False vacuum decay
in and out of equilibrium

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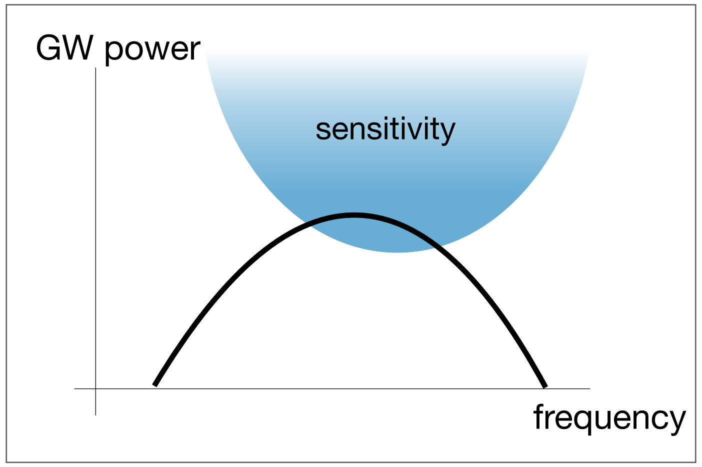
Motivation

The decay of a metastable state (false vacuum) plays an important role in many branches of physics, including particle physics and cosmology.

- **First order phase transitions in the early universe**

  Colliding bubbles generate gravitational waves

  see, e.g., **Caprini et al, 1910.13125**



- **EW baryogenesis**

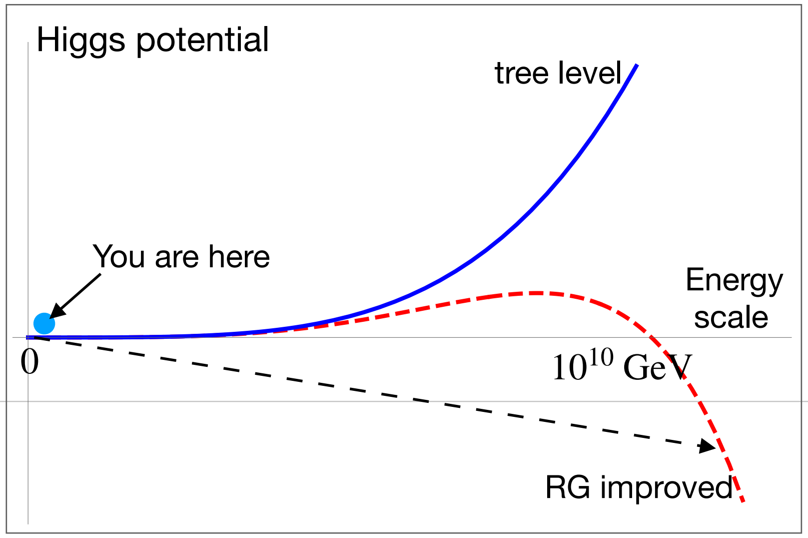
  Bubble wall moving through plasma generates baryon number

  for review see **Bodeker, Buchmuller, 2009.07294**

- **Higgs vacuum metastability**

  In the present-day Universe, the decay probability is small enough.
  But this can change in extreme environments.

  for reviews see **Espinosa et al, 1505.04825**
  Andreassen et al, 1604.06090



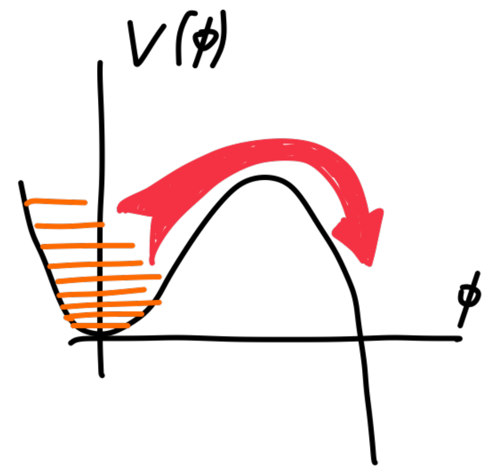
- **Experimental tests of nucleation theory**

  Zenesini et al, Nature Physics 20, 558–563 (2024) — first experimental result

Methods 1

Studies of decay of metastable state have more than a century long history.

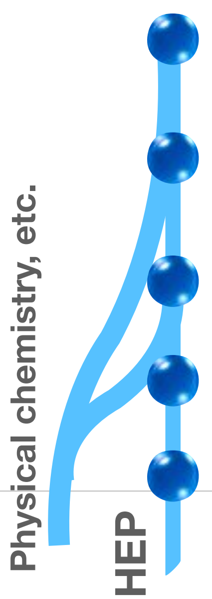
We are interested in the developments in the context of high-energy physics, field theory.



Euclidean — equilibrium — approach

It applies to systems whose metastable state is in **local equilibrium** — it can be assigned a temperature.

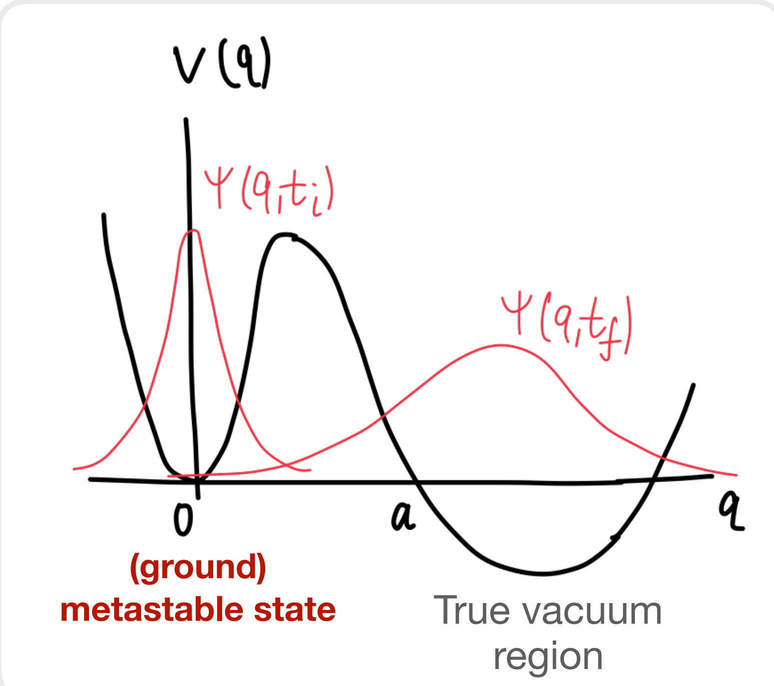
Milestones:



- Gibbs 1875 — first discussion of the critical bubble, its energy in the thin-wall approximation
- Wigner 1937 — Transition State Method for chemical reactions: saddle point, negative mode, zero modes
- Langer 1969 — Classical-statistical theory of metastability: many d.o.f. + external heat bath
- Affleck 1980 — Quantum-statistical theory of metastability: 1 d.o.f., no external heat bath
- Linde 1982 — Decay of false vacuum at finite temperature: field theory, different regimes

Decay of metastable state

Consider the quantum-mechanical system with the Hamiltonian $H = \dfrac{p^2}{2m} + V(q)$ and the “tunnelling” potential.



- $P_{surv} = 1 - e^{-\Gamma t}$ — survival probability in the metastable state

  at times not very short and not very long

  $\Gamma \sim e^{-B}$ — decay rate in the WKB approximation

  it shows the main exponential suppression;
  we will talk about the prefactor later

  $B = 2\int_0^a dq\sqrt{2mV(q)}$ — suppression exponent

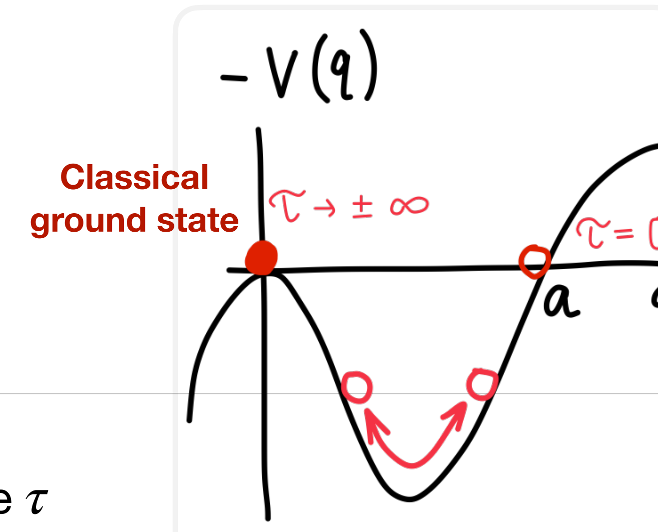
- Interestingly, $B = S_E[q_b]$, where q_b is the **bounce** trajectory.

$$S_E = \int d\tau \left[\frac{m}{2}\left(\frac{dq}{d\tau}\right)^2 + V(q)\right]$$ — **Euclidean** action associated with H

$$m\frac{d^2q}{d\tau^2} = -\frac{\partial(-V)}{\partial q}$$ — **classical** equation of motion in the “imaginary” time τ

$$q_b(\pm\infty) = 0\ , \quad \dot{q}_b(0) = 0$$ — boundary conditions selecting the bounce

they are uniquely associated with the false vacuum state



Classical particle moves
along the bounce trajectory

So one can solve Newton’s equation instead of the Schrödinger equation!

Price to pay: WKB approximation; vacuum boundary conditions.

not relevant here, but for non-equilibrium states…

Decay of metastable state

Consider the scalar field theory with the Lagrangian $L = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$ and the "tunnelling" potential.

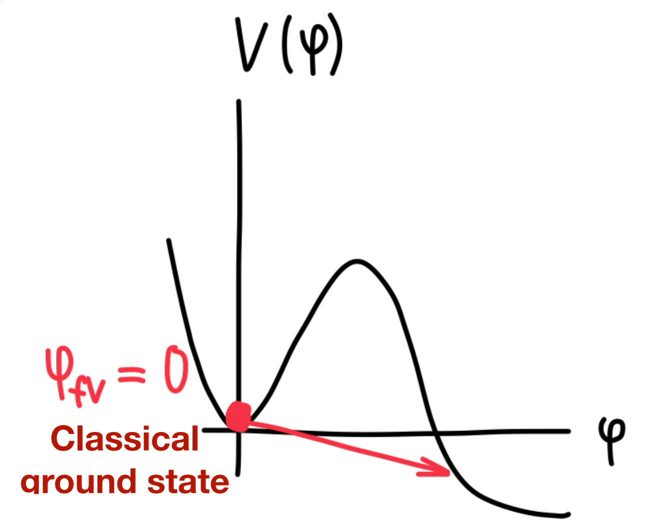
- Again, $P_{surv} = 1 - e^{-\Gamma t}$, where $\Gamma \sim e^{-Bt}$ is the decay rate per unit space.

  Again, $B = S_E[\phi_b]$, where S_E is the Euclidean action associated with L:

$$S_E = \frac{1}{g^2}\int d\vec{x}\,d\tau \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\frac{\partial\phi}{\partial x_i}\right)^2 + V(\phi)\right]$$

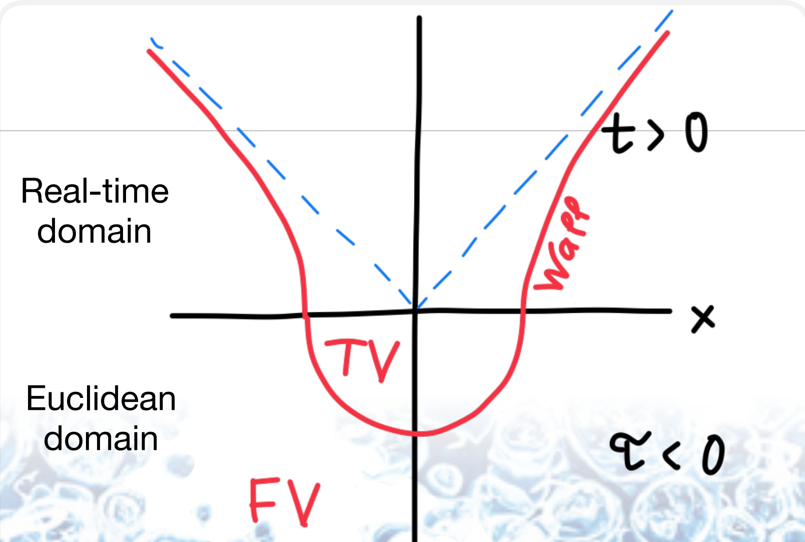
small coupling constant justifying WKB; in QM it is $\hbar p'/p^2$

in the configuration space



Tunnelling from the ground state

The bounce is found by solving the (classical) Klein-Gordon equation $(\partial_\tau^2 + \partial_i\partial^i)\phi - V'(\phi) = 0$ in the "imaginary" time.



