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THE N-JETTINESS SOFT FUNCTION AT 2-LOOP ORDER

Rudi Rahn

Based on [2312.11626] with Guido Bell, Bahman Dehnadi, and Tobias Mohrmann

Nikhef theory seminar, 25th April 2024

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OUTLINE

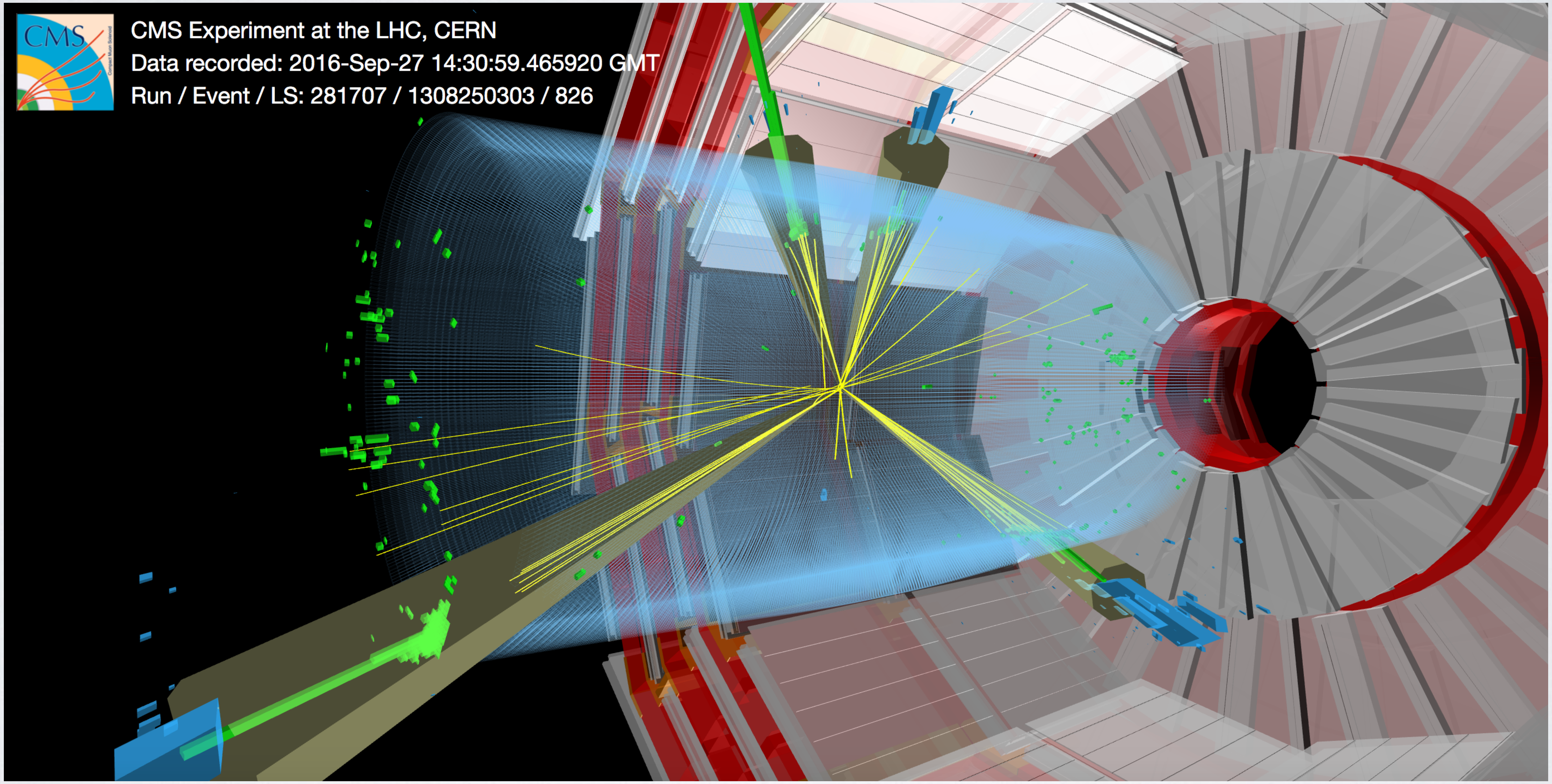
- Background:
  N-jettiness and the Method of Regions

- Calculation:
  SoftSERVE with N jets

- Results

- Endpoints

HOW MANY JETS ARE THERE?



N-JETTINESS

[Stewart, Tackmann, Waalewijn, '10]

- General definition:

$$\mathcal{T}_N = \sum_m \min_{\{q_j\}} \frac{2q_j \cdot k_m}{Q_j}$$

k_i^μ : Momenta of particles in the event

q_j^μ : Momenta of signal jets/beams

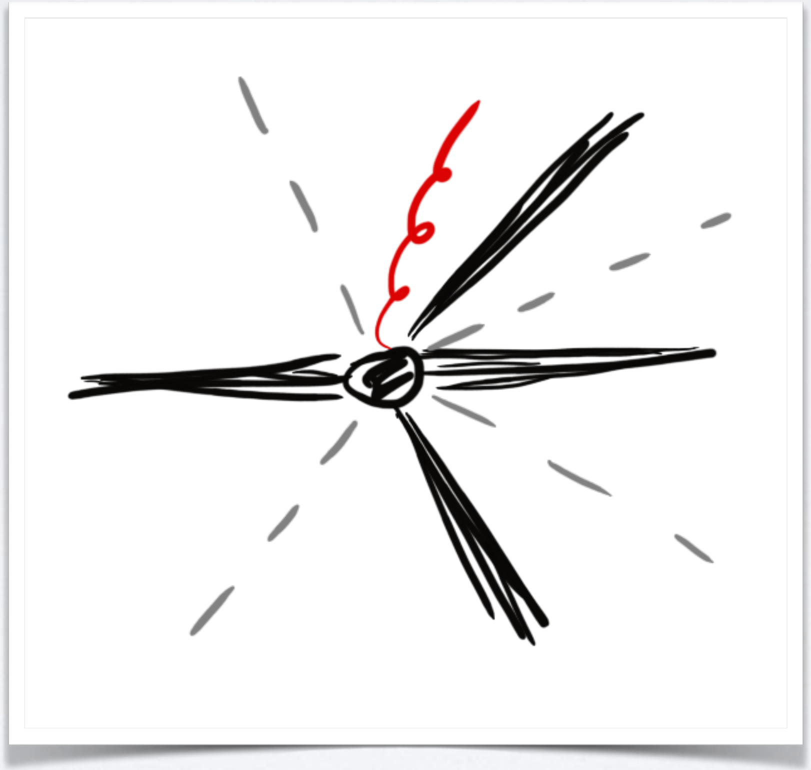
Q_j : Weight factors

- We choose $Q_j = 2E_j$, then $n_j = (1, \vec{n}_j)$, and

$$\mathcal{T}_N = \sum_m \min_j n_j \cdot k_m$$

FRAMES

- The minimisation splits the detector into sectors:

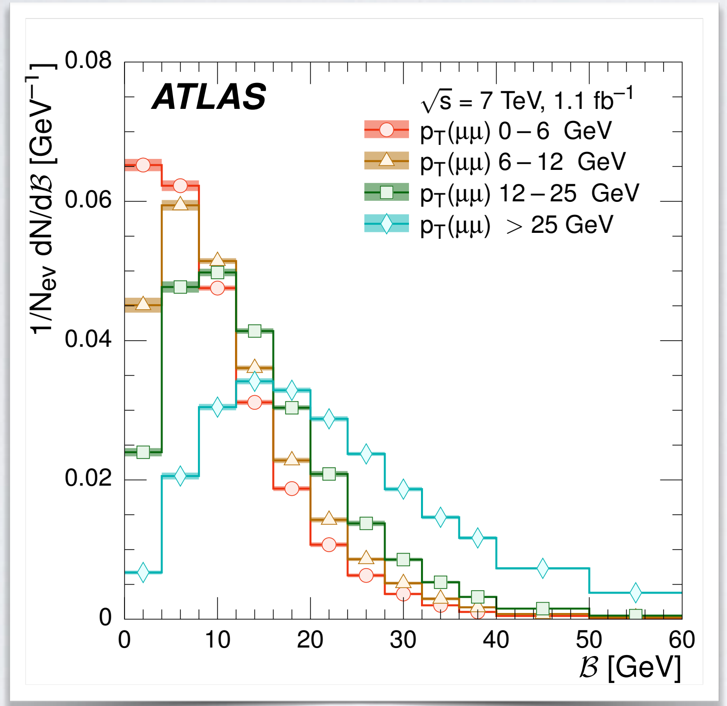


- Choices of Q_j are related by boosts

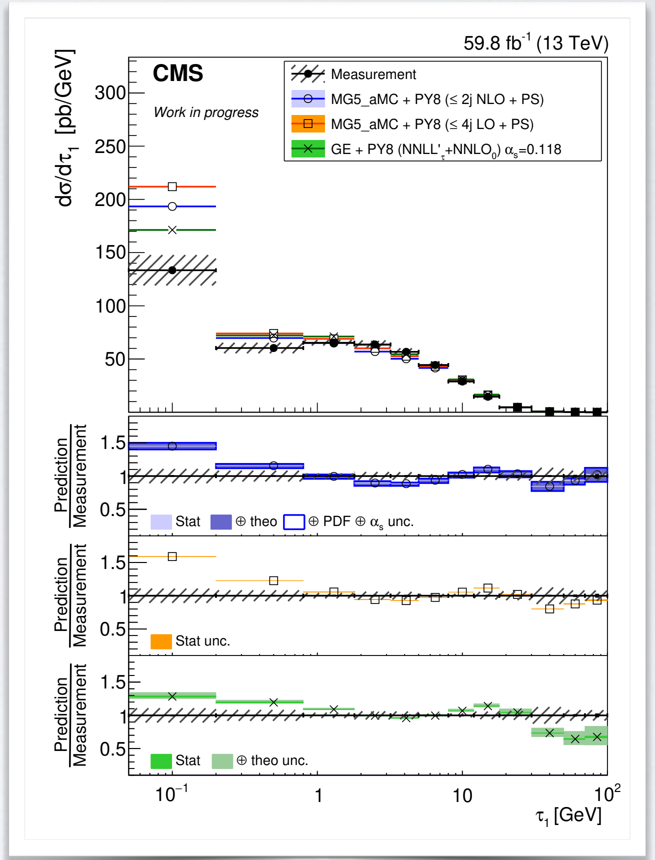
$\Rightarrow$ Frame dependent!

