(QCD) Resummation 101

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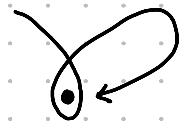
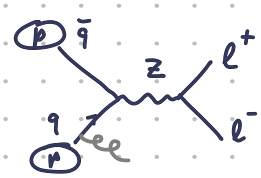
Johannes Michel (Nikhef/UvA)



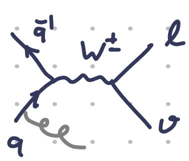
I. Intro

$$f(a,b) = e^{-a/b} \xrightarrow[\text{in } a]{\text{pert. theory}} 1 - \frac{a}{b} + \frac{1}{2}\frac{a^2}{b^2} + \mathcal{O}(a^3) \xrightarrow{b\sim a} ?$$

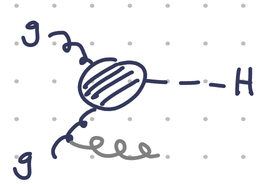
"re-sum" for $b \sim a \ll 1$ (think: $a = \alpha_s$!)



↳ proton content



↳ m_W

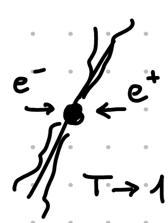


↳ y_b, y_c

Jet veto/0-jet bin:

$\vec{p}_T^{\,jet} = -\vec{p}_T^{\,H}$ $\quad |\vec{p}_T^{\,jet}| \leq p_T^{cut}$

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{\sum_i |\vec{p}_i|}$$

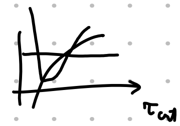


↳ $\tau \equiv \frac{p_T^{Z,W,H}}{Q}, \frac{p_T^{jet}}{Q}$ $(Q \equiv m_{Z,W,H})$, $1-T$ sensitive to additional radiation

II. The problem

$$\sigma(\tau < \tau_{cut}) = \sigma_{tree}\left[1 + \frac{\alpha_s}{\pi} f_1(\tau_{cut}) + \left(\frac{\alpha_s}{\pi}\right)^2 f_2(\tau_{cut}) + \dots\right]$$

$\tau_{cut} \to 0$: $\quad f_1(\tau_{cut}) \approx f_1(0)$? No!



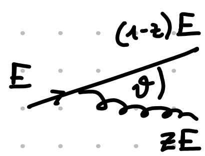
$$f_1(\tau_{cut}) = C_{1,2} \ln^2 \tau_{cut} + C_{1,1} \ln \tau_{cut} + C_{1,0} + \mathcal{O}(\tau_{cut})$$

$$\sigma(\tau < \tau_{cut}) = \underline{\sigma_{tree} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n} C_{n,m} \ln^m \tau_{cut}} + \underline{\mathcal{O}(\tau_{cut})}$$

<u>What's going on?</u> QCD radiates a <u>lot</u> in soft & collinear limits

$C_F = \frac{4}{3}$

$$dz\, d\vartheta^2 P(z,\vartheta^2) = \left| \quad \right|^2 \underset{\vartheta \to 0}{\overset{z \to 0}{\sim}} \frac{\alpha_s}{\pi} C_F \frac{dz}{z} \frac{d\vartheta^2}{\vartheta^2}$$



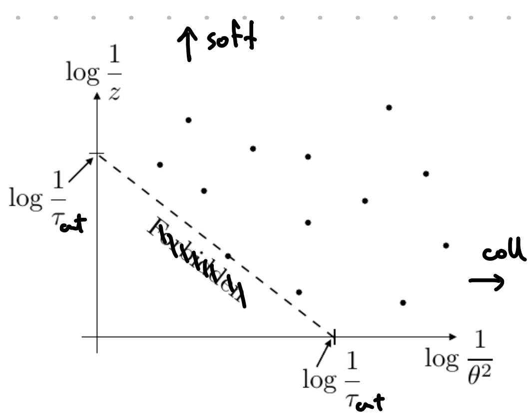
$\tau = 1 - T \approx z\vartheta^2$

$$= \frac{\alpha_s}{\pi} C_F\, d\left(\ln\frac{1}{z}\right) d\ln\frac{1}{\vartheta^2}$$

(Lund plane)