Bubble of true vacuum

- Again, one needs to know the boundary conditions.

  Assuming spherical symmetry of the bubble,

$$\phi_b(r \to \infty) = 0\,, \quad r = \sqrt{\tau^2 + \vec{x}^2}$$

So one can solve the KG equation instead of the (infinite-dimensional) Schrödinger equation!

Coleman 77; Callan, Coleman 77

Price to pay…

Decay of metastable state

Consider the scalar field theory with the Lagrangian $L = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$ and the "tunnelling" potential.

in the configuration space

Consider tunnelling from a **thermally excited** metastable state, with temperature T.

As usual, $\Gamma \sim e^{-B}$, $B = S_E[\phi_b]$, where ϕ_b is the **thermal bounce**.

The temperature should not be too high — Boltzmann suppression

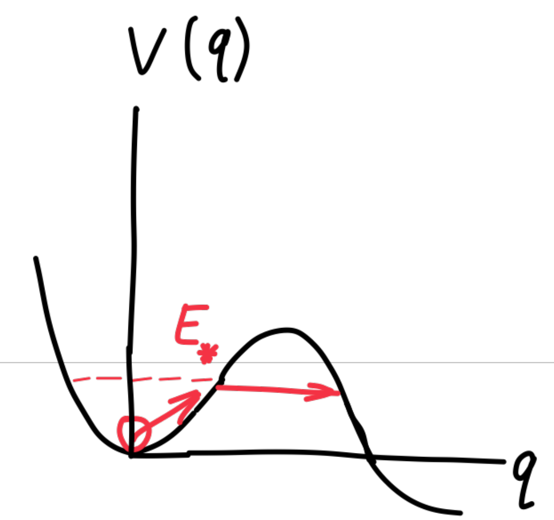
- Boundary conditions for the thermal bounce? Thermal partition function implies periodicity in the imaginary time.

Linde 82
Brown, Weinberg 07

bounce solution corresponding to tunnelling from the state with energy E

$$\Gamma \sim \int dE\, e^{-\frac{E}{T}} e^{-S_E[q_{b,E}]} \sim e^{-\frac{E_*}{T} - S_E[q_{b,E_*}]}$$

$$\longrightarrow \quad q_b(\tau + 1/T) = q_b(\tau)$$



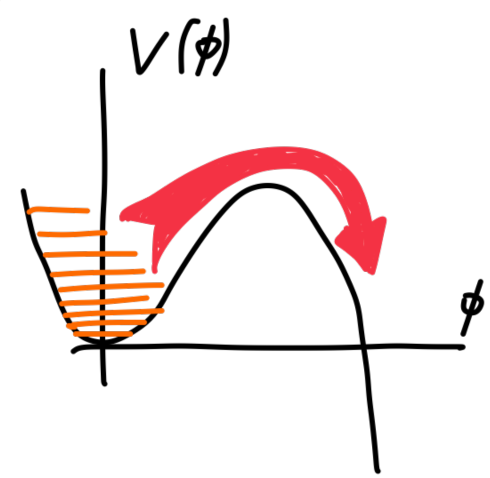
Tunnelling from an excited state

Decay of metastable state

Consider the scalar field theory with the Lagrangian $L = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$ and the "tunnelling" potential.

in the configuration space

At sufficiently high *T*, the decay occurs classically, through the formation of special thermodynamic fluctuation: **critical bubble**.

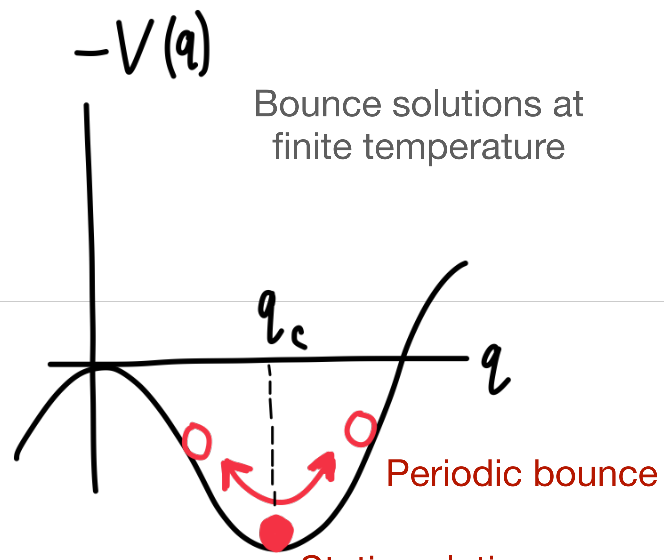


Decay in the classical regime

- The critical bubble is described by the static solution of the equations of motion.

the energy of the critical bubble — barrier energy

$$\Gamma \sim e^{-B}\,,\quad B = \frac{E_b}{T}$$ — Boltzmann suppression



Decay of metastable state

The typical picture of false vacuum decay at finite temperature is as follows.

Periodic thermal bounces dominate at low T ⇔ quantum tunnelling

Static critical bubble dominates at large T ⇔ classical thermal jumps

$$\Gamma = A(T)e^{-B(T)}$$

exp. suppression — “LO”

prefactor — “NLO”

Affleck 80

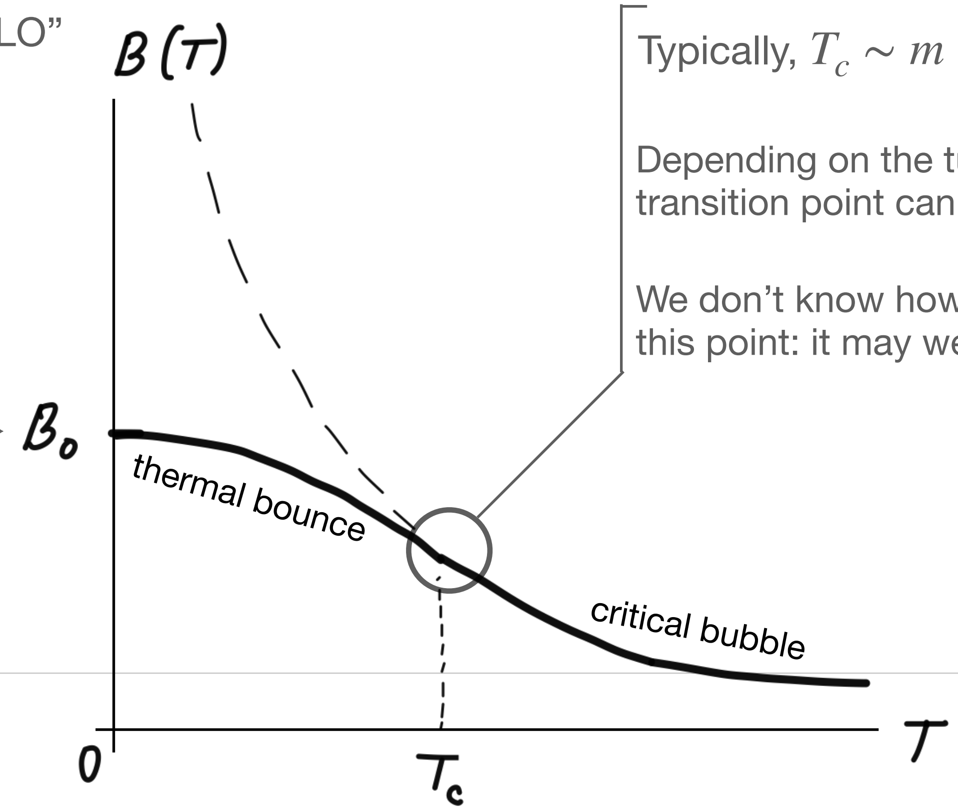
Chudnovsky 92

Typically, $T_c \sim m \equiv \hbar\omega_m$

Depending on the tunnelling potential, the transition point can be smooth or only continuous.

We don’t know how the **prefactor** behaves around this point: it may well be discontinuous.

vacuum suppression →



Exponential suppression of vacuum decay as a function of temperature

Going out of equilibrium

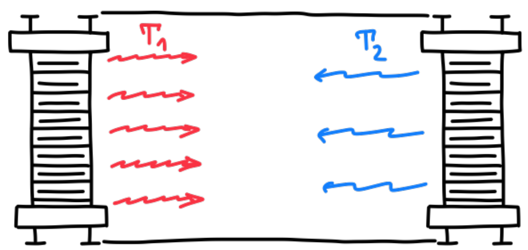
- The Euclidean formalism is, in general, not applicable to the decay of non-equilibrium initial states.

  In particular, it does not determine vacuum boundary conditions for the semiclassical solution.

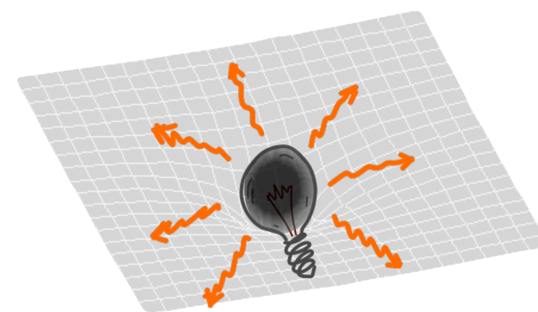
  These can be found in a more general “in-in” formalism. **2105.09331**

  In many physical systems the initial state is not in thermal equilibrium.

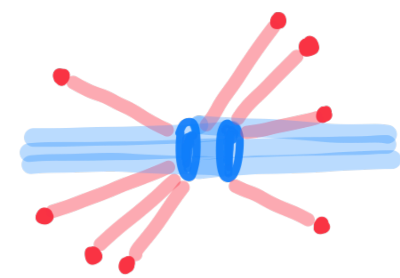
Anisotropic flux of radiation



Multicomponent radiation



A black hole in the **Unruh** vacuum
2105.09331, 2111.08017



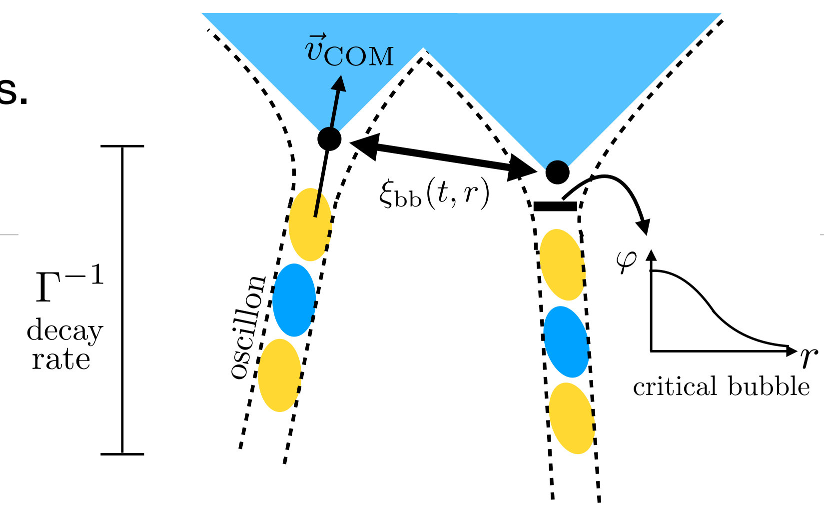
Particle collisions
Kuznetsov, Tinyakov 97
Levkov, Sibiryakov 05
Demidov, Levkov 15

- The Euclidean formalism does not capture real-time dynamics of vacuum decay.

  This dynamics contains many interesting features that may be relevant for observations.

Gleiser, Kolb et al, hep-ph/0409179, 0708.3844
Pirvu, Johnson, Sibiryakov 23

  This may be important for cosmological first order phase transitions or in table-top experiments.



from 2312.13364

- Actually, vacuum decay is, by definition, an out-of-equilibrium process!

  The validity of the Euclidean formalism should not be taken as granted.

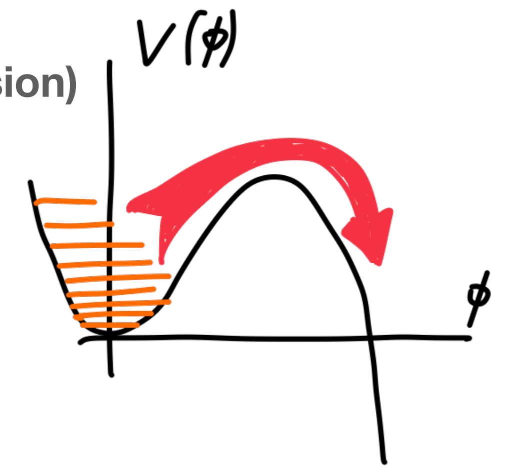
2407.06263, 2408.06411

Methods 2

Consider vacuum decay at finite temperature via **classical thermal** jumps of the field over the barrier.

i.e. at temperatures high (classical regime) but not too high (exponential — Boltzmann — suppression)

The decay happens through the formation of special thermodynamic fluctuation: **critical bubble**.



