Exploring Hadronic B decays through $SU(3)$ symmetry

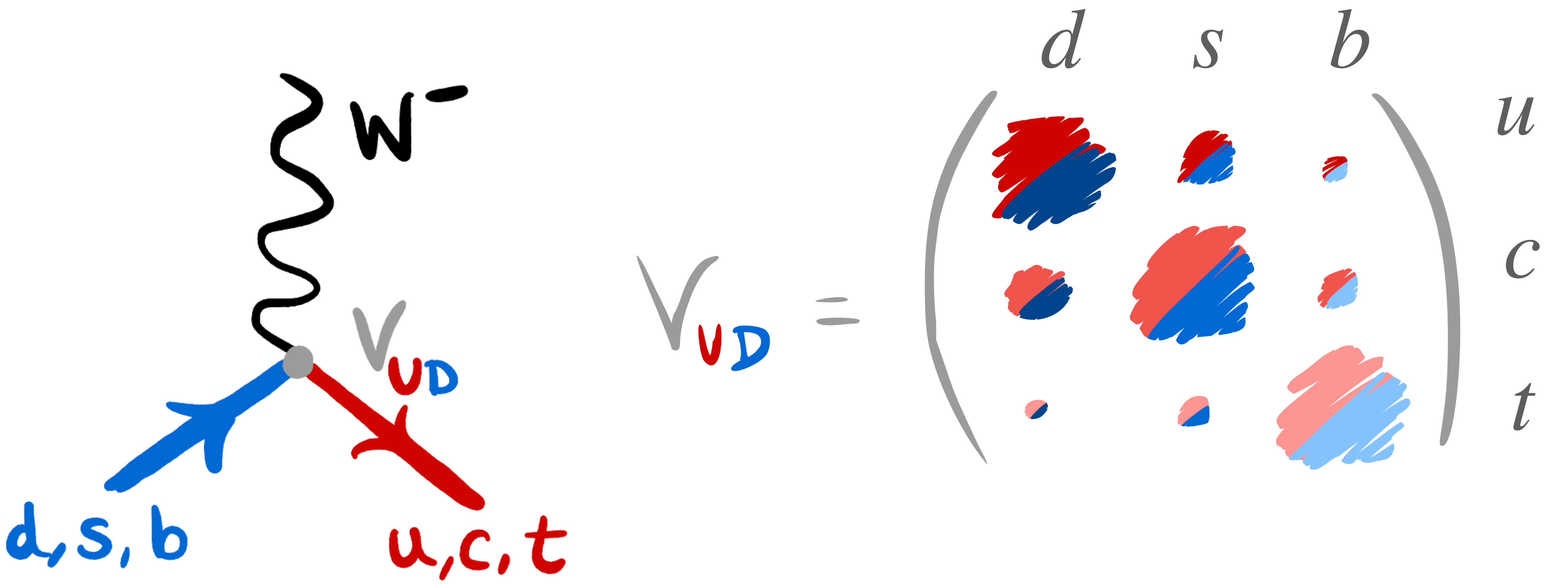
Marta Burgos Marcos

In collaboration with: **Keri Vos (Maastricht University - Nikhef)**

Méril Reboud (Université Paris - Saclay, IJCLab, Orsay)

Standard Model and CKM mechanism

Down-type to **up-type** quark transitions are parameterized by the CKM matrix V_{UD}



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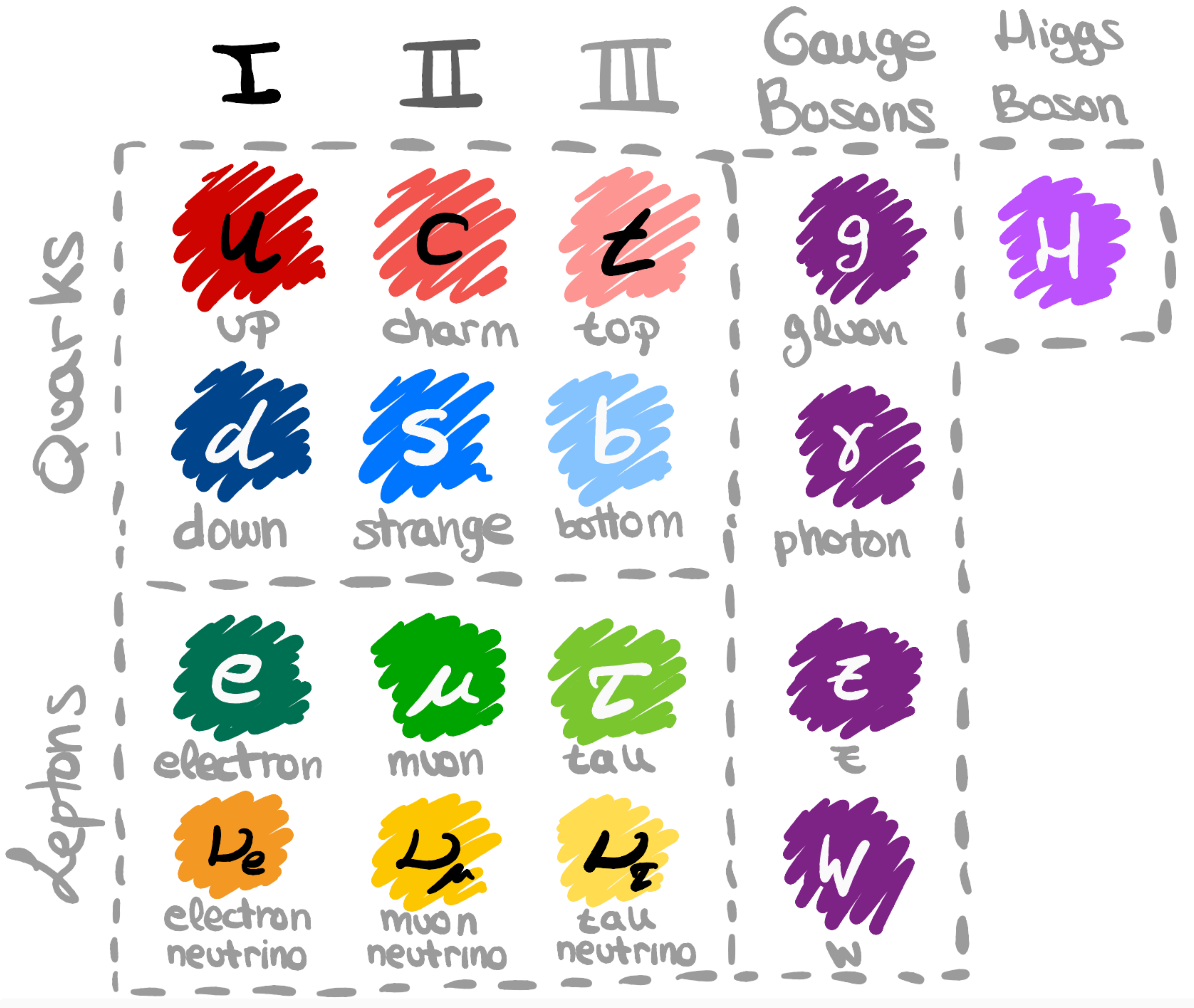

