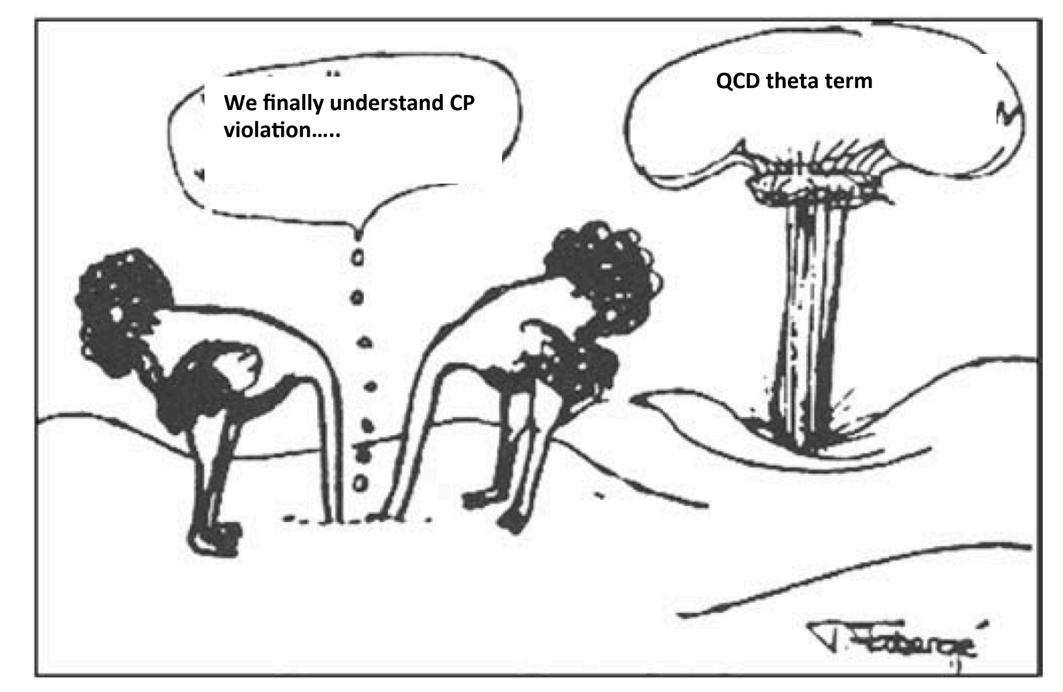
Electric Dipole Moments and the strong CP problem



Jordy de Vries, Nikhef, Amsterdam

Topical Lectures on electric dipole moments, Dec. 14-16

Introductory remarks

- Strong CP violation is a technical subject. **Here outline the main ideas.**
- Start by considering the QED Lagrangian

$$L = \overline{q}(i\gamma^{\mu} D_{\mu} - m_q)q - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad D_{\mu} = \partial_{\mu} - iQ_q A_{\mu}$$

- Why these terms? They are **almost all** gauge-invariant terms (U(1) gauge) with terms up to 'dimension 4'.
- **SM = all renormalizable terms that obey SU(3)xSU(2)xU(1) gauge invariance involving known degrees of freedom (quarks , leptons,..)**

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- However one term is missing.....
- $F_{\mu\nu}$ is a gauge-invariant quantity.... So we could have added a term:

$$\theta \frac{e^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F^{\mu\nu} \equiv \theta \frac{e^2}{32\pi^2} \tilde{F}_{\mu\nu} F^{\mu\nu}$$

$\varepsilon^{\alpha\beta\mu\nu}$

= 4 dimensional Levi-Civita tensor

= 0 if 2 indices are equal

Total derivatives

- So why don't we have this term in QED ?

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- First of all, what does it even describe ?

$$\theta \frac{e^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F^{\mu\nu} \sim \vec{E} \cdot \vec{B}$$

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Parity or Time-reversal: $\vec{E} \cdot \vec{B} \rightarrow -\vec{E} \cdot \vec{B}$

Note: $F_{\mu\nu} F^{\mu\nu} \sim \vec{E}^2 - \vec{B}^2 \rightarrow \vec{E}^2 - \vec{B}^2$

- So it describes a CP-odd interaction! Whoho !
- But.... This term has no physical consequences ! Why ?
- **Cause its a total derivative:** $\varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F^{\mu\nu} = \partial_\mu (\varepsilon^{\alpha\beta\mu\nu} A_\nu F_{\alpha\beta})$

QCD makes life complicated …

- This explains why we do not have a QED theta term
- QCD is more complicated, non-Abelian group, still total derivative

$$\varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} = \partial_{\mu} \varepsilon^{\alpha\beta\mu\nu} (A_{\nu} F_{\alpha\beta} + A_{\alpha} A_{\beta} A_{\nu})$$

- For a long time it was though that this has no consequences as well....

- Now I am going to wave hands .....