- **Real-time, classical, lattice simulations**

  They are applicable if occupation numbers of all relevant for the decay modes are big.

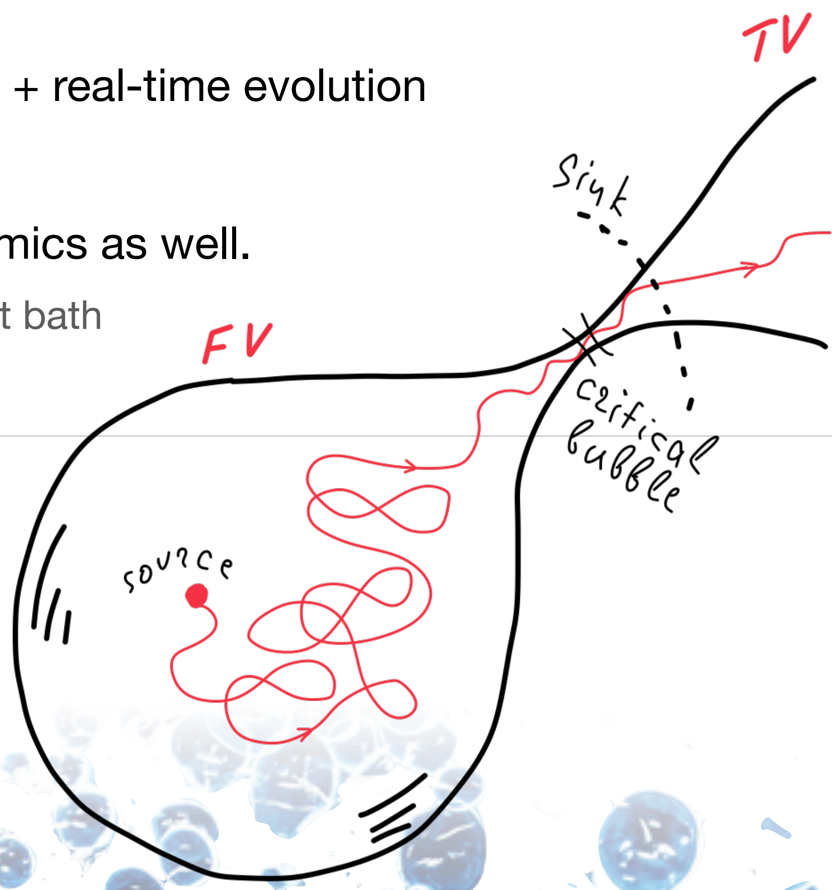
  Milestones:

  - Grigoriev, Rubakov, Shaposhnikov — Sphaleron transitions, kink-antikink pair production, Hamiltonian dynamics
  - Alford, Feldman, Gleiser — Vacuum decay, Langevin dynamics
  - Gould, Moore, Rummukainen — Vacuum decay, “multi-canonical sampling” + real-time evolution

First, we focus on the Hamiltonian evolution of a single field; later, on the Langevin dynamics as well.

i.e. no external heat bath

i.e. with external heat bath

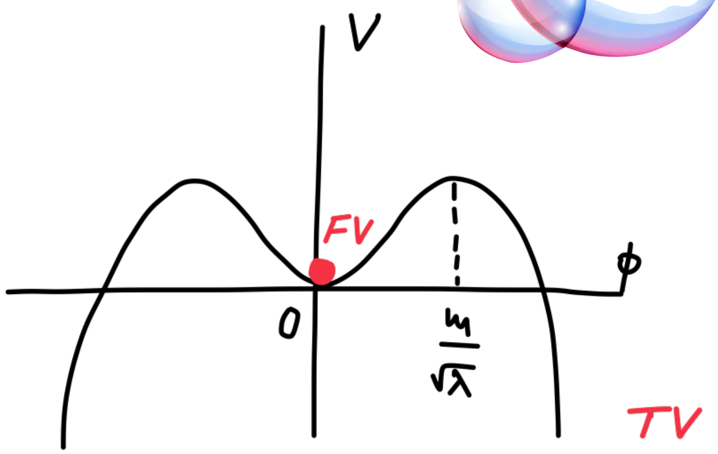


The goal is to test the predictions of the Euclidean theory and to see if anything interesting happens before and during the critical bubble nucleation.

Pîrvu, AS, Sibiryakov 2407.06263, 2408.06411

Simplest Setup

Scalar field theory in 1+1 dimensions: $S = \int \mathrm{d}t\, \mathrm{d}x \left(-\frac{(\partial_\mu \phi)^2}{2} - \frac{m^2\phi^2}{2} + \frac{\lambda\phi^4}{4} \right)$



- **Euclidean theory predicts:** $E_b = \frac{4m^3}{3\lambda}$, $\Gamma_E = \frac{6m^2}{\pi}\sqrt{\frac{E_b}{2\pi T}}\,\mathrm{e}^{-E_b/T}$

  barrier (critical bubble) energy

- **We want to measure the decay rate (among other things) in “first-principle” classical lattice simulations**

We prepare a suite of simulations with the initial thermal Rayleigh-Jeans spectrum:

Fourier modes of the field and momentum

$$\langle |\tilde{\phi}_j|^2 \rangle = T/\Omega_j^2 \,, \quad \langle |\tilde{\pi}_j|^2 \rangle = T$$

$$\Omega_j^2 = 2(1 - \cos k_j a)/a^2 + m_{th}^2 \,, \quad k_j = 2\pi j/L$$

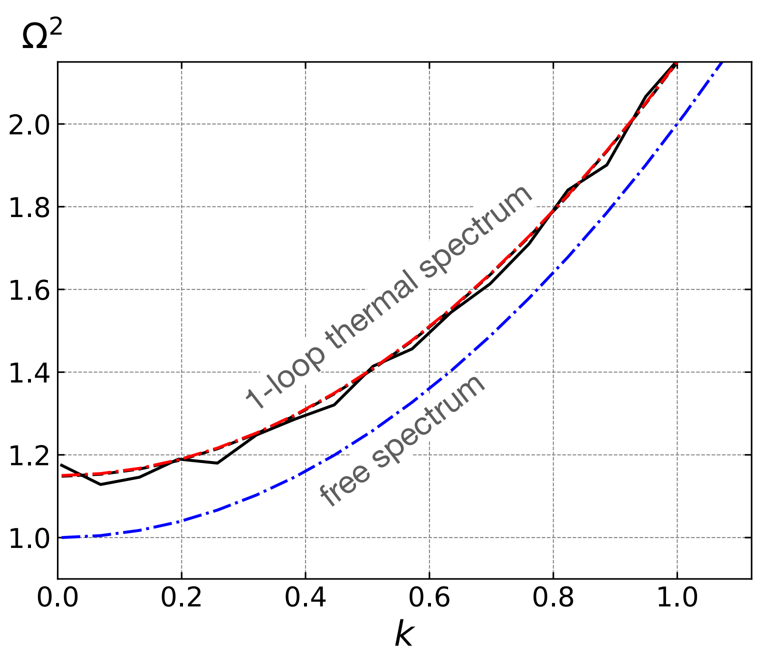
lattice spacing

box size

$$m_{th}^2 = m^2 - \boxed{\frac{3\lambda T}{2m}}$$

thermal correction to the mass, $\ll m^2$

…and evolve them until they decay (or simulation times out).



Measurement of the dispersion relation in the theory with the stable potential

We checked that this is an equilibrium state by evolving the theory with the stable potential using the Langevin equation.

What does it mean "decay rate"?

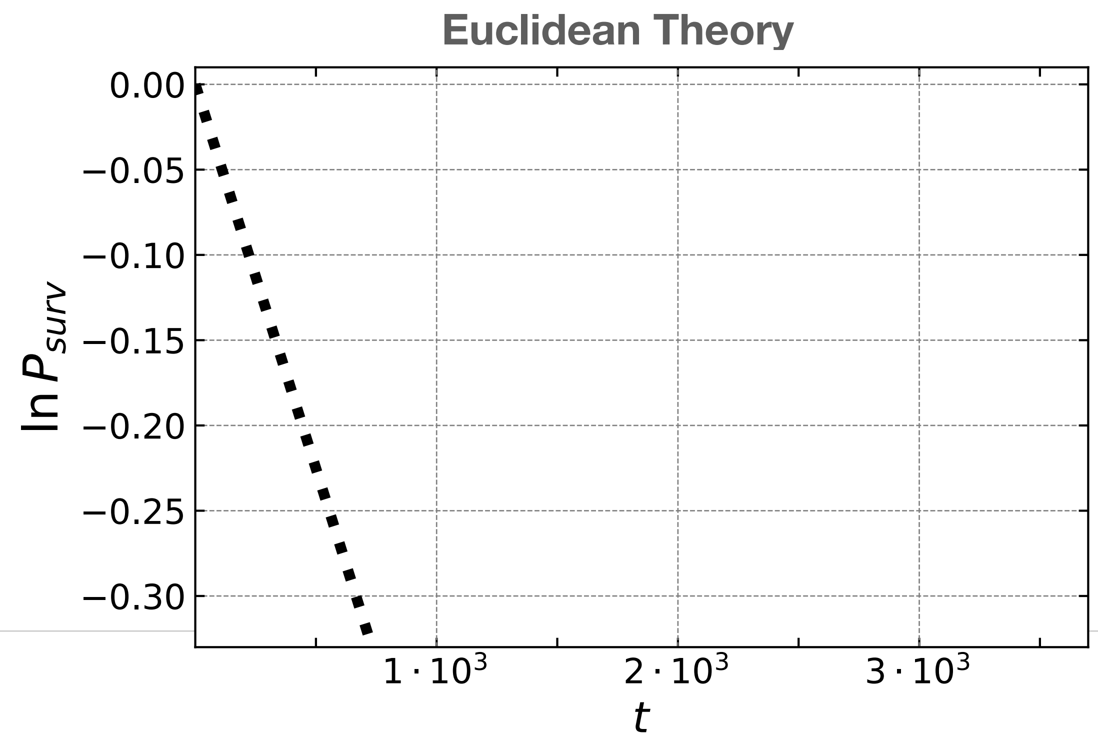
- Introduce **survival probability** $P_{surv}(t)$

  For decays obeying the exponential distribution, it follows the law:

  (we exclude early-time transients)