UN-OBSERVED OBSERVABLE

- Few measurements



[Aad et al, '16]



[Mijušković, '22]

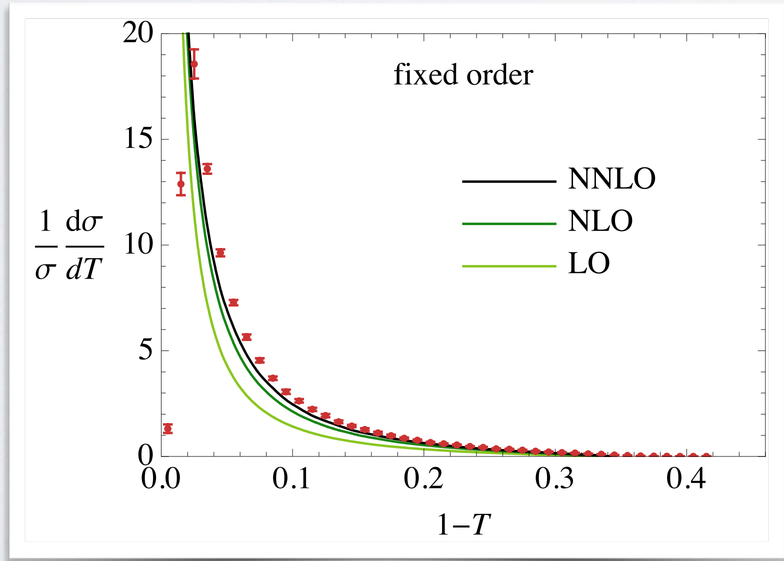
- Reason: underlying event, pileup

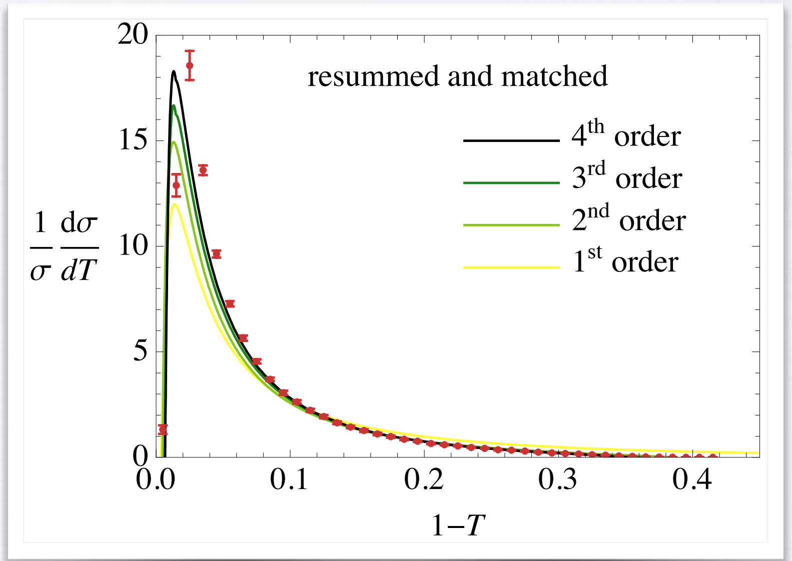
- Modifications: Leptons (Thrust), DIS, tracks,…

THRUST

[Farhi, '77]

- Lepton collider observable $$\tau = Q(1-T) = \sum_m \min(n\cdot k_m, \bar{n}\cdot k_m)$$ $$n = (1, \vec{n}_T) \quad \bar{n} = (1, -\vec{n}_T)$$
- Exhibits large logarithms: $\alpha_s^n \ln^{2n} \tau$
- These can be resummed to all orders





[Becher, Schwartz, '08]

PHASE SPACE SLICING

[Boughezal, Focke, Liu, Petriello, '15]
[Gaunt, Stahlhofen, Tackmann, Walsh, '15]

- How to get finite NNLO cross section predictions?

$$\sigma_{VV} + \sigma_{RV} + \sigma_{RR} = \text{finite}$$

- *Slice* phase space to add unresolved reals to virtuals

- N-jettiness to resolve additional emissions

$$\sigma_{\rm NNLO} = \sigma_{\rm NNLO}(\mathcal{T}_N < \mathcal{T}_N^{\rm cut}) + \sigma_{\rm NLO+1}(\mathcal{T}_N > \mathcal{T}_N^{\rm cut})$$

approximate using small $\mathcal{T}_N^{\rm cut}$

THE METHOD OF REGIONS

[Beneke, Smirnov, '97]

- Isolates dominant singular contributions to integrals

- Toy example:

[Becher, Broggio, Ferroglia, '14]

$$I = \int_0^\infty \mathrm{d}k \, \frac{k}{(k^2+m^2)(k^2+M^2)}$$

$$m \ll M$$

- Analytically:

$$I = \frac{\ln \frac{m}{M}}{m^2 - M^2} \approx -\frac{1}{M^2} \ln \frac{m}{M}$$

- Try to expand before integrating:

$$I \neq \int_0^\infty \mathrm{d}k \, \frac{1}{k(k^2+M^2)}$$

INTRODUCE REGIONS

- Use cutoff / factorisation scale: $m \ll \Lambda \ll M$

$$[0,\infty[= [0,\Lambda[\cup [\Lambda,\infty[$$

"Soft"
"Hard"

$$I_S = \int_0^\Lambda \mathrm{d}k \frac{k}{(k^2+m^2)M^2} \qquad I_H = \int_\Lambda^\infty \mathrm{d}k \frac{k}{k^2(k^2+M^2)}$$

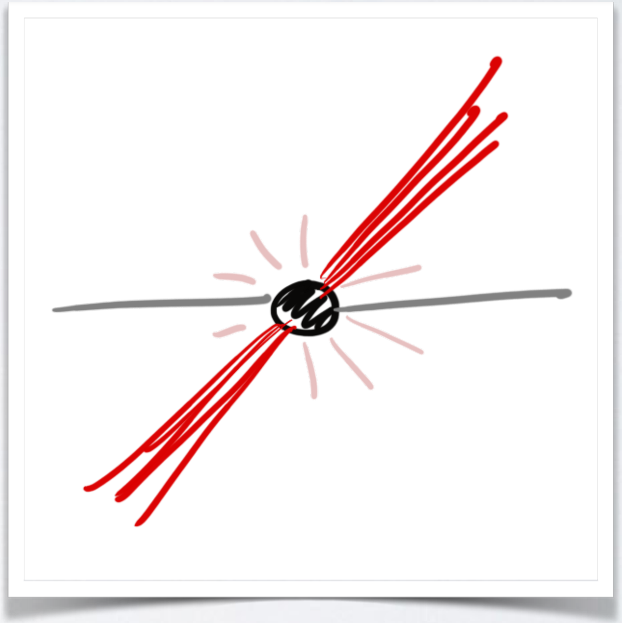
- Solve and add:

$$I_S + I_H = \frac{\ln\frac{\Lambda^2}{m^2}}{2M^2} + \frac{\ln\frac{M^2}{\Lambda^2}}{2M^2} = -\frac{1}{M^2}\ln\frac{m}{M}$$

MODES FOR N-JETTINESS

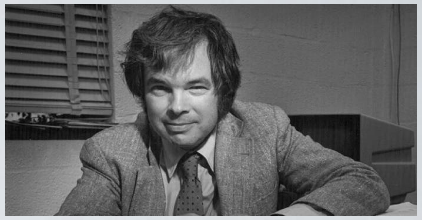
- Look at thrust, for small τ $$\tau = \sum_m \min(n \cdot k_m, \bar{n} \cdot k_m)$$
- Possible modes: $n \cdot k$ or $\bar{n} \cdot k$ must be of $\mathcal{O}(\tau)$:

	$(n \cdot k, \bar{n} \cdot k, k_\perp)$
$\bar{n}$-collinear	$(Q, \tau, \sqrt{Q\tau})$
n-collinear	$(\tau, Q, \sqrt{Q\tau})$
soft	(τ, τ, τ)



- Virtual corrections: also hard mode (Q, Q, Q) allowed
- For N-Jettiness: N+2 collinear modes

WILSONIAN EFT



[Wilson, '65+]

- Associate each region with a dedicated field

$$\psi(k) = \underbrace{\psi_l(k)}_{\text{Support on } [0,\Lambda]} + \underbrace{\Psi_h(k)}_{\text{Support on } [\Lambda,\infty]}$$

- Perform high energy mode path integral

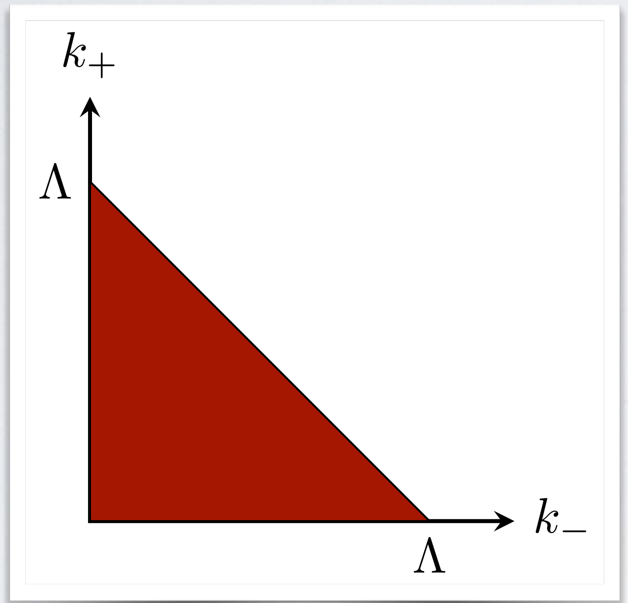
$$\int \mathcal{D}\psi\, e^{\mathrm{i}S[\psi]} = \int \mathcal{D}\psi_l \mathcal{D}\Psi_h\, e^{\mathrm{i}S[\psi_l,\Psi_h]} = \int \mathcal{D}\psi_l\, e^{\mathrm{i}S_{\mathrm{EFT}}[\psi_l]}$$

