

In Pursuit of Multiple Scalars @ Colliders



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Kennesaw State University, GA, USA
@ Nikhef, NL [May 23rd 2024]



Disambiguation: “Multiple Scalars”

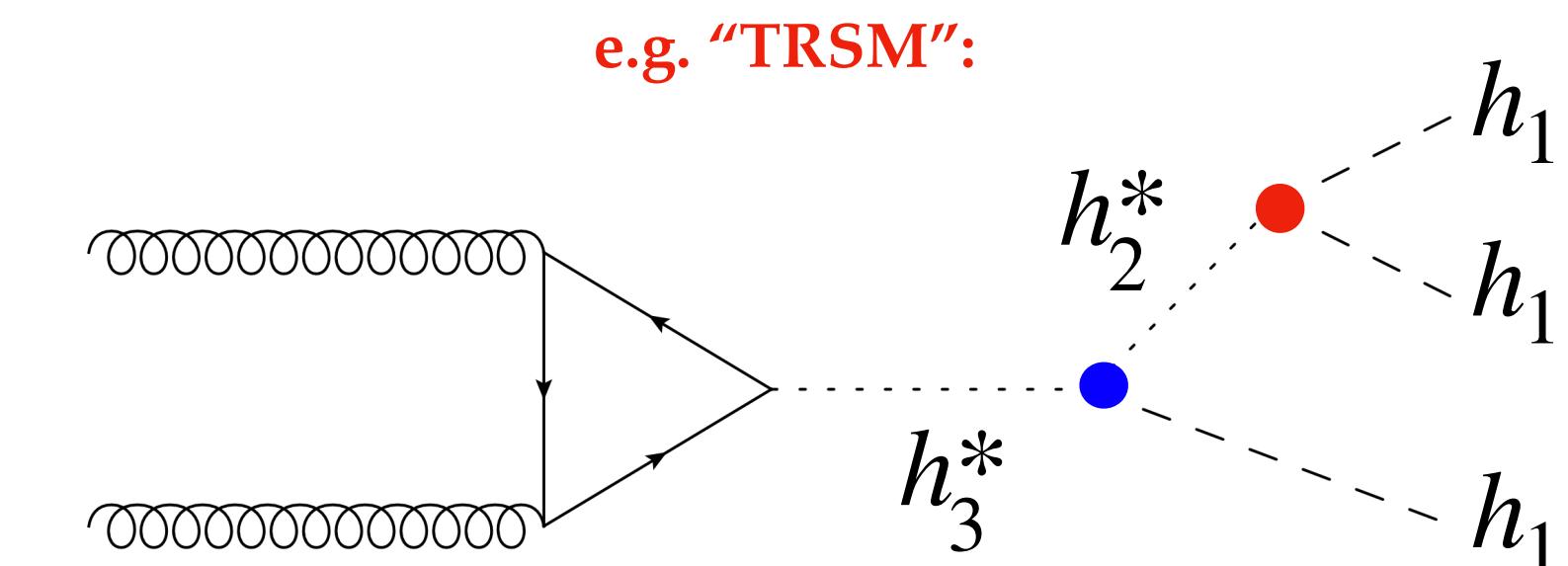
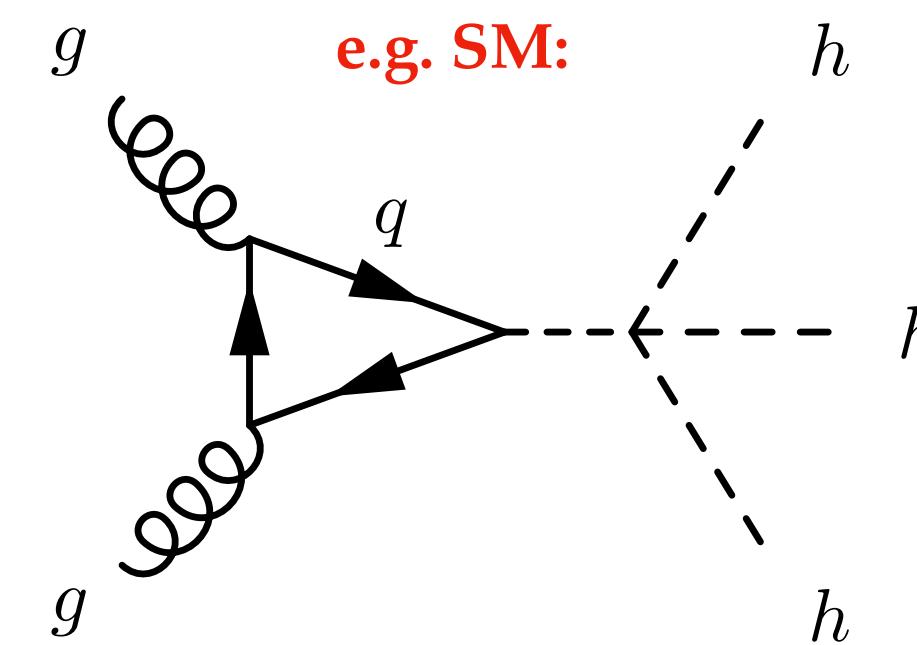


= New scalar fields added to the Standard Model,

e.g. SM + one singlet scalar field: $\mathcal{V}(\phi, S) = \bullet |\phi|^2 + \blacksquare |\phi|^4 + \bullet S^2 + \blacktriangle S^3 + \blacksquare S^4 + \blacktriangle |\phi|^2 S + \blacksquare |\phi|^2 S^2$

and/or

= The production of multiple physical scalar states at colliders.



The Plan:



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1 High Energy Physics Today

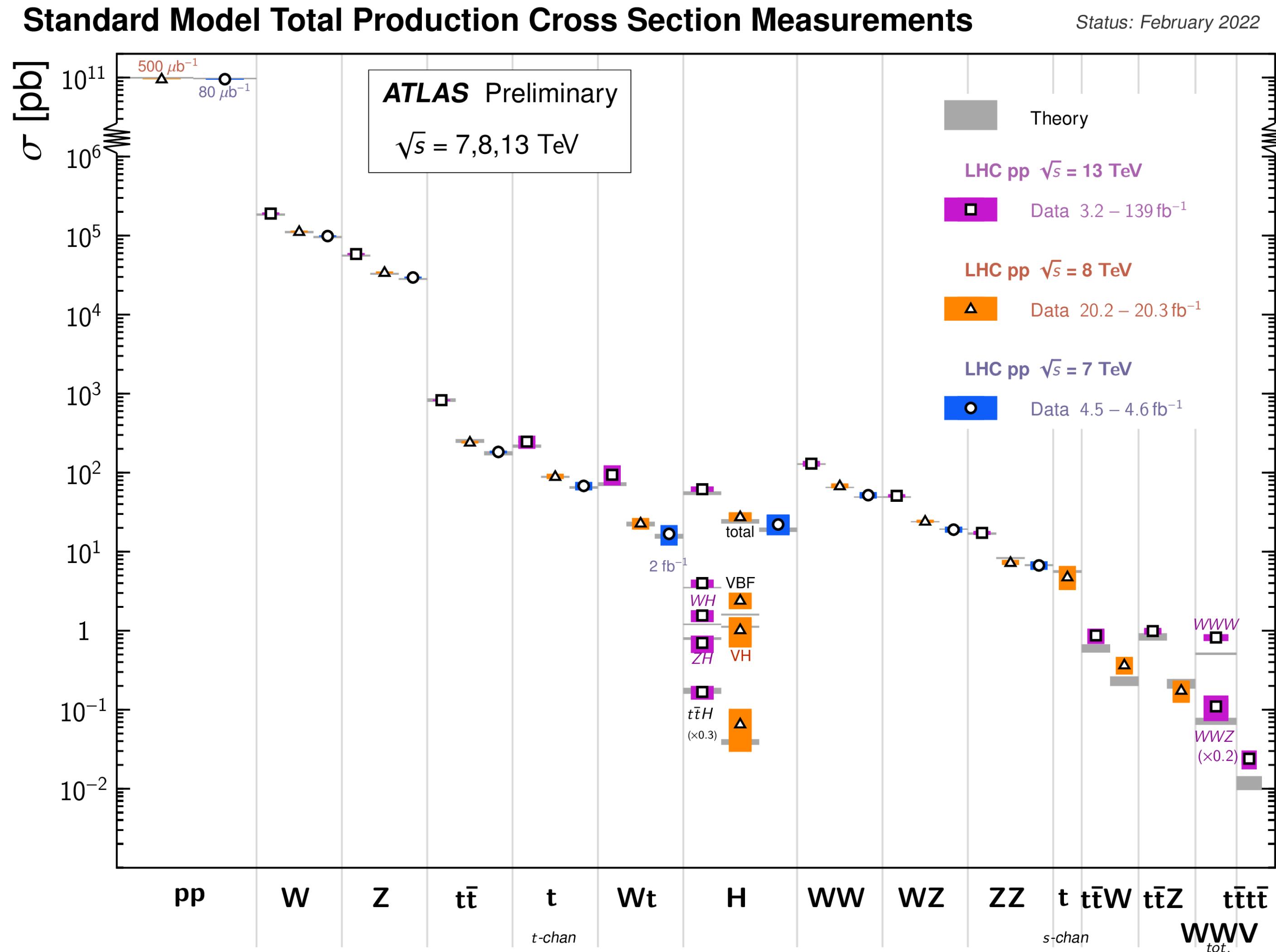
2 → The Breaking of Symmetry,

3 → Multi-scalar production.



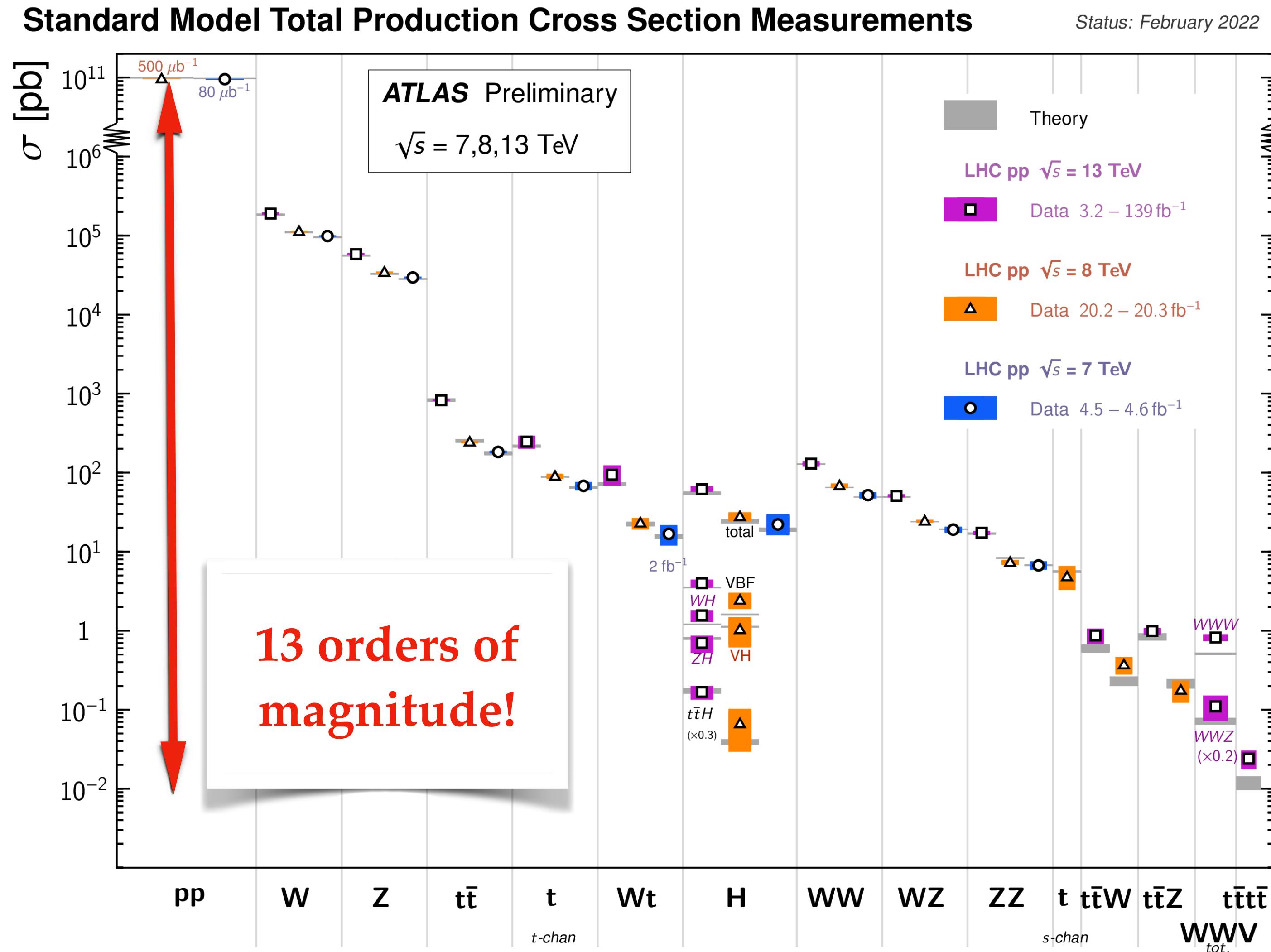
Experiment *VS* Theory

Cross Sections
[pb]



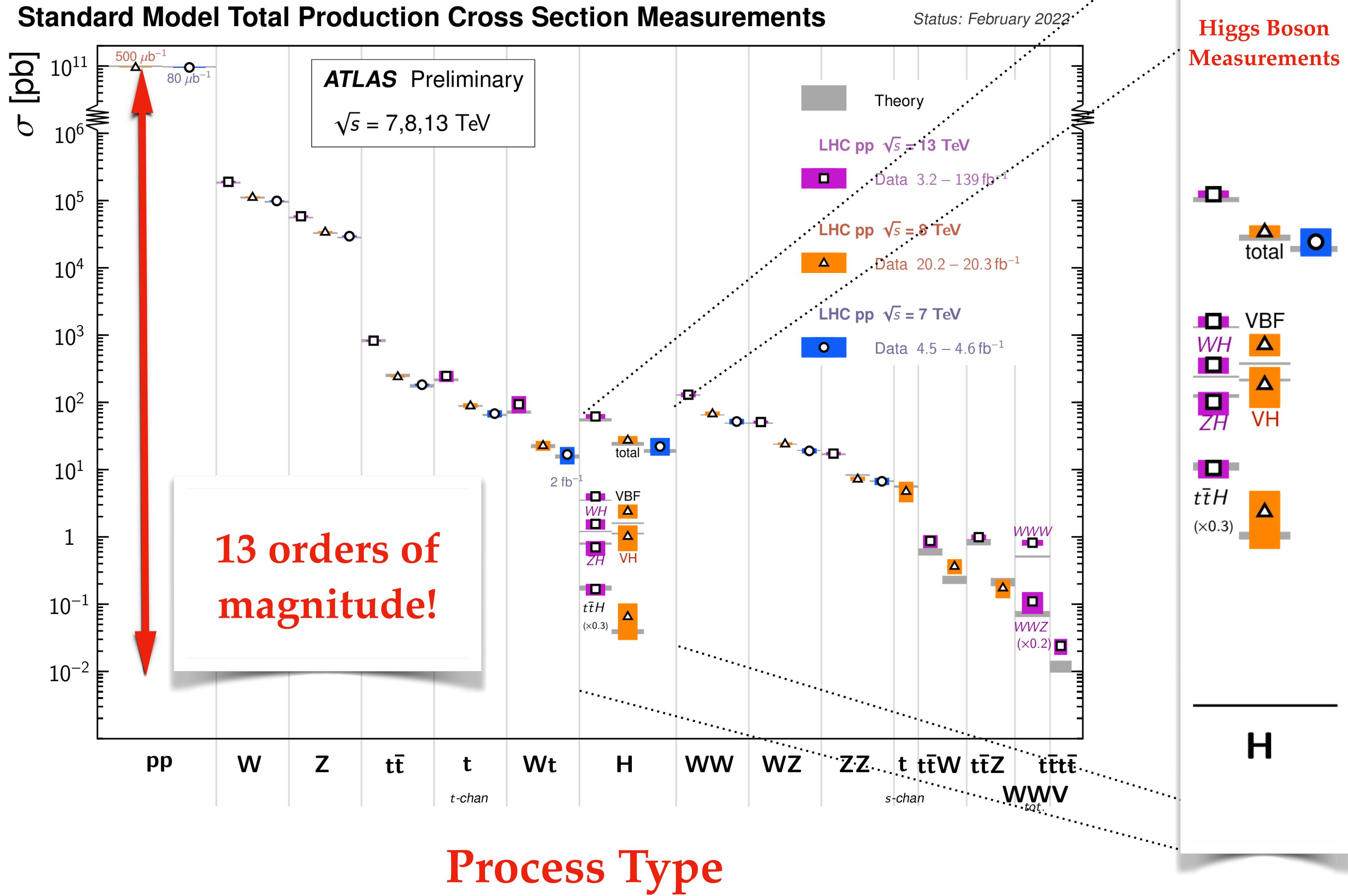
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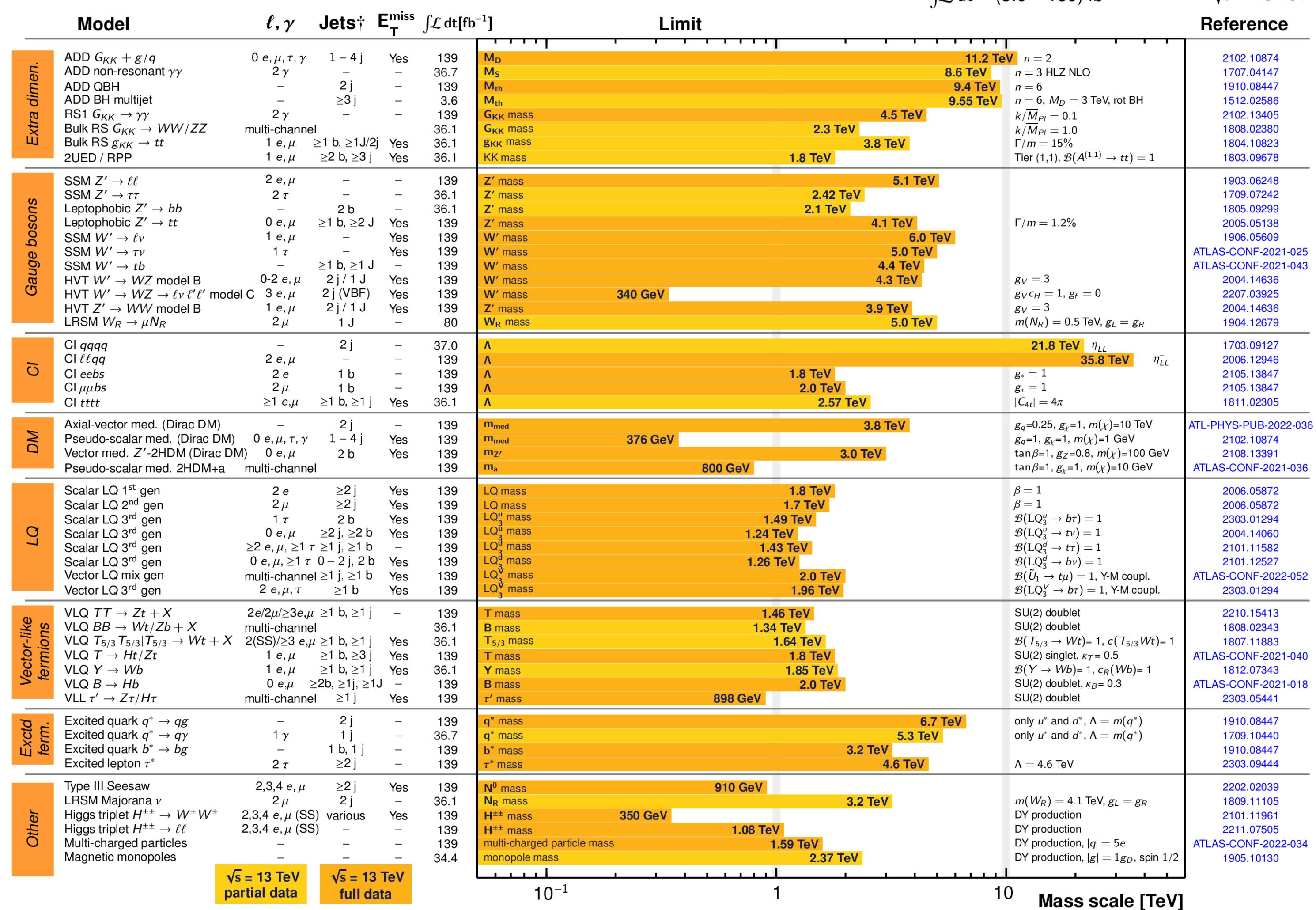
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Experiment VS Exotic New Phenomena

ATLAS Heavy Particle Searches* - 95% CL Upper Exclusion Limits

Status: March 2023

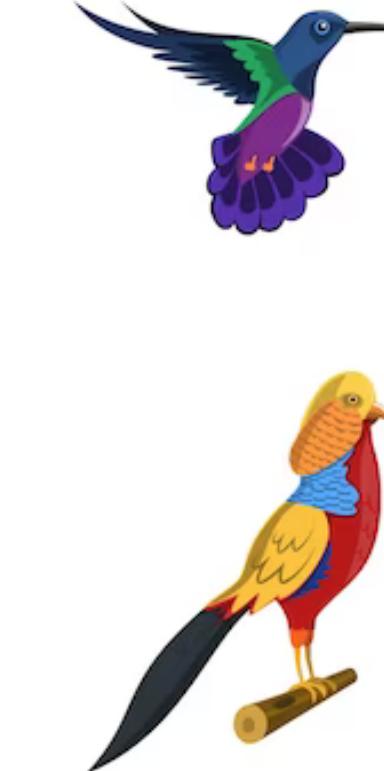


$\sqrt{s} = 13 \text{ TeV}$
partial data

$\sqrt{s} = 13 \text{ TeV}$
full data

ATLAS Preliminary
 $\int \mathcal{L} dt = (3.6 - 139) \text{ fb}^{-1}$

$\sqrt{s} = 13 \text{ TeV}$



Kinds of
New
Physics

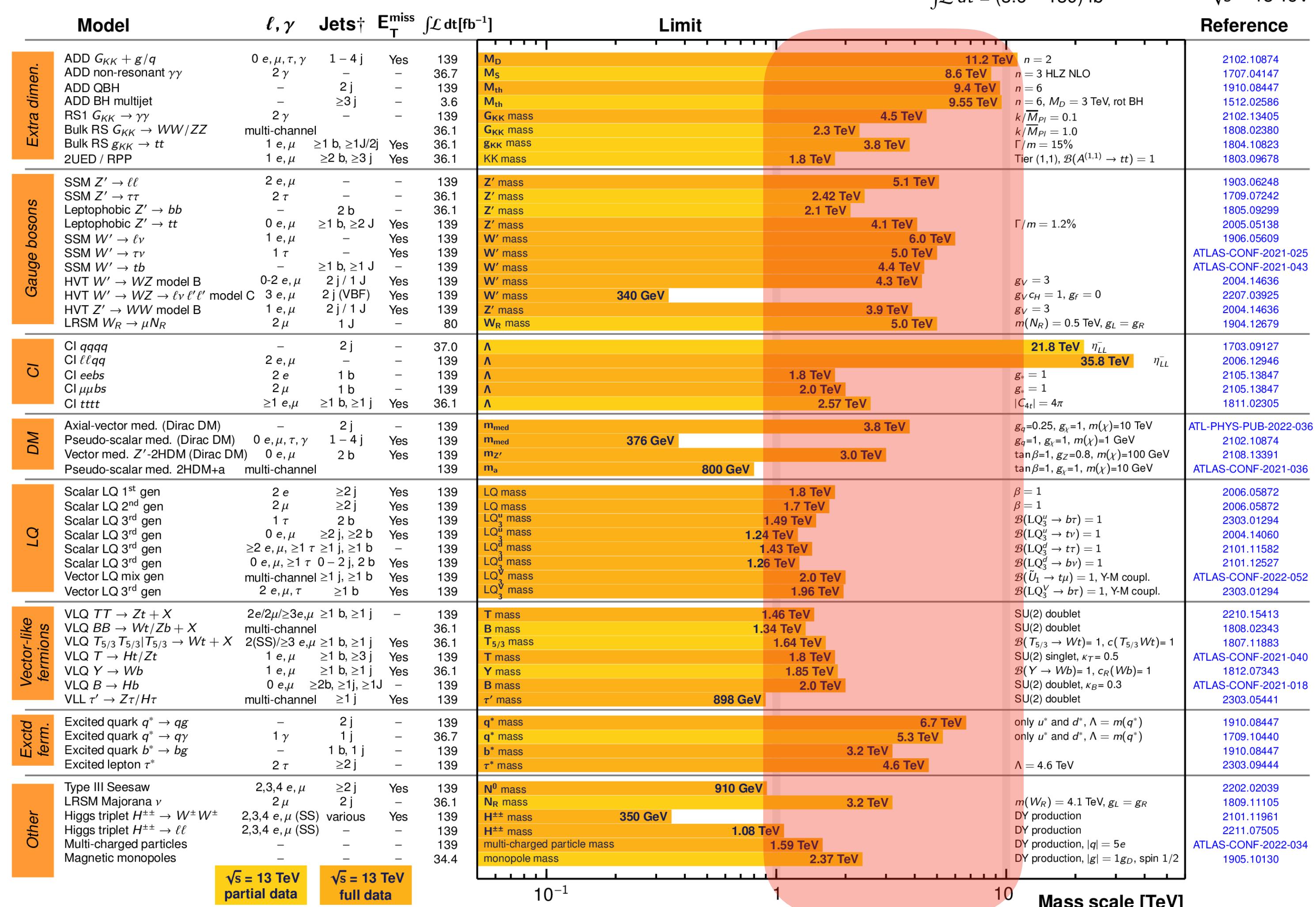
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†Small-radius (large-radius) jets are denoted by the letter j (J).

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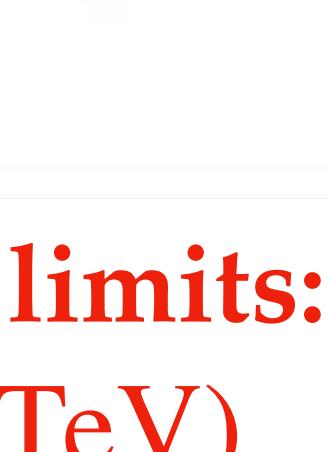
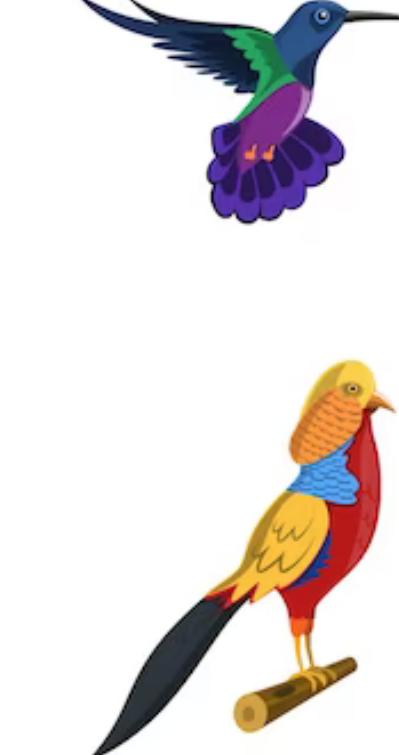
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ATLAS Preliminary

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$\sqrt{s} = 13 \text{ TeV}$



Mass Scale limits:
 $\mathcal{O}(1 - 10 \text{ TeV})$

STANDARD MODEL WINS!



⇒ **NO NEW PHENOMENA?**



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Questions



Questions



Answers



Questions



Answers



Questions

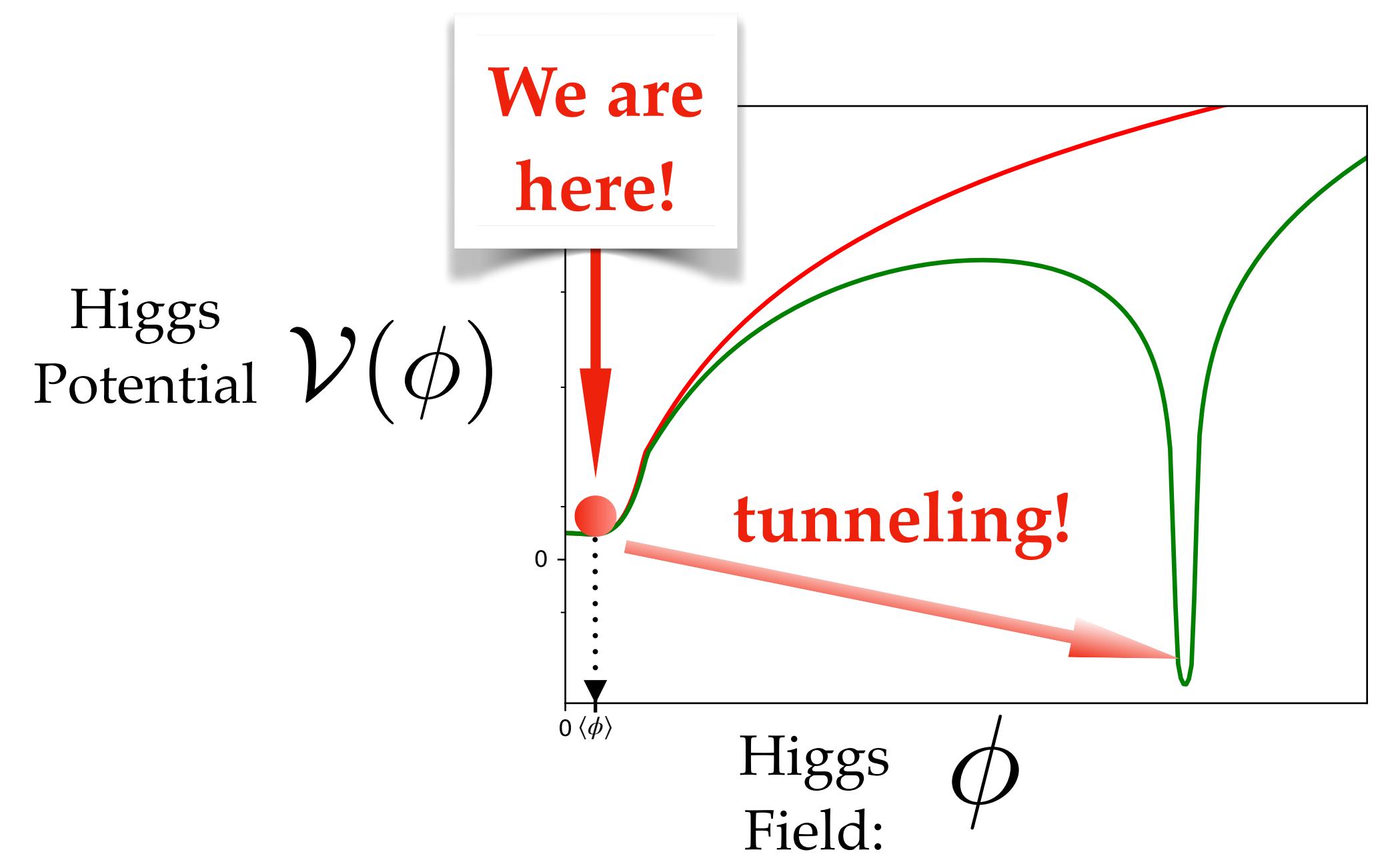


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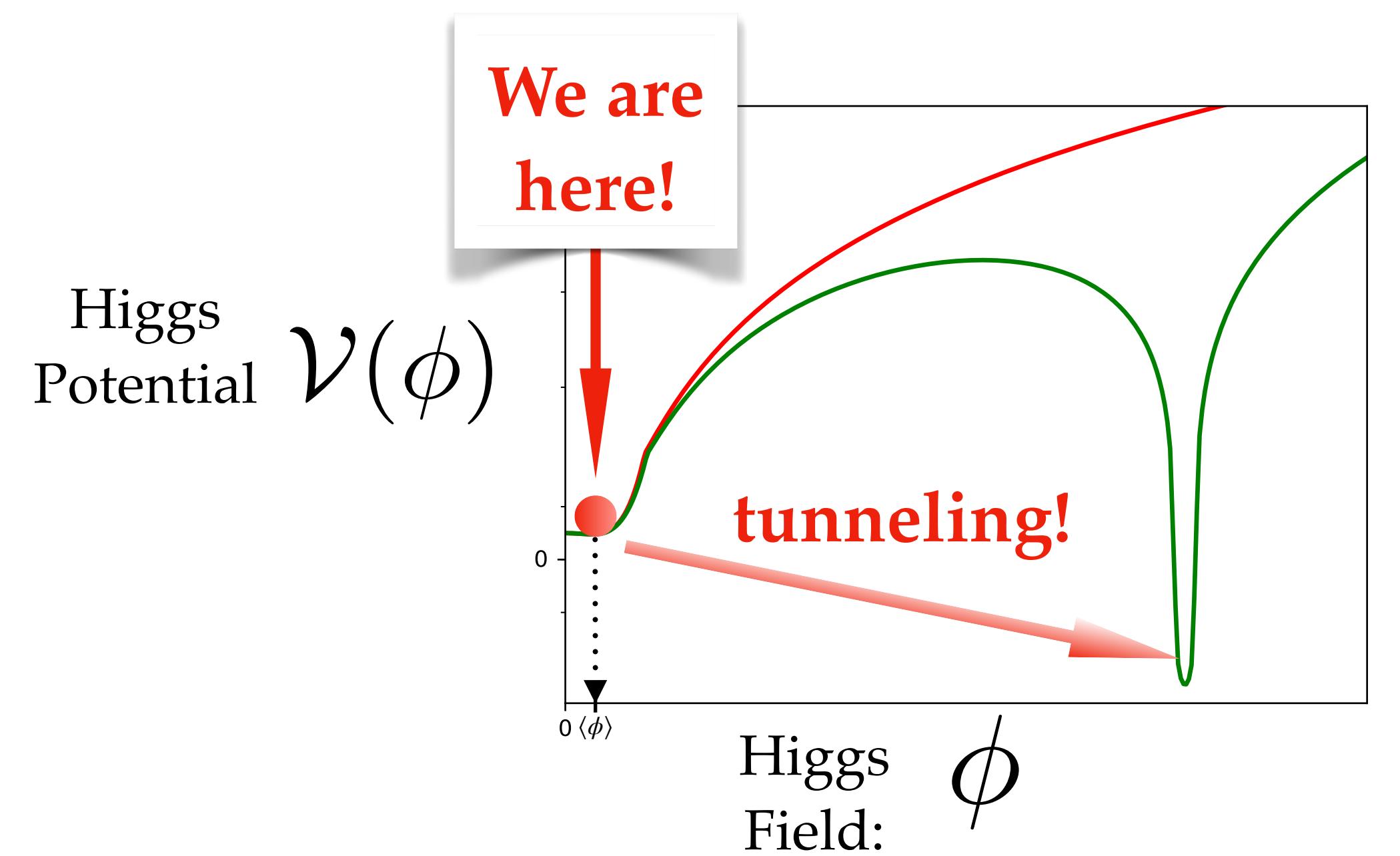


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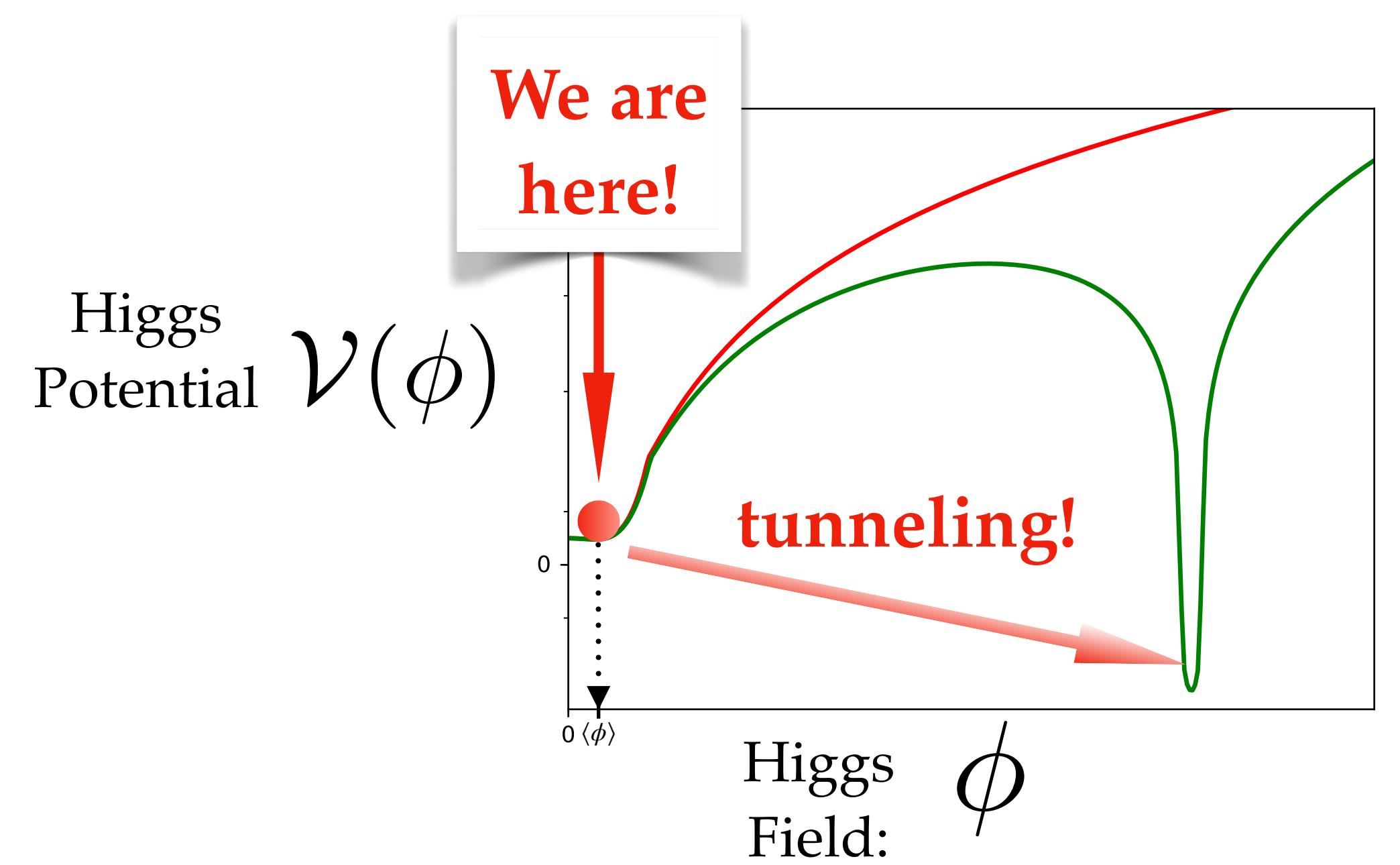


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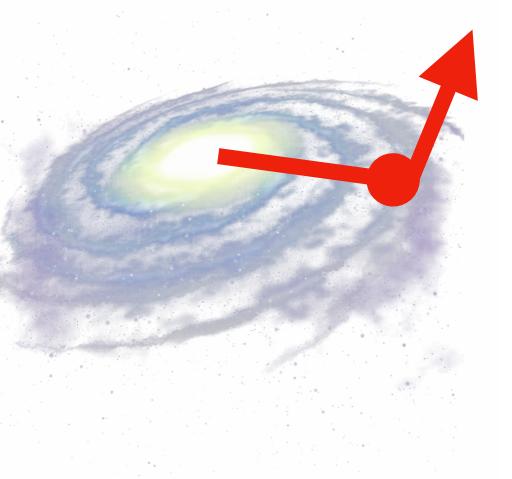
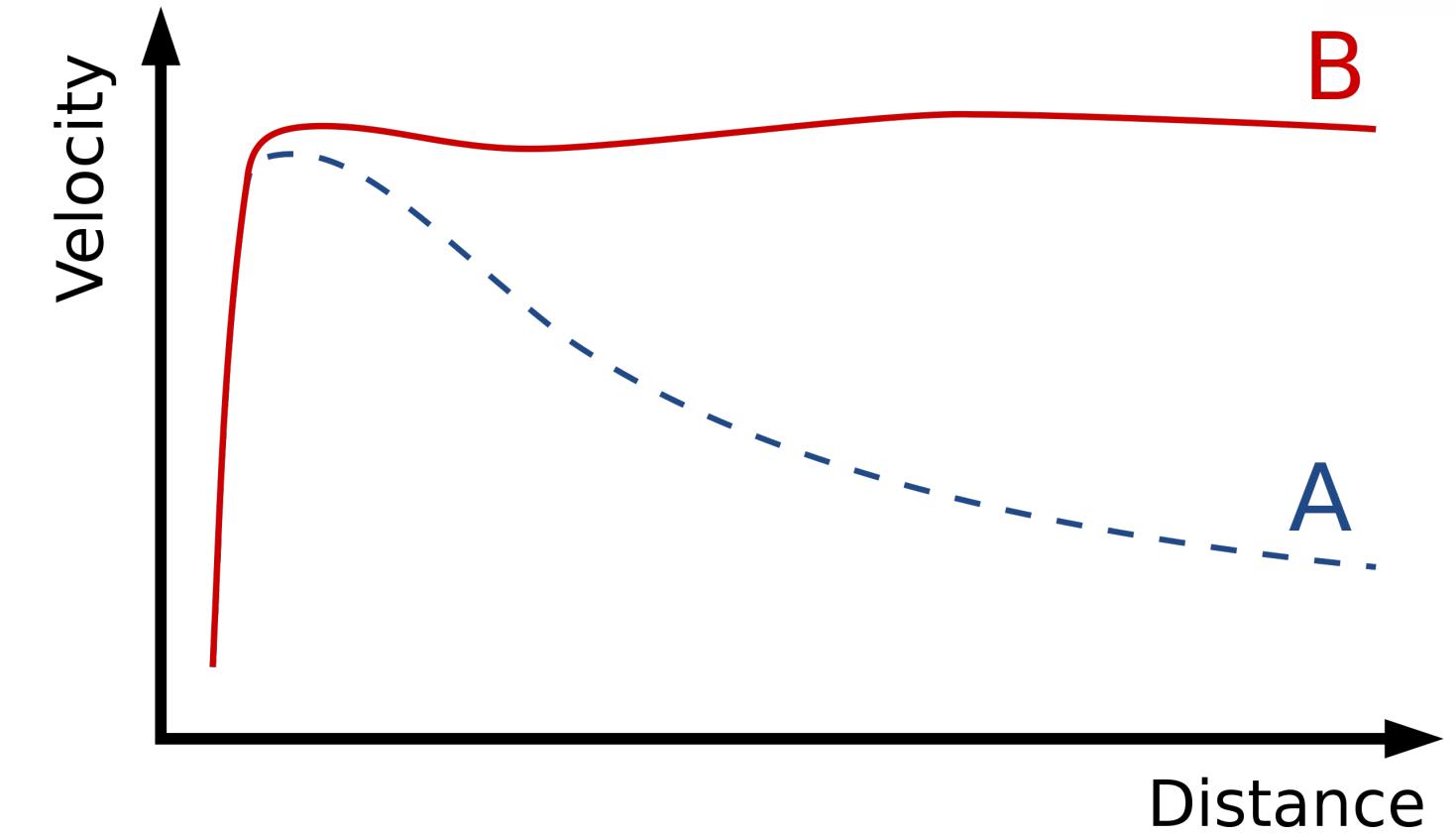
Vacuum Stability

Questions



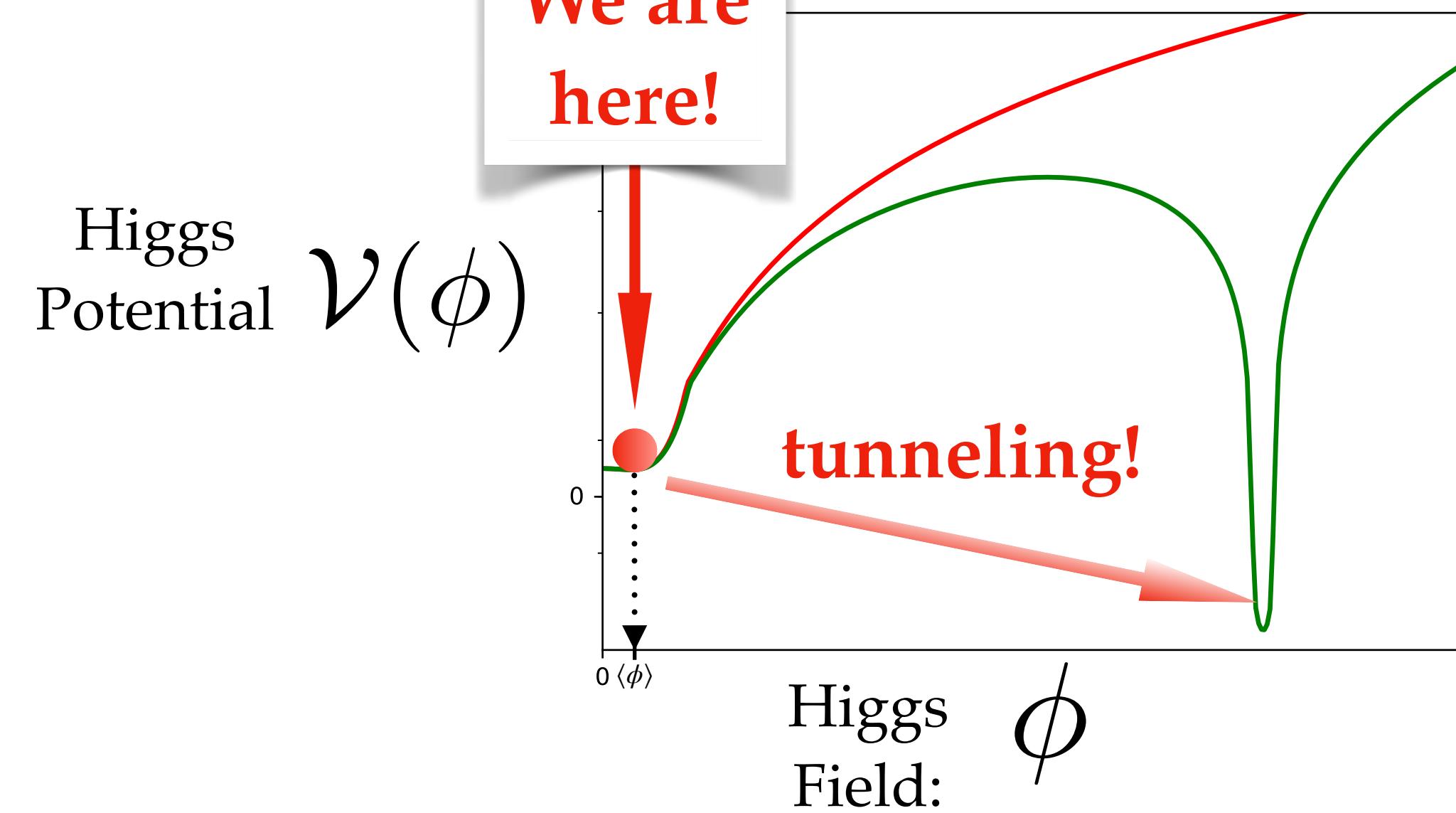
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Rotation curve of a typical spiral galaxy:
predicted (A) and observed (B).



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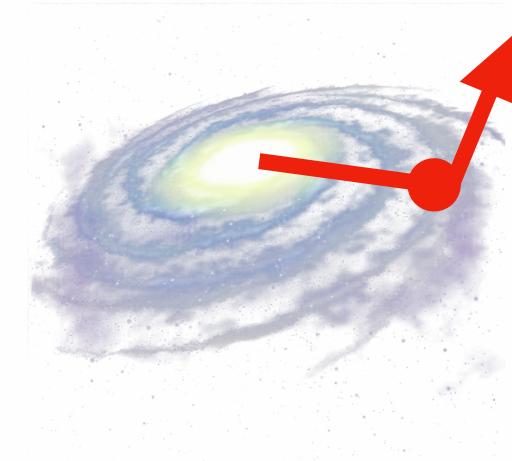
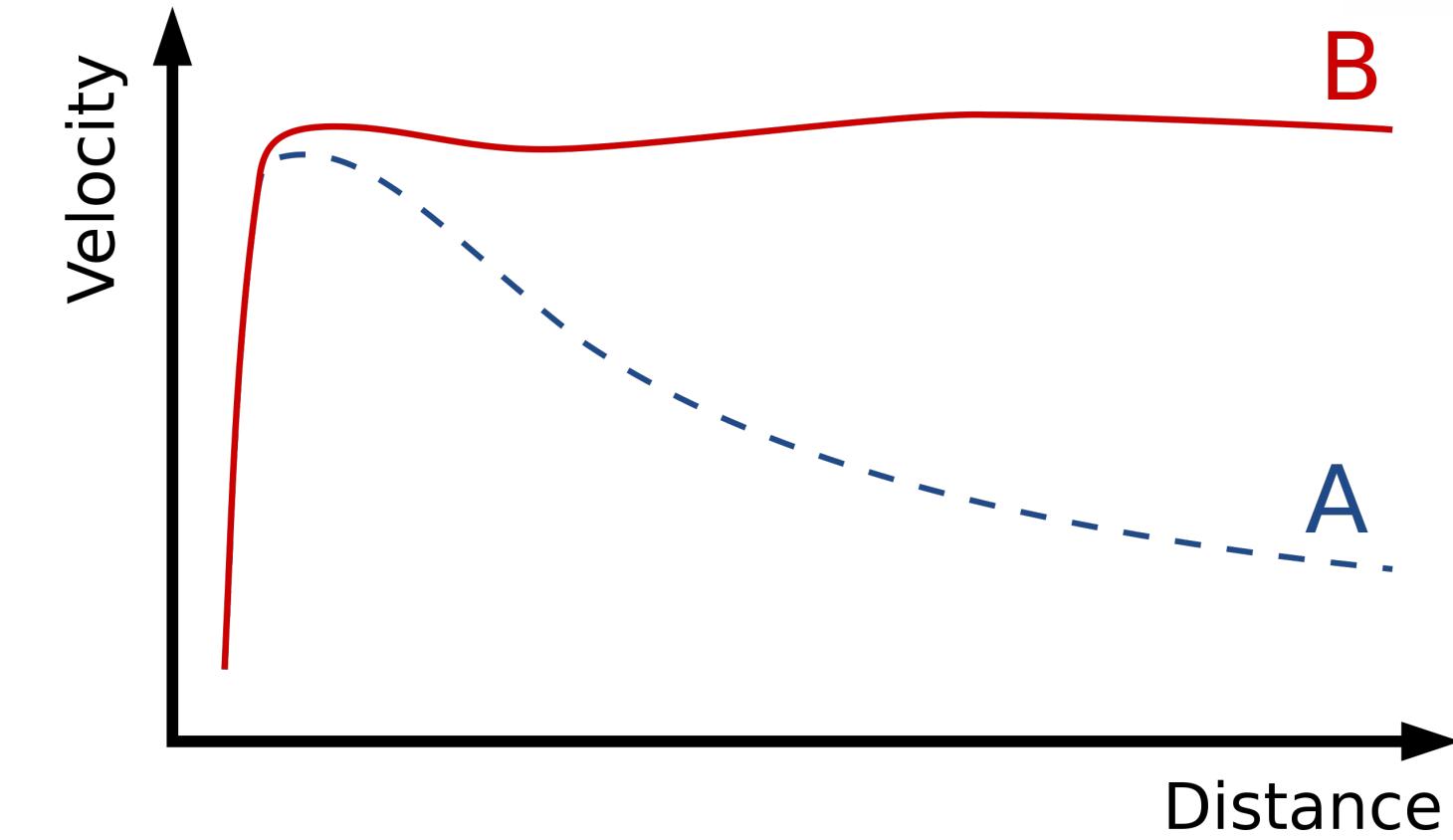
We are here!



Vacuum Stability

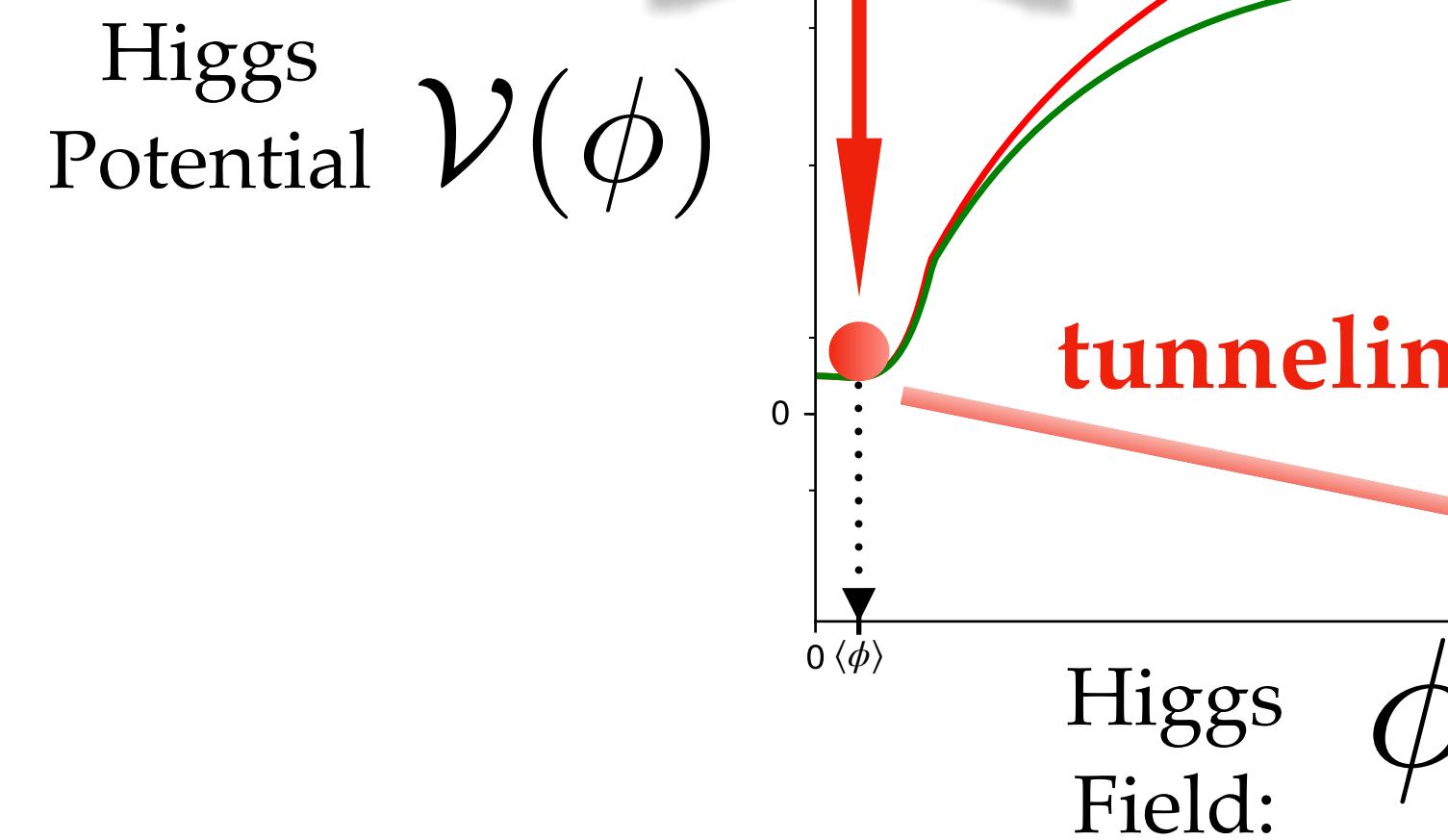
Dark Matter

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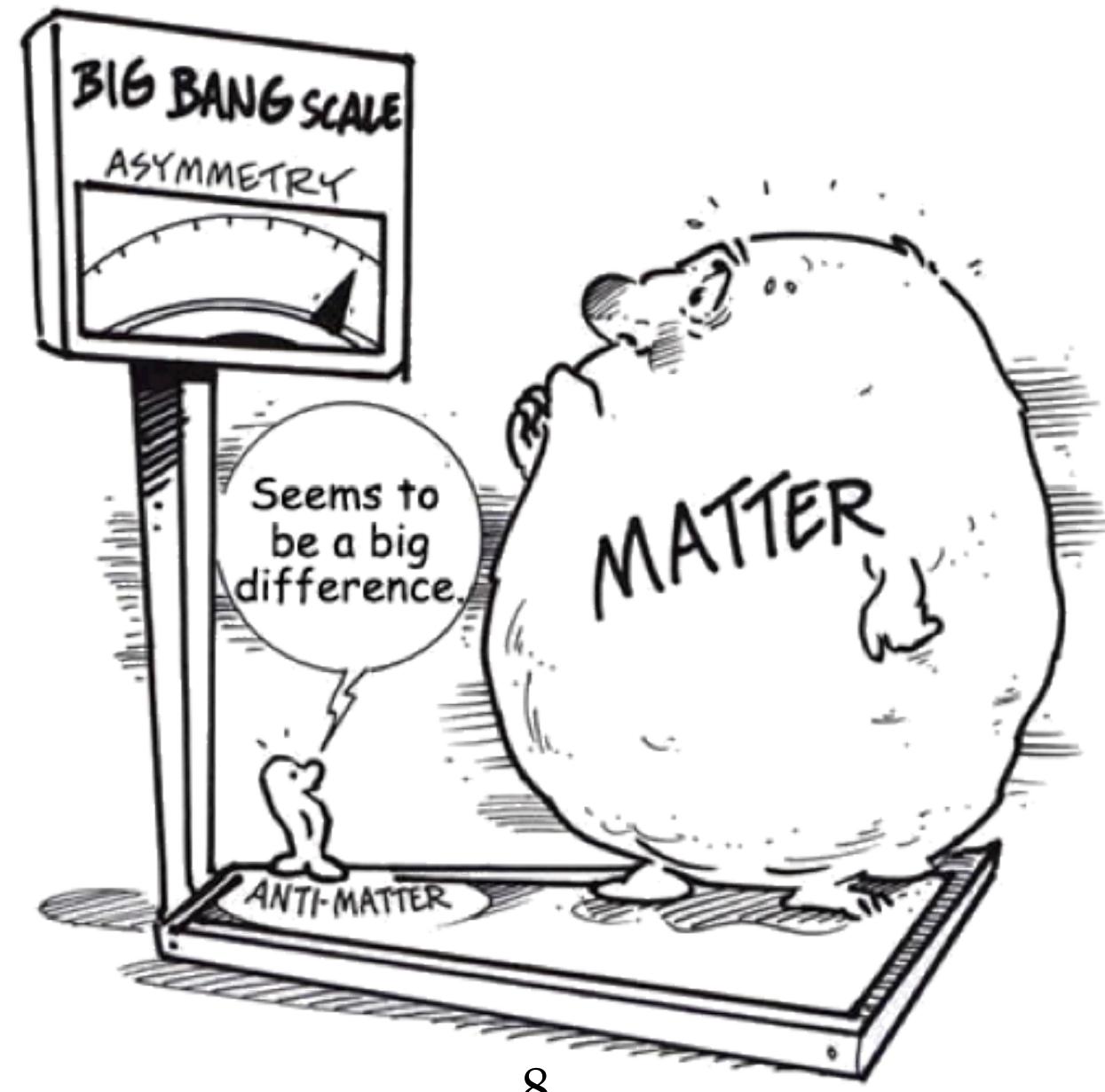


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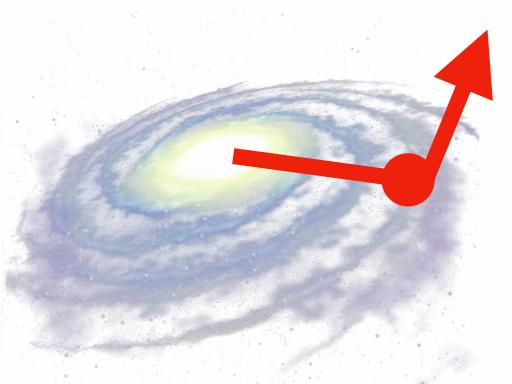
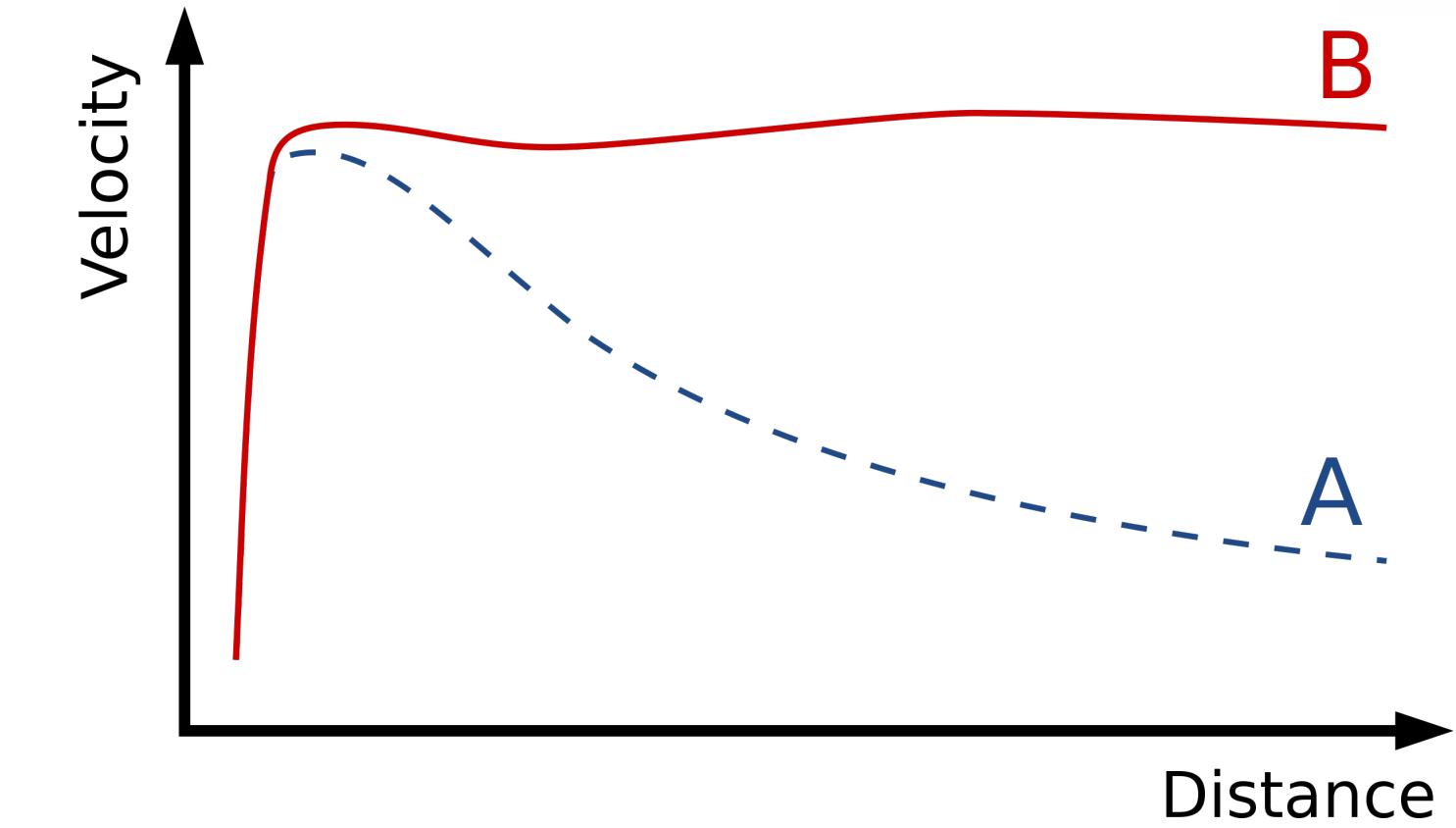


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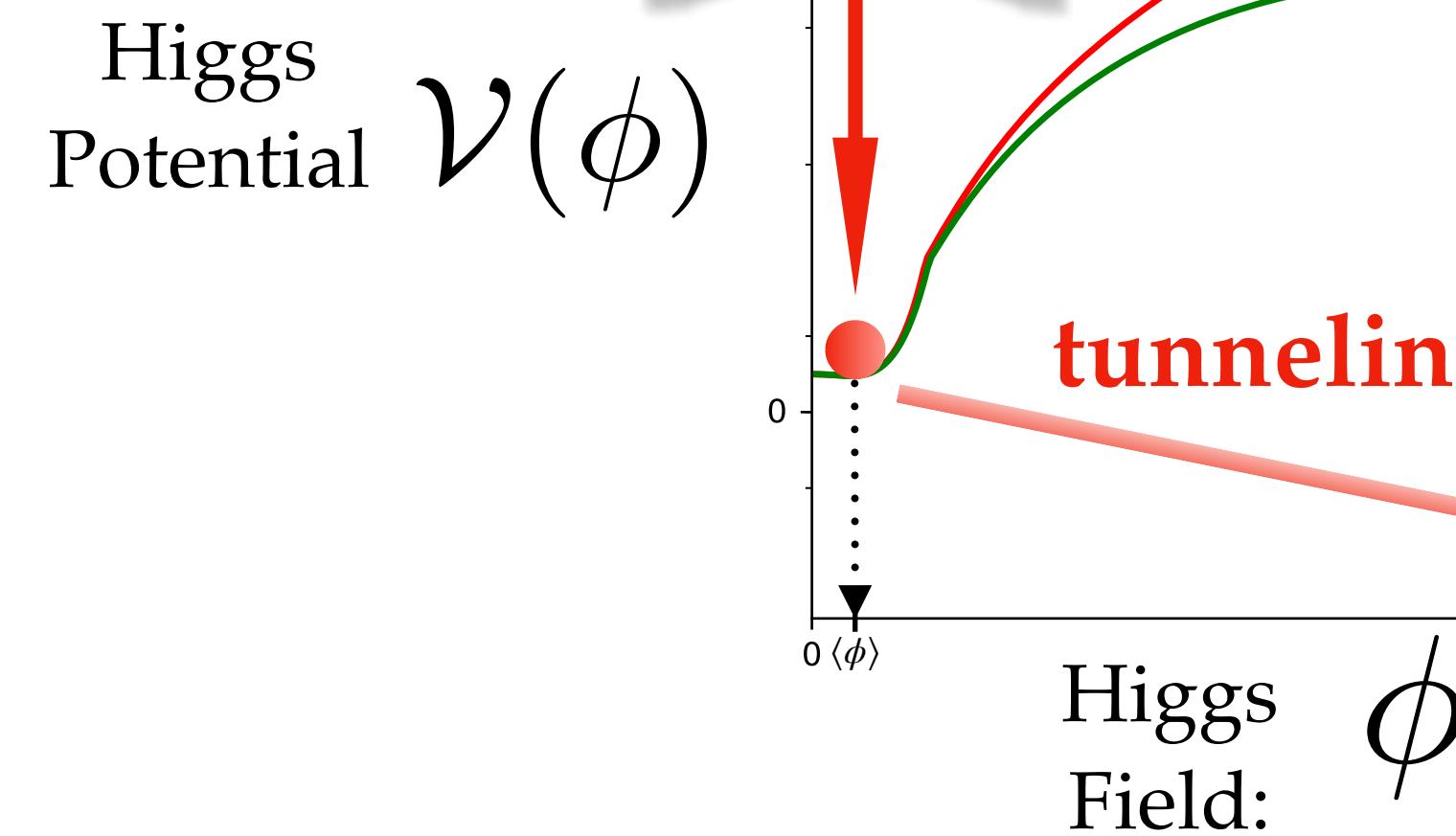
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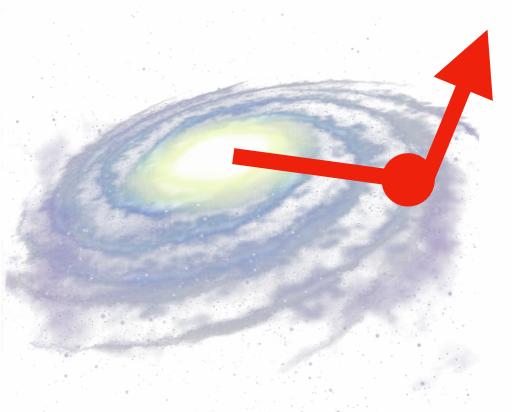
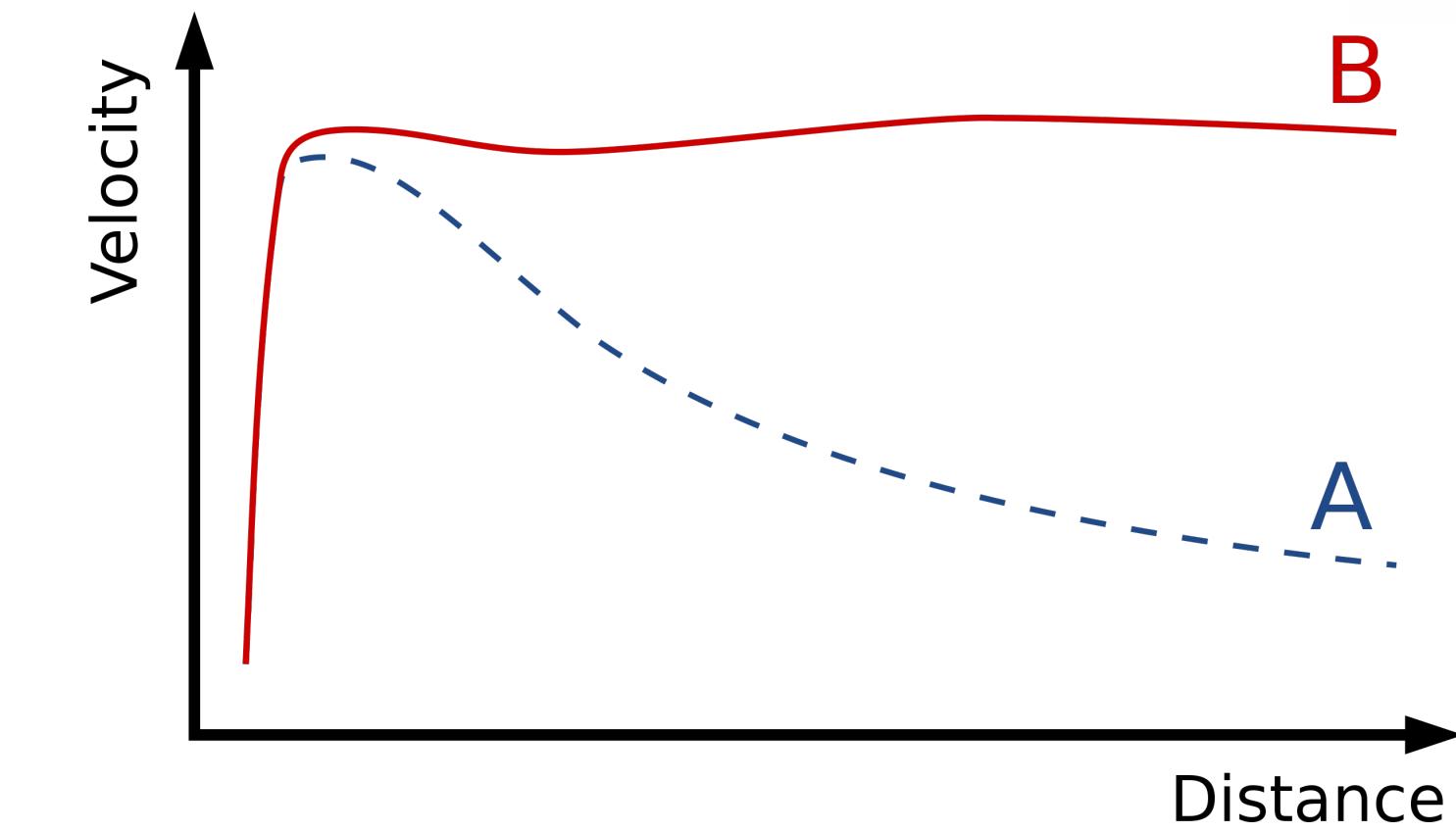


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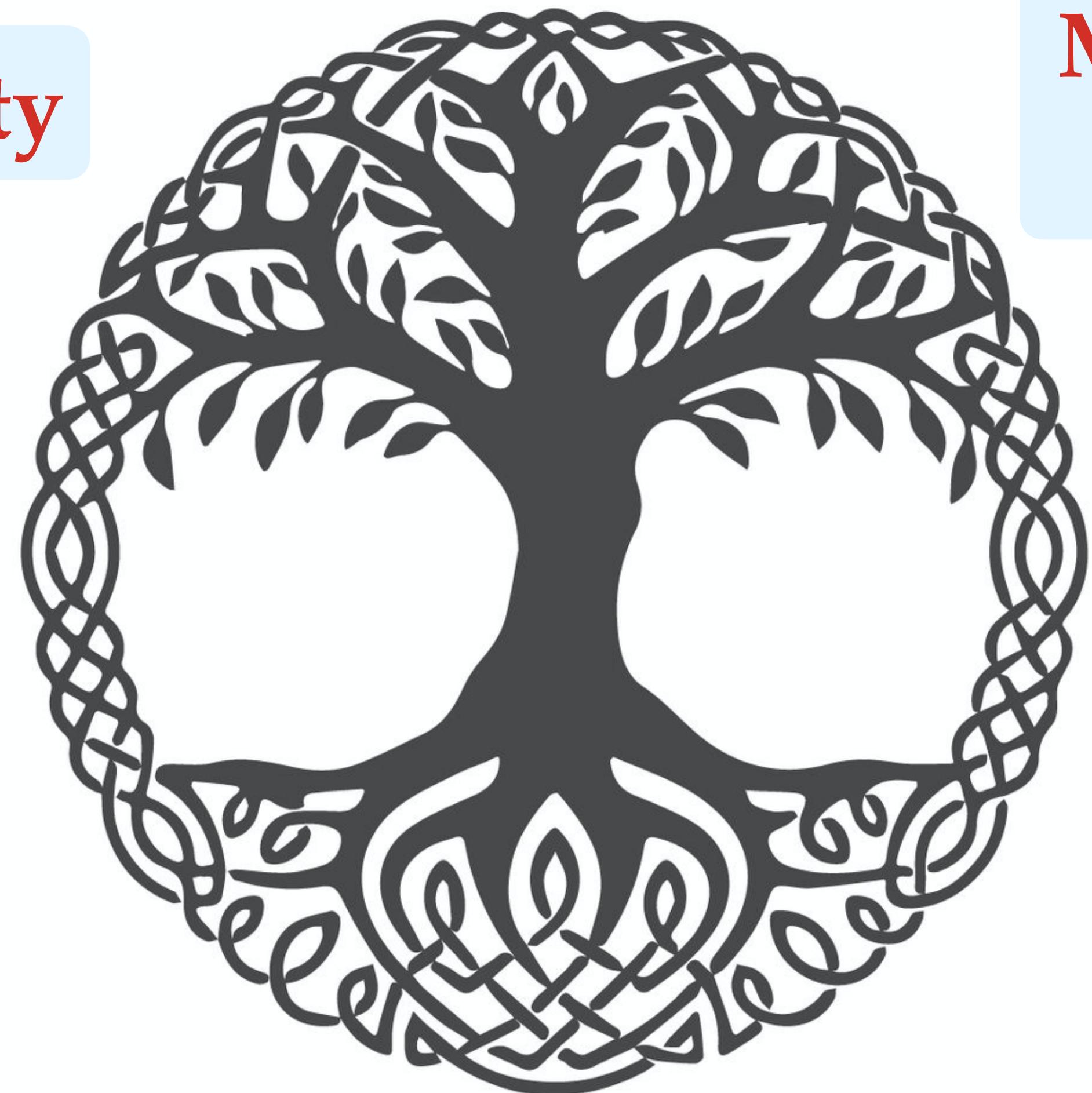
Matter-anti-matter Asymmetry



Vacuum Stability

Dark Matter

Matter-anti-matter
Asymmetry



The Higgs Field & Symmetry Breaking



Vacuum Stability

Dark Matter

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e.g. Future Circular Collider:
 $pp@100 \text{ TeV}, e^+e^-.$

e.g. "High-Energy" LHC:
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e.g. Muon Collider.