$$\ln P_{surv}(t) = \text{const} - \Gamma L \cdot t$$

This is decay rate



First surprise

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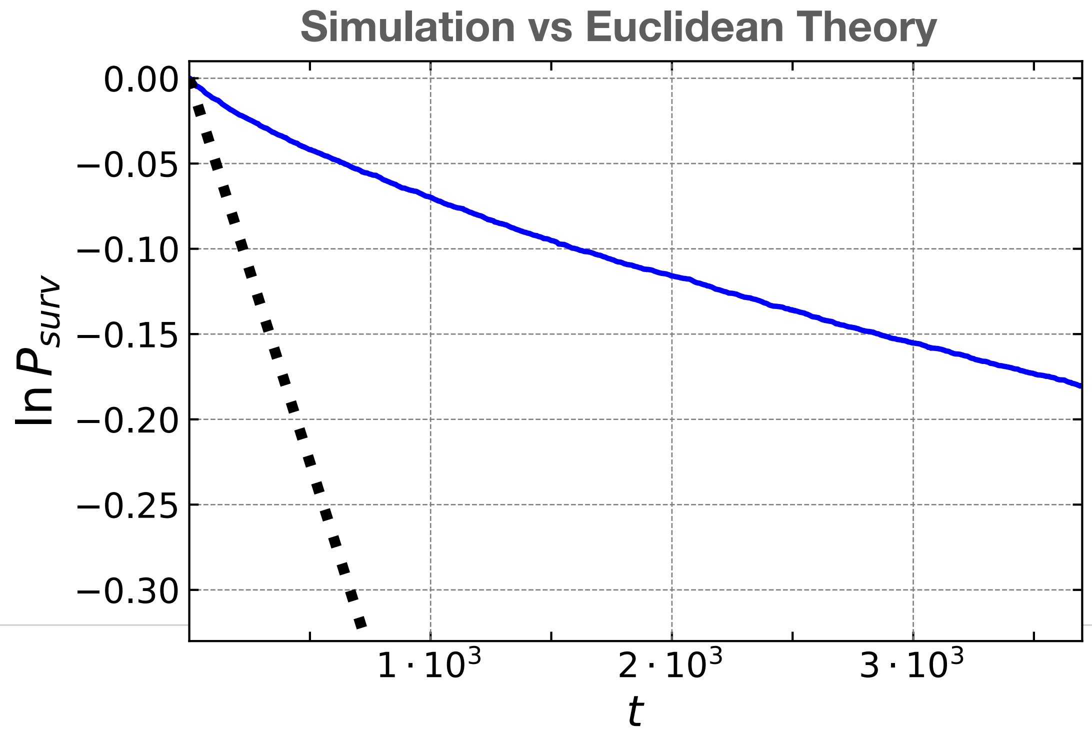
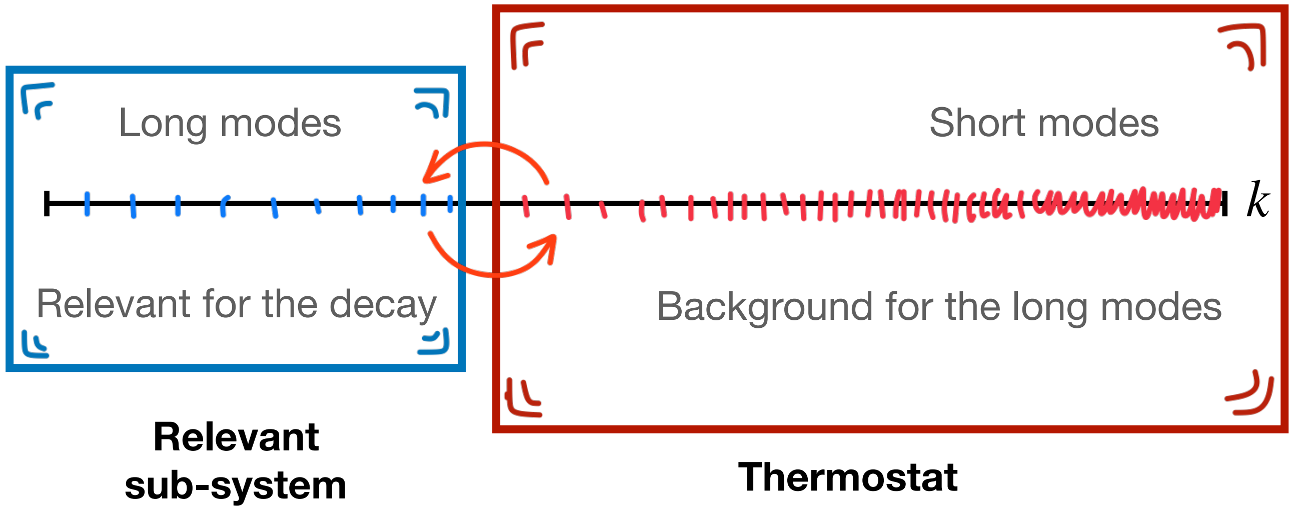


- Decay rate found in simulations is smaller than the Euclidean prediction
- It is, moreover, time-dependent, getting even smaller with time

What does it mean "thermal"?

- For the **Hamiltonian** evolution, it means the following:



But thermalisation in the theory is very **inefficient**: for modes with $\omega \sim m \sim$ (bubble size)$^{-1}$, the thermalisation time is

$$t_{th} \sim \frac{(2\pi)^3}{\tilde{T}^4}, \qquad \tilde{T} = \frac{\lambda T}{m^3} \ll 1$$

(due to $2 \to 4$ and $3 \to 3$ scattering processes)

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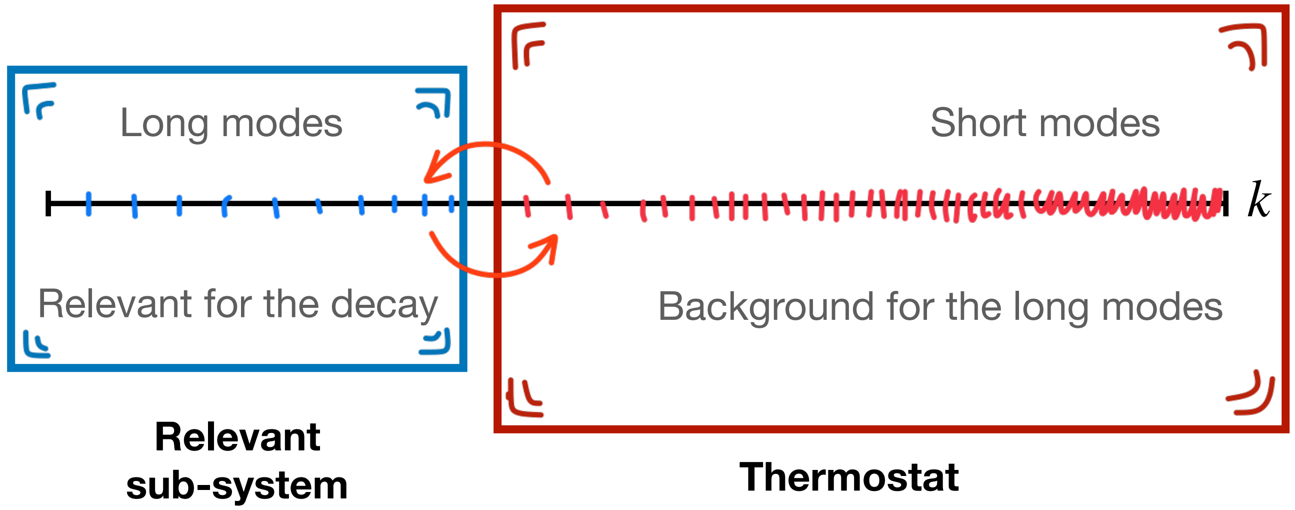

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- Compare this with the decay time: $t_{dec} \sim (\Gamma L)^{-1}$

In our simulations it happens that $t_{th} > t_{dec}$ (hardly relevant for cosmology, but can be relevant for experiments)

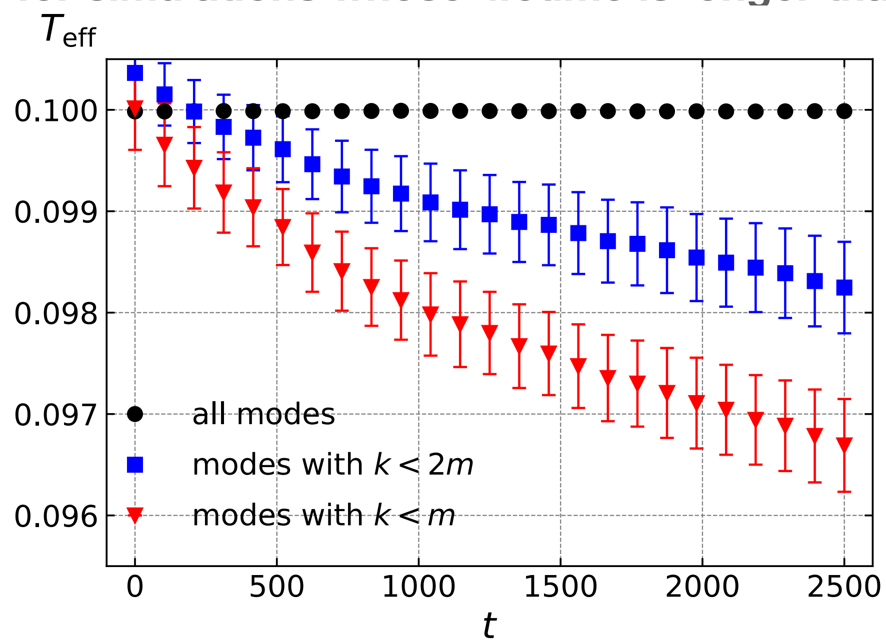
This leads to the interesting effect.

Classical Zeno effect

- Because of inefficient thermalisation, the initial power contained in the long modes is preserved during the simulation.
- The configuration which, due to a statistical fluctuation, has a higher initial long-mode power decays faster. The one with lower power lives longer.

→ Statistical properties of the ensemble change with time: long modes cool down.

Effective temp. of long modes for simulations whose lifetime is longer than *t*



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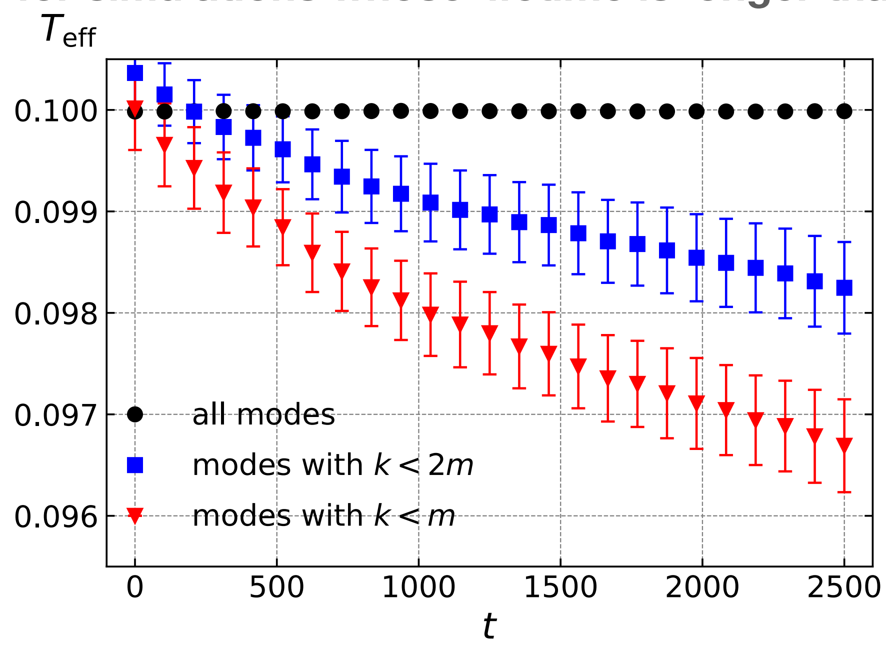


- Effective temp. of long modes drops by a few per cent during the run: enough to visibly suppress the decays.

Decay is a non-Markovian process (in this regime).

The longer we observe the system, the less chance it has to decay in the future: classical Zeno effect.

To find the unbiased rate, we extrapolate the slope of the survival probability curve to zero.

Second surprise

- We measure the (unbiased) decay rate at different temps. and fit with the formula (recall that $\Gamma_E = A_E \exp(-E_b/T)$)

$$\ln\Gamma(T) = -\frac{1}{2}\ln T + \ln A - \frac{B}{T}$$

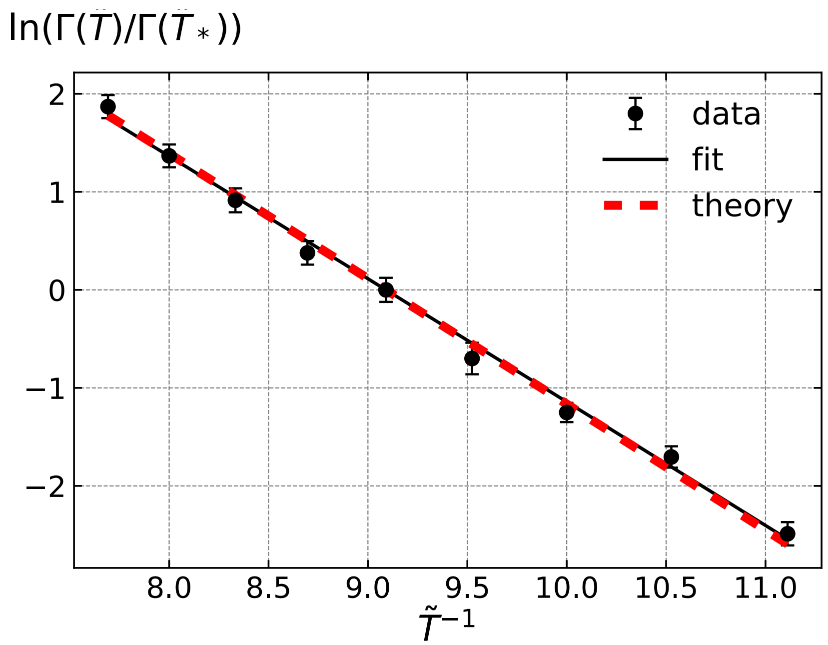
critical bubble energy

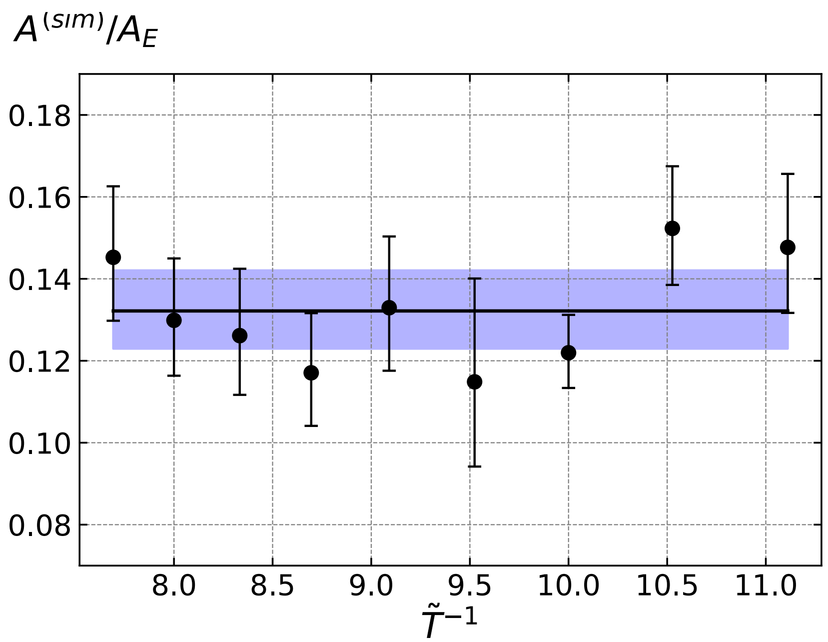
prefactor (with the zero mode excluded)

from the zero mode in the prefactor

One can measure *A* and *B* separately, using the ratio $\Gamma(T)/\Gamma(T_*)$ to find *B,* with some reference temp. T_*

Or one can make the 2-parameter fit, the result is the same (within the errorbars).