- High- and low energy contributions factorise

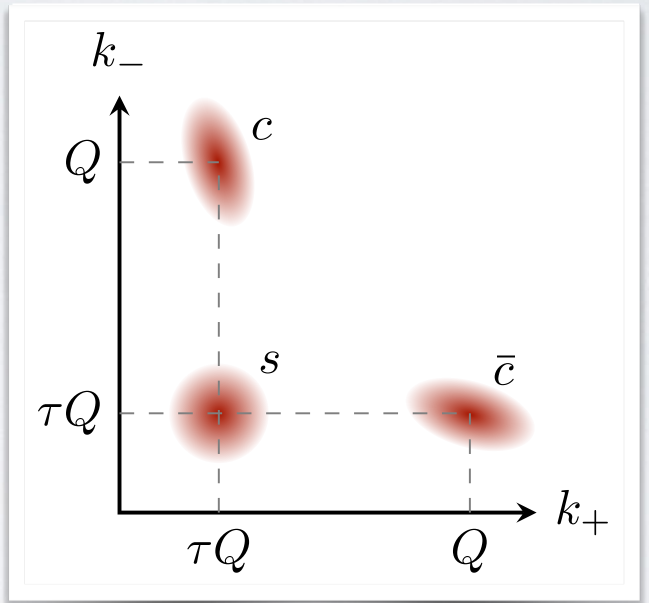
$$\mathcal{M} \approx c_W\, \mathcal{M}_{\mathrm{EFT}}$$

SCET FOR THRUST

- Slightly different mode picture



Classic
Wilsonian EFT



Soft-Collinear
Effective Theory

- Still factorises

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = H(Q^2,\mu)\int \mathrm{d}k_L \mathrm{d}k_R\, J(k_L,\mu) J(k_R,\mu) S(\tau - k_L - k_R,\mu)$$

STATE OF THE ART

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} = \sum_{i,j,\{k_n\}} B_i \otimes B_j \otimes \prod_{n=1}^{N} J_{k_n} \otimes \mathrm{tr}[H_{ij\to\{k_n\}} * S_{ij\to\{k_n\}}] + \mathcal{O}(\mathcal{T}_N)$$

- Hard, beam, and jet functions known to NNLO+
  [Broggio et al., '14]
  [Stewart et al., '10; Berger et al., '11; Gaunt et al., '14; Ebert et al., '20; Baranowski et al. '22]
  [Bauer et al, '04; Bosch et al., '04; Becher et al., '06, '10, '11; Brüser et al., '18; Banerjee et al., '18]

- NLO soft function, NNLO for 0-, 1-, partial 2-jettiness known
  [Jouttenous et al., '11]
  [Kelley et al., '11; Monni et al., '11; Hornig et al., '17]
  [Campbell et al., '18; Boughezal et al., '15]
  [Bell et al., '18; Jin et al., '19]

$\Rightarrow$ Calculate NNLO Soft function for at least N=2

- Recently: recipe for NNLO N-jettiness (confirming/-ed)
  [Agarwal, Melnikov, Pedron, '24]

Soft function
Simulation and
Evaluation of
Real and
Virtual
Emissions

[Bell, RR, Talbert, '18, '20]

- Soft functions can be calculated from

$$S(\tau,\epsilon) = \int \mathrm{d}\Pi_i \, |\mathcal{A}(\{k_i\},\epsilon)|^2 \, \delta(\tau - \tau(\{k_i\}))$$

- Amplitude is divergent, measurement harmless

$\Rightarrow$ Isolate divergences analytically,
do all the rest numerically

QUICK EXAMPLE: NLO

- Matrix element $$|\mathcal{A}(k)|^2 \sim \frac{1}{k\cdot n\;\; k\cdot\bar{n}}$$
- Expose divergences $$k^\mu = k_T\sqrt{y}\,n^\mu + \frac{k_T}{\sqrt{y}}\bar{n}^\mu + \ldots$$
- Divergences exposed: $$\int \mathrm{d}^d k\,|\mathcal{A}|^2 \sim \int \mathrm{d}k_T\,\mathrm{d}y\,k_T^{-1-\epsilon}y^{-1}$$ (Angles and observable omitted)
- Classify observable behaviour:

$$\tau(k) = k_T\,y^{\frac{n}{2}}\;f(y,\{\vartheta\})$$

Leading behaviour

Mass dimension

QUICK EXAMPLE: NLO

- Master formula $$S_0(\tau) = 1 + \left(\frac{Z_\alpha \alpha_s}{4\pi}\right)(\mu^2\bar{\tau}^2)\, S_R(\epsilon) + \mathcal{O}(\alpha_s^2)$$

$$S_R(\epsilon) = \frac{16 C_F e^{-\gamma_E \epsilon}}{\sqrt{\pi}} \frac{\Gamma(-2\epsilon)}{\Gamma(1/2-\epsilon)} \int_0^1 \mathrm{d}t \int_0^1 \mathrm{d}y\, y^{-1+n\epsilon}\, f(y,t)^{2\epsilon} \big(4t(1-t)\big)^{-1/2-\epsilon}$$

- Leads to a (bare) soft function involving integrals of *f*

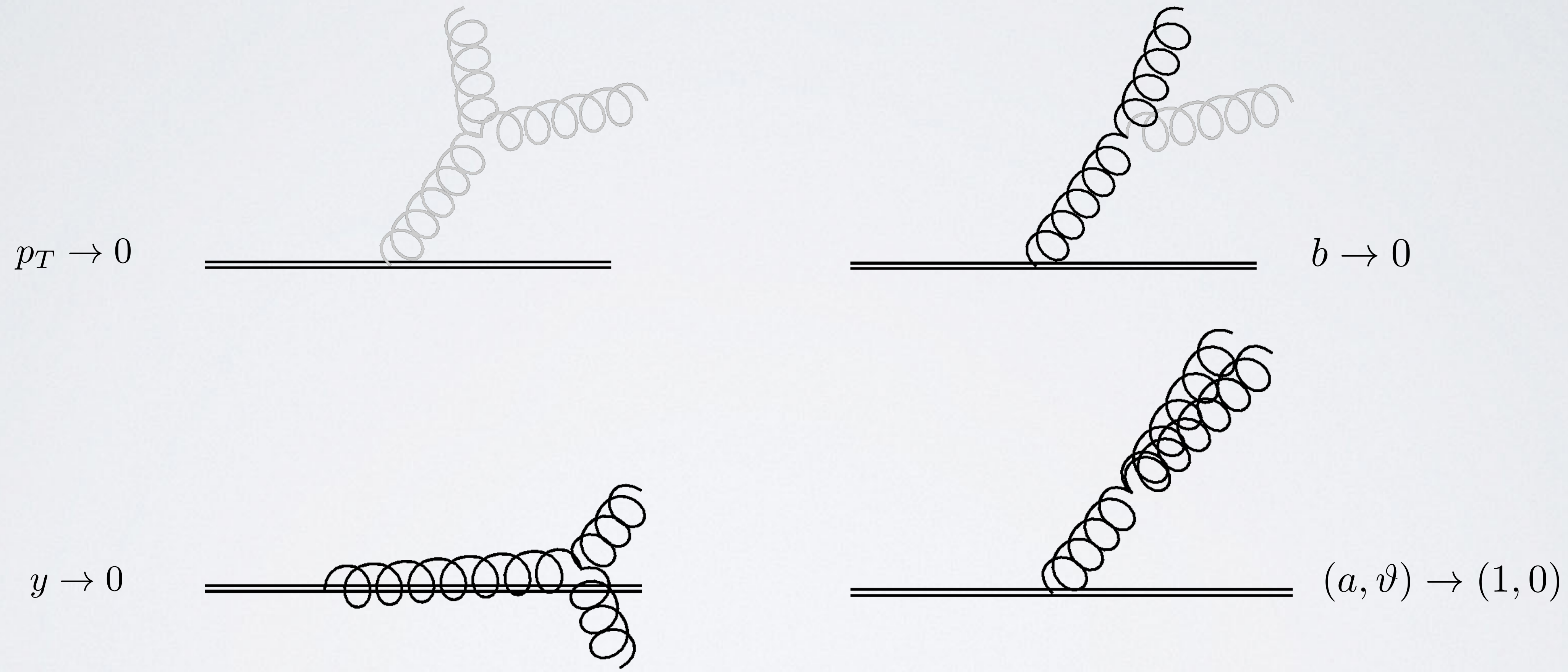
$$S_R(\epsilon) = \frac{-4C_F}{n\epsilon^2} - \frac{8C_F}{n\epsilon}\left[\int_0^1 \mathrm{d}t \frac{\ln f(0,t)}{\pi\sqrt{t(1-t)}} + \ldots\right] + \ldots$$

- SoftSERVE wraps this in C++, and integrates using Cuba

[Hahn, '05]

TWO EMISSIONS

- Four divergence cases:



TWO EMISSIONS

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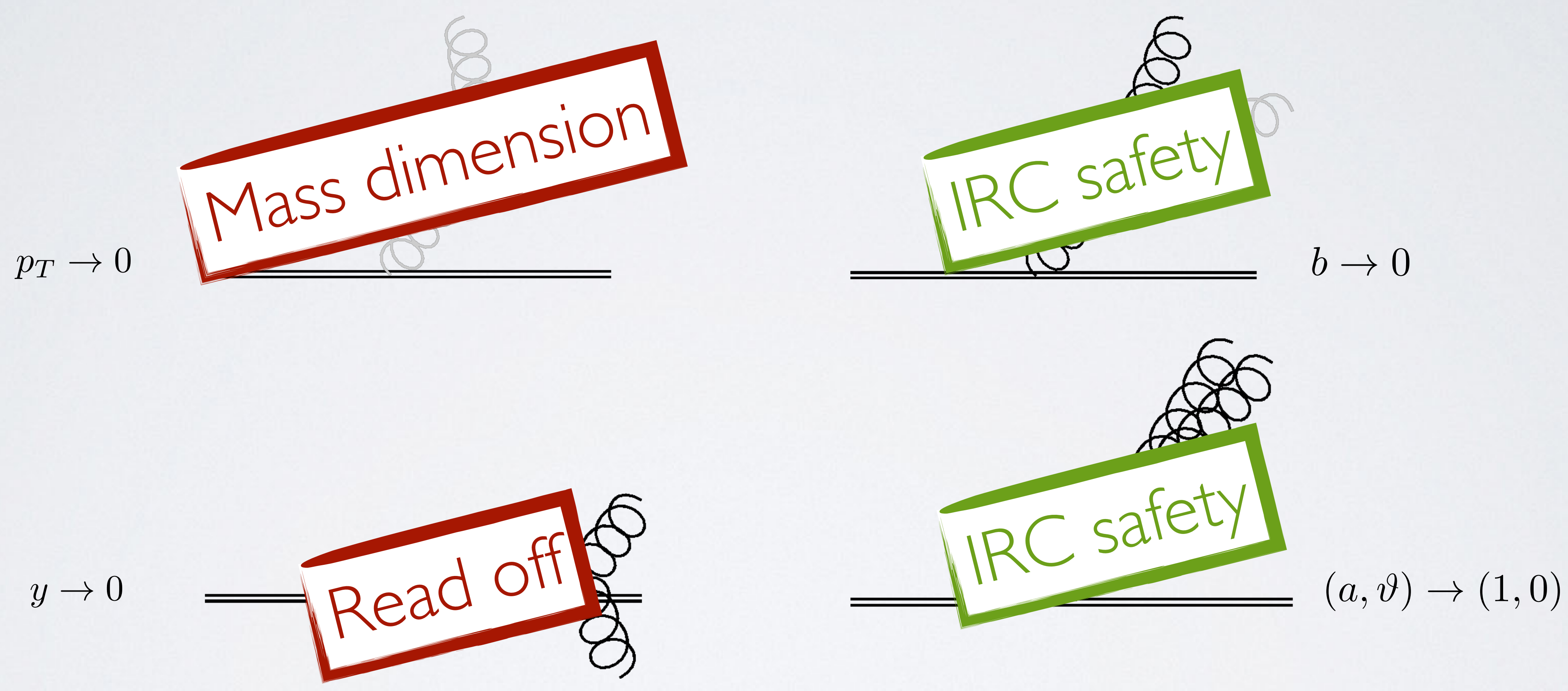
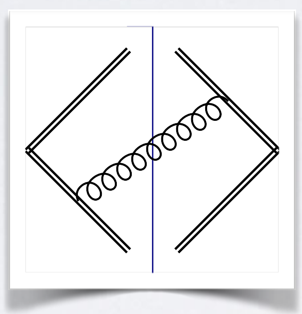


- Observable: $\tau(k,l) = p_T\, y^{\frac{n}{2}}\, F(a,b,y,\{\vartheta\})$

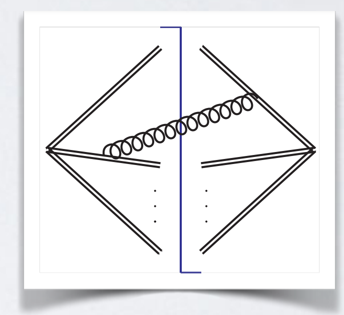
- Extension to N-jet cases:
  - More emitters
  - Nontrivial colour
  - Complicated geometry
  - Integration dimensions

- More emitters

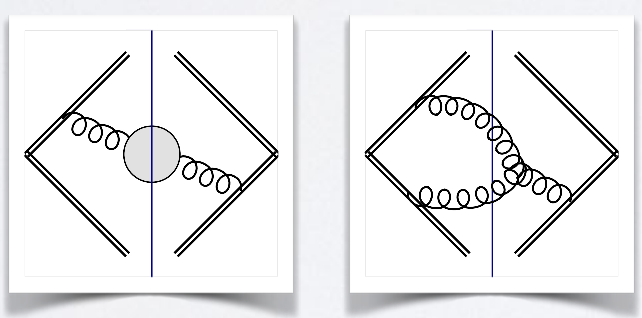
- NLO:

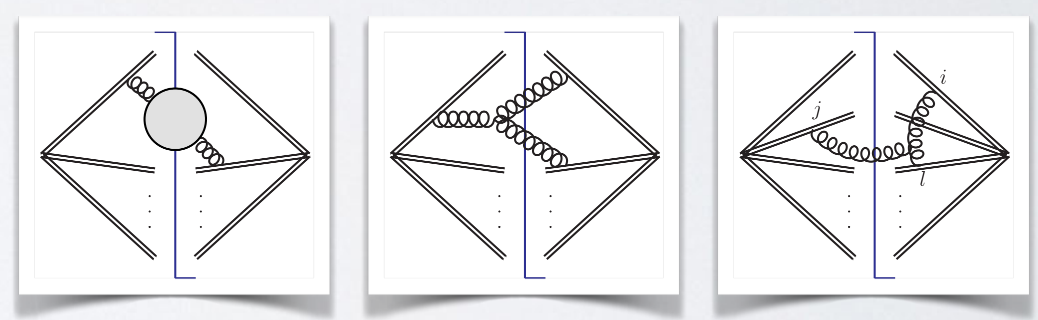


$\Rightarrow$



- NNLO:





- $\frac{(N+2)(N+1)}{2}$ dipoles, $(N+2)(N+1)N$ RV tripoles, many RR tri-/quadupoles

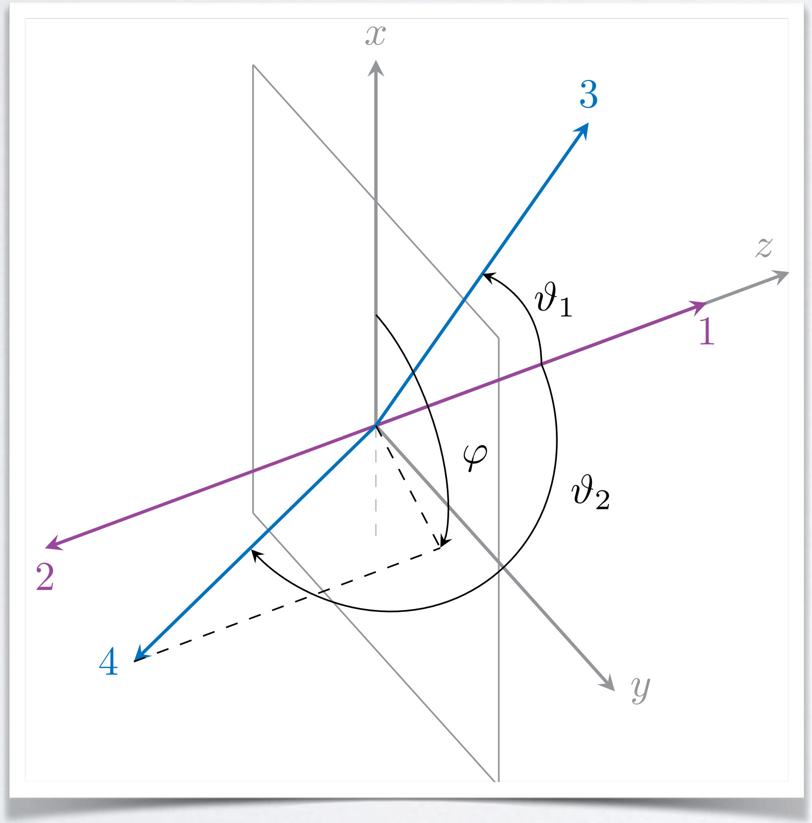
Soft Serve

- Nontrivial colour:
  - Dijet: $\mathbf{S}^{(\mathrm{NLO})} = \mathbf{T}_1 \cdot \mathbf{T}_2 \, S_R = -\mathbf{T}_1^2 \, S_R = -C_F \mathbb{1} \, S_R$
  - From N=2, explicit colour matrices $\mathbf{T}_i \cdot \mathbf{T}_j \, S_{ij}$ appear
    - Tripoles and quadrupoles: e.g. $\mathbf{T}_i^a \cdot \mathbf{T}_j^b \cdot \mathbf{T}_k^a \cdot \mathbf{T}_l^b \cdot \, S_{ijkl}$

      NAE accounts for most, except for real-virtual tripoles

      (They appear as $f_{abc} \mathbf{T}_i^a \mathbf{T}_l^b \mathbf{T}_j^c \, S_{ilj}^{\mathrm{tri}}$)

- Complicated geometry, see e.g. 2-jettiness



1-jettiness: 1 angle
2-jettiness: 3 angles
3-jettiness: 5 angles
N-jettiness: 2N-1 angles

- Grids grow very large

- Integration dimensions, dijet:
    - Three physical dimensions
      (two beams, one transverse reference)
    - One emission can probe 4th dimension,
      Two emissions can probe 4th and 5th dimension:

$$\frac{1}{\Gamma(-\epsilon)}\int_0^1 \mathrm{d}t_5\, t_5^{-1-\epsilon} \sim \mathcal{O}(\epsilon^0)$$

- Integration dimensions, N-jet:

  - Four physical dimensions
    (all the beams and jets)

  - One emission can probe 5th dimension,
    Two emissions can probe 5th and 6th dimension:

$$\frac{1}{\Gamma(-1/2-\epsilon)}\int_0^1 \mathrm{d}t_7\, t_7^{-3/2-\epsilon} \sim \mathcal{O}(\epsilon^0)$$

The “awful angle”

  - For N-jettiness: analytic integration

Soft Serve ALMOND

(Adapted for Wilson Lines in Multiple Occurring n-Directions)

- Added support for nontrivial geometry, RV tripoles, external geometry sampling, cluster deployment

THE CALCULATIONS

- Sample in steps of $\frac{\pi}{25}$, yielding 34476 samples (29904 2-jettiness, rest 0- and 1-jettiness)

- Fourfold symmetry reduces this to 8619

- 24 tripole and 18 dipole programs per point (NLO, C_A, and n_f for 6 dipoles)

- 900GB binaries and log files, 30min per point (NNLO programs, particularly C_A, the slowest)