The Higgs Field & Symmetry Breaking



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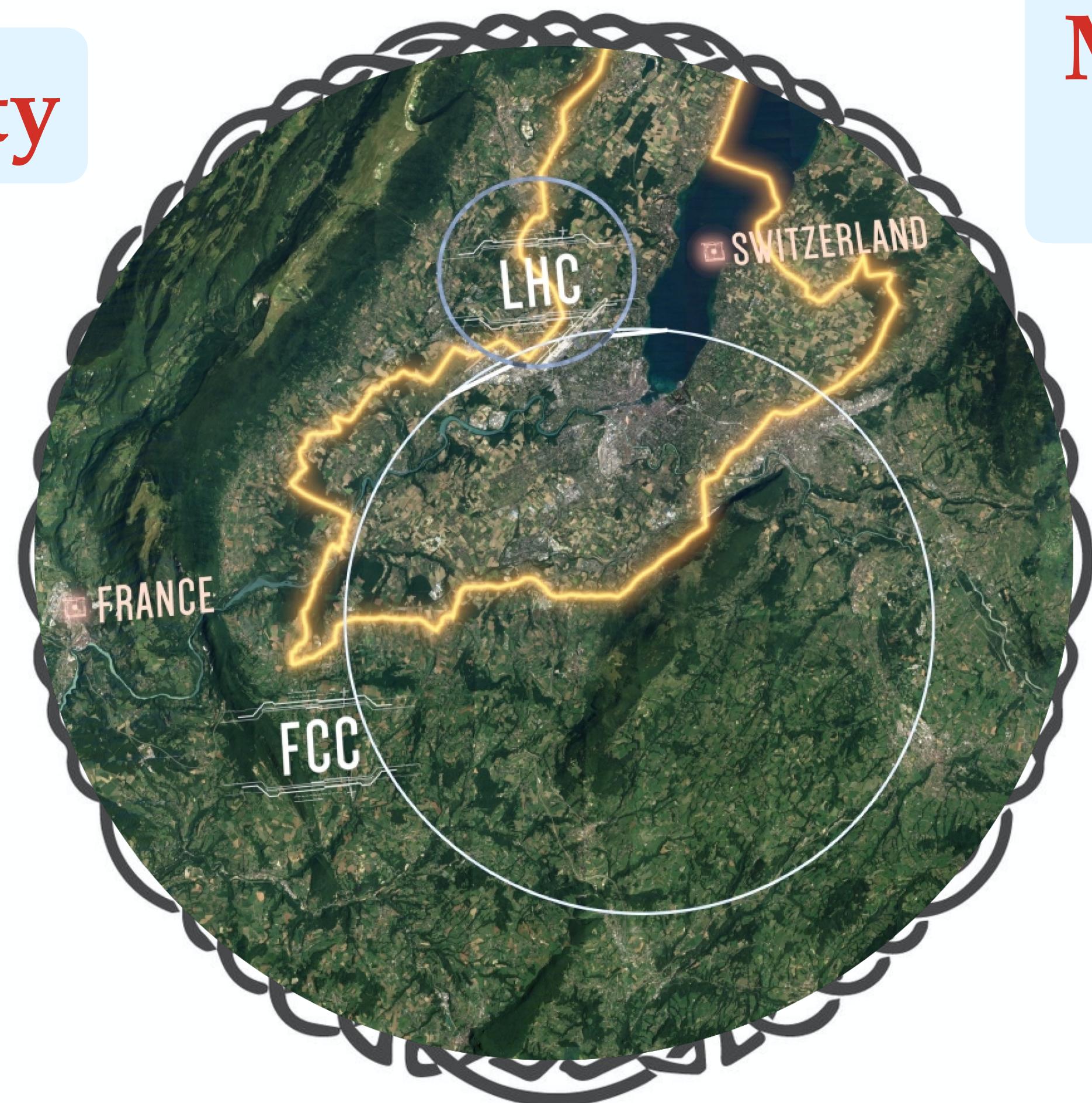
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Breaking the Symmetry

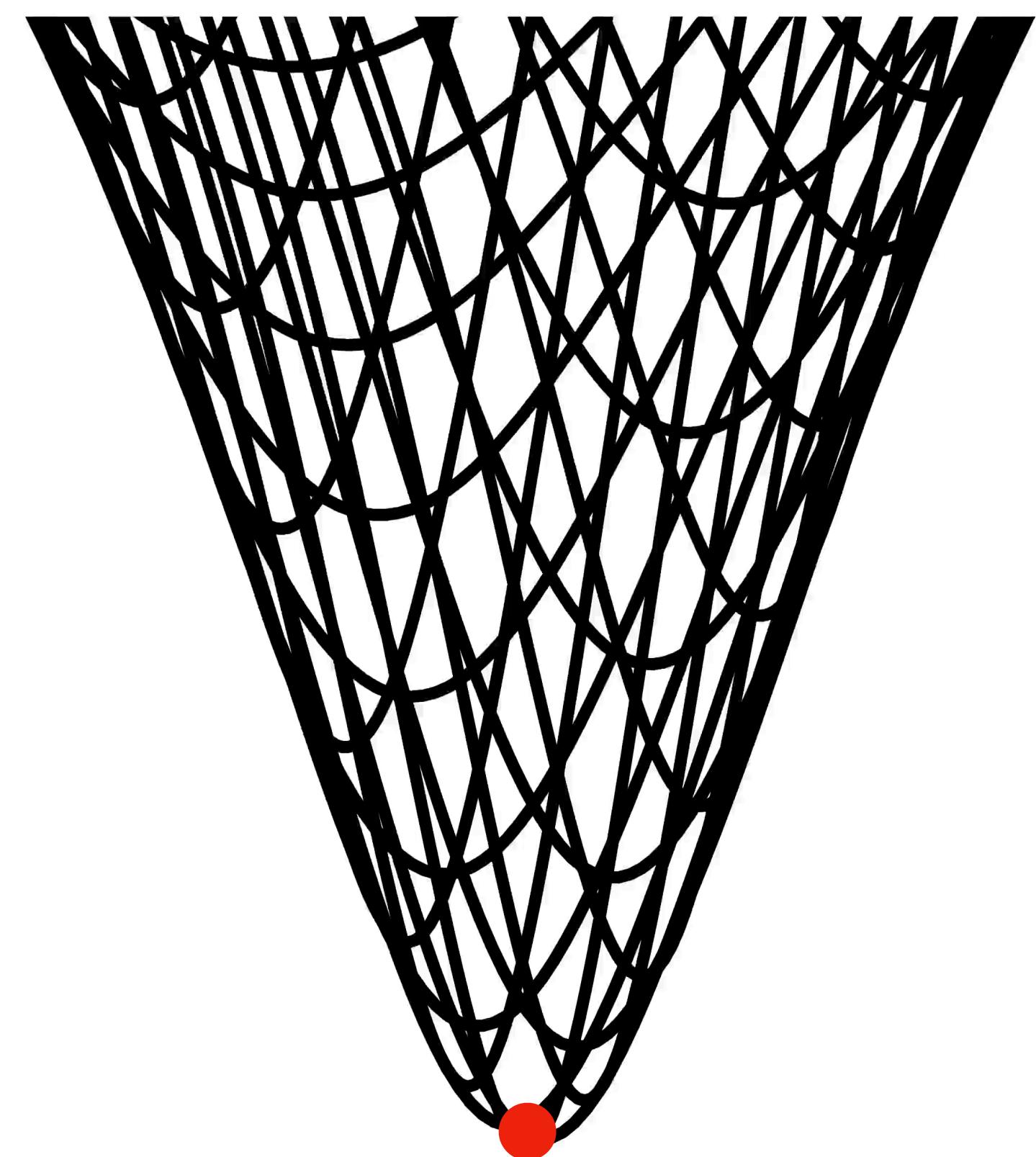


Breaking the Symmetry in the Standard Model

The potential of the Higgs field (ϕ), a complex doublet:

$$\phi = \begin{pmatrix} \phi_1 + i\phi_3 \\ \phi_2 + i\phi_4 \end{pmatrix}$$

(Arbitrarily) Set $\phi_3 = \phi_4 = 0$
to illustrate potential in
 (ϕ_1, ϕ_2) plane.



$\mathcal{V}(\phi)$
Higgs Field Potential

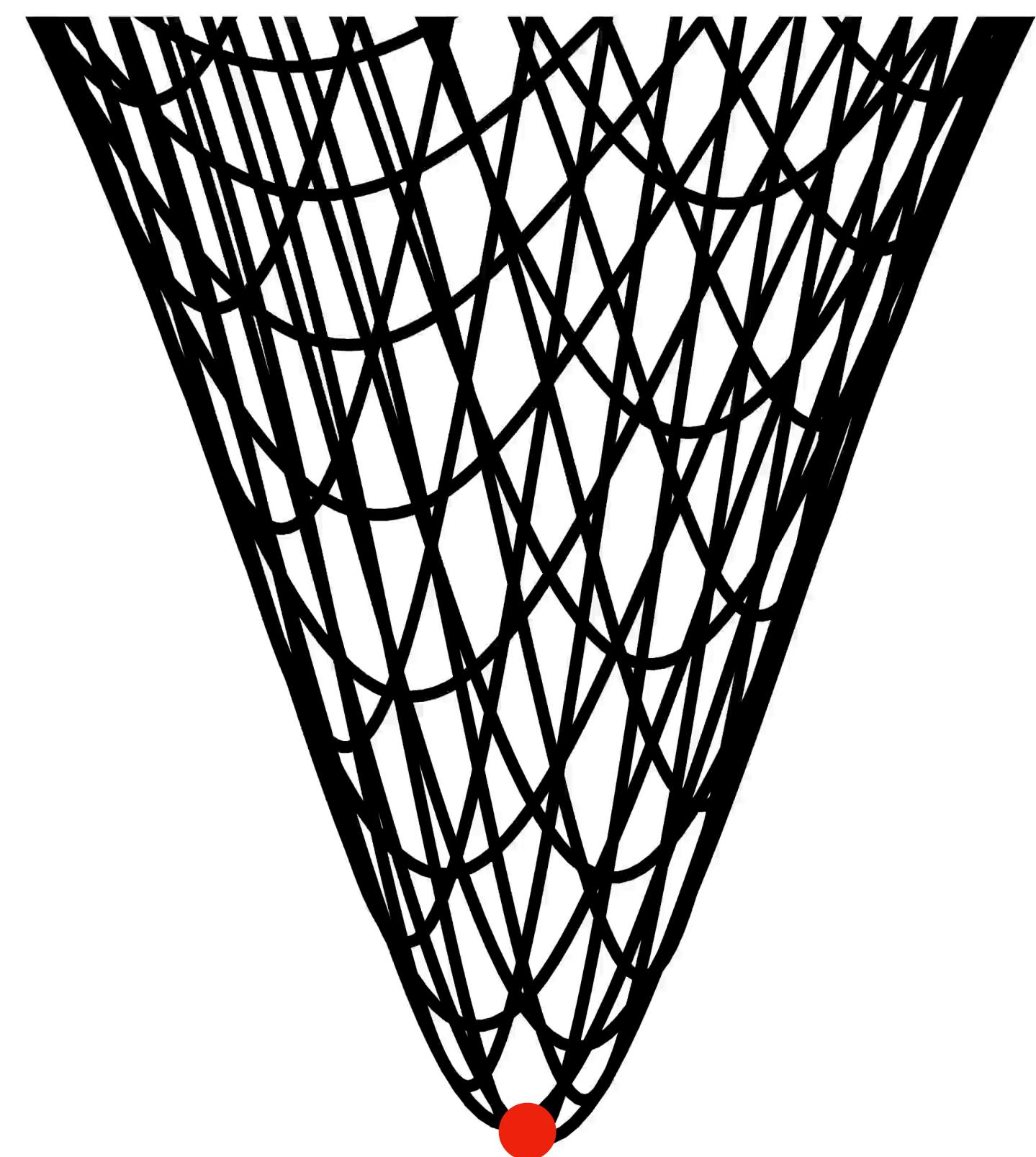
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Early universe \rightarrow Today:

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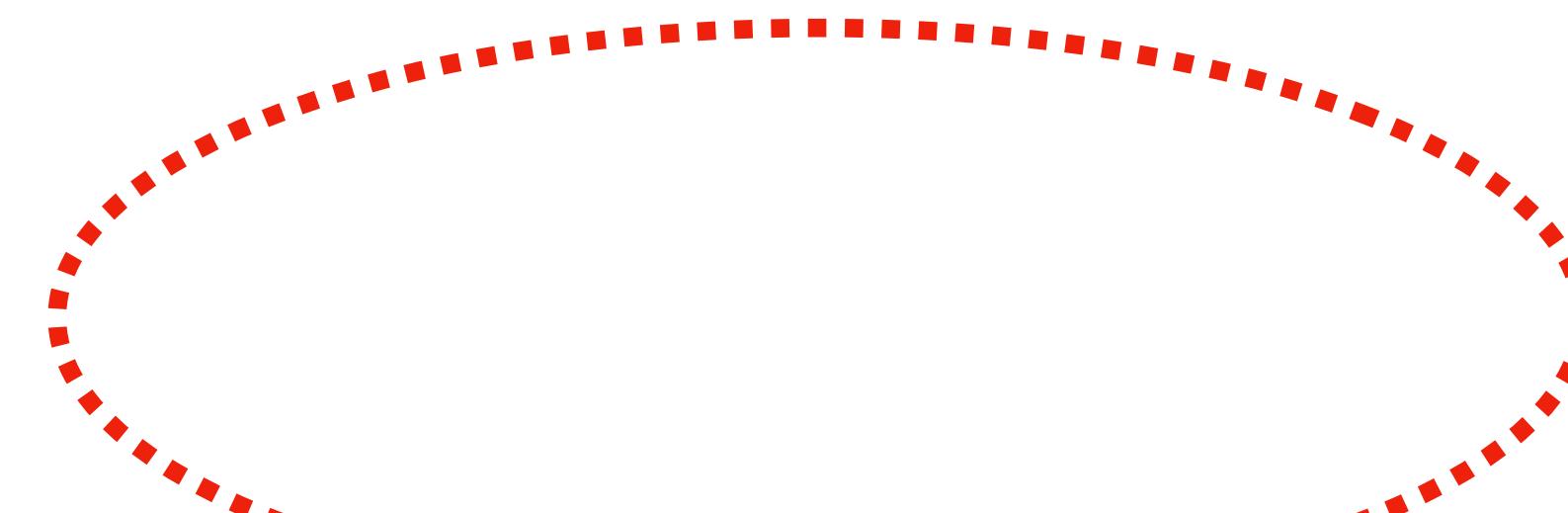
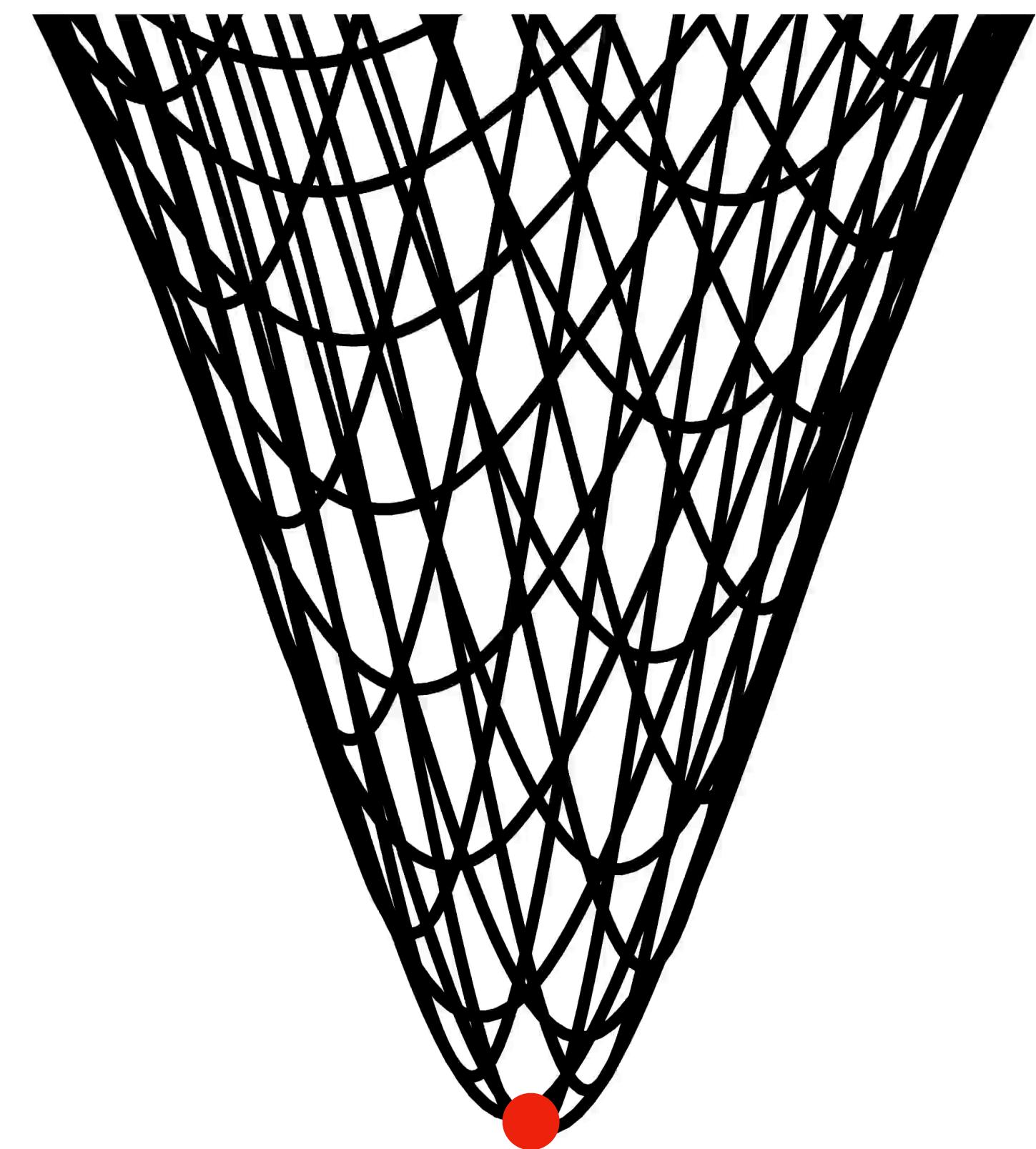
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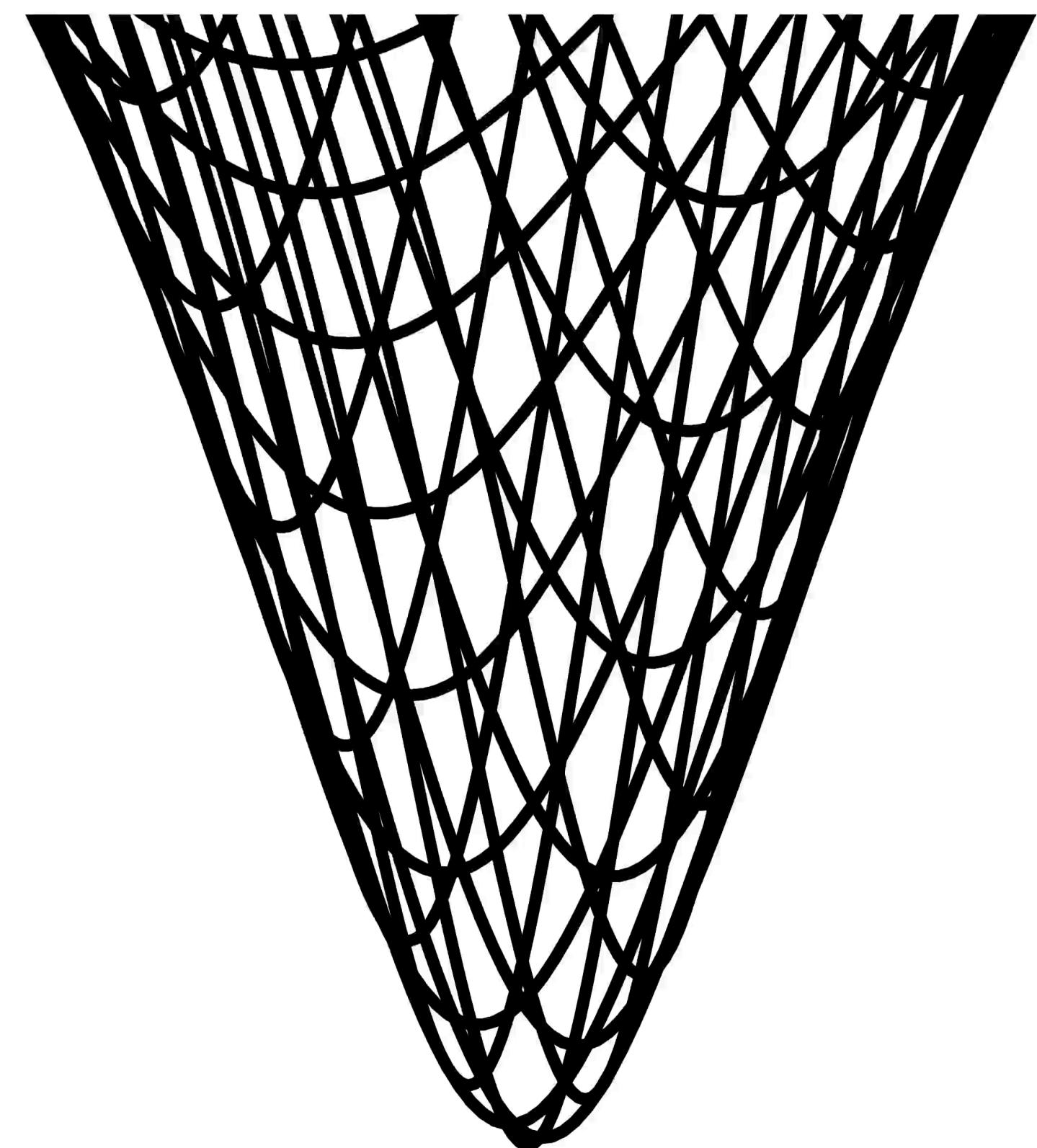
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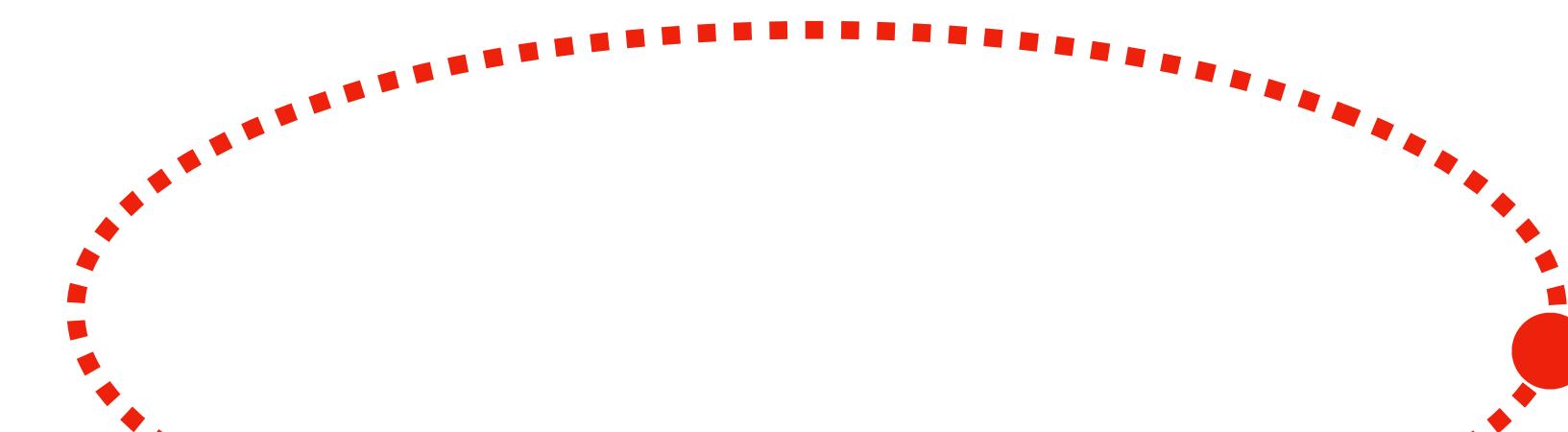
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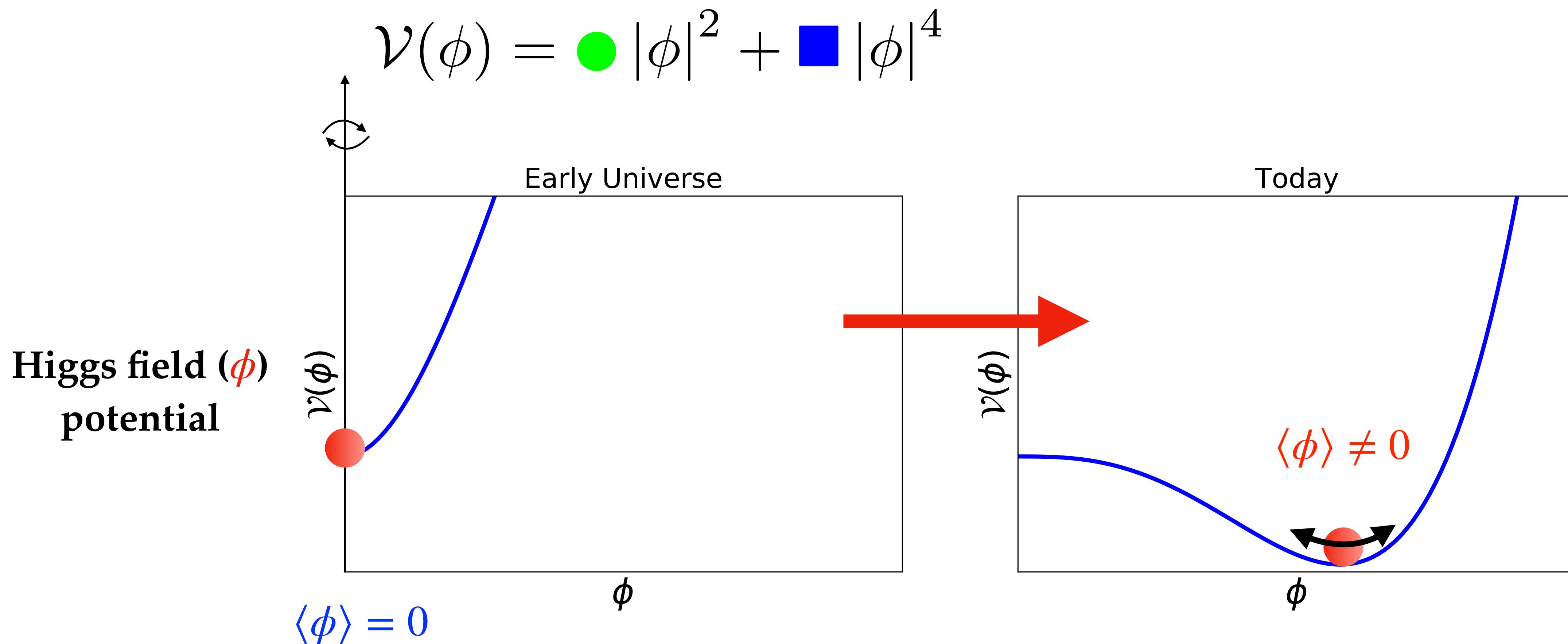
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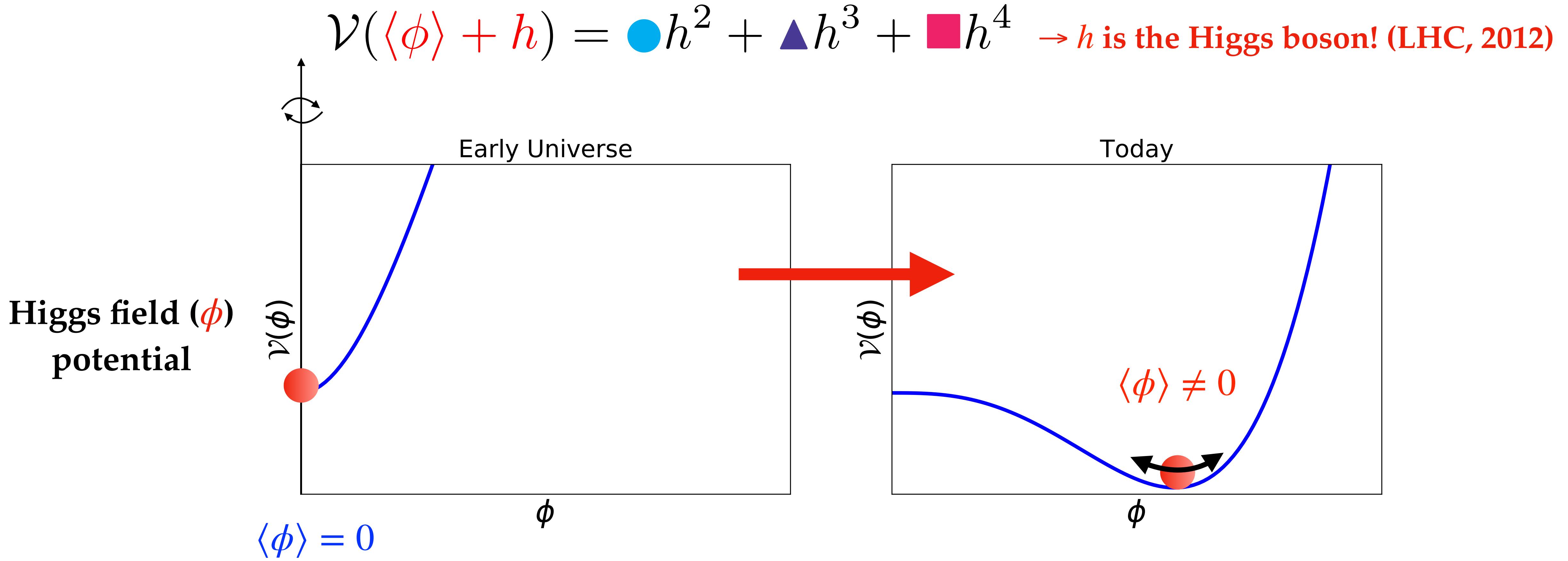
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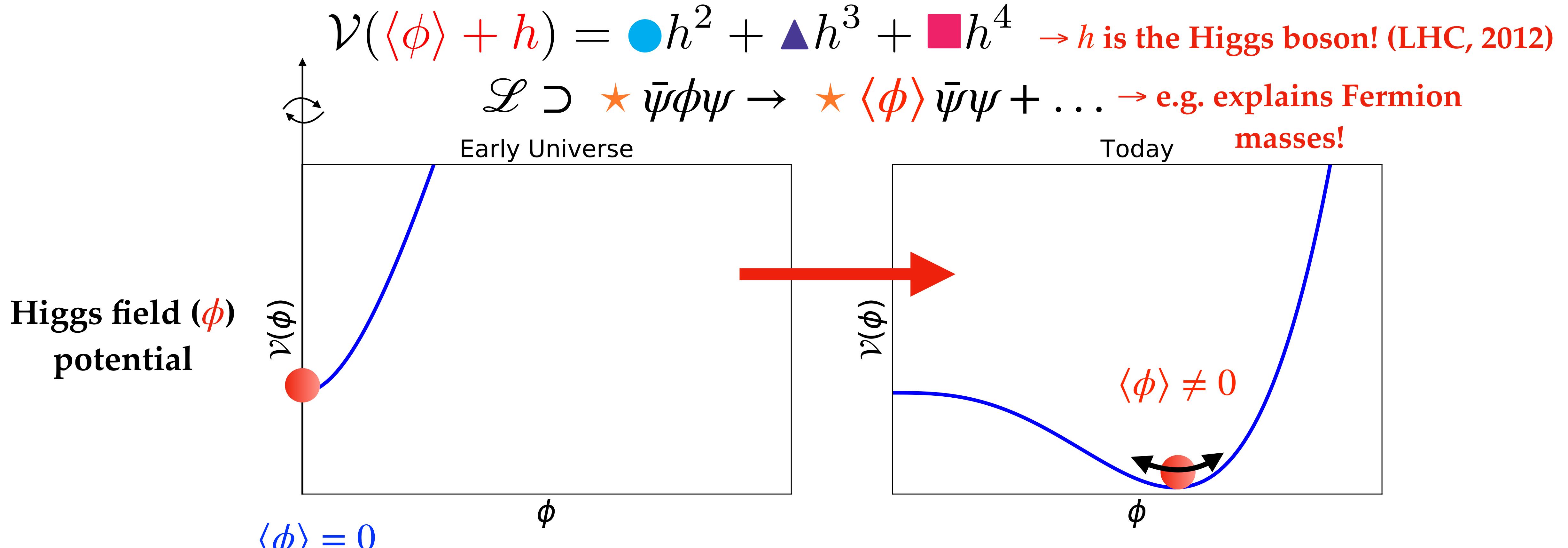
More Symmetry \rightarrow Less (obvious) Symmetry



Breaking the Symmetry in the Standard Model



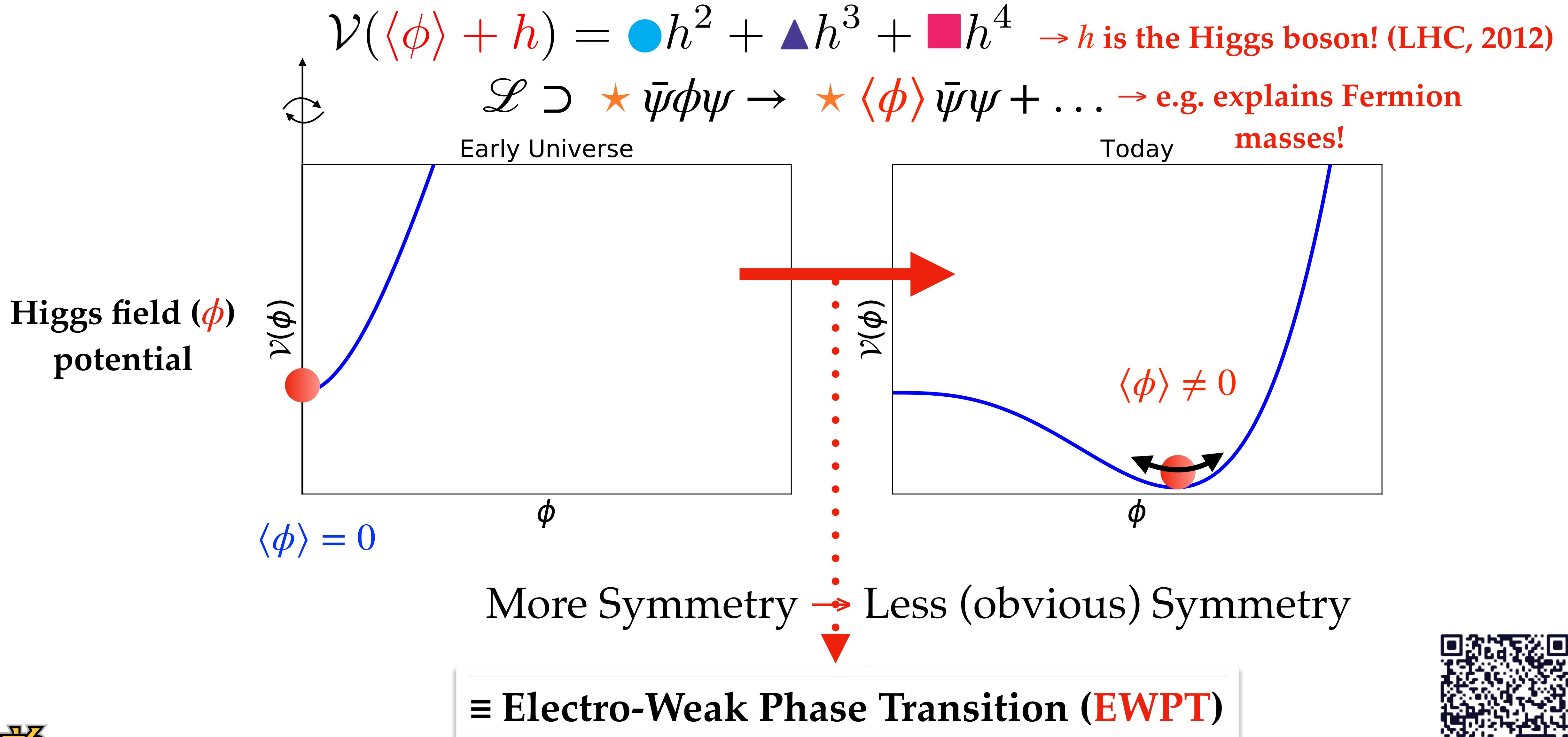
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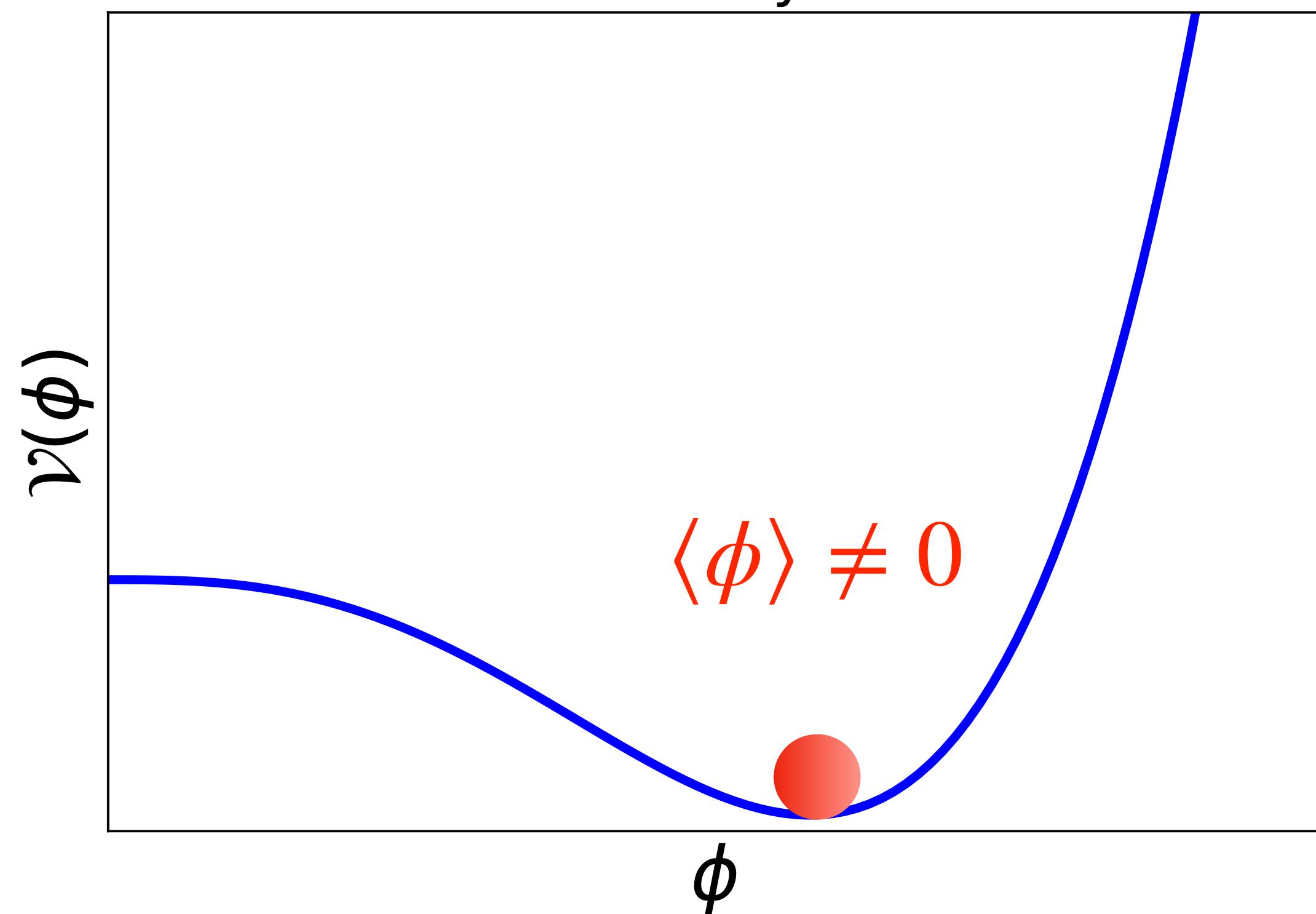
The Higgs Boson's Potential

$$\mathcal{V}(\langle\phi\rangle + h) = \textcolor{blue}{\bullet} h^2 + \textcolor{purple}{\Delta} h^3 + \textcolor{red}{\blacksquare} h^4 \rightarrow \text{the Higgs boson's self-interactions.}$$

Today

→ Determine shape of potential by measuring:

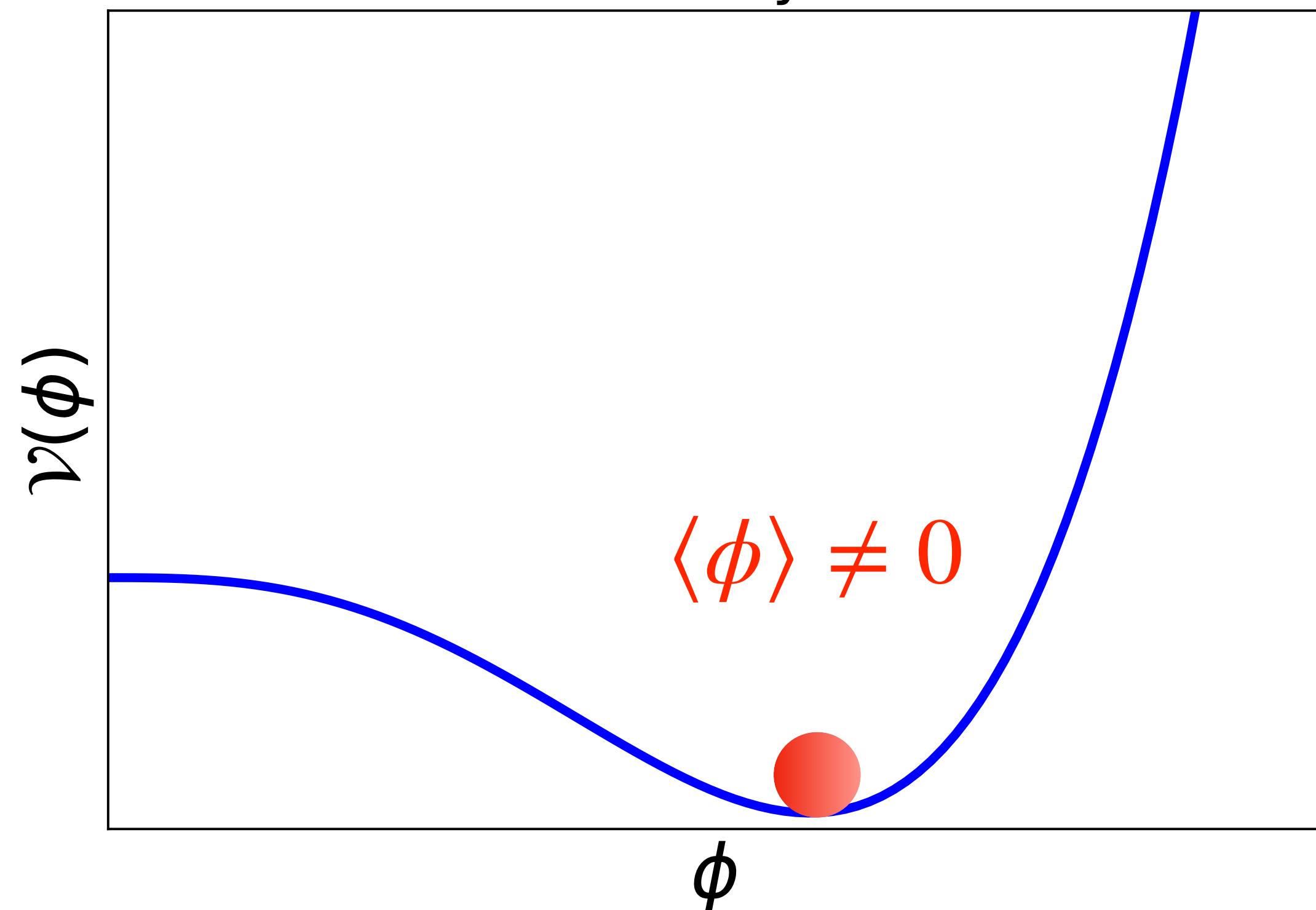
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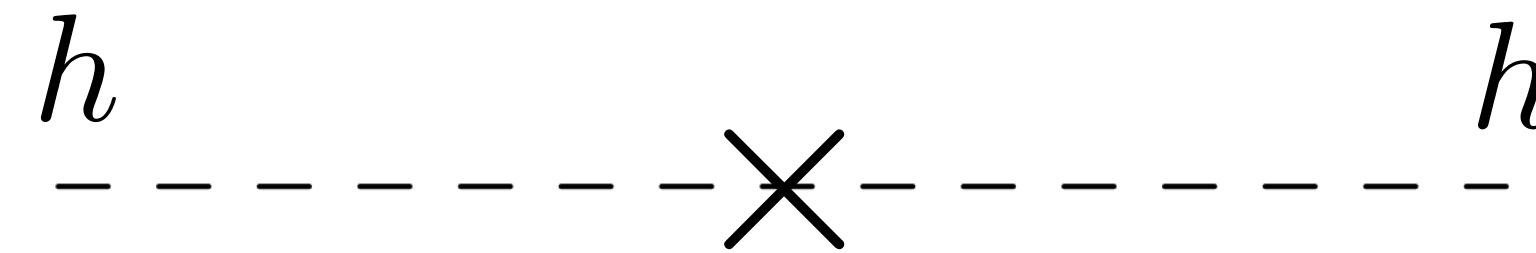
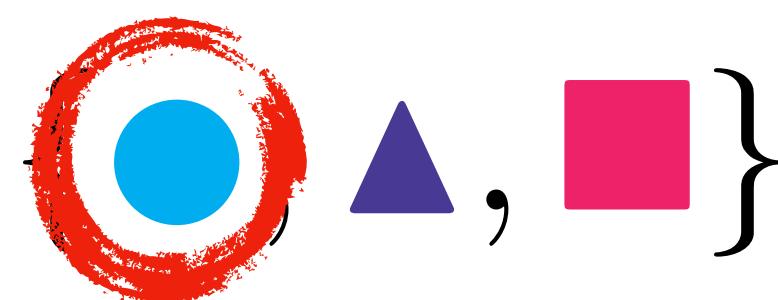
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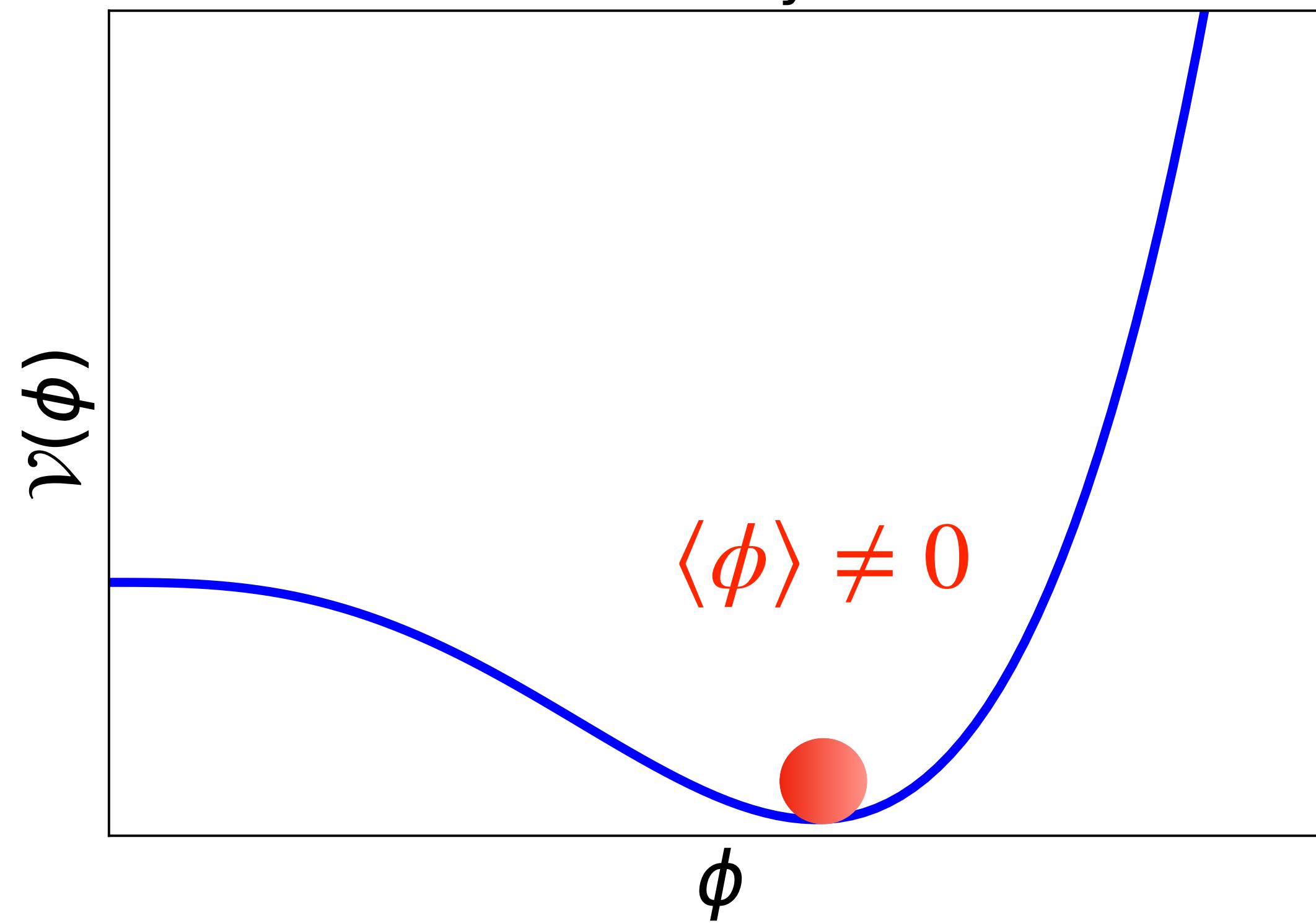
Higgs boson discovery @ LHC, 2012



