

Gravitational waves – detection and analysis

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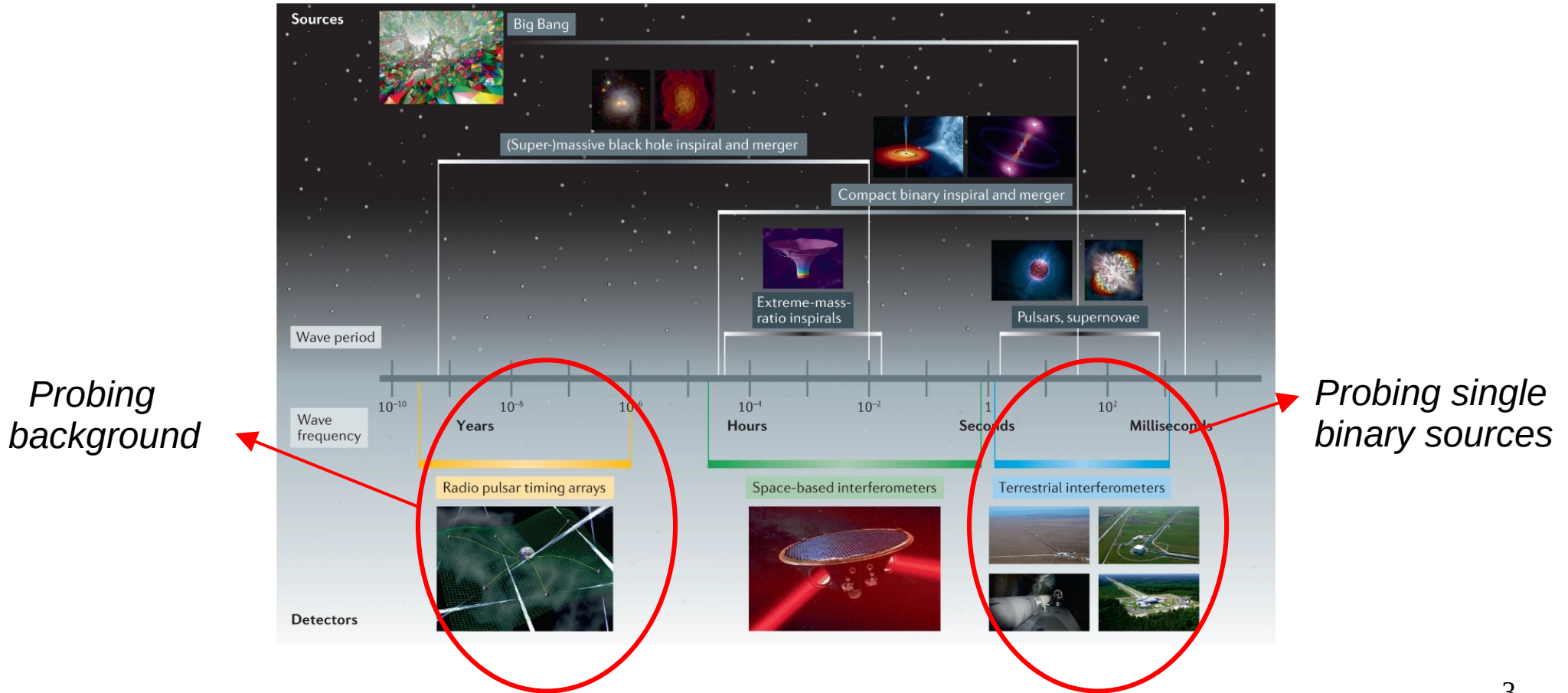
Einstein toolkit workshop
11th July, 2024



Plan of talk

- Introduction
- Parameter estimation in current era of gravitational waves
- Parameter estimation for Einstein Telescope

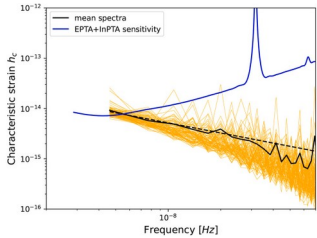
Introduction



Introduction



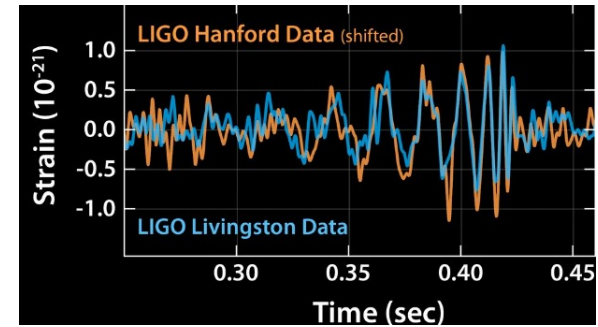
- Pulsar timing:
 - *Looking for superposition of population of binaries*



