False vacuum decay in and out of equilibrium

Andrey Shkerin

Perimeter Institute

Nikhef, Amsterdam

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Motivation

The decay of a metastable state (false vacuum) plays an important role in many branches of physics, including particle physics and cosmology.

First order phase transitions in the early universe

Colliding bubbles generate gravitational waves

see, e.g., **Caprini et al, 1910.13125**

EW baryogenesis

Bubble wall moving through plasma generates baryon number for review see **Bodeker, Buchmuller, 2009.07294**

Higgs vacuum metastability

In the present-day Universe, the decay probability is small enough. But this can change in extreme environments.

for reviews see **Espinosa et al, 1505.04825 Andreassen et al, 1604.06090**

Experimental tests of nucleation theory

Zenesini et al, [Nature Physics](https://www.nature.com/nphys) 20, 558–563 (2024) — first experimental result

³ **Methods 1**

 $V(\phi)$

Studies of decay of metastable state have more than a century long history. We are interested in the developments in the context of high-energy physics, field theory.

Euclidean — equilibrium — approach

It applies to systems whose metastable state is in **local equilibrium** — it can be assigned a temperature.

Milestones:

Consider the quantum-mechanical system with the Hamiltonian $H = \dfrac{p^2}{2} + V(q)$ and the "tunnelling" potential. 2*m* + *V*(*q*)

∫

0

a

 $P_{\textit{surv}} = 1 - e^{-\Gamma t}$ $-$ survival probability in the metastable state

at times not very short and not very long

 $\Gamma \sim e^{-B}$ $-$ decay rate in the WKB approximation

it shows the main exponential suppression; we will talk about the prefactor later

 $B=2$ \mid $\ d q \sqrt{2mV(q)}$ \mid \mid suppression exponent

Interestingly, $B = S_E[q_b]$, where q_b is the **bounce** trajectory.

$$
S_E = \int d\tau \left[\frac{m}{2} \left(\frac{dq}{d\tau} \right)^2 + V(q) \right] \qquad - \text{Euclidean action as}
$$

 $m\frac{d^{2}q}{dt^{2}} = -\frac{d^{2}q^{2}}{dt^{2}}$ – **classical** equation of motion in the "imaginary" time τ

 $dq\sqrt{2mV(q)}$

 $q_b(\pm \infty) = 0$, $\dot{q}_b(0) = 0$

 $=-\frac{\partial(-V)}{\partial}$

∂*q*

 d^2q

*dτ*²

— boundary conditions selecting the bounce they are uniquely associated with the false vacuum state

So one can solve Newton's equation instead of the Schrödinger equation! Price to pay: WKB approximation; vacuum boundary conditions. not relevant here, but for non-equilibrium states…

Consider the scalar field theory with the Lagrangian $L = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$ and the "tunnelling" potential. 1 2 $(\partial_\mu \phi)^2 - V(\phi)$ in the configuration space

Again, $P_{surv} = 1 - e^{-\Gamma t}$, where $\Gamma \sim e^{-B t}$ is the decay rate per unit space.

Again, $B = S_E[\phi_b]$, where S_E is the Euclidean action associated with L:

$$
S_E = \frac{1}{g^2} \int d\vec{x} d\tau \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi}{\partial x_i} \right)^2 + V(\phi) \right]
$$

small coupling constant justifying WKB; in QM it is $\hbar p'/p^2$

The bounce is found by solving the (classical) Klein-Gordon equation $(\partial_{\tau}^2 + \partial_i \partial^i)\phi - V'(\phi) = 0$ in the "imaginary" time.

Again, one needs to know the boundary conditions. Assuming spherical symmetry of the bubble,

$$
\phi_b(r \to \infty) = 0 \ , \ r = \sqrt{\tau^2 + \overrightarrow{x}^2}
$$

So one can solve the KG equation instead of the (infinitedimensional) Schrödinger equation!