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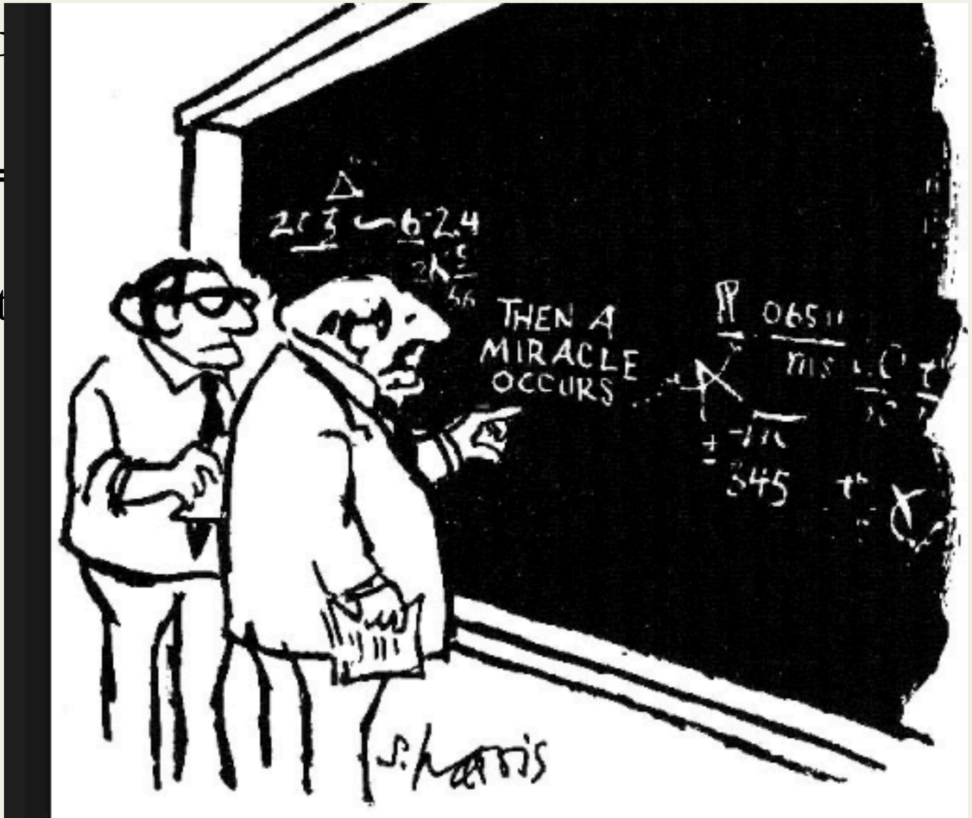

"I think you should be more explicit here in step two."

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- There are instanton solutions where A ~ 1/r only for very large r
- These solutions do not drop off fast enough to ignore the surface terms

$$\int d^4x\, \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} \neq 0$$

- **The QCD theta term has physical consequences !**

The eta-eta' puzzle

- Let us look at 3-flavor QCD

$$L = \sum_{u,d,s} \bar{q}(i\gamma^{\mu} D_{\mu} - m_q)q - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

- The u,d,s quark masses are much smaller than the hadron masses, so let us consider the 'chiral limit' $m_q \rightarrow 0$
- The very simple Lagrangian has a number of **global** symmetries

$$q \rightarrow e^{i\alpha} q$$ U(1) symmetry

$$q \rightarrow e^{i\beta^a \lambda^a} q$$ SU(3) symmetry

$$q \rightarrow e^{i\alpha_5 \gamma^5} q$$ $U_A(1)$ symmetry

$$q \rightarrow e^{i\beta^a \lambda^a \gamma^5} q$$ $SU_A(3)$ symmetry

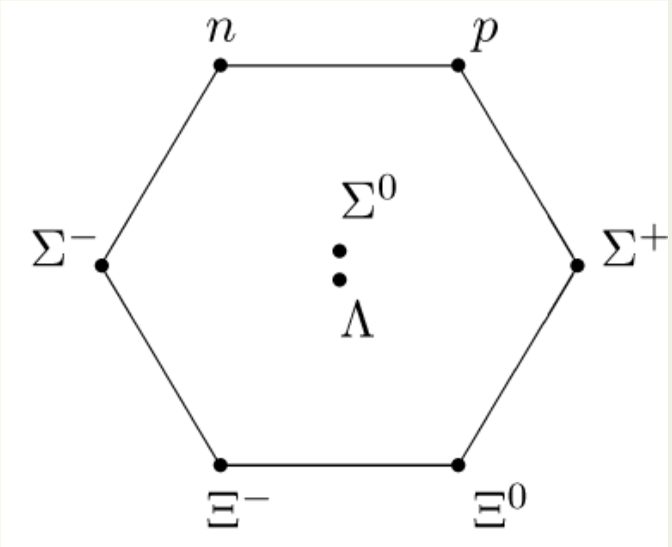
- **But do we actually see any of these symmetries in QCD ?**

Symmetry hunting

- Let's hunt these symmetries in the spectrum of mesons and baryons

$$q \rightarrow e^{i\alpha} q$$ U(1) symmetry **Check !** Baryon number conservation !

$$q \rightarrow e^{i\beta^a \lambda^a} q$$ SU(3) symmetry **Check !** Octet baryons with very similar properties (neutron, proton, Lambda, ...)



