Introduction to GRMHD



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See also my recent review arxiv:2405.10081

Kenta Kiuchi (CRA)



Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut)



Center for Gravitational Physics and Quantum Information

Yukawa Institute for Theoretical Physics, Kyoto University

Introduction Dawn of the gravitational wave astrophysics



Introduction Importance of electromagnetic counterpart



⇒ X-ray (9day→1600day)⇒ Radio (16day→700day)

Introduction Solved and unsolved problems in GW170817



Neutron rich matter are likely to be ejected (kilonova/macronova associated with the r-process nucleosynthesis)

Riddle on the detailed mass ejection process Origin of blue and red components?

Riddle on relativistic jet launching mechanism Driven by BH or massive neutron star?

Self-consistent NR modeling for BNS merger from inspiral to post-merger

Introduction

Toward physical modeling of GW sources

- ► Gravity (General Relativity)
- ► Strong interaction (Nuclear matter)
- ► Weak interaction (Neutrino)
- Electromagnetic interaction (Magnetic field)
- ► Highly dynamical system (GW!)
- ▶ Primarily no spatial symmetry (fully 3D+1 problem)





Slide courtesy of Y. Sekiguchi

To B or not to B in binary neutron star merger (by Victoria M. Kaspi)



Assumption : Rotational energy is dissipated by the magnetic dipole radiation $\Rightarrow B \propto (P\dot{P})^{1/2}$

To B or not to B in binary neutron star merger (by Victoria M. Kaspi)

► B-field in observed binary NSs : $10^{9.7} - 10^{12.2}$ G

Kinetic energy before the merger $\sim 10^{53}~{\rm g~cm^2~s^{-2}}~({\rm M}/{\rm 2.7M_{sun}})({\rm v}/{\rm 0.3c})^2$

B-field energy $\sim 10^{41}$ g cm² s⁻² (B/10¹²G)²(R/10⁶cm)³

B-field is irrelevant in BNS mergers ?

No $! \Rightarrow$ Several amplification mechanisms (Magneto Hydro Dynamical instabilities) could amplify the B-filed up to the dynamically important level

A couple of key ingredients I. Kelvin-Helmholtz Instability Kelvin Helmholtz instability (Rasio and Shapiro 99, Price & Rosswog 05)





A couple of key ingredients II. Magneto Rotational Instability

Magneto Rotational Instability (MRI) (Balbus & Hawley 91)



 \Rightarrow Angular momentum transport and viscous heating

A couple of key ingredients III. A large-scale dynamo mechanism Mean field dynamo theory $\mathbf{Q} = \bar{\mathbf{Q}} + \mathbf{q}, \ \bar{\mathbf{Q}} = \text{Axisym. Ave.}$ $\partial_t \bar{\mathbf{B}} = \nabla \times (\bar{\mathbf{U}} \times \bar{\mathbf{B}} + \bar{\mathcal{E}}),$ $\bar{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$ **u** & **b** : turbulence of the velocity and b-field. $\alpha \Omega$ dynamo $\mathcal{E}_i = \alpha_{ij} B_j + \beta_{ij} (\nabla \times B)_j$ In the current context, α_{ii} is dominant $\partial_t \bar{B}_\phi = R \bar{\mathbf{B}}_P \cdot \nabla_P \Omega \ (\Omega \text{ effect}),$

 $\partial_t \bar{B}_R = -\partial_z \mathcal{E}_\phi \approx -\partial_z (\alpha_{\phi\phi} \bar{B}_\phi) \ (\alpha \text{ effect}),$ $\partial_t \bar{B}_z = \partial_R \mathcal{E}_\phi \approx \partial_R (\alpha_{\phi\phi} \bar{B}_\phi)$

Relevance of the resolution study Magnetic winding timescale from the per-merger large-scale field

 $t_A \sim R/v_A \sim 100s (B/10^{12}G)^{-1} (\rho/10^{15}g/cm^3)^{1/2} (R/10^6 cm)$

Therefore, the magnetic winding originating from the per-merger field should be irrelevant in reality.

