Lambda} \frac{d|\ell_E|}{|\ell_E|} + \mathcal{O}(g^6)$$

$Q^2 \equiv (p_1+p_2)^2$

$\Lambda \approx$ Planck scale

<u>Idea</u>: Cut off loop integral at $|\ell_E| = \mu \ll \Lambda$, absorb the unknown UV contributions into the <u>renormalized</u> coupling:

$$= \underbrace{g^2(\Lambda)\left[1 + g^2\beta_0 \int_\mu^\Lambda \frac{d|\ell_E|}{|\ell_E|}\right]}_{\equiv\, \alpha_s(\mu)} + g^4\beta_0 \int_Q^\mu \frac{d|\ell_E|}{|\ell_E|} + \mathcal{O}(g^6)$$

$$= \alpha_s(\mu) + \alpha_s^2(\mu)\, \beta_0 \ln\frac{\mu}{Q} + \mathcal{O}(\alpha_s^3)$$

1) $\mu \frac{d}{d\mu} \mathcal{A} = 0 \quad \Rightarrow \quad \mu \frac{d}{d\mu} \alpha_s(\mu) = -\beta_0 \alpha_s(\mu)$

$$\Rightarrow \quad \boxed{\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \alpha_s(\mu_0)\beta_0 \ln\frac{\mu}{\mu_0}}} \qquad (\beta_0 > 0)$$

2) Want: $\mu \sim Q$ to avoid large logs

$\rightarrow$ $\alpha_s(Q)$ <u>resums</u> (large) logs of Q/μ_0

$\rightarrow$ $|\mathcal{A}(Q)|^2 \sim [\alpha_s(Q)]^2$ + small corrections

$\rightarrow$ β function shows up in <u>data</u>

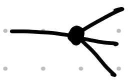
for dijet invar. mass spectrum

$+ \mathcal{O}(g^6)$

Back to factorization theorem (and one way to prove it):

- Fundamental issue: separation of scales $p_T^{cut} \ll Q$

  $\Rightarrow$ EFT (Soft Collinear Effective Theory)

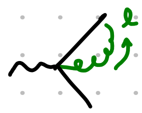
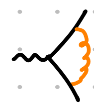
$\to C \times$ 

Fermi	m_W	m_μ	W	Leptons
SCET	Q	p_T^{cut}	hard	coll./soft

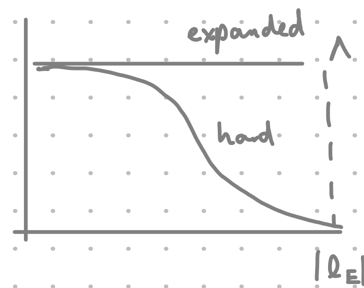
"matching coefficient"

- $H(Q) = |C(Q)|^2$ with $\mathcal{L}_{SCET} \supset C(Q)\, Z_\mu \bar{\Psi}_{coll,b} \gamma^\mu \Psi_{coll,a}$

$\Rightarrow$ coupling in Lagrangian like α_s

$C(Q) = 1 +$  $-\,2$  $-$  $+ \mathcal{O}(\alpha_s^2)$

$C_F \ln \frac{\Lambda}{Q}$ $\qquad C_F \ln^2 \frac{\Lambda}{Q}$ $\qquad - C_F \ln^2 \frac{\Lambda}{Q}$



$$\to \mu \frac{d}{d\mu} C(Q,\mu) = \frac{1}{2}\Big[\underbrace{\Gamma_q(\alpha_s)}_{\text{cusp anom. dim.} \,=\, \frac{\alpha_s}{\pi} C_F + \mathcal{O}(\alpha_s^2)} \ln\frac{Q}{\mu} + \gamma_H(\alpha_s) \Big] C(Q,\mu)$$

replace by H

erase

Putting the RGE to work:

$$\sigma(Q,p) = H(Q,\mu)\, F(p,\mu) \qquad p \equiv p_T^{cut}$$

$\mu \sim Q$

$\downarrow$ RG

$\mu \sim p$

$\rightarrow$ split up log as $\ln\frac{Q}{p} = \ln\frac{Q}{\mu} + \ln\frac{\mu}{p}$