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$$q \to e^{i\beta^a \lambda^a \gamma^5} q$$ $SU_A(3)$ symmetry ???? ????? No P-odd neutron …

$$q \to e^{i\alpha_5 \gamma^5} q$$ $U_A(1)$ symmetry ???? ?????

- What is going on? Does QCD even describe hadrons ?

Nambu-Goldstone to the rescue !

- But wait ! Maybe these symmetries are **spontaneously** broken !
- What does this imply? Goldstone theorem: for each **spontaneously broken global symmetry** there appears a **massless** boson .

- Well, QCD has no massless bosons.....

Nambu-Goldstone to the rescue !

- But wait ! Maybe these symmetries are **spontaneously** broken !
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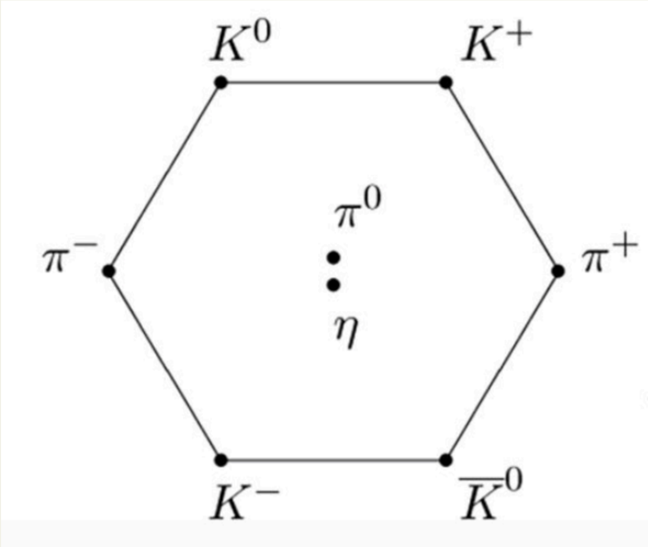
- Well, QCD has no massless bosons..... But there are very light bosons !
- Pions (3), Kaons (4), eta (1) are much lighter than ~ 1 GeV (baryon mass)
- They are **Pseudo-Goldstone** bosons of $SU_A(3)$ (because of quark mass)



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$q \to e^{i\beta^a \lambda^a} q$ SU(3) symmetry **Check !** Octet baryons with very similar Masses (neutron, proton, Lambda, ...) !

$q \to e^{i\beta^a \lambda^a \gamma^5} q$ $SU_A(3)$ symmetry **Check!** Spontaneously broken !

$q \to e^{i\alpha_5 \gamma^5} q$ $U_A(1)$ symmetry ???? ?????

- **What about the last one ?** There are no more light mesons....
- The eta' is there but it is massive ~ 1 GeV >> $m_{pion,Kaon,eta}$
- This was a big problem in the early days of QCD

Anomalous symmetry

- Anomalous symmetries are classical symmetries that are broken by QM
- Action (or Lagrangian) invariant but path-integral not
- **We forgot about the Jacobian !**

$$\int [d\psi][d\bar{\psi}]\; e^{iS[\psi,\bar{\psi}]}$$

$$S[\psi,\bar{\psi}] = S[\psi',\bar{\psi}']$$

$$\int [d\psi][d\bar{\psi}] = \int [d\psi'][d\bar{\psi}']\,\mathcal{J} \qquad \mathcal{J} \neq 1$$

$$\psi \to \psi' \qquad \bar{\psi} \to \bar{\psi}'$$

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- The $U_A(1)$ symmetry is not a symmetry at al !

Axial transformation induces a shift in the θ term

$$\log \mathcal{J} = -\alpha \int d^4x\; \frac{g_s^2}{32\pi^2}\, G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}$$

Total derivative but nonzero !

- No reason to expect another Goldstone Boson ! **Eta' mass 'explained'**

Complex masses

- Let us look at 2-flavor QCD with masses + theta term

$$L = \sum_{u,d} \bar{q}(i\gamma^\mu D_\mu - m_q)q \qquad +\theta \frac{g_s^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu}$$

- We assumed the masses to be real, but there is **no** reason to do so
- To **make** the masses real, we do a $U_A(1)$ transformation

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$$q \rightarrow e^{i\alpha_5 \gamma^5} q$$

- But this induces a theta term from the anomaly ! $\theta \rightarrow \theta + 2i \ Arg(m_q)$
- So we can **'trade'** the theta term for a complex mass or vice versa

$$+\theta \frac{g_s^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} \longleftrightarrow -\left(\frac{m_u m_d}{m_u + m_d}\right) \theta \, \bar{q} i \gamma^5 q$$