But, many GRMMHD simulations conducted so far assumed ${\sim}10^{15}\text{-}10^{17}\text{G}$ as the pre-merger large-scale field

Why? A: To compensate the high computational cost to resolve the KHI/MRI ⇒ Trade off is to shorten the winding timescale originating from the premerger large scale field

Relevance of the resolution study

With a single resolution, it is impossible to disentangle the large-scale field generated by the non-trivial process from the pre-merger large-scale field effect

<u>Summary for key ingredients</u>:

(i) Kelvin-Helmholtz instability at the merger(ii) Magneto Rotational Instability inside the merger remnant(iii) A large-scale dynamo mechanism inside the merger remnant

Numerically resolving them is necessary condition. To do so, the direct high resolution or effective high resolution with a sub-grid model is essential.

Electromagnetic emission in compact binary mergers

R(paid)-process nucleosynthesis and EM (Lattimer & Schramm 74, Metzger et al. 10, Li & Paczynski 98)

Role of the r-process elements

► Heating source via radio-active decay (Kasen et al. 17) $\dot{\epsilon} \approx 10^{10} \text{ erg s}^{-1} \text{ g}^{-1} \left(\frac{t}{\text{day}}\right)^{-1.3}$

 Opacity source (Lanthanide elements) (Barnes & Kasen 13, Tanaka & Hotokezaka 13)

 $\kappa \approx 10 \ {\rm cm}^2 \ {\rm g}^{-1}$



Slide courtesy of M. Tanaka

Properties of electromagnetic emission (Optical-IR)
Peak time (diffusion time = dynamical time)

$$t_{\rm peak} \approx 5.7 \,\mathrm{day} \left(\frac{\kappa}{10 \,\mathrm{cm}^{-2} \,\mathrm{g}^{-1}}\right)^{1/2} \left(\frac{M_{\rm eje}}{0.03 M_{\odot}}\right)^{1/2} \left(\frac{v_{\rm ej}}{0.2c}\right)^{-1/2}$$

► Peak Luminosity

$$\mathcal{L} \approx \dot{\epsilon} M_{\rm ej} \approx 6 \times 10^{41} \,\mathrm{erg \ s^{-1}} \left(\frac{M_{\rm eje}}{0.03 M_{\odot}}\right) \left(\frac{t}{\mathrm{day}}\right)^{-1.3}$$

R-process nucleosynthesis and its opacity



► Electron fraction Y_e (# of electron/# of baryon) is a key quantity ► $Y_e \gtrsim 0.25$ produces negligible / small amount of lanthanide \Rightarrow low opacity in optical

- ► $Y_e \leq 0.25$ produces lanthanide \Rightarrow high opacity in IR
- ► Neutrino reaction determines Y_e of the ejecta

Numerical modeling of BNS: Short-lived case

Shibata & Hotokezaka 19



► Short-lived case is inferred in GW170817, due to non-direction of strong radio flare (Shibata et al. 17, Margalit & Metzger 17, +)

Numerical modeling of BNS: Short-lived case Downside of the previous works

► Short-term simulation of O(0.1)s at most (Radice et al. 18, Zappa et al. 18, Foucart et al. 22, and many)

 Non-self-consistent model of the merger remnants, e.g., BH+torus (Fernandez et al. 19, Siegel & Metzger 18, Fujibayashi et al. 20, and many)
Phenomenological prescription to model the MRI-driven turbulent viscosity

(Fujibayashi et al. 20a,b, 22, Radice et al. 18)

We are tackling the problem using Japanese supercomputer Fugaku (400PFLOPS).



Methodology Ab initio Numerical Relativity simulation

- Einstein's solver (Shibata & Nakamura 95, Baumgarte & Shapiro 98, Barker et al. 06, Campanelli et al. 06, Hilditch et al. 13)
- ► Nuclear theory-based equation of state for the NS matter (SFHo) (Steiner et al. 13)
- ► Relativistic magnetohydrodynamics solver (KK et al. 22, Migone et al. 09, Gardiner & Stone 08)
- ► Neutrino-radiation transfer solver (Sekiguchi et al. 12)

+ for more technical issues e.g., conservative mesh-refinement, see KK et al. 22

We performed a BNS simulation for 1.2s on Fugaku (KK et al. PRL 23).