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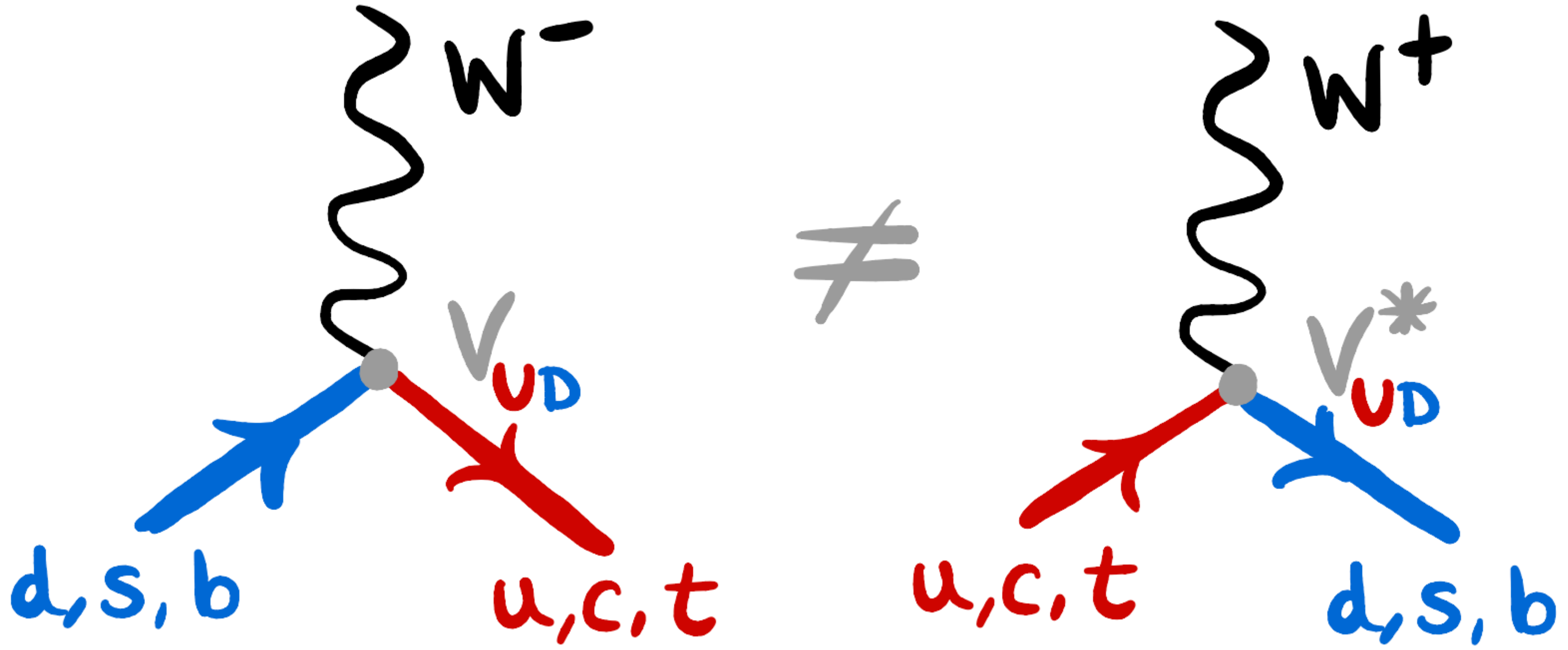
Complex phases in the CKM matrix are responsible for **CP violation**