- One 'physical' combination

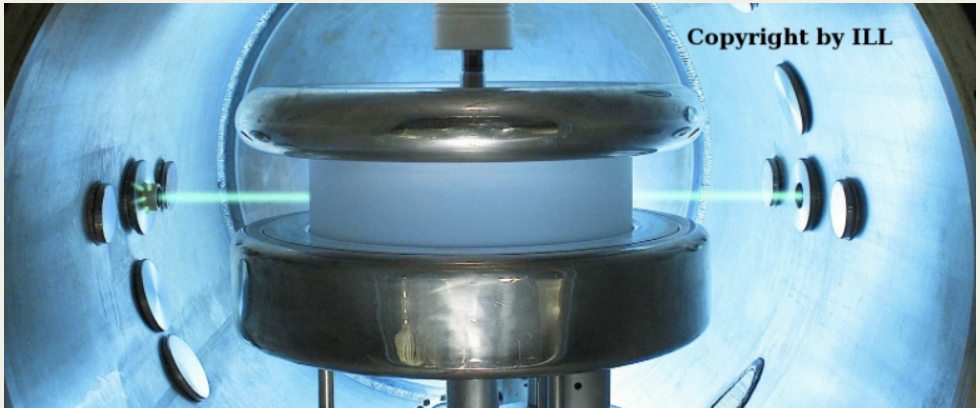
$$\bar{\theta} = \theta + n_f Arg(m_q)$$

Let's take a breath…..

- QCD contains a theta term, a CP-violating interaction
- Total derivative but still contributes via 'instantons'
- The effect is real because we are missing a Goldstone boson

$$+\theta \frac{g_s^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} \quad \longleftrightarrow \quad -\left(\frac{m_u m_d}{m_u + m_d}\right) \theta\, \bar{q} i \gamma^5 q$$

- We can trade the theta term for a complex quark mass
- Theta itself is unknown! One of the SM parameters !
- **We should measure it ! Let's do it !**



How do we measure the theta term ?

- Difficult problem. Not unlike measuring the normal quark mass…
- Only **two** feasible ways that I know and one is much better
- **Electric dipole moments of hadrons and nuclei**

- **Problem: low-energy QCD is nonperturbative.**
- How to calculate the nucleon EDM from CPV at quark-gluon level ?

$$L_{dip} = -\frac{d_n}{2}\bar{\Psi}_n \sigma^{\mu\nu} i\gamma^5 \Psi F_{\mu\nu} \qquad \text{from} \quad -\left(\frac{m_u m_d}{m_u + m_d}\right)\bar{\theta}\ \bar{q} i\gamma^5 q \ = -\left(m^*\right)\bar{\theta}\ \bar{q} i\gamma^5 q$$

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- Let's guess something: d_n should be proportional to ~ $(m^*\theta)$
- There should be a coupling to a photon somewhere ~ e
- To get dimensions right we need 1/mass^2, let's say nucleon mass....

$$d_n \sim e\frac{m^*}{m_N^2}\bar{\theta} \sim e\frac{10\ MeV}{(1\,GeV)^2}\bar{\theta} \sim 10^{-3}\bar{\theta}\ e\ \ fm$$