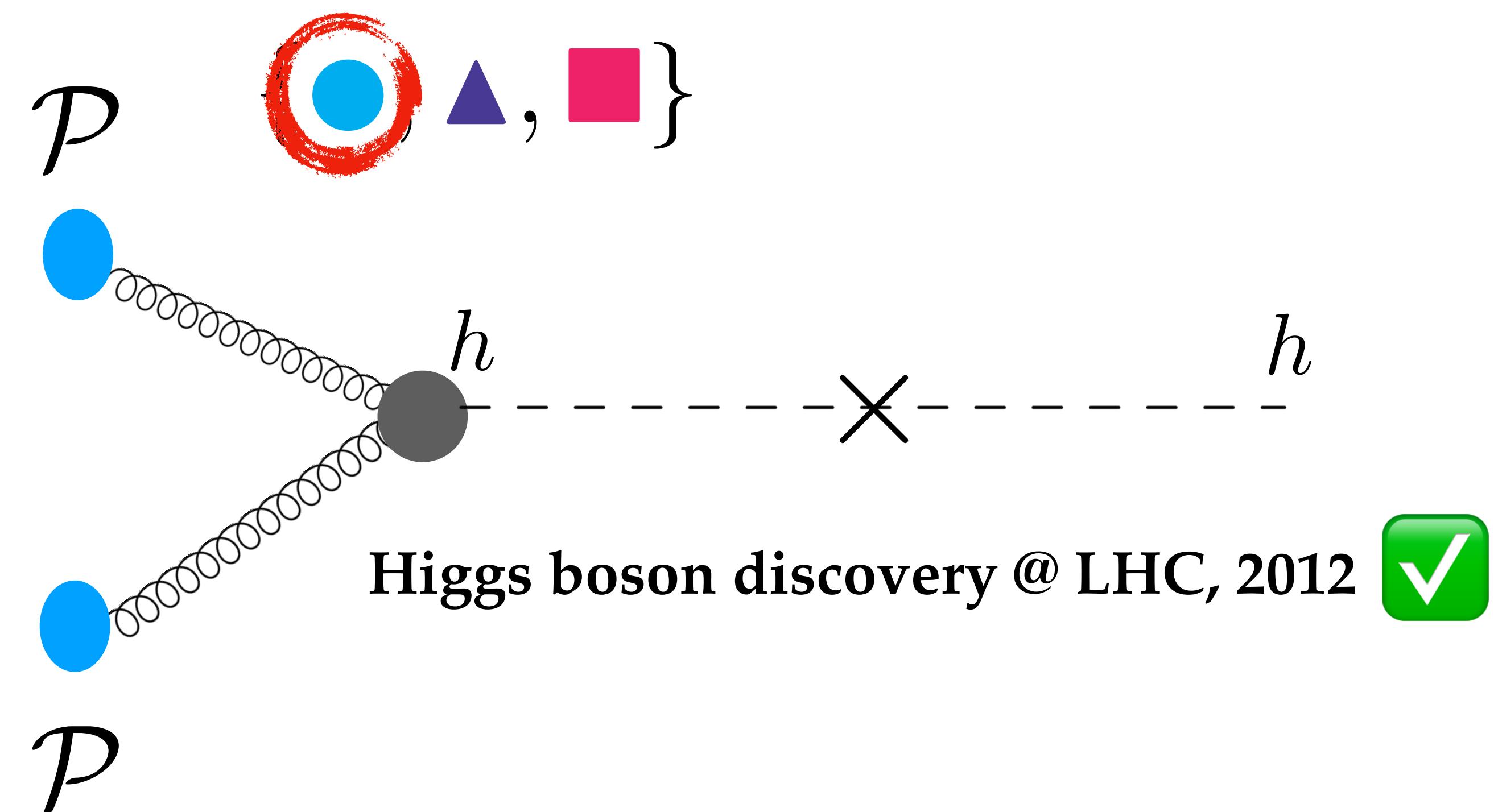
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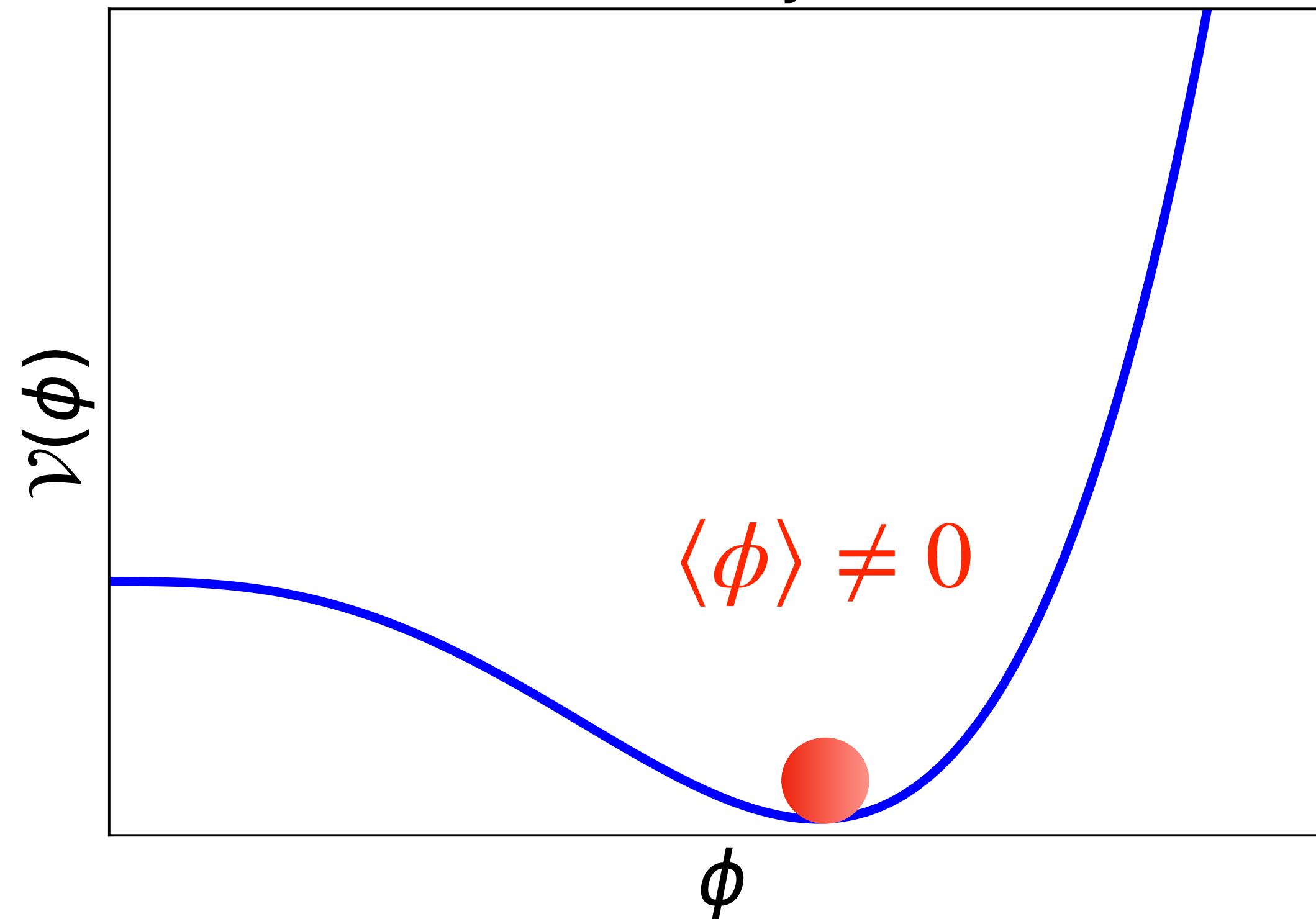
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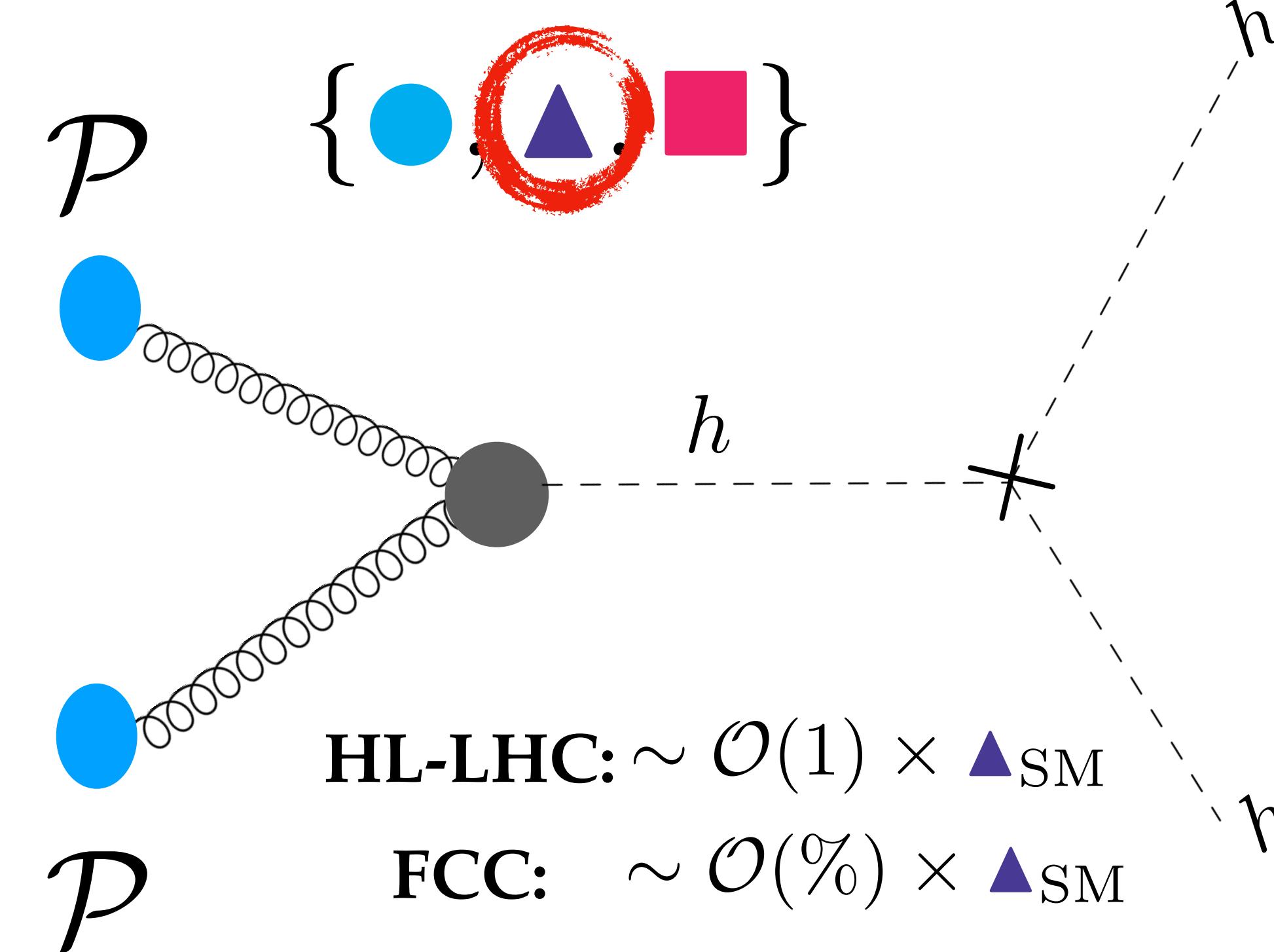
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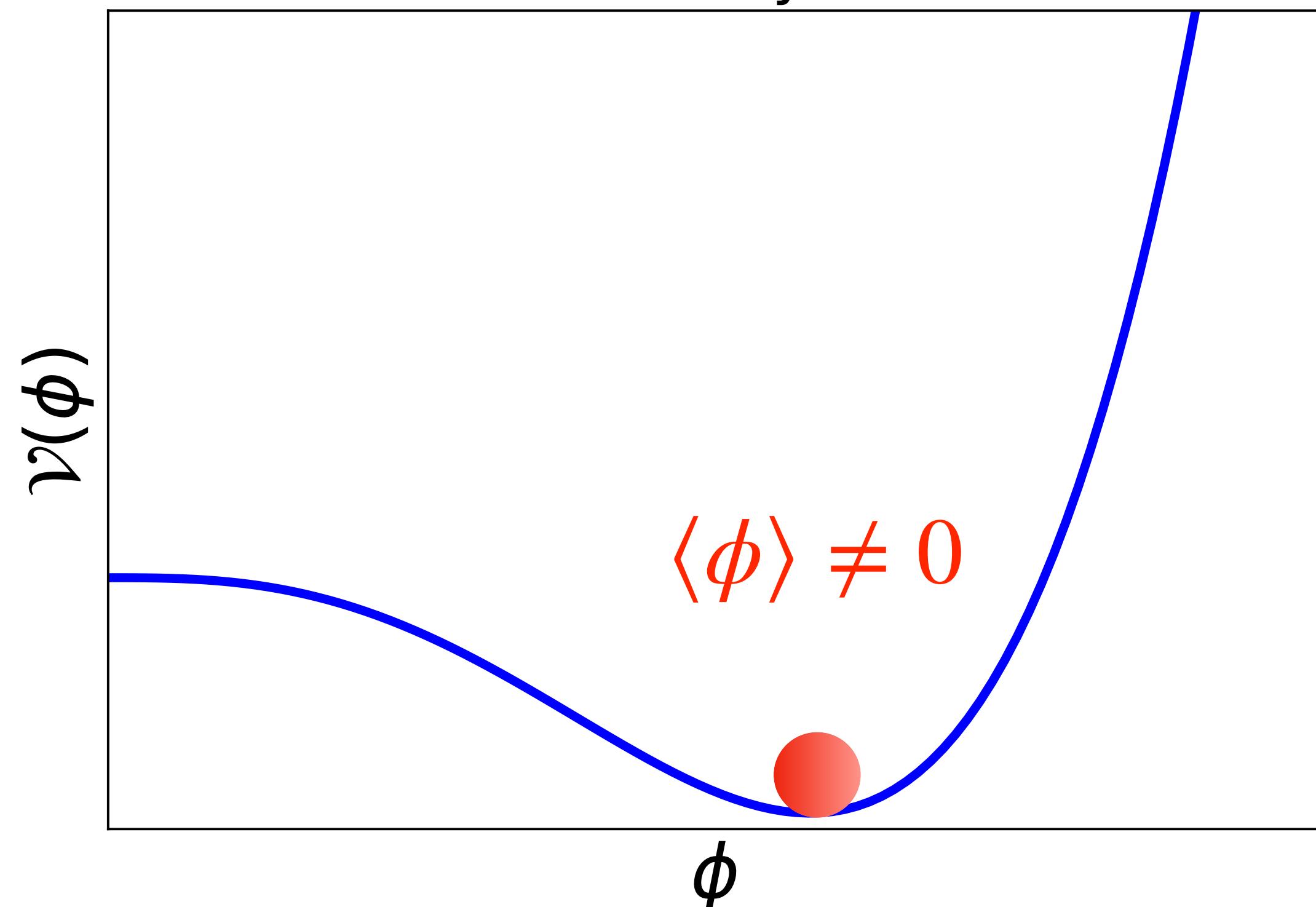
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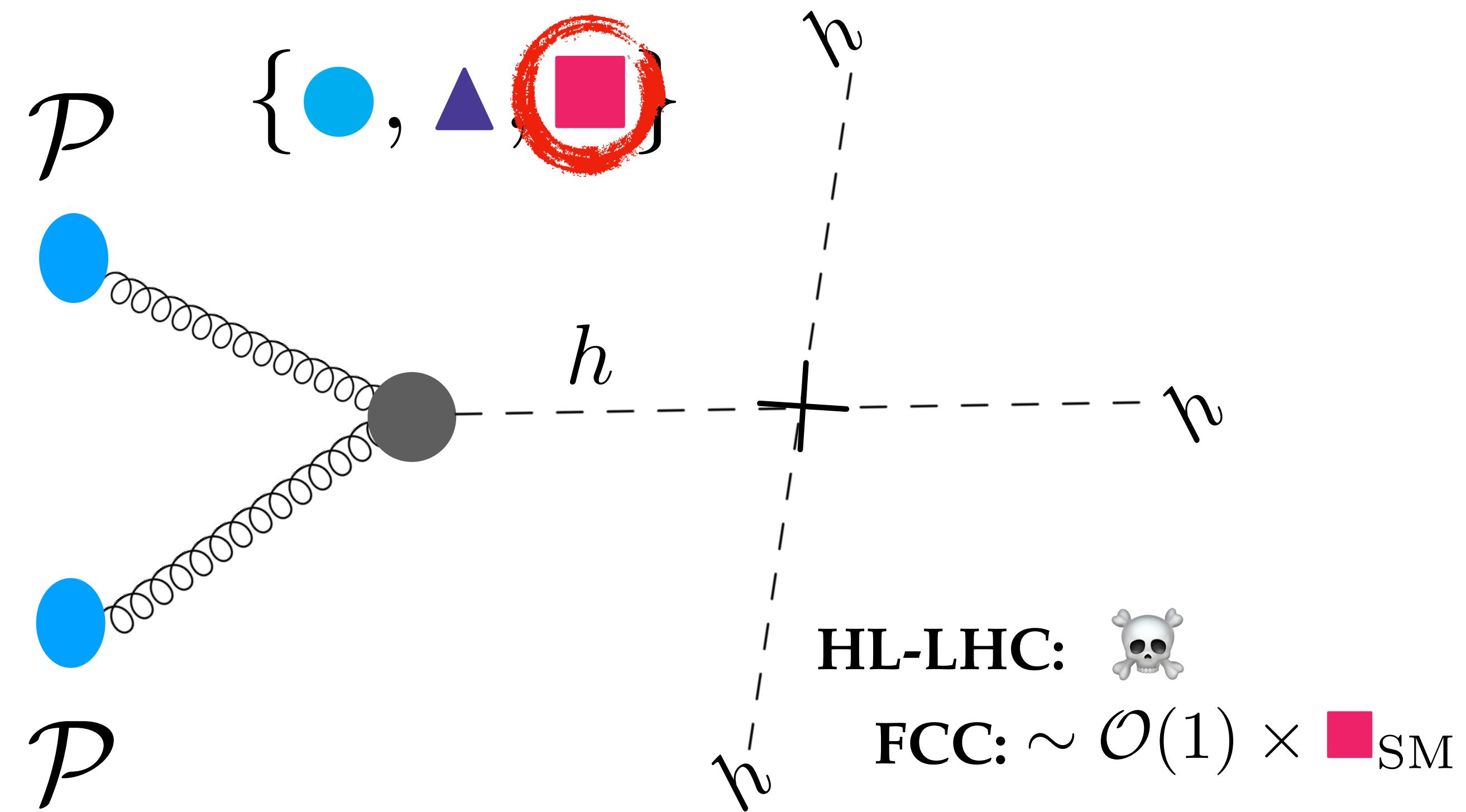
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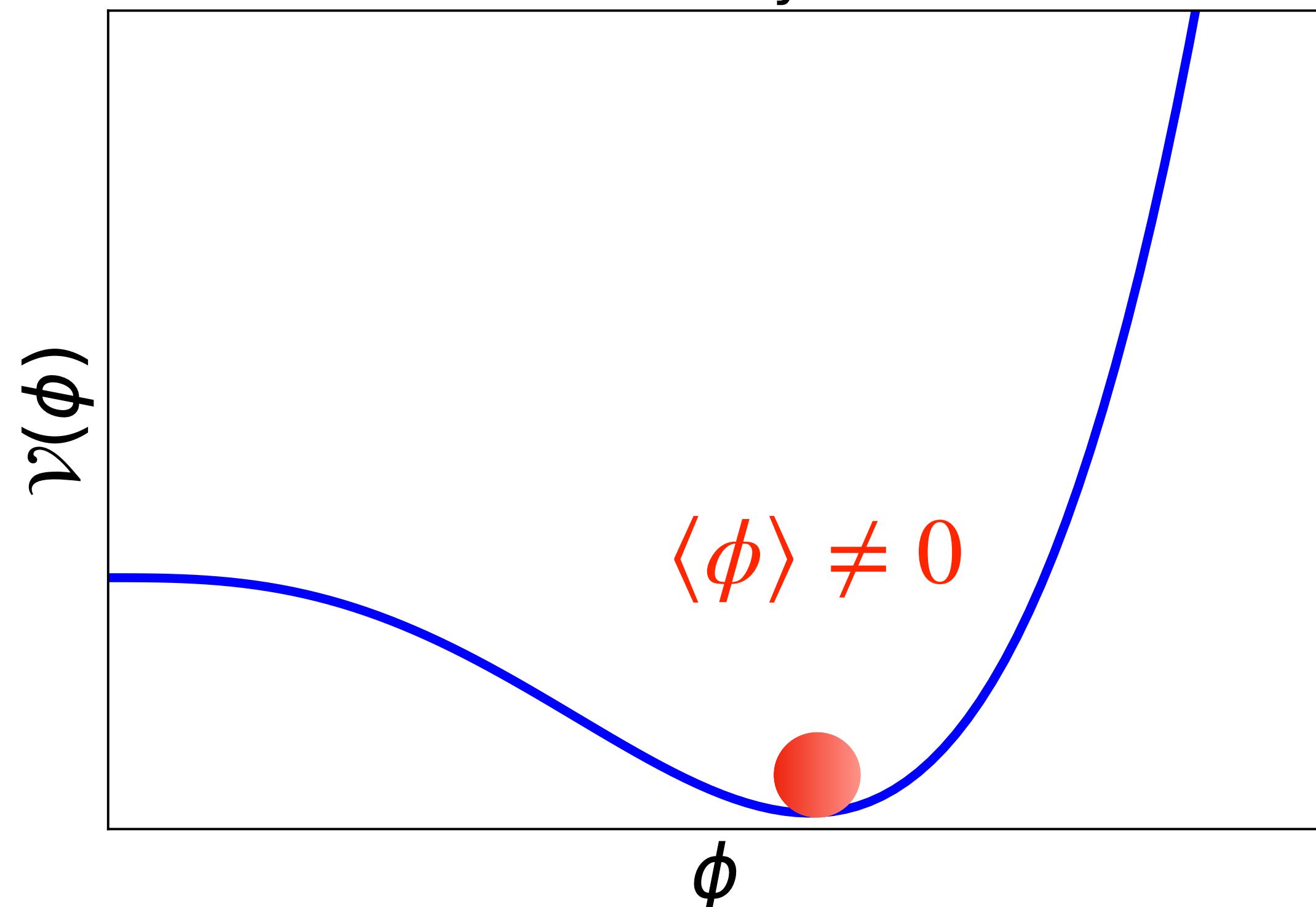


[e.g. AP, Sakurai, arXiv:1508.06524, AP,
Tetlalmatzi-Xolocotzi, Zaro,
arXiv:1909.09166]

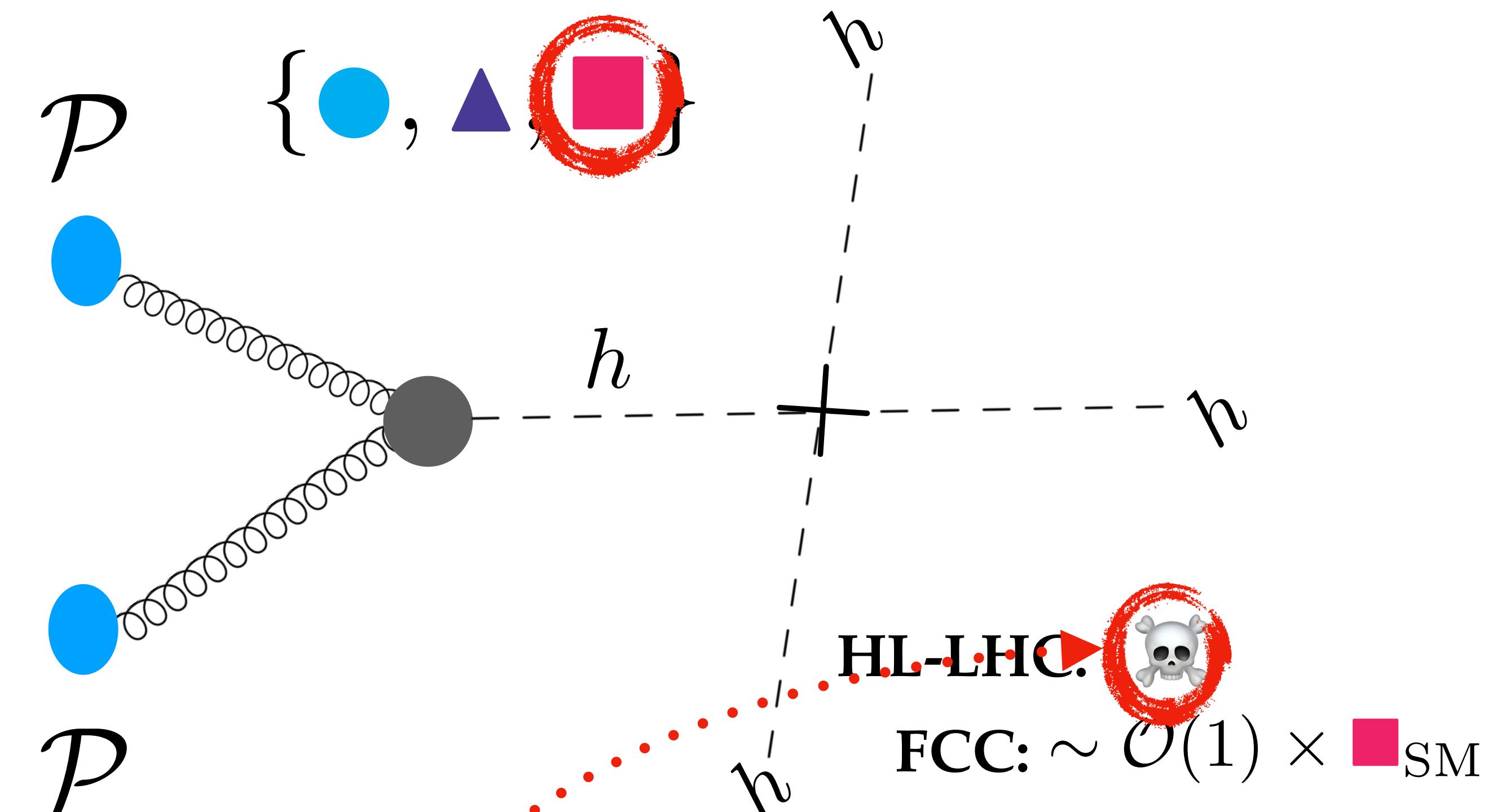
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[[AP](#), Robens, Tetlamatzi-Xolocotzi, arXiv:2101.00037 +
Karkout, [AP](#), Postma, Tetlamatzi-Xolocotzi, van de Vis, du Pree,
arXiv:2404.12425, [AP](#), Tetlamatzi-Xolocotzi, arXiv:2312.13562]

SEE LATER!

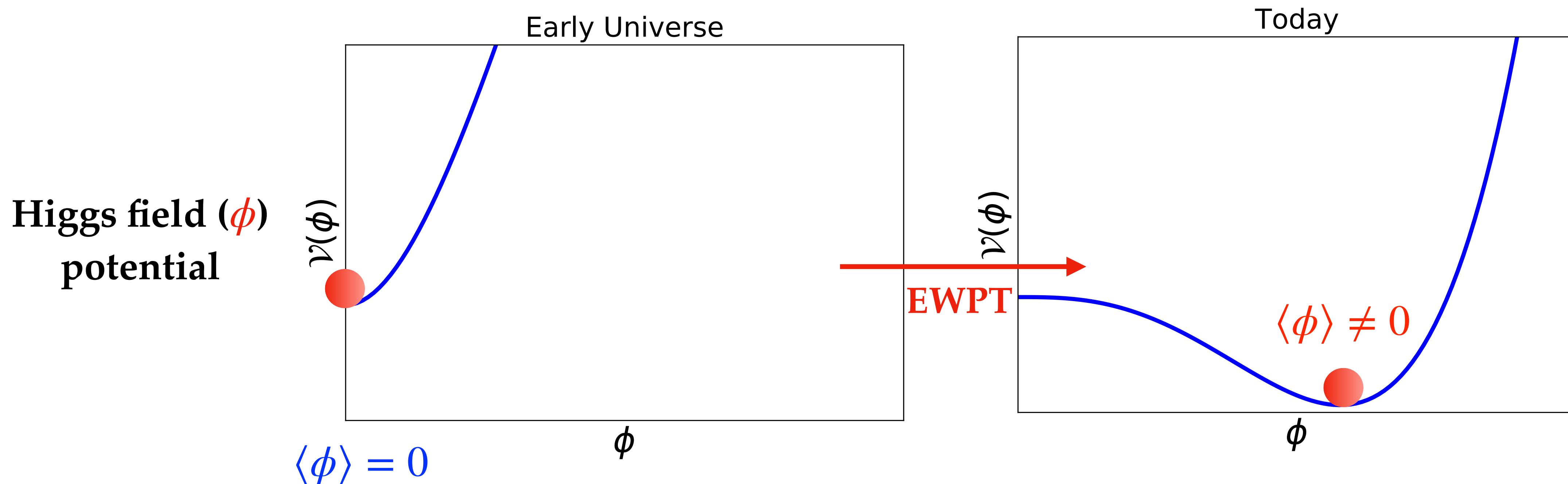
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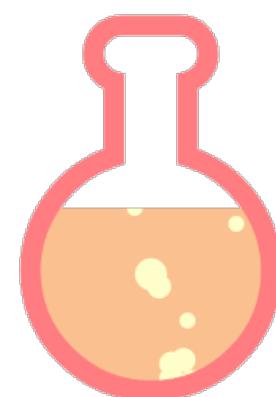
**TO BE
CONTINUED... ➔**

Breaking the Symmetry in the SM



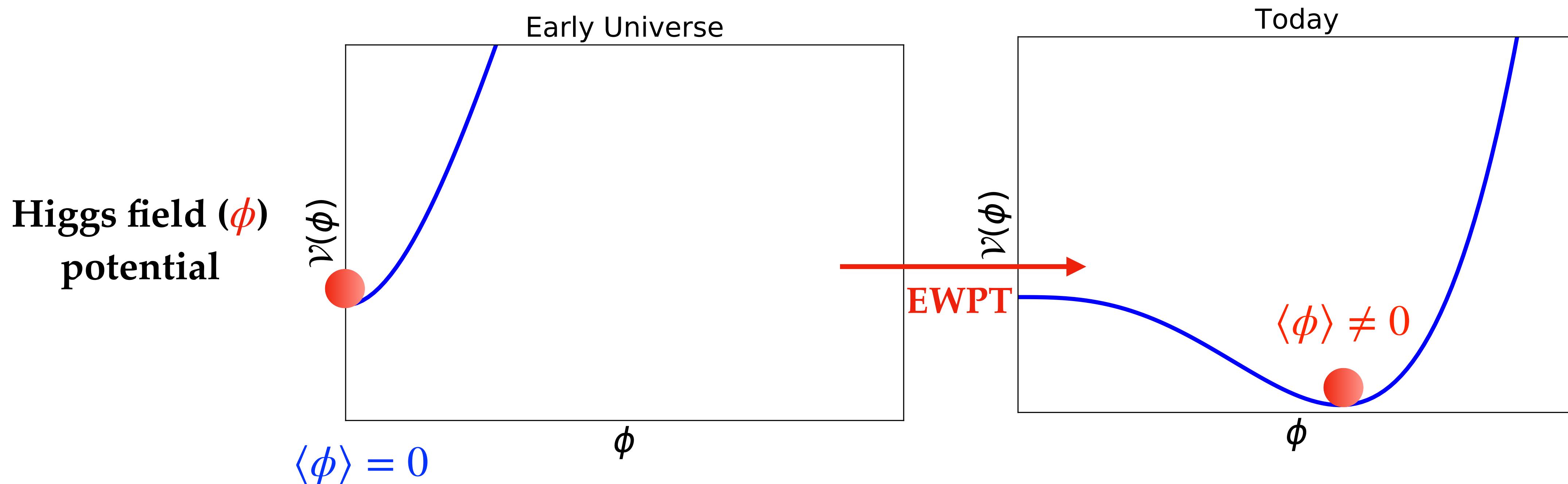
- **Nature of EWPT** → Important open question, e.g. its order:

- A **First-Order transition** (e.g. the boiling of water)?



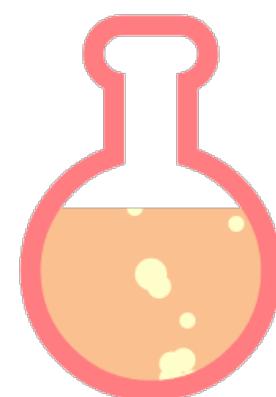
- or a **Second-Order transition** (e.g. the superfluid transition) or **cross-over**?

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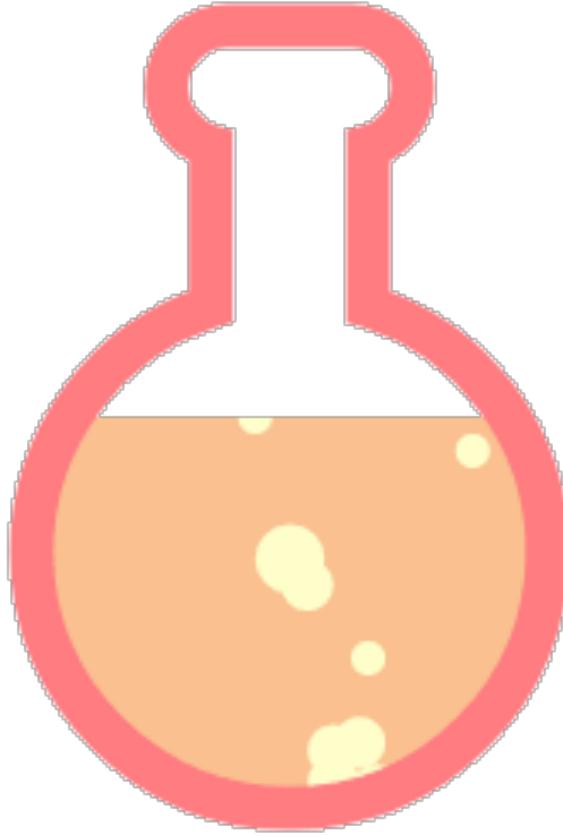
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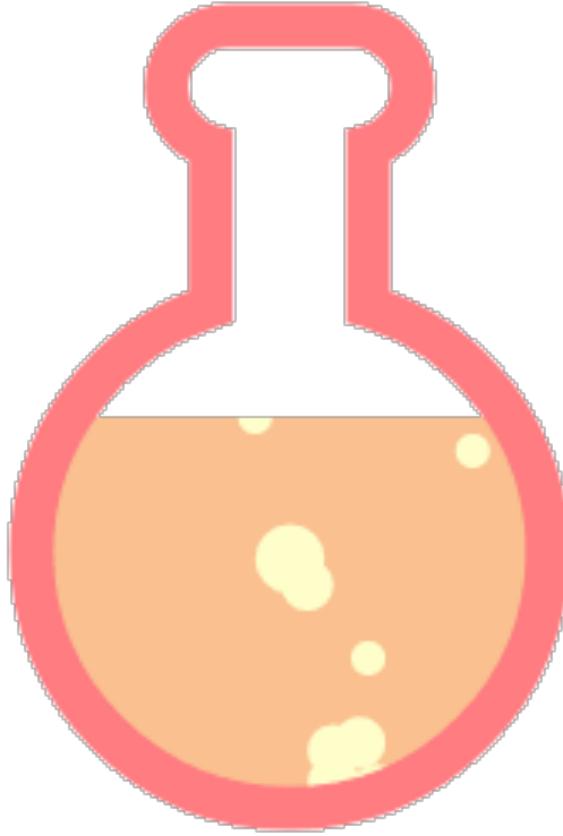
The Nature of the Phase Transition

- Clues to the origin of **matter-anti-matter asymmetry**.
- Was the asymmetry created during the EWPT?
 - “Electro-Weak Baryogenesis” (**EWBG**).
- Pre-requisite: **a First-Order transition**. → 
- Note: This does not occur in the SM!

[Kajantie, Laine, Rummukainen, Shaposhnikov hep-ph/9605288]



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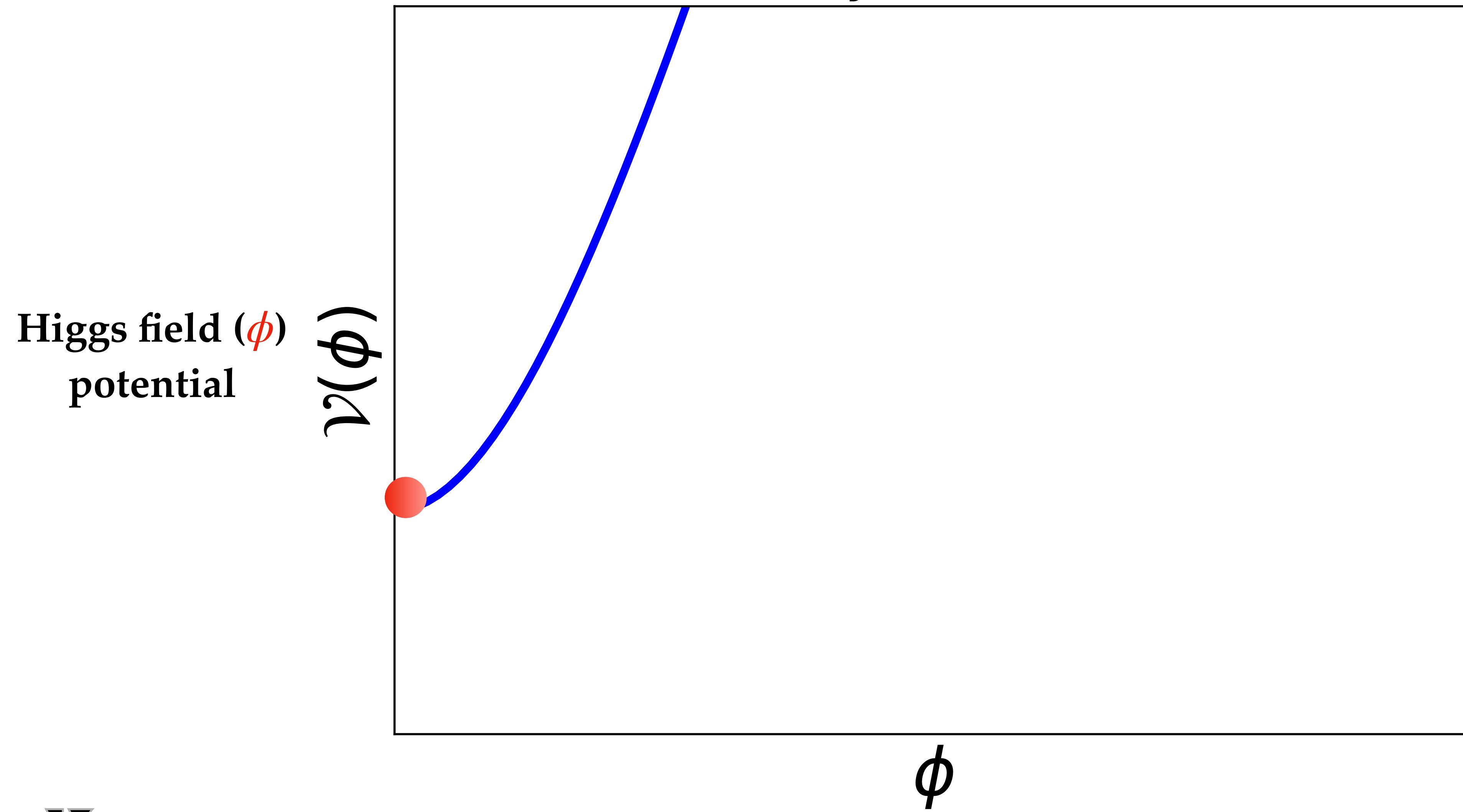
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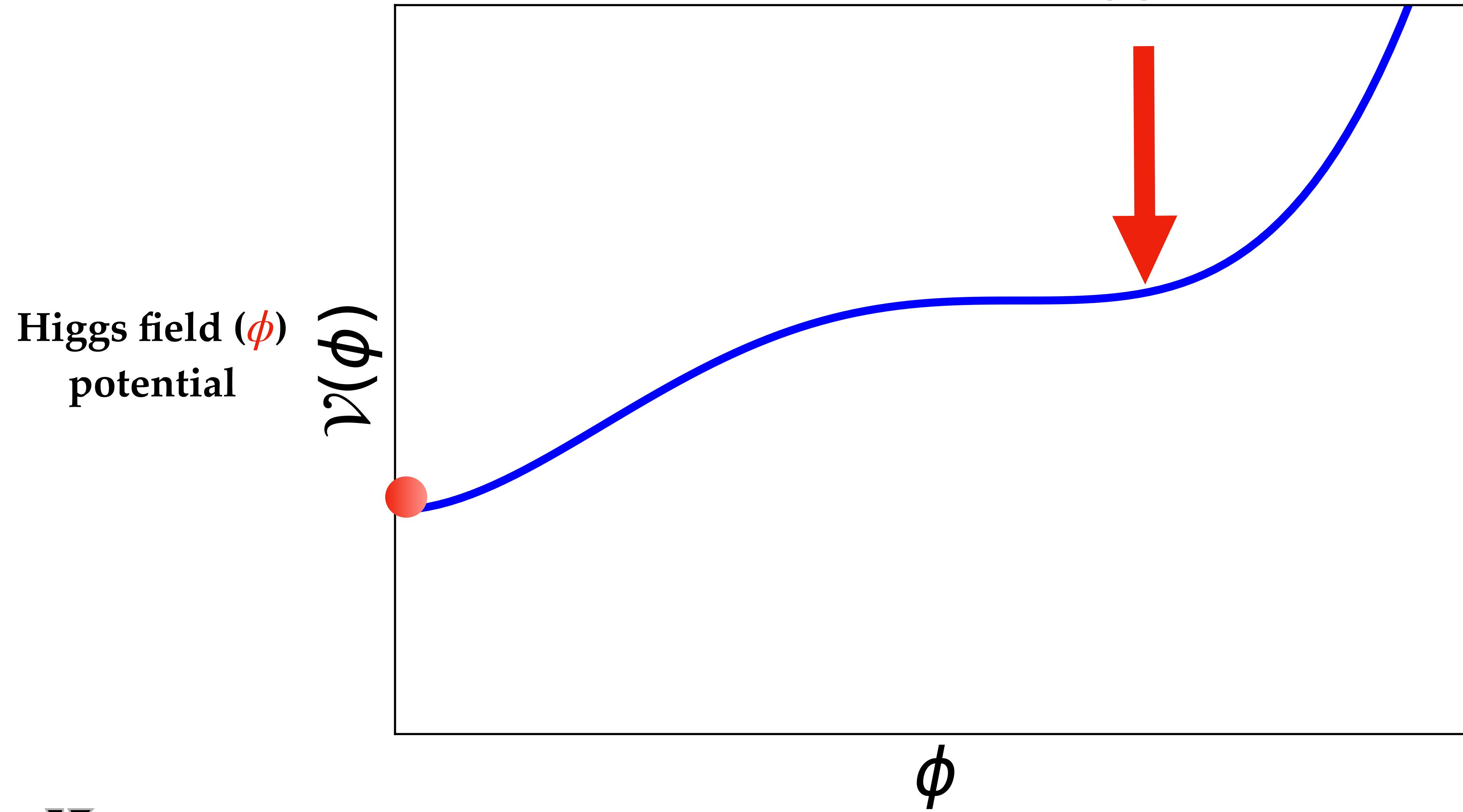
A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Early Universe



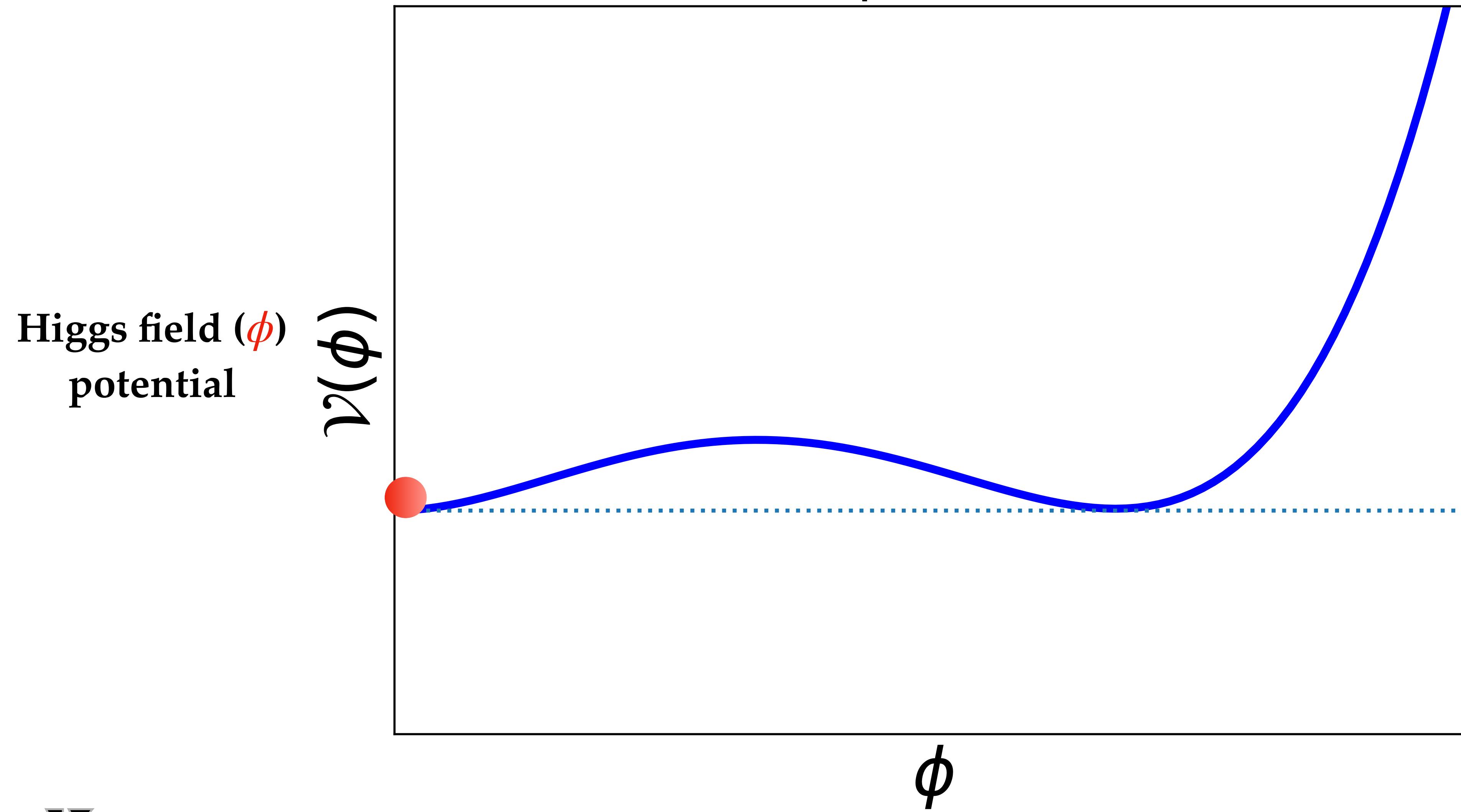
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Second minimum appears



A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

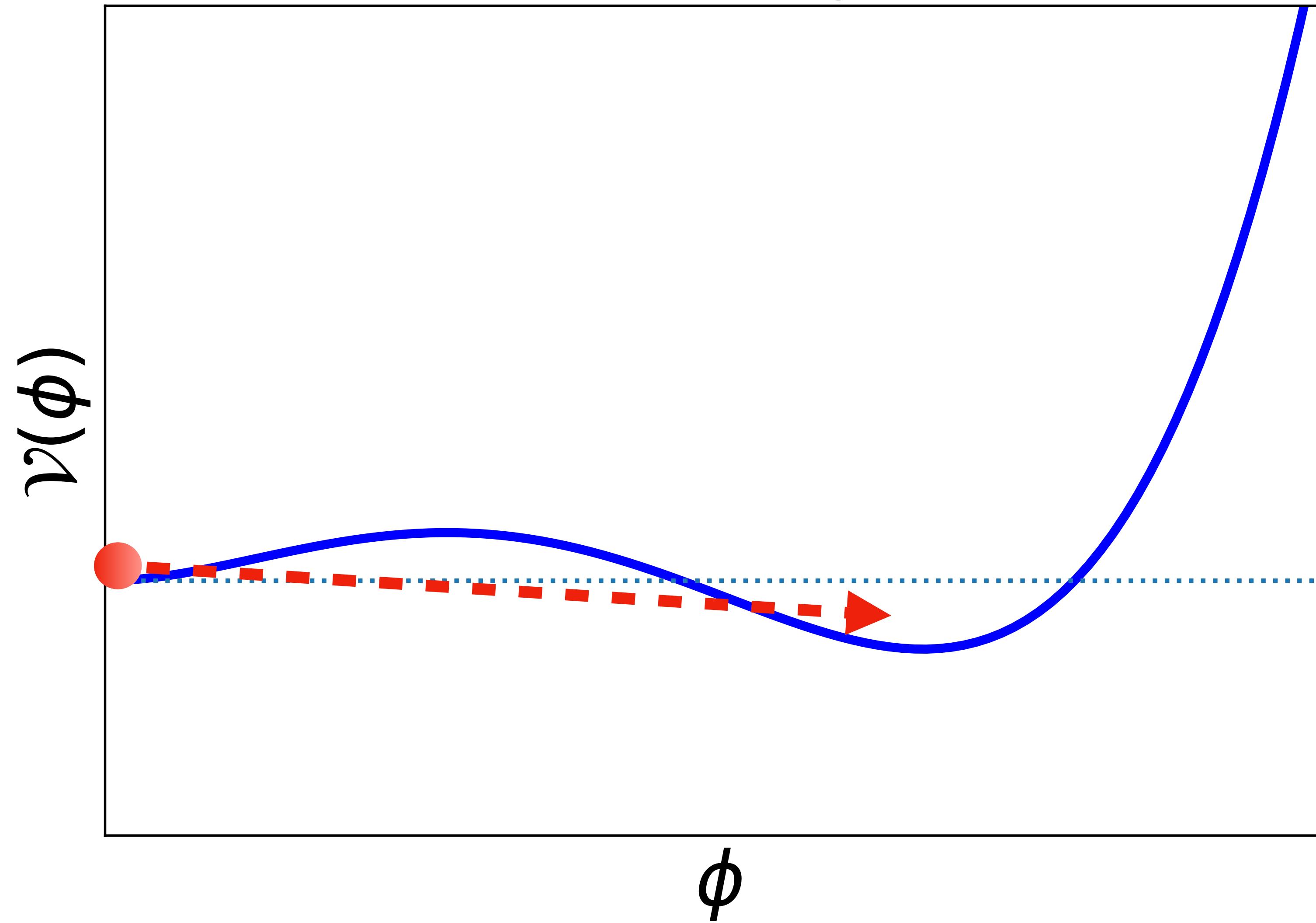
Critical temperature reached



A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Tunneling!

Higgs field (ϕ)
potential

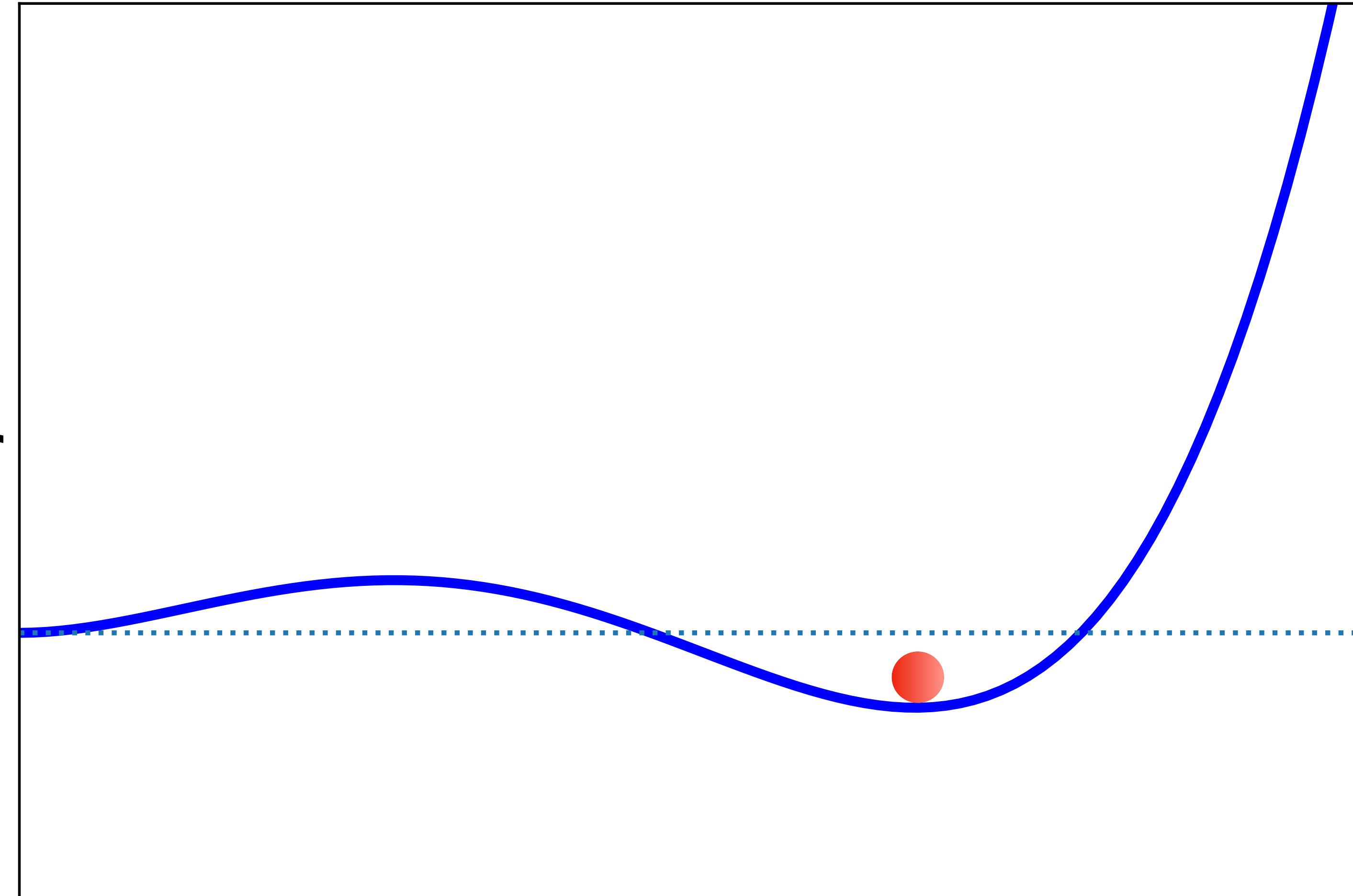


A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Tunneling!

Higgs field (ϕ)
potential

$V(\phi)$



ϕ

16

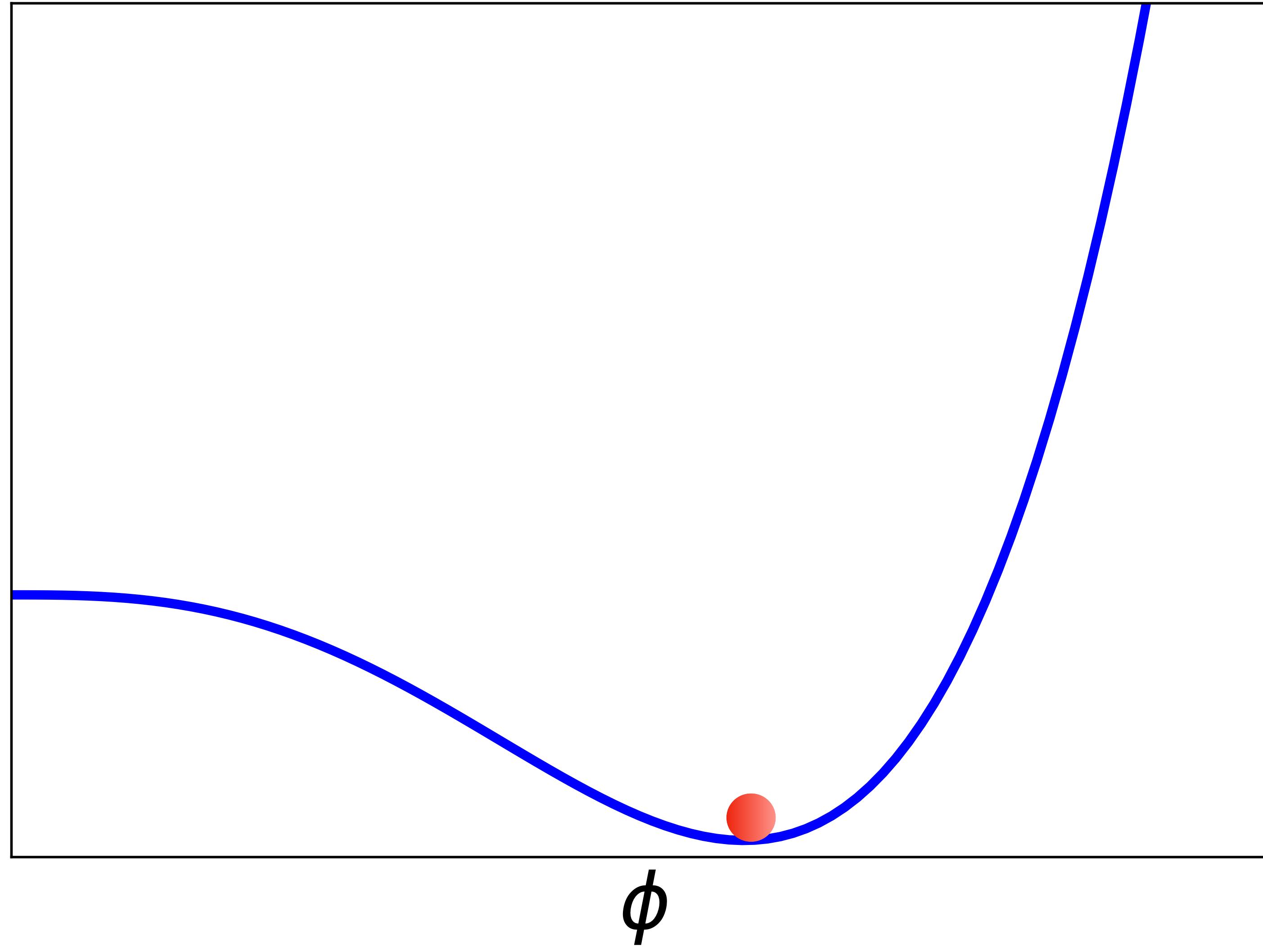


A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Today

Higgs field (ϕ)
potential

$\mathcal{V}(\phi)$



ϕ

16

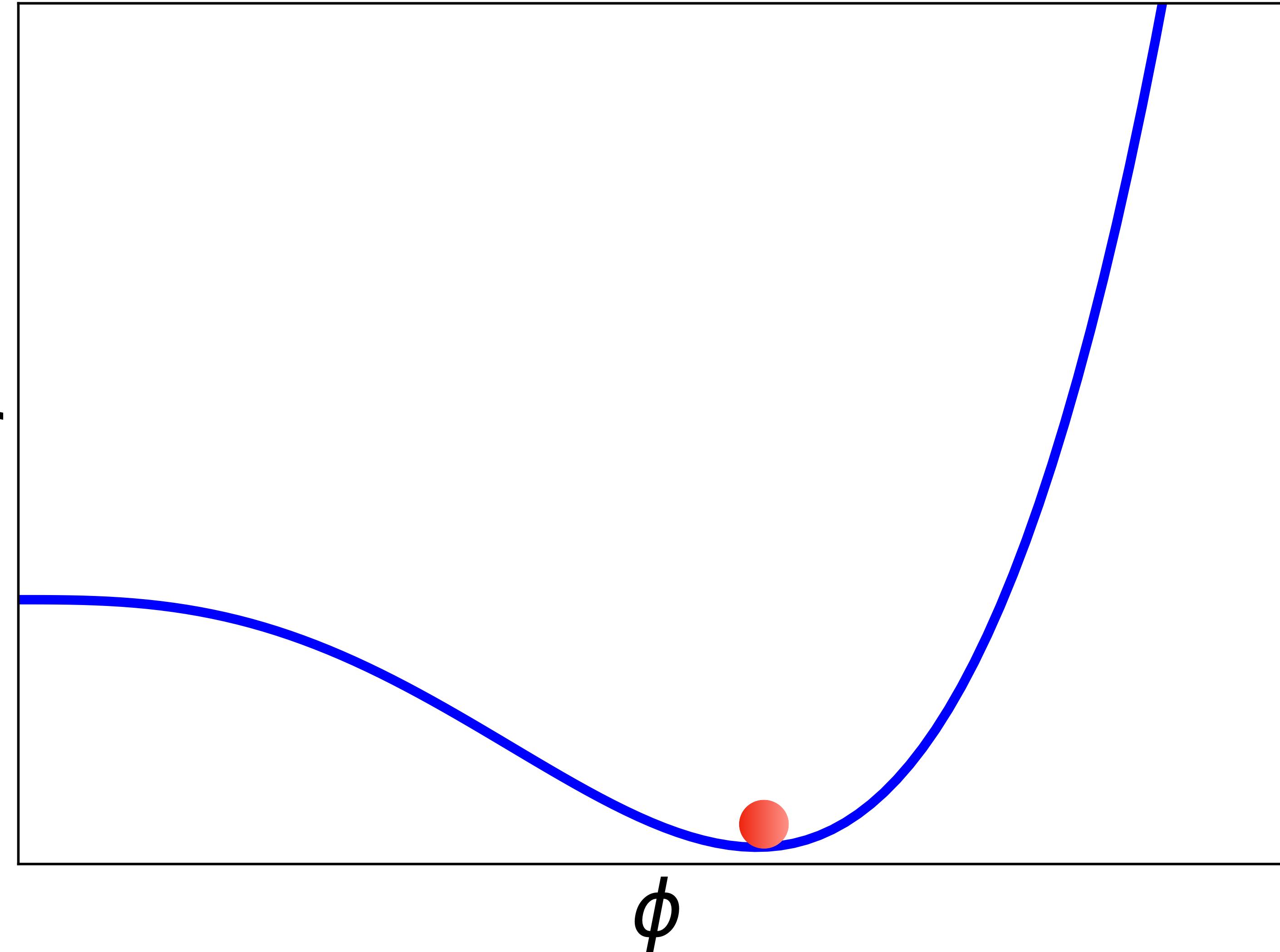


A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Today

Higgs field (ϕ)
potential

$\mathcal{V}(\phi)$



$$\langle \phi \rangle \neq 0$$

VEV
 \Rightarrow

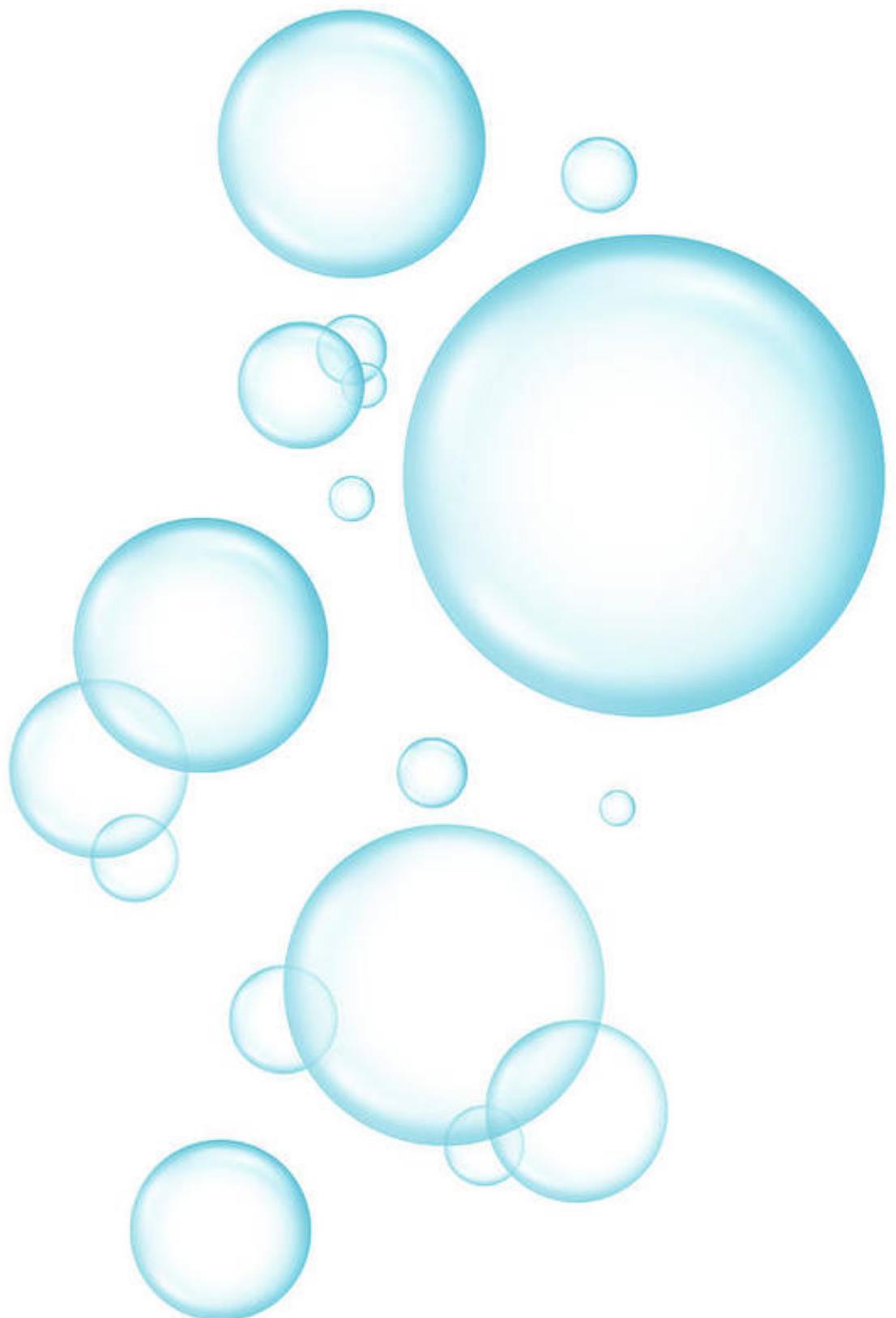
Symmetry
Breaking.



A First-Order EWPT

Some time after critical temperature is reached:

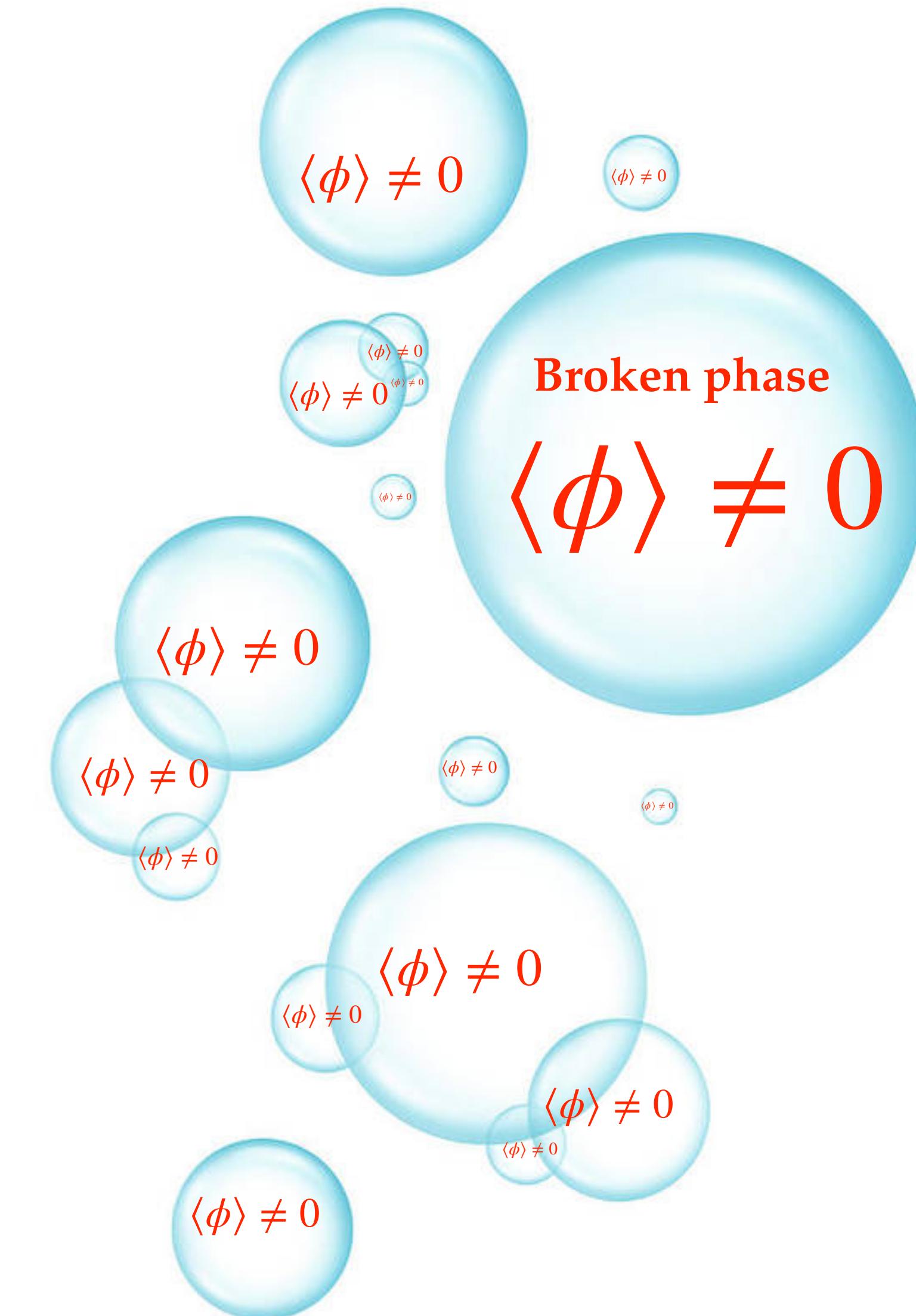
→ **Bubbles** of the broken phase nucleate and expand.



A First-Order EWPT

Some time after critical temperature is reached:

→ Bubbles of the broken phase nucleate and expand.