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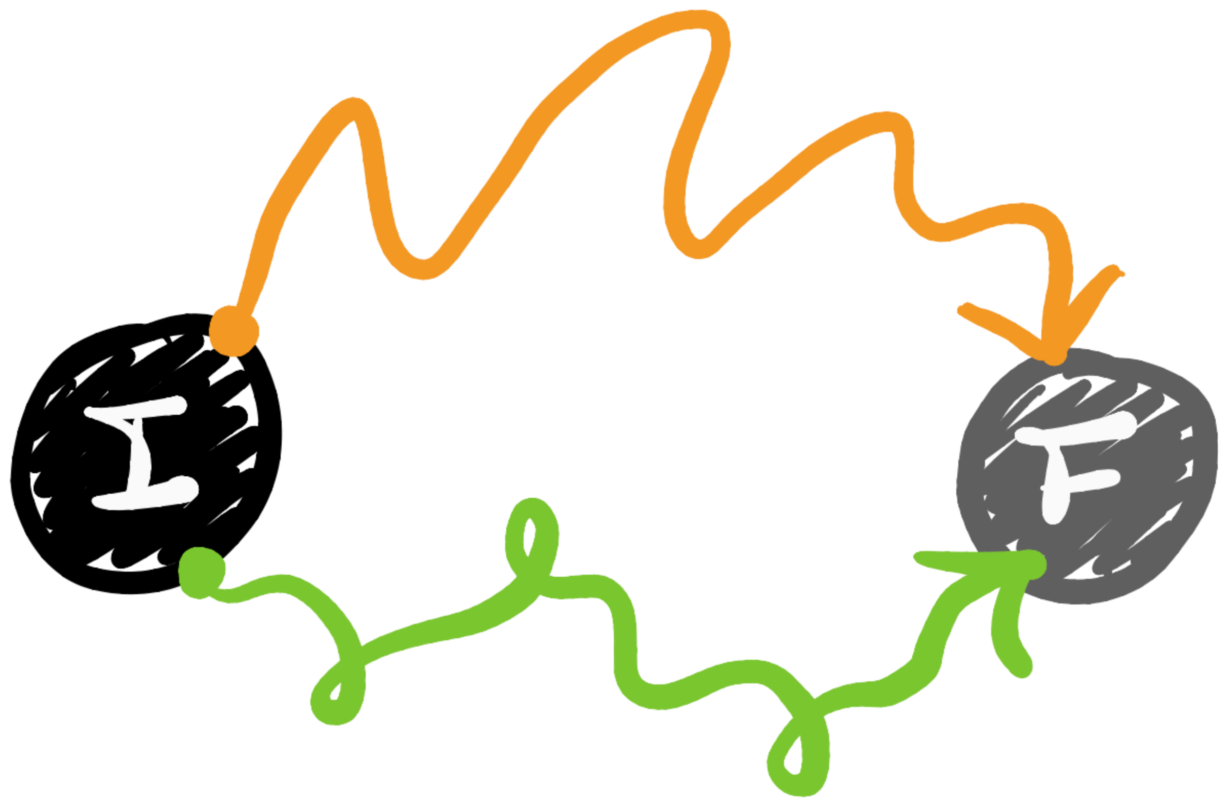

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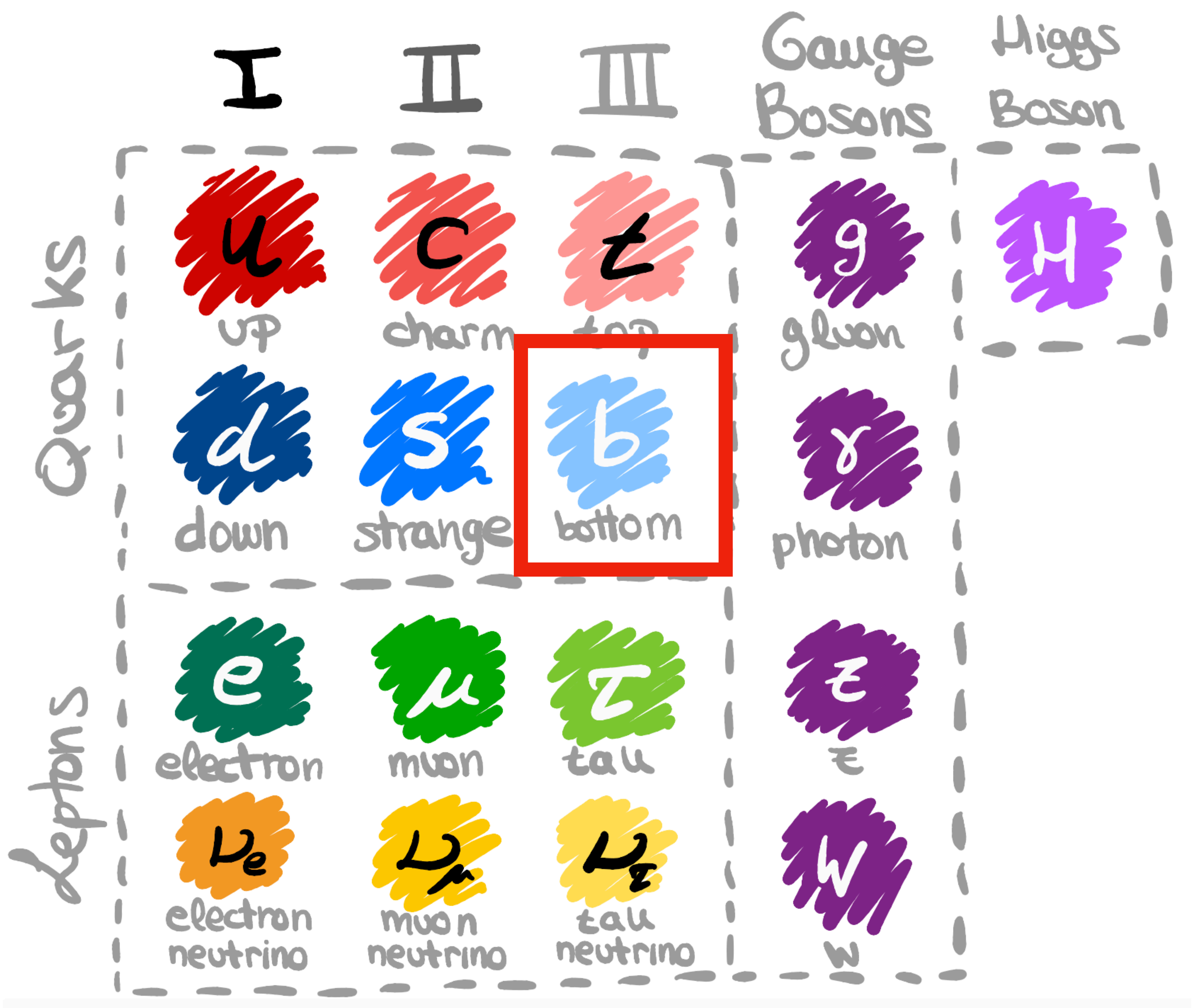