Coleman 77; Callan, Coleman 77

Consider the scalar field theory with the Lagrangian $L = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$ and the "tunnelling" potential. 1 2 $(\partial_\mu \phi)^2 - V(\phi)$ in the configuration space

Consider tunnelling from a **thermally excited** metastable state, with temperature *T*.

As usual, $\Gamma \sim e^{-B}$, $B = S_E[\phi_b]$, where ϕ_b is the **thermal bounce**.

The temperature should not be too high — Boltzmann suppression

Boundary conditions for the thermal bounce? Thermal partition function implies periodicity in the imaginary time.

Brown, Weinberg 07

bounce solution corresponding to tunnelling from the state with energy *E*

$$
\Gamma \sim \int dE \, e^{-\frac{E}{T}} e^{-S_E[q_{b,E}]} \sim e^{-\frac{E_*}{T} - S_E[q_{b,E_*}]}
$$

$$
q_b(\tau + 1/T) = q_b(\tau)
$$

Consider the scalar field theory with the Lagrangian $L = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$ and the "tunnelling" potential. 1 2 $(\partial_\mu \phi)^2 - V(\phi)$ in the configuration space

At sufficiently high *T*, the decay occurs classically, through the formation of special thermodynamic fluctuation: **critical bubble**.

The critical bubble is described by the static solution of the equations of motion.

the energy of the critical bubble — barrier energy

 $\Gamma \sim e^{-B}$, $B=$ *Eb T* — Boltzmann suppression Decay in the classical

The typical picture of false vacuum decay at finite temperature is as follows.

Periodic thermal bounces dominate at low τ \Leftrightarrow quantum tunnelling Static critical bubble dominates at large τ \Leftrightarrow classical thermal jumps

Going out of equilibrium

The Euclidean formalism is, in general, not applicable to the decay of non-equilibrium initial states.

In particular, it does not determine vacuum boundary conditions for the semiclassical solution.

These can be found in a more general "in-in" formalism. **2105.09331**

In many physical systems the initial state is not in thermal equilibrium.

Multicomponent radiation

A black hole in the **Unruh** vacuum **2105.09331, 2111.08017**

Particle collisions **Kuznetsov, Tinyakov 97 Levkov, Sibiryakov 05 Demidov, Levkov 15**

 \mathcal{L}

The Euclidean formalism does not capture real-time dynamics of vacuum decay.

This dynamics contains many interesting features that may be relevant for observations.

Pirvu, Johnson, Sibiryakov 23 Gleiser, Kolb et al, hep-ph/0409179, 0708.3844

This may be important for cosmological first order phase transitions or in table-top experiments.

Actually, vacuum decay is, by definition, an out-of-equilibrium process!

The validity of the Euclidean formalism should not be taken as granted.

2407.06263, 2408.06411

 $F = \{x_i\}_{i=1}^n$, observables in vacuum decay. The basic observables in vacuum decay rate that decay rate the decay rate of \mathbb{R}^n and the critical bubble profile '(*r*), which can both be predicted from the instanton techniques and measured from bubbles for \mathcal{L} real-time simulations. Observables beyond the decay rate \mathcal{L} include the bubble-bubble-bubble-bubble-bubble-bubble-bubble-bubble-bubble-bubble-bubble-bubble-bubble-bubblecenter-of-mass velocity \mathcal{P} and oscillon precursors that we investigate investigate in \mathcal{P}

Methods 2

Consider vacuum decay at finite temperature via **classical thermal** jumps of the field over the barrier. **i.e. at temperatures high (classical regime) but not too high (exponential — Boltzmann — suppression)** The decay happens through the formation of special thermodynamic fluctuation: **critical bubble**.

Real-time, classical, lattice simulations

They are applicable if occupation numbers of all relevant for the decay modes are big.

Milestones:

- Grigoriev, Rubakov, Shaposhnikov Sphaleron transitions, kink-antikink pair production, Hamiltonian dynamics
-
- Alford, Feldman, Gleiser Vacuum decay, Langevin dynamics
	- Gould, Moore, Rummukainen Vacuum decay, "multi-canonical sampling" + real-time evolution

First, we focus on the Hamiltonian evolution of a single field; later, on the Langevin dynamics as well.

i.e. no external heat bath i.e. with external heat bath

 FV

 50^{V^2C}

The goal is to test the predictions of the Euclidean theory and to see if anything interesting happens before and during the critical bubble nucleation.