Problem: *Timing a network of pulsars, resulting background amplitude?*



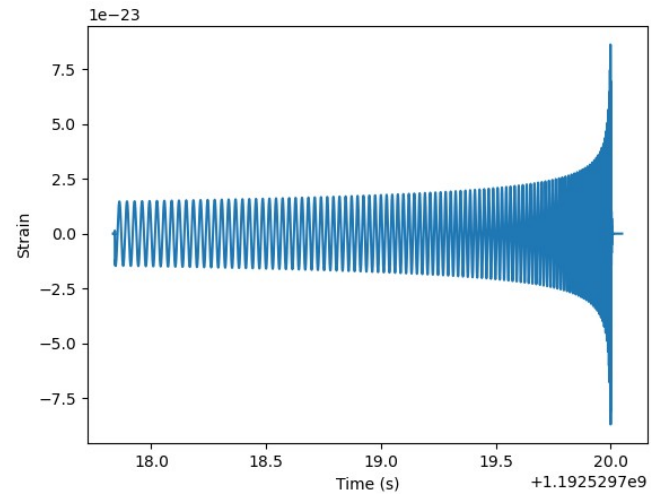
- Ground-based (LIGO/Virgo):
 - *Detecting and analysing merger of single binaries*



Problem: *Individual source properties?*

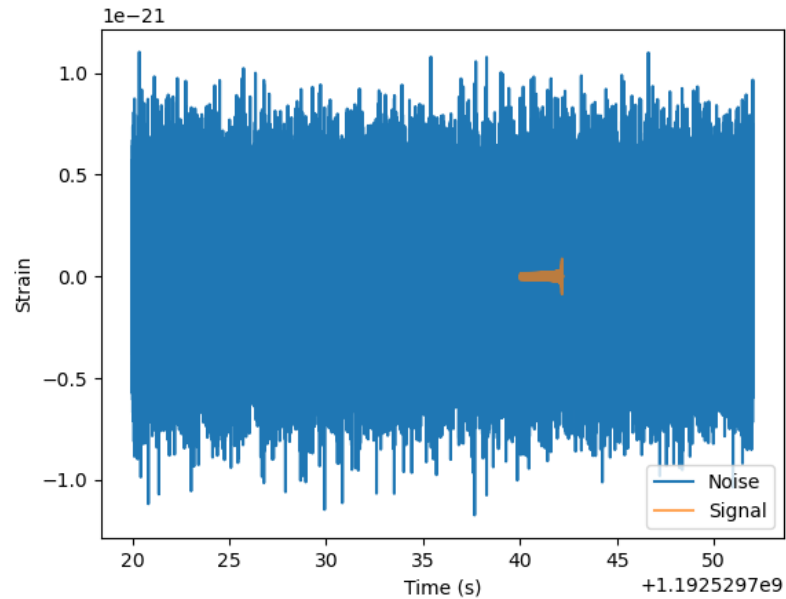
Introduction

Problem statement: Having detected a gravitational-wave signal, what are the source properties?



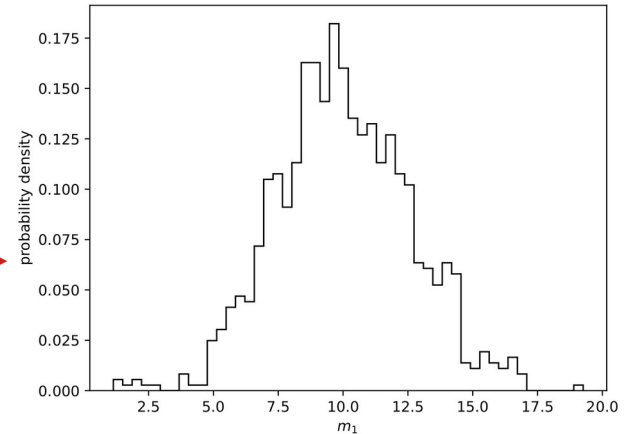
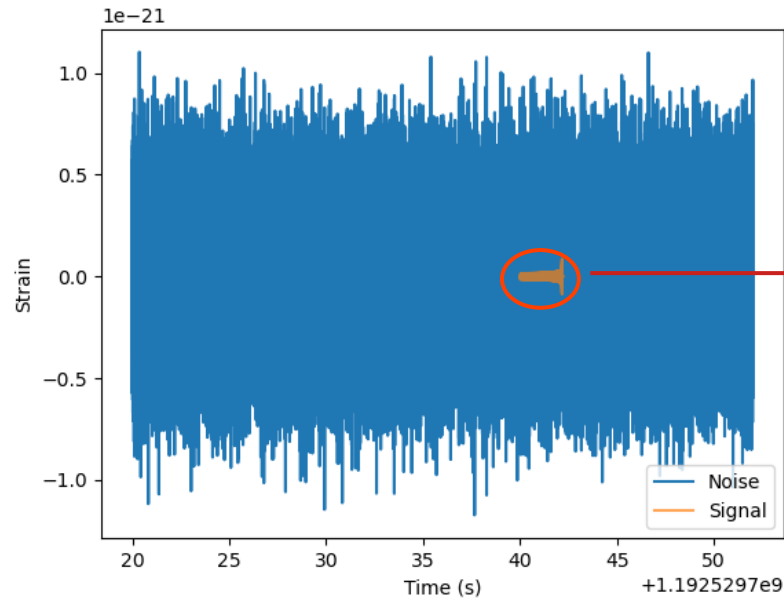
Introduction