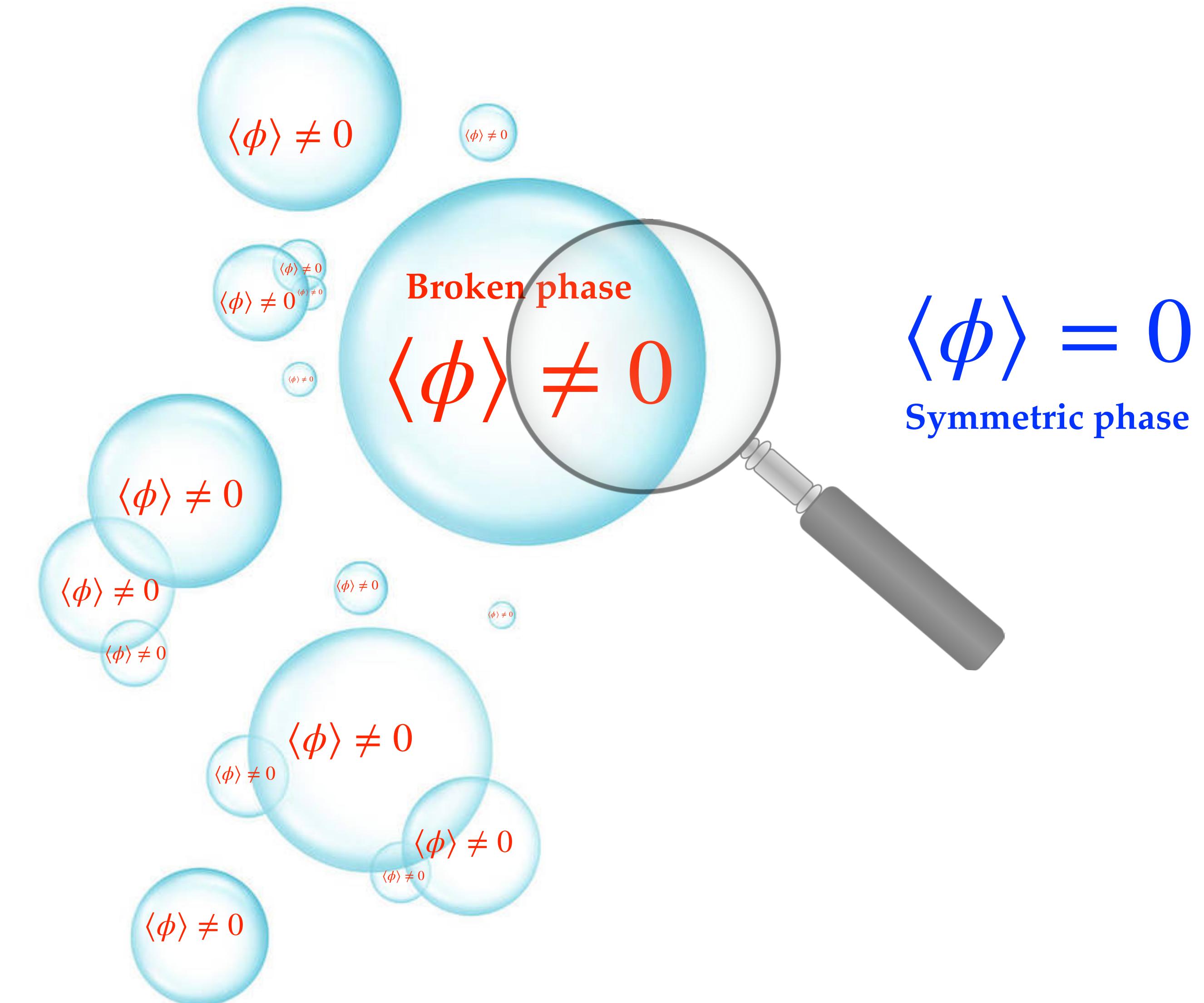
$\langle \phi \rangle = 0$
Symmetric phase



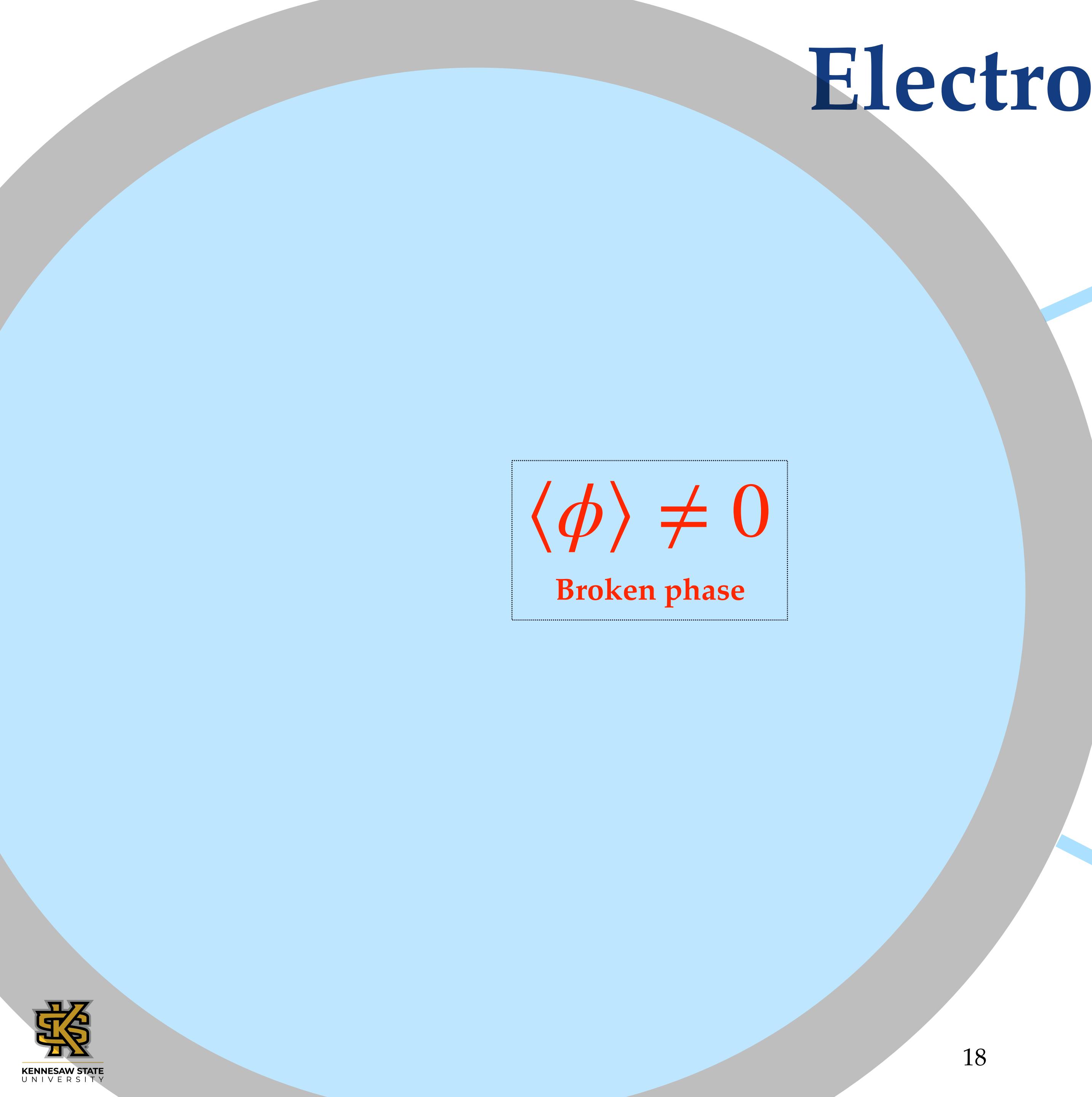
A First-Order EWPT

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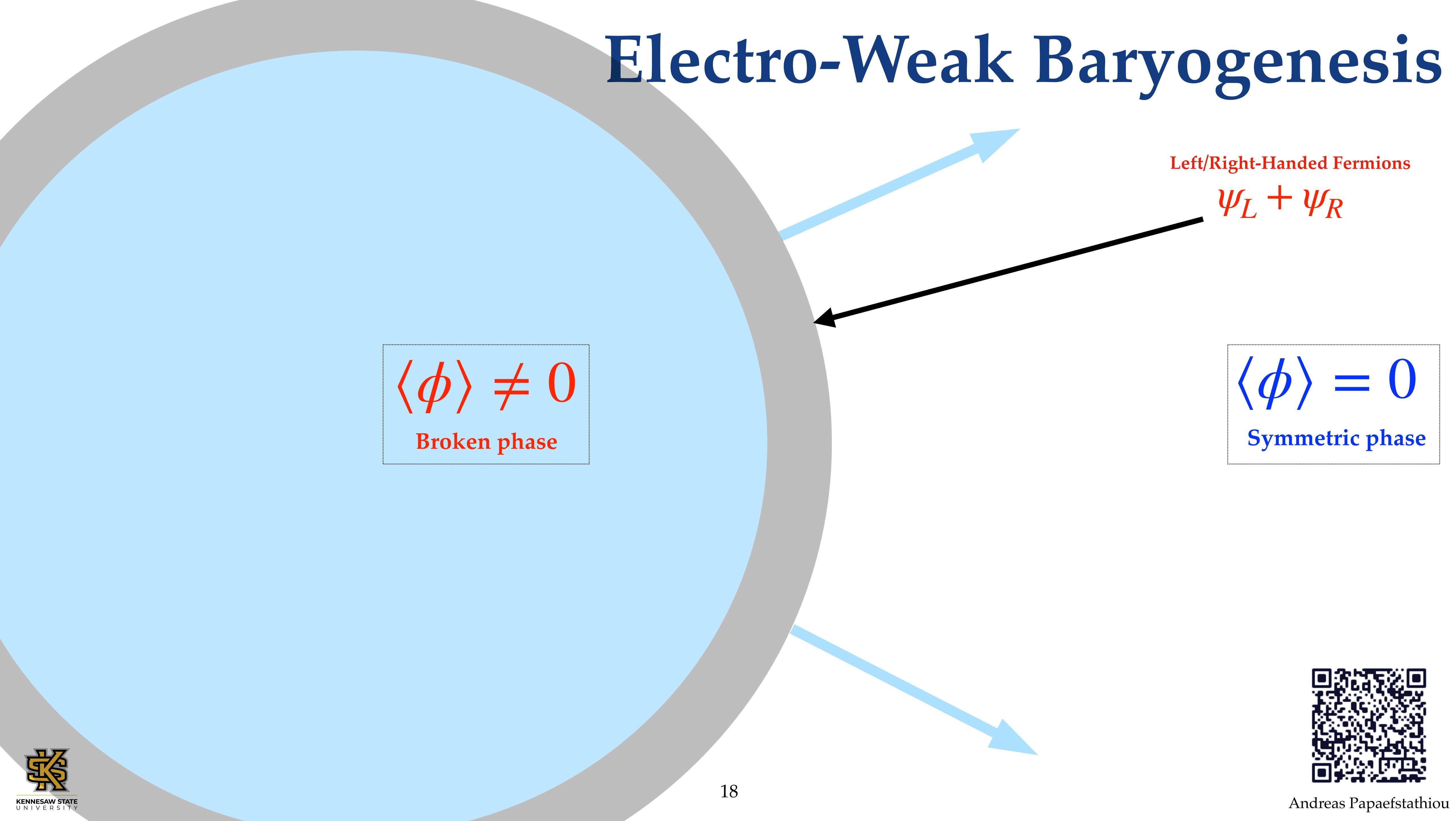
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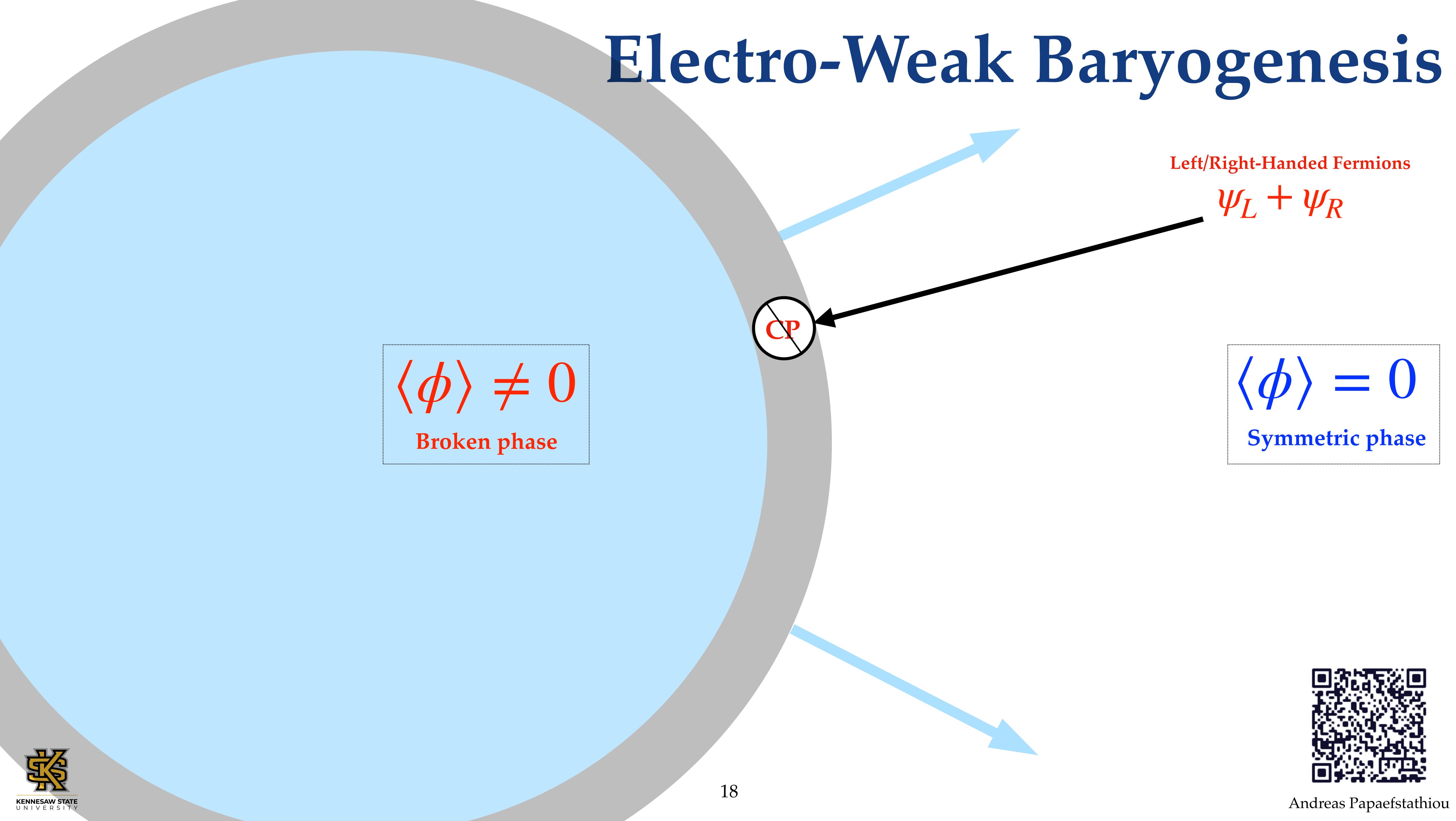
Electro-Weak Baryogenesis



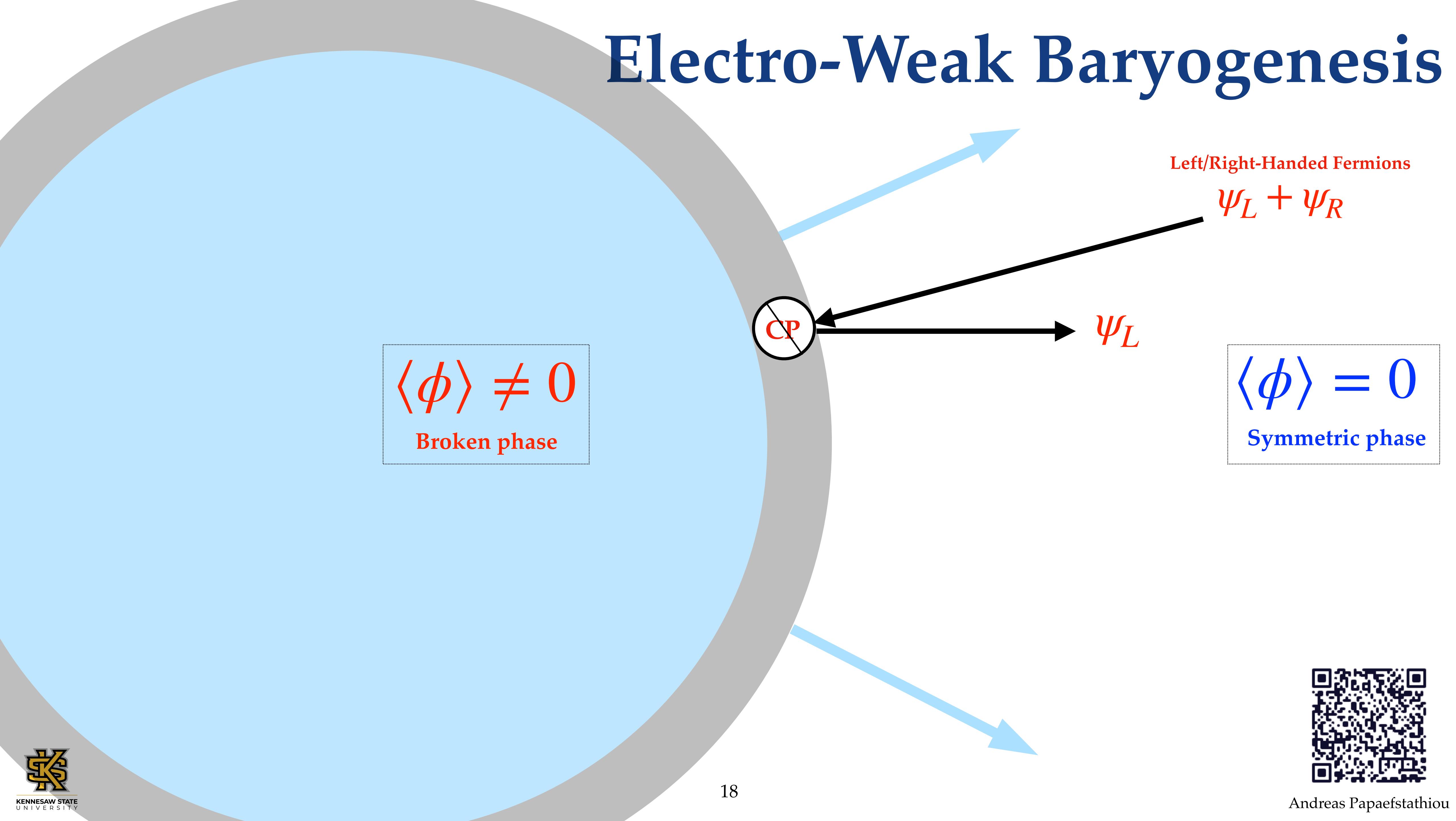
Electro-Weak Baryogenesis



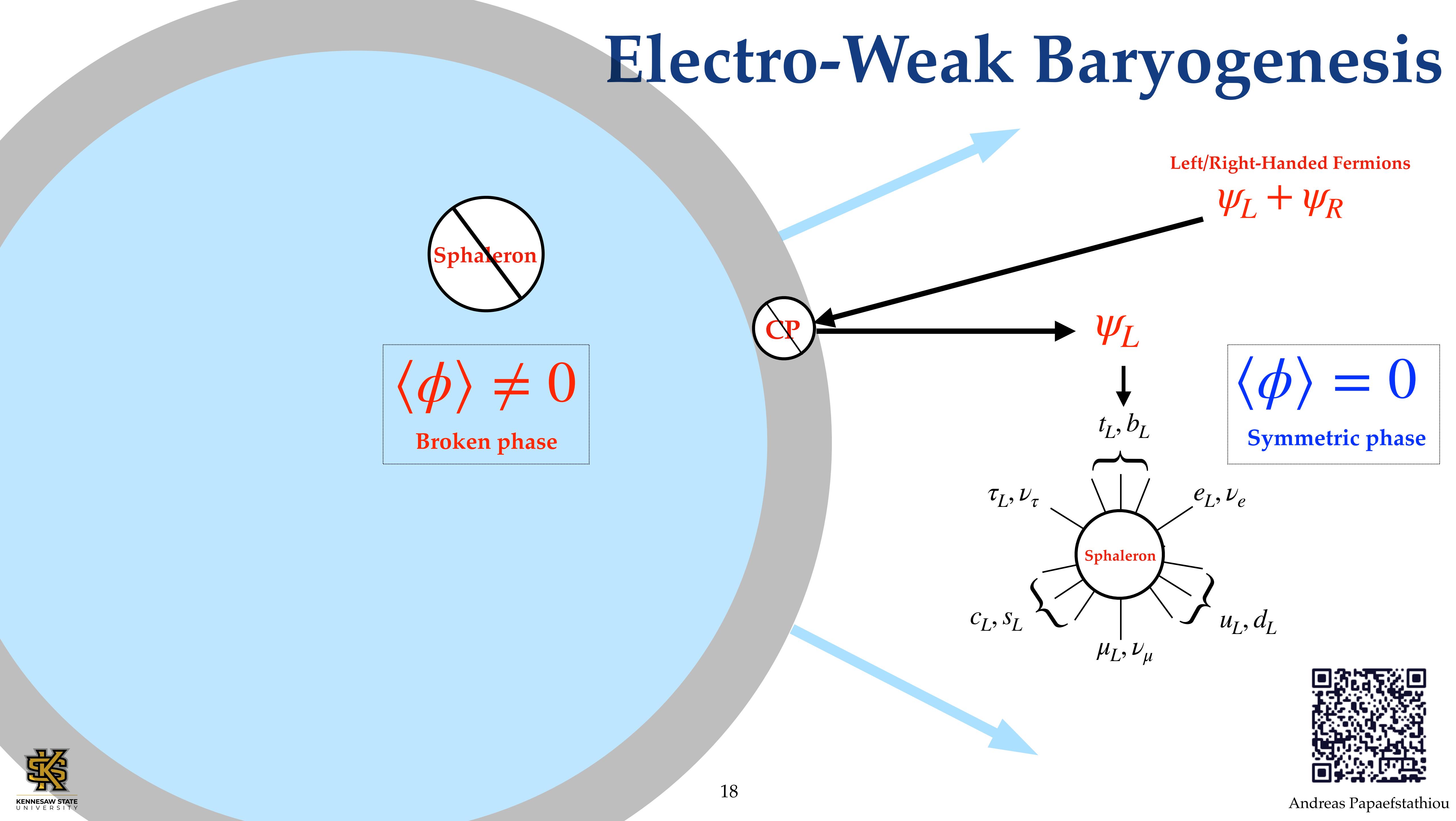
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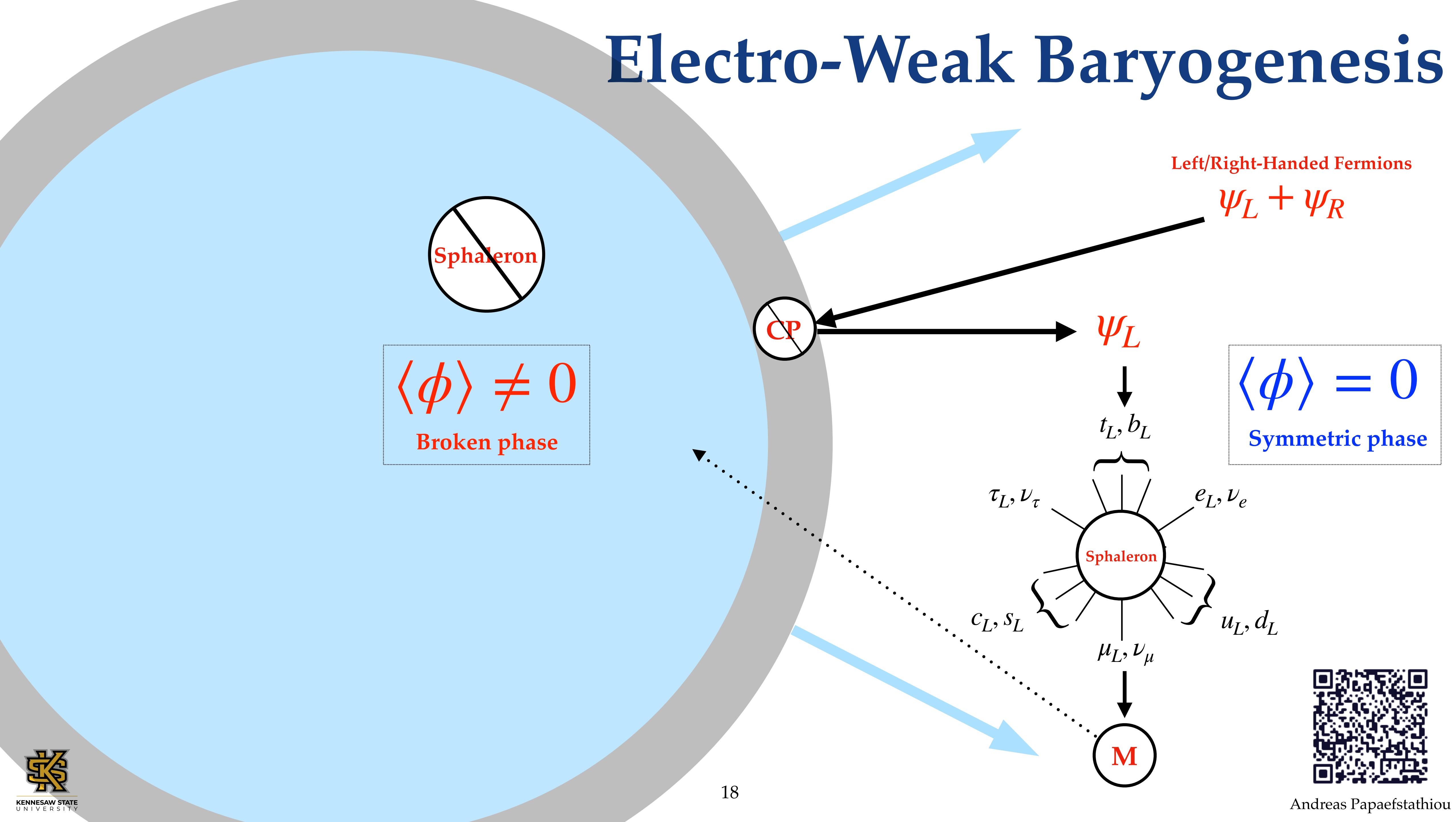
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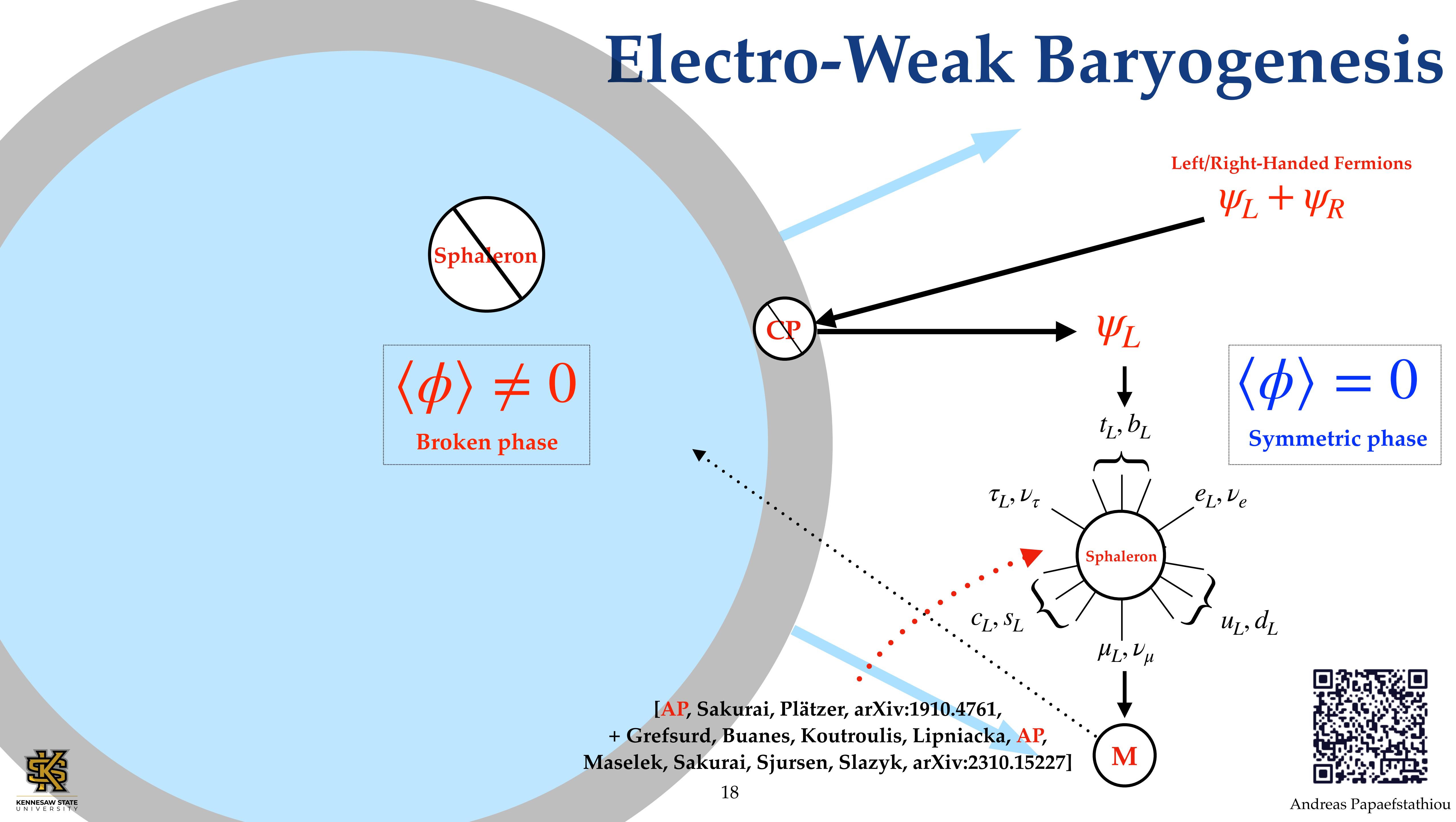
Electro-Weak Baryogenesis



Electro-Weak Baryogenesis

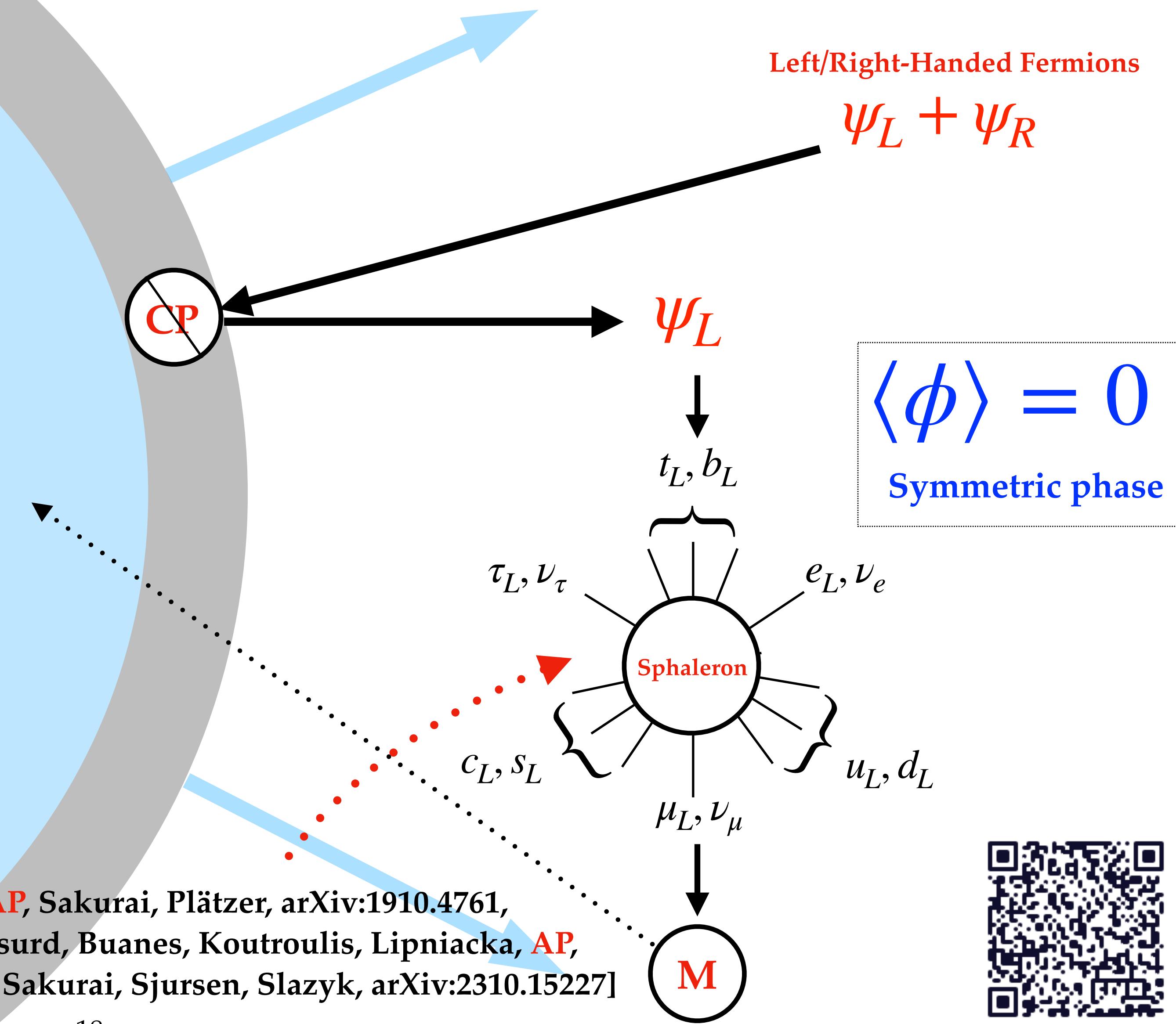


Electro-Weak Baryogenesis



Electro-Weak Baryogenesis

A First-Order
Transition requires
New Phenomena
beyond the SM!



[AP, Sakurai, Plätzer, arXiv:1910.4761,
+ Grefsurd, Buanes, Koutroulis, Lipniacka, AP,
Maselek, Sakurai, Sjursen, Slazyk, arXiv:2310.15227]



A Note on Sphaleron Suppression

- Suppression of sphaleron rate inside bubble,
⇒ Baryon Asymmetry “swept in” broken phase and “frozen in”.
- Rate $\sim \exp[-\langle \phi(T_C) \rangle / T_C \times \dots]$,
[T_C : the critical temperature.]
⇒ Require: $\langle \phi(T_C) \rangle / T_C \geq 1$,
⇒ a “Strong” First-Order EWPT \equiv SFO-EWPT.



Electro-Weak Archaeology



We live here!



$$\langle \phi \rangle \neq 0$$

Broken phase



Electro-Weak Archaeology



We live here!



$$\langle \phi \rangle \neq 0$$

Broken phase

→ What are the imprints of
Electro-Weak Baryogenesis
at Colliders?



Andreas Papaefstathiou

Electro-Weak Archaeology



We live here!



$$\langle \phi \rangle \neq 0$$

Broken phase

→ What are the imprints of
Electro-Weak Baryogenesis
at Colliders?

→ Let's explore this in
explicit New Physics models!



Extending the Scalar Sector [AP, White, arXiv:2010.00597]

- A First-Order EWPT dictates new phenomena. [Kajantie, Laine, Rummukainen, Shaposhnikov hep-ph/9605288]
- Consider first the **simplest possible extension to the SM!**

$$\mathcal{V}(\phi, S) = \textcolor{red}{\bigcirc} |\phi|^2 + \textcolor{blue}{\blacksquare} |\phi|^4$$

Add: S , a **new scalar field**,
No SM “charges” \equiv Singlet.

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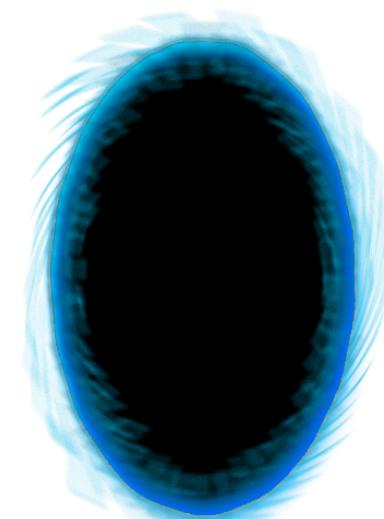
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$$+ \textcolor{red}{\triangle} |\phi|^2 S + \textcolor{purple}{\blacksquare} |\phi|^2 S^2 \leftarrow \text{“Portal” interactions.}$$

[$|\phi|^2$ is also an SM singlet!]

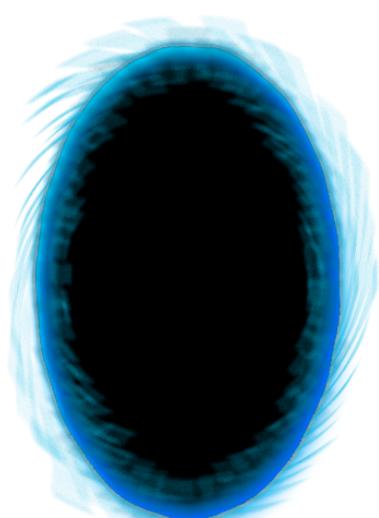
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$$+ \textcolor{red}{\triangle} |\phi|^2 S + \textcolor{purple}{\square} |\phi|^2 S^2 \leftarrow \text{“Portal” interactions.}$$

[$|\phi|^2$ is also an SM singlet!]

$$+ S \times (\text{Hidden Sector}) + \dots \leftarrow \text{Dark Matter?}$$

Extending the Scalar Sector [AP, White, arXiv:2010.00597]

$$\mathcal{V}(\phi, S) = \textcolor{red}{\bullet} |\phi|^2 + \textcolor{blue}{\blacksquare} |\phi|^4 + \textcolor{magenta}{\bullet} S^2 + \textcolor{cyan}{\blacktriangle} S^3 + \textcolor{brown}{\blacksquare} S^4 + \textcolor{red}{\blacktriangle} |\phi|^2 S + \textcolor{violet}{\blacksquare} |\phi|^2 S^2$$

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EWSB \leftrightarrow VEVs:

$$\phi \rightarrow \langle \phi \rangle + h$$

$$S \rightarrow \langle S \rangle + \chi$$

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EWSB \leftrightarrow VEVs:

$$\begin{aligned}\phi &\rightarrow \langle \phi \rangle + h & \longrightarrow \mathcal{V} \supset \textcolor{red}{\bullet} h^2 + \textcolor{cyan}{\circ} h\chi + \textcolor{blue}{\circ} \chi^2 \\ S &\rightarrow \langle S \rangle + \chi\end{aligned}$$

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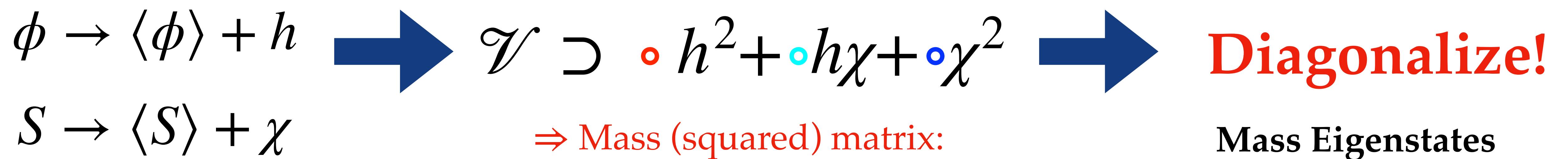
⇒ Mass (squared) matrix:

$$M^2 = \begin{pmatrix} \frac{\partial^2 \mathcal{V}}{\partial h^2} & \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} \\ \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} & \frac{\partial^2 \mathcal{V}}{\partial \chi^2} \end{pmatrix}$$

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$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ \chi \end{pmatrix}$$

θ : mixing angle

Extending the Scalar Sector [AP, White, arXiv:2010.00597]

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Mass Eigenstates

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ \chi \end{pmatrix}$$

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Extending the Scalar Sector [AP, White, arXiv:2010.00597]

$$\mathcal{V}(\phi, S) = \textcolor{green}{\bullet} |\phi|^2 + \textcolor{blue}{\blacksquare} |\phi|^4 + \textcolor{magenta}{\bullet} S^2 + \textcolor{cyan}{\blacktriangle} S^3 + \textcolor{brown}{\blacksquare} S^4 + \textcolor{red}{\blacktriangle} |\phi|^2 S + \textcolor{violet}{\blacksquare} |\phi|^2 S^2$$

Mass Eigenstates

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ \chi \end{pmatrix} \rightarrow \begin{array}{l} h_1 \rightarrow \text{"SM-like" Higgs boson.} \\ h_2 \rightarrow \text{new scalar resonance.} \end{array}$$

θ : mixing angle

i.e. choose: $|\theta| \gtrsim 0$, and:

$$h_1 = h \cos \theta + \chi \sin \theta$$

$$h_2 = -h \sin \theta + \chi \cos \theta$$



Extending the Scalar Sector [AP, White, arXiv:2010.00597]

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$h_1 \rightarrow$ “SM-like” Higgs boson.

$h_2 \rightarrow$ new scalar resonance.

i.e. choose: $|\theta| \gtrsim 0$, and:

$$h_1 = h \cos \theta + \chi \sin \theta$$

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Primary targets for collider studies!

$h_1 \Rightarrow$ Reductions in Higgs boson rates.

$h_2 \Rightarrow$ New resonance searches.



SM+Singlet: Current Collider Constraints

(i) LHC searches for **scalar resonances** in $h_2 \rightarrow h_1 h_1, ZZ, W^+W^-, \dots$, since:

$$g_{h_2 XX} \sim g_{h XX}^{\text{SM}} \times \sin \theta \text{ and } h_2 \rightarrow h_1 h_1 \text{ (if allowed),}$$

(ii) **Reductions in Higgs boson signal strengths** (LHC), since:

$$g_{h_1 XX} \sim g_{h XX}^{\text{SM}} \times \cos \theta,$$

[(i)+(ii) → e.g. through HiggsBounds/HiggsSignals]

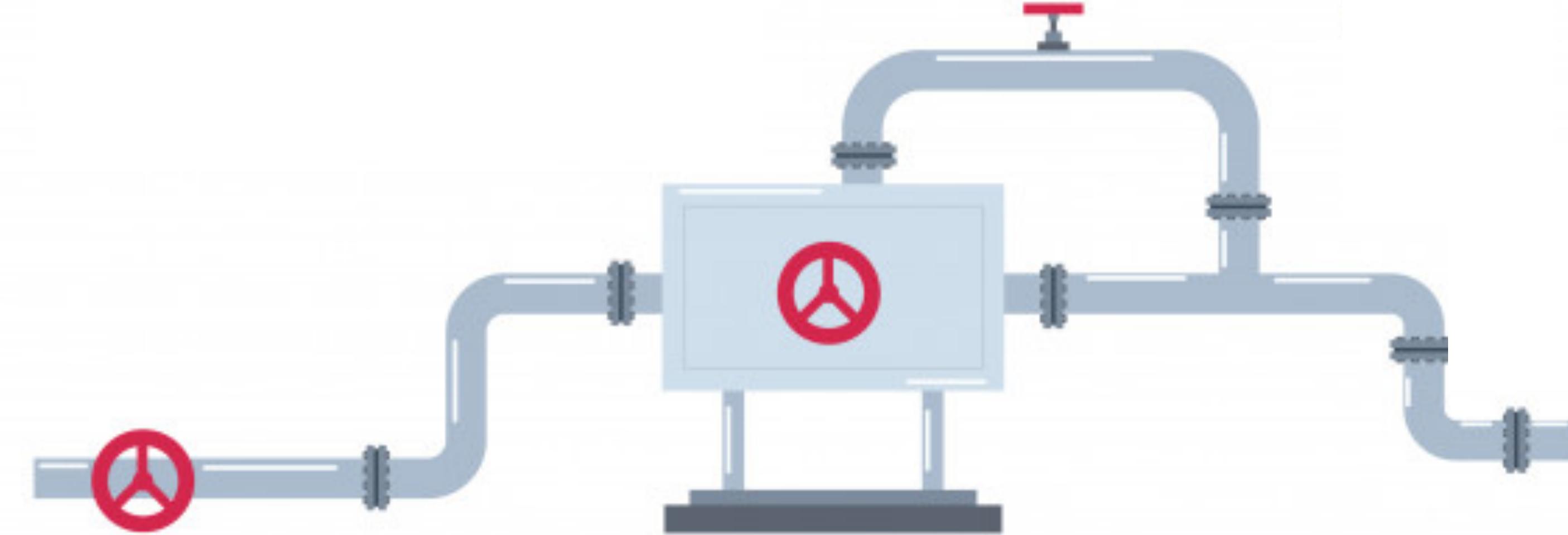
(iii) **Electro-Weak Precision Observables** (LEP). [e.g. Profumo, Ramsey-Musolf, Wainwright, Winslow, arXiv:1407.5342]

[+ **W mass constraints**. [e.g. López-Val, Robens arXiv:1406.1043, AP, Robens, White, arXiv:2205.14379]



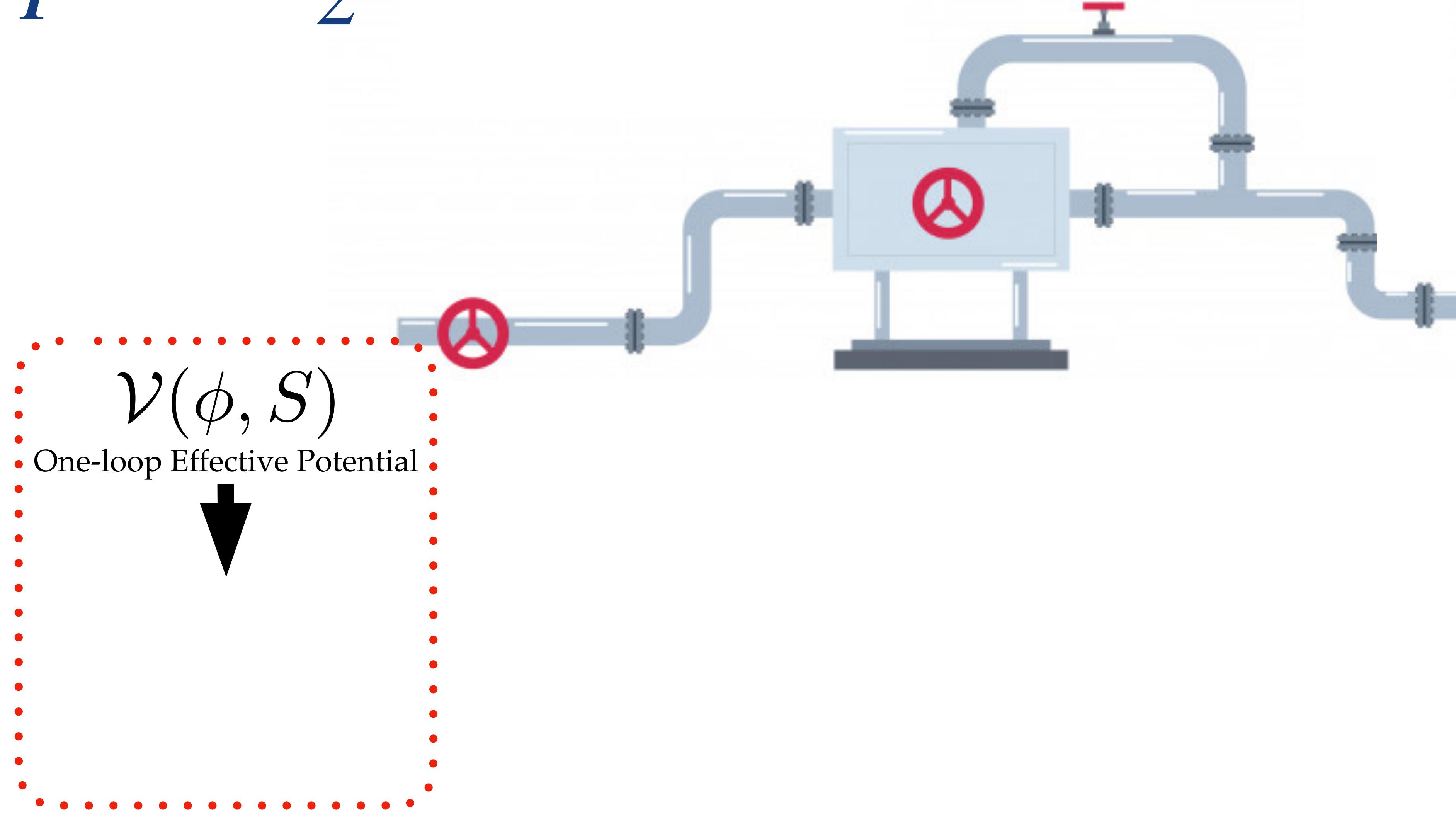
$pp \rightarrow h_2$ @ Future Colliders

[AP, White, arXiv:2010.00597]



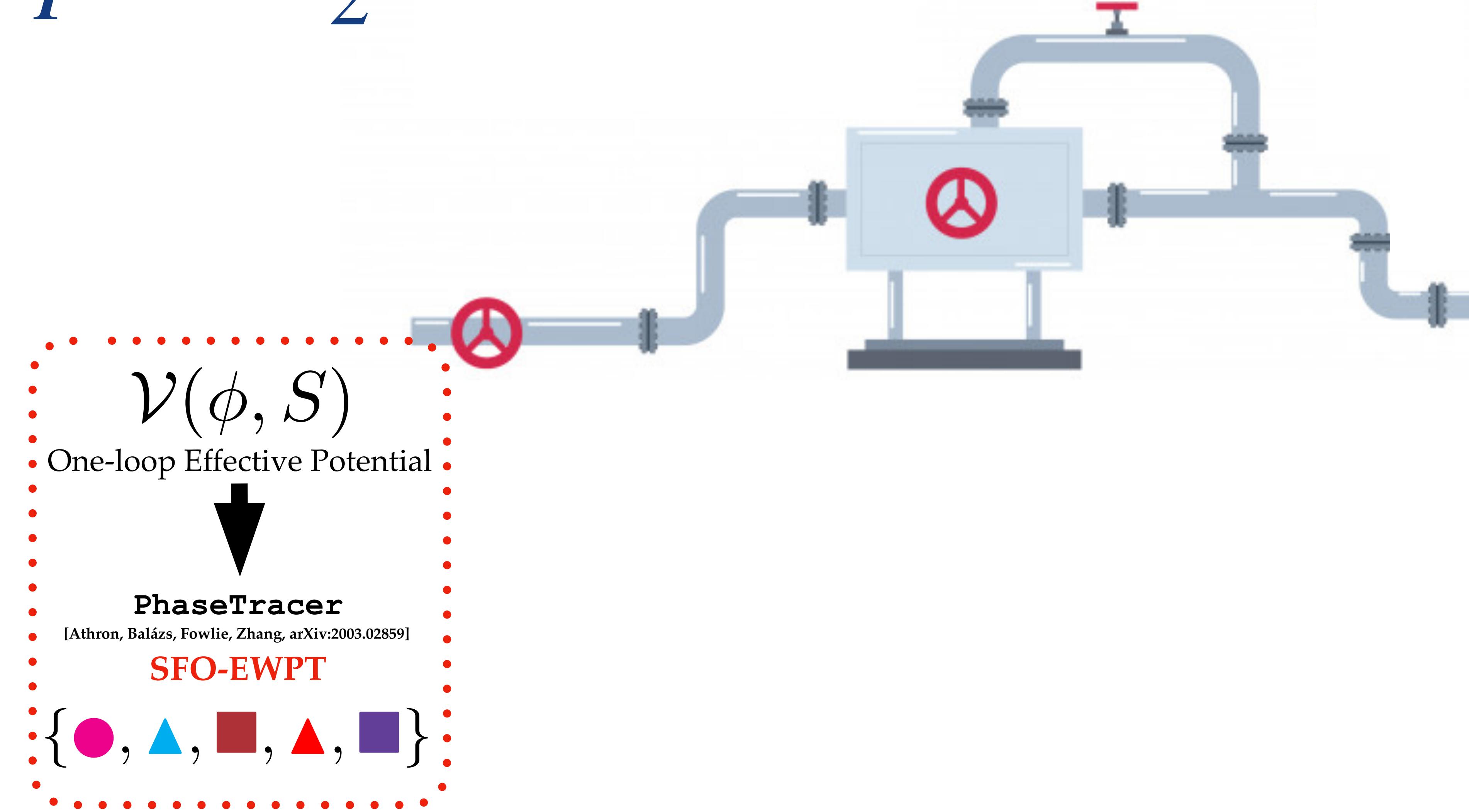
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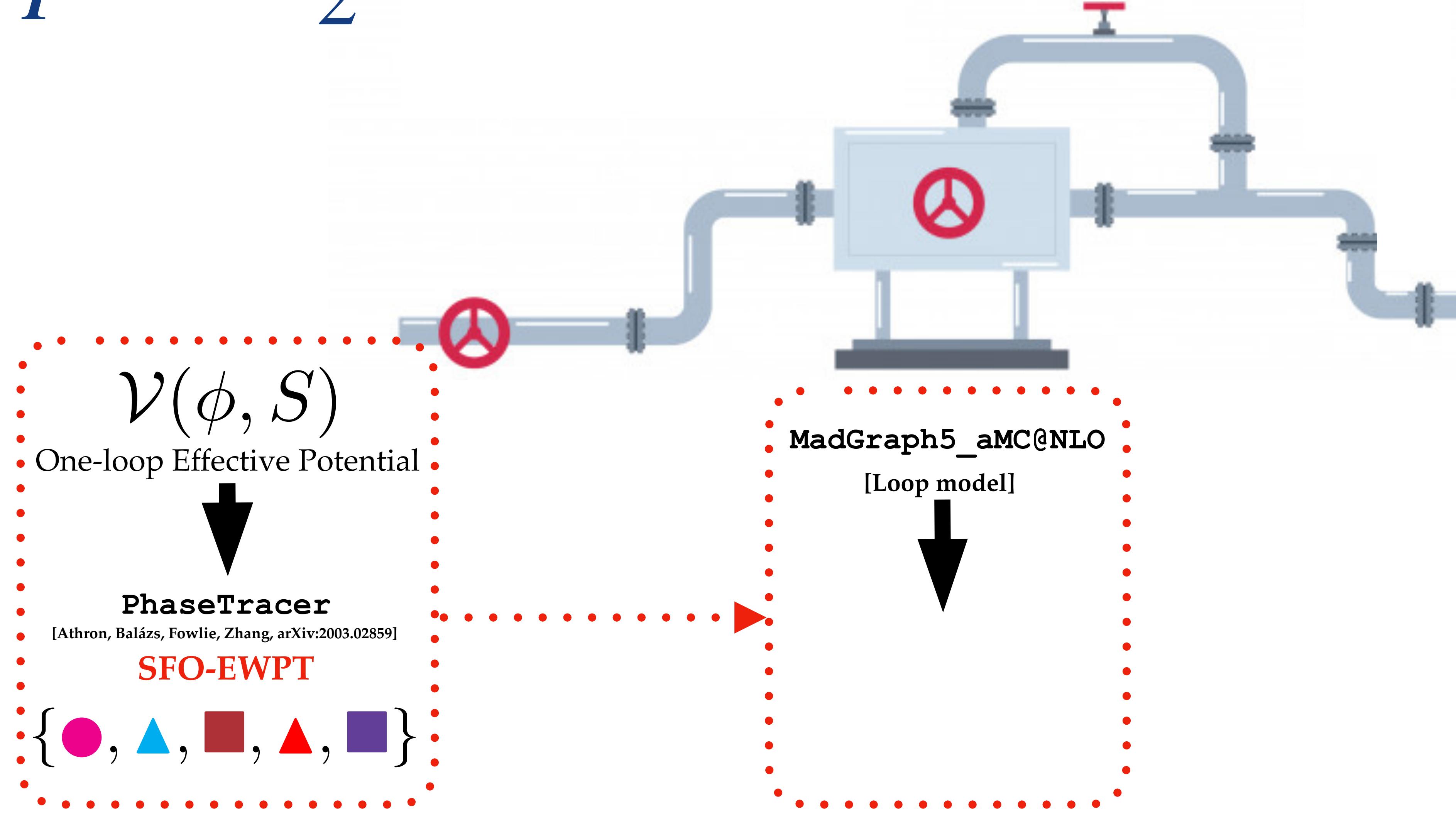
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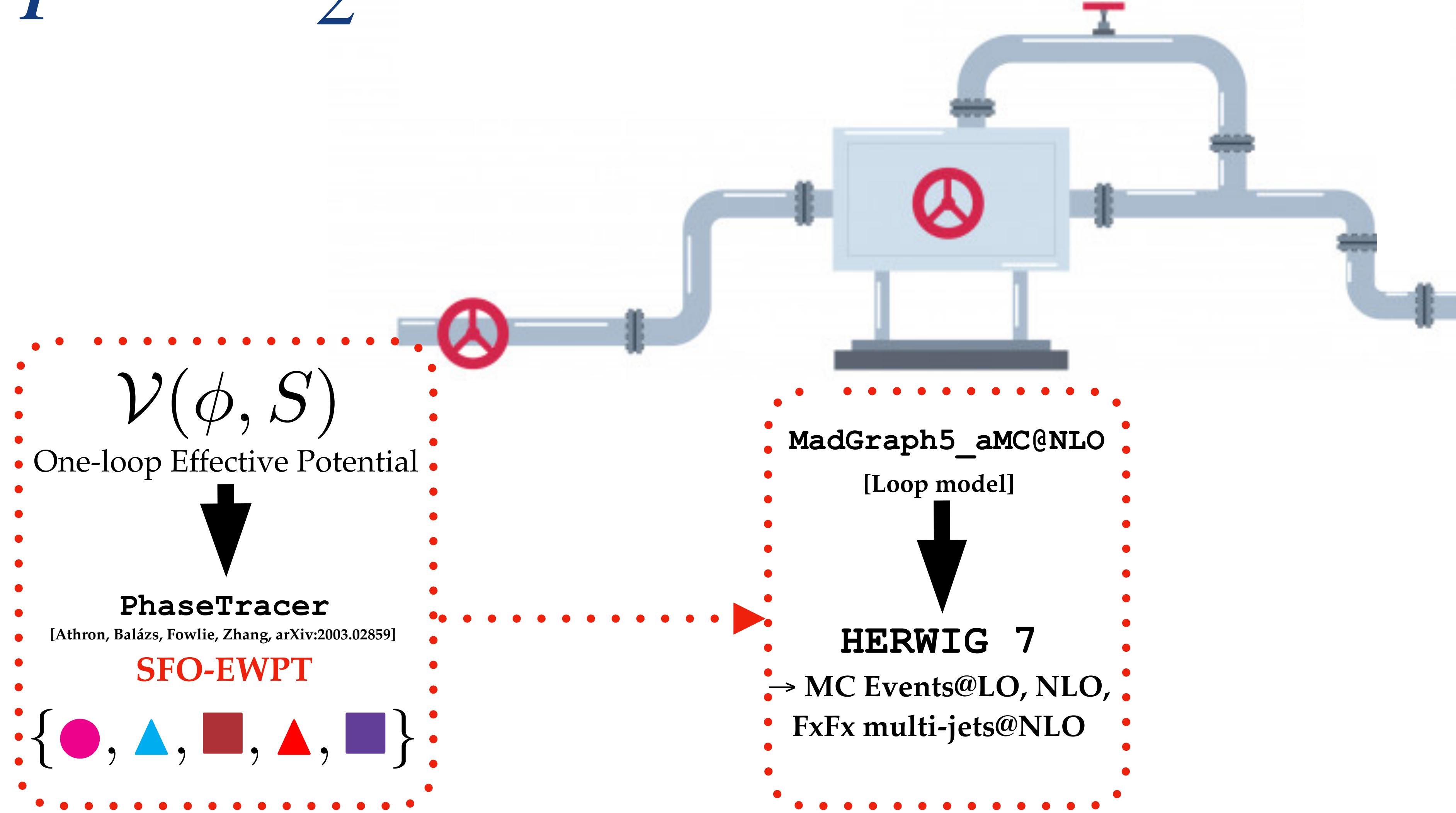
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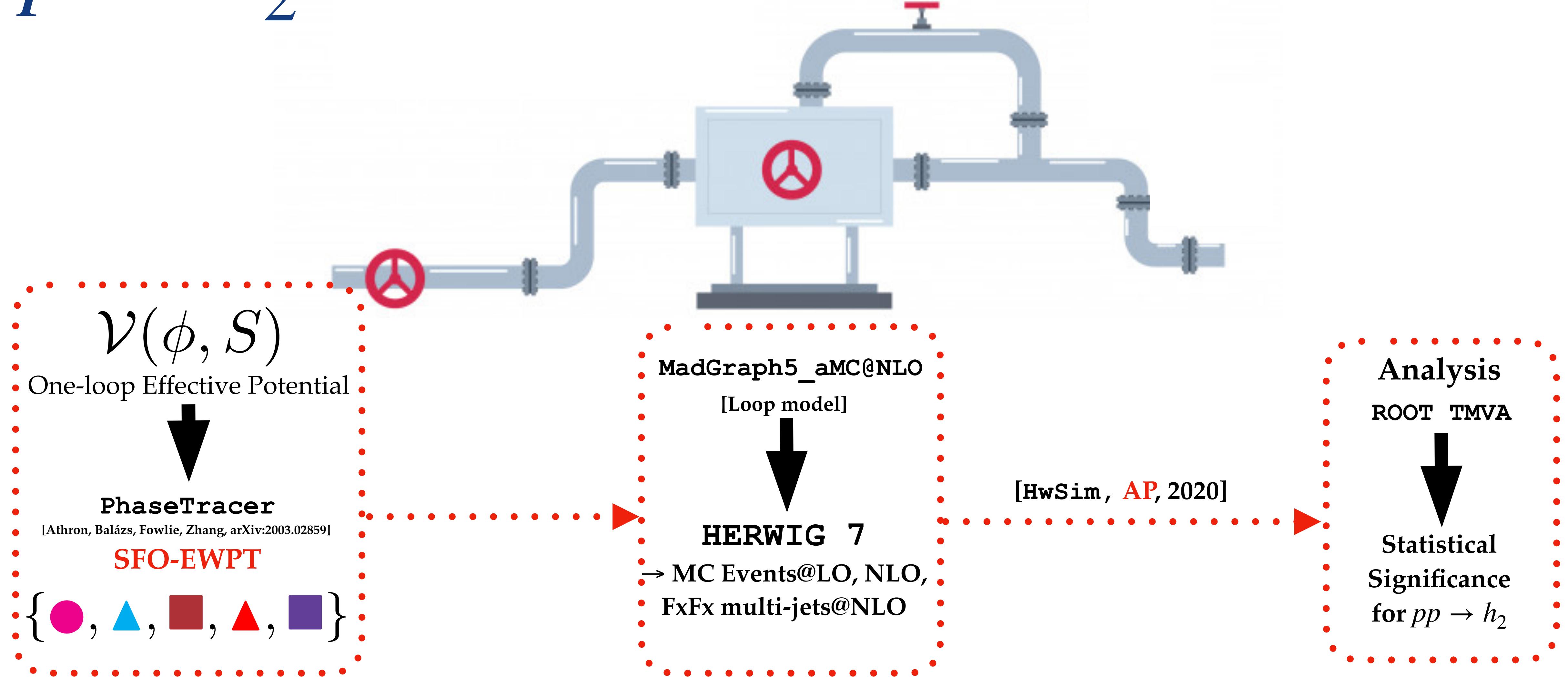
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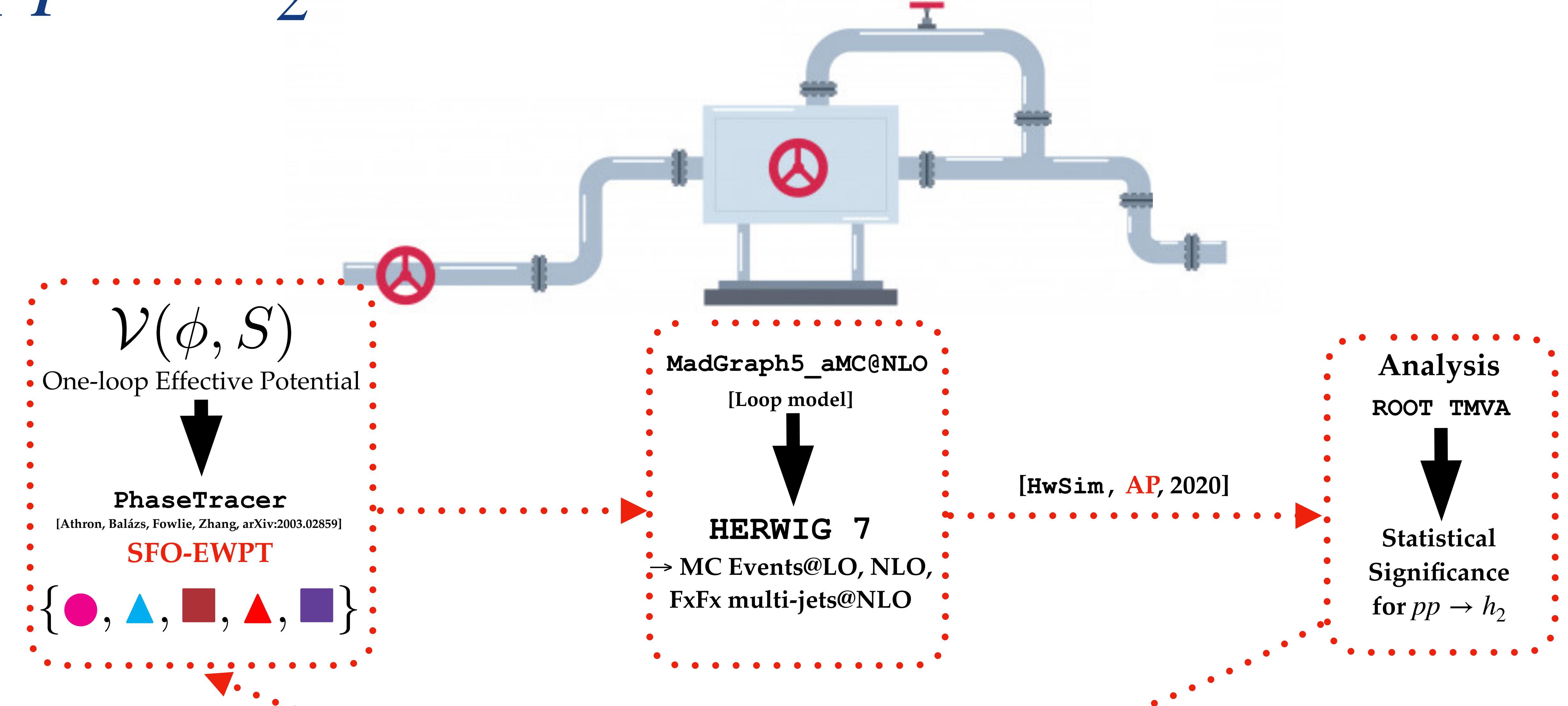
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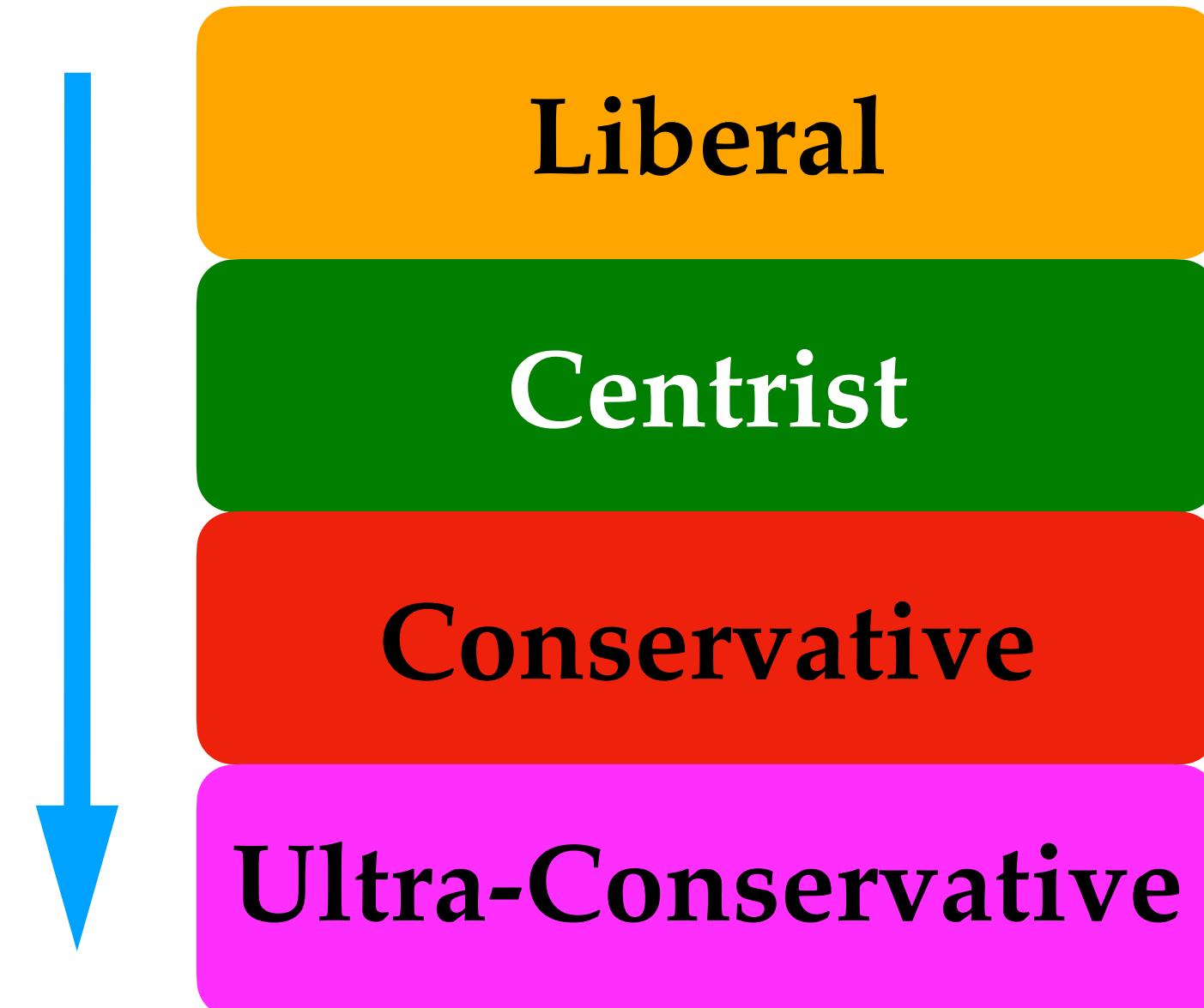
[AP, White, arXiv:2010.00597]



Degrees of Theoretical Uncertainty

[AP, White, arXiv:2010.00597]

Lower uncertainty
⇒
SFO-EWPT
more certain to
have occurred.