Pîrvu, AS, Sibiryakov 2407.06263, 2408.06411

TV

Simplest Setup allipiest ae point of the potential barrier separating the two vacua. *Setup* — We consider a real scalar field in (1 + 1) dimensions with the action \mathbf{C} **dimplest s** pation and noise provided by an *external* heat bath and \mathbf{r}

 $\overline{}$ ⇡*T ·* uclidean theory predicts: **Euclidean theory predicts:** ¹ We use the system of units *c* = ~ = *k^B* = 1 and define the rate **D** Euclidean theory predicts: $E_b = \frac{4m^3}{3^3}$, $\Gamma_E = \frac{6m^2}{5^3}$

ble energy, Im*^F* / ^e*Eb/T* , as well as the determinant of

We prepare a suite of simulations with the initial thermal Rayleigh-Jeans spectrum:

are sampled at 100 time moments evenly distributed in the interval 0 *<t<* 100. The simulation $d = 3$ data are shown by the solid black dashed line shown by the shown by \mathbb{Z}^3 I_{c} is in excellent agreement with the one-loop prediction (red dot-dashed curve). The tree-level \mathbf{r}

 $S = \frac{1}{\sqrt{2\pi}} \int_{0}^{2\pi} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}}$

 $H = H \cdot \mathbb{R}$ choose a strong dissipation ⌘ˆ = 1 which quickly brings the system to thermal equilibrium,

dispersion relation (blue dot-dashed curve) does not describe the data.

In the Fourier modes of the field and momentum and the growth rate of its unstable mode are \mathbb{Z} $Q_1^2 = 2(1 - \cos k \cdot a)/a^2 + m$ 3 *m .* (4) $\begin{array}{|c|c|c|c|c|}\n\hline\n\text{1} & \text{1} &$ $m_{th}^2 = m^2 - \frac{3\pi R}{\epsilon}$ and substituting its integrating in the substitution of $m_{th}^2 = m^2 - \frac{3\pi R}{\epsilon}$ into the Euclidean formula ($\angle m$) is the nucleon to the nucleon to the nucleon to $\angle m$ is the nucleon to the nucleon to $\angle m$ $\langle |\tilde{\phi}_j|^2 \rangle = T/\Omega_j^2$, $\langle |\tilde{\pi}_j|^2 \rangle = T$ $\Omega_j^2 = 2(1 - \cos k_j a)/a^2 + m_{th}^2$, $k_j = 2\pi j/L$ $m_{th}^2 = m^2 - \left| \frac{3\lambda T}{2m} \right|$ $\boxed{2m}$ thermal correction to the mass, $\ll m^2$ lattice spacing box size

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…and evolve them until they decay (or simulation times out).

ed that this is an equili We checked that this is an equilibrium state by evolving the
theory with the stable potential using the Langevin equation. We checked that this is an equilibrium state by evolving the

What does it mean "decay rate"? $(5^{12}$ the number of configurations of continuous that h The *survival probability Psurv*(*t*) is then defined as the \mathbf{v} is simulated and controls its lifetime. which, due to a statistical fluctuation, has a higher local fluctuation, \mathbf{r} mode power decays faster, while the one with lower power tion is monoto t does It mean "decav ature *T* and monitor them until they decay into the true For the temperatures in our simulations *tth* & 106*/m* ate^{"?} than the typical decay time \mathbf{d} (*L*)¹ ⇠ ¹⁰4*/m*. The initial power contained in the

 \bullet lotroduce ounival orabobility D (t) **0** Introduce survival probability $P_{surv}(t)$

upper curve in Formal Fig. 1. For decays obeying the exponential distribution, the long-mode power decreases. The e↵ect is apparent in For decays obeying the exponential distribution, it follows the law:

(we exclude early-time peared in the series of the temperature in the range of the range α exclude early time transients, (we exclude early-time transients)

> $\ln P_{surv}(t) = \text{const} - \Gamma L \cdot t$ $U(t) = \text{const} \quad \Gamma I$ $\mathbf{F}(\mathbf{v}) = \text{const}$ the exponential distribution, the exponential distribution, the exponential distribution, the exponential distribution of \mathbb{R} in this is decay rate

long modes is then essentially preserved for each indi-

First surprise 13 the number of configurations that have no only \mathbf{r} is a configuration of \mathbf{r} The *survival probability Psurv*(*t*) is then defined as the t_{in} and cannot be neglected. We generate an ensemble of simulations with temperature *T* and monitor them until they decay into the true

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 $\ln P_{surv}(t) = \text{const} - \Gamma L \cdot t$ This is decay rate $U(t) = \text{const} \quad \Gamma I$ $\mathbf{F}(\mathbf{v}) = \text{const}$ the exponential distribution, the exponential distribution, the exponential distribution, the exponential distribution of \mathbb{R}

- Decay rate found in simulations is smaller than the Euclidean prediction
- It is, moreover, time-dependent, getting even smaller with time

What does it mean "thermal"? Assumed that all particles have compared to the simulation of $\frac{14}{3}$ what does it mean "thermal" *?*

 \bigodot

To check this estimate, we make this estimate, we make the following numerical experiment. First, we switch α

 32.32 scattering preserves the energy distribution due to (1)

 $\frac{1}{2}$

But thermalisation in the theory is very **inefficient**: for modes with $\omega \thicksim m \thicksim$ (bubble size) $^{-1}$, the thermalisation time is the interaction with modes of comparable energy. Substituting !*^p* ⇠ *m* to eq. (3.9) and But thermalisation in the theory is very **inefficient**: for modes with α

$$
t_{th} \sim \frac{(2\pi)^3}{\tilde{T}^4}
$$
, $\tilde{T} = \frac{\lambda T}{m^3} \ll 1$ (due to 2 \rightarrow 4 and 3 \rightarrow 3 scattering processes)

Assume for simplicity that all particles have comparable momenta of order *p*. Then the **What does it mean "thermal"?**

the interaction with modes of comparable energy. Substituting !*^p* ⇠ *m* to eq. (3.9) and But thermalisation in the theory is very **inefficient**: for modes with $\omega \sim m \sim$ (bubble size)⁻¹, the thermalisation time is

$$
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$$
, $\tilde{T} = \frac{\lambda T}{m^3} \ll 1$ (due to 2 \rightarrow 4 and 3 \rightarrow 3 scattering processes)

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Compare this with the decay time: $\;t_{dec}\sim (\Gamma L)^{-1}\;$ To check this estimate, we make the following numerical experiment. First, we switch the following numerical experiment. First, we switch the following numerical experiment. First, we switch the following numerical experim

In our simulations it happens that $t_{th} > t_{dec}$ (hardly relevant for cosmology, but can be i This leads to the interesting effect. (hardly relevant for cosmology, but can be relevant for experiments)

Classical Zeno effect 16

- Because of inefficient thermalisation, the initial power contained in the long modes is preserved during the simulation.
- The configuration which, due to a statistical fluctuation, has a higher initial longmode power decays faster. The one with lower power lives longer.
- Statistical properties of the ensemble change with time: long modes cool down.

Effective temp. of long modes drops by a few per cent during the run: enough to visibly suppress the decays.

Classical Zeno effect

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- The configuration which, due to a statistical fluctuation, has a higher initial longmode power decays faster. The one with lower power lives longer.
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 \mathbb{R}^n for \mathbb{R}^n , the lowering with time decay probability can be associated with the associated with the set of \mathbb{R}^n

lowering effective temperature of long modes. The latter can be expanded as *^T*e↵ ⇡ *^T*˜(1↵*t*)

where the one expects and the *T* by *T* by *T* and the *T* by *T* by *T* by *T* and the *T* by *T* and the *T* by *T*

Effective temp. of long modes drops by a few per cent during the run: enough to visibly suppress the decays.

evolution of *T*e↵, averaged over surviving simulations, with time.