Limiting theta

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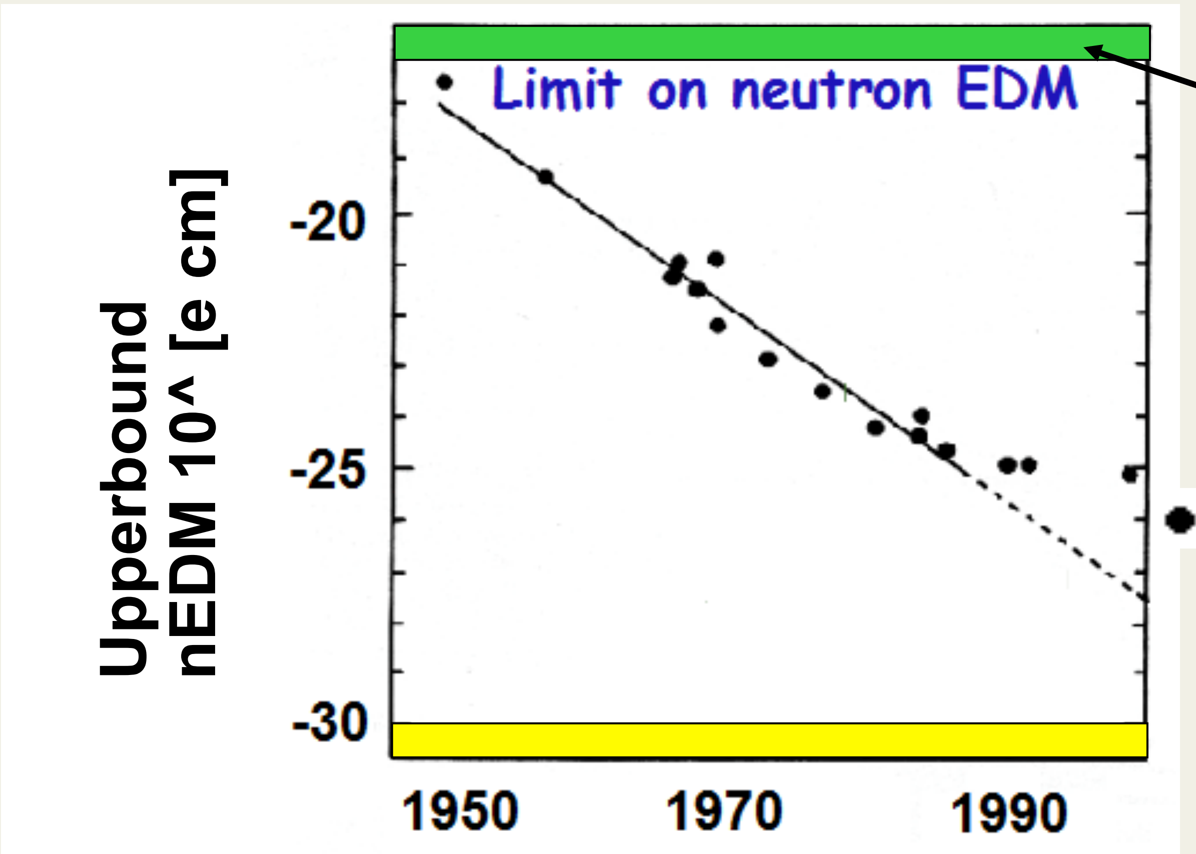

If $\theta \sim 1$, just like the CKM phase

Sets θ upper bound: $\theta < 10^{-10}$

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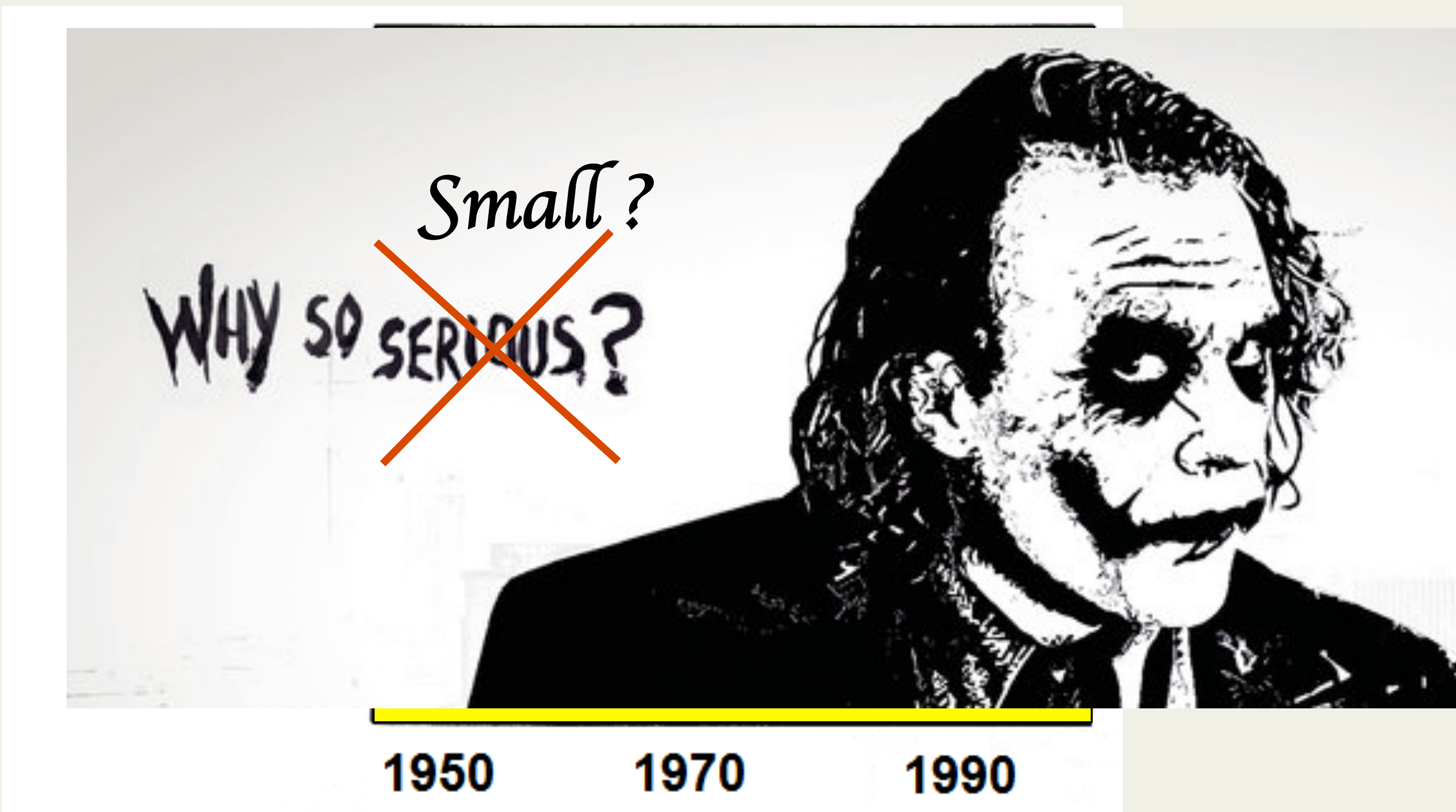