THE TARGET

- What I'm about to plot:

$$S(\tau,\mu) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\left(\frac{\Gamma_0}{n}\,L_{ij}^2 + \frac{2\gamma_0^S}{n}\,L_{ij} + c_{ij}^{(1)}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2$$

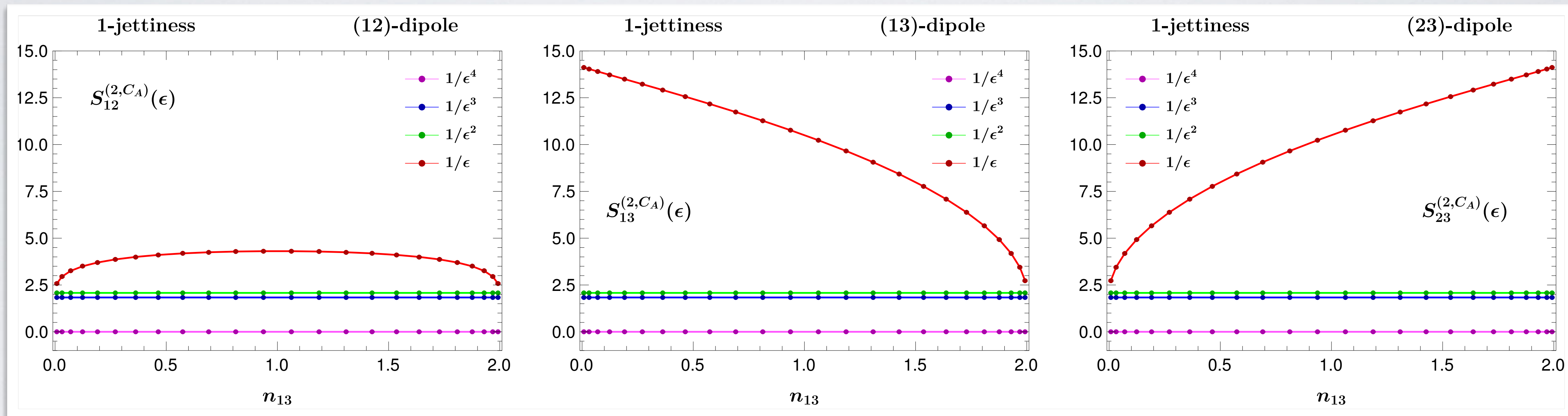
$$\times\Bigg\{\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\left(\frac{2\beta_0\Gamma_0}{3n}\,L_{ij}^3 + \left(\frac{\Gamma_1}{n} + \frac{2\beta_0\gamma_0^S}{n}\right)L_{ij}^2 + 2\left(\frac{\gamma_1^S}{n} + \beta_0 c_{ij}^{(1)}\right)L_{ij} + c_{ij}^{(2)}\right)$$

$$+ 2\pi\sum_{i\neq j\neq k} f_{ABC}\,\mathbf{T}_i^A\,\mathbf{T}_j^B\,\mathbf{T}_k^C\left(\Gamma_0\lambda_{ij}\left(\frac{\Gamma_0}{3n}L_\mu^3 + \left(\frac{\Gamma_0}{n}\tilde{L}_{jk} + \frac{\gamma_0^S}{n}\right)L_\mu^2 + \left(\frac{\Gamma_0}{n}\tilde{L}_{jk}^2 + \frac{2\gamma_0^S}{n}\tilde{L}_{jk} + c_{jk}^{(1)}\right)L_\mu\right) + \tilde{c}_{ijk}^{(2)}\right)$$

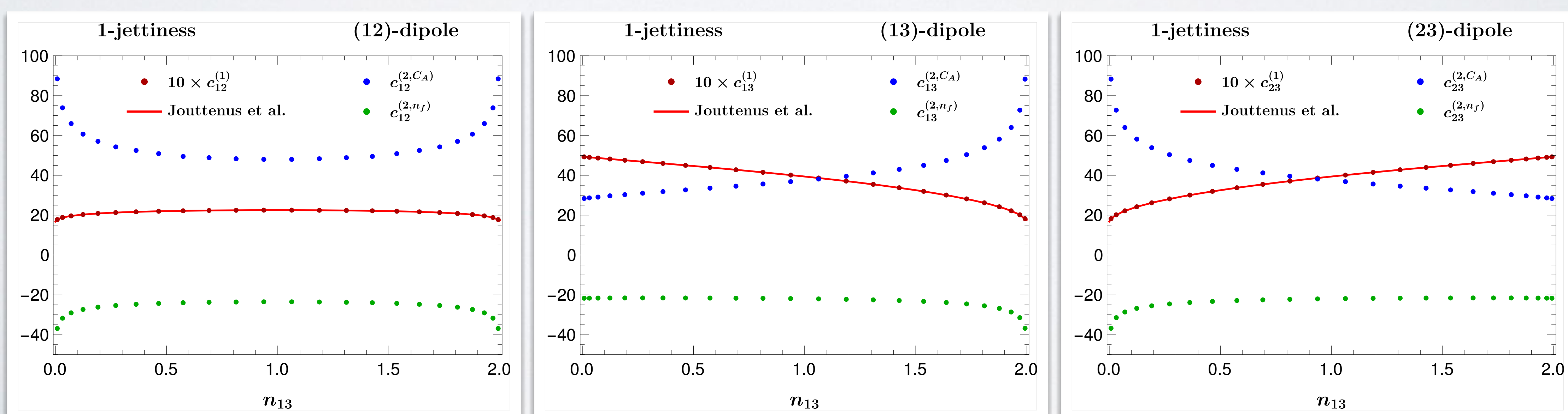
$$+ \frac{1}{4}\sum_{i\neq j}\sum_{k\neq l}\{\mathbf{T}_i\cdot\mathbf{T}_j\,,\,\mathbf{T}_k\cdot\mathbf{T}_l\}\left(\frac{\Gamma_0}{n}\,L_{ij}^2 + \frac{2\gamma_0^S}{n}\,L_{ij} + c_{ij}^{(1)}\right)\left(\frac{\Gamma_0}{n}\,L_{kl}^2 + \frac{2\gamma_0^S}{n}\,L_{kl} + c_{kl}^{(1)}\right)\Bigg\}$$

1-JETTINESS

- Pole cancellations



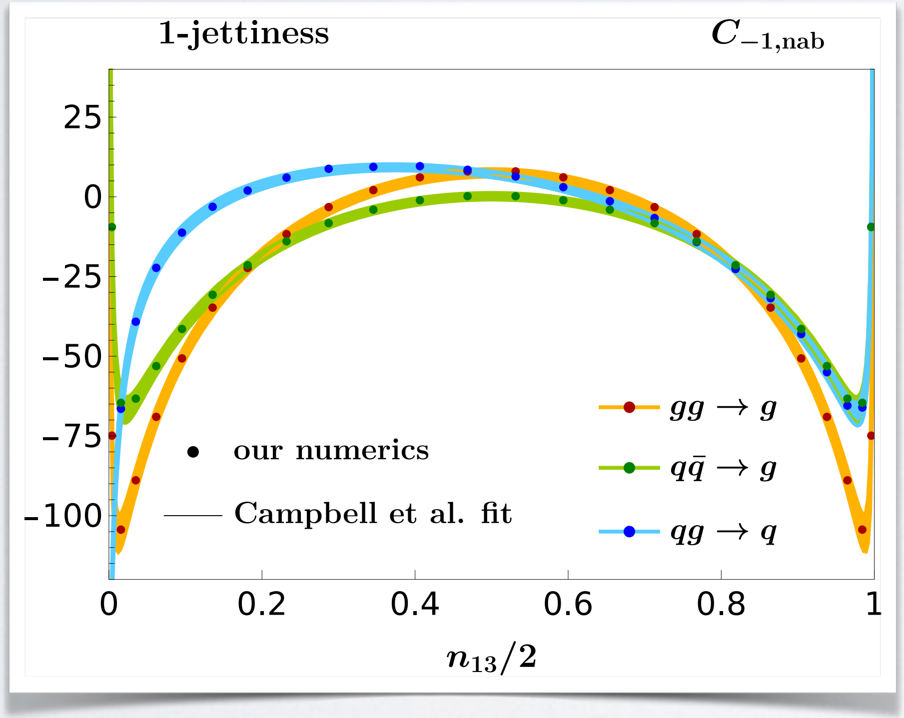
- Finite results

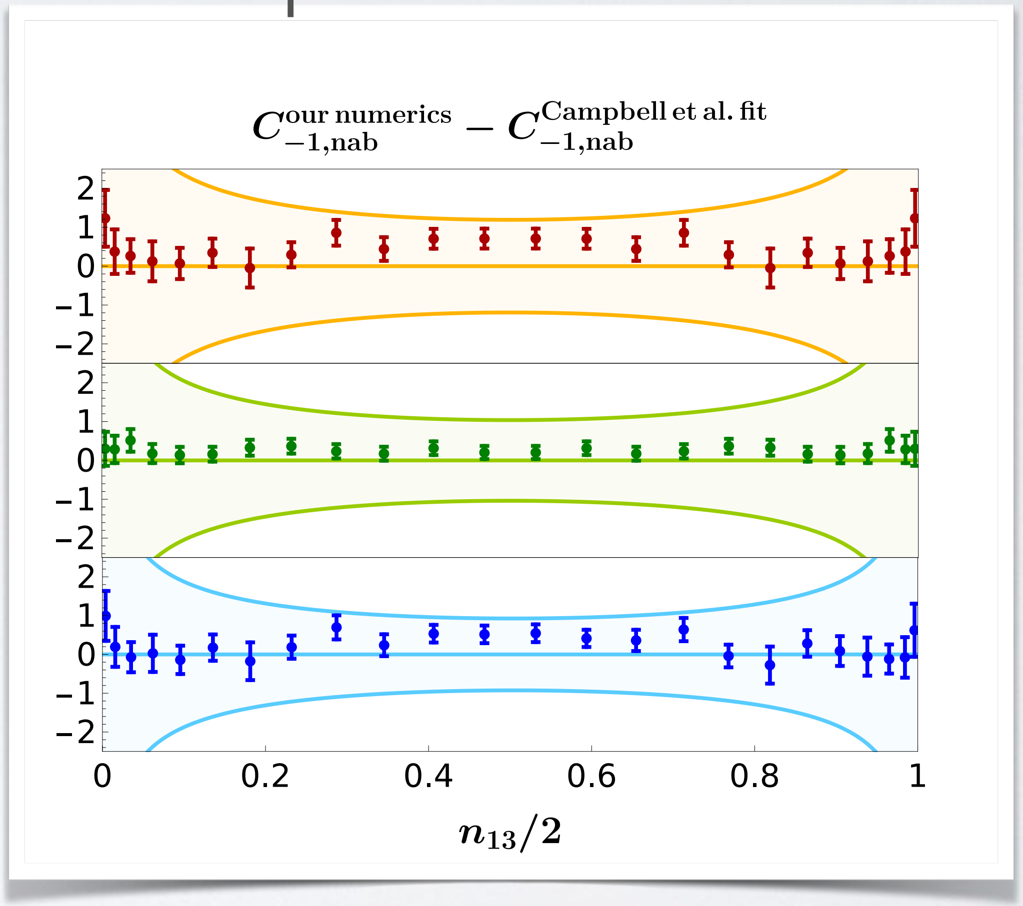


1-JETTINESS COMPARISON

[Campbell, Ellis, Mondini, Williams, '18]

- Compare with result from Campbell et al.