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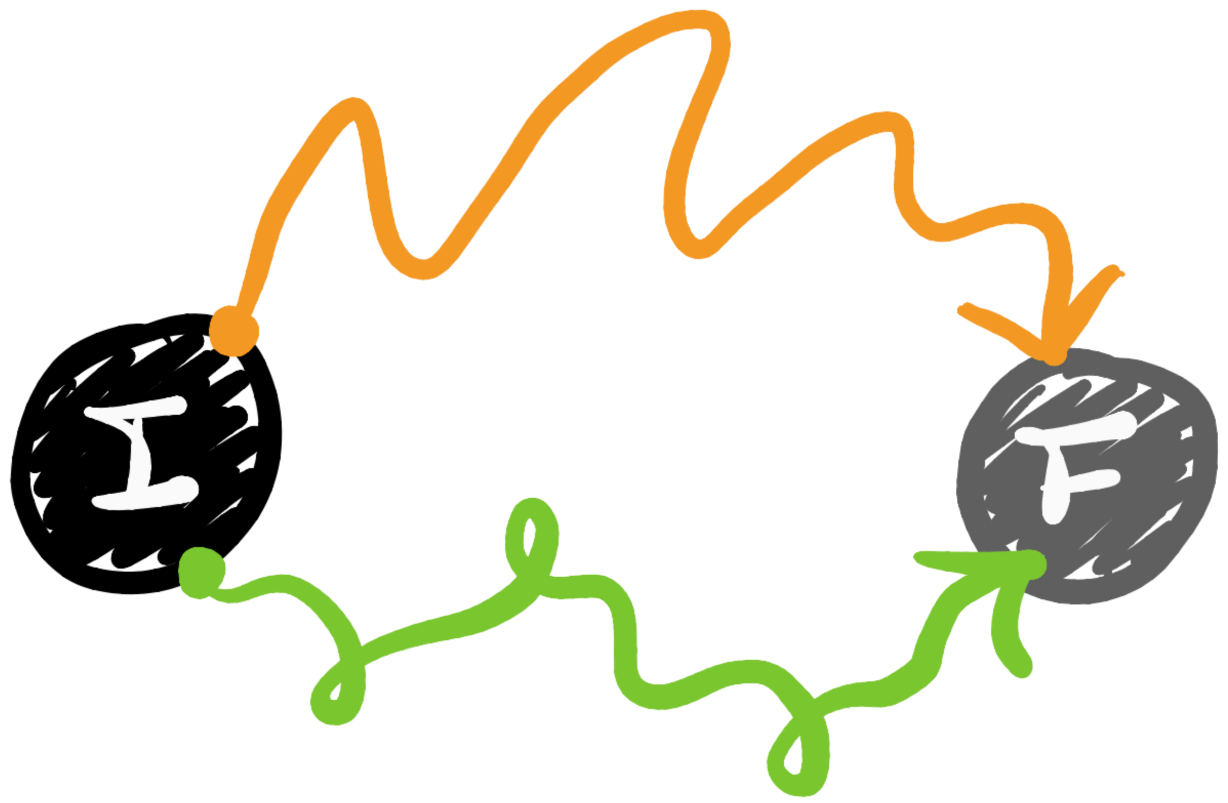