Decay is a non-Markovian process (in this regime).

Extrapolation of the time-dependent decay rate *^t* to *t* = 0, at several values of temperature. Solid $\frac{1}{2}$ and read origination it has to decay in the fature. crassical Zenio encorrect. The longer we observe the system, the less chance it has to decay in the future: classical Zeno effect.

with smaller in the more likely to remain the more likely to remain the shows that the more likely the shows that shows the shows the shows that the shows the shows the shows that is now the shows that is now the shows tha To find the unbiased rate, we extrapolate the slope of the survival probability curve to zero.

Second surprise 18 **Conondeurnica** uncertainty. Repeating the procedure at di↵erent tem-

 \bullet We m $\sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n}$ We measure the (unbiased) decay rate at different temps. and fit with the formula (recall that $\Gamma_E = A_E \exp(-E_b/T)$)

 $\ln \Gamma(T) = -\frac{1}{2}\ln T + \ln A - \frac{B}{T}$ for critical bubbing $\Gamma(T) = -\frac{1}{2}\ln T + \ln A - \frac{B}{T}$ with the zetal prefactor (with the zetal \overline{P} prefactor (with the zero mode excluded) prefactor (with the zero mode excluded) dissipation theorem: critical bubble energy

 ρ Sho can measure *A* and *B* beparatory, doing the ratio Or one can make the 2-parameter fit, the result is the same (within the errorbars). One can measure A and B separately, using the ratio $\Gamma(T)/\Gamma(T_*)$ to find B, with some reference temp. T_*

 $L_{\text{L}} = 0.001$ rate measurement. For each probability curve, $L_{\text{L}} = 0.001$ and $L_{\text{L}} = 0.001$

 $m_{\rm m}$ is the measurement. The measurement with $m_{\rm m}$

 s_{max} , it can be estimated by statistics; it can be estimated as α

- Critical bubble energy agrees with the Euclidean theory (<2% error bar)
- The measured prefactor is smaller by a factor ~8.

details) and the contract of the contract of the contract of the contract of the contract of

Something wrong with thermalisation again? Violation of thermal equilibrium near the critical bubble?

More evidence: Langevin evolution (19) where condensed fang

We can reduce artificially the thermalisation time by coupling the system to an external heat bath. e can reduce artificially the thermalisation time by coupling the sy

$$
\begin{cases}\n\ddot{\phi} + \eta \dot{\phi} - \phi'' + m^2 \phi - \lambda \phi^3 = \xi \\
\langle \xi(t, x) \rangle = 0, \ \langle \xi(t, x) \xi(t', x') \rangle = 2\eta T \delta(t - t') \delta(x - x')\n\end{cases}
$$
\n
$$
t_{th} \sim \eta^{-1}
$$

Noise and dissipation change the dynamics of vacuum decay.

They don't change the critical bubble.

Survival probability from Langevin dynamics (Eq. (13)

curve correspond to Poisson fluctuations. *Black dotted:* Pre-

 \sim 0.000 \geq

*.*01

t = 0. *Red thin:*

m. Wiggles in the

 $\mathcal{L}(\mathcal{S})$ is the previous curve at \mathcal{S} . The previous curve at \mathcal{S}

 t_{max}

 $\mathcal{L}(\mathbb{C}[\mathbb{$

. \mathbf{y} $\frac{1}{2}$ ϵ independent code as iding as $\eta \approx$ No Zeno e ffect as long as *η* ≳ Γ *L*