- Critical bubble energy agrees with the Euclidean theory (<2% error bar)
- The measured prefactor is smaller by a factor ~8.

? Something wrong with thermalisation again? Violation of thermal equilibrium near the critical bubble?

More evidence: Langevin evolution

We can reduce artificially the thermalisation time by coupling the system to an external heat bath.

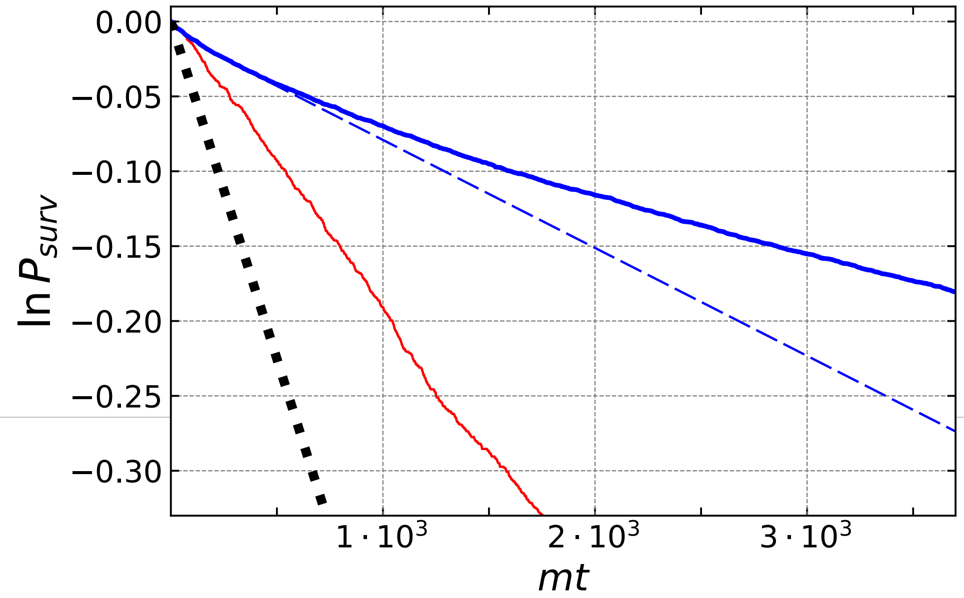
$$\begin{cases} \ddot{\phi} + \eta\dot{\phi} - \phi'' + m^2\phi - \lambda\phi^3 = \xi \\ \langle \xi(t,x) \rangle = 0 \ , \ \ \langle \xi(t,x)\xi(t',x') \rangle = 2\eta T \delta(t-t')\delta(x-x') \end{cases}$$

$\rightarrow \quad t_{th} \sim \eta^{-1}$

Noise and dissipation change the dynamics of vacuum decay.

They don't change the critical bubble.

Simulation at zero vs non-zero noise ($\eta = 10^{-2}m$)

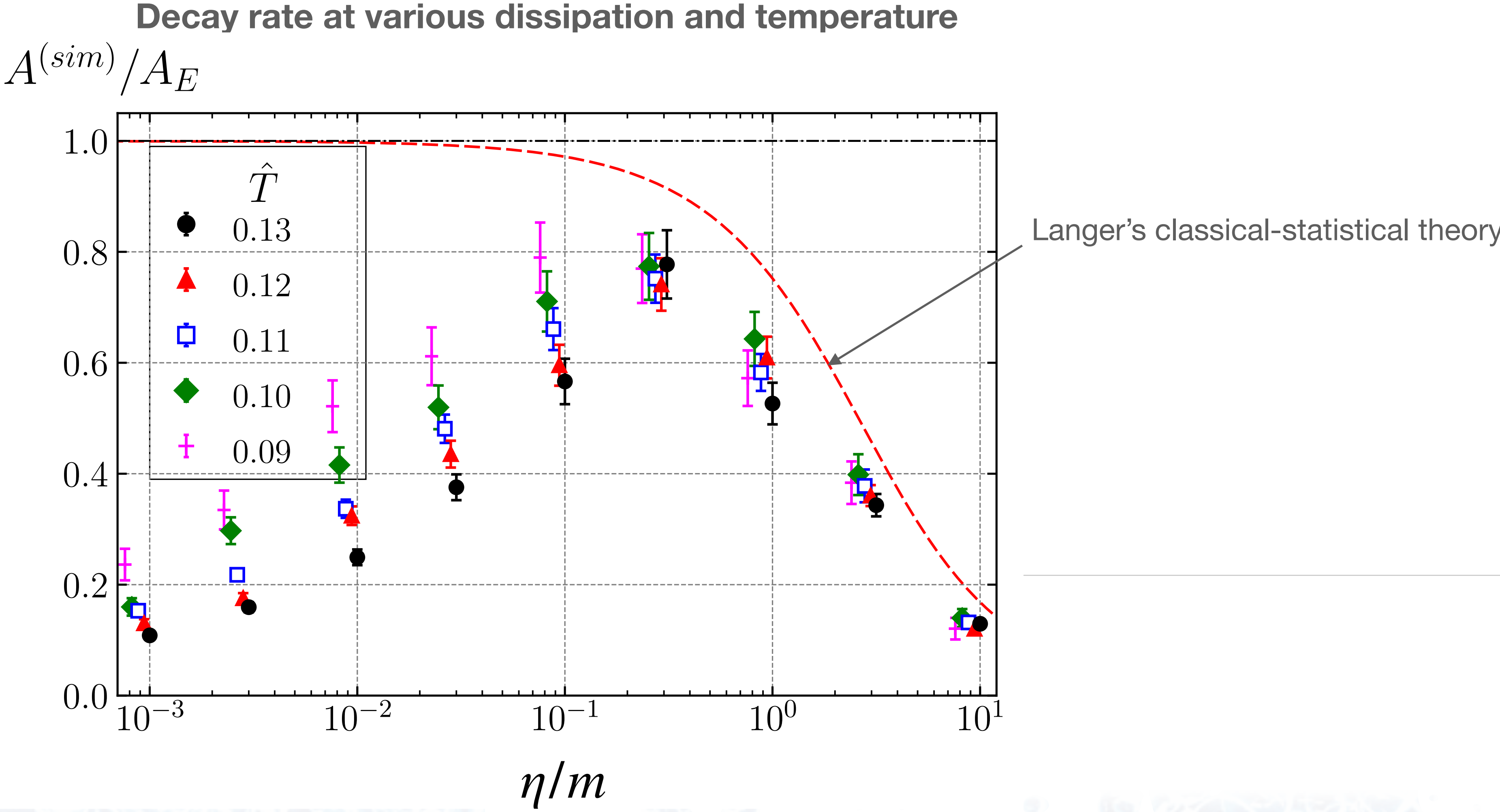


- No Zeno effect as long as $\eta \gtrsim \Gamma L$
- Decay rate increases, but still below the Euclidean bound

Langevin dynamics: decay rate

We observe the following behavior:

- As dissipation increases, Γ increases as well. It reaches maximum at $\eta \simeq 3 \cdot 10^{-1} m$, then starts decreasing due to over-damping.
- Γ tends to increase when T goes down.



Violation of equilibrium condition

In Physical Chemistry, the analog of Euclidean Theory is Transition State Theory (TST).

for review see **Hanggi, Talkner, Borkovec, Rev.Mod.Phys. 62 (1990)**

TST deals with particles (one or few d.o.f.) in the external heat bath, $\eta > 0$.

It is known that TST is violated if there is no equilibrium around the barrier.
The following condition must be satisfied:

$$\eta \gg \frac{\omega\_T}{E_b}$$

We can generalise this condition to **Langevin** dynamics of field theory.

This is done by careful examination of Langer's work.

For the **Hamiltonian** dynamics of field theory, we suggest the following condition:

All our current and future results are consistent with it.

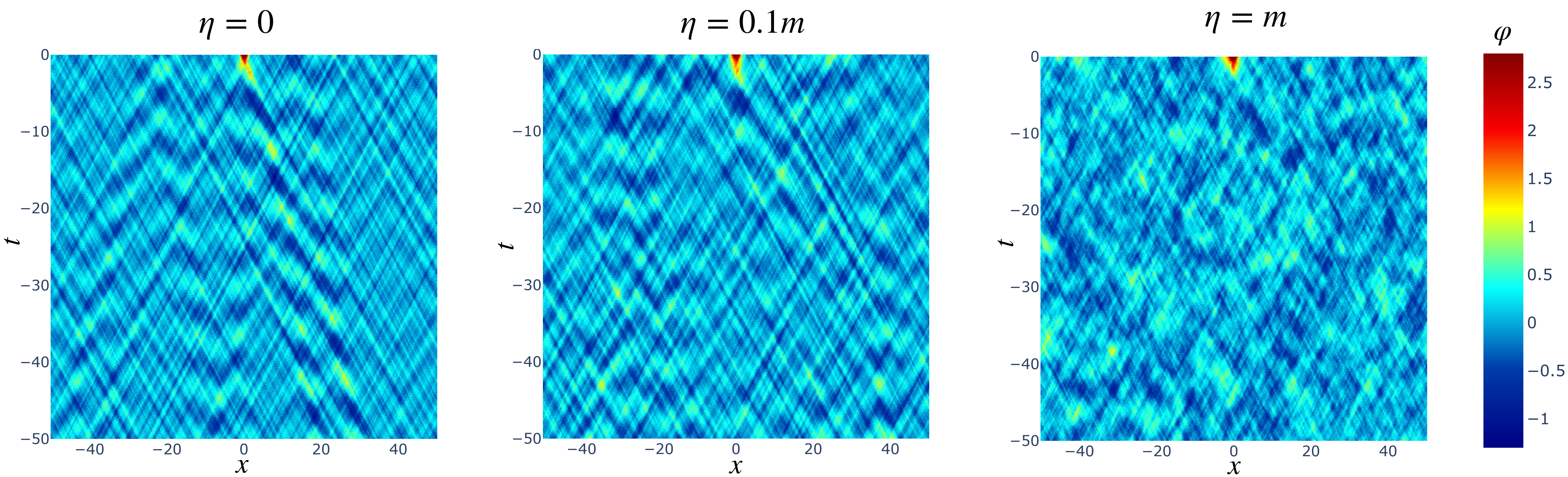
Effective free energy of the critical bubble

$$t_{th} \lesssim \frac{\mathcal{F}_b}{\omega\_T}$$

- It is generally violated for weakly-coupled theories with one coupling (one field)
- In theories with many fields, it must be examined on a case-by-case basis.

Dynamics of vacuum decay

When equilibrium is violated, interesting features appear in the field evolution prior to the decay.



At small dissipation, we observe a population of nonlinear waves with $\omega < m$ — **oscillons**.

They disappear when $\eta > 0.1m$ and the system evolves due to the stochastic terms.