~ 1,
like
CKM
ase

Sets θ upper bound: θ < 10^{-10}

Perhaps the estimate is stupid….

- The estimate was based on inserting some quasi-random factors

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm kin} - \bar{m}\bar{q}q - \varepsilon\bar{m}\,\bar{q}\tau^3 q \;\; + m_\star\,\bar{\theta}\,\bar{q}i\gamma^5 q$$

$$\bar{m} = \frac{m_u + m_d}{2}$$

$$\varepsilon = \frac{m_u - m_d}{m_u + m_d}$$

- We use ‘chiral perturbation theory’ to match this to the hadronic level. Skip all details.

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$$\mathcal{L}'_\chi = \mathcal{L}_\chi - \boxed{\frac{m_\pi^2}{2}\pi^2} - \delta m_N\,\bar{N}\tau^3 N \quad +\bar{g}_0\,\bar{N}\tau\cdot\pi N$$

Pion mass

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$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{kin}} - \bar{m}\bar{q}q + \varepsilon\bar{m}\,\bar{q}\tau^3 q + m_\star\,\bar{\theta}\,\bar{q}i\gamma^5 q$$

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Strong proton-neutron mass splitting

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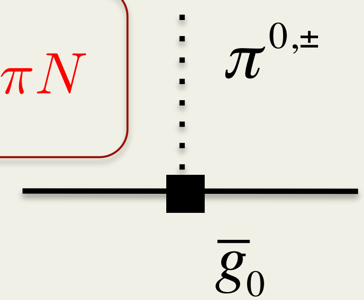

CP-odd pion-nucleon interaction

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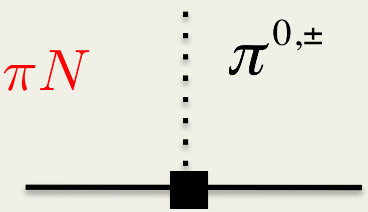
Linked via SU$_A$(2) rotation

$$m_\star = \frac{m_u m_d}{m_u + m_d}$$

$$\mathcal{L}'_\chi = \mathcal{L}_\chi - \frac{m_\pi^2}{2}\pi^2 - \delta m_N\,\bar{N}\tau^3 N \;\; + \bar{g}_0\,\bar{N}\tau\cdot\pi N$$

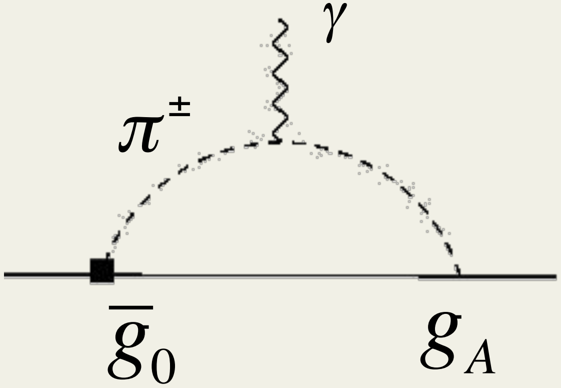


Nucleon mass splitting (strong part, no EM!) ⟷ **CP-odd pion-nucleon interaction**

Use **lattice** for mass splitting

$$g_0 = -\frac{\delta m_N}{2f_\pi}\frac{m_*\bar{\theta}}{\bar{m}\varepsilon} = -(15.5 \pm 2.5)\cdot 10^{-3}\;\bar{\theta}$$

The Nucleon EDM



Neutron EDM

$$d_n = -\frac{e g_A \bar{g}_0}{8\pi^2 f_\pi} Log \frac{m_\pi^2}{m_N^2} \longrightarrow d_n \simeq -2.5 \cdot 10^{-16}\, \bar{\theta}\, e\,\text{cm}$$

- **Very close to the naïve estimate**
- Very recent developments. Use lattice-QCD (difficult !)