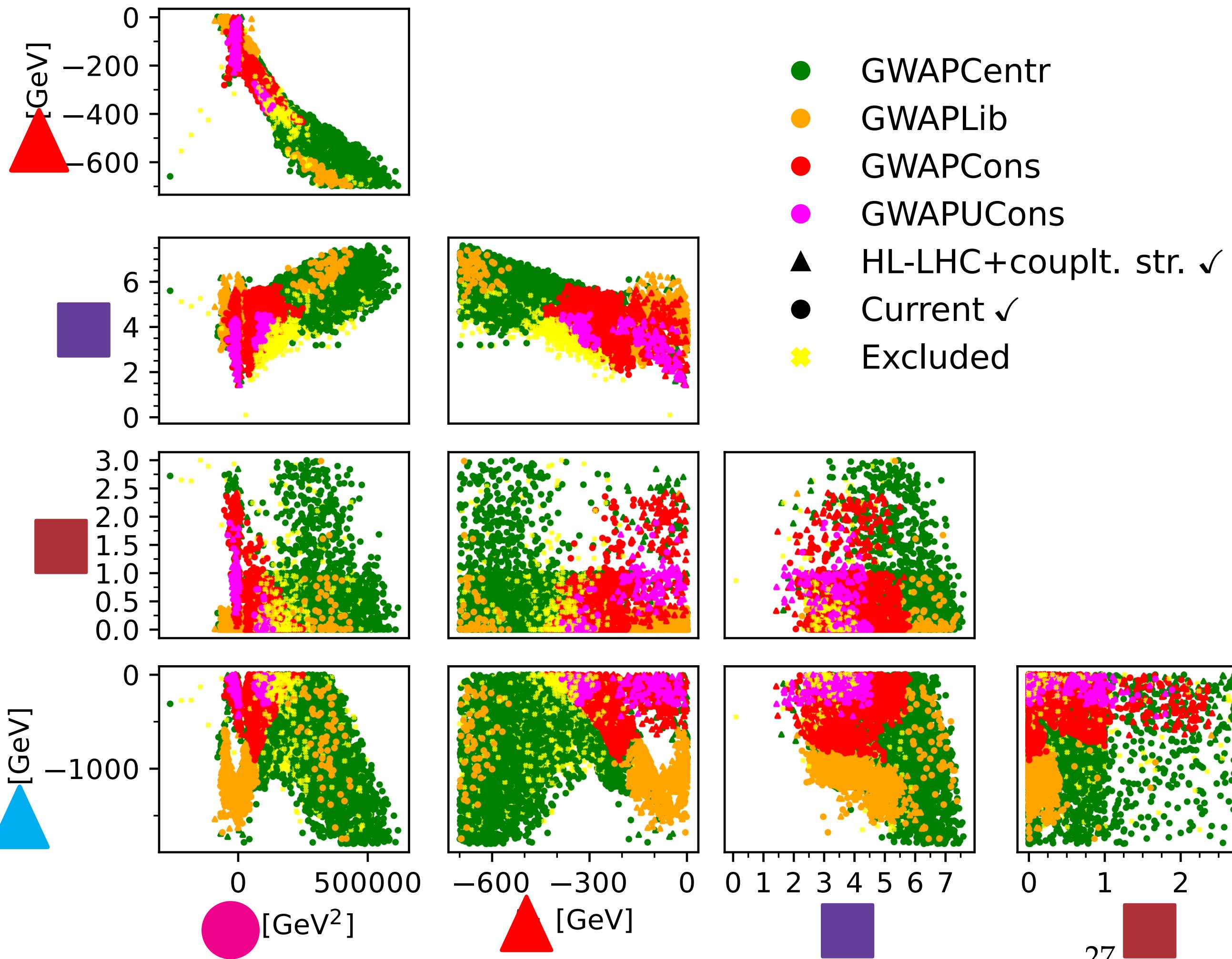
⇒ Colour-coded parameter-space points, denoting **theoretical uncertainty**.

& if: = Point is excluded today.



SFO-EWPT Parameter Space

$$\mathcal{V}(\phi, S) = \bullet |\phi|^2 + \blacksquare |\phi|^4 + \bullet S^2 + \blacktriangle S^3 + \blacksquare S^4 + \blacktriangle |\phi|^2 S + \blacksquare |\phi|^2 S^2$$

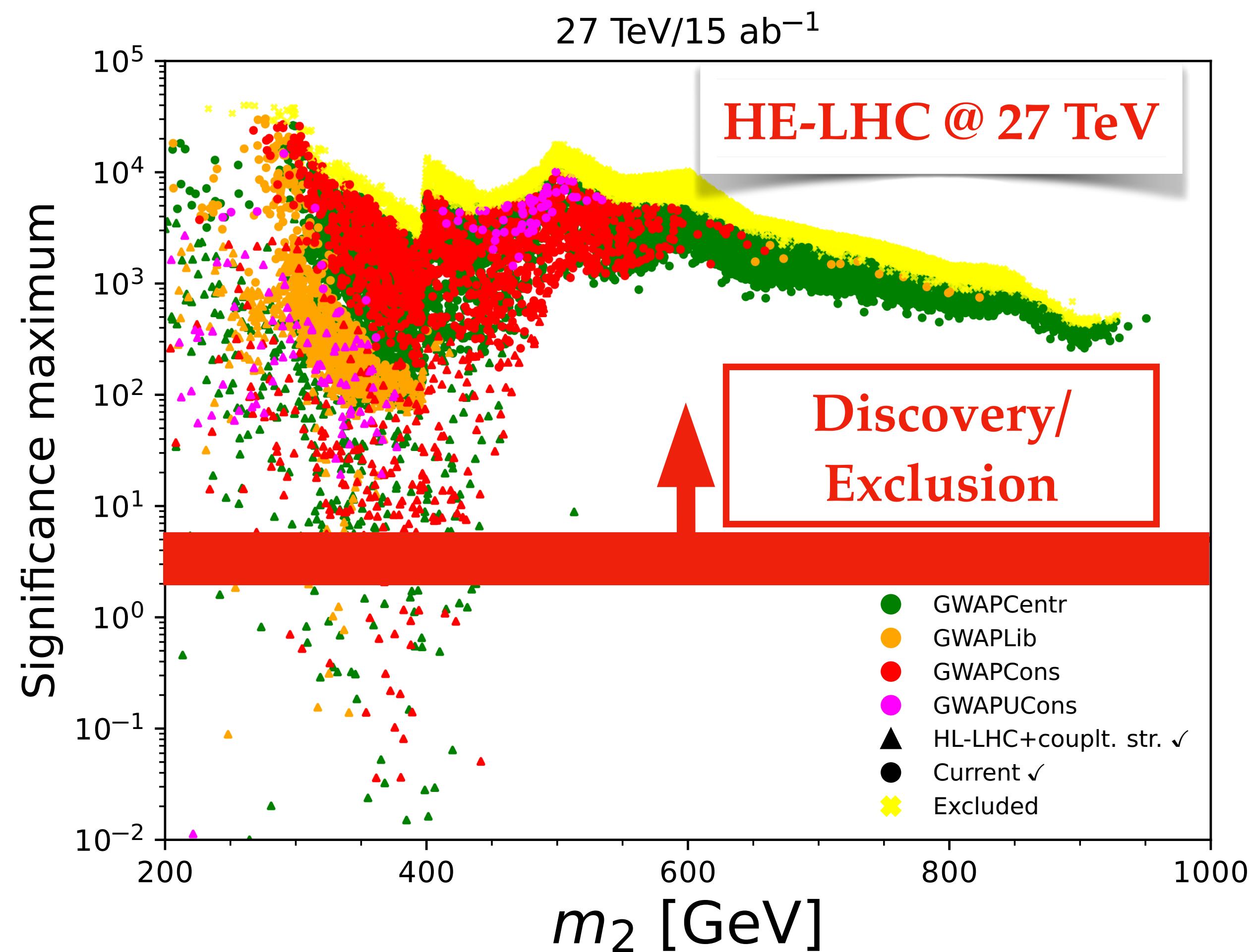
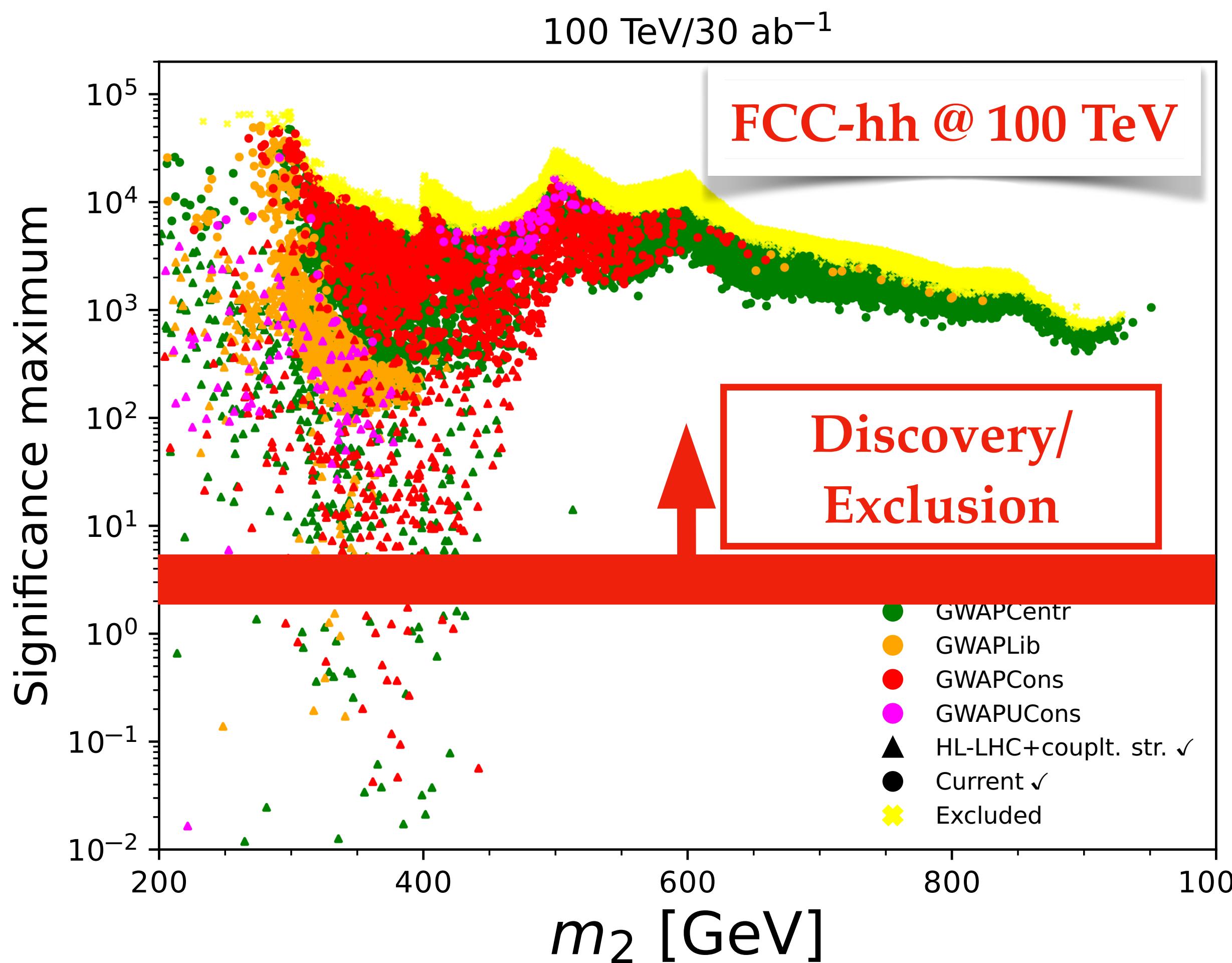


5 free parameters.



$pp \rightarrow h_2$ Significance @ Future Colliders

Color-coding of parameter-space points denotes **theoretical uncertainty**.



✗ = Point is excluded today.

Discovery Post-Mortem

“With 4 parameters I can fit an elephant and with 5 I can make him wiggle his trunk.”

– John Von Neumann

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Can we indeed fit the shape of an elephant with 4 parameters?

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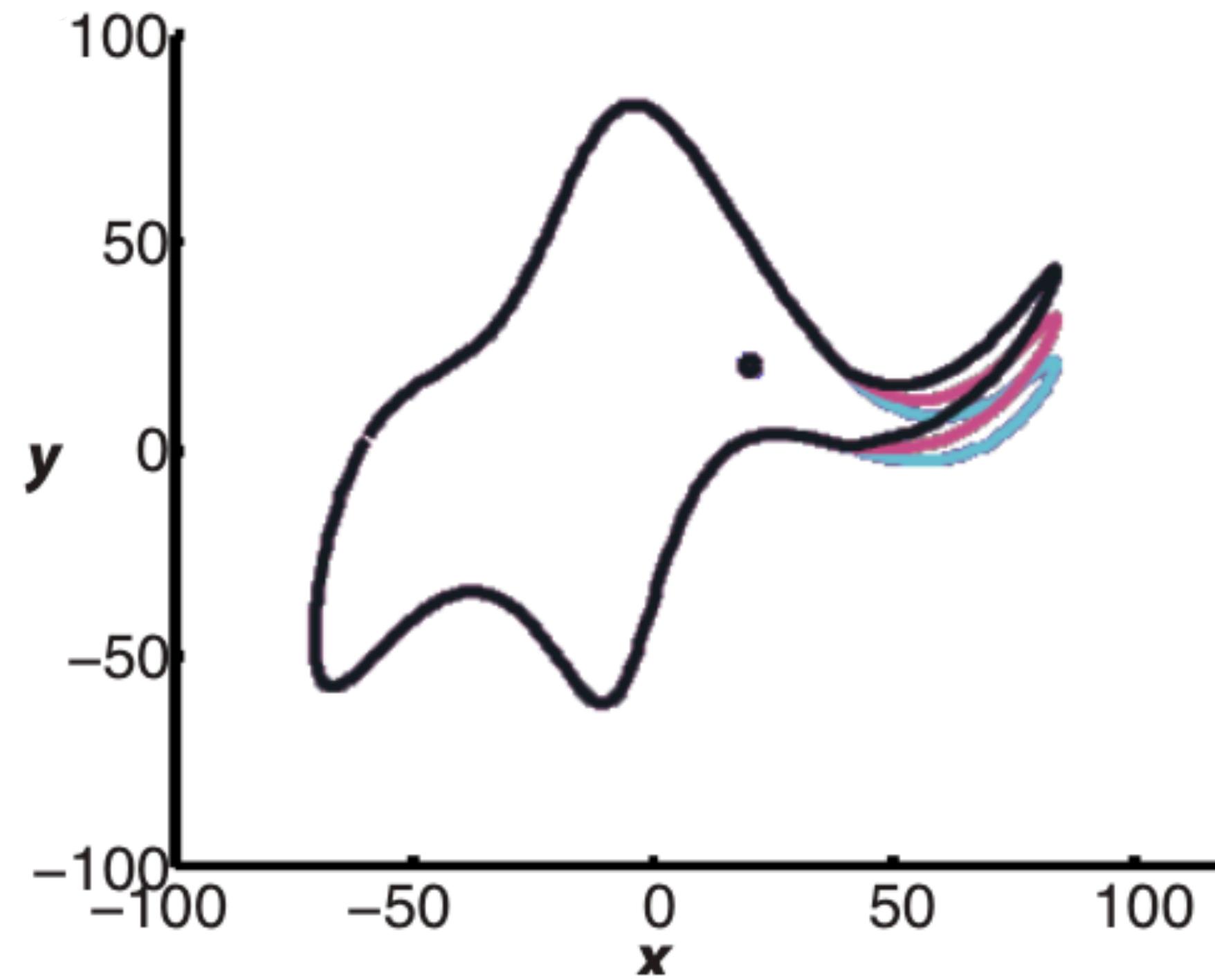
— John Von Neumann

Can we indeed fit the shape of an elephant with 4 parameters?

→ Yes! With four complex parameters,

[and with five we can make it wiggle its trunk.]

[Mayer, Khairy, Howard, Am. J. Phys., Vol. 78, No. 6, June 2010]



Discovery Post-Mortem

“With 4 parameters I can fit an elephant and with 5 I can make him wiggle his trunk.”
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If we discover e.g. a new scalar particle at colliders,

→ Can we verify that it is indeed the remnant of a singlet field that generates a SFO-EWPT?

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- ⇒ The “Inverse Problem”.

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⇒ The “Inverse Problem”. → Experimental observations → $\{\textcolor{magenta}{\bullet}, \textcolor{cyan}{\blacktriangle}, \textcolor{brown}{\blacksquare}, \textcolor{red}{\blacktriangle}, \textcolor{purple}{\blacksquare}\}$

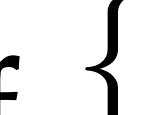
Discovery Post-Mortem Example [AP, White, arXiv:2108.11394]

Combine possible measurements:

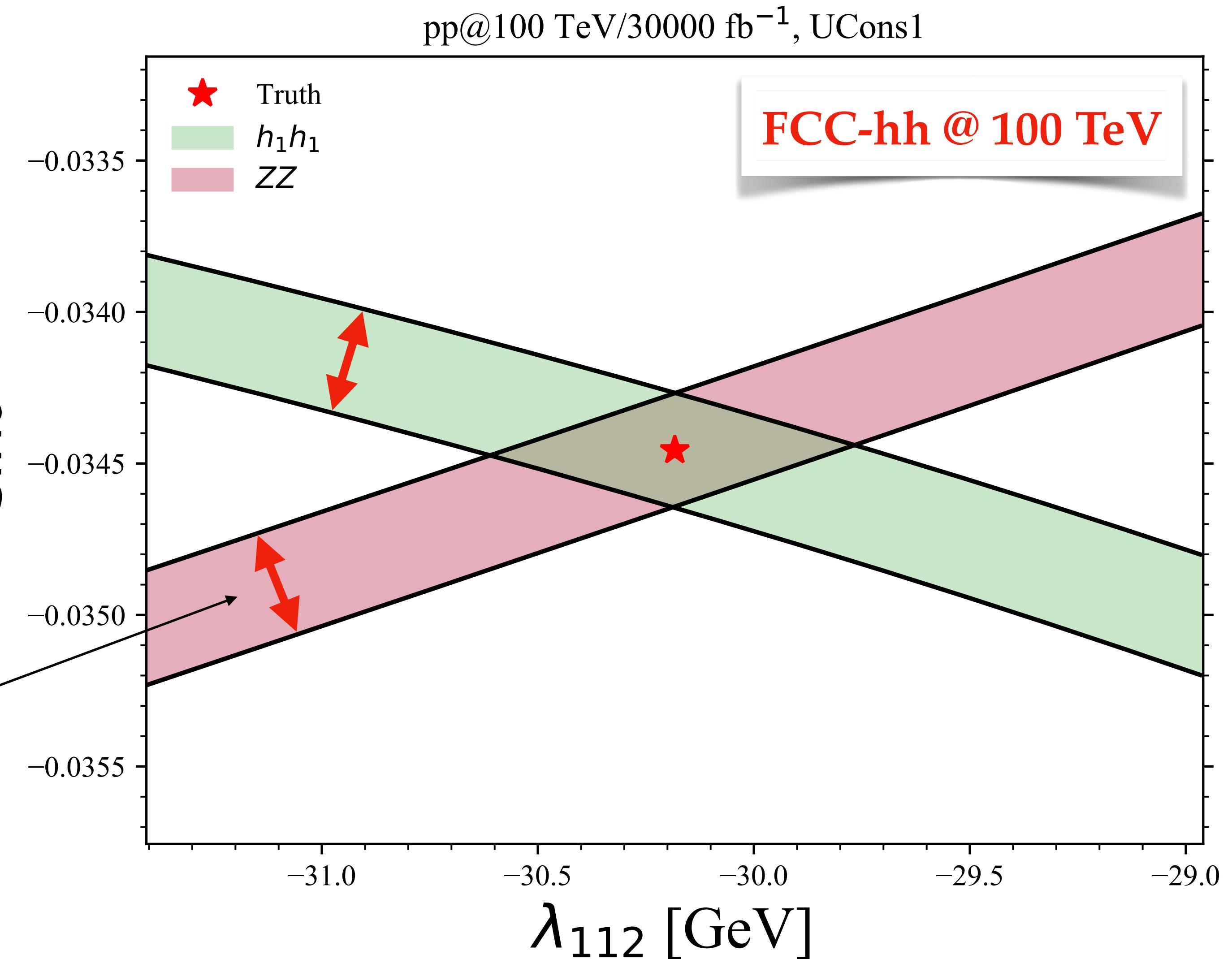
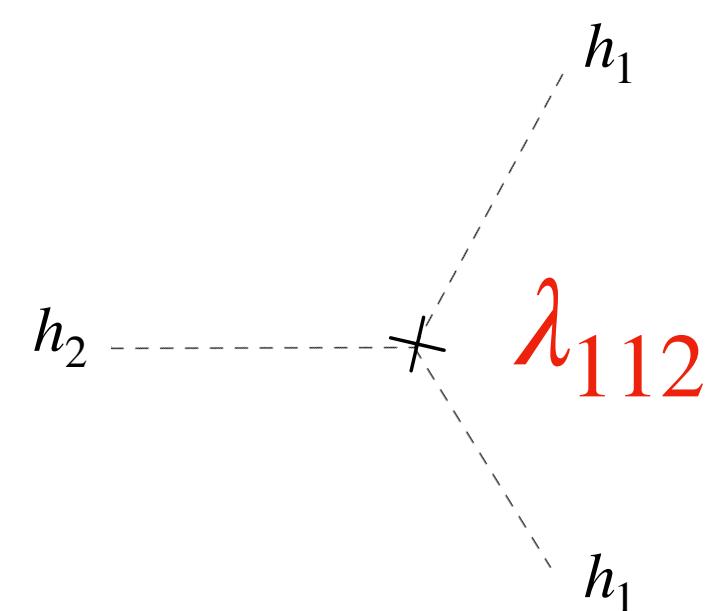
$pp \rightarrow h_2 \rightarrow ZZ$

$pp \rightarrow h_2 \rightarrow h_1 h_1$

→ both functions of $\sin \theta$ & λ_{112} !

→ in turn fns of {, , , , }.

[Width of bands represents expected measurement uncertainty].



Discovery Post-Mortem Example

[AP, White, arXiv:2108.11394]

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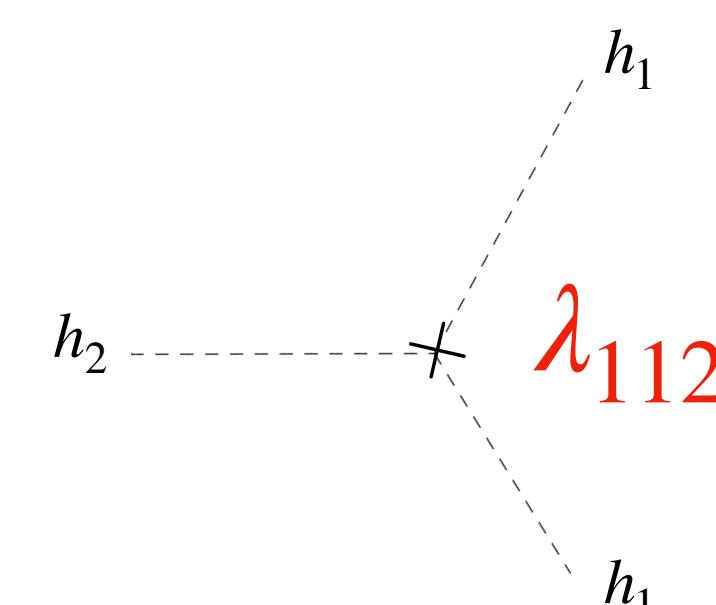
$pp \rightarrow h_2 \rightarrow ZZ$

$pp \rightarrow h_2 \rightarrow h_1 h_1$

→ both functions of $\sin \theta$ & λ_{112} !

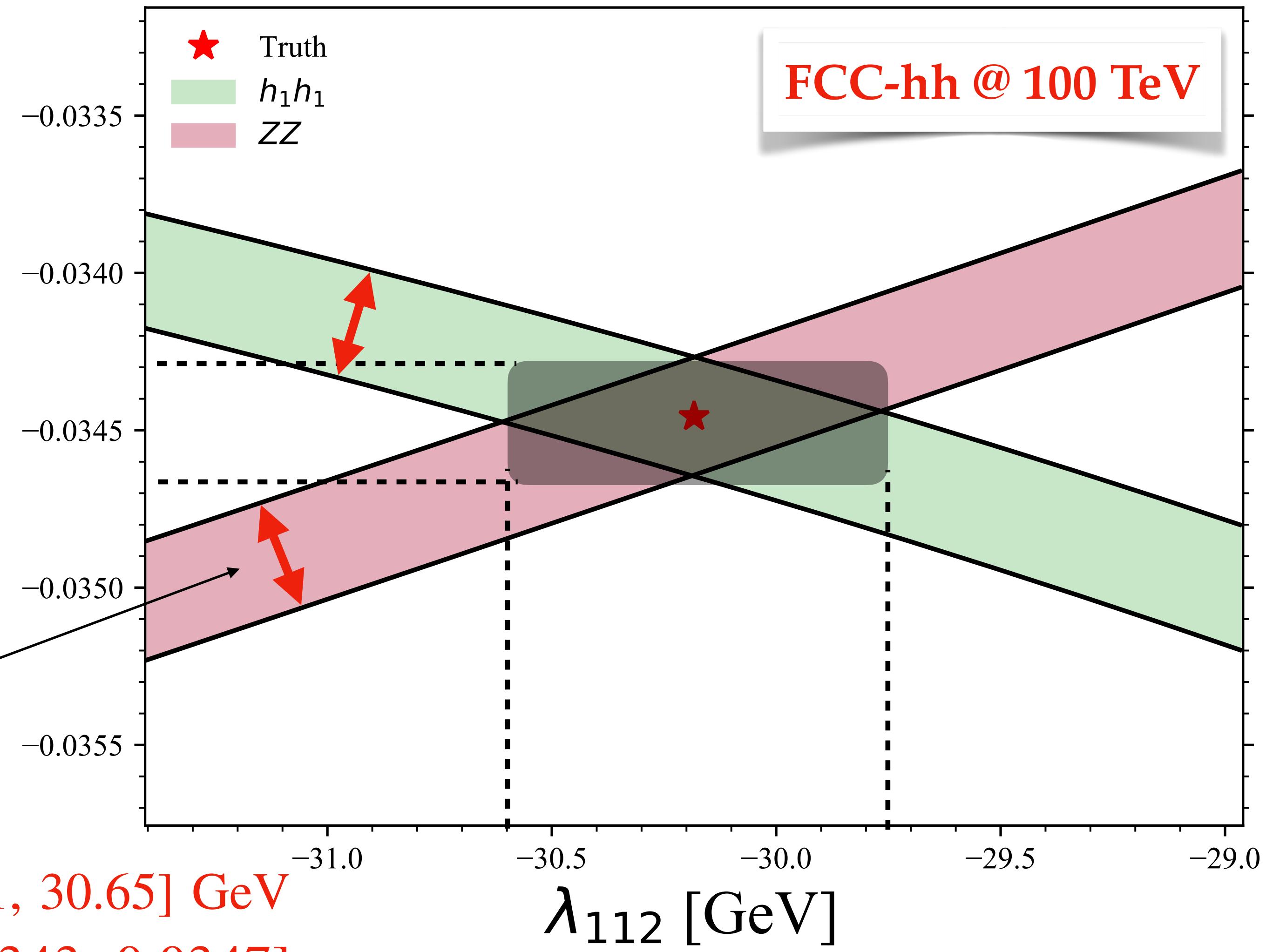
→ in turn fns of $\{\text{●}, \text{▲}, \text{■}, \text{▲}, \text{■}\}$.

[Width of bands represents expected measurement uncertainty].



$$\begin{aligned}\lambda_{112} &\in [29.81, 30.65] \text{ GeV} \\ \sin \theta &\in [0.0343, 0.0347]\end{aligned}$$

pp@100 TeV/30000 fb $^{-1}$, UCons1



The Inverse Problem in Extended Scalar Sectors:

Multi-scalar processes should play a **crucial rôle**:

$$pp \rightarrow h_1 h_2$$

$$|\mathcal{M}|^2 \sim \lambda_{122}^2, \lambda_{112}^2 + \dots$$

$$pp \rightarrow h_2 h_2$$

$$|\mathcal{M}|^2 \sim \lambda_{222}^2, \lambda_{122}^2 + \dots$$

$$pp \rightarrow h_1 h_1 h_1$$

$$|\mathcal{M}|^2 \sim f[\lambda_{1111}, \lambda_{1112}, \lambda_{111}, \lambda_{112}]$$

[...]

The Inverse Problem in Extended Scalar Sectors:

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[...]

$pp \rightarrow hhh$



SM Multi-Higgs Boson Production “Fun” Facts

- \exists factor of $\mathcal{O}(10^{-3})$ each time you “draw” an extra Higgs boson @ pp colliders.

SM, 14 TeV



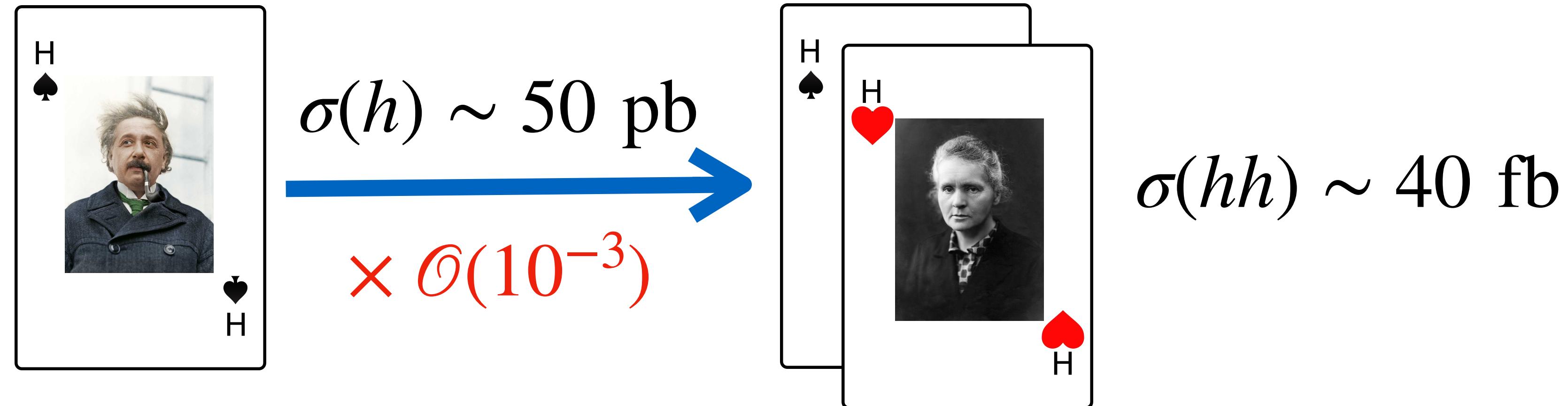
$$\sigma(h) \sim 50 \text{ pb}$$



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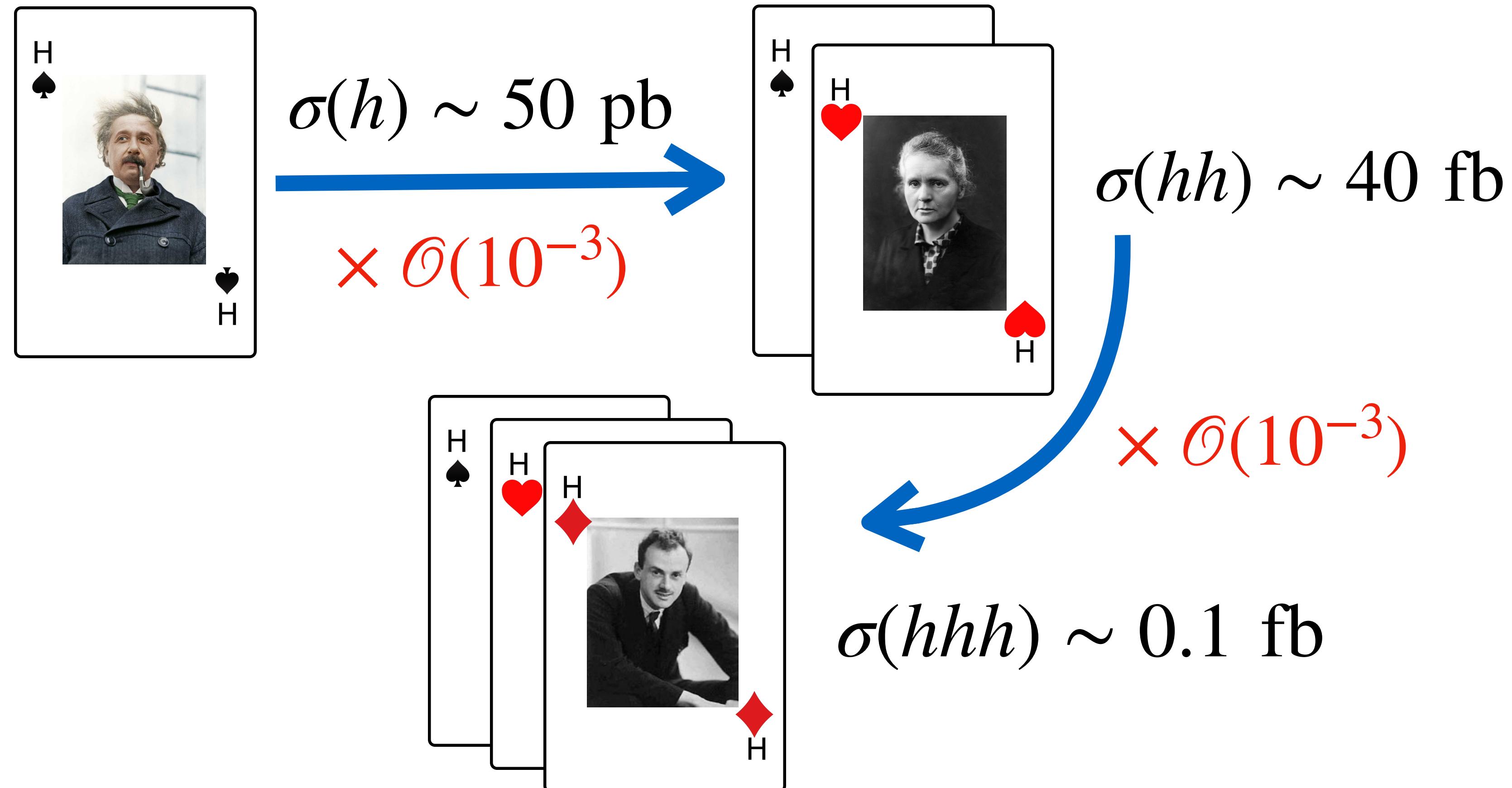
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SM Multi-Higgs Boson Production “Fun” Facts

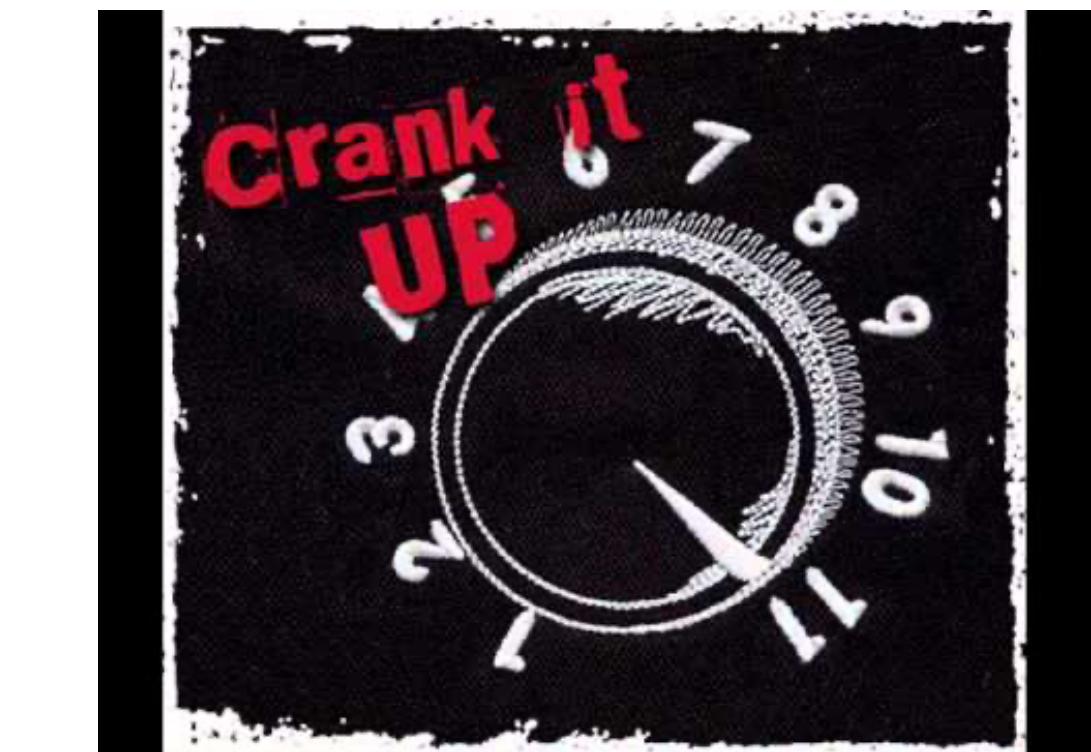
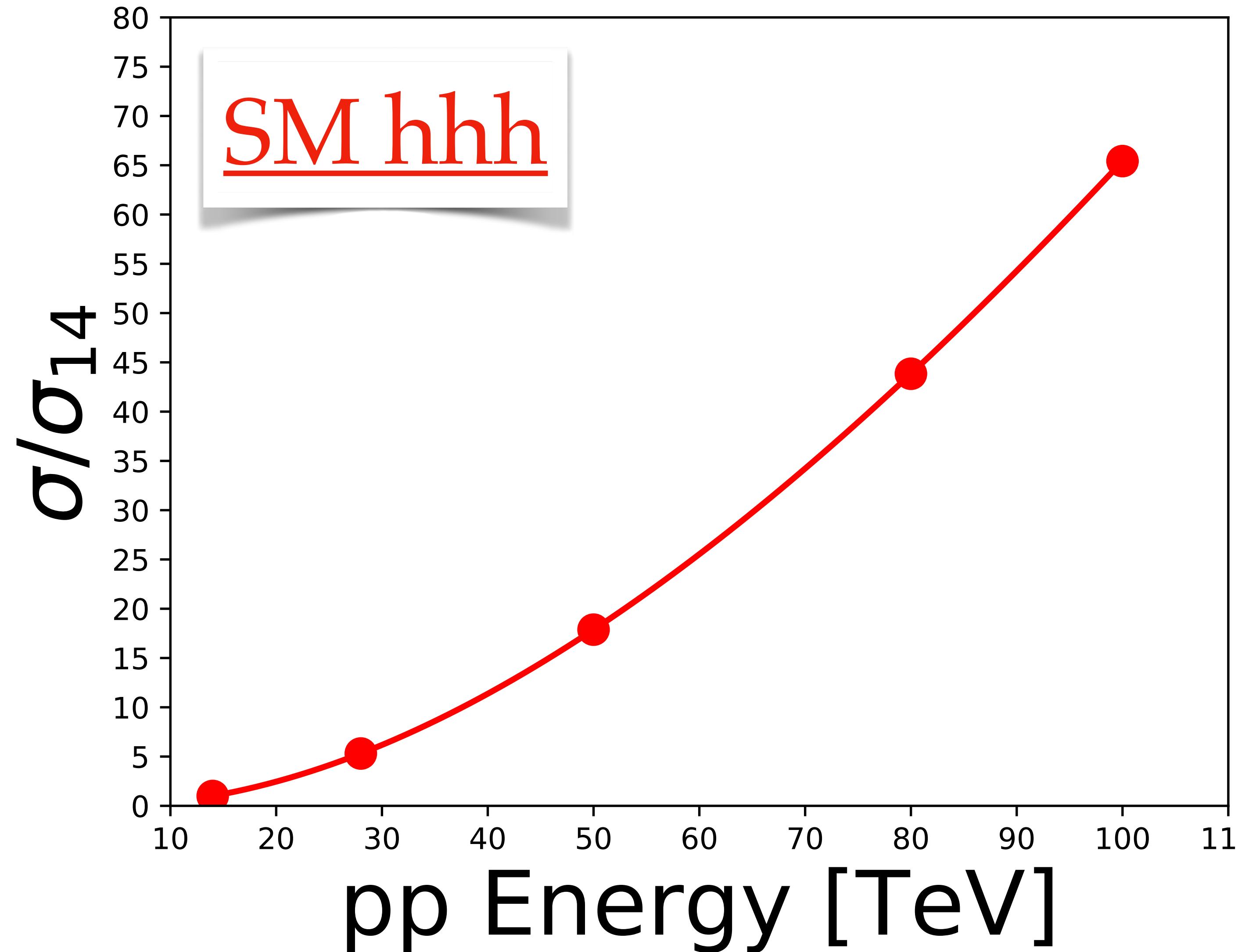
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SM, 14 TeV



SM Multi-Higgs Boson Production “Fun” Facts

- Cranking up the pp energy could help!

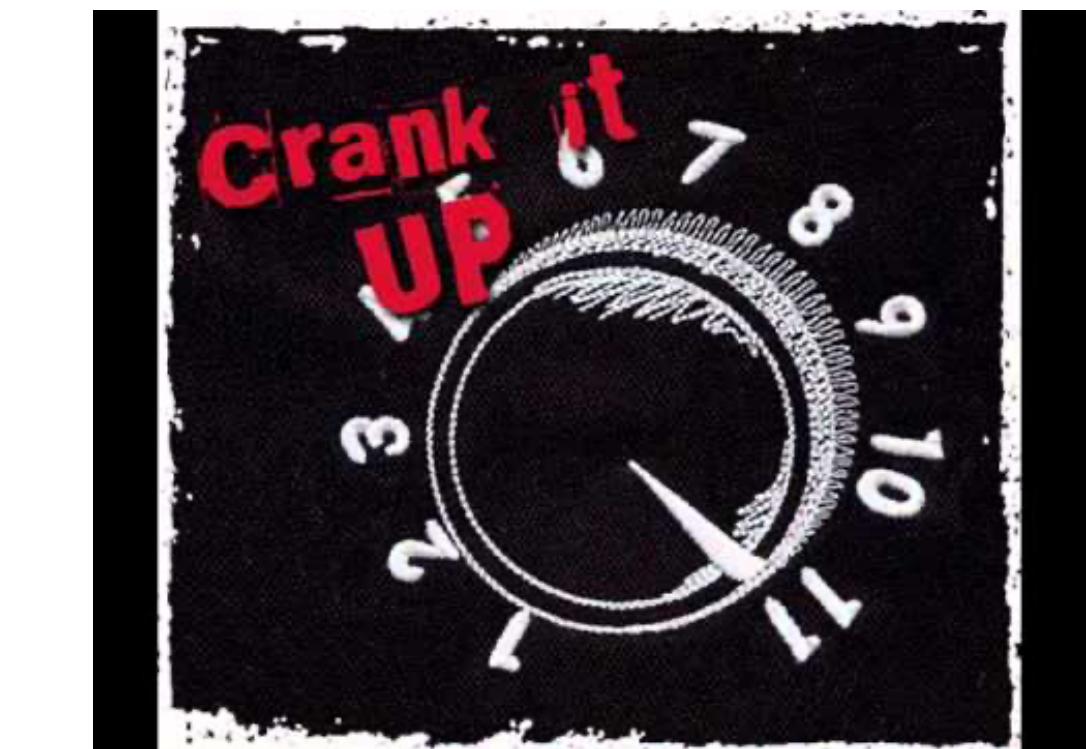
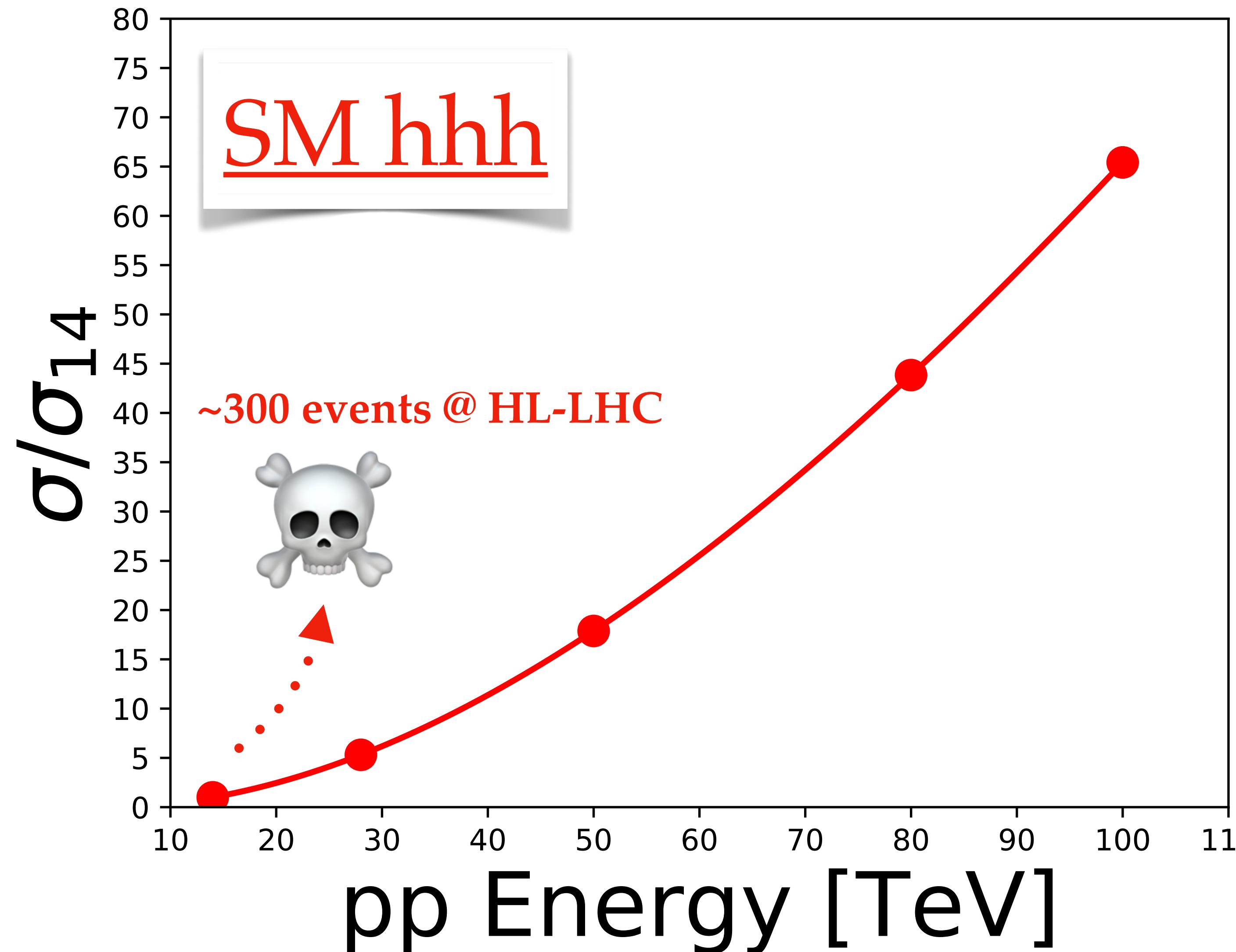


~ $\times 60$ increase in
cross section
 $14 \text{ TeV} \rightarrow 100 \text{ TeV}$.



SM Multi-Higgs Boson Production “Fun” Facts

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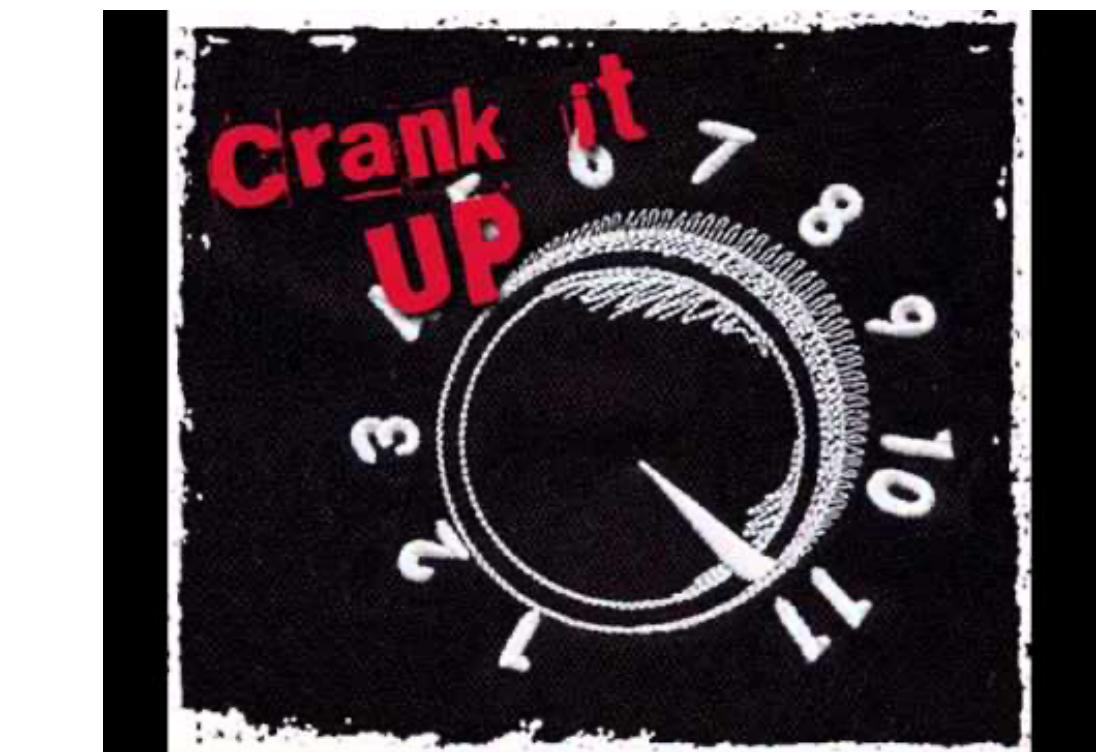
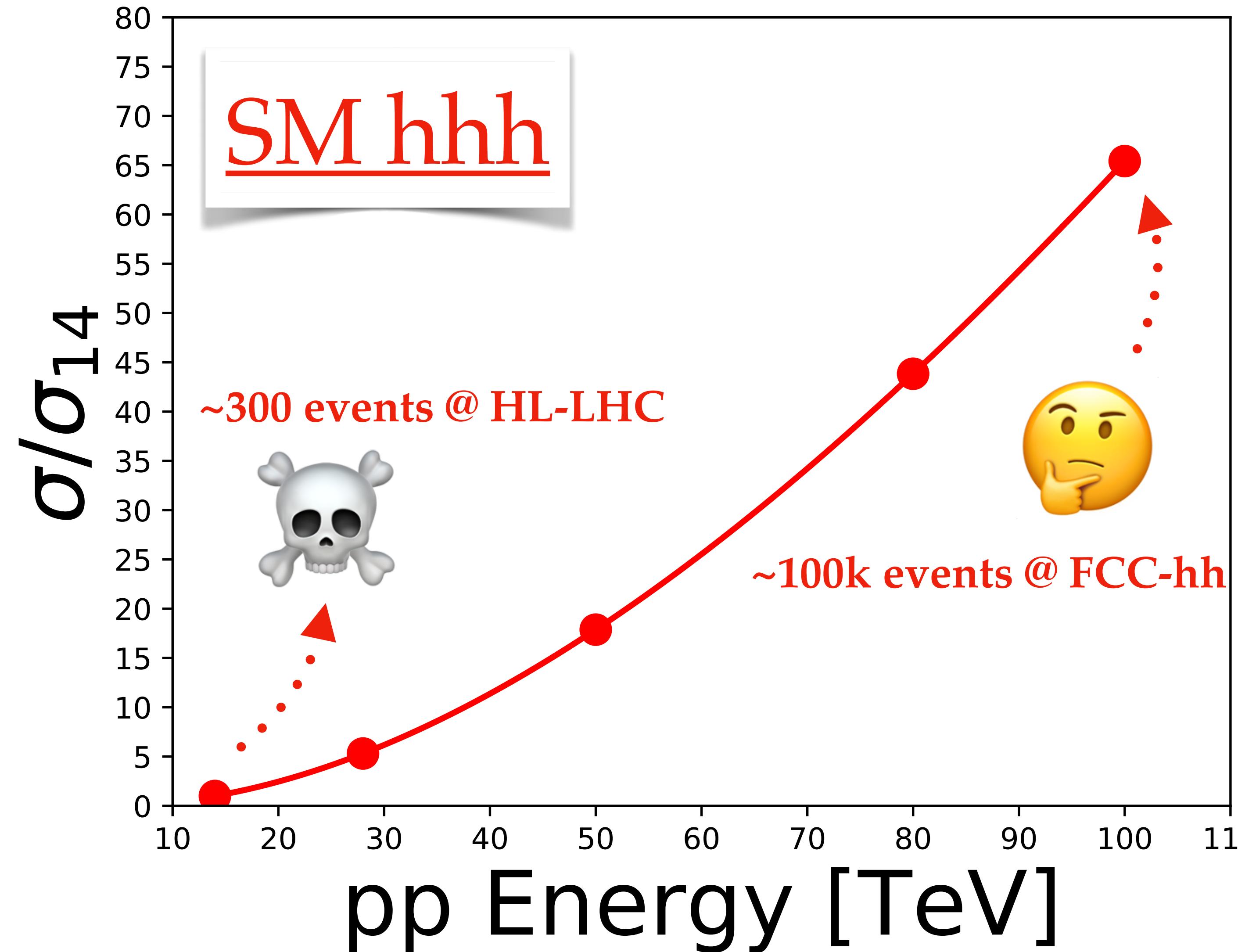


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SM Multi-Higgs Boson Production “Fun” Facts

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**THE SECRET
iNGREDiENT
is ALWAYS
LoVE**



KENNESAW STATE
UNIVERSITY



THE SECRET
iNGREDiENT
is ALWAYS
~~LOVE~~ NEW PHYSICS



THE SECRET
iNGREDiENT
is ALWAYS
~~LOVE~~ NEW PHYSICS

Here:

- A. hhh in $SM+2$ singlet scalar fields,
- B. hhh with anomalous couplings.



A. hhh in SM+2 singlet scalar fields,



SM + Two Real Singlet Scalars [= TRSM]

- Consider adding **two real singlet scalar fields** $S, X \rightarrow$ the **TRSM**.

- And: impose **discrete \mathbb{Z}_2 symmetries**: $\mathcal{Z}_2^S : S \rightarrow -S, X \rightarrow X$

$$\mathcal{Z}_2^X : X \rightarrow -X, S \rightarrow S$$

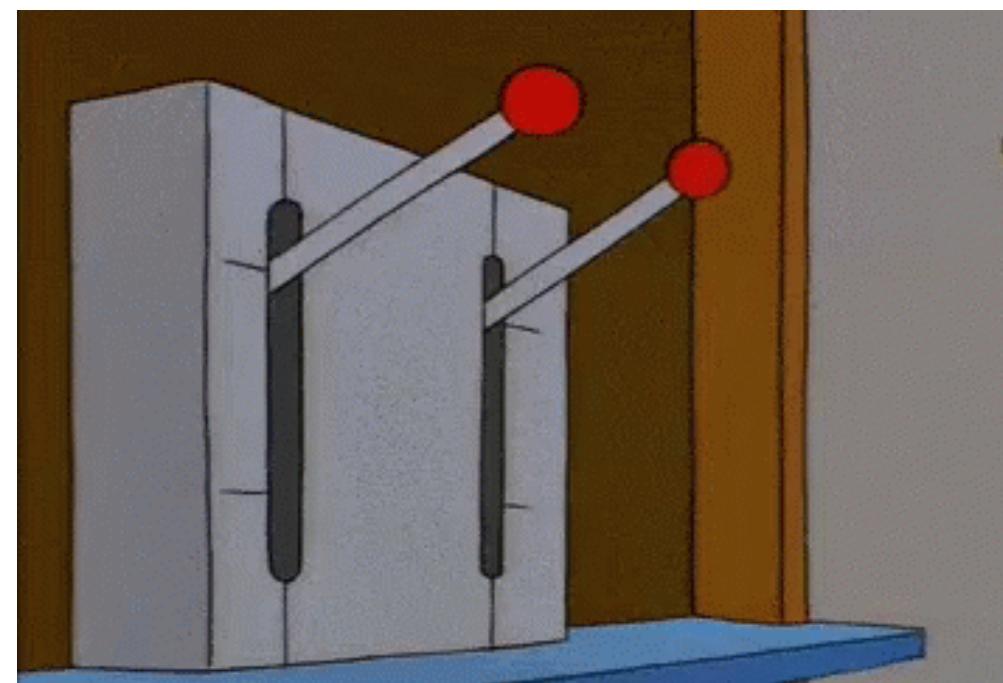
⇒ **TRSM** scalar potential:

$$\begin{aligned} \mathcal{V}(\phi, S, X) = & \bullet |\phi|^2 + \blacksquare |\phi|^4 + \bullet S^2 + \square S^4 + \bullet X^2 + \square X^4 \\ & + \blacksquare S^2 X^2 \\ & + \blacksquare |\phi|^2 S^2 + \blacksquare |\phi|^2 X^2 \end{aligned}$$



SM + Two Real Singlet Scalars [= TRSM]

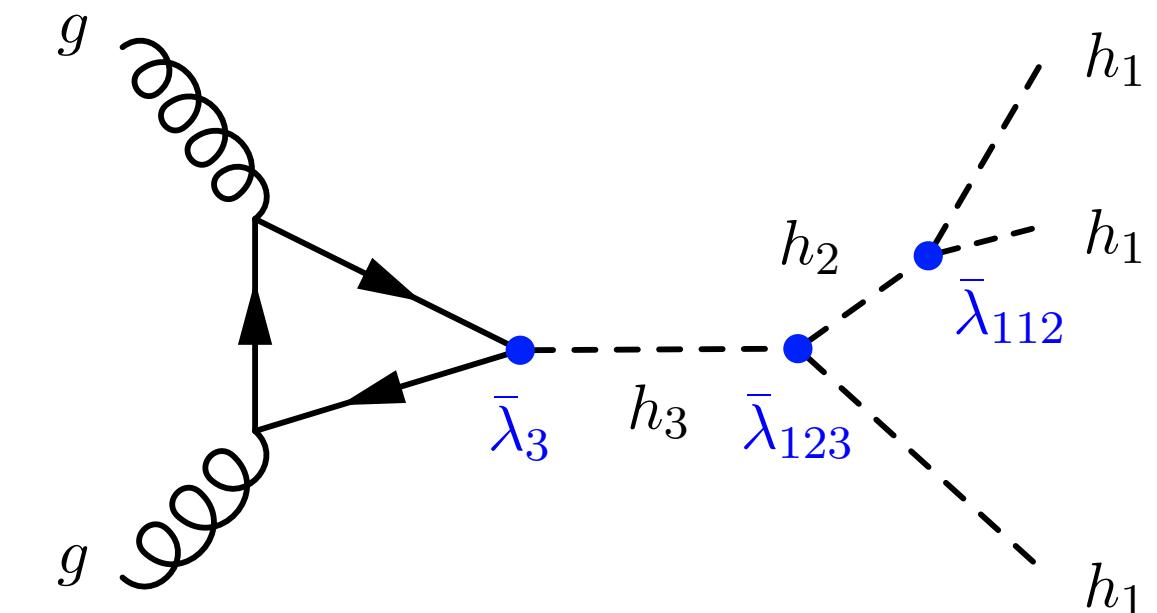
- Go through EWSB...



- ⇒ Get **three** scalar bosons: $h_1, h_2, h_3 \rightarrow h_1 \approx \text{SM-like Higgs boson}$.
- ⇒ Seven independent parameters: M_2, M_3 + three mixing angles + two VEVs.
- ⇒ Modified / Additional interactions between scalars.
- ⇒ **hhh** that may even be detectable at the LHC! [AP, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]

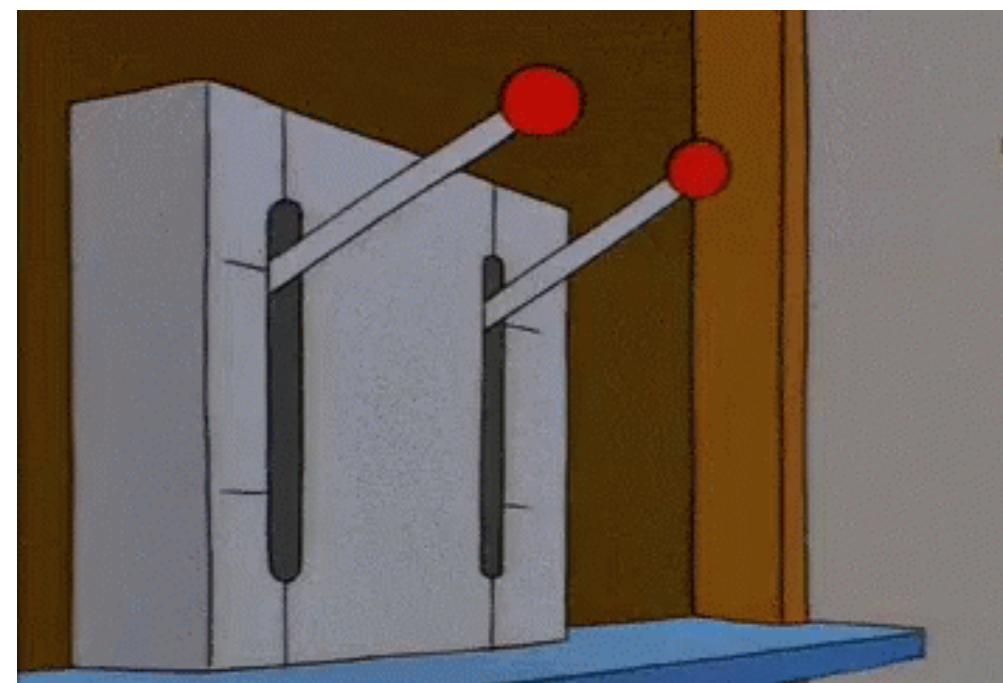
e.g.: $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$

→ Double-resonant enhancement!



SM + Two Real Singlet Scalars [= TRSM]

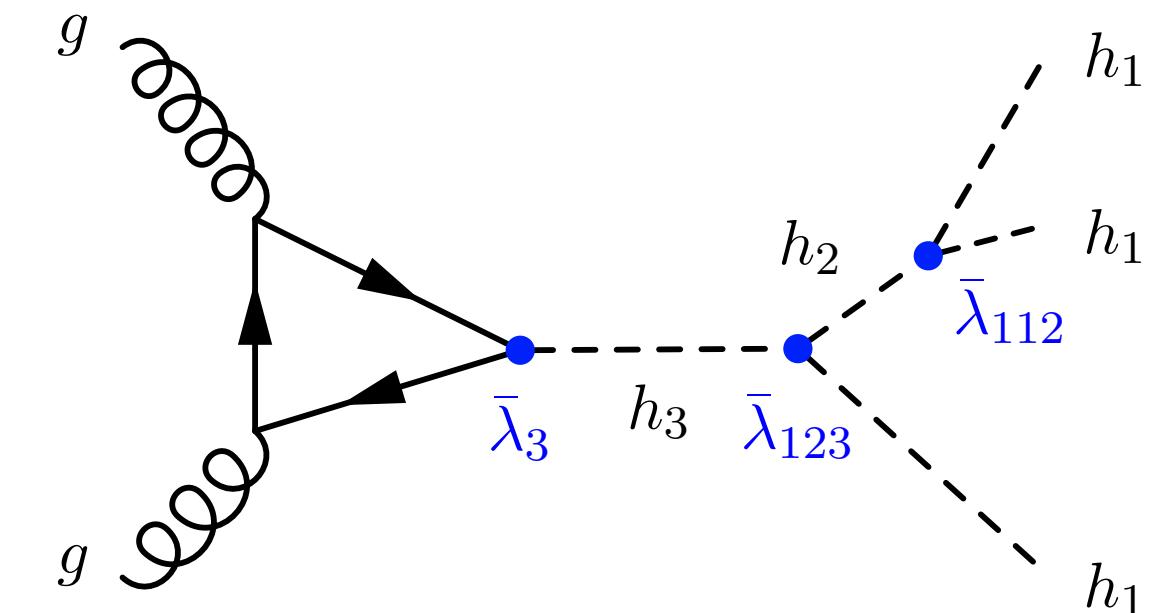
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e.g.: $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$

→ Double-resonant enhancement!



hhh in the TRSM [14 TeV]

- Focus on a particular family of benchmark points: “Benchmark Plane 3” = “BP3” in [Robens, Stefaniak, Wittbrodt, arXiv:1908.08554].

Label	(M_2, M_3) [GeV]	$\sigma(pp \rightarrow h_1 h_1 h_1)$ [fb]
A	(255, 504)	32.40
B	(263, 455)	50.36
C	(287, 502)	39.61
D	(290, 454)	49.00
E	(320, 503)	35.88
F	(264, 504)	37.67
G	(280, 455)	51.00
H	(300, 475)	43.92
I	(310, 500)	37.90
J	(280, 500)	40.26

- In BP3: All params fixed except M_2, M_3 !

Cross section can be much higher than in the SM! 😲

→ c.f. SM: $\sigma \sim 0.1 \text{ fb}$ @ 14 TeV.