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Parameter estimation

- Match data to model $\sim 10^6 - 10^8$ times
- 15 dimensional parameter space for a binary black hole system:

$$\vec{\theta} = \{m_1, m_2, \vec{S}_1, \vec{S}_2, \alpha, \delta, \psi, \iota, D_L, t_c, \phi_c\}$$

$$s = n + h$$

assumption: noise is Gaussian

$$p(s|\theta) \propto \exp\left(-\frac{1}{2} \sum_I (s_I - h_I(\theta) | s_I - h_I(\theta))\right)$$

$$(a|b) = 2 \int_0^\infty df \frac{\hat{a}(f)\hat{b}(f)^* + \hat{a}(f)^*\hat{b}(f)}{S_n(f)}$$

detector noise power spectral density (PSD)

likelihood

prior

Bayes' theorem

$$p(\theta|s) = \frac{p(s|\theta)p(\theta)}{p(s)}$$

evidence (normalizing factor)

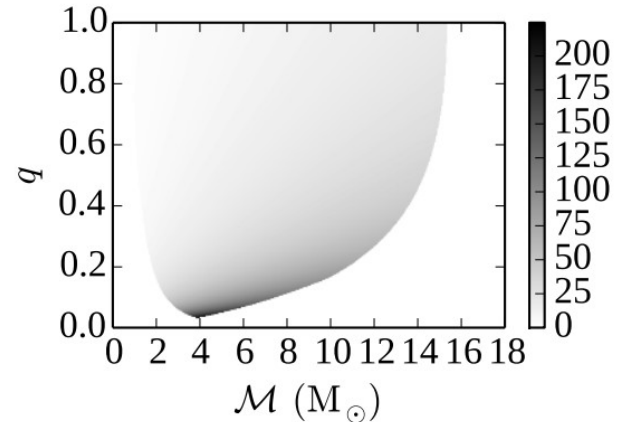
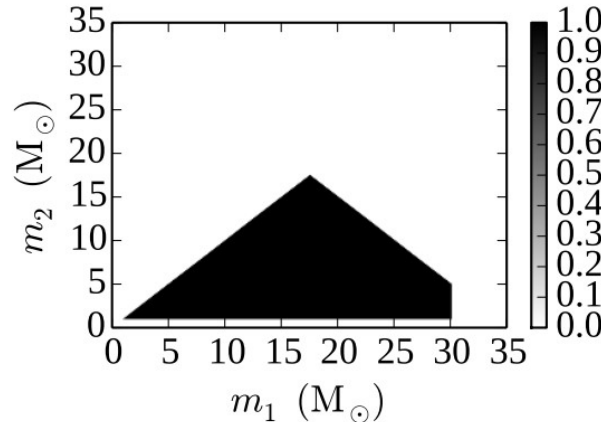
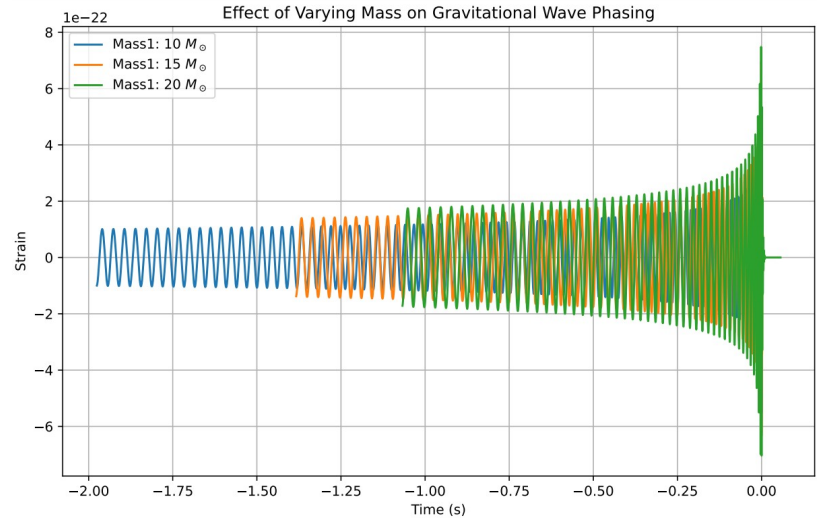
Parameter estimation