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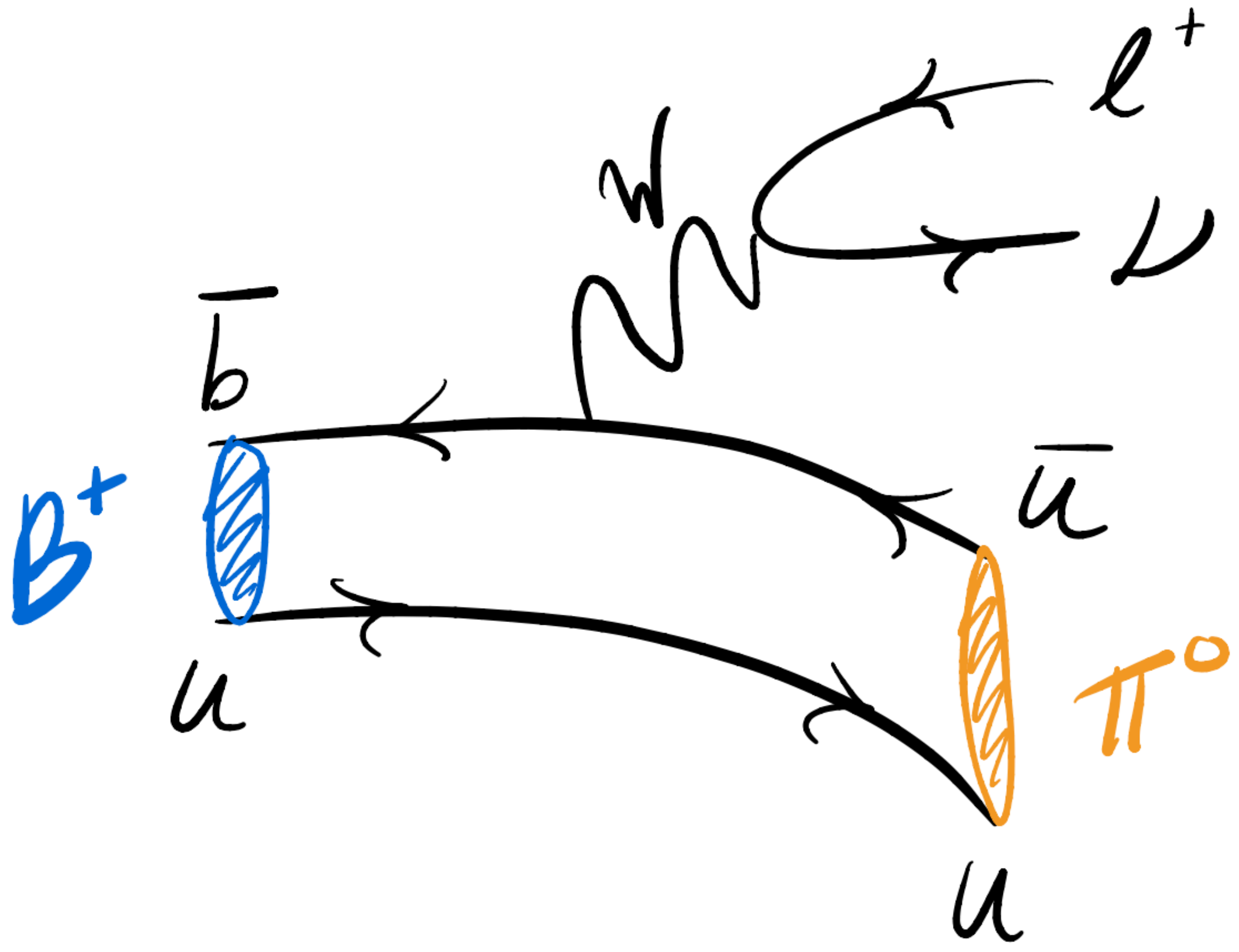
hadronically?

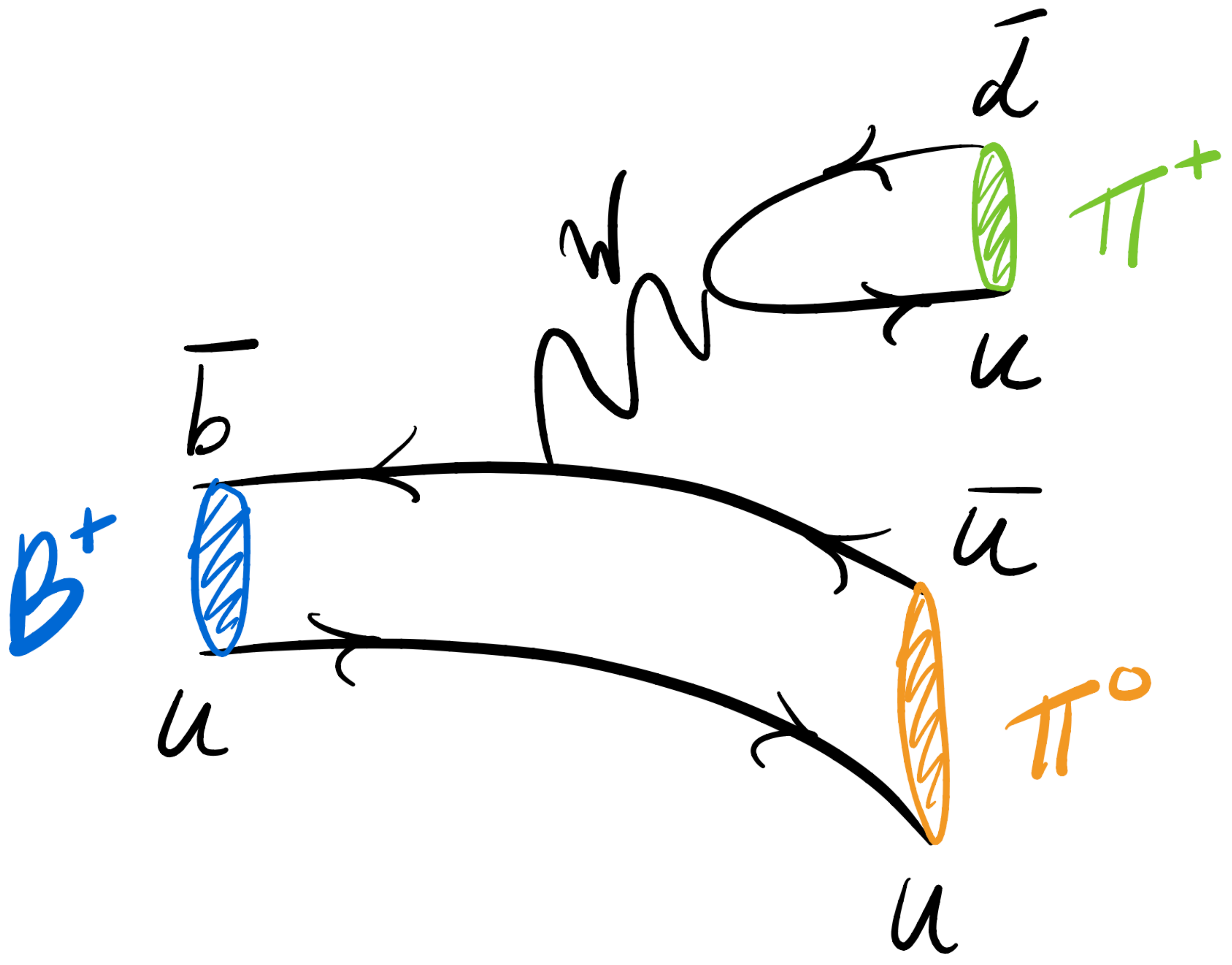
Purely **perturbative** techniques **no longer valid**

Current predictions governed by **uncertainties**

What is so complicated?