*.*012*/m*

,

a

The initial conditions are picked up for an ensemble σ Gaussian perturbations around the false vacuum with σ t_{max} the thermal Rayleigh-Jeans spectrum. Namely, we define \mathcal{L}

compose the field and its canonical momentum ⇡ ⌘ ˙ at *^t* = 0 in Fourier modes,

0

Most runs are performed with

choosing *ma* = 0

 $\overline{\mathcal{L}}$

10th-order Gaussian Decay rate increases but still *.*04, *mL* 2 [80 *,* 100] and *t* 0 *.*17 *a* . FIG. 1. *Blue thick:* Survival probability in real-time simula-Decay rate increases, but still below the Euclidean bound

L = 100*/m*

,

Langevin dynamics: decay rate $(1)^{20}$

We observe the following behavior:

- As dissipation increases, Γ increases as well. It reaches maximum at $\eta \simeq 3 \cdot 10^{-1} m$, then starts decreasing due to over-damping.
- Γ tends to increase when *T* goes down.

Violation of equilibrium condition

In Physical Chemistry, the analog of Euclidean Theory is Transition State Theory (TST).

for review see **Hanggi, Talkner, Borkovec, Rev.Mod.Phys. 62 (1990)**

TST deals with particles (one or few d.o.f.) in the external heat bath, $\eta > 0$.

It is known that TST is violated if there is no equilibrium around the barrier. The following condition must be satisfied: – For !*T /E^b* & ⌘ & *t* 1

$$
\eta \gg \frac{\omega_{-} T}{E_b}
$$

We can generalise this condition to **Langevin** dynamics of field theory. eralise this condition to **Langevin** dynamics of field theory.

This is done by careful examination of Langer's work.

For the **Hamiltonian** dynamics of field theory, we suggest the following condition:

All our current and future results are consistent with it.

$$
t_{th} \lesssim \frac{\mathcal{F}_b}{\omega_- T}
$$

 \angle Effective free energy of the critical bubble

 $\frac{1}{\omega_{-}T}$

- It is generally violated for weakly-coupled theories with one coupling (one field)
- In theories with many fields, it must be examined on a case-by-case basis.

Dynamics of vacuum decay

When equilibrium is violated, interesting features appear in the field evolution prior to the decay.

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 λ t small dissipation, we observe a population of nonlinear waves with $\omega < m -$ **oscillons**. all dissipation, we observe a population of nonlinear waves with $\omega < m -$ **oscillons**. α ipation, we observe a population of nonlinear waves with $\omega < m -$ **oscillons**. Figure 12: Typical evolution of the field preceding the critical bubble formation α pation, we observe a population of norminear waves with $\omega < m -$ **oscilions**. At small dissipation, we observe a population of nonlinear waves with $\omega < m -$ **oscillons**.

They disappear when $\eta > 0.1m$ and the system evolves due to the stochastic terms.

Dynamics of vacuum decay

In the Hamiltonian dynamics, every critical bubble is preceded by an oscillon.

Johnson, Pîrvu, Sibiryakov, 2312.13364

We can track its trajectory.

Thanks to Dalila's smart numerical routine.

Stacking many oscillons together, we get the average **oscillonic precursor** to the critical bubble:

 \mathcal{L}^{in} interest, e.g. the configurations shown in Fig. 12. Next, we take the Fourier transformation of \mathcal{L}^{in}

along the time dimension. We set the amplitude of the negative-frequency $\mathcal{L}_\mathcal{F}$

 $t \geq 2$ and double the amplitude of the positive-frequency modes to conserve the energy.

 T is inverse temporal-domain F the inverse temporal F the Hilbert transform, we arrive at the Hilbert transform.

Finally, the signal envelope is simply the absolute value of this transform.

Discussion

How general are these results?

In 3+1 the thermalization is faster, but still no equilibrium in the Hamiltonian dynamics of a single field.

Adding more species does **not** automatically improve the thermalization condition.

Adding gauge fields? We don't know yet.

How important are these results e.g. in cosmology?

If you need an accurate prediction of the decay rate or for the effects pertaining to the dynamics of bubble nucleation — these results are important.

If you need an order of magnitude estimate or parametric dependence — these results are (likely) not important.