Dynamics of vacuum decay

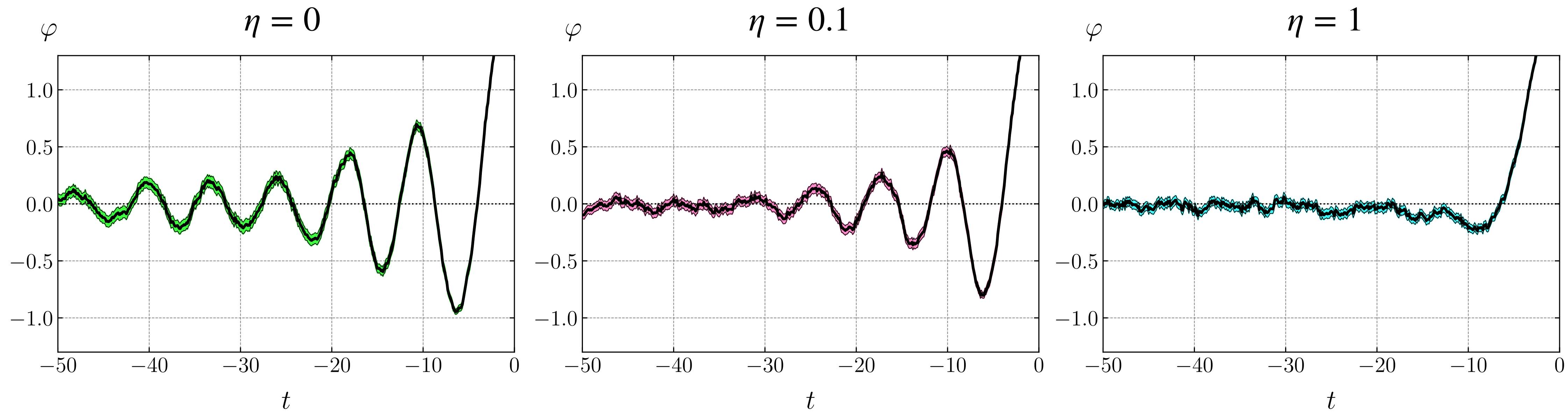
In the Hamiltonian dynamics, every critical bubble is preceded by an oscillon.

Johnson, Pîrvu, Sibiryakov, 2312.13364

We can track its trajectory.

Thanks to Dalila's smart numerical routine.

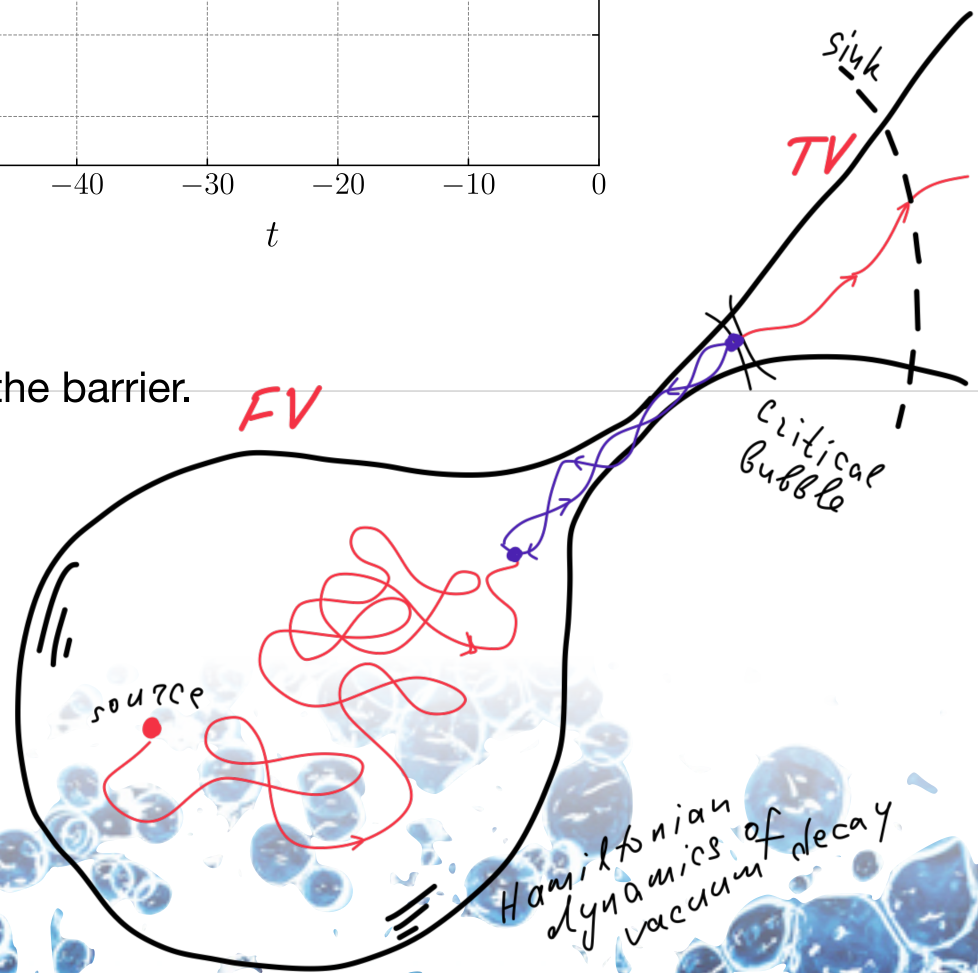
Stacking many oscillons together, we get the average **oscillonic precursor** to the critical bubble:



In our system, the presence of oscillons indicate violation of thermal equilibrium near the barrier.

Thus, they are correlated with the diminishing decay rate.

But how deep is this correlation?..



Discussion

- How general are these results?

  In 3+1 the thermalization is faster, but still no equilibrium in the Hamiltonian dynamics of a single field.

  Adding more species does **not** automatically improve the thermalization condition.

  Adding gauge fields? We don't know yet.

- How important are these results e.g. in cosmology?

  If you need an accurate prediction of the decay rate or for the effects pertaining to the dynamics of bubble nucleation — these results are important.

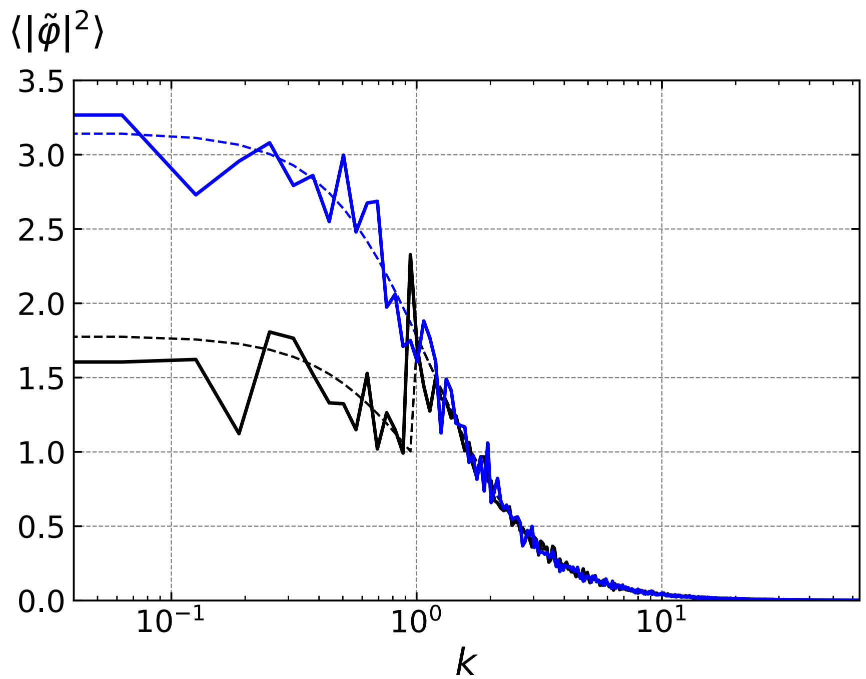
  If you need an order of magnitude estimate or parametric dependence — these results are (likely) not important.

  Our results are not **directly** applicable to sphaleron transitions or e.g. production and collision of kinks. But we're looking into this now.

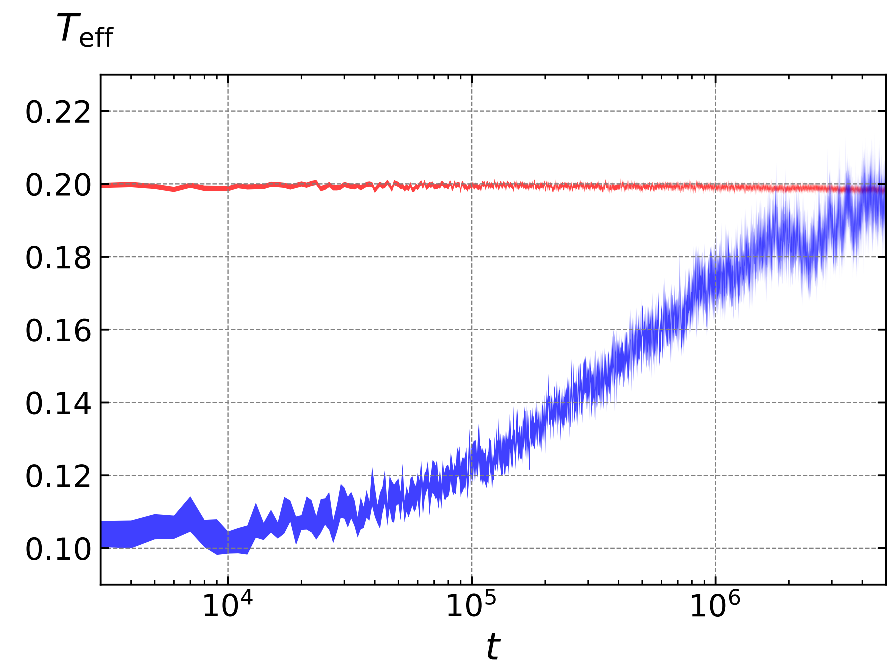
backup slides

Thermalisation time

We perform the numerical experiment estimating the thermalisation time of long modes in the Hamiltonian system.



Initial and final spectra

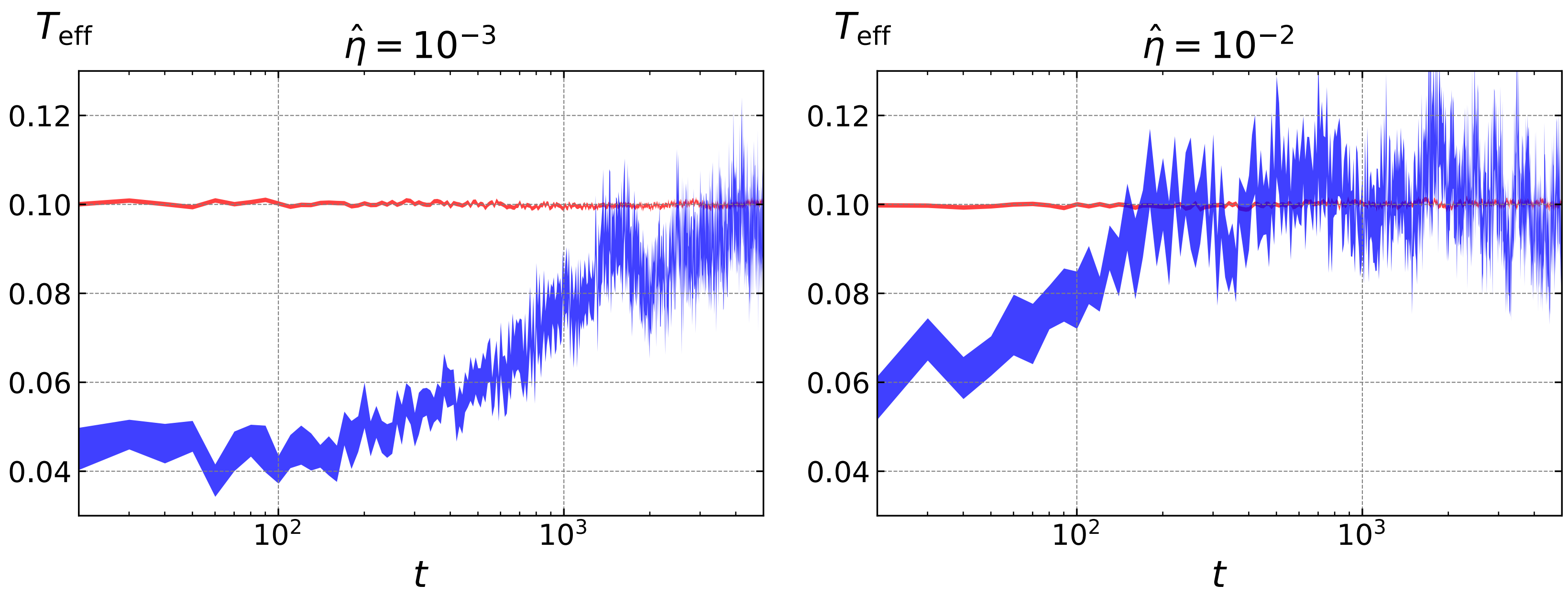


Effective temp. of long modes ($k < m$, blue) vs temp. of all modes (red)

The result agrees with the theoretical estimate $t_{th} \sim \dfrac{(2\pi)^3}{\tilde{T}^4}$

Thermalisation with external heat bath

We perform the numerical experiment estimating the thermalisation time of long modes with the Langevin evolution.