Semileptonic | **vs.** | **Hadronic**





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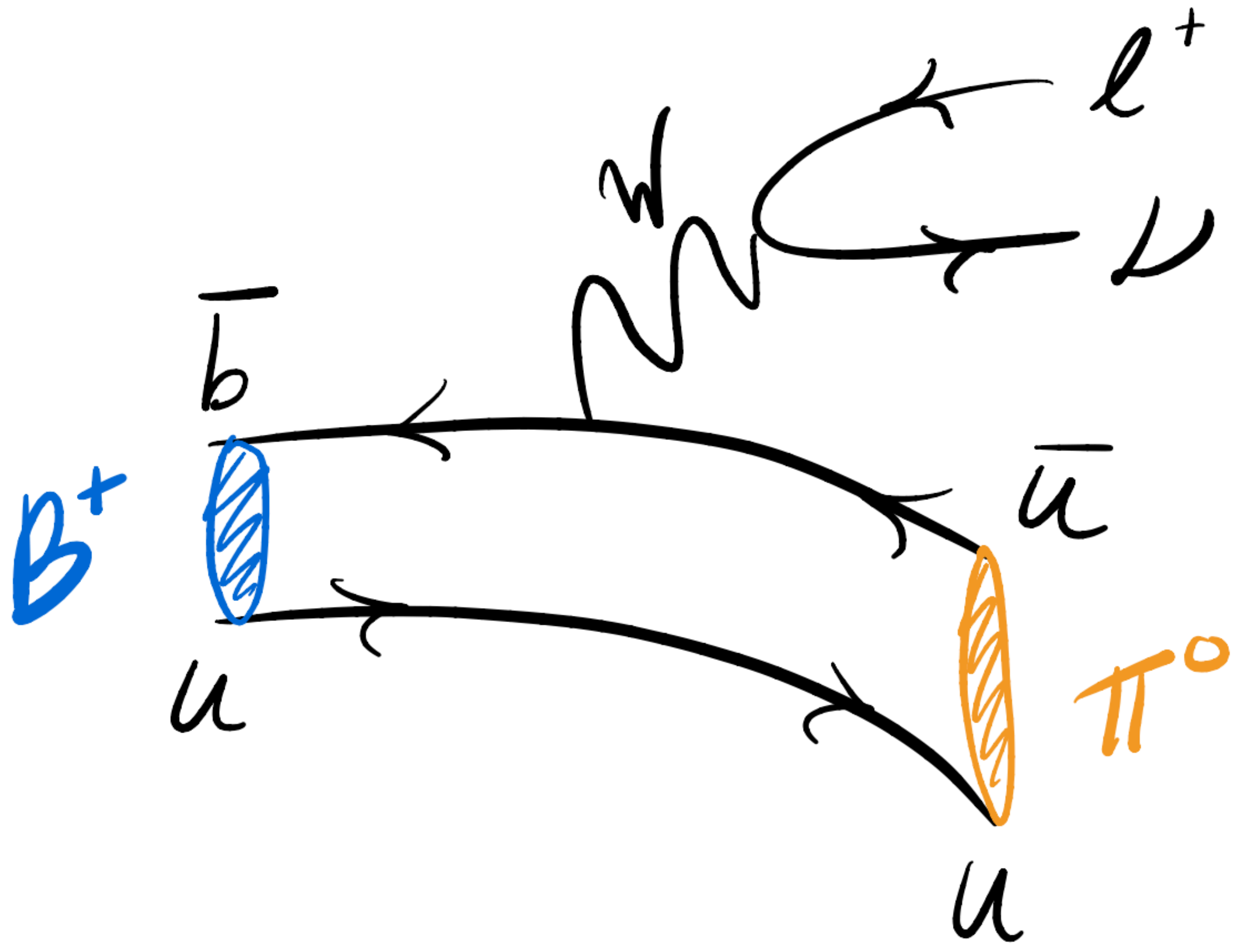
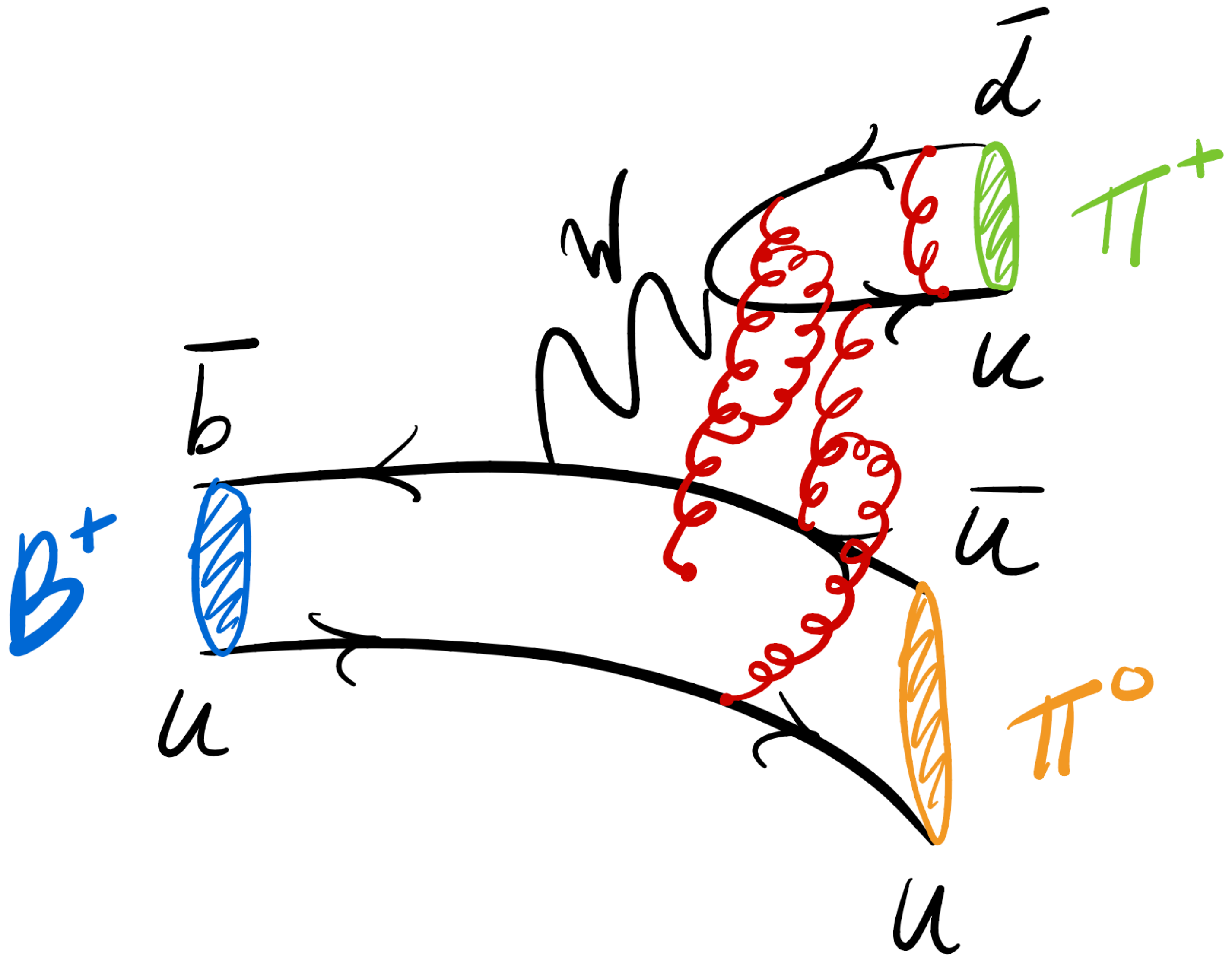
Semileptonic vs. **Hadronic**