Lattice + **ChPT** $\quad d_n = -(3.9 \pm 1.0) \cdot 10^{-16}\, \bar{\theta}\, e\,\text{cm}$

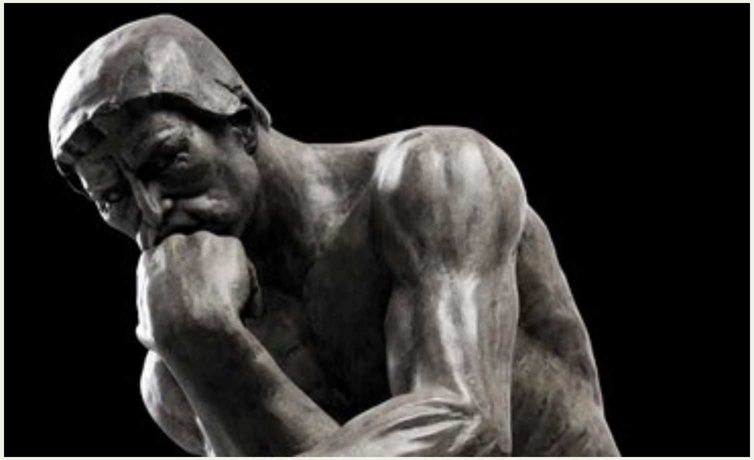
- So everything agrees it seems $\longrightarrow \bar{\theta} < 10^{-10}$

- Why is theta term so small ? **'Strong-CP Problem'**

Philosophy of theta

- The smallness of theta is considered as one of the outstanding problems of the Standard Model **'Strong-CP problem'**

- **First of all, is it really a problem ?**

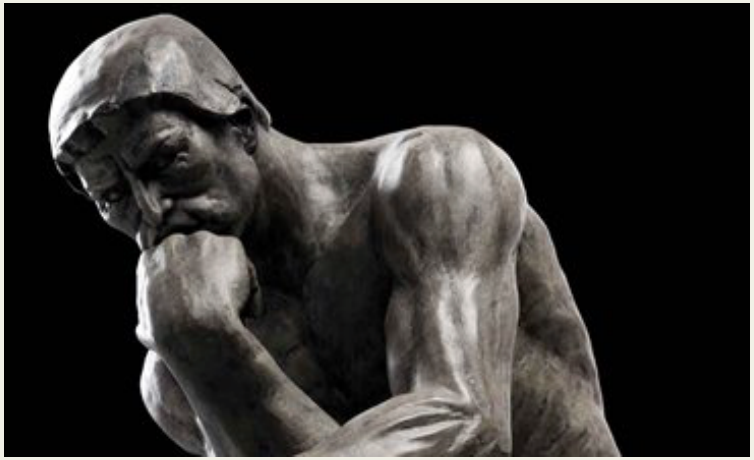


Philosophy of theta

- The smallness of theta is considered as one of the outstanding problems of the Standard Model **'Strong-CP problem'**

- **First of all, is it really a problem ?**
- Small parameters already appear in the SM: $Y_u \sim m_u/v \sim 10^{-5}$, $|V_{ub}| \sim 10^{-3}$, neutrino masses (not fair perhaps)

- Note that there is no **'anthropic'** reason for theta to be so small….



- **I don't know….**

Some possible ways to make theta small

- **What if the lightest quark is massless ?** $-\left(\frac{m_u m_d}{m_u + m_d}\right)\bar{\theta}\ \bar{q} i\gamma^5 q$

- This could be, masses of hadrons do not really care.
- But precision phenomenology of mesons + lattice

$$\frac{m_u}{m_d} = 0.46 \pm 0.02 \pm 0.02$$

- Reincarnation of this idea by Wilczek '16
  "Superheavy Light Quarks and the Strong P,T problem"

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- **Other easy solutions**: demand P or CP is an exact symmetry
- But then difficult to get large CKM phase and theta < 10^{-10}

- Arguably most popular solution: **axions**

Axions

- **Lecture set on its own**, but the idea is not too hard
- We saw: theta term connected to $U_A(1)$ anomaly

- Add to the SM some new SU(3)-charged fields which have a new $U_A(1)$ symmetry
- Assume this symmetry is spontaneously broken (axion=goldstone)

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a(x)}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

- The theta term becomes a pseudoscalar 'field'