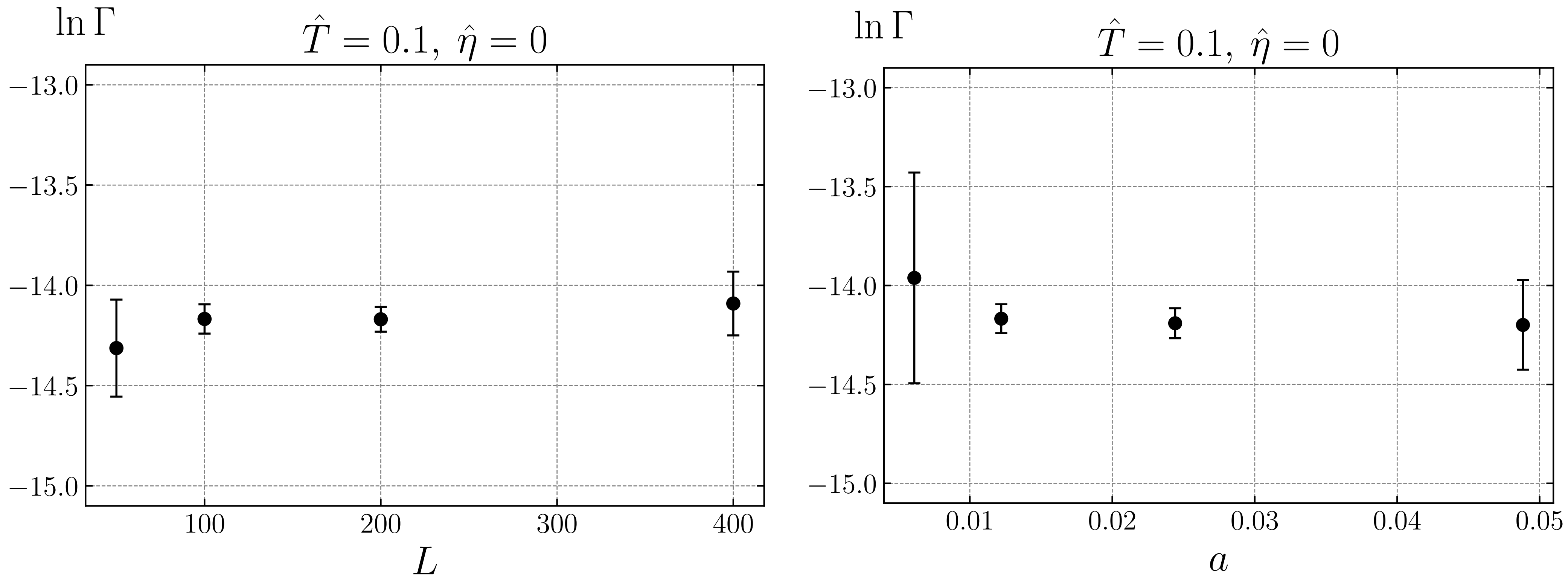
Effective temperature of long modes ($k < m$, blue) and the temperature of the ensemble ($k > m$, red)

The result agrees with the estimate $t_{th} \sim \eta^{-1}$.

Box size and lattice spacing

In simulations, we take $L = 100$ and $a \simeq 0.01$ (in units of mass).

The plots below demonstrate insensitivity of the decay rate to L and a.



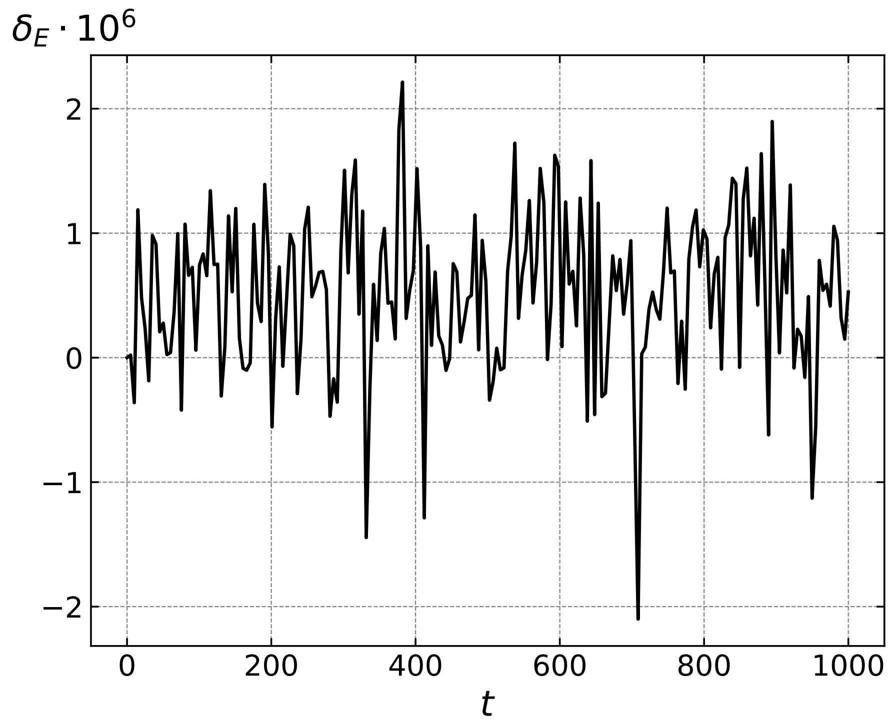
We use this to put the upper bound on the systematic error of the decay rate measurement.

Accuracy of numerical scheme

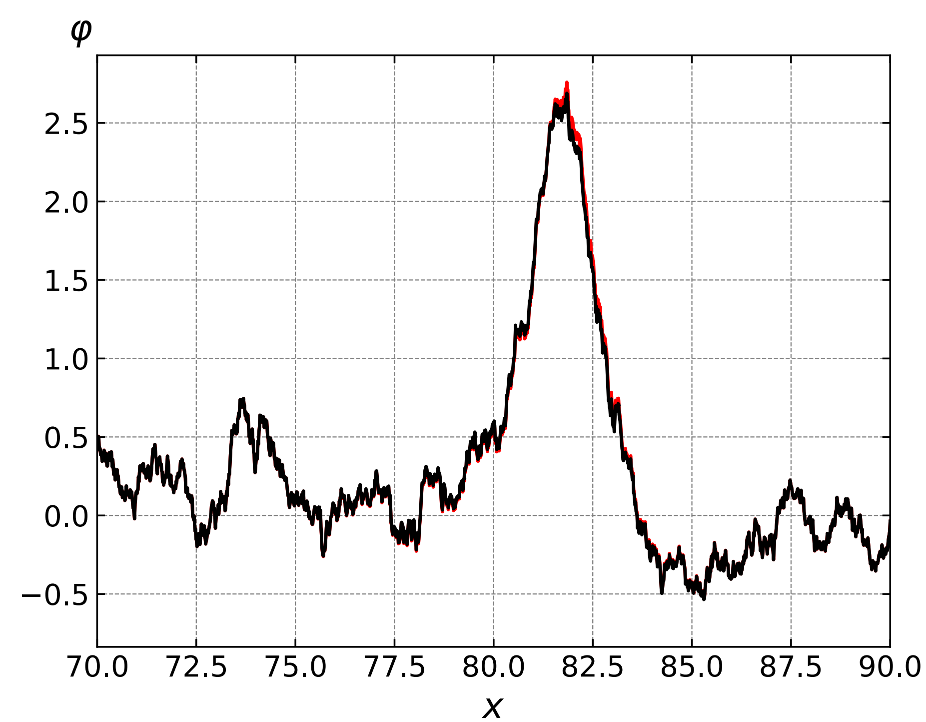
Hamiltonian dynamics

We use the 4th order pseudo-spectral, operator-splitting scheme.

The plots below show that it is enough to take $h/a \simeq 0.8$ to achieve the relative energy non-conservation $\lesssim 10^{-6}$.



Relative energy variation



Two decaying configurations evolved from the same initial state, with $h/a = 0.4, 0.8$.

Accuracy of numerical scheme

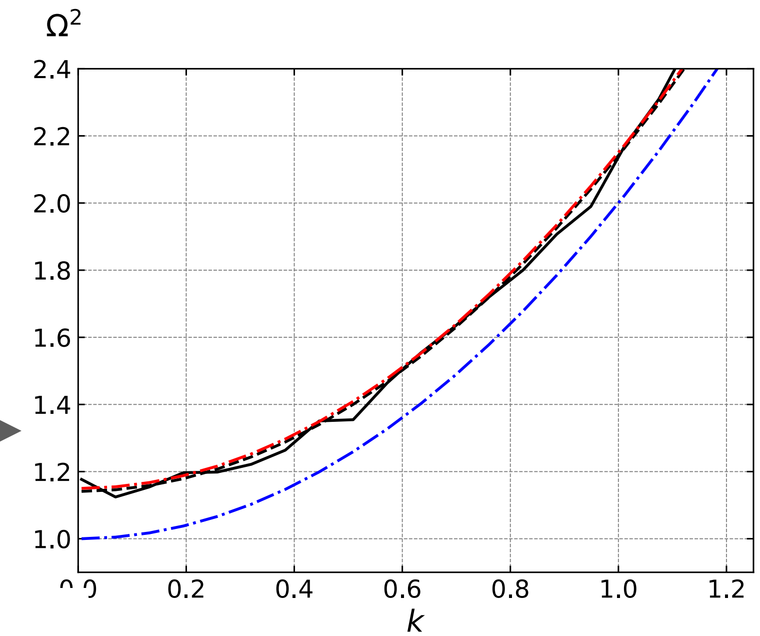
Langevin dynamics

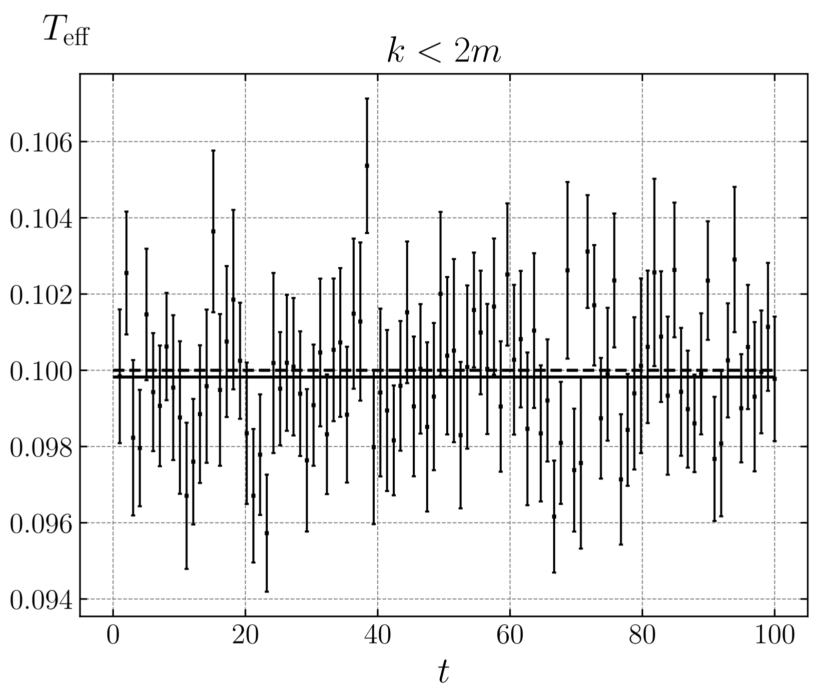
We use the 3rd strong order pseudo-spectral, operator-splitting scheme.