Leptonic and hadronic parts **factorize**

Strong interaction **confined** to the $B \to P$ transition

Non-perturbative interactions between the final state hadrons

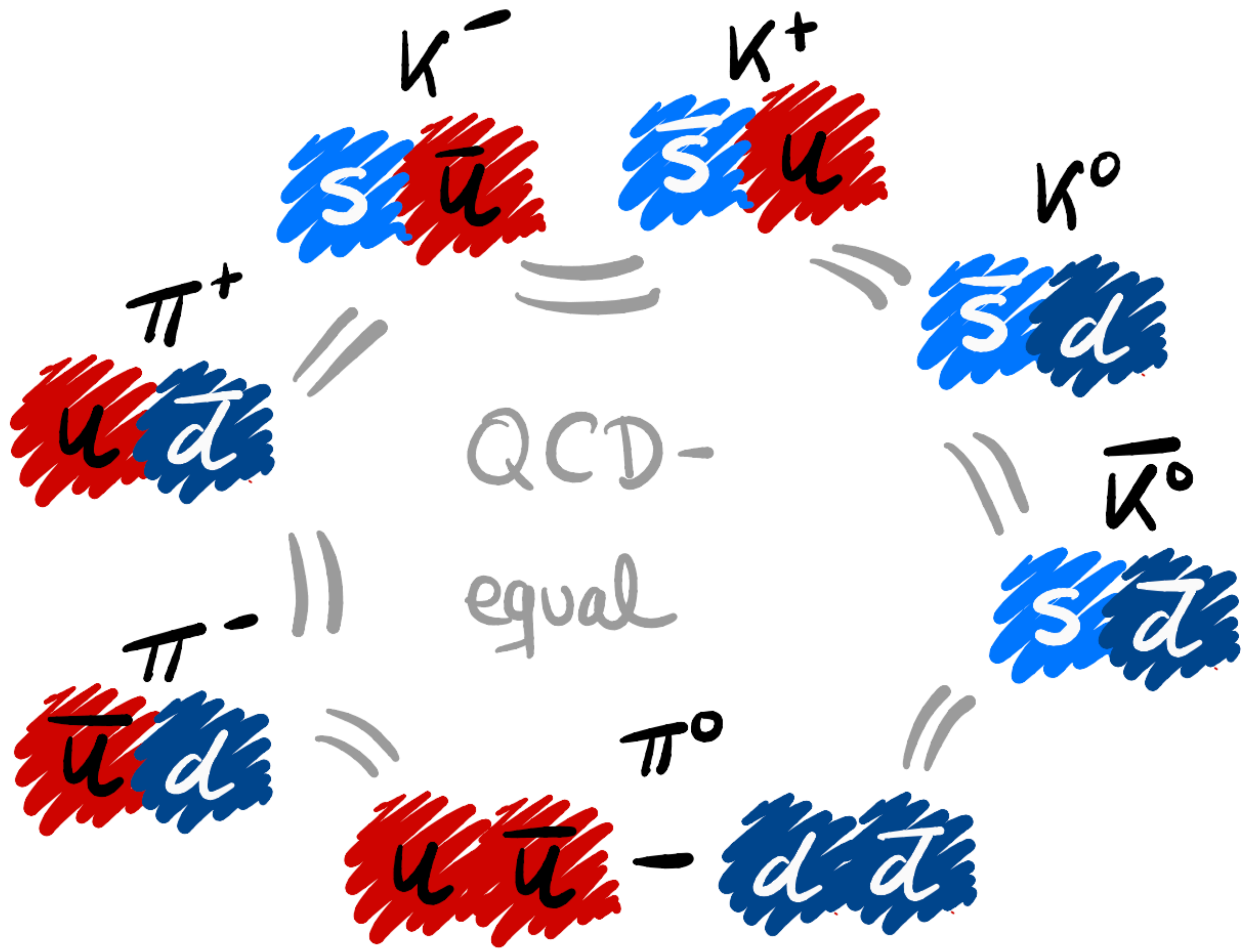
There is currently **no strict theoretical approach** possible

How can we describe $B \to PP$ decays?