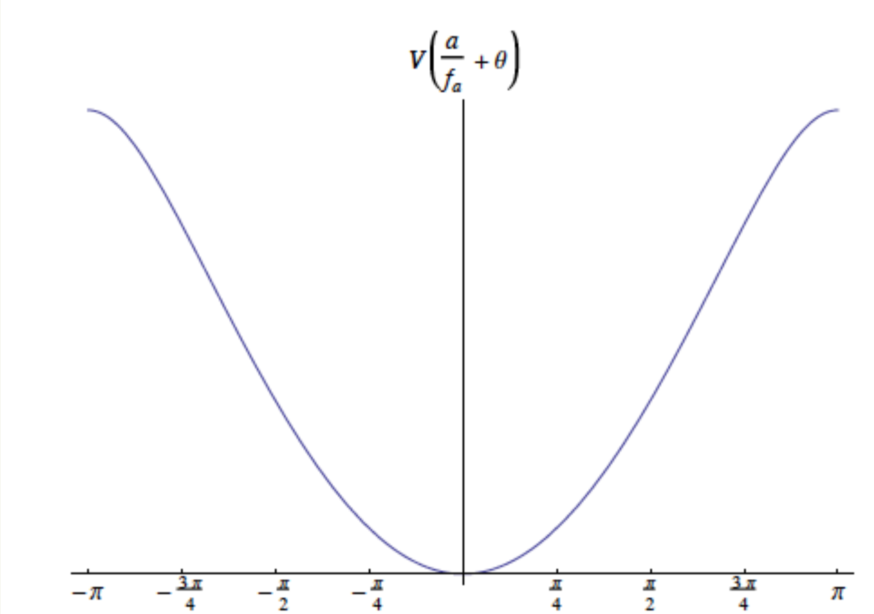
$$\bar{\theta} \to \bar{\theta} + \frac{a}{f_a}$$

- This field has a potential, just like the Higgs

Axions

- Potential gets a minimum

$$\theta_{total} = \frac{<a>}{f_a} + \bar{\theta} = 0$$



- This solves the CP-problem **independent** of the starting value of the theta term !

- **Dynamical solution** (Peccei-Quinn mechanism)

- **Very nice, but where is the axion?**

- Mass inversely proportional to f_a which can be huge ! So axion could be **very very light**

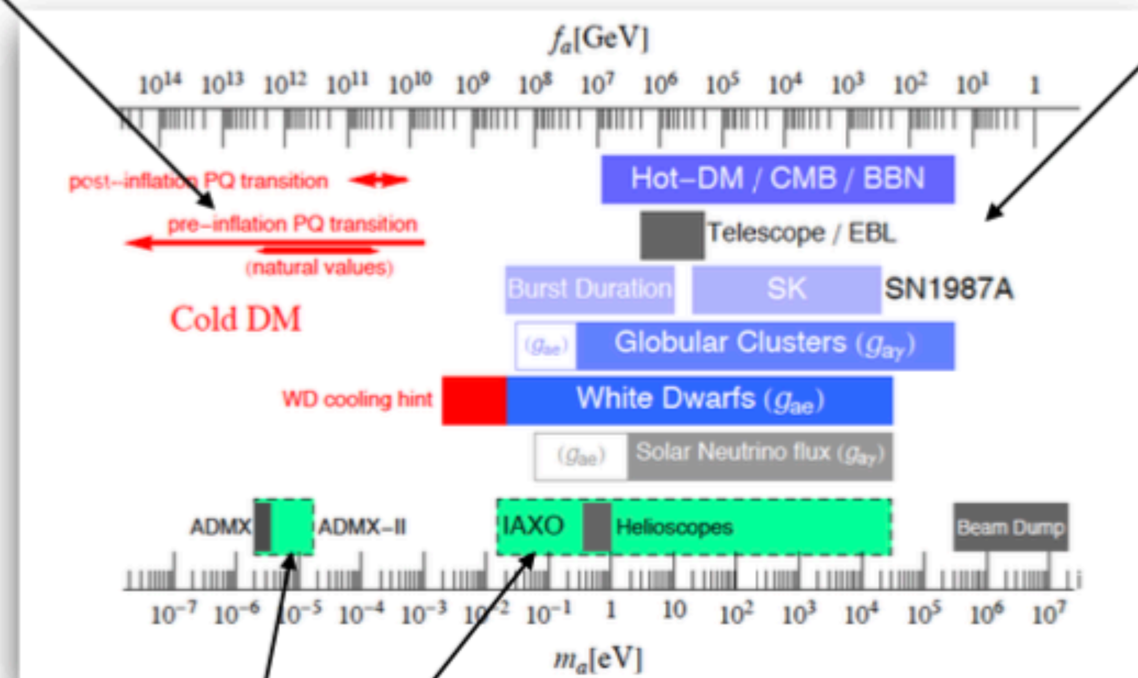
$$m_a^2 \propto \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

Status of axion searches

Axion as cold dark matter lives here

$$m_a \sim 6 \text{ meV } (10^9 \text{ GeV}/f_a)$$

Disfavored by astrophysics / cosmological observations (grey) or argument (blue)



Sensitivity of planned experiments

Slide from Cirigliano, '16

ArXiv: 1311.0029

Summary

- QCD is difficult…..
- Because it is non-Abelian (self-interacting gluons) there appears a new term that violates P and T: the theta term

- Theta term is one of SM parameters (must measure it)
- Neutron electric dipole moment limits theta < 10^{-10}

- **At the moment we do not understand why**
- Perhaps theta is simply small for no fundamental reason, in that case it might be that theta will be found in **future experiments**

- Very attractive solution**: the axion mechanism**
- However, no axions have been found for over 30 years....
- Axions are also **a Dark Matter candidate**