$\rightarrow$ Solve the RGE ("running") between Q and p

$$\frac{d}{dx} f(x) = \gamma(x) f(x) \quad \Rightarrow \quad f(b) = \exp\left[\int_a^b dx\, \gamma(x)\right] f(a)$$

$$\sigma_{res}(Q,p) = H(Q,\mu=Q) \exp\left\{\int_Q^p \frac{d\mu}{\mu}\left[\Gamma_q(\alpha_s)\ln\frac{Q}{\mu} + \gamma_H(\alpha_s)\right]\right\} F(p,\mu=p)$$

- Bound. cond.s free of large logs
- Resummed to all orders by exponential instead
- Keep only $\Gamma = \frac{\alpha_s}{\pi} C_F + \mathcal{O}(\alpha_s^2)$, ignore running of α_s

$$\rightarrow \int_Q^p \frac{d\mu}{\mu}[\ldots] = \frac{\alpha_s}{\pi} C_F \int_Q^p \frac{d\mu}{\mu} \ln\frac{Q}{\mu} = -\frac{\alpha_s}{\pi} C_F \frac{1}{2}\ln^2\frac{p}{Q} \quad \square$$

LL result

- NLL, NLL', NNLL etc. $\leftrightarrow$ specific loop orders in Γ_q, γ_H and bound. cond.s

$$\begin{aligned}
\Gamma_q(\alpha_s) &= \Gamma_0\alpha_s + \Gamma_1\alpha_s^2 + \Gamma_2\alpha_s^3 + \Gamma_3\alpha_s^4 \\
\gamma(\alpha_s) &= \qquad\quad\;\; \gamma_0\alpha_s + \gamma_1\alpha_s^2 + \gamma_2\alpha_s^3 \\
H(Q,Q) &= 1 \qquad\qquad\quad + h^{(1)}\alpha_s + h^{(2)}\alpha_s^2 \\
F(p,p) &= 1 \qquad\qquad\quad + f^{(1)}\alpha_s + f^{(2)}\alpha_s^2
\end{aligned}$$

LL NLL NNLL N^3LL

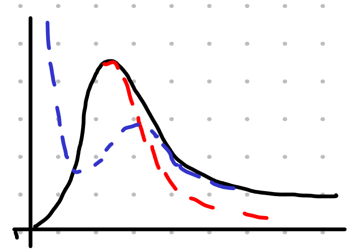
iii) <u>TNPs</u> $\rightarrow$ F. Tackmann, 2411.18606

- Scale variations $\Rightarrow$ not the true of (parametric) syst. uncert.
- Instead: Next unknown coeff. in the $\mathcal{O}(6-10)$ pert. series entering the anom. dim.s / boundary cond.s
- Get complete theory covariance matrix for all bins in resummation region

$\hookrightarrow$ Used by CMS m_W (see slides)

iv) Bonus: Matching & nonp. physics

$$\sigma_{\text{matched}} = \sigma_{\text{res}} + \underbrace{\left[\sigma_{\text{FO}} - \sigma_{\text{res@FO}}\right]}_{\mathcal{O}(\tau)}$$



$$B_i(x, \vec{b}_T) = \Big[\underbrace{I(x, \vec{b}_T)}_{\text{perturbative}} \otimes_x f(x)\Big] \otimes_{\vec{b}_T} \underbrace{f_i^{\text{nonp.}}(x, \vec{b}_T)}_{\substack{\text{e.g. Gaussian} \\ \text{with width} \sim \Lambda_{QCD} \sim 1\,\text{GeV}}}$$

$$\hookrightarrow \frac{\Lambda_{QCD}^2}{(p_T^Z)^2}$$