Numerical modeling of BNS: Short-lived case



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Final snapshot with a meridional cut



B-field amplification and MRI sets in

B-field energy



B-field is amplified by the Kelvin-Helmholts instability, winding, non-axisymmetric MRI in a hypermassive neutron star phase (KK et al. 14,15,18)
Winding and axisymmetric MRI after the BH formation

B-field amplification and MRI sets in MRI quality factor with the cut-off density



MRI is completely resolved in a bulk region of the torus after 0.1s.
MRI-driven turbulent state is established.



► MRI-turbulent viscosity is produced and it is 0.01-0.03. ⇒ $t_{vis} \sim O(0.1s)$

MRI dynamo to sustain the MRI-driven turbulence Butterfly Diagram for the toroidal B-field (R=50km)



► It clearly suggests the sign flip pattern which lasts until end of the simulation \Rightarrow MRI dynamo sustains the turbulent state.

Neutrino luminosity evolution

► MRI-driven turbulent viscosity facilitates the angular momentum transport \Rightarrow The torus expands and the temperature drops.

Neutrino luminosity



► Neutrino luminosity decreases, and it becomes steep around \approx 0.7s. \Rightarrow All the turbulent viscous heating is consumed by the torus expansion.

Mass ejection (Dynamical and Post-merger)

Mass ejection rate measured on R=3,000 km



▶ Dynamical ejecta starts to appear at \approx 0.01s and peaks around \approx 0.03-0.04s (Fast tail and mildly relativistic ejecta).

▶ Post-merger ejecta due to the MRI-driven turbulence emerges at \approx 0.3s.

▶ The ejection rate exceeds the accretion rate at \approx 1.1s.

Ejecta properties



▶ Electron fraction distribution has two distinct peaks at \approx 0.03 (dynamical) and \approx 0.24 (post-merger). The latter is determined when the weak interaction freezes out.

► The low-Ye component corresponds to the s/k_B ≈ 3 (tidal) and 10 (shocked) components. The high-Ye corresponds to the s/k_B ≈ 20 with $v_{\infty}/c \approx 0.1$ (post-merger).

Self-consistent picture of the mass ejection from a BNS merger



► $M_{\text{post,eje}} \approx 8 \times 10^{-3} M_{\odot}$, $M_{\text{dyn,eje}} \approx 6 \times 10^{-3} M_{\odot}$ A simple kilonova modeling requires $0.05 M_{\odot}$ (Hotokezaka & Nakar 20, +) Jet launching in short-lived case

Absence for the launching jet in our simulation

- ► Mild BH spin? $\chi \approx 0.6-0.65$
- Shortness of simulation? Still strong ram pressure due to the fall-back matter. \Rightarrow Maybe matter of the time?



R-process nucleosynthesis MHD vs viscous hydrodynamics



Numerical modeling of BNS: Long-lived case

Shibata & Hotokezaka 19



Can a long-lived remnant drive a relativistic outflow to be observed as a short-gamma ray burst? (Metzger et al. 11, Zhang & Meszaros 01)
Different jet launching mechanism from BH case, i.e., the BZ mechanism. Computationally challenging (Mösta et al. 2020 etc.)

Numerical modeling of BNS: Long-lived case



Electron fraction of ejecta Density + B-field lines

B-field strength

Methodology Ab initio numerical relativity simulation

Einstein's solver (Shibata & Nakamura 95, Baumgarte & Shapiro 98, Barker et al, 06, Campanelli et al. 06, Hilditch et al. 13)

▶ Nuclear theory-based equation of state for the NS matter (DD2) (Banik et al. 14)

► Relativistic magnetohydrodynamics solver (KK et al. 22, Migone et al. 09, Gardiner & Stone 08)

► Neutrino-radiation transfer solver (Sekiguchi et al. 12)

+ for more technical issues (see KK et al. 22)

We performed a BNS simulation with Δx_{finest} =12.5m and B_{0,max}=10^{15.5}G for 0.2s (KK et al. Nature Astronomy 24)