[AP, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]



hhh in the TRSM “BP3” [14 TeV]

- Search for **hhh** via: $pp \rightarrow (b\bar{b})(b\bar{b})(b\bar{b}) \rightarrow \textbf{6 b-jets}$.
- About **20%** of the **hhh** final state!
- Significances **large**, even when including systematic uncert.:

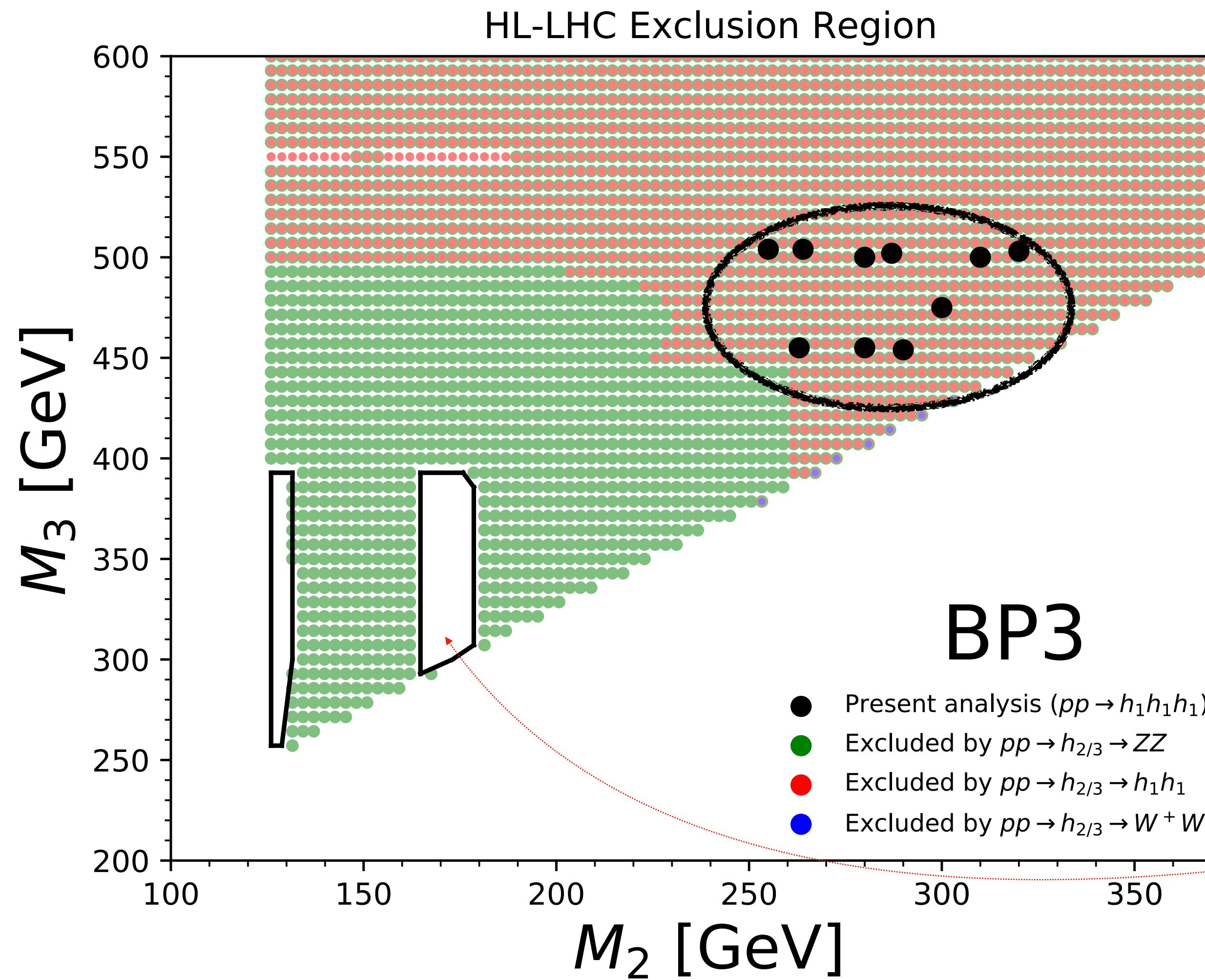
Label	$\text{sig} _{300\text{fb}^{-1}}$ (syst.)	$\text{sig} _{3000\text{fb}^{-1}}$ (syst.)
A	2.92 (2.63)	9.23 (5.07)
B	4.78 (4.50)	15.10 (10.14)
C	4.01 (3.56)	12.68 (6.67)
D	5.02 (4.03)	15.86 (6.25)
E	3.76 (2.87)	11.88 (4.18)
F	3.56 (3.18)	11.27 (5.98)
G	5.18 (4.16)	16.39 (6.45)
H	4.64 (3.47)	14.68 (4.94)
I	4.09 (2.88)	12.94 (3.87)
J	4.00 (3.56)	12.65 (6.66)

[AP, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]



hhh in the TRSM “BP3” [14 TeV]

- hhh will (**probably?**) not be a discovery channel,
- but could be important in determining the parameters of the model, if scalars are discovered!



Could help solve the “**inverse problem**” in the TRSM?

[AP, Robens, Tetlalmatzi-Xolocotzi,
arXiv:2101.00037]

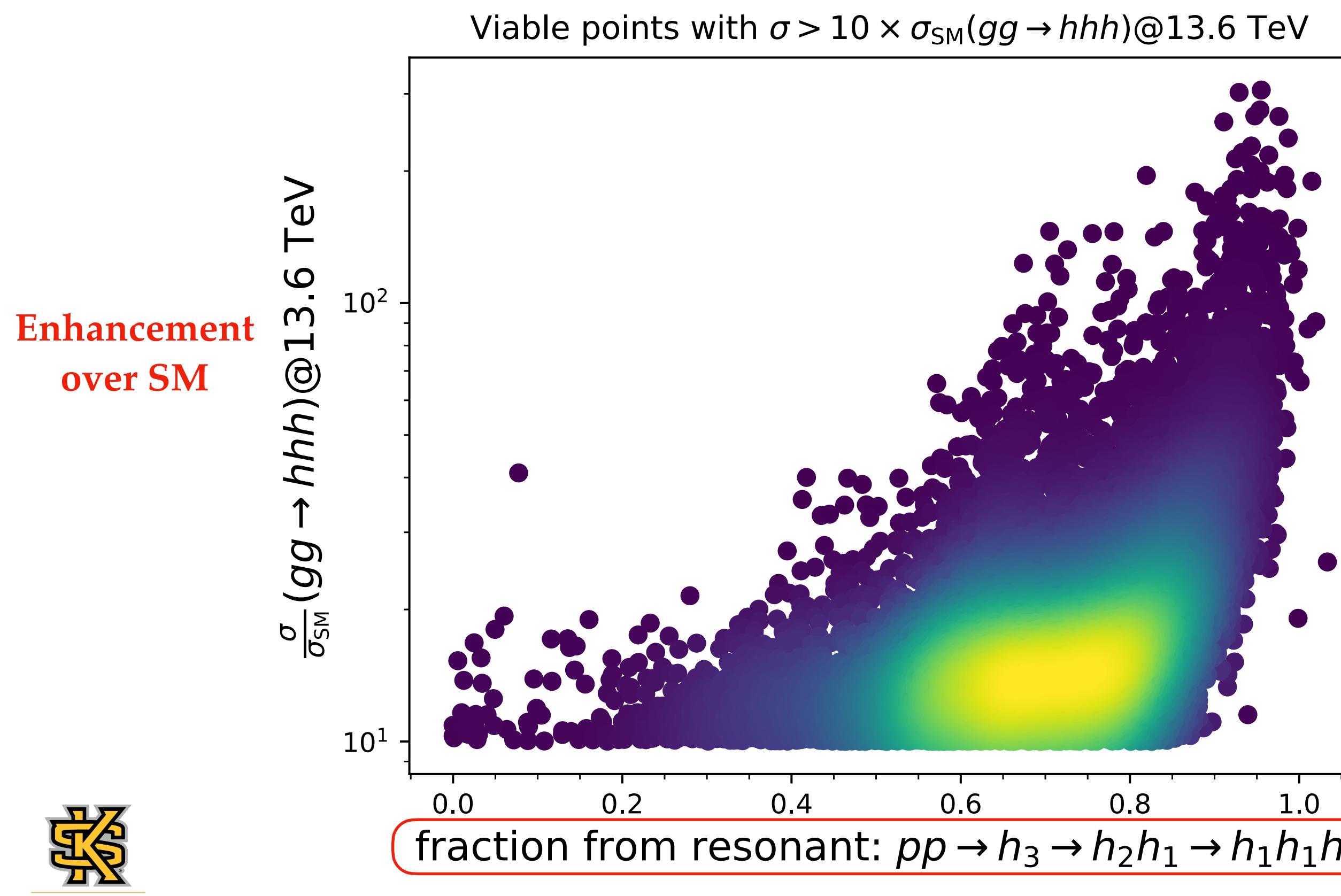
Note: regions near $M_2 \sim 130$ GeV and $M_2 \sim 170$ GeV will remain viable at the end of the HL-LHC.



SFO-EWPT and hhh in the TRSM

- $\textcolor{red}{Q}$: Can there be a SFO-EWPT in the TRSM, related to **electro-weak baryogenesis**?
- and if so, will this lead to **enhanced hhh at the LHC?**

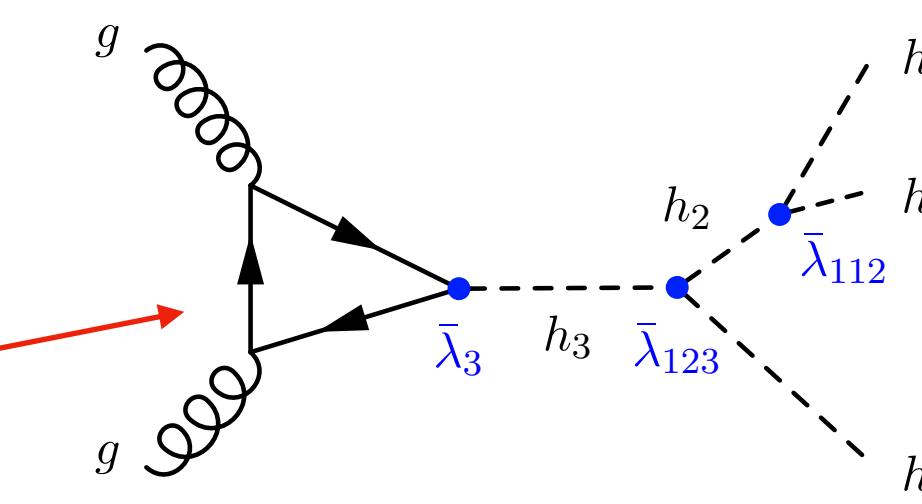
[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi,
van de Vis, du Pree, arXiv:2404.12425]



- Updated TRSM scan with additional TH+EXP constraints.

- Enhancements $\mathcal{O}(100) \times \text{SM}!$

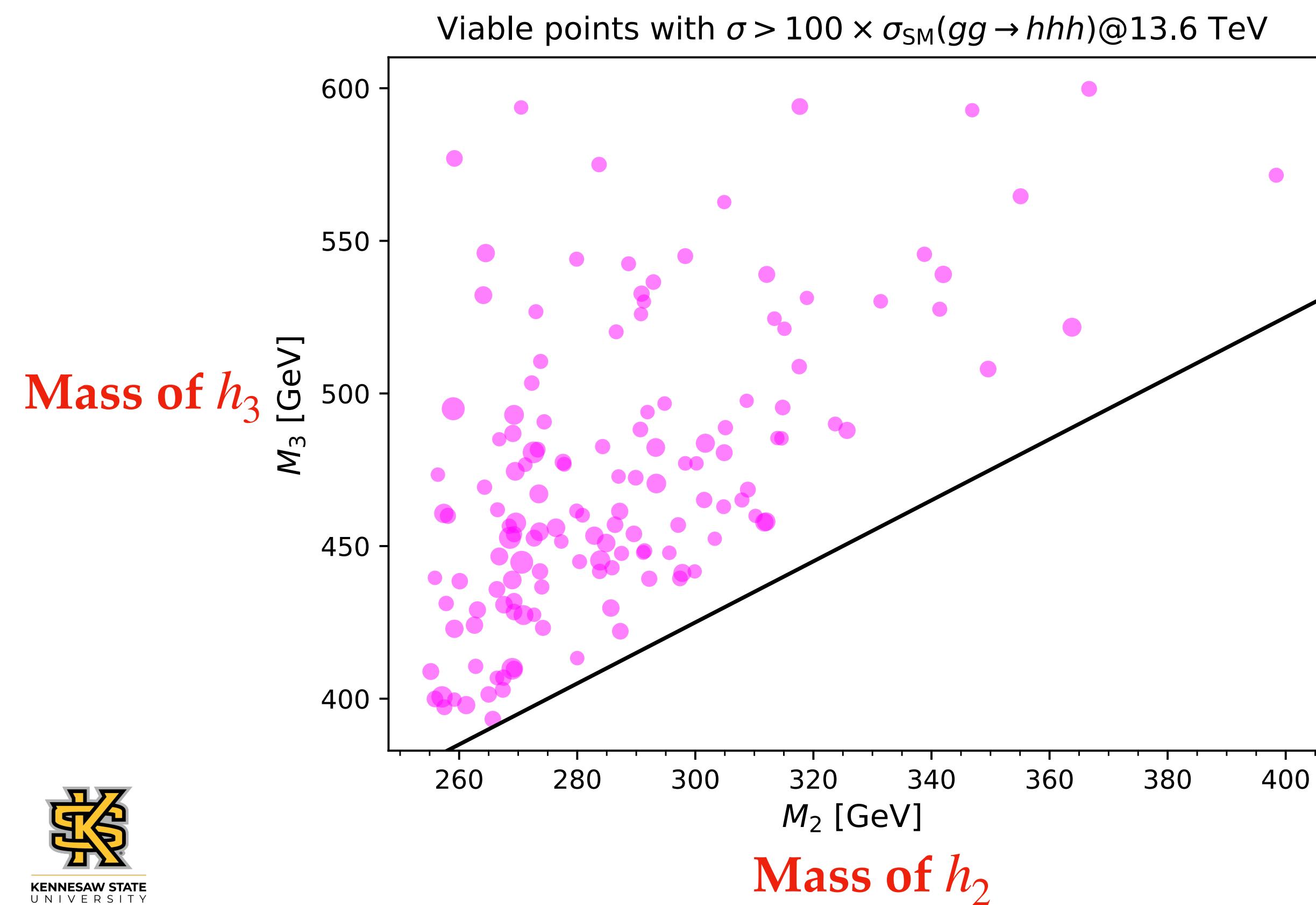
How much of the total cross section comes from... ?



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[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi,
van de Vis, du Pree, arXiv:2404.12425]



- An updated set of benchmark points,
- they go *beyond* BP3: all params. varied!
- Enhancements $\mathcal{O}(100) \times \text{SM} @ 13.6 \text{ TeV}!$



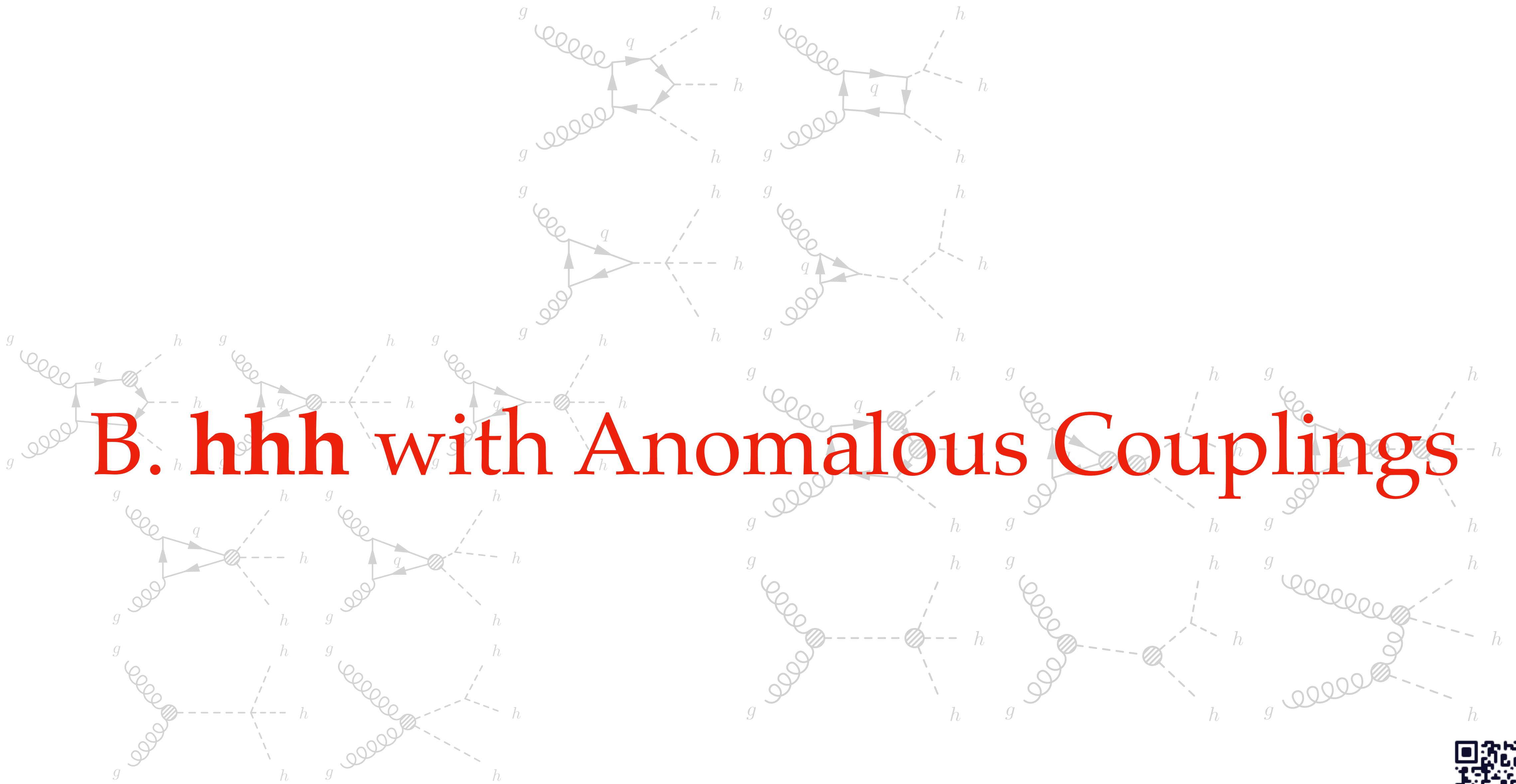
SFO-EWPT and hhh in the TRSM

- $\textcolor{red}{Q}$: Can there be a SFO-EWPT in the TRSM, related to electro-weak baryogenesis?
- and if so, will this lead to enhanced hhh at the LHC?
- **Unfortunately...**
 - ▶ SFO-EWPT & enhanced hhh are mutually exclusive!
 - ▶ barrier not generated if both new scalars attain a non-zero VEV,
 - ▶ and non-zero VEVs are necessary for sufficient mixing!
- Removing the \mathcal{Z}_2 restrictions might help!

[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi,
van de Vis, du Pree, arXiv:2404.12425]



B. hhh with Anomalous Couplings



D=6-Inspired Anomalous Couplings

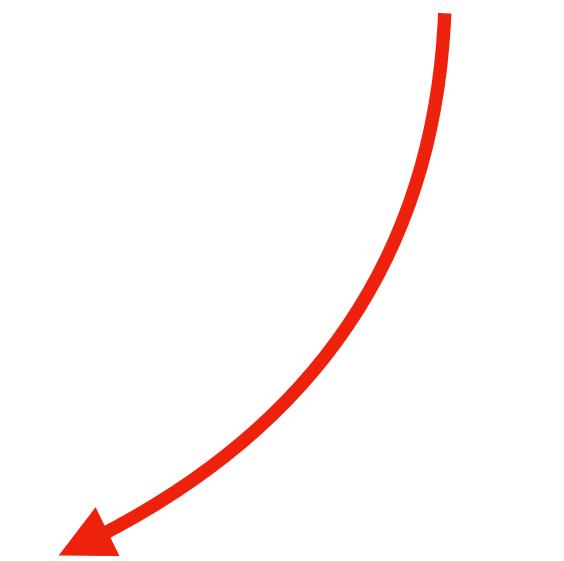
- Add higher-dimensional operators to the SM Lagrangian!
 - Capture the effects of new particles at scales \gg collision energies.

- e.g. Add D=6 operators relevant to multi-Higgs boson production, of the form $\frac{\mathcal{O}_6}{\Lambda^2}$:

$$\mathcal{L}_{h^n} \supset -\mu^2 |\phi|^2 - \lambda |\phi|^4 - (y_t \bar{Q}_L \phi^c t_R + y_b \bar{Q}_L \phi b_R + \text{h.c.})$$

$$+ \frac{c_H}{2\Lambda^2} (\partial^\mu |\phi|^2)^2 - \frac{c_6}{\Lambda^2} \lambda_{\text{SM}} |\phi|^6 + \frac{\alpha_s c_g}{4\pi \Lambda^2} |\phi|^2 G_{\mu\nu}^a G_a^{\mu\nu}$$

$$- \left(\frac{c_t}{\Lambda^2} y_t |\phi|^2 \bar{Q}_L \phi^c t_R + \frac{c_b}{\Lambda^2} y_b |\phi|^2 \bar{Q}_L \phi b_R + \text{h.c.} \right)$$

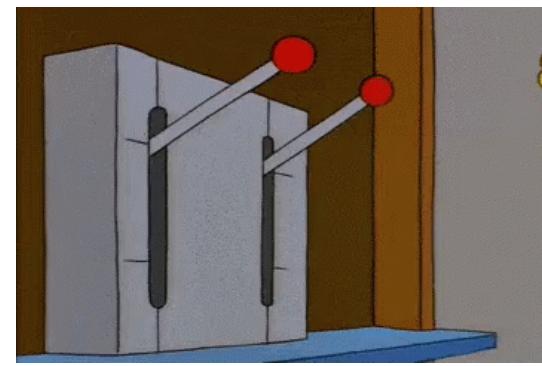


[see e.g. Goertz, AP, Yang, Zurita, arXiv:1410.3471 for similar hh study]

D=6-Inspired Anomalous Couplings

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Go through EWSB...



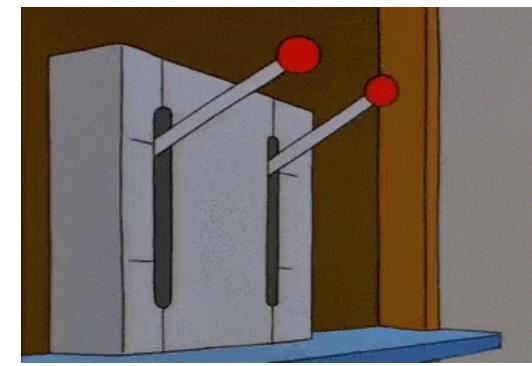
⇒ in terms of the physical scalar Higgs boson h :

$$\begin{aligned} \mathcal{L}_{D=6} \supset & -\frac{m_h^2}{2v} (1+c_6) h^3 - \frac{m_h^2}{8v^2} (1 + 6c_6) h^4 \\ & + \frac{\alpha_s c_g}{4\pi} \left(\frac{h}{v} + \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\ & - \left[\frac{m_t}{v} (1+c_t) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_b) \bar{b}_L b_R h + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^2} \left(\frac{3c_t}{2} \right) \bar{t}_L t_R h^2 + \frac{m_b}{v^2} \left(\frac{3c_b}{2} \right) \bar{b}_L b_R h^2 + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^3} \left(\frac{c_t}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_b}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right], \end{aligned}$$

D=6-Inspired Anomalous Couplings

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

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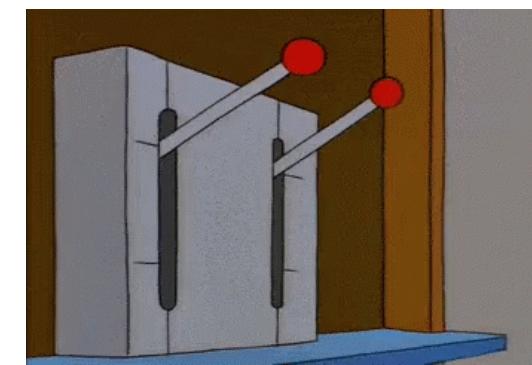
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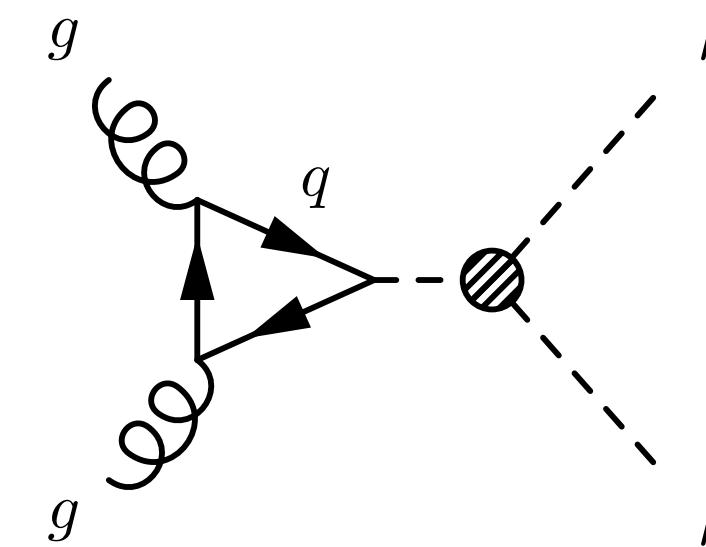
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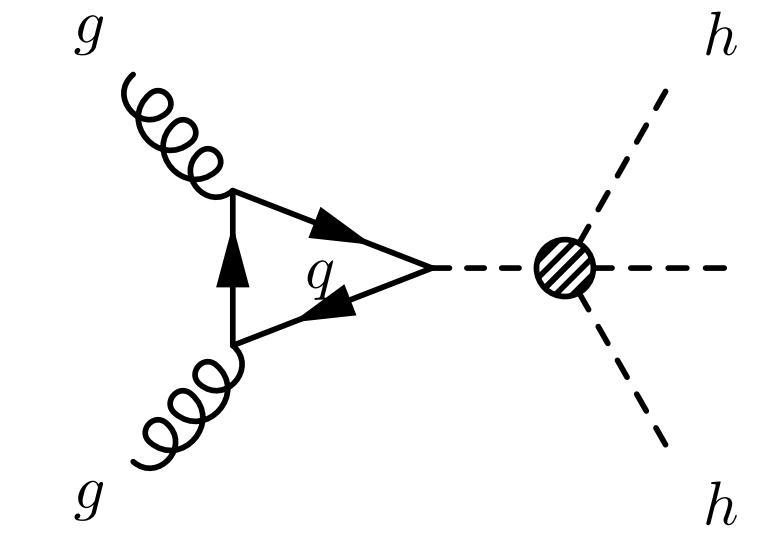
- Go through EWSB...



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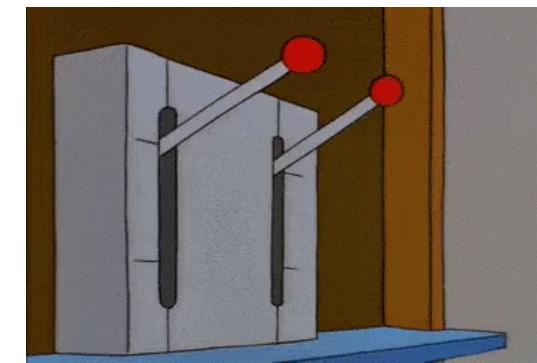
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[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

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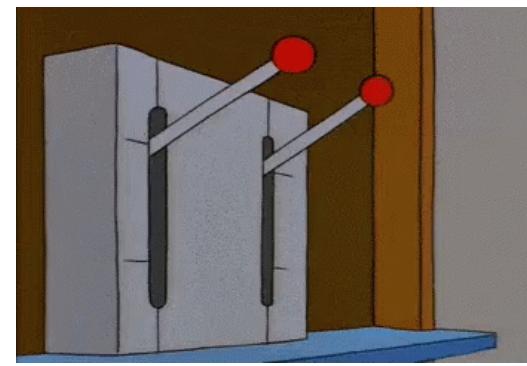
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D=6-Inspired Anomalous Couplings

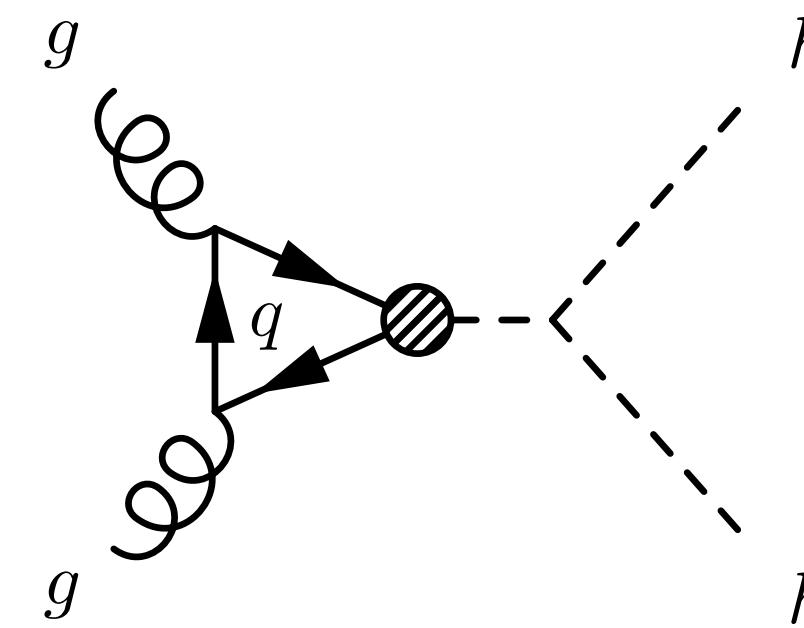
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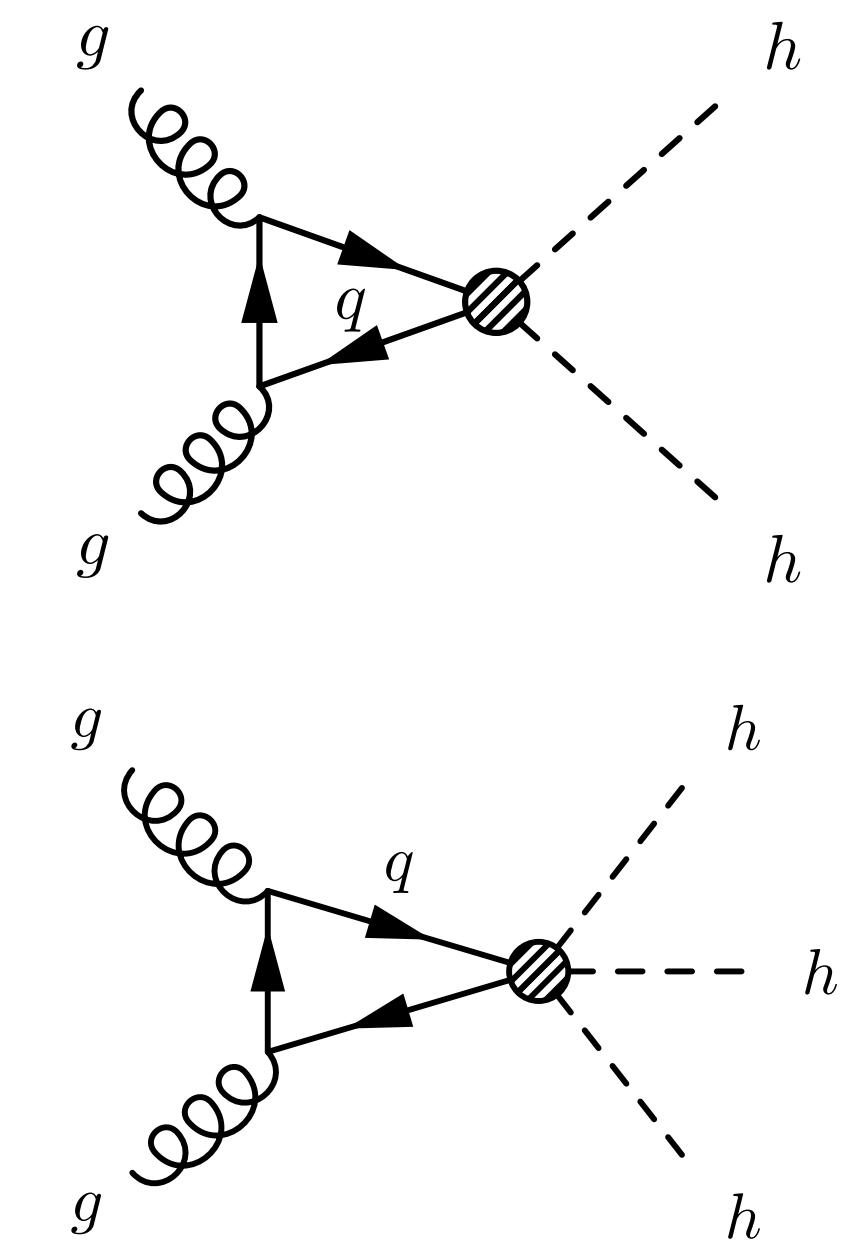


⇒ in terms of the physical scalar Higgs boson h :

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$$\begin{aligned}& - \left[\frac{m_t}{v} (1+c_t) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_b) \bar{b}_L b_R h + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^2} \left(\frac{3c_t}{2} \right) \bar{t}_L t_R h^2 + \frac{m_b}{v^2} \left(\frac{3c_b}{2} \right) \bar{b}_L b_R h^2 + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^3} \left(\frac{c_t}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_b}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right],\end{aligned}$$



D=6-Inspired Anomalous Couplings

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- A slightly more “general” picture is obtained by “dissociating” the operators as:

$$\begin{aligned} \mathcal{L}_{\text{Pheno}} \supset & -\frac{m_h^2}{2v} (1+d_3) h^3 - \frac{m_h^2}{8v^2} (1+d_4) h^4 \\ & + \frac{\alpha_s}{4\pi} \left(c_{g1} \frac{h}{v} + c_{g2} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\ & - \left[\frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^2} \left(\frac{3c_{t2}}{2} \right) \bar{t}_L t_R h^2 + \frac{m_b}{v^2} \left(\frac{3c_{b2}}{2} \right) \bar{b}_L b_R h^2 + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^3} \left(\frac{c_{t3}}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_{b3}}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right], \end{aligned}$$

Recover D=6 by setting:
 $d_3 = c_6$,
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 $c_{g1} = c_{g2} = c_g$,
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Note: This can be also be motivated via the [Electro-weak Chiral Lagrangian](#),

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instead of c_t

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D=6-Inspired Anomalous Couplings

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Further modify to match more closely LHC experiments' definitions:

$$\begin{aligned}
 \mathcal{L}_{\text{PhenoExp}} \supset & -\lambda_{\text{SM}} v (1+d_3) h^3 - \frac{\lambda_{\text{SM}}}{4} (1+d_4) h^4 \\
 & + \frac{\alpha_s}{12\pi} \left(c_{g1} \frac{h}{v} - c_{g2} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\
 & - \left[\frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\
 & - \left[\frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.} \right] \\
 & - \left[\frac{m_t}{v^3} \left(\frac{c_{t3}}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_{b3}}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right],
 \end{aligned}$$

Defined: $\lambda_{\text{SM}} = m_h^2/2v^2$.

Obtain CMS-like parametrization by:

$$\begin{aligned}
 \kappa_\lambda &= (1+d_3), \\
 k_t &= c_{t1}, \\
 c_2 &= c_{t2}, \\
 c_g &= c_{g1}, \\
 c_{gg} &= c_{2g}.
 \end{aligned}$$

And ATLAS-like parametrization by:

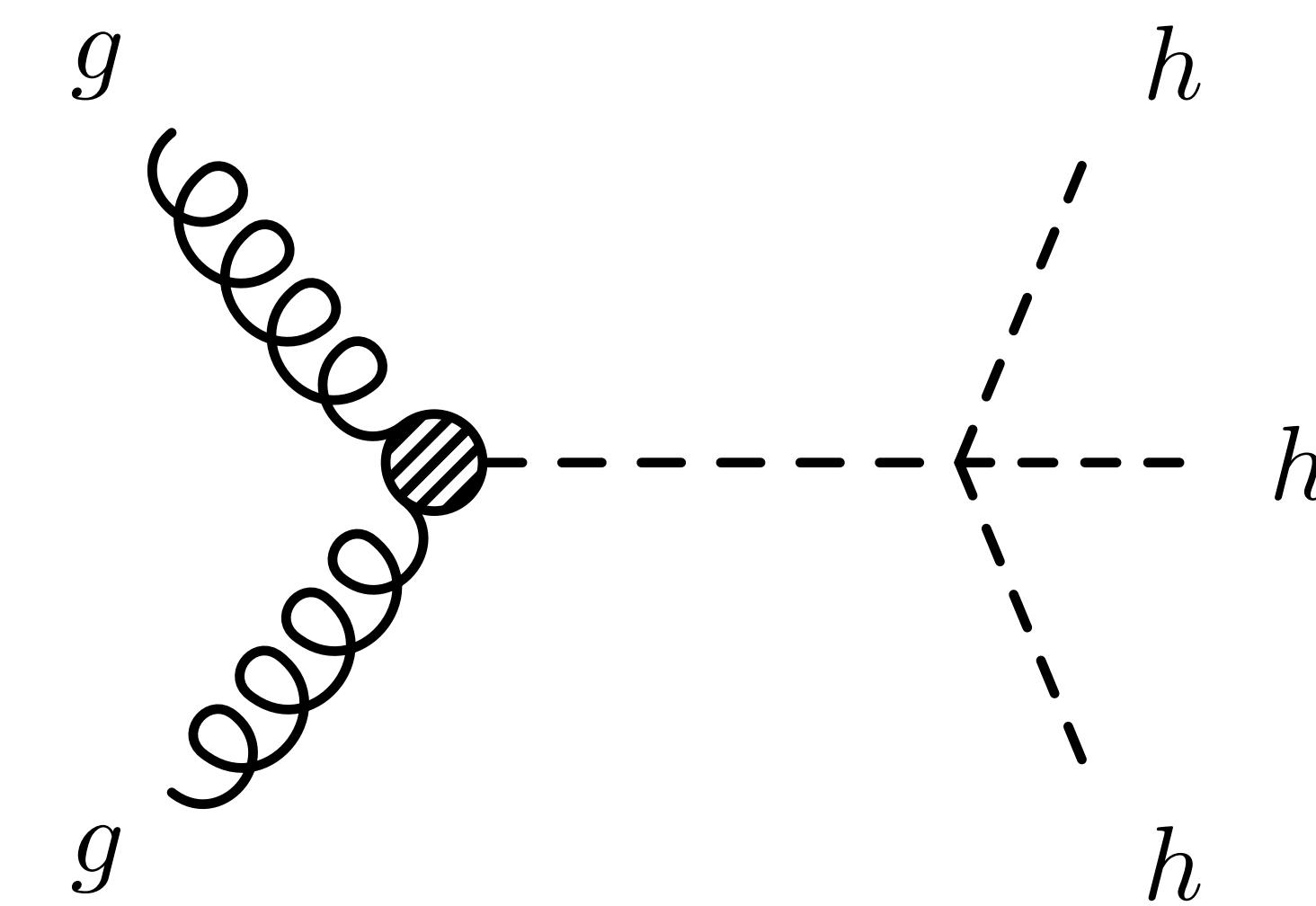
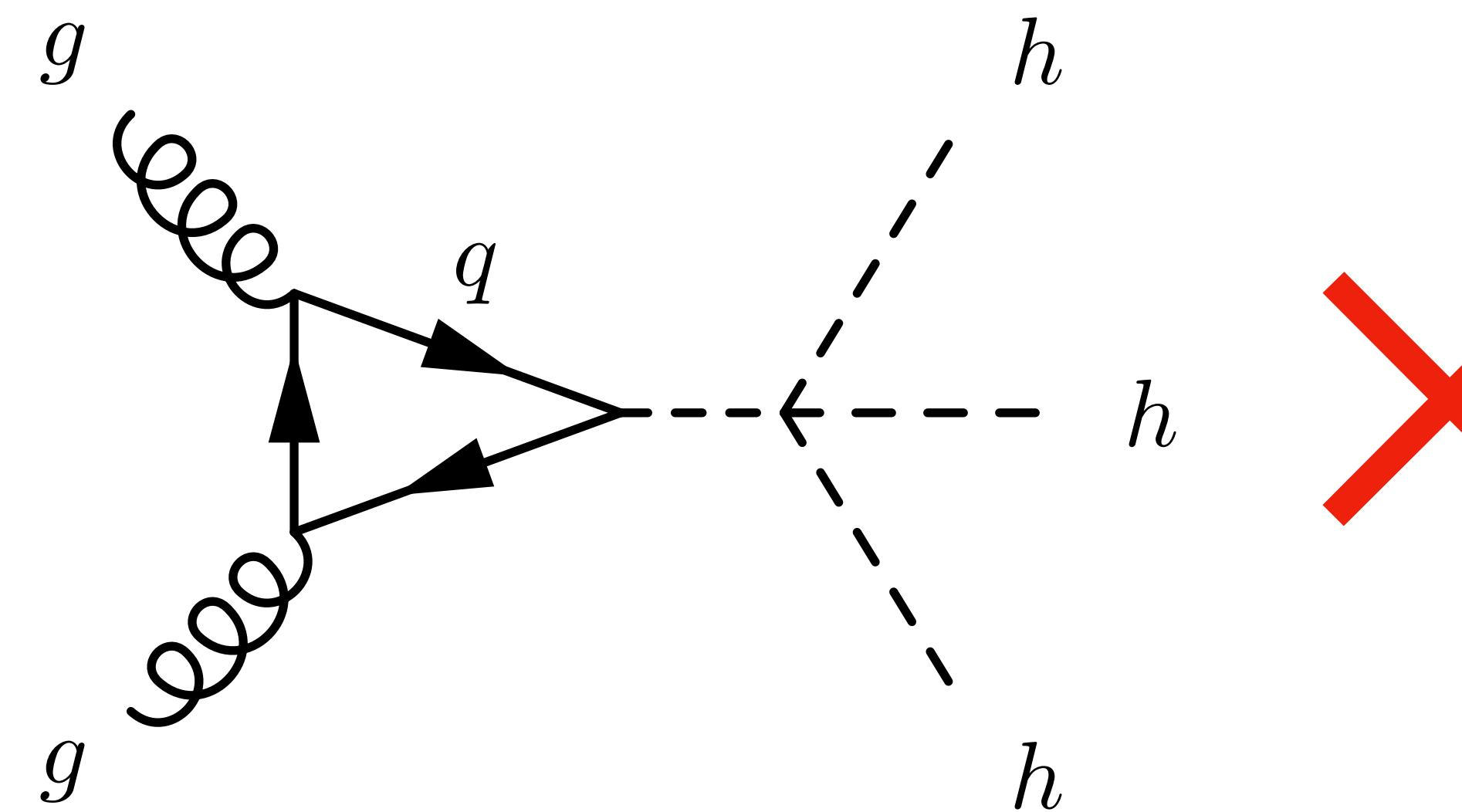
$$\begin{aligned}
 c_{hh} &= (1+d_3), \\
 c_{gh} &= 2c_{g1}/3, \\
 c_{ggh} &= -c_{g2}/3.
 \end{aligned}$$

Monte Carlo Implementation of Anomalous Couplings

- We have implemented a **MadGraph5_aMC@NLO** “loop” model for $\mathcal{L}_{\text{PhenoExp}}$.
- Includes Loop \times Tree level interference between the various diagrams.

[see: Hirschi, <https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/LoopInducedTimesTree>].

- e.g.:



[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

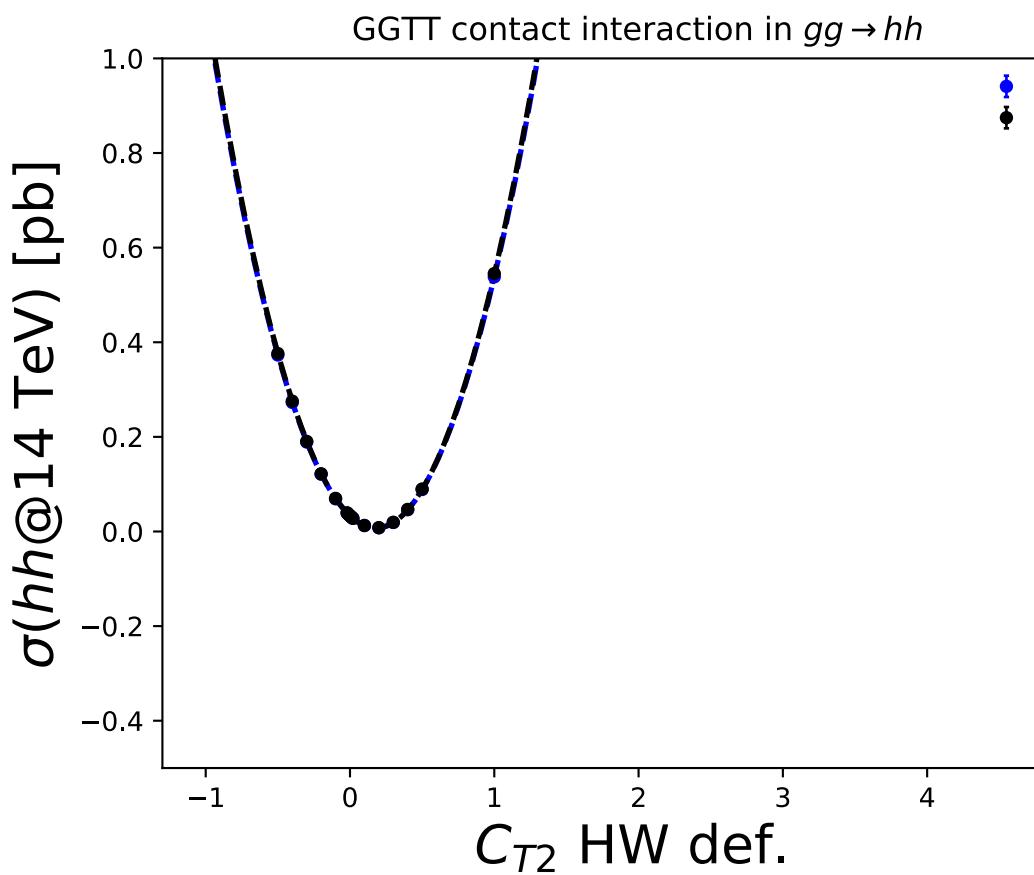
[Get model at: https://gitlab.com/apapaefs/multihiggs_loop_sml]



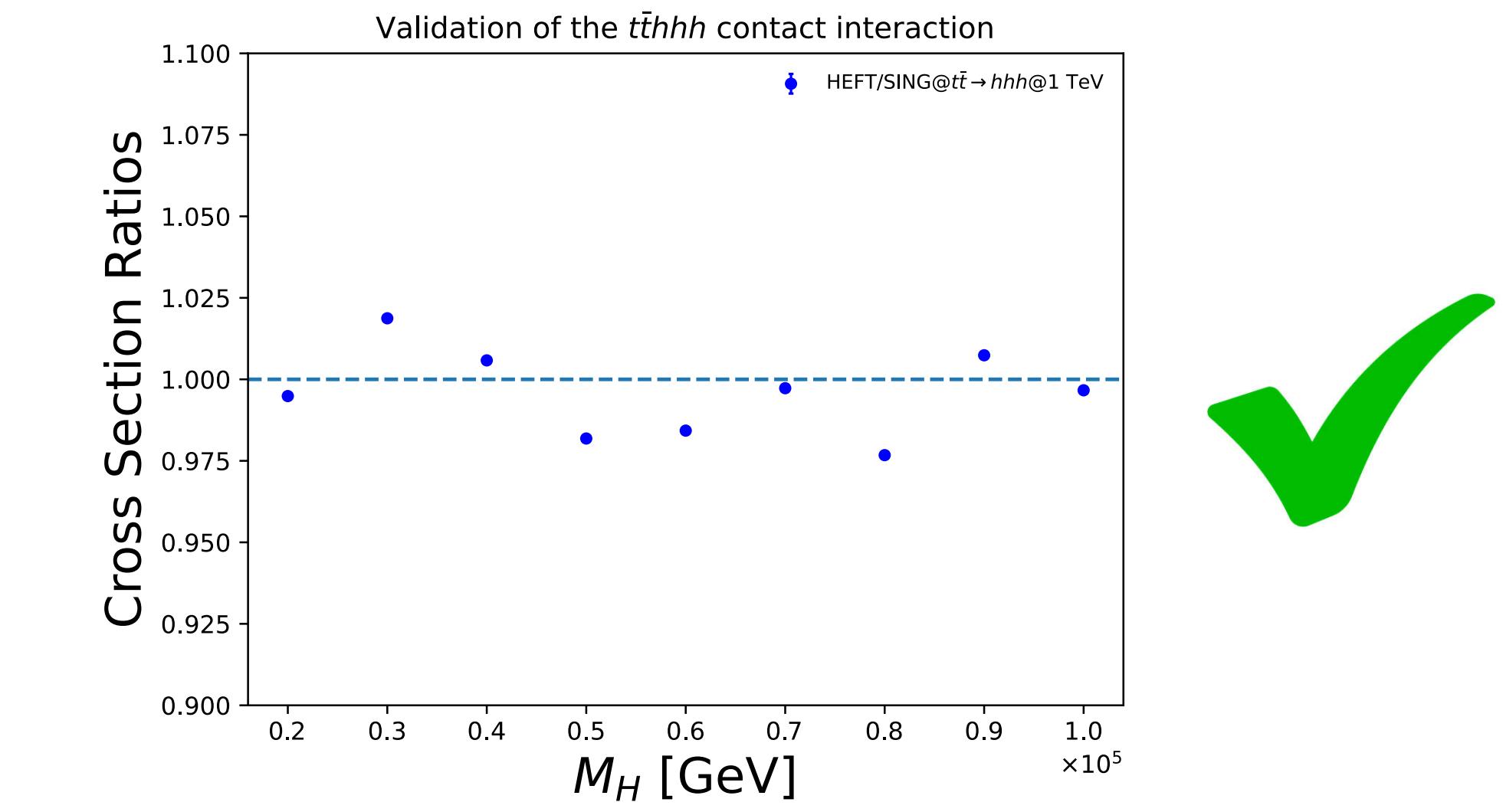
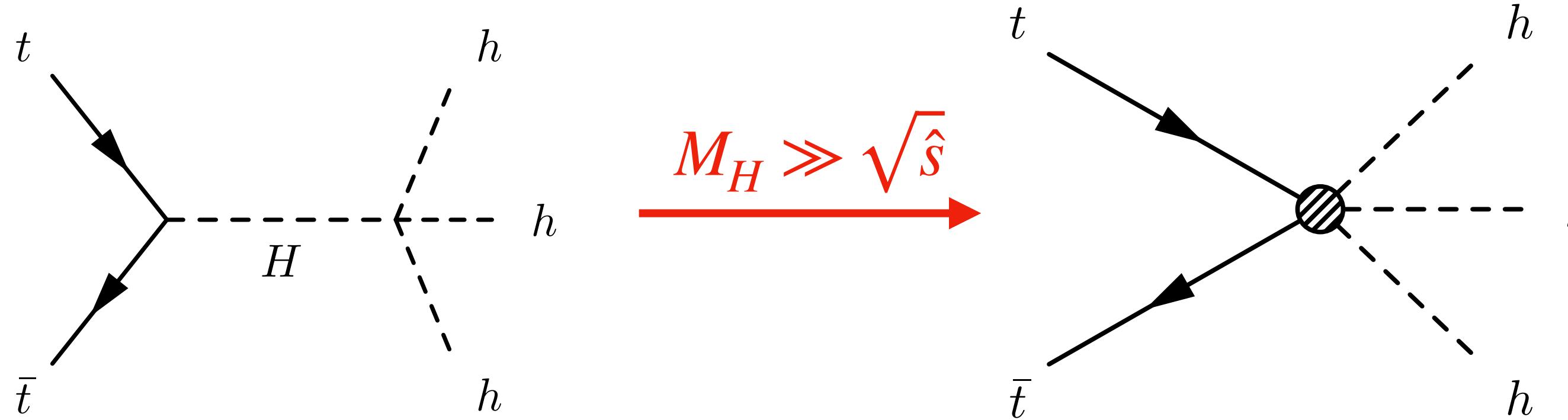
Model Validation

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Most couplings validated vs. a **Herwig 7** $pp \rightarrow hh$ implementation, e.g.:



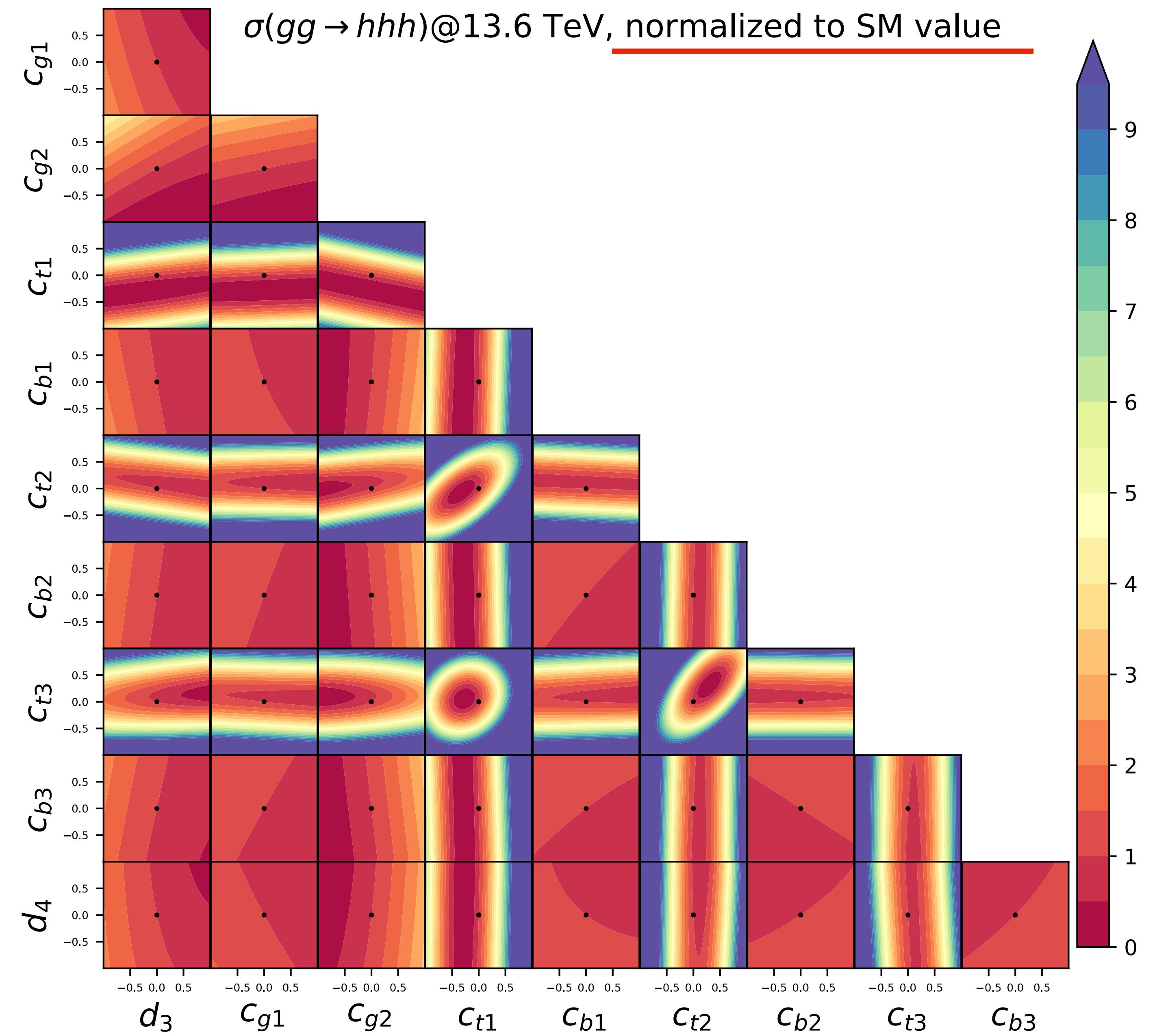
- The one “new” non-trivial coupling that appears, $\propto c_{t3} t\bar{t}h^3$ has been validated via an “EFT” limit, in the $t\bar{t} \rightarrowhhh$ process:



hhh Cross Sections @ 13.6 TeV

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Cross section as a multiple of the SM
- ($\sigma_{\text{SM}} \sim 0.04 \text{ fb}$ at LO@13.6 TeV).
- In each 2D panel shown: all other coefficients set to zero!



Anomalous Couplings Constraints

- Other processes constrain (at LO) all coefficients except $\{c_{t3}, d_4\}$ (only in hh).
- **Projected constraints:**

Percentage uncertainties			
	HL-LHC	FCC-hh	Ref.
$\delta(d_3)$	50	5	[145] (table 12)
$\delta(c_{g1})$	2.3	0.49	[145] (table 3)
$\delta(c_{g2})$	5	1	[140] (Figure 12, right)
$\delta(c_{t1})$	3.3	1.0	[145] (table 3)
$\delta(c_{t2})$	30	10	[140] (Figure 12, right)
$\delta(c_{b1})$	3.6	0.43	[145] (table 3)
$\delta(c_{b2})$	30	10	assumed same as c_{t2}

[See AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562 for the references]



Anomalous Couplings Constraints

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

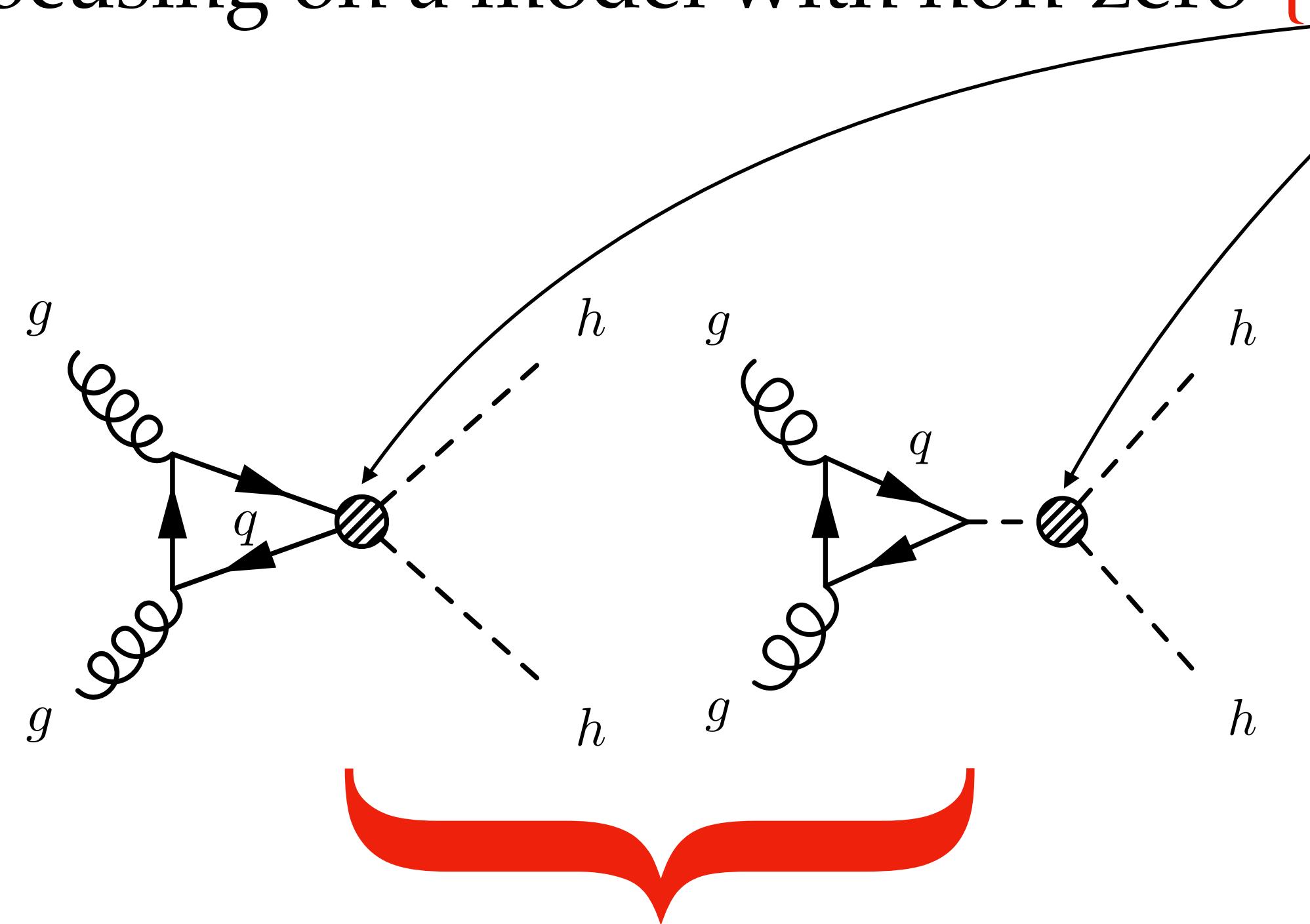
- Focusing on a model with non-zero $\{c_{t2}, d_3, c_{t3}, d_4\}$:



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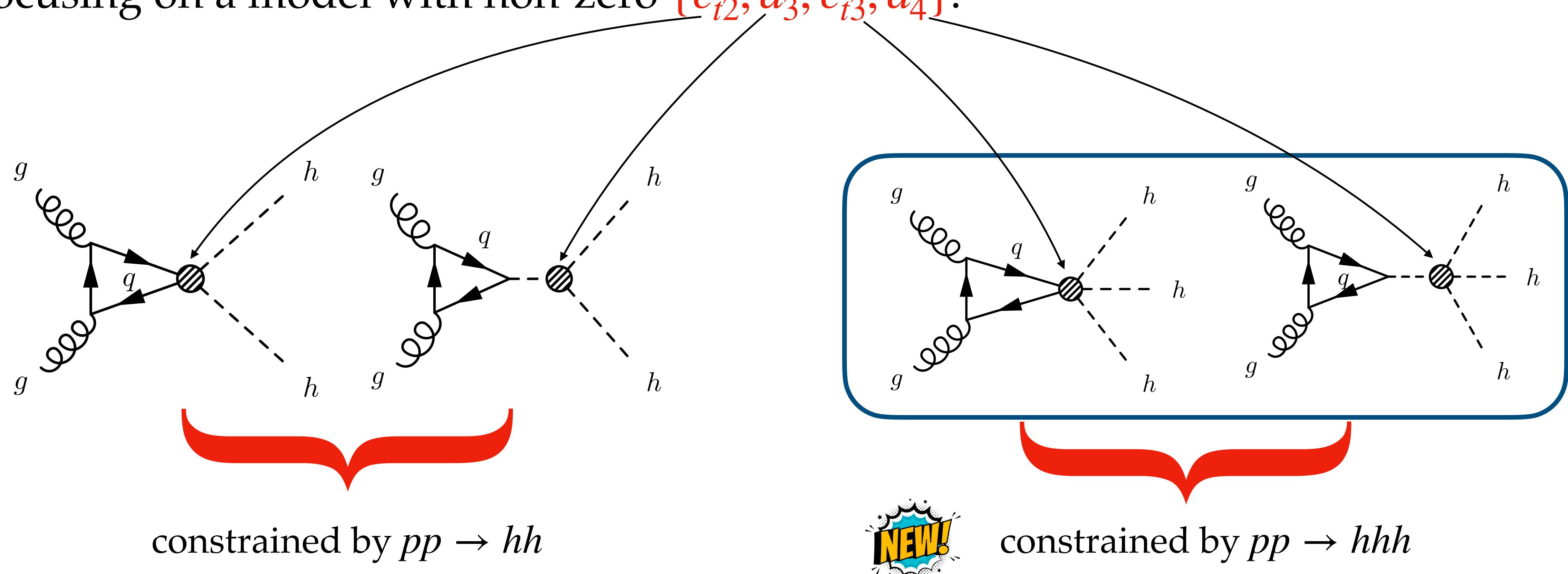
constrained by $pp \rightarrow hh$