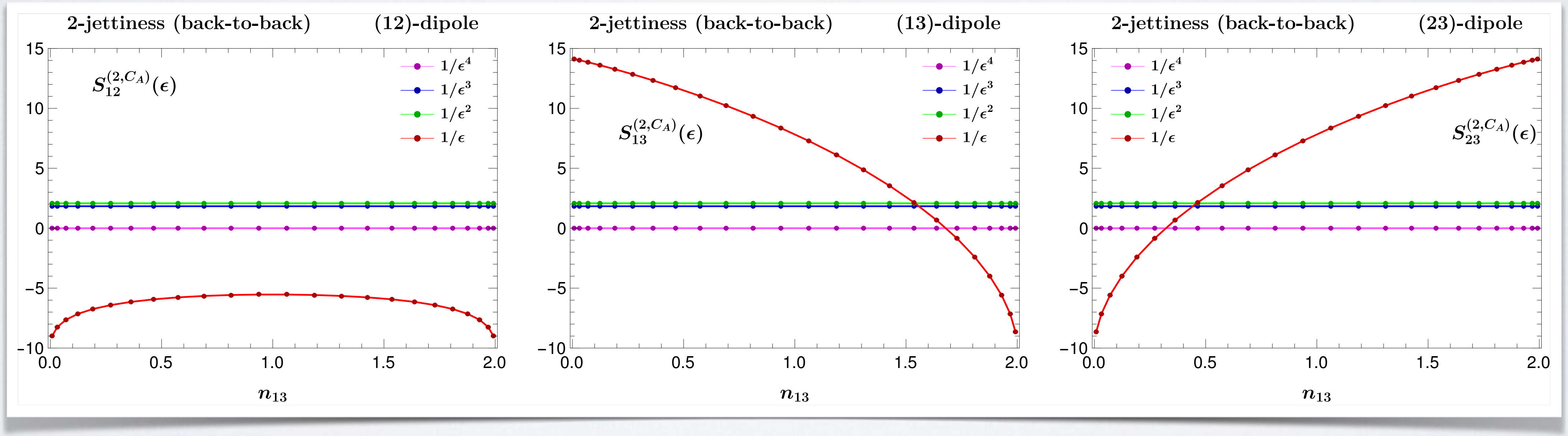




- Only fit functions for physical channels provided

2-JETTINESS - B2B

- Pole cancellations



2-JETTINESS - B2B

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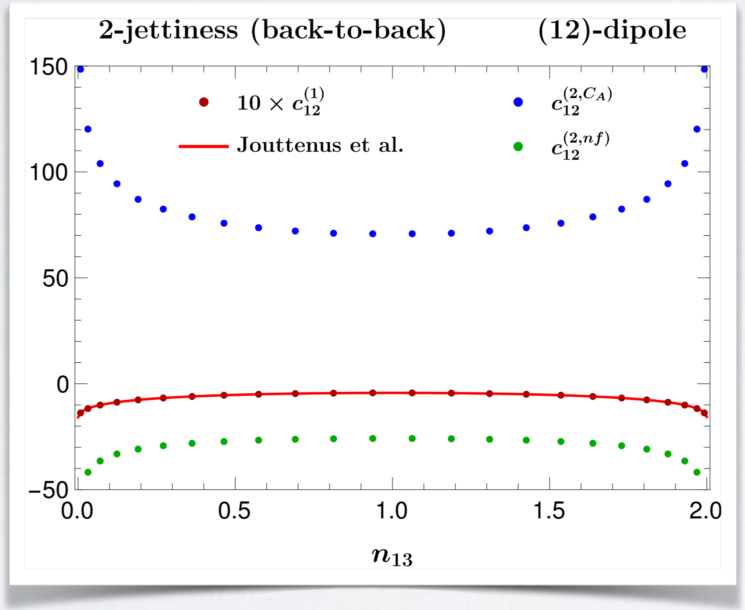
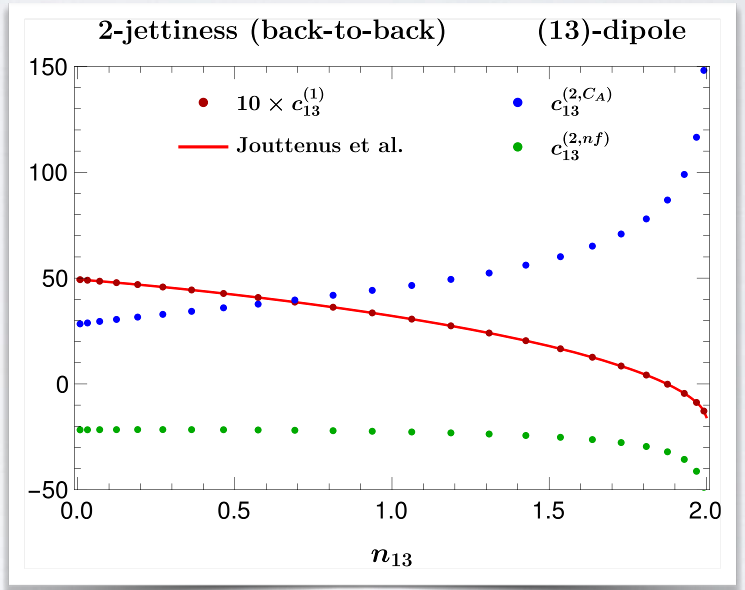
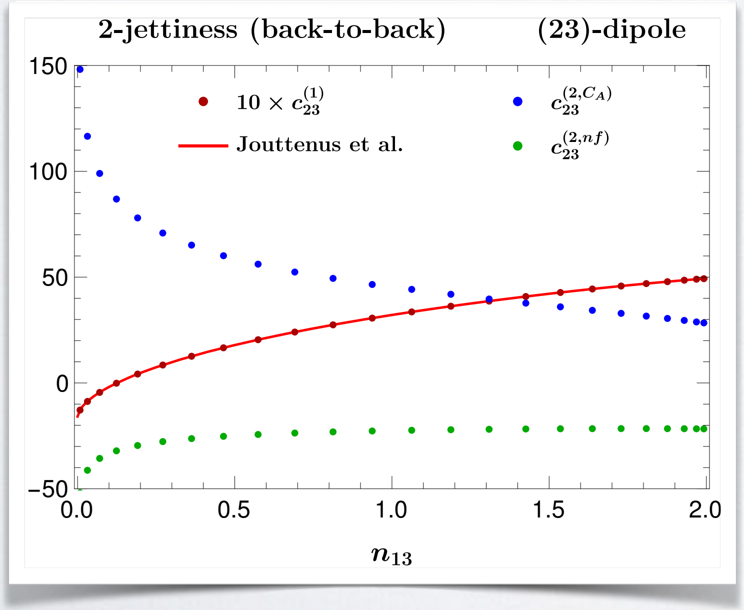
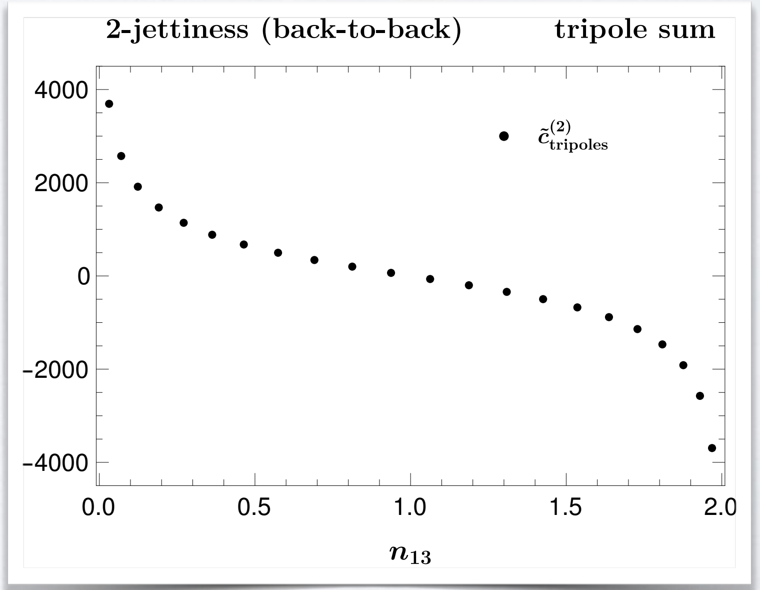
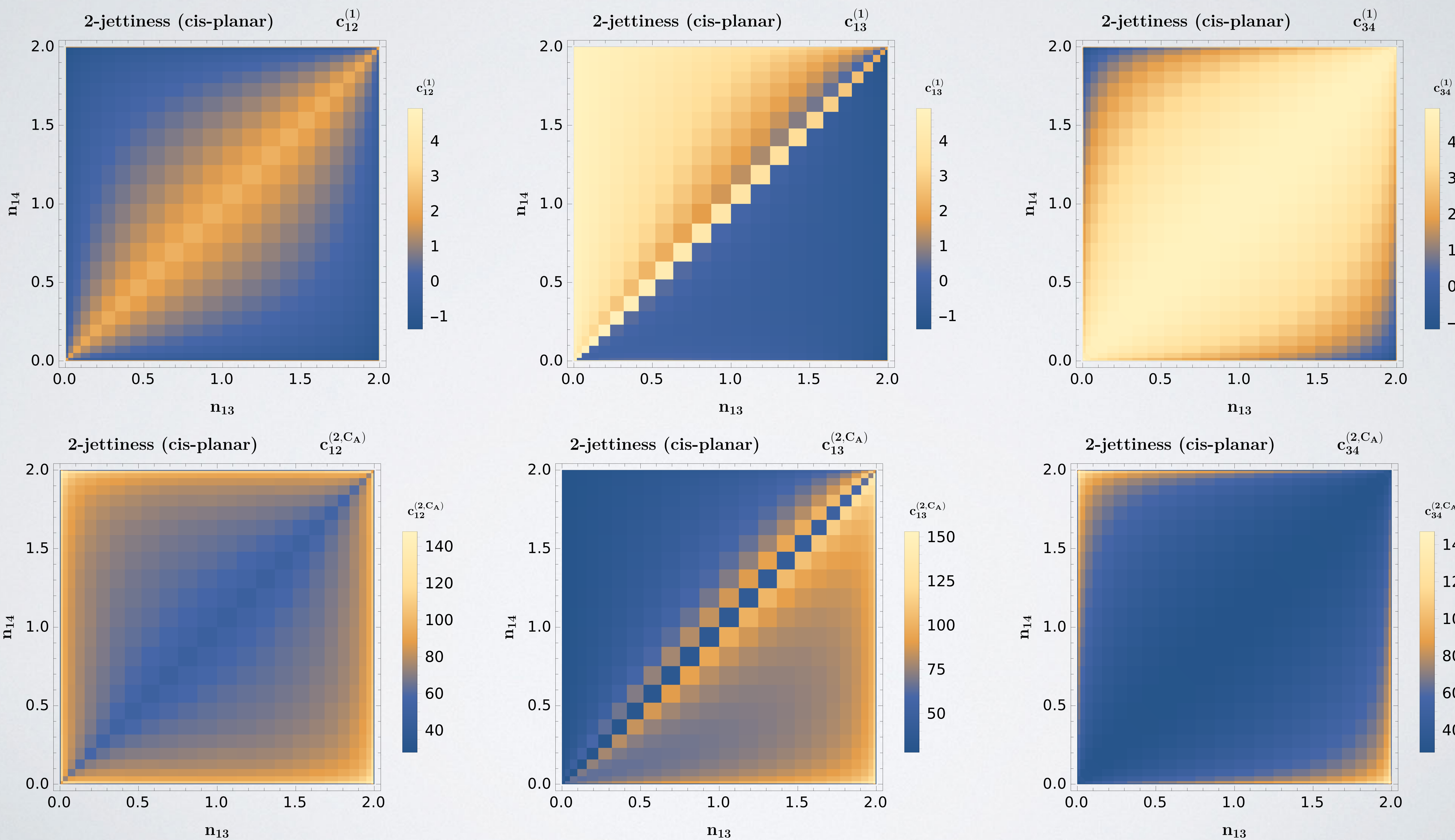