B-field amplification inside the merger remnant B-field energy 10^{52} $\propto t^{2}$ 10^{51} 10^{50} $[10^{10}]{\text{Grg}} \stackrel{10}{=} 10^{49}$ $\Delta x = 12.5 \text{ m}$ 10^{48} 10^{49} Poloidal 10^{48} Toroidal $\Delta x = 12.5 \text{ m}$ 10^{47} = 100 m 10^{47} $\Delta x = 12.5 \text{ m} \rightarrow 50 \text{ m}$ 10^{46} = 200 m x = 12.5 m, B15 $\Delta x = 50 \text{ m} \rightarrow 100 \text{ m}$ 10^{45} $\Delta x = 12.5 \text{ m}, B14$ $\Delta x = 200 \text{ m}$ 10^{46} 257510015050 125() $t - t_{\text{merger}} \pmod{1}$

▶ B-field is amplified by the Kelvin-Helmholts instability at the merger (Rasio & Shapiro 95, Price & Rosswog 06, KK et al. 14,15,18)

⇒ Efficient magnetic winding with the coherent poloidal field for t – t $_{merger} \ge$ 20 ms



Kelvin-Helmholtz dynamo at the merger



► In reality, the KH dynamo would produce a strong, but small-scale magneto turbulence (see also Palenzuera et al. 22, Aguilera-Miret et al. 22, 23). A mechanism to generate a globally coherent B-field is necessary.

Mean B-field in MRI-active region



Deep inside (Outside) core is MRI-inactive (active) region
Bulk EM energy is contained in the MRI-inactive region.

Mean B-field in MRI-active region Electromagnetic energy in MRI-active region



Mean B-field in MRI-active region

Mean-Poloidal magnetic flux on a sphere



Mean-Poloidal flux stays an approximately constant in MRI-inactive region
Mean-Poloidal flux increases in MRI-active region
High resolution is key

A large scale $\alpha \Omega$ dynamo driven by MRI Mean field dynamo theory $\mathbf{Q} = \bar{\mathbf{Q}} + \mathbf{q}, \ \bar{\mathbf{Q}} = \text{Axisym. Ave.}$ $\partial_t \bar{\mathbf{B}} = \nabla \times (\bar{\mathbf{U}} \times \bar{\mathbf{B}} + \bar{\mathcal{E}}),$ $\bar{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$ **u** & **b** : turbulence of the velocity and b-field. $\alpha \Omega$ dynamo $\mathcal{E}_i = \alpha_{ij} B_j + \beta_{ij} (\nabla \times B)_j$ In the current context, α_{ii} is dominant $\partial_t \bar{B}_\phi = R \bar{\mathbf{B}}_P \cdot \nabla_P \Omega \ (\Omega \text{ effect}),$ $\partial_t \bar{B}_R = -\partial_z \mathcal{E}_\phi \approx -\partial_z (\alpha_{\phi\phi} \bar{B}_\phi) \ (\alpha \text{ effect}),$ $\partial_t \bar{B}_z = \partial_R \mathcal{E}_\phi \approx \partial_R (\alpha_{\phi\phi} \bar{B}_\phi)$

Generation of a large-scale field via $\alpha~\Omega$ dynamo

<u>MRI-driven</u> α Ω dynamo theory prediction

- 1. B_{ϕ} should be anticorrelated with B_{p}
- 2. E_{ϕ} should be correlated or anti-correlated with B_{ϕ}
- 3. Dynamo cycle period P_{theory} = 2 π ($\alpha_{\phi \phi} d\Omega / dlnR k_z/2$)^{-1/2}
- 4. Dynamo wave propagation direction according to the Yoshimura-Parker rule $\alpha_{\phi\phi} \nabla \, \Omega x \, e_{\phi}$

Prerequisite:

Magneto Rotational Instability (MRI) should be resolved to generate EMF

 $\bar{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$

Generation of a large-scale field via $\alpha \ \Omega$ dynamo



MRI is well resolved in $\Delta x=12.5m \text{ run} \Rightarrow \text{Turbulence}$ is developed MRI is not resolved in $\Delta x=200m \text{ run} \Rightarrow \text{No turbulence}$



Generation of a large-scale field via $\alpha \ \Omega$ dynamo



Table 1 The $\alpha\Omega$ dynamo period prediction and simulation data at several radii