Anomalous Couplings Constraints

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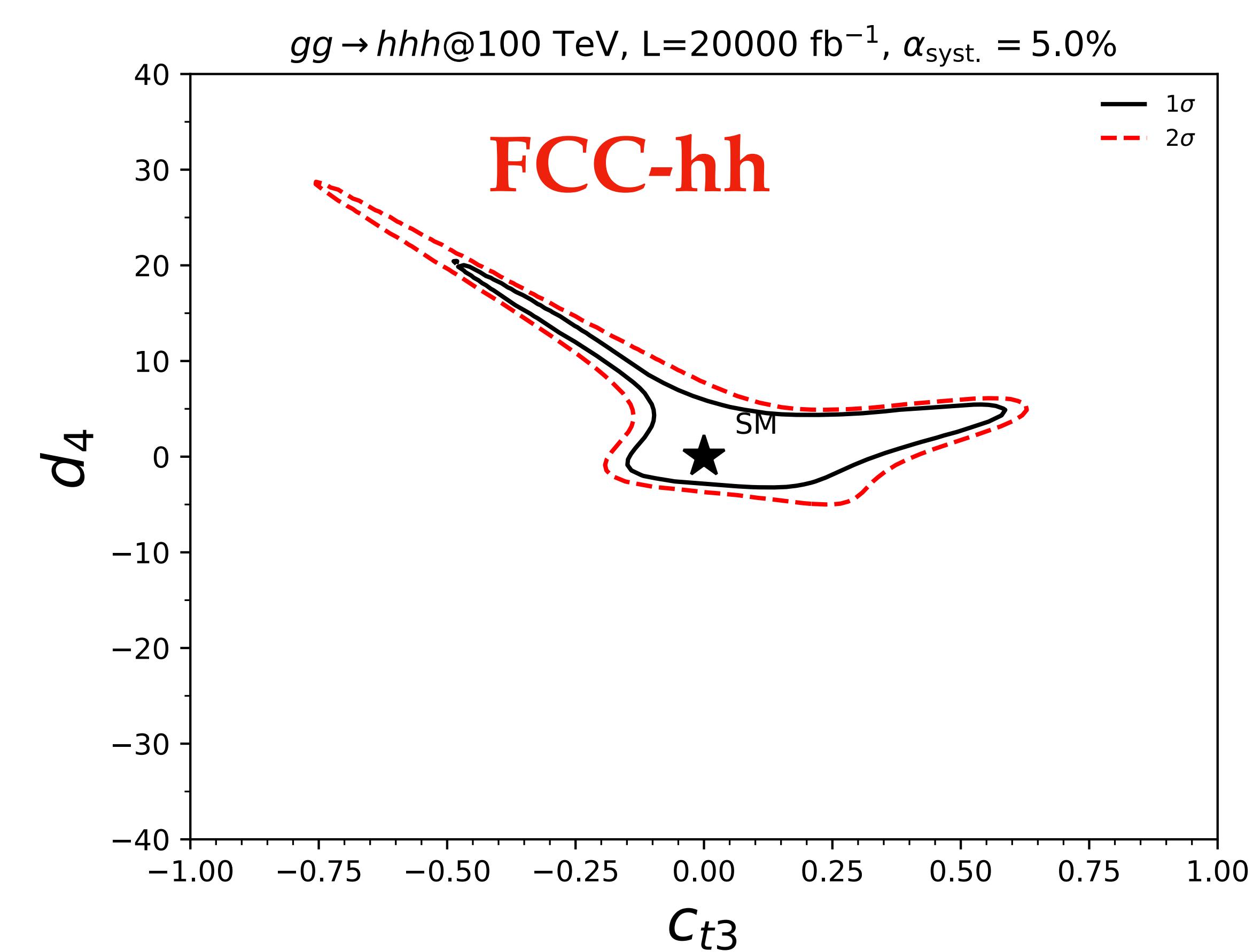
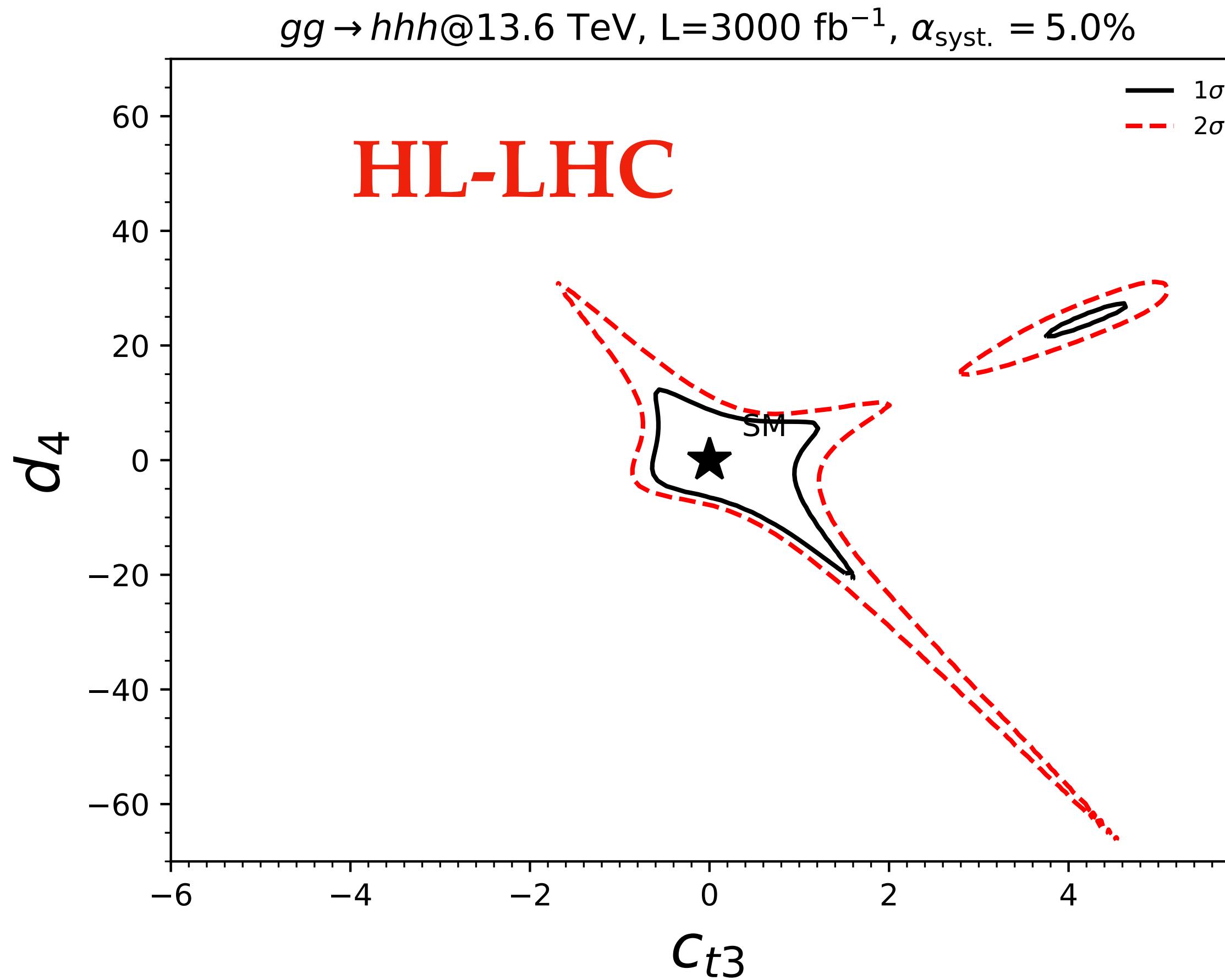
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Anomalous Couplings Constraints

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- Focusing on a model with only $\{c_{t2}, d_3, c_{t3}, d_4\}$,
- Using the **6 b-jet final state**, and marginalizing over $\{c_{t2}, d_3\}$ within projected constraints:

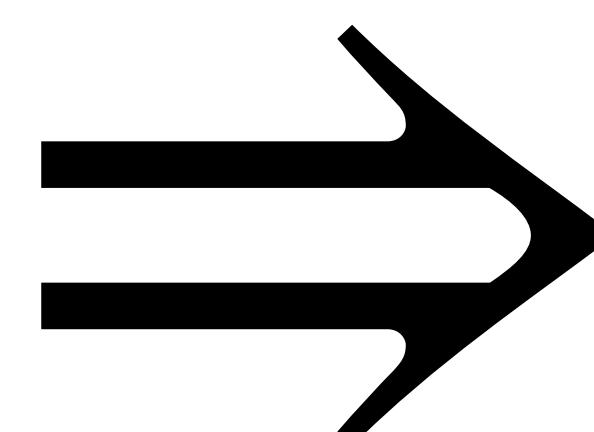


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	HL-LHC 68%	HL-LHC 95%	FCC-hh 68%	FCC-hh 95%
d_4	[-6.6, 12.4]	[-10.0, 21.3]	[-3.9, 10.5]	[-10.6, 18.8]
c_{t3}	[-0.6, 1.1]	[-0.9, 3.6]	[-0.1, 0.3]	[-0.4, 0.6]



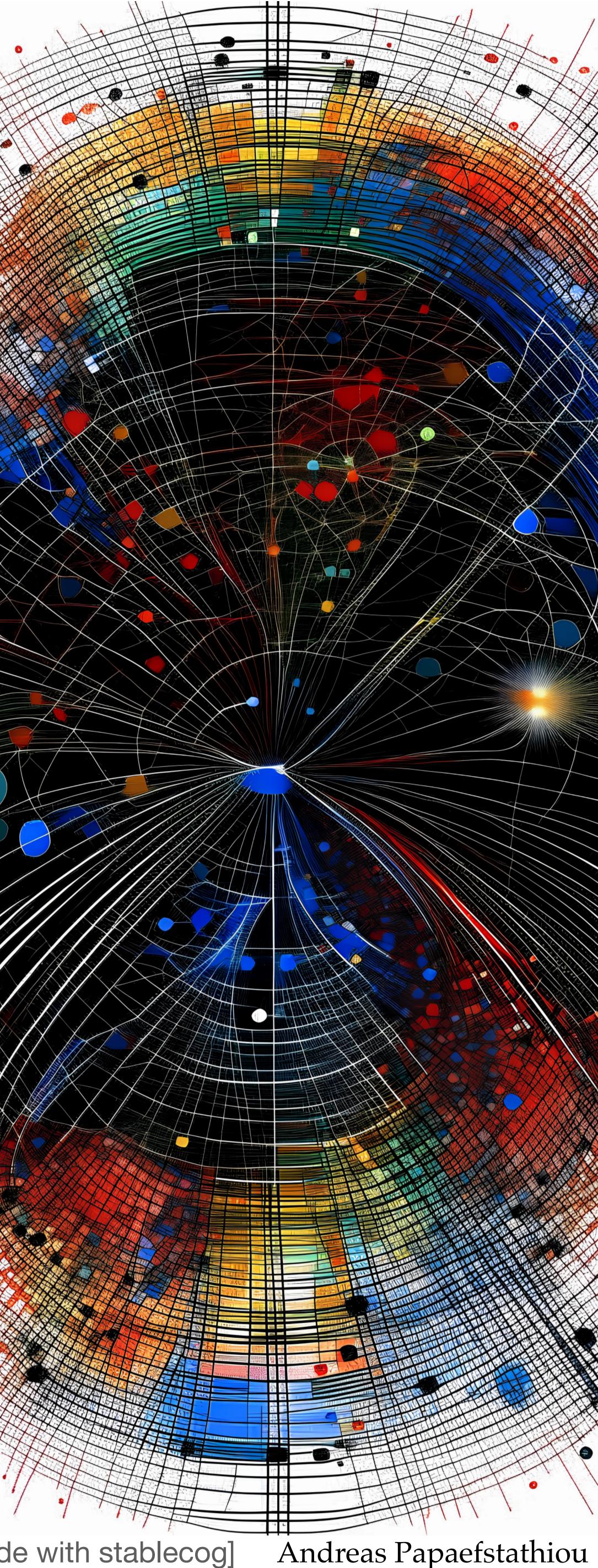
$$c_{t3} \sim \mathcal{O}(0.1 - 1)$$

$$d_4 \sim \mathcal{O}(10)$$



Summary & Outlook (I)

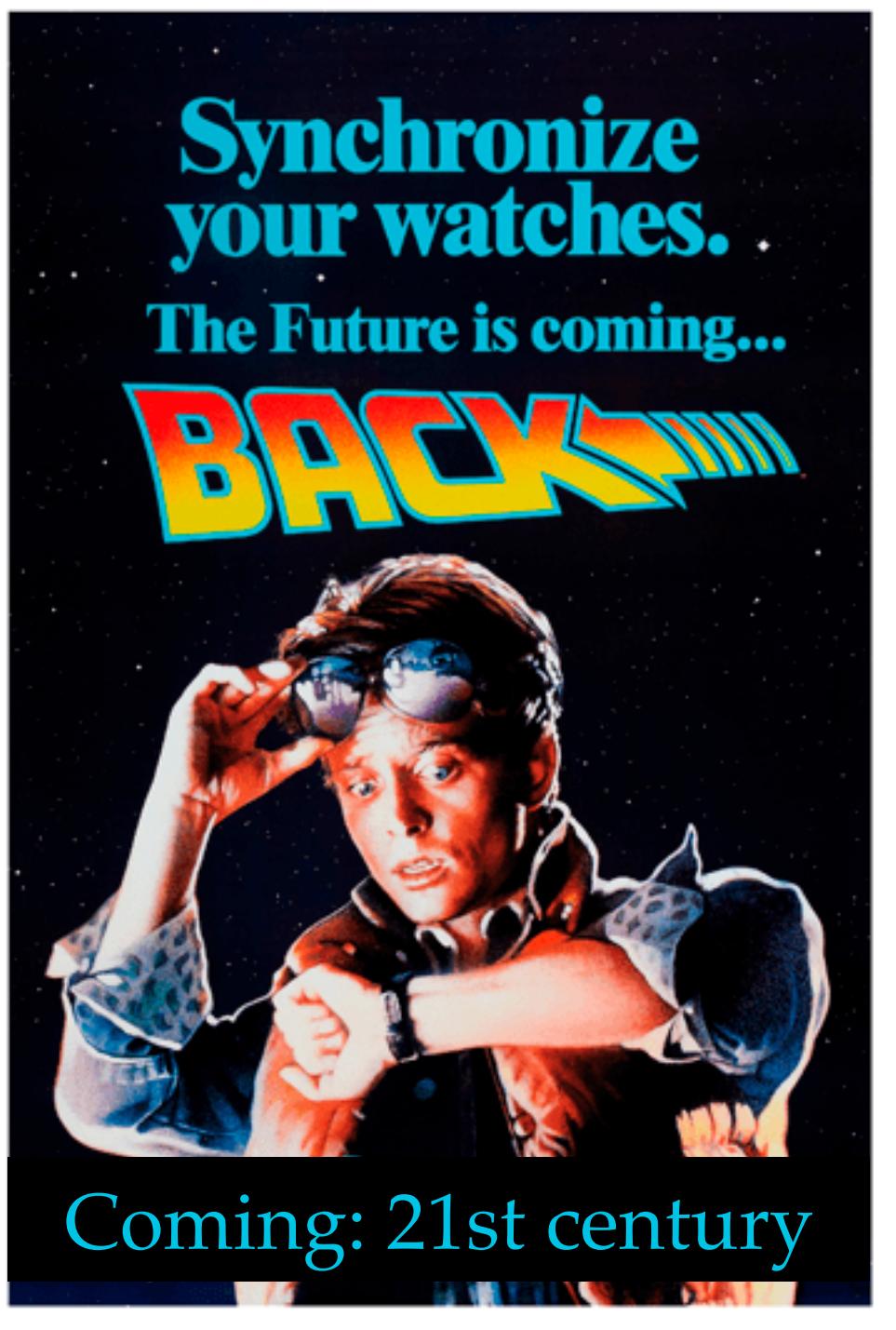
- Despite the successes of the SM, there remains a multitude of **open questions**,
 - Some may be linked via the **Electro-Weak Phase Transition**.
- The **Nature of the Electro-Weak Phase Transition** is an important scientific enquiry.
 - ▶ **(Strong) First-Order EWPT [not in SM!] → Matter-Anti-Matter asymmetry.**
 - ▶ Extending the **scalar sector of the SM could be the necessary catalyst.**
 - ▶ Future particle colliders have the **potential to probe this mechanism.**



[made with stablecog] Andreas Papaefstathiou

Summary & Outlook (II)

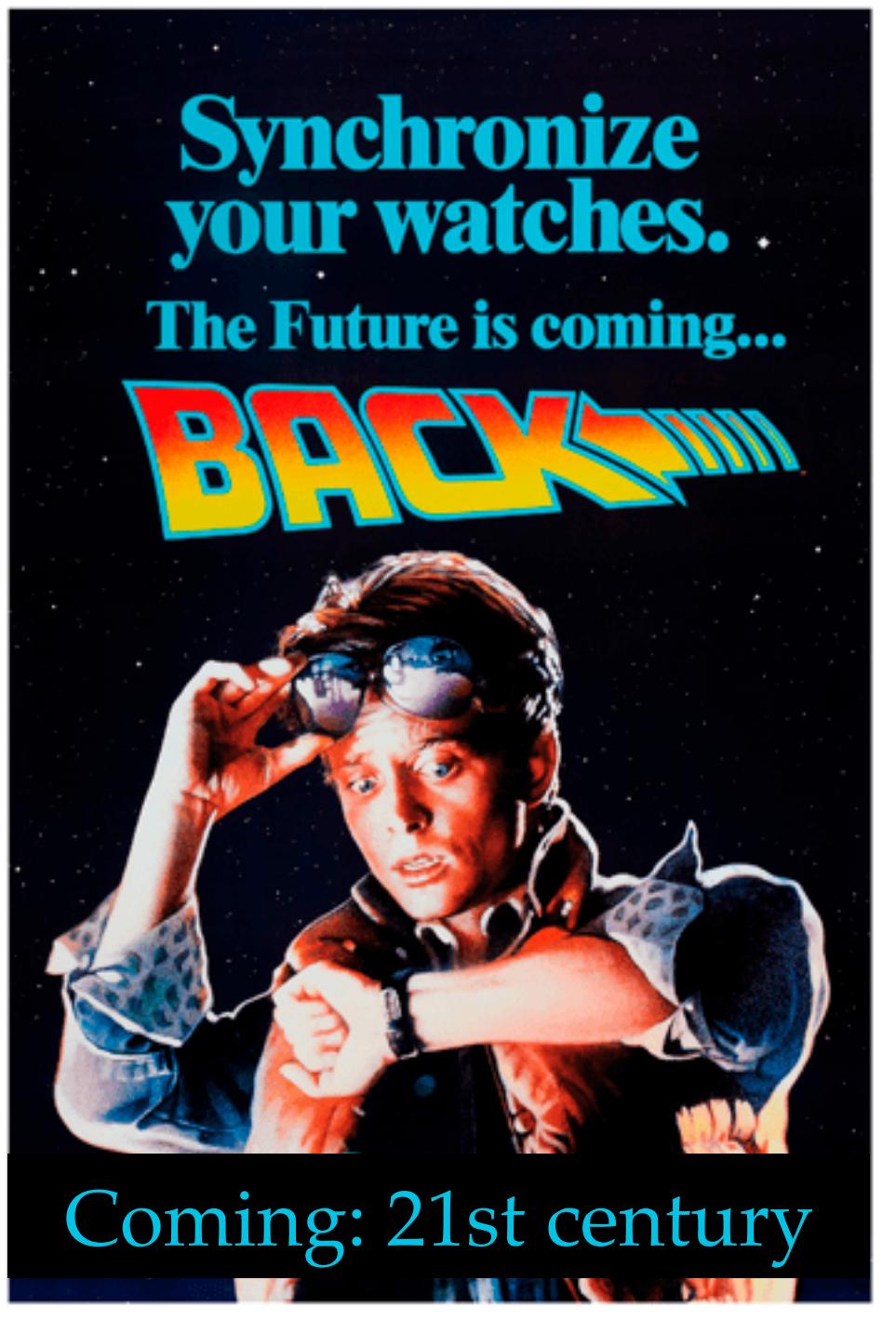
- Following *any* discovery, solving the inverse problem would be the crucial next step.
- Multi-scalar production processes (e.g. hhh production) may play a crucial role in models with extended scalar sectors.
- hhh may be enhanced in models with extended scalar sectors.
Could we see hints at the LHC?
- hhh production will probe modifications to the Higgs quartic self-coupling, within the anomalous coupling picture.
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Andreas Papaefstathiou

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Dank je wel!

Thanks!

Questions?



Appendices

Theoretical Uncertainties

- Uncertainties → Can affect e.g. the strength of the transition, $\langle \phi(T) \rangle / T$.
- Due to:
 - **gauge dependence**, [Patel, Ramsey-Musolf, arXiv:1101.4665]
 - **scale dependence** → Linde's IR problem: expansion parameter is $gn_B \sim gT/m$, (n_B mode occupation), diverges as $m \rightarrow 0 \Rightarrow$ perturbativity breaks down.
[Linde, Phys. Lett. 96B (1980) 289.]
- ⇒ To make reliable and sensible statements on colliders prospects:
 - Crucial to take uncertainties into account.

Theoretical Uncertainty Bands

- Define “uncertainty band” by:

1. Deriving 1-loop effective potential in the covariant gauge,

[Arnold, Espinosa, hep-ph/9212235], [Andreassen, MSc, Norwegian U. Sci. Tech., 2013]

2. Run couplings $\lambda \rightarrow \lambda(\mu)$, μ is RGE scale, [SARAH, Staub, arXiv:0806.0538]

3. Scan parameter space of Lagrangian,

4. Vary $\mu \in [\frac{1}{2} \times m_Z, 5 \times m_Z]$ & gauge params. $\xi_i \in [0,3]$ → **band of 8 pts.**

5. Use **PhaseTracer** for each point in band → Get phase transitions, $\langle \phi(T_c) \rangle / T_c$.

[Athron, Balázs, Fowlie, Zhang, arXiv:2003.02859]

Parameter-space Categories

1. Define two conditions:
 - i. VEV at 1-loop: $\langle \phi(T = 0) \rangle = 246 \pm 30$ GeV & deepest minimum.
 - ii. $\langle \phi(T_c) \rangle / T_c > 1$ & no other transition with higher T_c .

2. Define four mutually-exclusive categories*:



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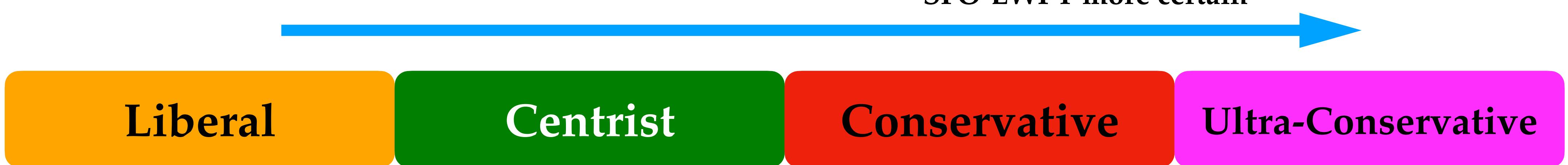
e.g.:

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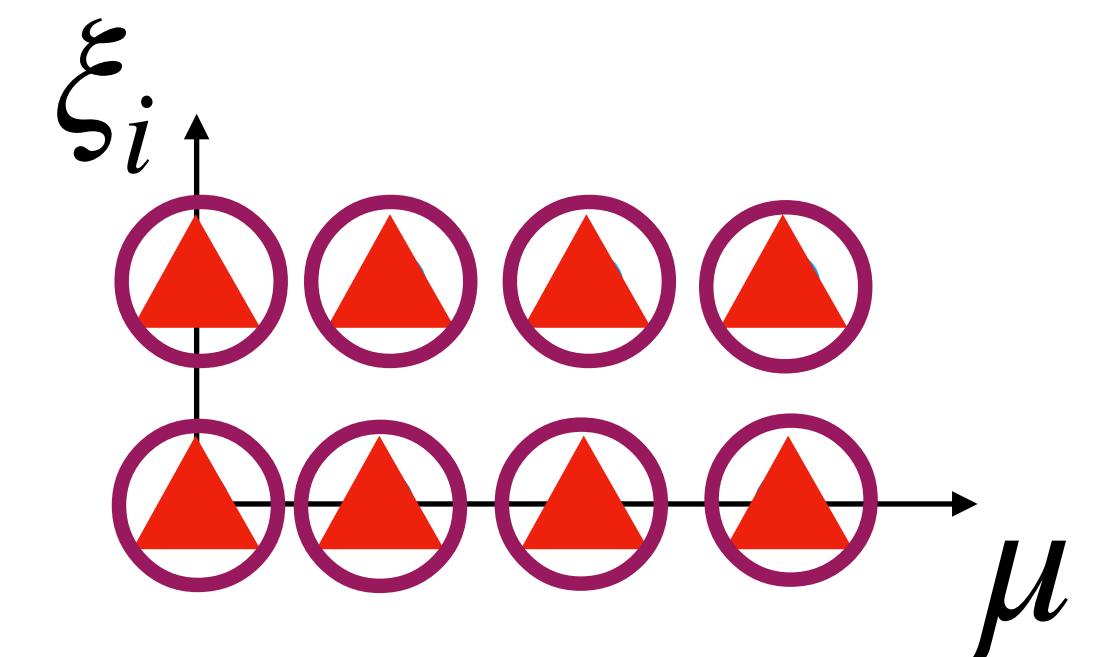
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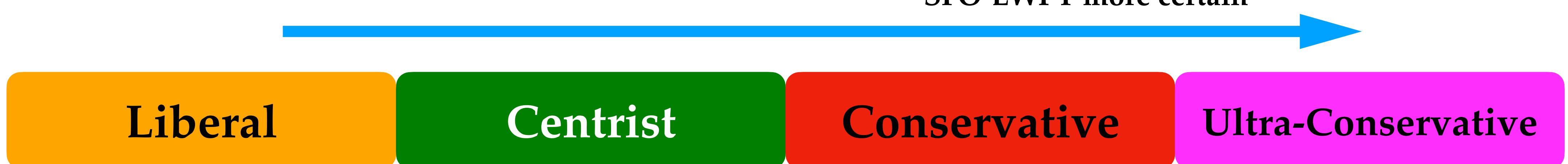


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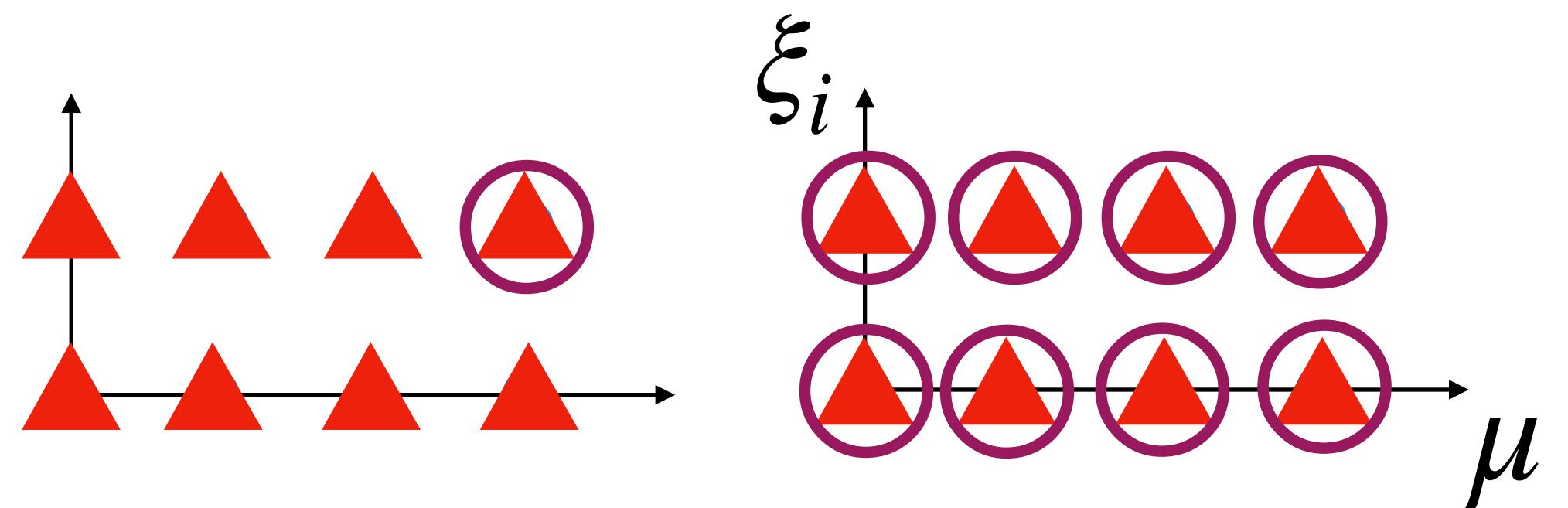
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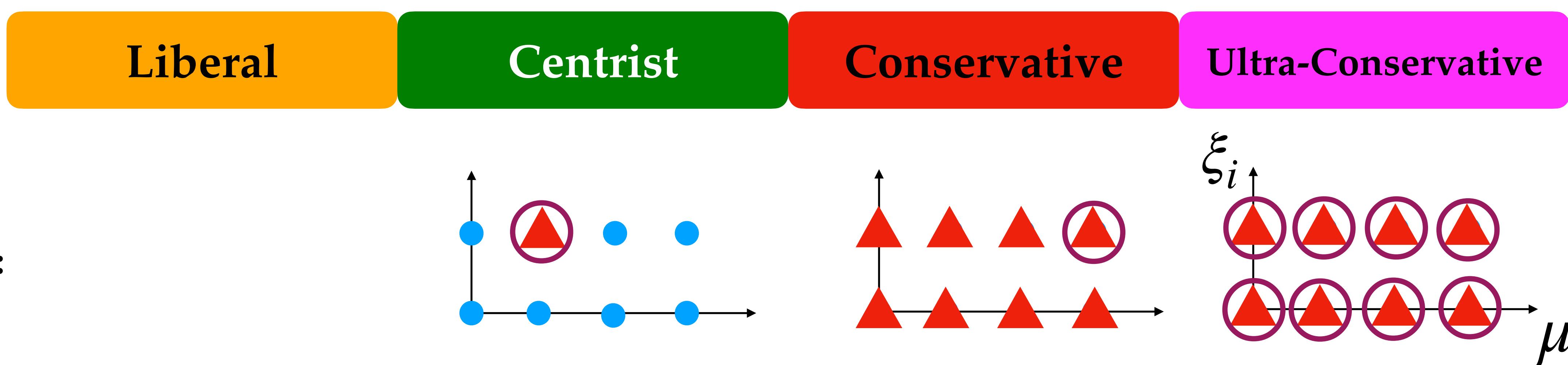
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SFO-EWPT more certain



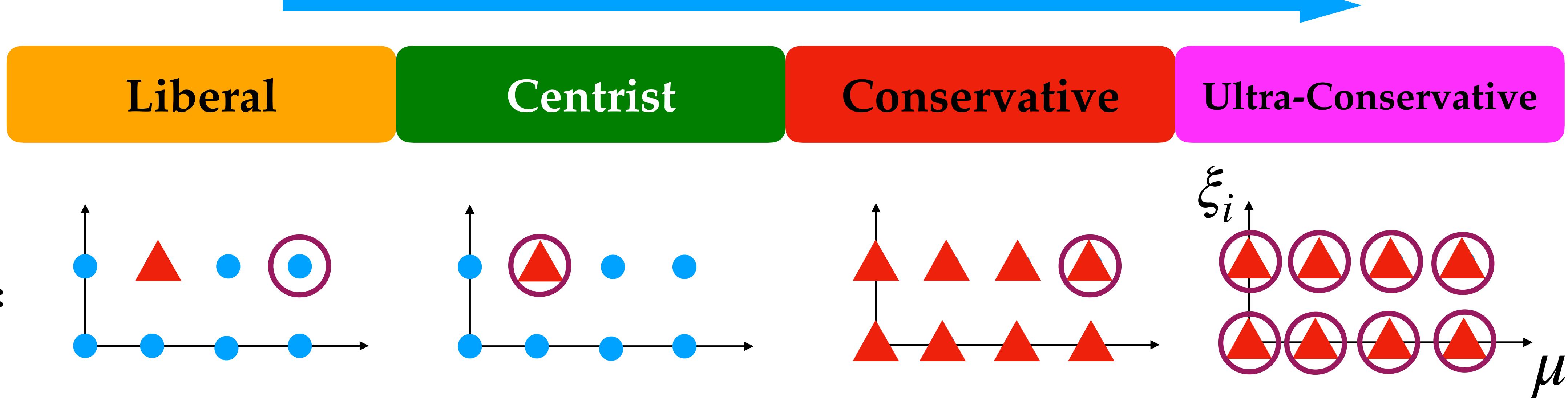
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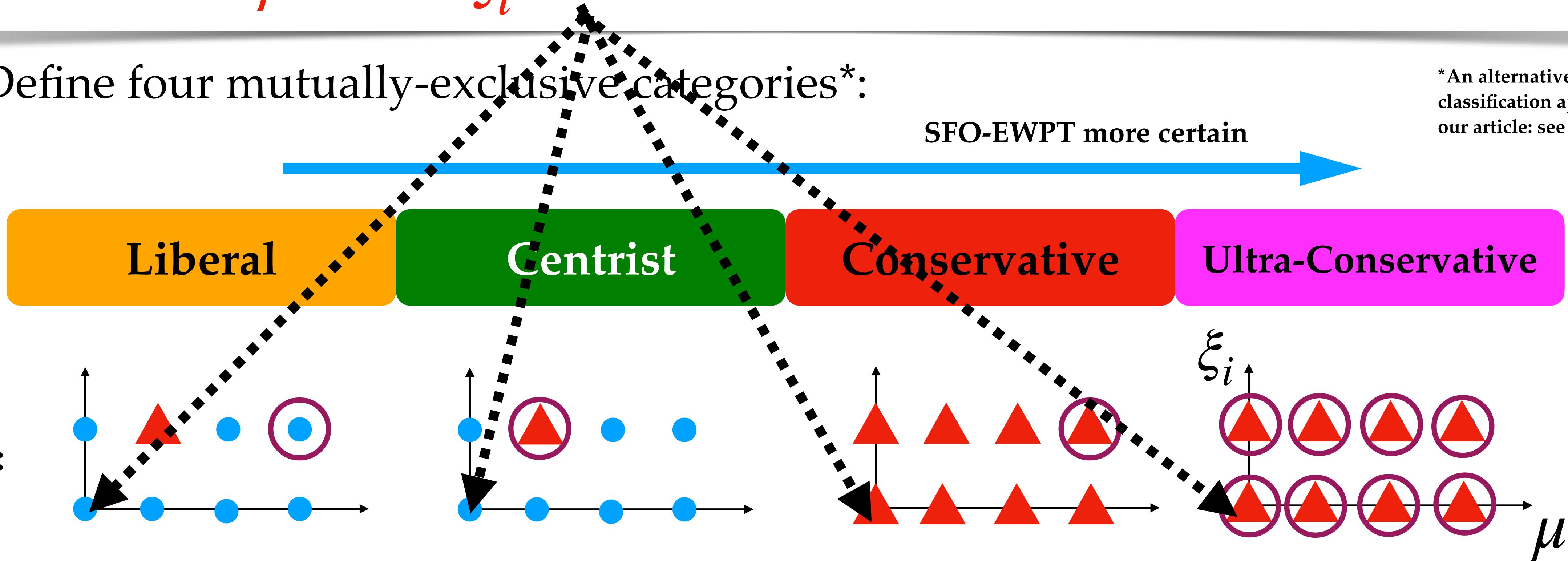


Parameter-space Categories

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Note: For phenomenological analyses, take “central” μ and ξ_i .

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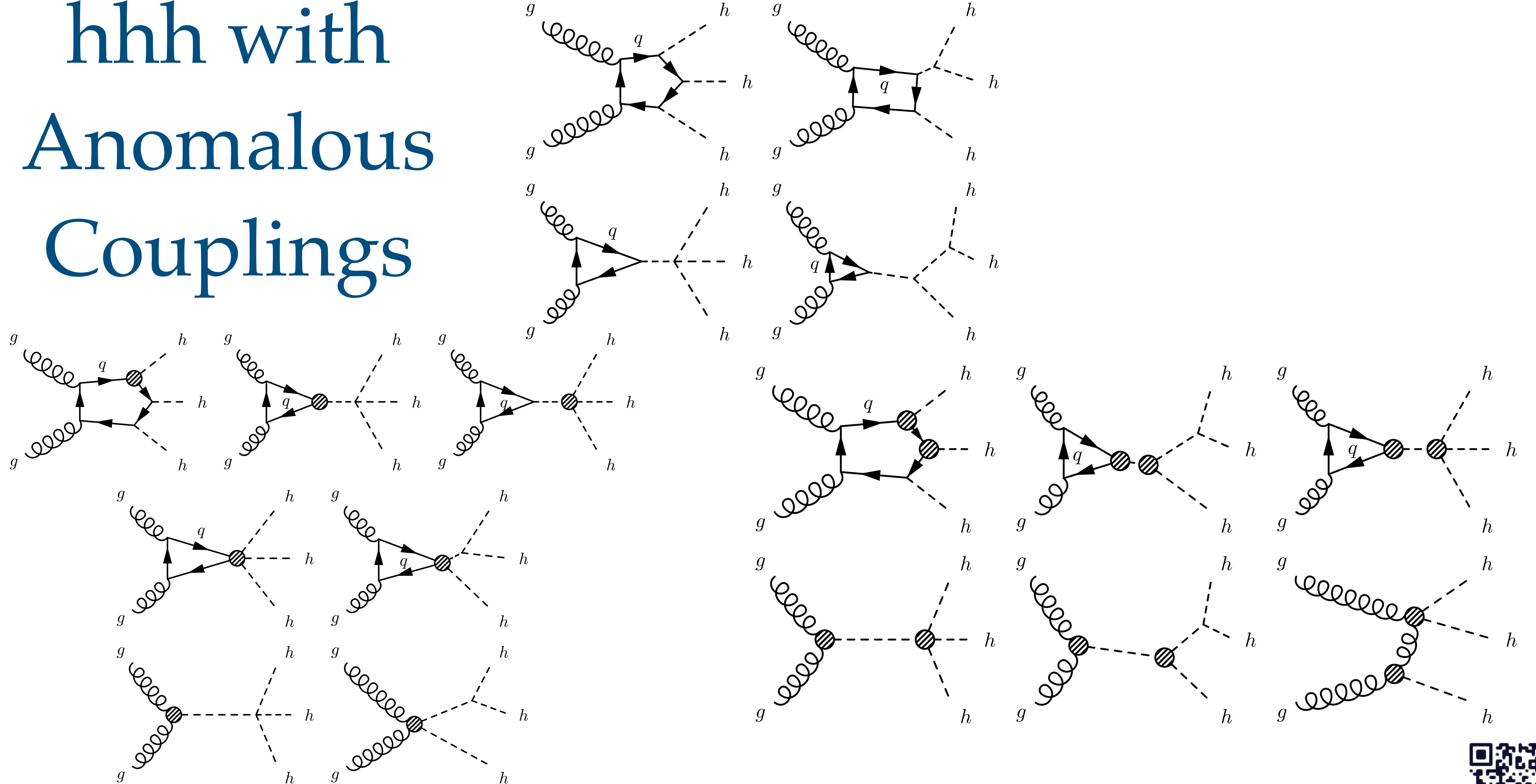
TRSM Monte Carlo Event Generation

- We have implemented a **MadGraph5_aMC@NLO (MG5_aMC)** “loop” model for the TRSM:
 - **MG5_aMC input parameters:** the three mixing angles, two masses/widths and **all** the scalar couplings (only 7 are independent in TRSM).
 - Comes with a **Python script** that:
 - allows conversion of M_2, M_3 + three mixing angles + two VEVs to the **MG5_aMC** model input,
 - calculates several single-production cross sections, branching ratios, widths,
 - and writes associated **MG5_aMC** parameter card (**param_card.dat**) automatically.
 - Get it at: <https://gitlab.com/apapaefs/twosinglet>.

[[AP](#), Tania Robens, Gilberto Tetlalmatzi-Xolocotzi, arXiv:2101.00037]



hhh with Anomalous Couplings



Electro-Weak Precision Observables

- Real singlet scalar field
 - → modifies Higgs contributions to diagonal weak gauge boson vacuum polarisation diagrams,
 - & introduces additional contributions.
- Quantify via S, T, U parameters. [Hagiwara, Matsumoto, Haidt, Kim, hep-ph/9409380]
- Change in EWPO \mathcal{O} ($= S, T, U$):

$$\Delta \mathcal{O} = (\mathcal{O}(m_2^2) - \mathcal{O}(m_1^2)) \times \sin^2 \theta$$

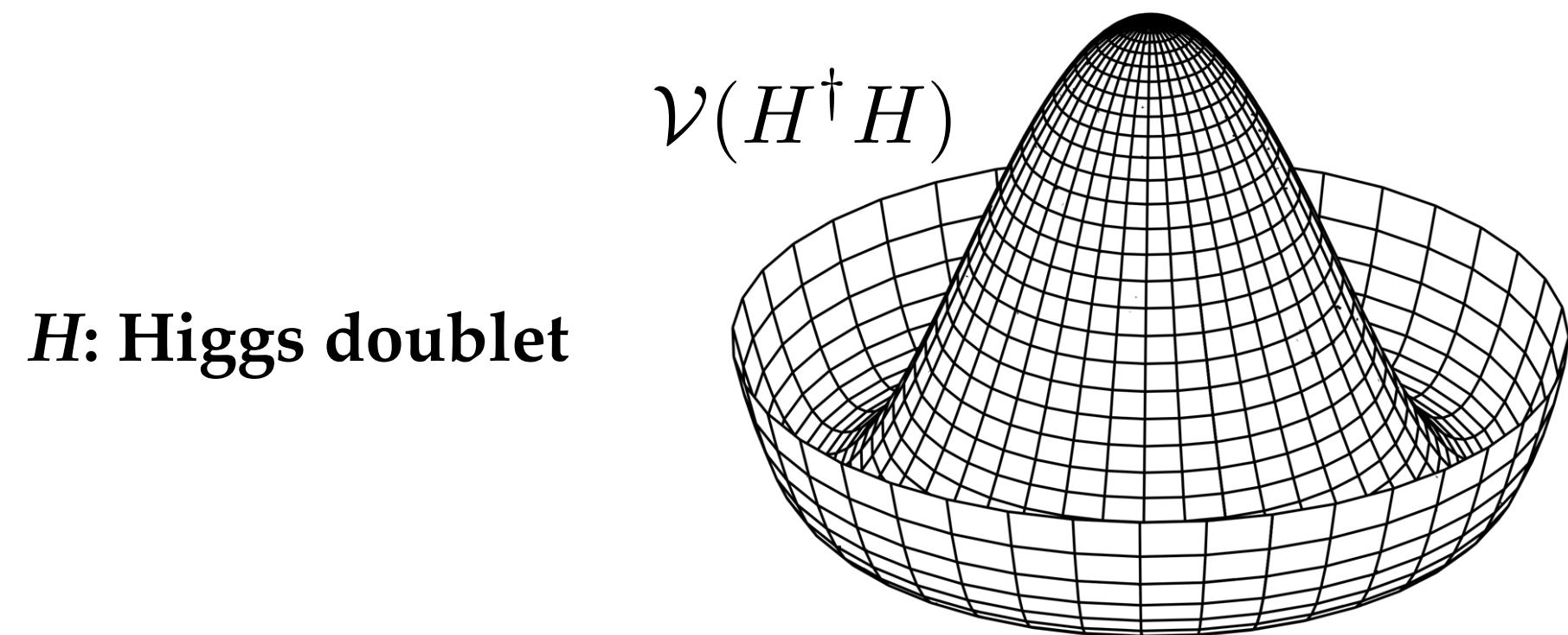
⇒ calculate compatibility with experimental measurement $\Delta \mathcal{O}^{\text{EXP}}$.



The Higgs Potential & Vacuum Stability

The Importance of the Higgs sector

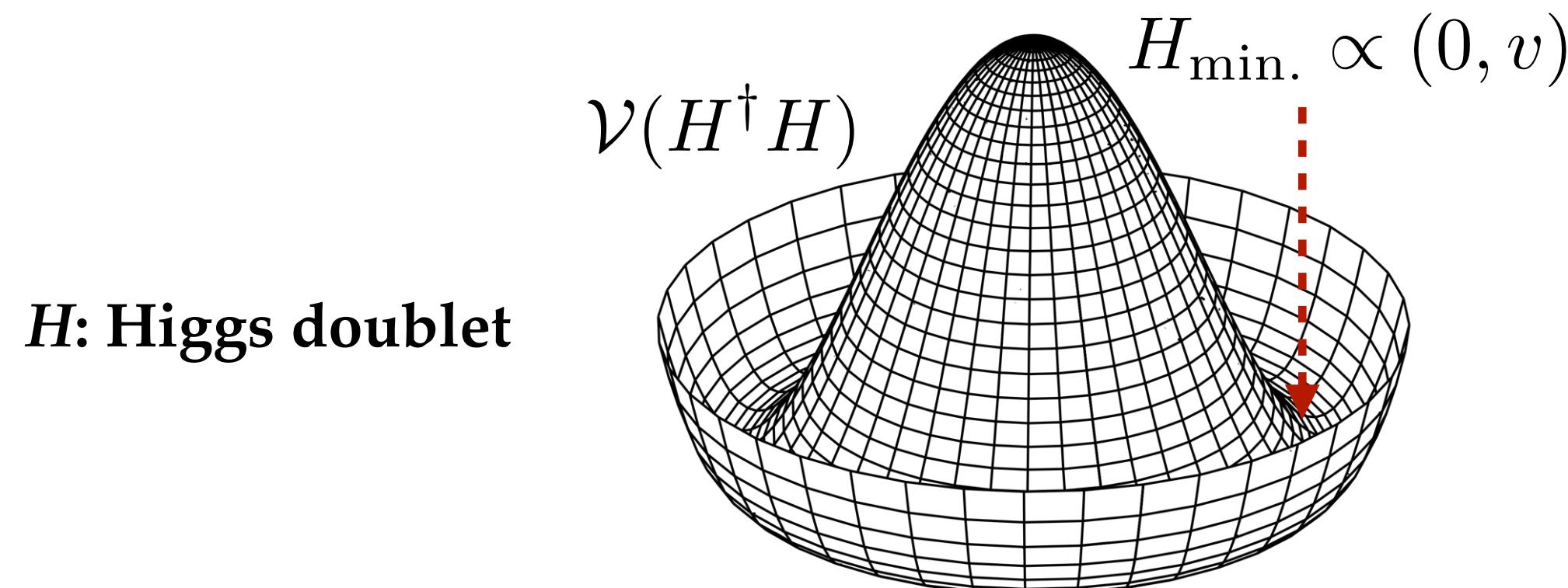
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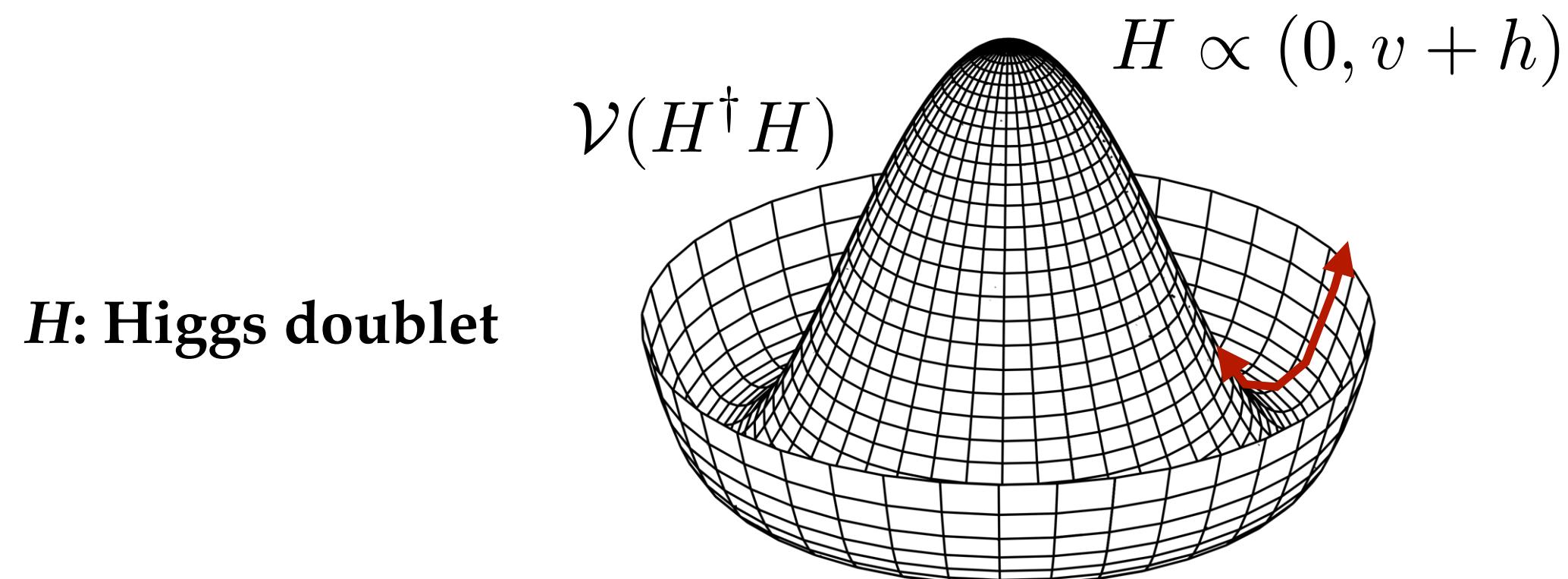
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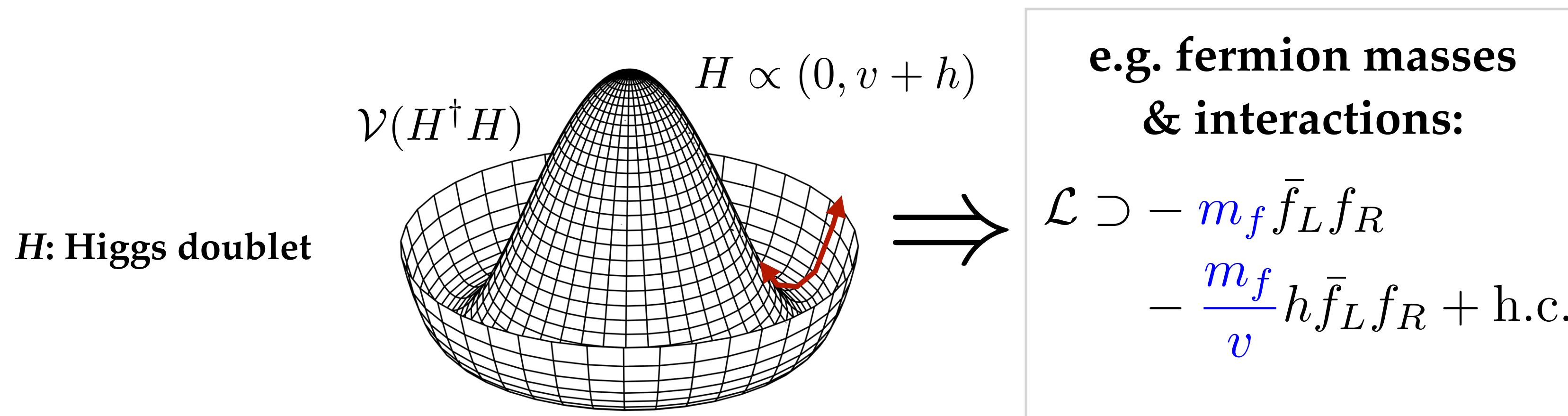
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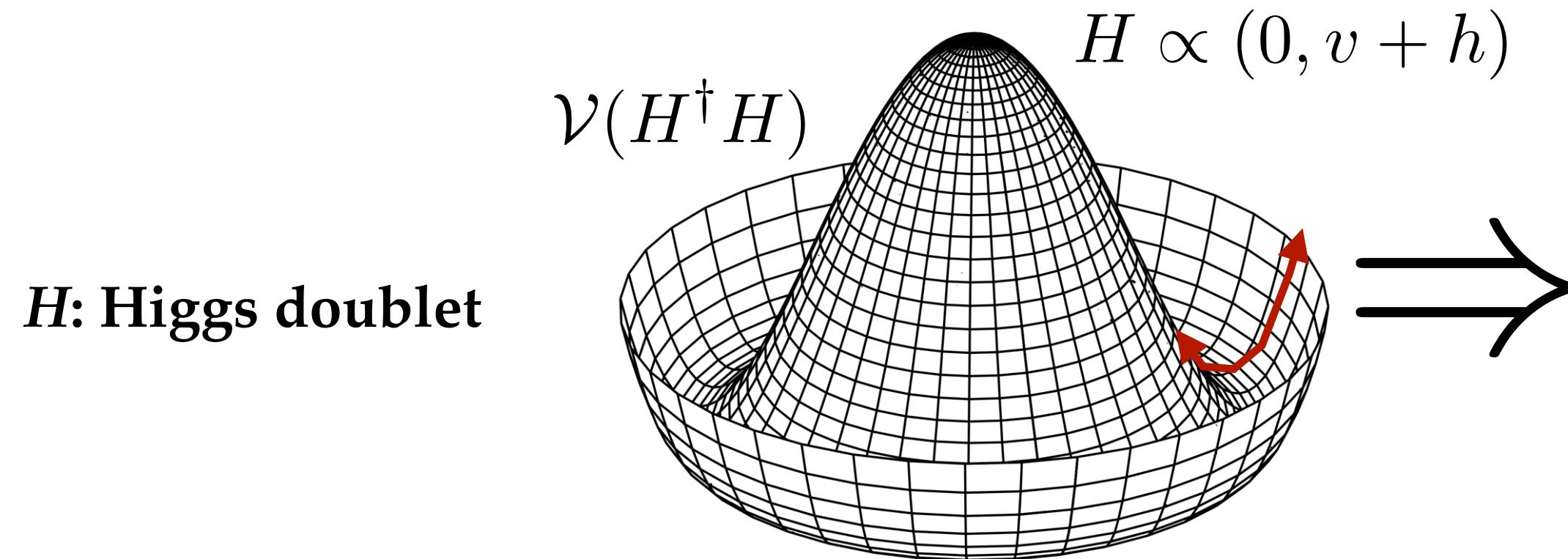
- the Higgs boson: the central protagonist of EWSB:



- an important characteristic of the Higgs boson is the way it couples to itself:

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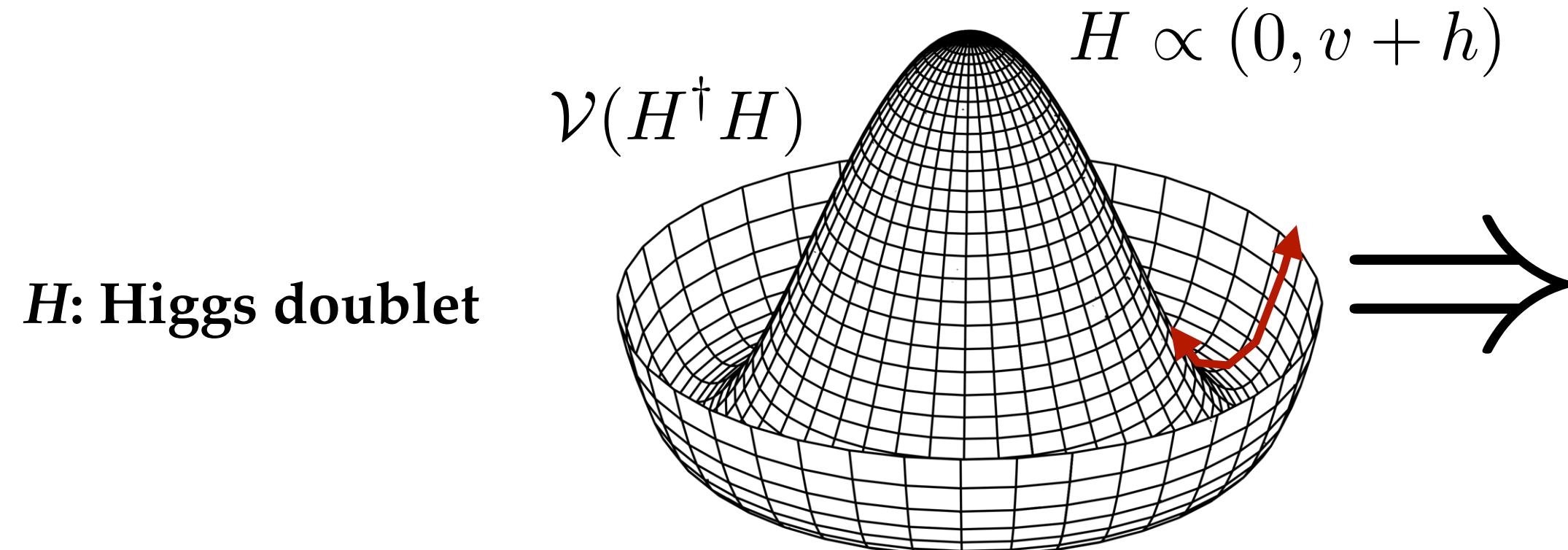
e.g. gauge boson masses
& interactions:

$$\begin{aligned}\mathcal{L} \supset & [m_W^2 W^{\mu+} W_{\mu}^{-} + \frac{1}{2} m_Z^2 Z^{\mu} Z_{\mu}] \\ & \times \left(1 + \frac{h}{v}\right)^2\end{aligned}$$

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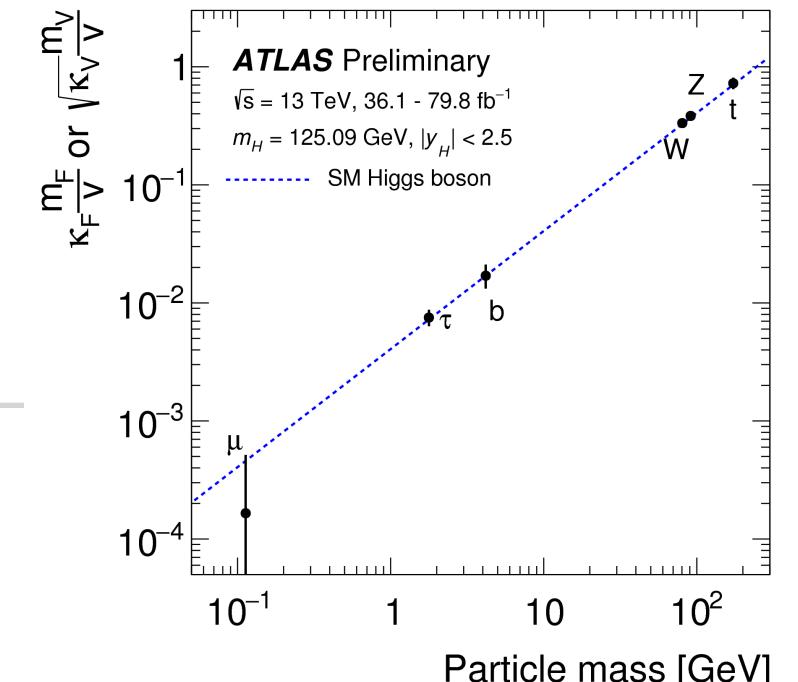
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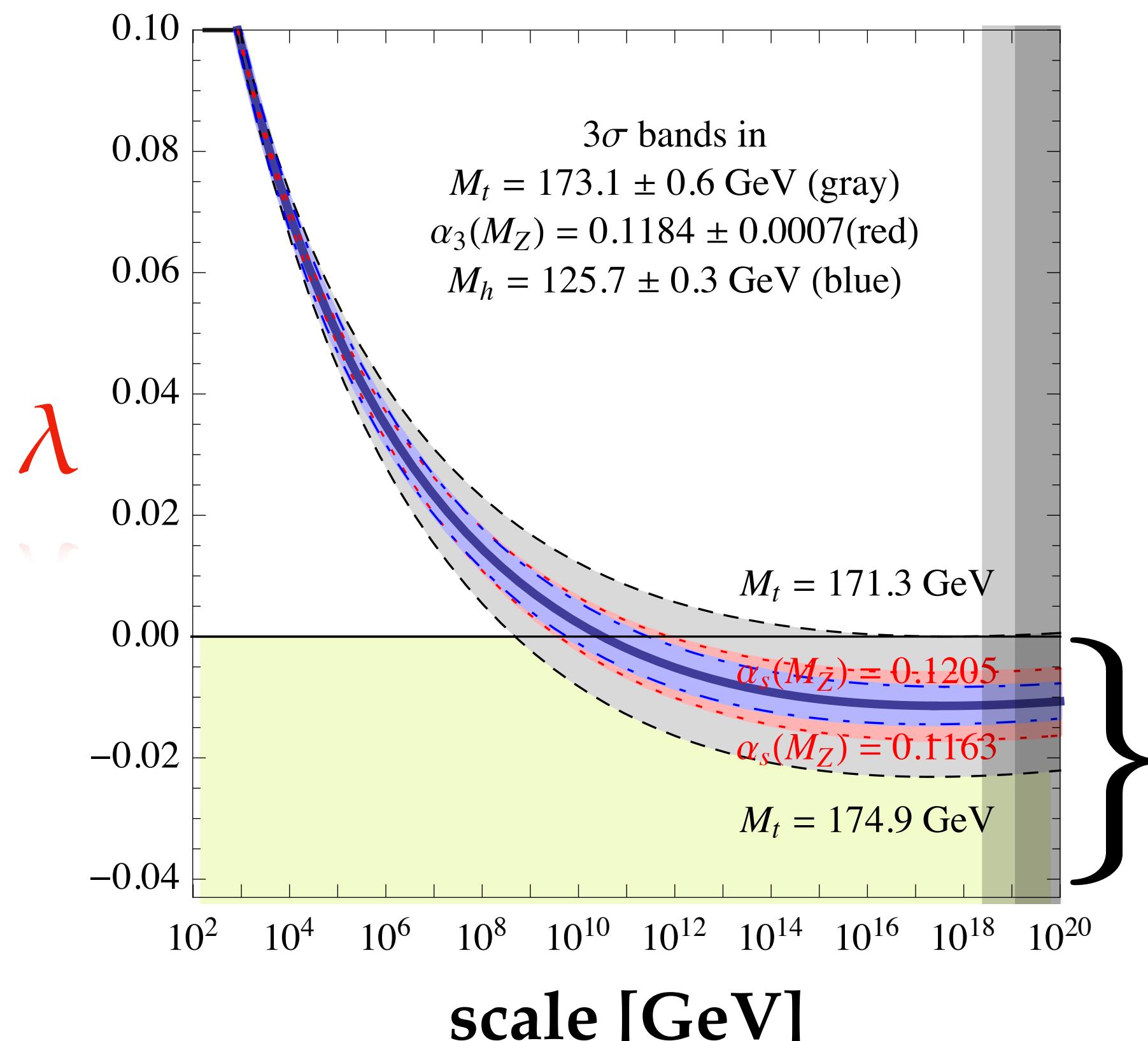
- an important characteristic of the Higgs boson is the way it couples to itself:

Vacuum Stability

- SM potential for the Higgs doublet:

$$\mathcal{V}(H^\dagger H) = -m^2(H^\dagger H) + \lambda(H^\dagger H)^2$$

- renormalisation group evolution of the coupling λ :

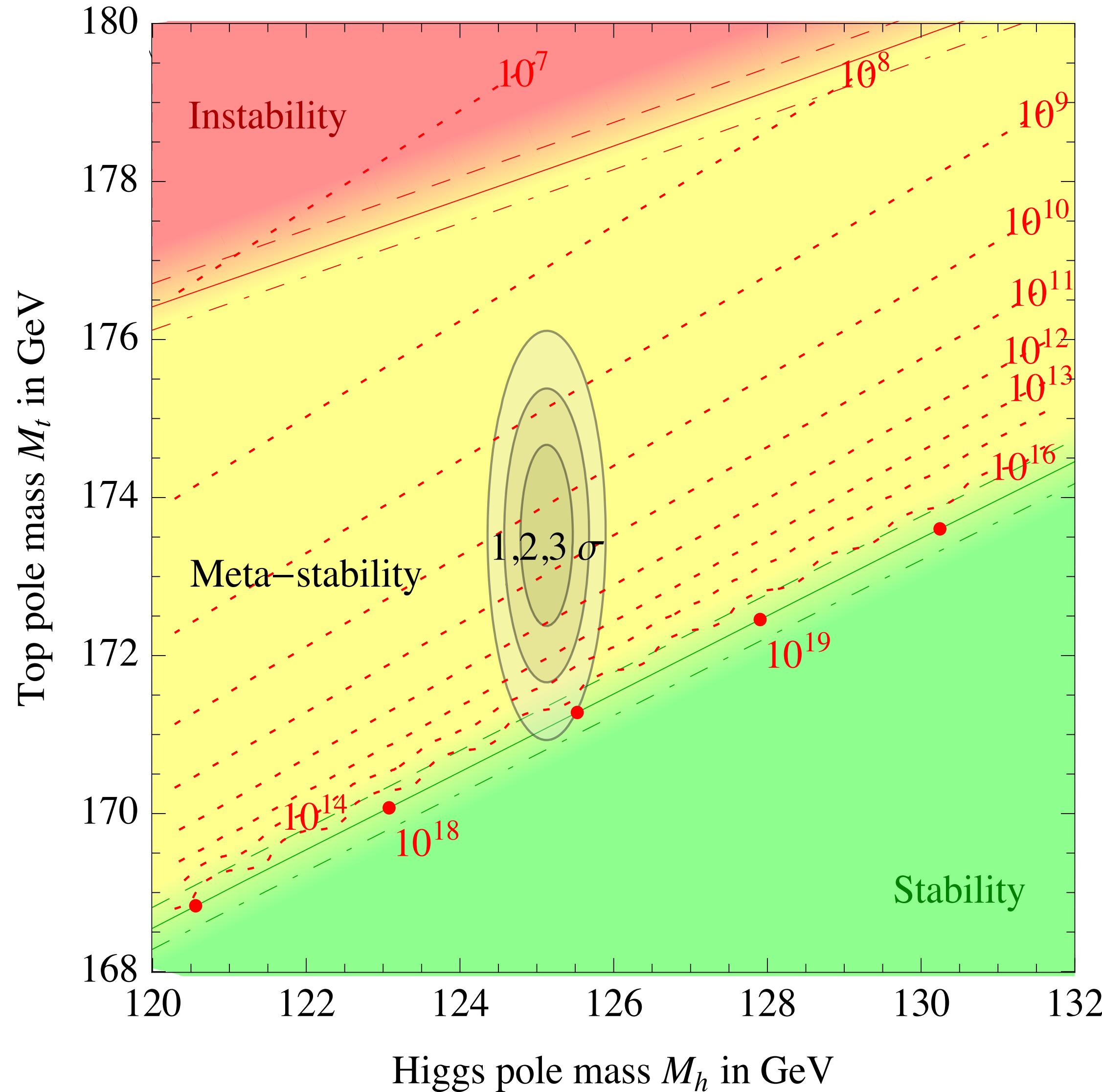


[Cabibbo, Maiani, Parisi, Petronzio, 1979,
Hung, 1979, ...,
Degrassi, Di Vita, Elias-Miró, Giudice,
Isidori, Strumia, 1205.6497, Buttazzo,
Degrassi, Giardino, Giudice, Sala, Salvio,
Strumia, 1307.3536 ..., Espinosa,
1512.01222]

potentially
unstable or
meta-stable
vacuum!

vacuum stability

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, 1307.3536, Espinosa, 1512.01222]



in deriving this: **assumed Standard Model.**

→ a hint for a non-standard Higgs sector?

→ further investigation necessary.

Sphaleron/Instanton Processes

Instantons and Baryon-# Violation

- toy model:
(1+1)-dimensions, Abelian gauge field A^μ , complex scalar Φ^μ , Dirac fermion of unit charge Ψ .
- Euclidean space action:

$$S = \int d^2x \left[\frac{1}{4} F_{\mu\nu}^2 + |(\partial_\mu - ieA_\mu)\Phi|^2 + V(\Phi) + i\bar{\Psi}(\partial_\mu - ieA_\mu)\gamma^\mu\Psi \right]$$

“Higgs potential”: $V(\Phi) = \lambda(\Phi^*\Phi - v^2)^2 \implies \text{“EWSB”} \implies M_A, M_h$

Instantons and Baryon-# Violation

- consider the current:

$$K_\mu = \frac{e}{2\pi} \epsilon_{\mu\nu} A_\nu$$

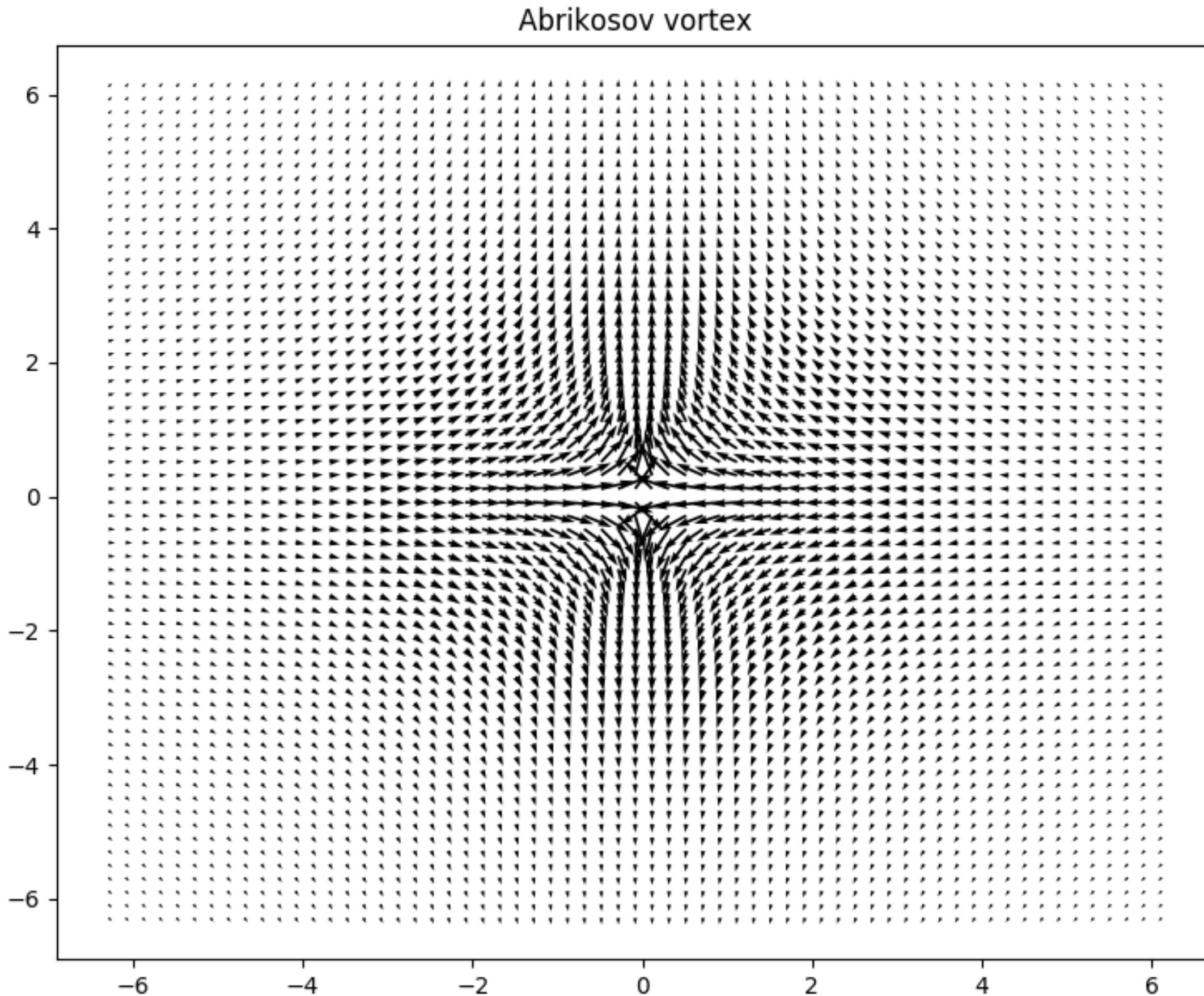
- corresponds to “charge density”:

$$N_{\text{CS}} = \int dx \ K_0 = \frac{e}{2\pi} \int dx \ A_1$$

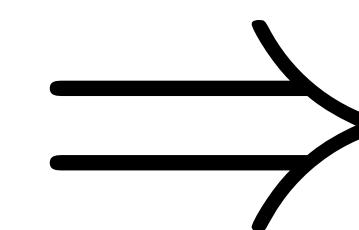
- known as the “winding” or “Chern-Simons” number.