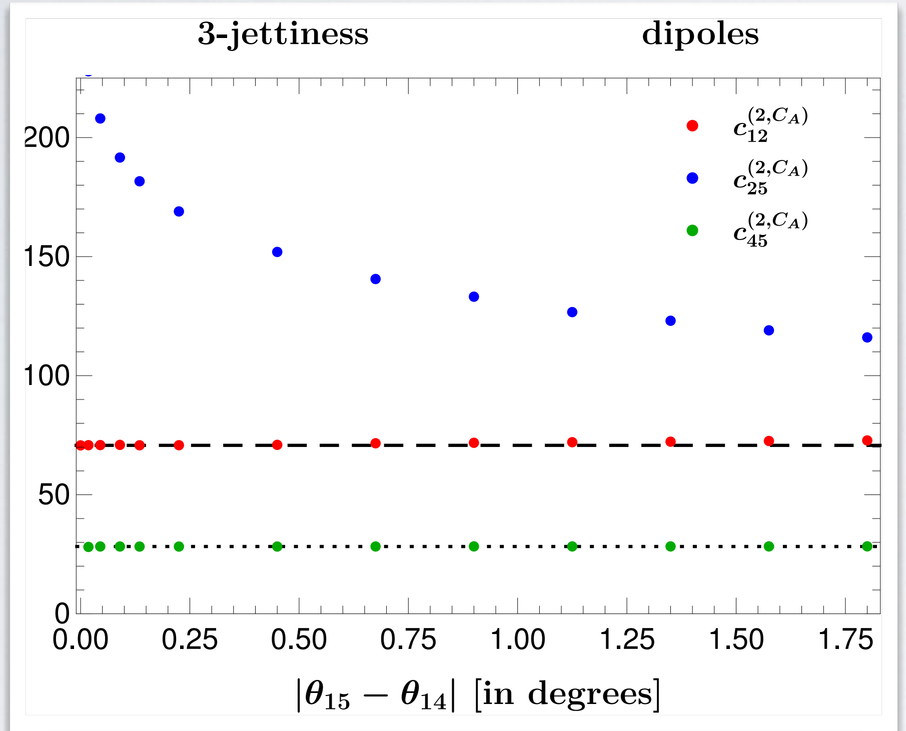


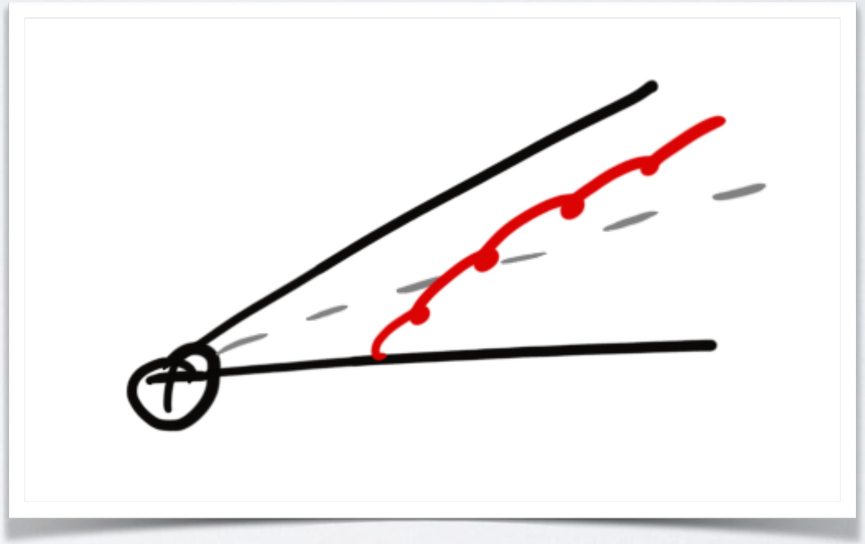
2-JETTINESS - PLANAR



EDGE CASES

- When jets become collinear, some results diverge





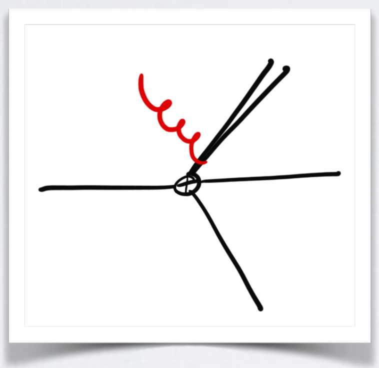
- Non-dipole jet collinear to dipole jet: divergence

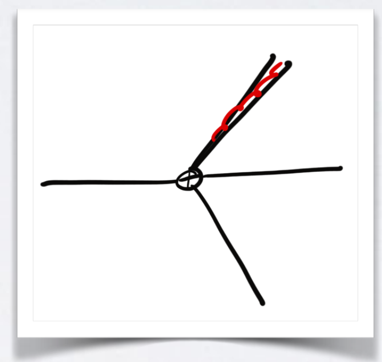
- Origin: contribution from one jet enhanced by collinearity to dipole jet

METHOD OF REGIONS!

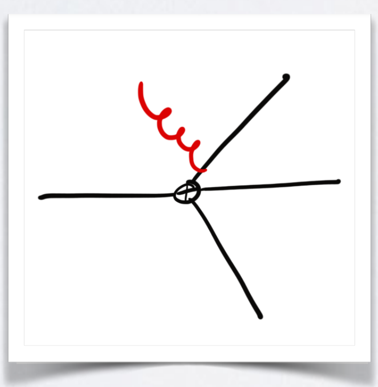
- Two regions:

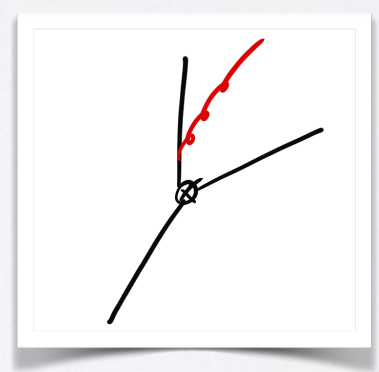
bulk and collinear





$\Downarrow$





$\Downarrow$

(N-1)-jet base + Universal correction

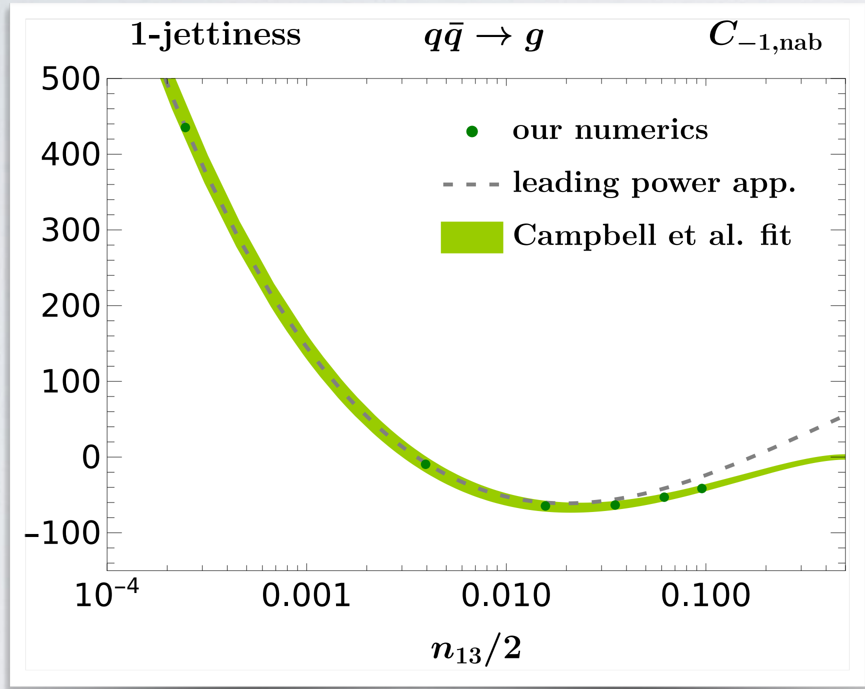
IMPLICATIONS

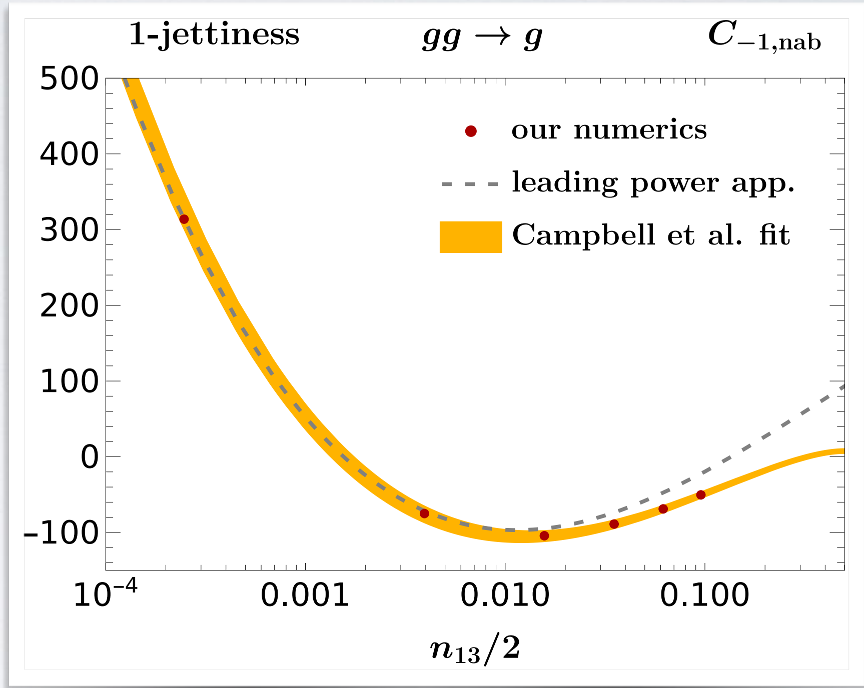
- A kind of refactorisation
  (global soft base and collinear soft correction)
- Pattern should be general
- Correction is observable and dipole dependent
- N-Jettiness:

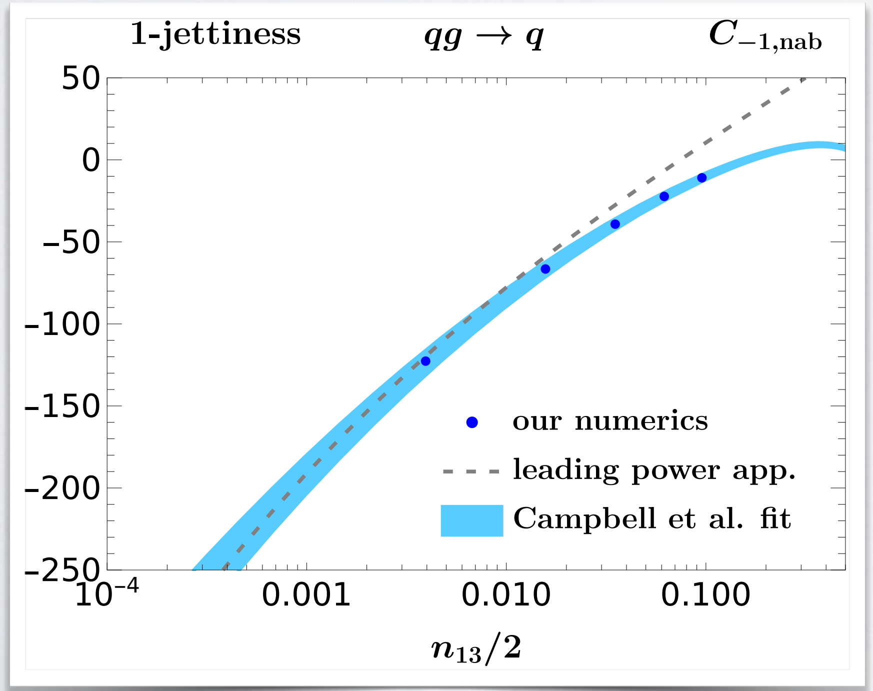
$$c_{ij}^{(1,\mathrm{corr})} = -\frac{\pi^2}{3}$$

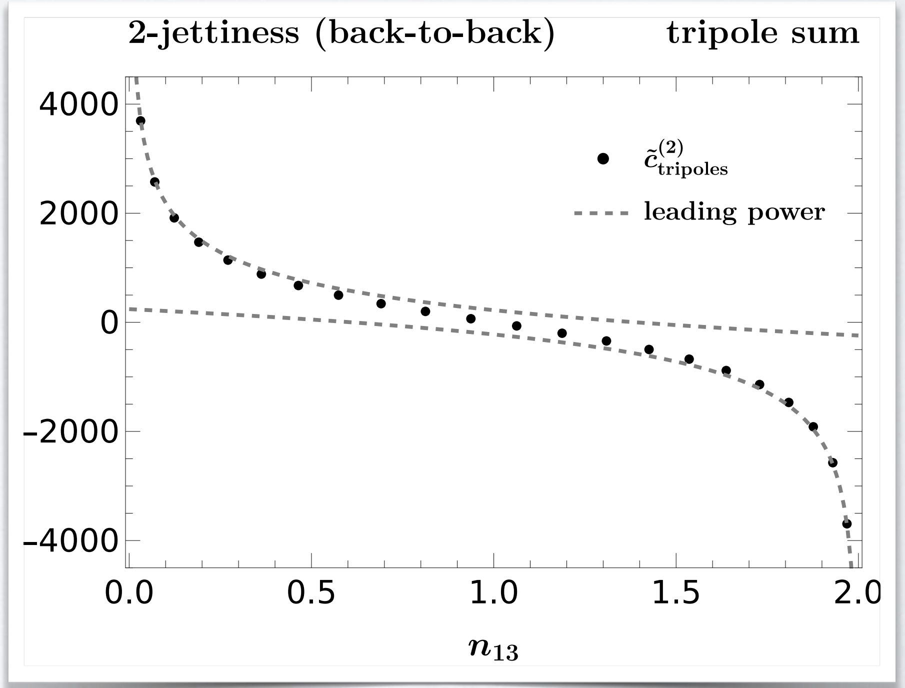
$$c_{ij}^{(2,\mathrm{corr})} = T_F n_f \left(\frac{4\pi^2}{9} \ln\left(\frac{2\delta}{n_{ij}}\right) - 8.023(2) \right) - C_A \left(\frac{11\pi^2}{9} \ln\left(\frac{2\delta}{n_{ij}}\right) - 10.0335(2) \right)$$

CONVERGENCE







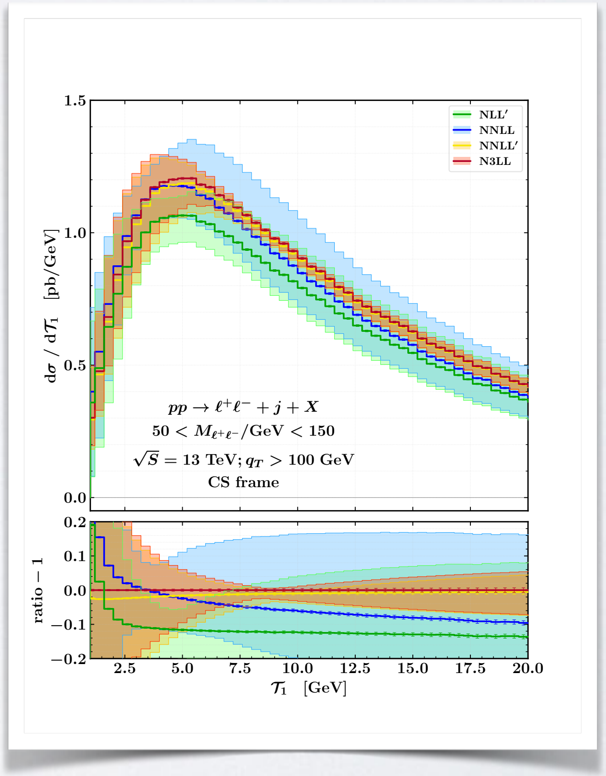


IS THIS USEFUL?

- 1-jettiness at N^3LL for GENEVA
  [Alioli, Bauer, Berggren, Tackmann, Walsh, '15]

- Z+jet production in lab frame vs CS-frame
  [Alioli, Bell, Billis, Broggio, Dehnadi, Lim, Marinelli, Nagar, Napoletano, RR '24]

- For some Q_j, $n_{13} \sim 10^{-12}$ has to be probed



CONCLUSIONS AND OUTLOOK

- We extended the SoftSERVE code to N-jet cases and applied it to the N-jettiness at NNLO
- Derived a grid for the 2-jettiness soft function
- Investigated logarithmic divergences at the edges of phase space using the Method of Regions
- Used our results for an N^3LL resummation
- Next: non-global logarithms, N-gularities, …

THANK YOU!