Measurable parameters: affecting waveform phasing

Combining information from `n` sources:

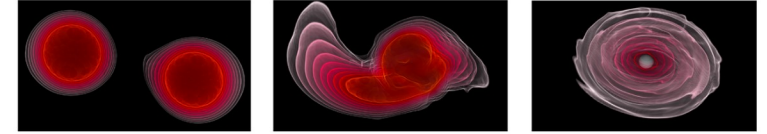
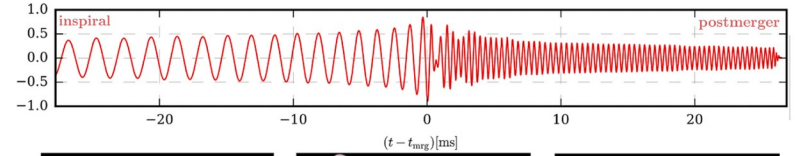
$$P(\theta|\{D_i\}) = \frac{P(\theta) \prod_{i=1}^n P(D_i|\theta)}{P(\{D_i\})}$$

$$P(\{D_i\}) = \int P(\theta) \prod_{i=1}^n P(D_i|\theta) d\theta$$



Preferred – weaker correlations

Measuring tidal effects from gravitational wave signals



Gen.Rel.Grav. 53 (2021) 3, 27: Dietrich, Hinderer, Samajdar

$$\Psi(f) = \Psi_{\text{PP}}(f) + \Psi_{\text{SO}}(f) + \Psi_{\text{SS}}(f) + \Psi_{\text{Tides}}(f)$$

Point-particle phasing

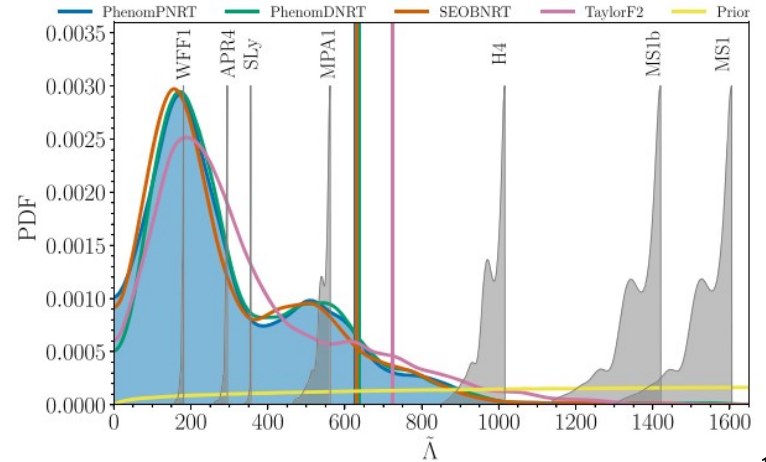
Tidal phasing

$$Q_{ij} = -\lambda(m) \mathcal{E}_{ij}$$

Parametrised by

$$\tilde{\Lambda} = \frac{16 (m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{3 (m_1 + m_2)^5}$$

$$\delta\tilde{\Lambda} = \frac{1}{2} \left[\sqrt{1 - 4\eta} \left(1 - \frac{13272}{1319}\eta + \frac{8944}{1319}\eta^2 \right) (\Lambda_1 + \Lambda_2) + \left(1 - \frac{15910}{1319}\eta + \frac{32850}{1319}\eta^2 + \frac{3380}{1319}\eta^3 \right) (\Lambda_1 - \Lambda_2) \right]$$



Parameter estimation in the third-generation era

General expectations of binary neutron star (BNS) sources in the third generation (3G) era:

	No. of detections	SNR_{net}	No. with $\text{SNR}_{\text{net}} > 250$	No. with $\text{SNR}_{\text{net}} > 100$	No. with $\text{SNR}_{\text{net}} > 50$	No. with $\text{SNR}_{\text{net}} > 20$
Low rate	98898	$19.2^{+22.1}_{-4.9}$	17 (0.017%)	298 (0.30%)	2712 (2.7%)	44350 (48%)
Median rate	396793	$19.1^{+22.0}_{-4.8}$	73 (0.018%)	1257 (0.32%)	10659 (2.7%)	177296 (45%)
High rate	1004525	$19.1^{+22.1}_{-4.8}$	196 (0.020%)	3255 (0.32%)	27135 (2.7%)	448610 (45%)

Samajdar+, PRD 104, 044003 (2021)