Instantons and Baryon-# Violation

- a classical solution to equations of motion is the “Abrikosov vortex”:



$$A_r = 0, \quad A_\theta = \frac{1}{er} f(r),$$
$$f(0) = 0, \quad 1 - f(r) \sim e^{-M_A r}$$



changes the Chern-Simons number by one unit:

$$\Delta N_{\text{CS}} = \int d^2x \partial_\mu K^\mu = \boxed{\frac{e}{4\pi} \int d^2x \epsilon_{\mu\nu} F^{\mu\nu}} = 1$$

Instantons and Baryon-# Violation

- “instanton” transition necessarily accompanied by change of chirality of fermions by two units:

$$j_\mu^5 = \bar{\Psi} \gamma_\mu \gamma_5 \Psi$$

$$\frac{1}{2} \partial_\mu j^{5\mu} = \frac{e}{4\pi} \epsilon_{\mu\nu} F^{\mu\nu}$$

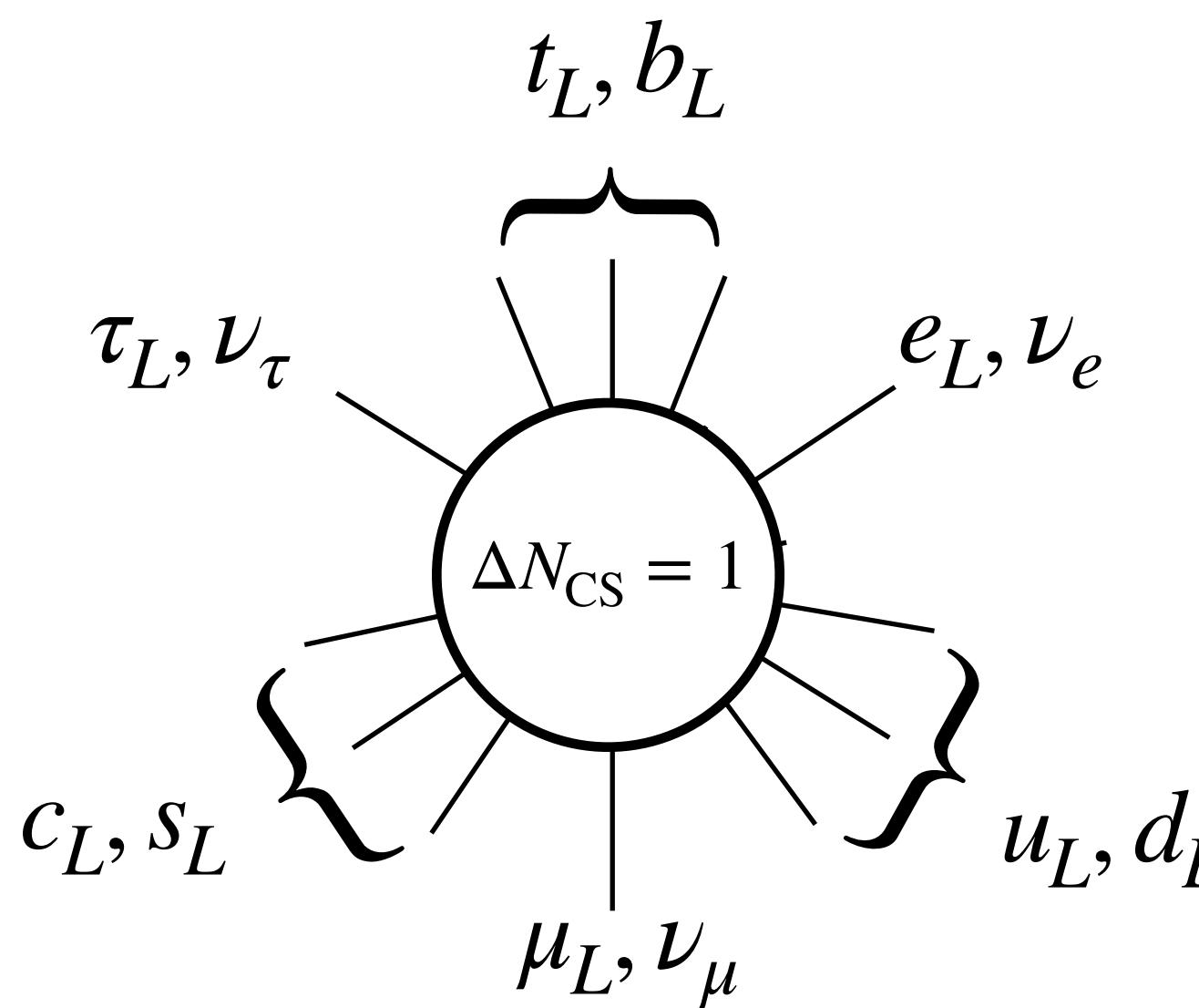
anomalous divergence of the axial-vector current.

$$\Delta N_{\text{CS}} = \int d^2x \partial_\mu K^\mu = \boxed{\frac{e}{4\pi} \int d^2x \epsilon_{\mu\nu} F^{\mu\nu}} = 1$$

$$\Rightarrow \Delta Q_5 = \Delta \int dx j_0^5 = 2$$

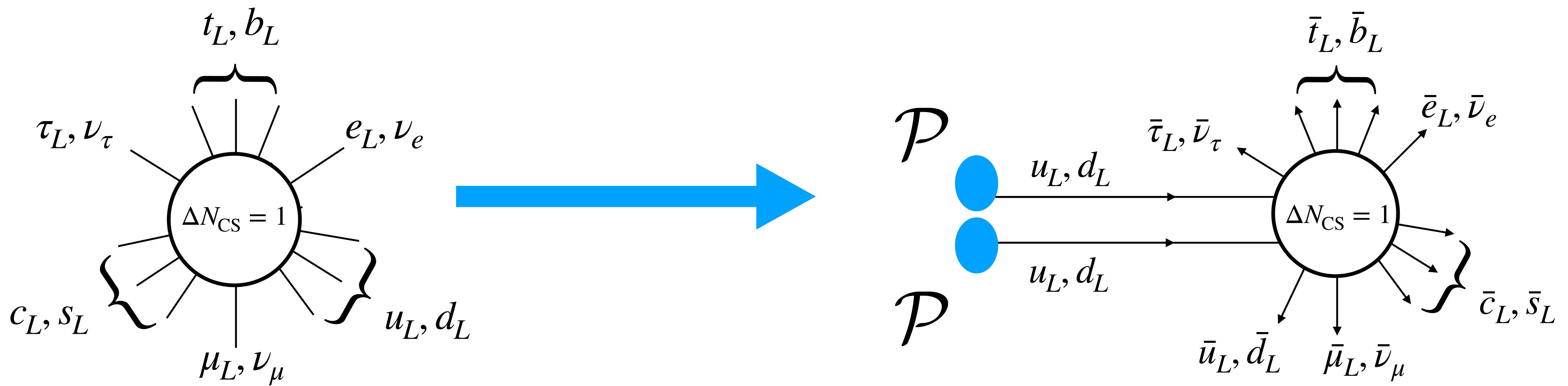
EW Sphalerons at colliders?

- Rate and observability of sphaleron processes at colliders debated.
e.g. [Bezrukov, Levkov, Rebbi, Rybakov, Tinyakov, hep-ph/0304180] VS. [Tye, Wong, 1505.0360, 1710.07223].
- Ponder: **Sphaleron-induced interactions at hadron colliders:**



EW Sphalerons at colliders?

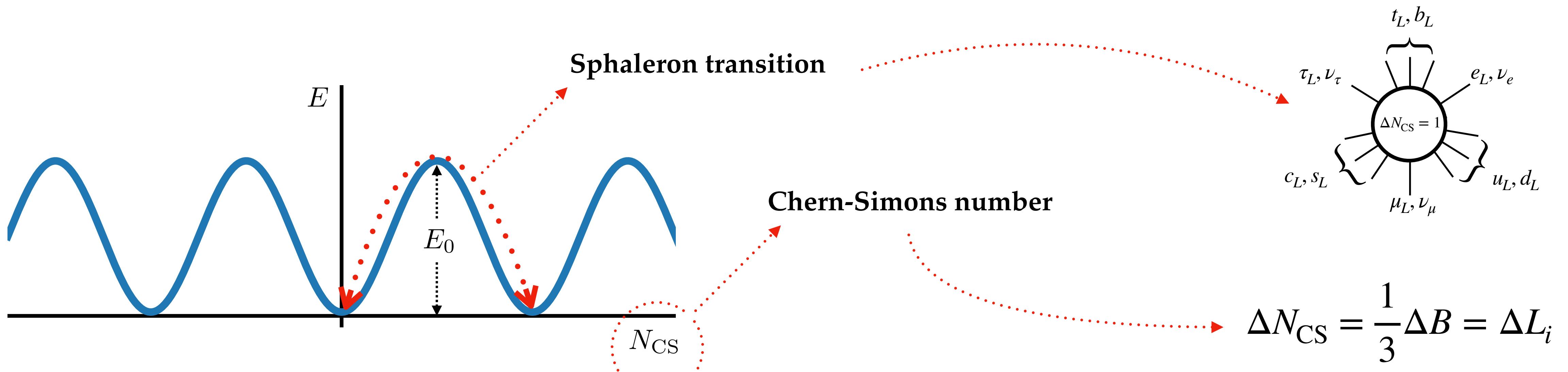
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What is the Sphaleron?



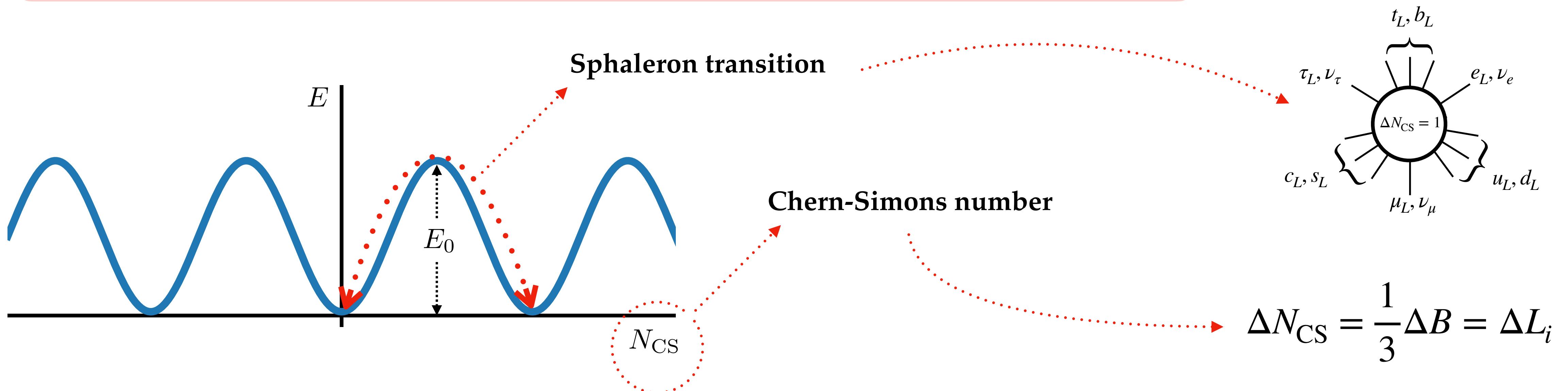
- $E_0 \sim \mathcal{O}(10)$ TeV, separates **degenerate** Electro-Weak vacua.



What is the Sphaleron?



- $E_0 \sim \mathcal{O}(10)$ TeV, separates degenerate Electro-Weak vacua.



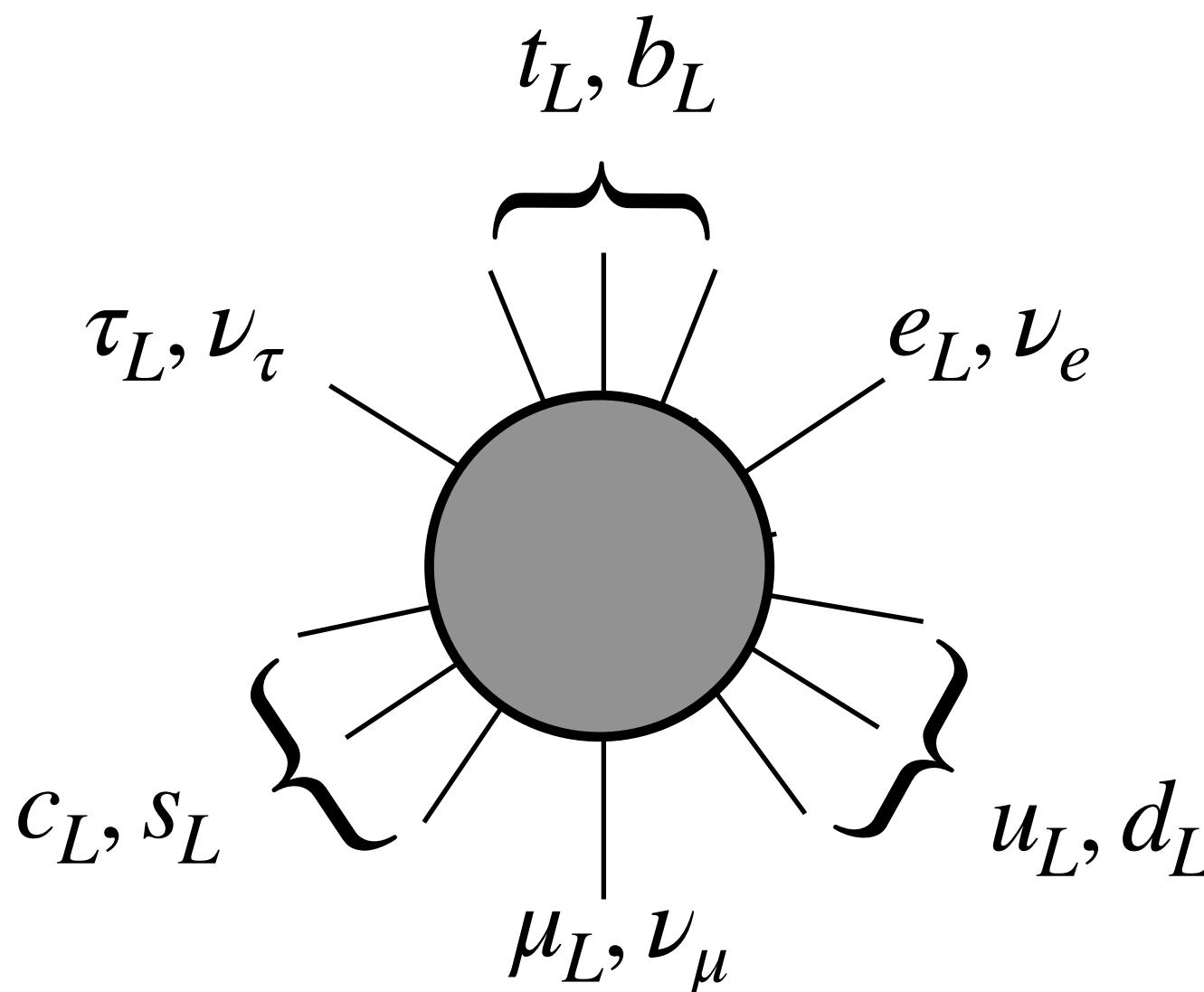
The Sphaleron energy depends
crucially on the Higgs sector!

A Note on Sphaleron Suppression

- Suppression of sphaleron rate inside bubble
⇒ Baryon Asymmetry “swept in” broken phase and “frozen in”.
- Rate $\sim \exp[-\langle \phi(T_C) \rangle / T_C \times \dots]$,
[T_C : the critical temperature.]
- ⇒ Require: $\langle \phi(T_C) \rangle / T_C \geq 1 \Rightarrow$ a “Strong” First-Order EWPT (SFO-EWPT).

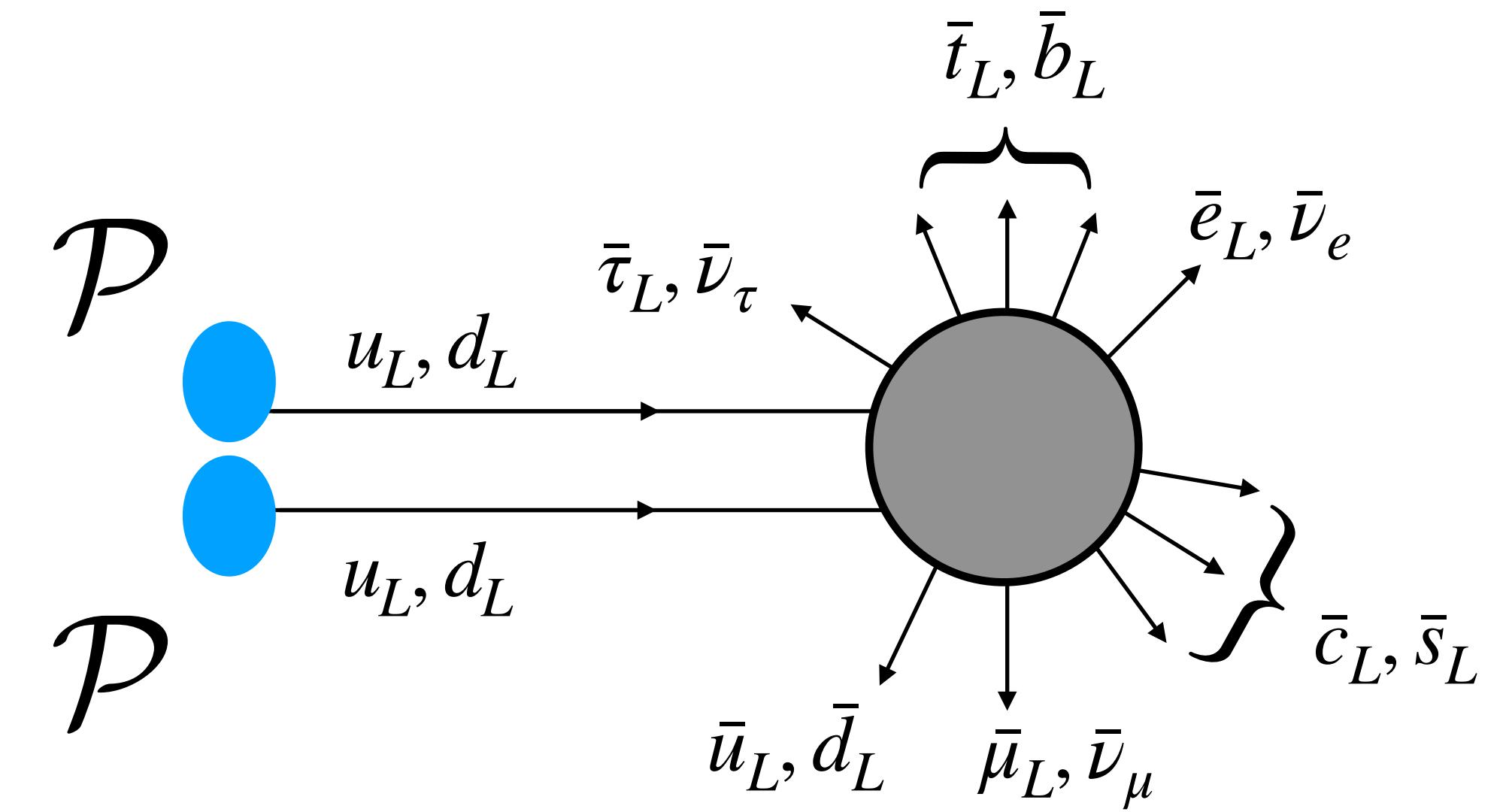
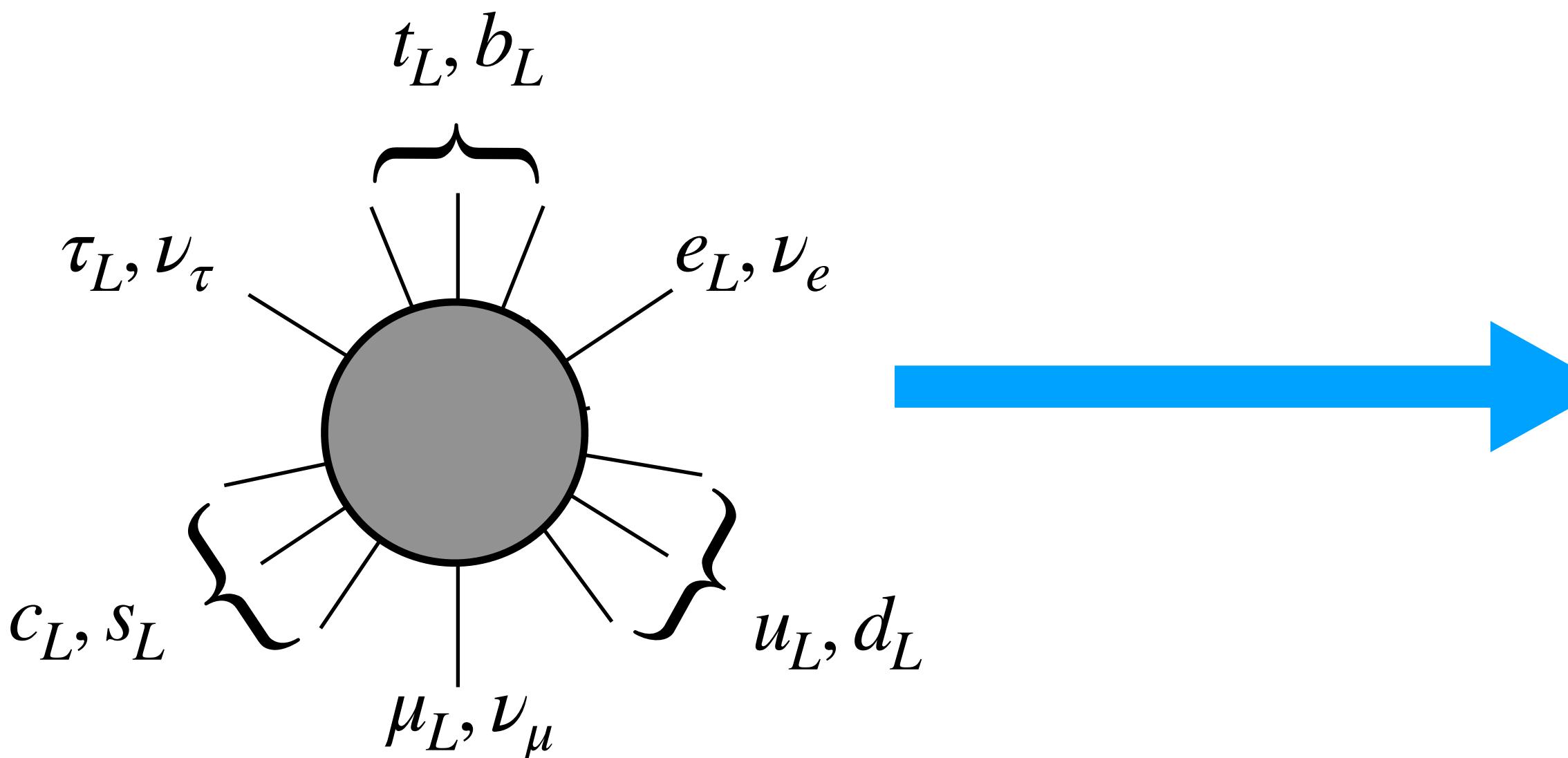
Sphaleron Suppression

- Inside the bubble: 
- Suppression requires “**Strong**” First-Order EWPT (**SFO-EWPT**).
- Despite suppression: Sphalerons @ colliders?

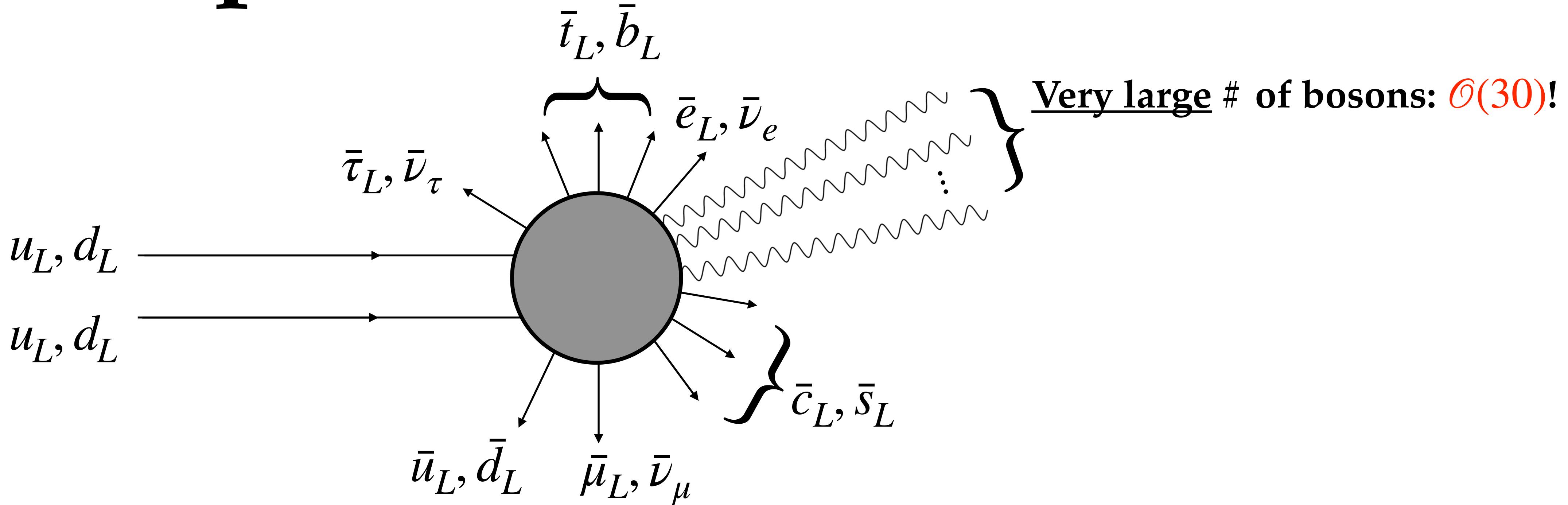


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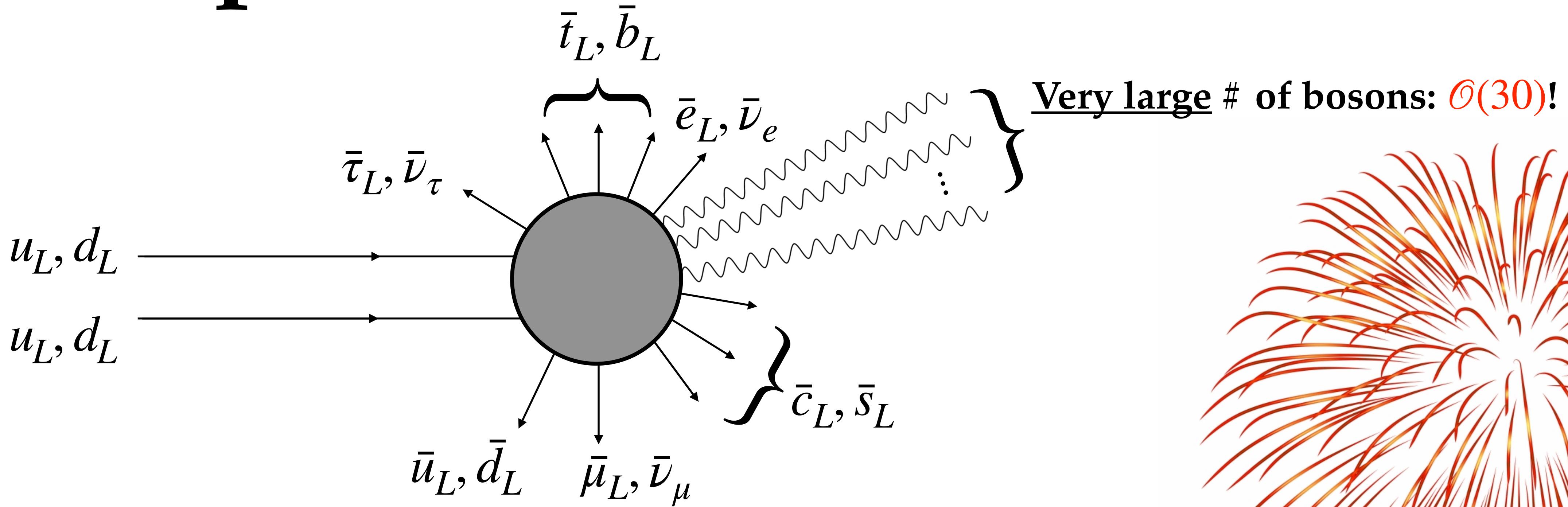


EW Sphalerons at Colliders?

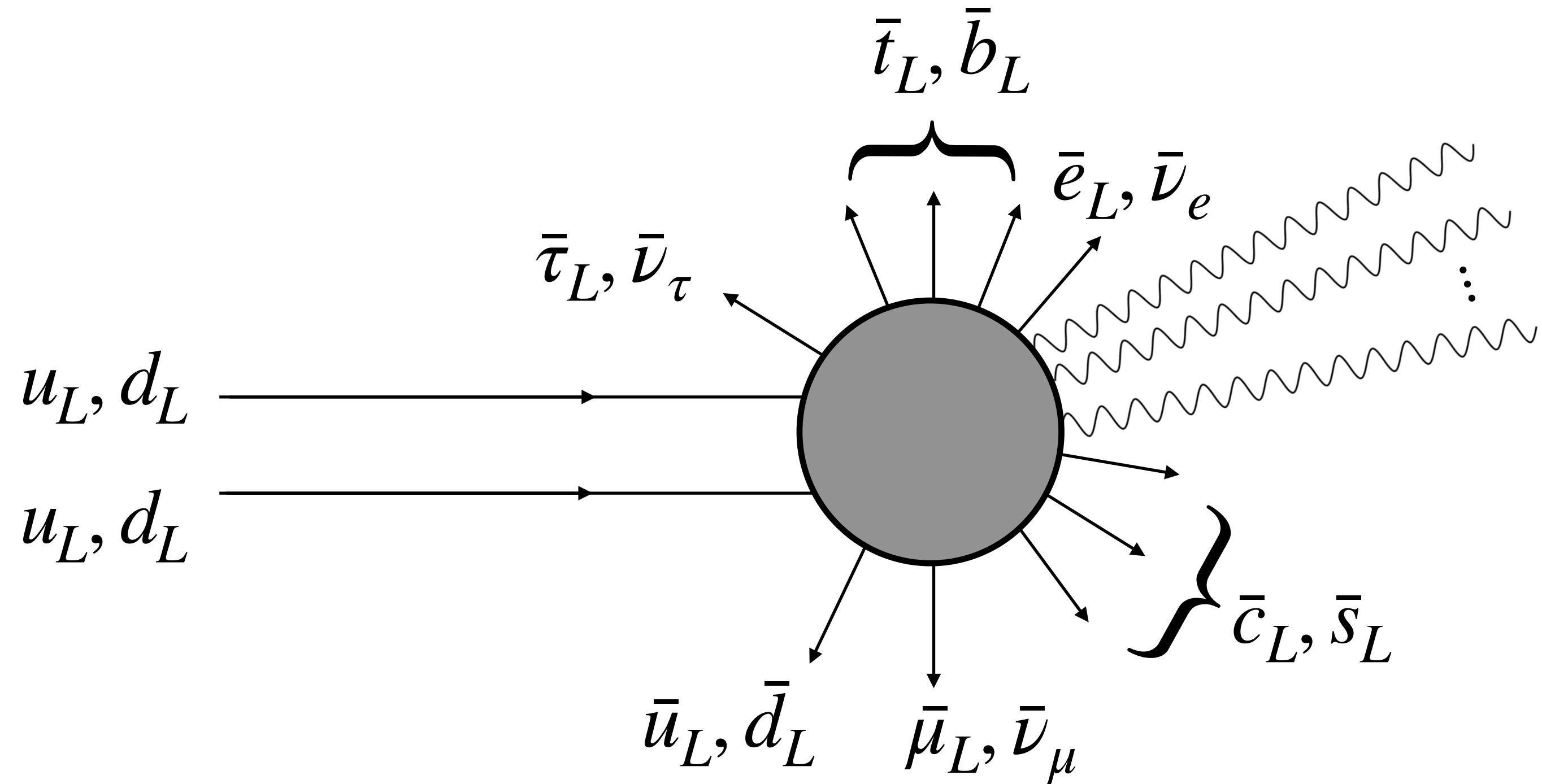


- Possible enhancement if large number of bosons,
⇒ Events would **spectacularly light up detectors** at experiments!

EW Sphalerons at Colliders?



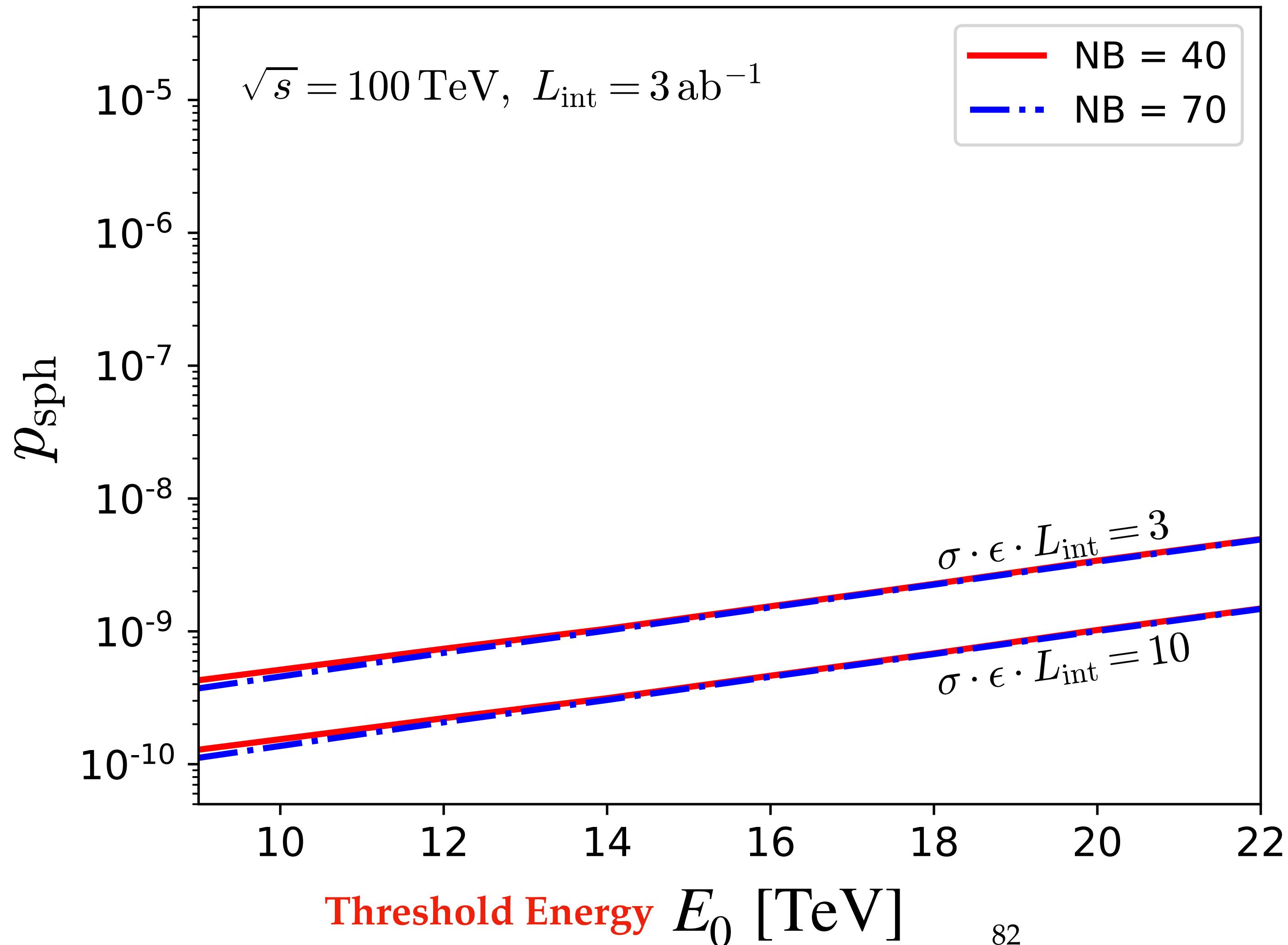
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Sphalerons at the FCC

- Parametrise parton-parton cross section by p_{sph} :

$$\hat{\sigma}(E) = \frac{p_{\text{sph}}}{m_W^2} \Theta(E - E_0)$$



→ Event Generator within HERWIG 7.

[AP, Sakurai, Plätzer, arXiv:1910.4761]

EW Sphalerons at Colliders?

[[AP](#), Sakurai, Plätzer, [arXiv:1910.4761](#)]

.

EW Sphalerons at Colliders?

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Homework:

- (i) What can we learn about the Higgs sector and EWBG?
- (ii) New theoretical features in Sphaleron MC.
- (iii) Model discrimination, e.g. VS micro-black holes.
- (iv) Collaboration with experimentalists for measurements.



hhh: Final states

Assume: K-factor = 2.

[Maltoni, Vryonidou, Zaro, 1408.6542]

$hh \rightarrow$ final state	BR (%)	$N_{20\text{ab}^{-1}}$	
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	22207	
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.20	8328	
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.31	7297	\rightarrow Fuks, Kim, Lee, 1510.07697,
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.58	1824	Fuks, Kim, Lee, 1704.04298.
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.98	1128	
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.90	1041	\rightarrow Kilian, Sun, Yan, Zhao, Zhao, 1702.03554.
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.69	799	
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.23	263	\rightarrow AP, Sakurai, 1508.06524, Chen, Yan, Zhao, Zhao, Zhong, 1510.04013, Fuks, Kim, Lee, 1510.07697.

[AP, Sakurai, 1508.06524]

Singlet model details

$$m_h^2 \equiv \frac{d^2 V}{dh^2} = 2\lambda v_0^2$$

$$m_s^2 \equiv \frac{d^2 V}{ds^2} = b_3 x_0 + 2b_4 x_0^2 - \frac{a_1 v_0^2}{4x_0}$$

$$m_{hs}^2 \equiv \frac{d^2 V}{dhds} = (a_1 + 2a_2 x_0) \frac{v_0}{2}.$$

$$m_{2,1}^2 = \frac{m_h^2 + m_s^2 \pm |m_h^2 - m_s^2| \sqrt{1 + \left(\frac{m_{hs}^2}{m_h^2 - m_s^2}\right)^2}}{2},$$

$$\begin{aligned} h_1 &= h \cos \theta + s \sin \theta \\ h_2 &= -h \sin \theta + s \cos \theta \end{aligned}$$

$$\sin 2\theta = \frac{(a_1 + 2a_2 x_0)v_0}{m_1^2 - m_2^2}$$

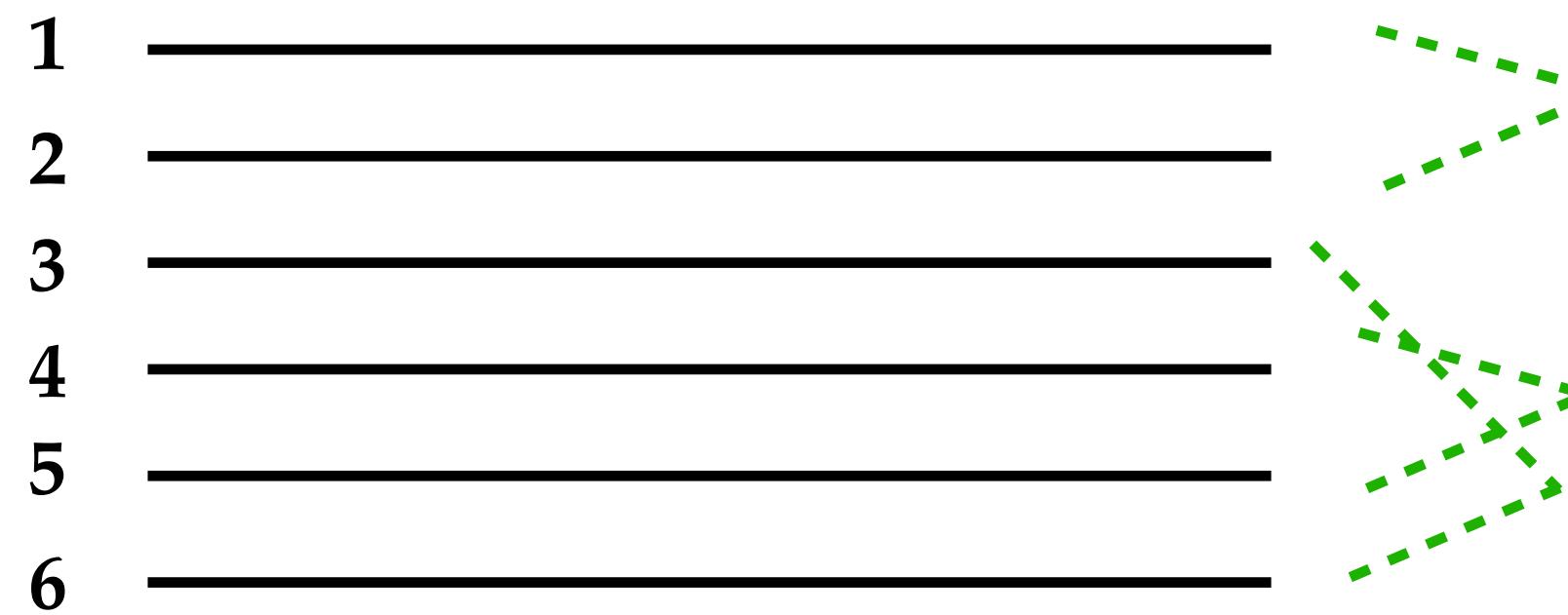
The 6b final state, analysis [[AP](#), Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro,
arXiv:1909.09166]

- What can we learn about the anomalous couplings via hhh at 13.6 TeV?
- Begin by using the 6 b-jet final state!
 1. Require 6 tagged b-jets.

1 _____
2 _____
3 _____
4 _____
5 _____
6 _____

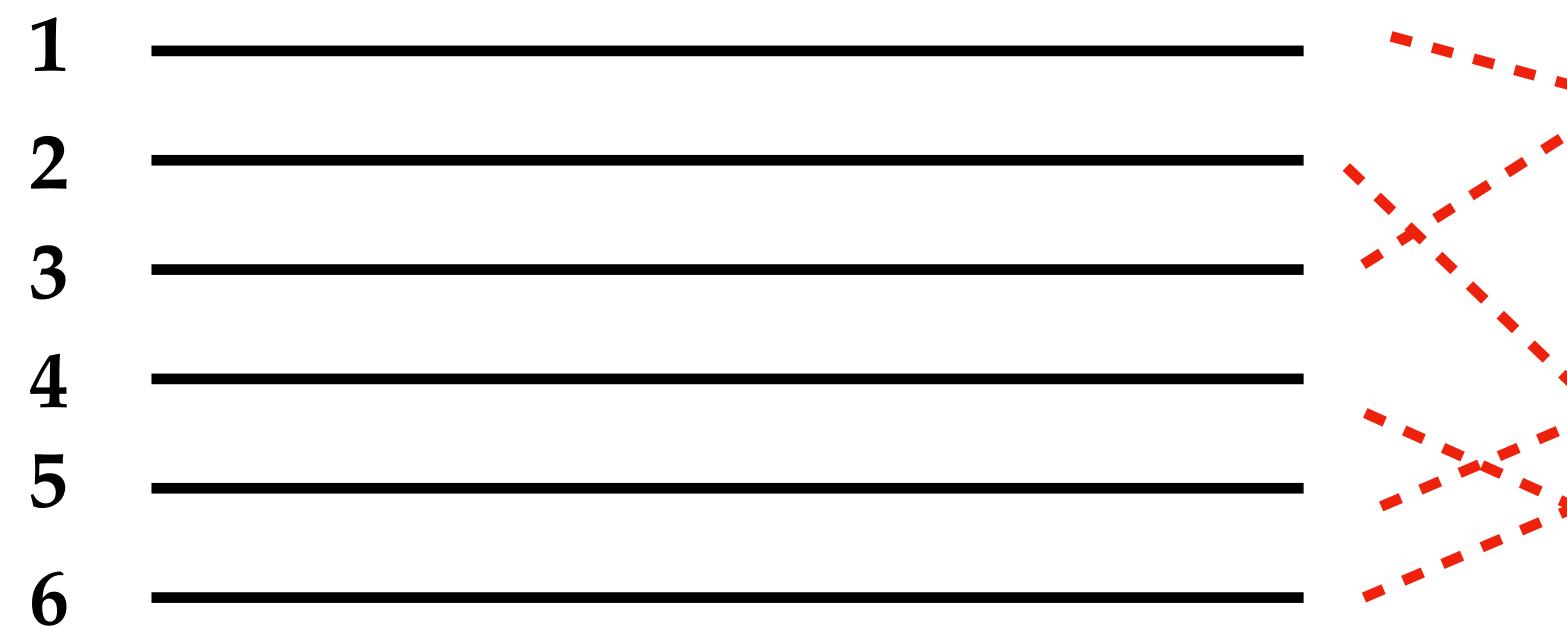
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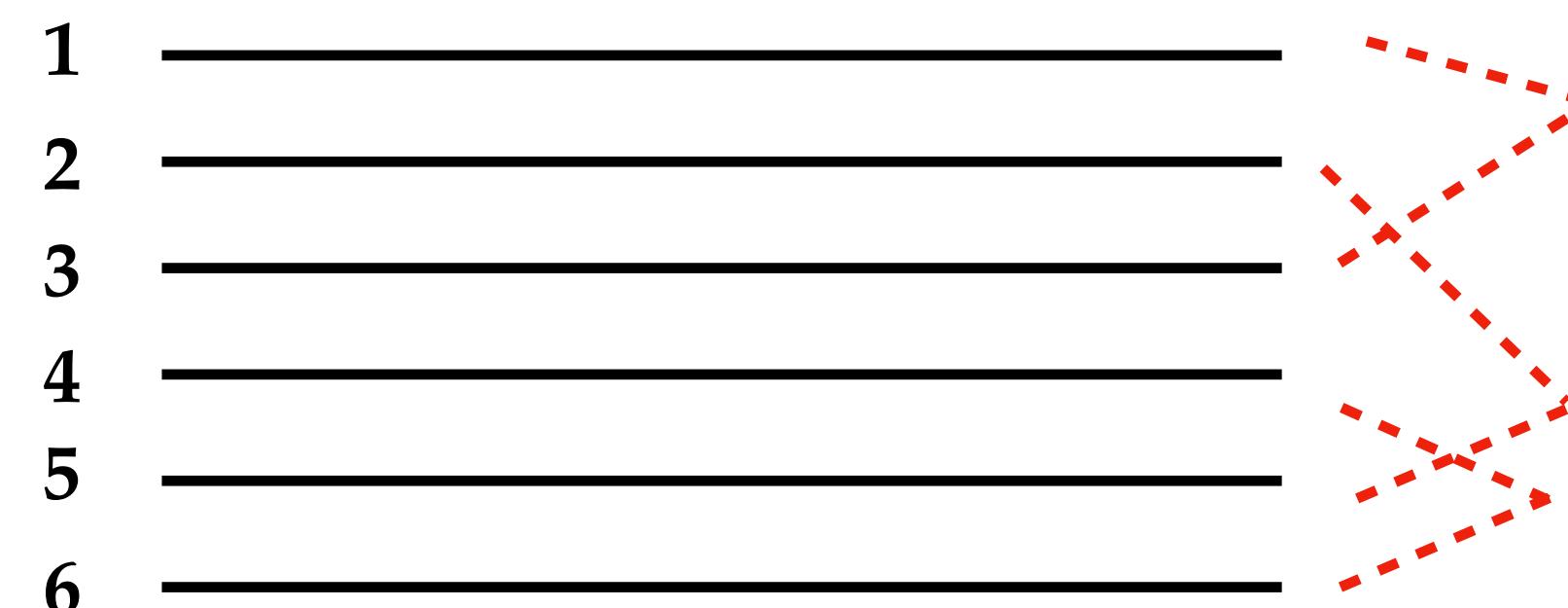


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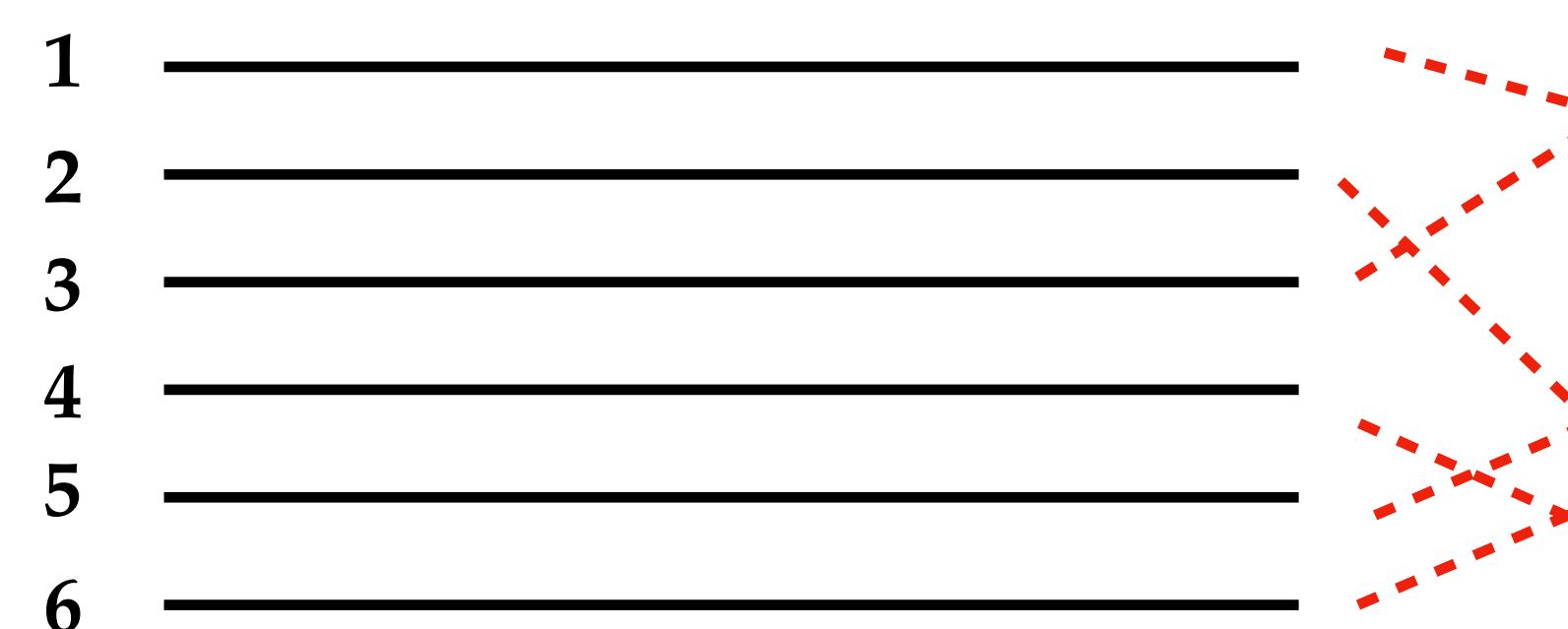
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 3. For each pairing construct:



$$\chi^2 = \sum_{qr \in \text{pairings } I} (M_{qr} - m_h^2)^2$$

≡ sum of squared differences from Higgs mass (~125 GeV)

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⇒ 4. Pairing that gives $\boxed{\min \chi^2}$ determines “reconstructed Higgs boson”.

$$\chi^2_{\min}$$

The 6b final state, analysis

observable	cut	$h_r^i \rightarrow$ Higgs boson candidates
$p_{T,b}$	$> 45 \text{ GeV}$	
$ \eta_b $	< 3.2	
$\Delta R_{b,b}$	> 0.3	
$p_T(h_r^i)$	$> [170, 120, 0] \text{ GeV}, i = 1, 2, 3$	
χ^2_{\min}	$< 17 \text{ GeV}$	
$\Delta m_{\min, \text{ mid, max}}$	$< 8, 8, 11 \text{ GeV}$	 the three terms in χ^2_{\min} .
$\Delta R(h_r^i, h_r^j)$	$< [3.5, 3.5, 3.5], (i, j) = [(1, 2), (1, 3), (2, 3)]$	
$\Delta R_{bb}(h_r^i)$	$< [3.5, 3.5, 3.5], i = 1, 2, 3$	

signal/backgrounds after analysis

Process	σ_{GEN} (pb)	$\sigma_{\text{NLO}} \times \text{BR}$ (pb)	$\epsilon_{\text{analysis}}$	$N_{20 \text{ ab}^{-1}}^{\text{cuts}}$
hh (SM)	2.88×10^{-3}	1.06×10^{-3}	0.0131	278
QCD $(b\bar{b})(b\bar{b})(b\bar{b})$	26.15	52.30	2.6×10^{-5}	27116
$q\bar{q} \rightarrow hZZ \rightarrow h(b\bar{b})(b\bar{b})$	8.77×10^{-4}	4.99×10^{-4}	1.8×10^{-4}	~ 2
$q\bar{q} \rightarrow ZZZ \rightarrow (b\bar{b})(b\bar{b})$	7.95×10^{-4}	7.95×10^{-4}	1.2×10^{-5}	< 1
ggF $hZZ \rightarrow h(b\bar{b})(b\bar{b})$	1.08×10^{-4}	1.23×10^{-4}	$\mathcal{O}(10^{-3})$	~ 2
ggF $ZZZ \rightarrow (b\bar{b})(b\bar{b})$	1.36×10^{-5}	2.73×10^{-5}	2×10^{-5}	$\ll 1$
$h(b\bar{b})(b\bar{b})$	1.46×10^{-2}	1.66×10^{-2}	5.4×10^{-4}	179
$hh(b\bar{b})$	1.40×10^{-4}	9.11×10^{-5}	2.8×10^{-4}	~ 1
$hhZ \rightarrow hh(b\bar{b})$	4.99×10^{-3}	1.61×10^{-3}	7.2×10^{-4}	23
$hZ(b\bar{b}) \rightarrow h(b\bar{b})(b\bar{b})$	9.08×10^{-3}	1.03×10^{-2}	1.4×10^{-4}	29
$ZZ(b\bar{b}) \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$	2.87×10^{-2}	5.74×10^{-2}	1×10^{-5}	11
$Z(b\bar{b})(b\bar{b}) \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$	0.93	1.87	3×10^{-5}	1121
\sum backgrounds				2.8×10^4

Reducible backgrounds

process	σ_{GEN} (pb)	$\sigma_{\text{GEN}} \times \mathcal{P}(6 b - \text{jets})$ (pb)
$(b\bar{b})(b\bar{b})(c\bar{c})$	76.8	0.768
$(b\bar{b})(c\bar{c})(c\bar{c})$	75.6	0.00756
$(c\bar{c})(c\bar{c})(c\bar{c})$	22.5	22.5×10^{-5}
$(b\bar{b})(b\bar{b})(jj)$	1.32×10^4	1.32
$(b\bar{b})(jj)(jj)$	9.79×10^5	0.00979
$(jj)(jj)(jj)$	1.37×10^6	1.37×10^{-6}

c.f. $\sigma_{\text{GEN}}(6b) = 26.15$ pb

↑
applied:
 $\mathcal{P}_{c \rightarrow b} = 0.1$
 $\mathcal{P}_{j \rightarrow b} = 0.01$

⇒ Assuming perfect b-tagging + identical analysis efficiency to QCD 6b:

→ ~10% contribution from reducible backgrounds.

for P(b-tagging) = 0.8:

→ ~30% contribution.

Scalar singlet model self-couplings

triple:

$$\begin{aligned}\lambda_{111} = & \lambda v_0 c_\theta^3 + \frac{1}{4}(a_1 + 2a_2 x_0) c_\theta^2 s_\theta , \\ & + \frac{1}{2} a_2 v_0 s_\theta^2 c_\theta + \left(\frac{b_3}{3} + b_4 x_0 \right) s_\theta^3 ,\end{aligned}$$

$$\begin{aligned}\lambda_{112} = & v_0(a_2 - 3\lambda) c_\theta^2 s_\theta - \frac{1}{2} a_2 v_0 s_\theta^3 \\ & + \frac{1}{2}(-a_1 - 2a_2 x_0 + 2b_3 + 6b_4 x_0) c_\theta s_\theta^2 + \frac{1}{4}(a_1 + 2a_2 x_0) c_\theta^3\end{aligned}$$

$$\begin{aligned}\lambda_{122} = & v_0(3\lambda - a_2) s_\theta^2 c_\theta + \frac{1}{2} a_2 v_0 c_\theta^3 \\ & + (b_3 + 3b_4 x_0 - \frac{1}{2}a_1 - a_2 x_0) s_\theta c_\theta^2 + \frac{1}{4}(a_1 + 2a_2 x_0) s_\theta^3 ,\end{aligned}$$

$$\begin{aligned}\lambda_{222} = & \frac{1}{12} [4(b_3 + 3b_4 x_0) c_\theta^3 - 6a_2 v_0 c_\theta^2 s_\theta \\ & + 3(a_1 + 2a_2 x_0) c_\theta s_\theta^2 - 12\lambda v_0 s_\theta^3] ,\end{aligned}$$

quartic:

$$\begin{aligned}\lambda_{1111} = & \frac{1}{4}(\lambda c_\theta^4 + a_2 c_\theta^2 s_\theta^2 + b_4 s_\theta^4) , \\ \lambda_{1112} = & -\frac{1}{2}[-b_4 + \lambda + (-a_2 + b_4 + \lambda)(2c_\theta^2 - 1)] c_\theta s_\theta , \\ \lambda_{1122} = & \frac{1}{16} \{ a_2 + 3(b_4 + \lambda) \\ & + 3(a_2 - b_4 - \lambda)[(c_\theta^2 - s_\theta^2)^2 - (s_\theta c_\theta)^2] \} , \\ \lambda_{1222} = & \frac{1}{4}[b_4 - \lambda + (-a_2 + b_4 + \lambda)(c_\theta^2 - s_\theta^2)] s_\theta c_\theta , \\ \lambda_{2222} = & \frac{1}{4}(b_4 c_\theta^4 + a_2 c_\theta^2 s_\theta^2 + \lambda s_\theta^4) .\end{aligned}$$

TRSM $hh \rightarrow 6b$ analysis details

Introduce two observables:

$$\chi^{2,(4)} = \sum_{qr \in I} (M_{qr} - M_1)^2$$
$$\chi^{2,(6)} = \sum_{qr \in J} (M_{qr} - M_1)^2$$

→ constructed from different pairings of 4 and 6 b-tagged jets, M_{qr} is the invariant mass of the pairing qr .

TRSM $hh \rightarrow 6b$ analysis details

Label	(M_2, M_3) [GeV]	$< P_{T,b}$ [GeV]	$\chi^{2,(4)} <$ [GeV 2]	$\chi^{2,(6)} <$ [GeV 2]	$m_{4b}^{\text{inv}} <$ [GeV]	$m_{6b}^{\text{inv}} <$ [GeV]
A	(255, 504)	34.0	10	20	-	525
B	(263, 455)	34.0	10	20	450	470
C	(287, 502)	34.0	10	50	454	525
D	(290, 454)	27.25	25	20	369	475
E	(320, 503)	27.25	10	20	403	525
F	(264, 504)	34.0	10	40	454	525
G	(280, 455)	26.5	25	20	335	475
H	(300, 475)	26.5	15	20	352	500
I	(310, 500)	26.5	15	20	386	525
J	(280, 500)	34.0	10	40	454	525

Table 3. The optimised selection cuts for each of the benchmark points within **BP3** shown in table 2. The cuts not shown above are common for all points, as follows: $|\eta|_b < 2.35$, $\Delta m_{\text{min}, \text{med}, \text{max}} < [15, 14, 20] \text{ GeV}$, $p_T(h_1^i) > [50, 50, 0] \text{ GeV}$, $\Delta R(h_1^i, h_1^j) < 3.5$ and $\Delta R_{bb}(h_1) < 3.5$. For some of the points a m_{4b}^{inv} cut is not given, as this was found to not have an impact when combined with the m_{6b}^{inv} cut.

TRSM $hh\rightarrow 6b$ analysis details (Signal vs Bkg)

Label	(M_2, M_3) [GeV]	$\varepsilon_{\text{Sig.}}$	$S _{300\text{fb}^{-1}}$	$\varepsilon_{\text{Bkg.}}$	$B _{300\text{fb}^{-1}}$	$\text{sig} _{300\text{fb}^{-1}}$ (syst.)	$\text{sig} _{3000\text{fb}^{-1}}$ (syst.)
A	(255, 504)	0.025	14.12	8.50×10^{-4}	19.16	2.92 (2.63)	9.23 (5.07)
B	(263, 455)	0.019	17.03	3.60×10^{-5}	8.12	4.78 (4.50)	15.10 (10.14)
C	(287, 502)	0.030	20.71	9.13×10^{-5}	20.60	4.01 (3.56)	12.68 (6.67)
D	(290, 454)	0.044	37.32	1.96×10^{-4}	44.19	5.02 (4.03)	15.86 (6.25)
E	(320, 503)	0.051	31.74	2.73×10^{-4}	61.55	3.76 (2.87)	11.88 (4.18)
F	(264, 504)	0.028	18.18	9.13×10^{-5}	20.60	3.56 (3.18)	11.27 (5.98)
G	(280, 455)	0.044	38.70	1.96×10^{-4}	44.19	5.18 (4.16)	16.39 (6.45)
H	(300, 475)	0.054	41.27	2.95×10^{-4}	66.46	4.64 (3.47)	14.68 (4.94)
I	(310, 500)	0.063	41.43	3.97×10^{-4}	89.59	4.09 (2.88)	12.94 (3.87)
J	(280, 500)	0.029	20.67	9.14×10^{-5}	20.60	4.00 (3.56)	12.65 (6.66)

Table 4. The resulting selection efficiencies, $\varepsilon_{\text{Sig.}}$ and $\varepsilon_{\text{Bkg.}}$, number of events, S and B for the signal and background, respectively, and statistical significances for the sets of cuts presented in table 3. A b -tagging efficiency of 0.7 has been assumed. The number of signal and background events are provided at an integrated luminosity of 300 fb^{-1} . Results for 3000 fb^{-1} are obtained via simple extrapolation. The significance is given at both values of the integrated luminosity excluding (including) systematic errors in the background according to Eq. (5.1) (or Eq. (5.2) with $\sigma_b = 0.1 \times B$).

TRSM BP3 Definition

Parameter	Value
M_1	125.09 GeV
M_2	[125, 500] GeV
M_3	[255, 650] GeV
θ_{hS}	-0.129
θ_{hX}	0.226
θ_{SX}	-0.899
v_S	140 GeV
v_X	100 GeV
κ_1	0.966
κ_2	0.094
κ_3	0.239

TRSM BP3 Benchmark Point Info

Label	(M_2, M_3)	Γ_2 [GeV]	Γ_3 [GeV]	$\text{BR}_{2 \rightarrow 11}$ [GeV]	$\text{BR}_{3 \rightarrow 11}$	$\text{BR}_{3 \rightarrow 12}$
A	(255, 504)	0.086	11	0.55	0.16	0.49
B	(263, 455)	0.12	7.6	0.64	0.17	0.47
C	(287, 502)	0.21	11	0.70	0.16	0.47
D	(290, 454)	0.22	7.0	0.70	0.19	0.42
E	(320, 503)	0.32	10	0.71	0.18	0.45
F	(264, 504)	0.13	11	0.64	0.16	0.48
G	(280, 455)	0.18	7.4	0.69	0.18	0.44
H	(300, 475)	0.25	8.4	0.70	0.18	0.43
I	(310, 500)	0.29	10	0.71	0.17	0.45
J	(280, 500)	0.18	10.6	0.69	0.16	0.47

Table 5. The total widths and new scalar branching ratios for the parameter points considered in the analysis. For the SM-like h_1 , we have $M_1 = 125$ GeV and $\Gamma_1 = 3.8$ MeV for all points considered. The other input parameters are specified in table 1. The on-shell channel $h_3 \rightarrow h_2 h_2$ is kinematically forbidden for all points considered here.

Monte Carlo Implementation of Anomalous Couplings

- Get the **MG5_aMC** model at: https://gitlab.com/apapaefs/multihiggs_loop_sm.
- [A patch to **MG5_aMC** to enable **Loop × Tree** is included].
- Can generate events either at:
 - **SM**^{^2} + interference of [**SM** × **One**-Insertion diagrams], i.e.:
$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{SM}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{1\text{-ins.}}\} \propto 1 + c_i$$
 - or
 - **SM**^{^2} + interference of [**SM** × **One or Two** insertion diagrams] + [**One Insertion**]^{^2}, i.e.:
$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{SM}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{1\text{-ins.}}\} + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{2\text{-ins.}}\} + |\mathcal{M}_{1\text{-ins.}}|^2 \propto 1 + c_i + c_j c_k + c_\ell^2$$