Our results are not **directly** applicable to sphaleron transitions or e.g. production and collision of kinks. But we're looking into this now.

backup slides

Thermalisation time $\frac{1}{26}$ h nalisation time and a contract of the contract

We perform the numerical experiment estimating the thermalisation time of long modes in the Hamiltonian system.
Example:

²⁷ **Thermalisation with external heat bath**

We perform the numerical experiment estimating the thermalisation time of long modes with the Langevin evolution.

Effective temperature of long modes ($k < m$, blue) and the ${\bf temperature\ of\ the\ ensemble\ } (k>m{\bf, red})$

The result agrees with the estimate $t_{th} \sim \eta^{-1}.$ $\hspace{3cm} \hspace{3cm}$

²⁸ **Box size and lattice spacing**

In simulations, we take $L = 100$ and $a \simeq 0.01$ (in units of mass).

The plots below demonstrate insensitivity of the decay rate to *L* and *a*.

 α on the every primarie expect of the decey rate measurement We use this to put the upper bound on the systematic error of the decay rate measurement.

²⁹ **Accuracy of numerical scheme**

Hamiltonian dynamics

We use the 4th order pseudo-spectral, operator-splitting scheme.

The plots below show that it is enough to take $h/a \simeq 0.8$ to achieve the relative energy nonconservation $\lesssim 10^{-6}$.

 C_1 or C_2 operator-splitting scheme for the Langevin dynamics scheme for the Langevin dynamics of C_1

³⁰ **Accuracy of numerical scheme Langevin dynamics**

Langevin vs Hamiltonian evolution

Let's make the following numerical experiment.

- Evolve the ensemble with non-zero η for $t \gg \eta^{-1}$ so that all surviving configurations reach equilibrium with the heat bath.
- Decouple the ensemble from the heat bath by setting $\eta = 0$.

 $\frac{1}{2}$ a. Decay rate measured in simulation of $\frac{1}{2}$ evolution at α is **the temperature is the temperature is** α Thus, the deviation of the rate from equilibrium is really due to the field dynamics near the barrier.

More observables

${\bf Shape}$ of the critical bubble $\,\displaystyle{\phi_{b}^{\mathrm{(}}x)}$

Should we compute the bubble using the bare potential or an effective potential? If effective, which fields to include and when?

Dynamics of bubble nucleation

Euclidean theory tells us little about how the critical bubble actually forms out of thermal fluctuations.

This dynamics is quite interesting: bubble velocities, oscillonic precursors…

Gleiser, Kolb… hep-ph/0409179, 0708.3844 Johnson, Pîrvu, Sibiryakov, 2312.13364

Critical bubble profile and the set of the

Take many simulations, **synchronise** them in space and time, produce the average.

Take many simulations, **synchronise** them in space and time, produce the average, pinpoint the critical bubble. Take many simulations **synchronise** them in space and time produce the aver

we employ two different reconstruction routines. They agree with each we employ two amerent reconstruction routines. They agreed
other and with the Euclidean prediction. $T_{\rm H}$ most important prediction of the tree-level $F_{\rm H}$

 $C_{\mathbf{z}}$ \mathbf{C}

L ·

^p*|*↵*[|]*

Here is the system and and and **in** the system and and **leaders** in the unstable mode around the \mathcal{A} is expression to 1-loop in the third in the third in the third in the theory (2.1). To find the theory (2.1). To find the theory (2.1). The theory (2.1) in the third in the theory (2.1). The theory (2.1) is the t

2.3 Euclidean decay rate $\mathcal{L}(\mathbf{Q})$ rate is related to the entropy part of free profile reconstructed from simulations using two different methods (solid \mathbf{Q}) \mathbf{Q} No surprise here: the critical bubble is determined by the bare potential; fluctuations contribute to the prefactor.

 $\mathcal{L} = \mathcal{L} \mathcal{L}$

blue and many fields!
Things can be different with many fields!

 c_1 is used instead of the the theory is used instead of the bare mass (2.10) is used in the bare mass. Fig:Sph(t)

 α

F, consider the partition function function function \mathbf{F}