Trick: $\sigma(\tau < \tau_{cut}) = \sigma - \sigma(\tau > \tau_{cut}) = -\sigma(\tau > \tau_{cut}) + \text{const.}$

$$\hookrightarrow \frac{\alpha_s}{\pi} f_1(\tau_{cut}) = -\int^1 dz \int^1 d\vartheta^2 P(z,\vartheta^2)\, \Theta(z\vartheta^2 > \tau_{cut}) + \text{const.}$$



$$= -\frac{\alpha_s}{\pi} C_F \frac{1}{2} \ln^2 \tau_{cut}$$

$+$ single log $+$ const.

Nested soft-coll. limit

$\Rightarrow$ two logs at each order

<u>N.B.</u>: Draw w/o dots at first.

III. Physics of LL Resummation

A. Larkoski
1709.06195

- Keep adding gluons according to P
  ↳ Deep reasons for independence, no need for quarks, etc....

- Emissions exp'ly far apart in z, ϑ "real space"
  ↳ Value of $\tau = \sum_{\text{gluons}} z_g \vartheta_g^2 \approx z\vartheta^2$ set by a single one
  ↳ on the line

- No emissions in "forbidden region" (otherwise $\tau > \tau_{cut}$)

$$\Rightarrow \quad \frac{\sigma(\tau < \tau_{cut})}{\sigma} = P(\text{no emissions in } \Delta)$$

$$= \prod_{i=1}^{N} P(\text{no emit in region } i)$$

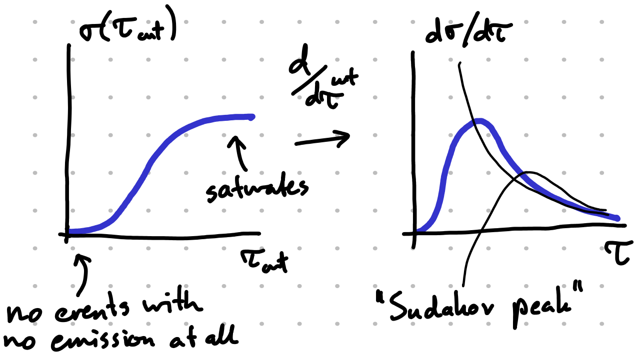
$$= \prod_{i=1}^{N} \Big[1 - \underbrace{P(\text{emit in region } i)}_{\frac{\alpha_s C_F}{\pi}(\text{Area of } i) = \frac{\alpha_s C_F}{\pi} \frac{\text{Area of } \Delta}{N}}\Big]$$

$$= \prod_{i=1}^{N} \Big[1 - \frac{1}{N} \frac{\alpha_s}{\pi} C_F \frac{1}{2} \ln^2 \tau_{cut}\Big]^N$$

$$\overset{N\to\infty}{\Rightarrow} \quad \boxed{\frac{\sigma(\tau < \tau_{cut})}{\sigma} = \exp\Big[-\frac{\alpha_s}{\pi} C_F \frac{1}{2} \ln^2 \tau_{cut}\Big]}$$

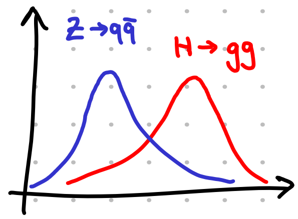
resums **L**eading **L**ogs $m = 2n$, $C_{n,m} = \frac{1}{n!}\left(-\frac{1}{2}\right)^n C_F^n$

(technically: double-log. approx. since coupling fixed)



Comments:

- Peak position and quark/gluon separation:



$C_F = \frac{4}{3}$ vs. $C_A = 3$
(or higher α_s!)

- No-emission probability P(no emissions in Δ) key building block for turning this into a Markov process yielding fully realistic event samples $\rightarrow$ Melissa

- LL behavior (up to derivatives) common for almost all observables up to coefficients Δ vs. Δ multiplying the universal "cusp anom. dim." $\Gamma_0^{q/g} = C_F/C_A$ ($\rightarrow$ only process dep., up to σ_{tree})

  $\rightarrow$ Deep differences beyond, see tomorrow.

- But how to <u>prove</u> this?

  $\rightarrow$ Also see tomorrow.