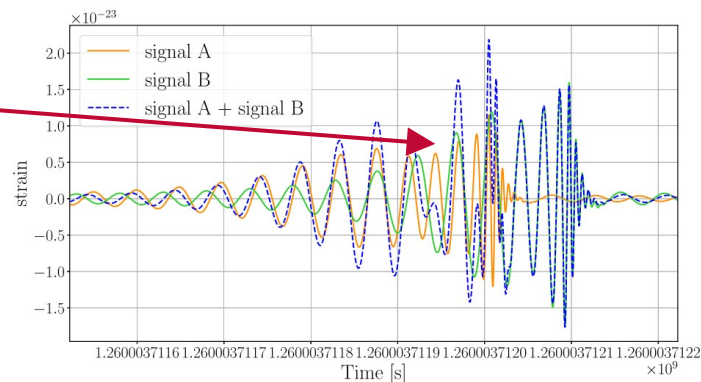
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Challenges from a data analysis point of view:

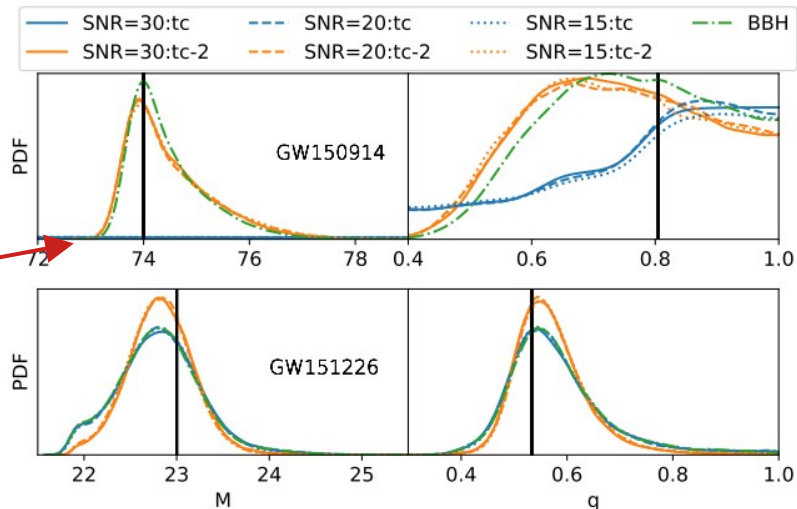
- **Computational resources**
- **Many signals in band at the same time**
- **Modelling inaccuracies**



Analysis of overlapping signals

When will overlapping signals will be an issue?

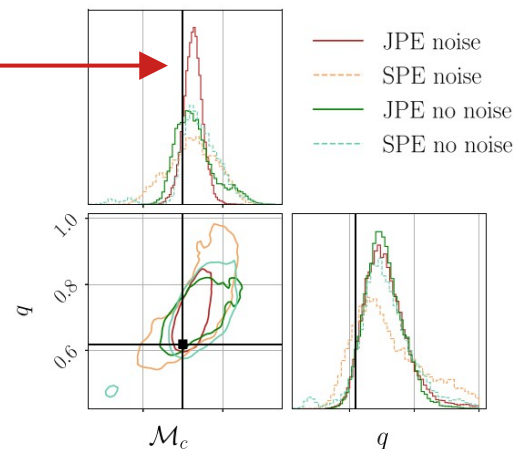
Short binary black hole signal overlapping with a long binary neutron star signal; **ending at the same time!**



Possible solutions:

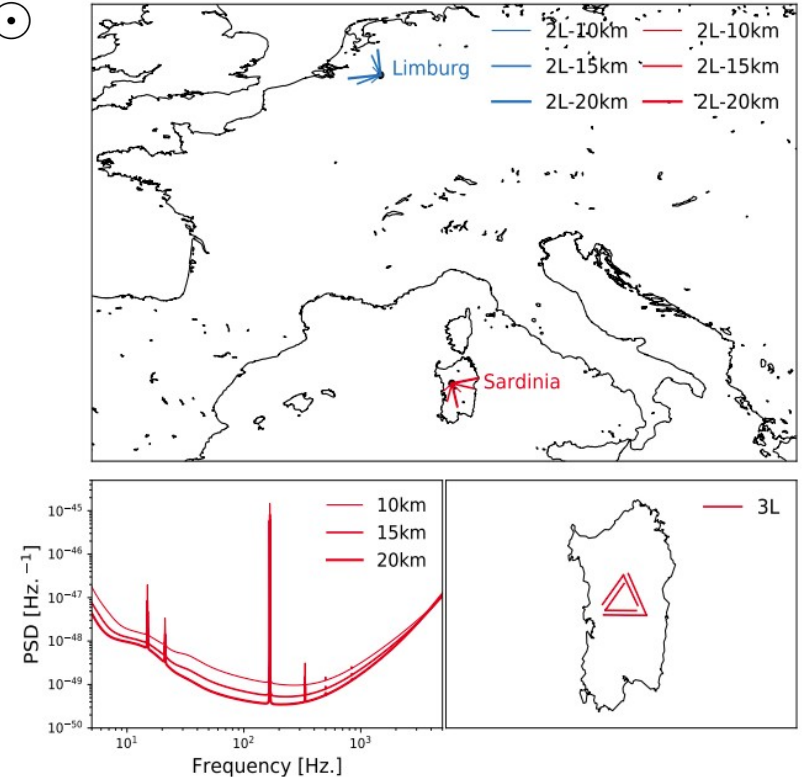
- Extend the likelihood to account for multiple signals (JPE).
- Subtract signal at a time and analyse remaining data – hierarchical subtraction.