R (km)	$\alpha_{\phi\phi}$ (cm/s)	Ω (rad/s)	Shear rate	k_z (/cm)	$P_{\mathrm{theory}}\left(\mathbf{s}\right)$	P_{sim} (s)
20 30 40 50	$\begin{array}{c} -8.1\times 10^6 \\ -1.0\times 10^7 \\ -1.0\times 10^7 \\ -4.4\times 10^6 \end{array}$	$4025 \\ 2515 \\ 1688 \\ 1200$	q = -1.0 q = -1.34 q = -1.44 q = -1.50	$6.3 imes 10^{-6}$ $4.2 imes 10^{-6}$ $3.3 imes 10^{-6}$ $2.6 imes 10^{-6}$	0.020 0.021 0.037 0.062	0.018 0.018-0.024 0.018-0.030 0.030-0.040

Generation of a large-scale field via $\alpha~\Omega$ dynamo



Dynamo wave propagates to the z direction according to the Yoshimura-Parker rule $\alpha_{\phi\phi} \nabla \Omega x e_{\phi}$

Generation of a large-scale field via $\alpha~\Omega$ dynamo



Waves generated in the MRI-active region propagates towards the polar
The B-field deep inside the core in the polar region stays buried throughout the simulation

Launching a relativistic outflow



▶ Poynting flux dominated outflow launched by the magnetic tower outflow is $L_{poy} \sim 10^{51} \text{ erg/s}$

Fixed Relativistic outflow is confined in a region with $\Theta\!\sim\!12^\circ$ 、 $\Gamma_{\infty}\!pprox\!10\text{--}20$

Lorentz force-driven mass ejection



Summary

► Direct numerical relativity modeling of GW sources is essential to interpret/predict GW event ⇒ Multimessenger (GW+EM+neutrino?)

For example, O4 runs now. If the information of the GW source is available,

Source parameter of binary neutron star: Mass of each NSs + unknown equation of state

Ab initio simulation on super computers

Multimessenger signals GW + EM + ?

Back up slide

Application II : Numerical modeling of BH-NS merger

Black hole – Neutron Star merger



LIGO-VIRGO-KAGRA collaboration 21

► GW200105, GW200115 *no EM counterpart

Application II : Numerical modeling of BH-NS merger

Numerical Relativity-Neutrino-Radiation-Magnetohydrodynamics simulation of BH-NS merger (Hayshi,, KK, et al. 21)

► Neutrino radiation transfer is necessary to predict Y_e of the ejecta

► Magnetohydrodynamics is necessary to reveal the massive torus evolution, in particular, the angular momentum transport

► Merger simulation is necessary to build a self-consistent model of the massive torus formation

Extremely long-term simulation (\approx 2 seconds)



Numerical modeling of BH-NS merger MRI works?

Butterfly diagram



► MRI-dynamo works \Rightarrow Effective turbulent viscosity ($\alpha \approx 0.01$) \Rightarrow Torus expands due to the angular momentum transport

Numerical modeling of BH-NS merger

Neutrino luminosity

Gravitational unbounded baryonic mass



A part of the viscous heating is consumed by the neutrino emission
⇒ Temperature decreases due to the torus expansion
⇒ At some point, the neutrino emission becomes inefficient.
⇒ All the viscous heating is used for the torus expansion

Numerical modeling of BH-NS merger

Electron fraction distribution of gravitationally unbounded material



► Two distinct peaks Low Y_e component ⇒ Dynamical ejecta ⇒ NIR band emission High Y_e component ⇒ Post-merger ejecta ⇒ Optical band emission

Time: 0.01 ms









_ 1.0e+10	
5e+9	
_ _ 2e+9	c]
_ 1e+9	[g/c
_ 5e+8	d
_ _ 2e+8	
_ 1.0e+08	





Numerical modeling of BH-NS merger Magnetically tower "jet"



Isotropic Poynting Luminosity



► Magnetically tower "jet" builds up magnetosphere ⇒ L_{iso} and θ_{jet} are roughly consistent with the observed values.

Numerical modeling of BH-NS merger Poynting flux distribution

t = 400.20 ms



t = 1000.26 ms



► After 1-2 seconds, the opening angle increases due to the torus expansion \Rightarrow Agree with the observed duration of the Short Gamma-Ray Bursts.