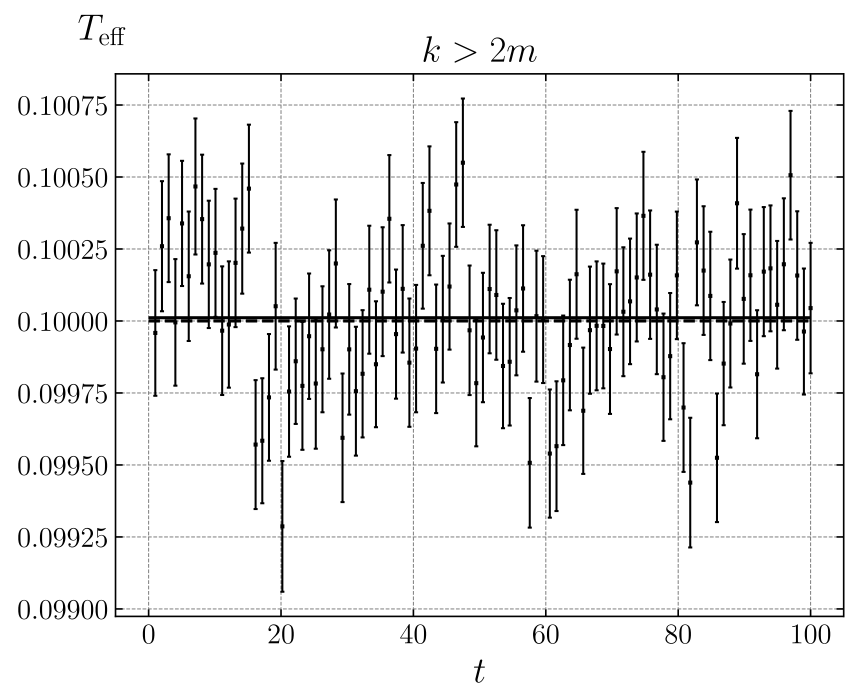
We took it from [Telatovich, Li, 1706.04237] but corrected their mistake.

The timestep is $h/a \simeq 0.25$ at $\eta \lesssim 1$ and $h/a \simeq 0.1$ at $\eta > 1$.

Dispersion relation measured in simulations (black), compared with the free (blue) and thermally-corrected (red) ones.







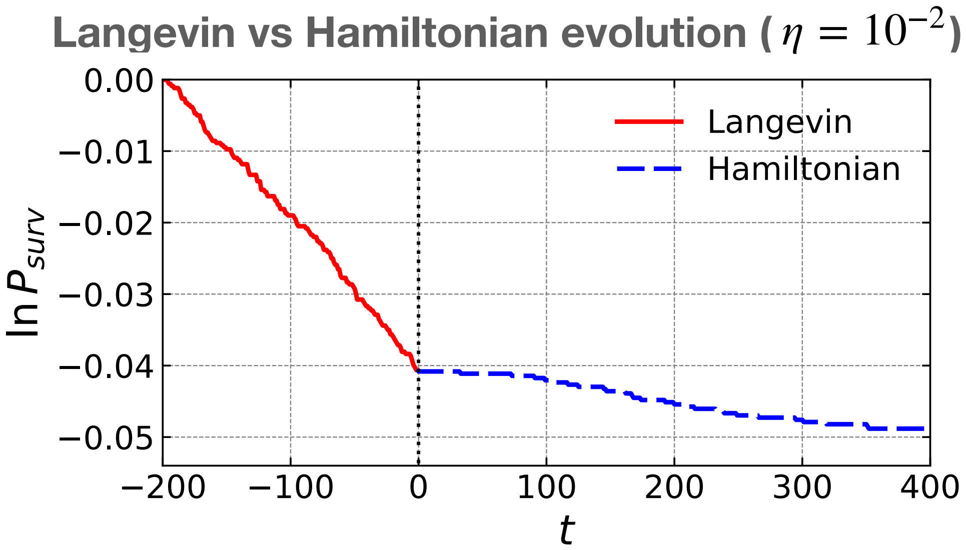
Effective temp. of long and short modes measured during the simulation.

Langevin vs Hamiltonian evolution

Let's make the following numerical experiment.

- Evolve the ensemble with non-zero η for $t \gg \eta^{-1}$ so that all surviving configurations reach equilibrium with the heat bath.

- Decouple the ensemble from the heat bath by setting $\eta = 0$.



The decay rate changes abruptly to the one that we got before for the Hamiltonian evolution.

Thus, the deviation of the rate from equilibrium is really due to the field dynamics near the barrier.

More observables

- **Shape of the critical bubble** $\phi_b(x)$

  Should we compute the bubble using the bare potential or an effective potential?

  If effective, which fields to include and when?

- **Dynamics of bubble nucleation**

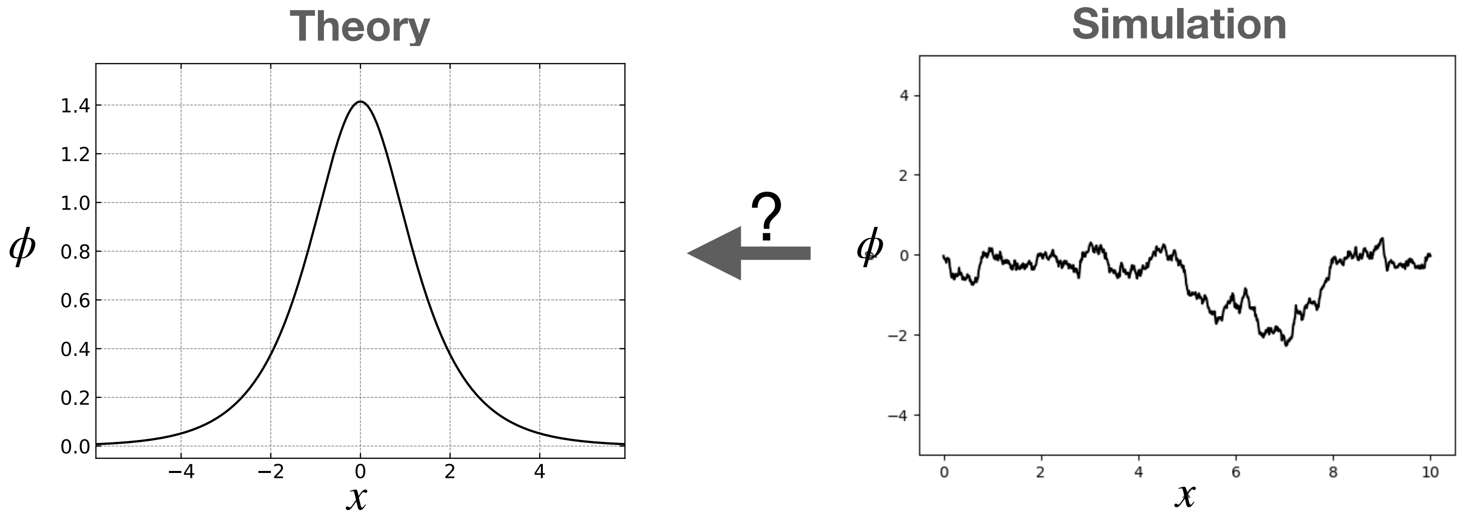
  Euclidean theory tells us little about how the critical bubble actually forms out of thermal fluctuations.

  This dynamics is quite interesting: bubble velocities, oscillonic precursors…

  Gleiser, Kolb… hep-ph/0409179, 0708.3844

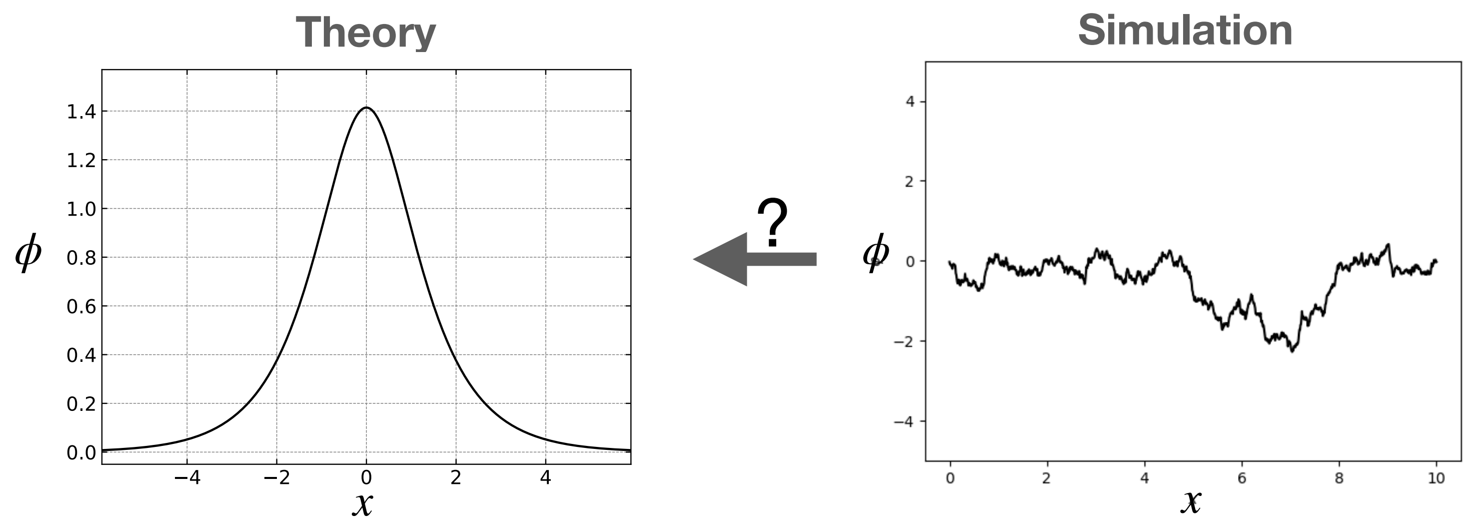
  Johnson, Pîrvu, Sibiryakov, 2312.13364

Critical bubble profile



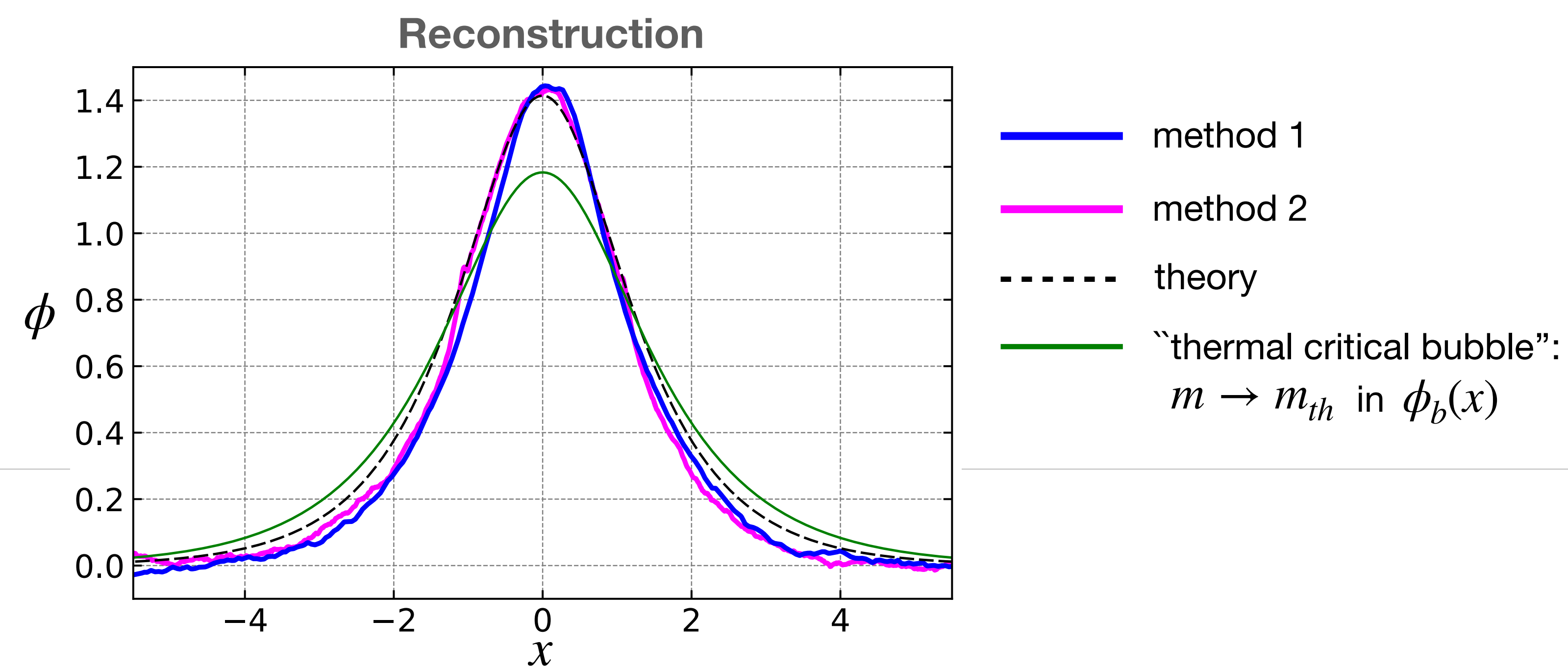
Take many simulations, **synchronise** them in space and time, produce the average.

Critical bubble profile



Take many simulations, **synchronise** them in space and time, produce the average, pinpoint the critical bubble.

We employ two different reconstruction routines. They agree with each other and with the Euclidean prediction.



No surprise here: the critical bubble is determined by the bare potential; fluctuations contribute to the prefactor.

Things can be different with many fields!