Samajdar+, Phys. Rev. D 104, 044003, 2021



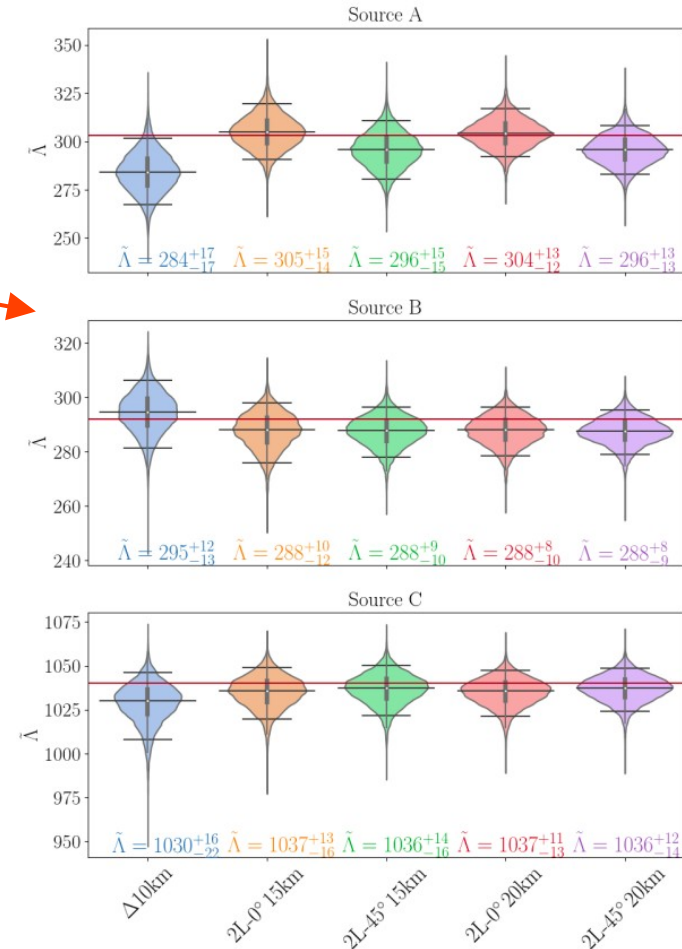
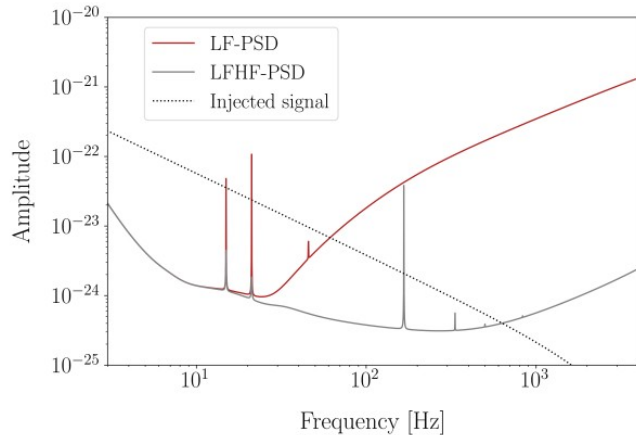
Instrumental noise with 3G

- Longer signals; $m_1 = m_2 = 1.4 M_{\odot}$
 - Duration in band in current era
~ **2 minutes**
 - With Einstein telescope,
duration ~ **2 hours**
- Non stationarity of noise:
 - Non-constant power spectral density over length of signal.
- Correlated noise.

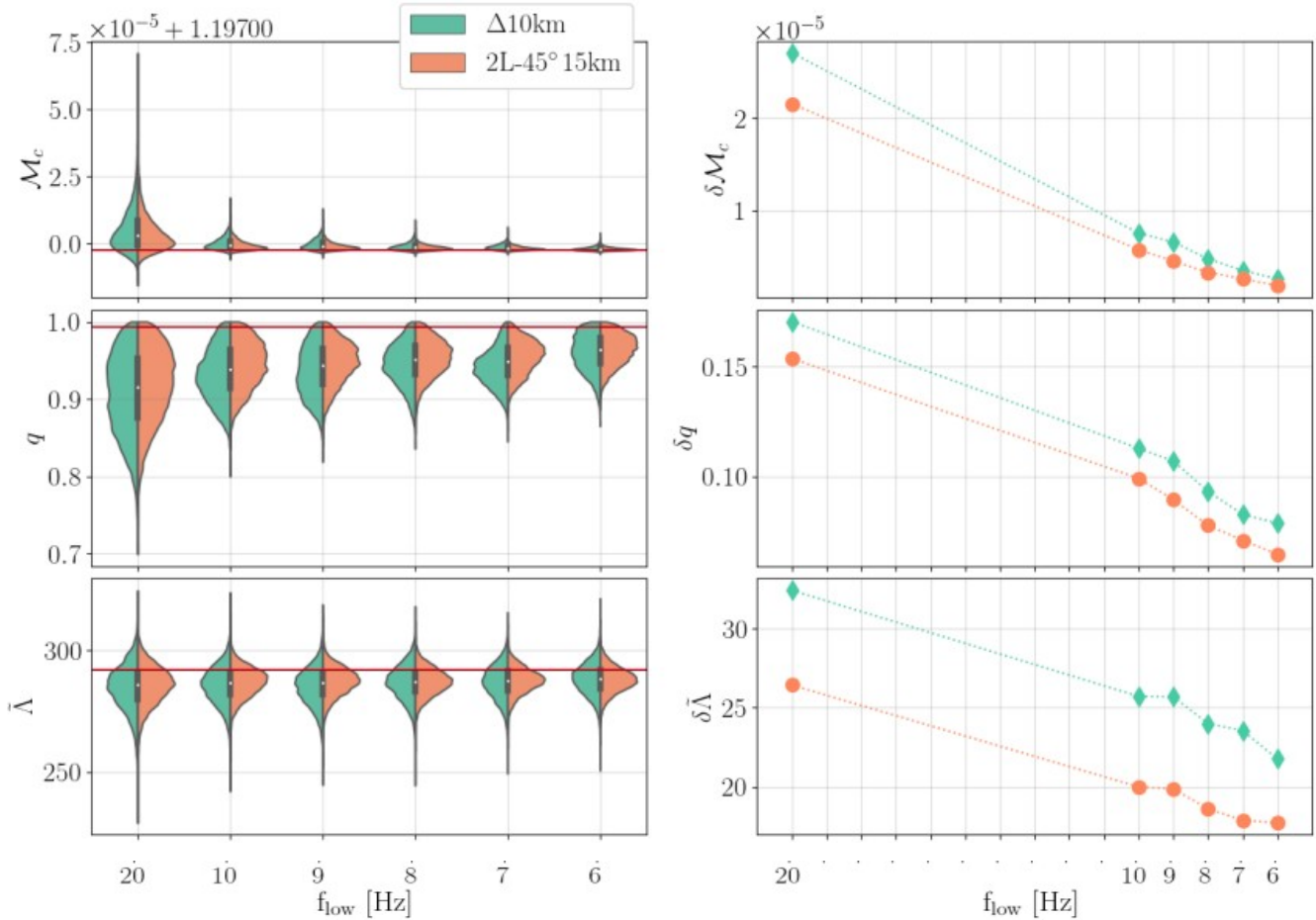


Tidal deformability estimates with 3G

- Main differences come from varying arm-lengths
- Other parameter: vary laser power:



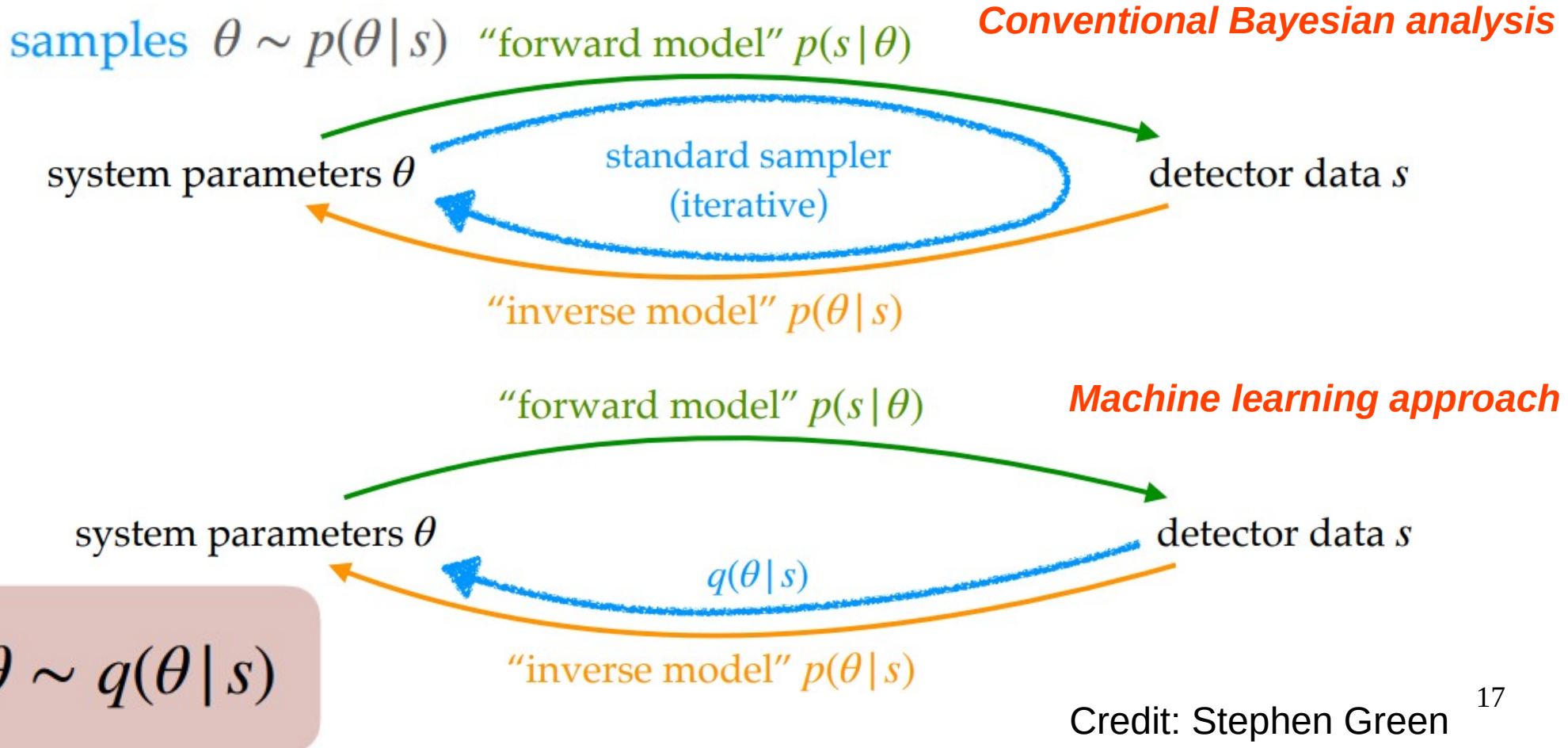
Improved bounds



Tidal information dominant from high frequencies



Improving efficiency



Improving efficiency

Nested sampling with normalizing flows for gravitational-wave inference

Michael J. Williams, John Veitch, and Chris Messenger
Phys. Rev. D **103**, 103006 – Published 5 May 2021

Real-Time Gravitational Wave Science with Neural Posterior Estimation

Maximilian Dax, Stephen R. Green, Jonathan Gair, Jakob H. Macke, Alessandra Buonanno, and Bernhard Schölkopf
Phys. Rev. Lett. **127**, 241103 – Published 8 December 2021

Neural Importance Sampling for Rapid and Reliable Gravitational-Wave Inference

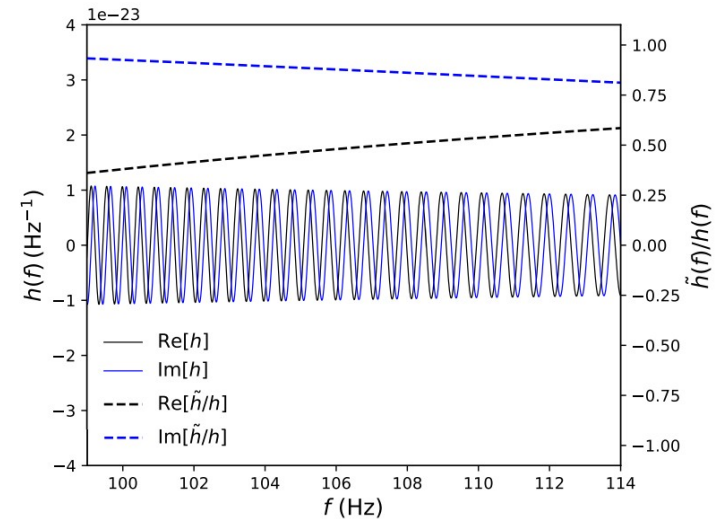
Maximilian Dax, Stephen R. Green, Jonathan Gair, Michael Pürrer, Jonas Wildberger, Jakob H. Macke, Alessandra Buonanno, and Bernhard Schölkopf
Phys. Rev. Lett. **130**, 171403 – Published 26 April 2023

Normalizing Flows as an Avenue to Studying Overlapping Gravitational Wave Signals

Jurriaan Langendorff, Alex Kolmus, Justin Janquart, and Chris Van Den Broeck
Phys. Rev. Lett. **130**, 171402 – Published 26 April 2023

Many many more works!

Relative binning:



Zackay et al, 2018: arXiv 1806.08792

Summary

- Wealth of information from 3G era!
- Important effects to take into account:

Summary

- Wealth of information from 3G era!
- Important effects to take into account:
 - Glitches
 - Speed and efficiency
 - Overlapping signals

GW170817 signal