1. $SU(3)$ Flavor Symmetry

Assume quarks up, down and strange are degenerate and massless under the **strong interaction**

Under $SU(3)$ symmetry, all $B \to PP$ are **related**, with $P = \pi, K$ because all interact the same way (under QCD)



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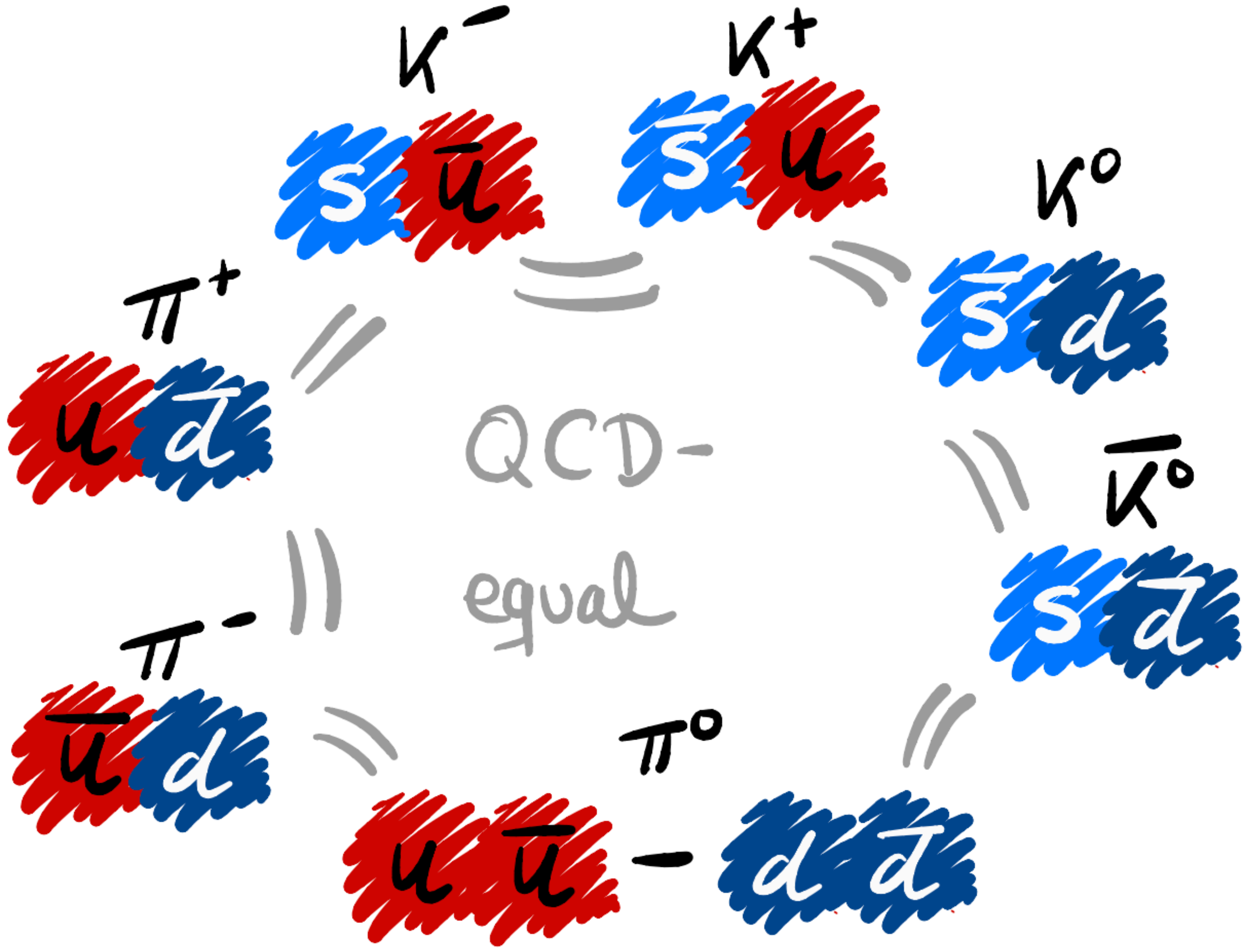

Note! This symmetry is broken in nature $m_u \neq m_d \neq m_s$ but it is a useful approximation

2. Topological parameterization

Parameterize all $B \to PP$ decays in terms of topological coefficients

→

Each topological coefficient represents a different Feynman diagram

Any two body B decay can be expressed as: $A^{TDA}(B \to PP) = \lambda_u^{(q)} A_u + \lambda_c^{(q)} A_c + \lambda_t^{(t)} A_t$

$$\lambda_i^{(q)} = V_{ib}^* V_{uq}$$

$$q = d, s$$

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↓ **CKM unitarity!**

$\lambda_u^{(q)} + \lambda_c^{(q)} + \lambda_t^{(q)} = 0$

$\lambda_i^{(q)} = V_{ib}^* V_{uq}$

$q = d, s$

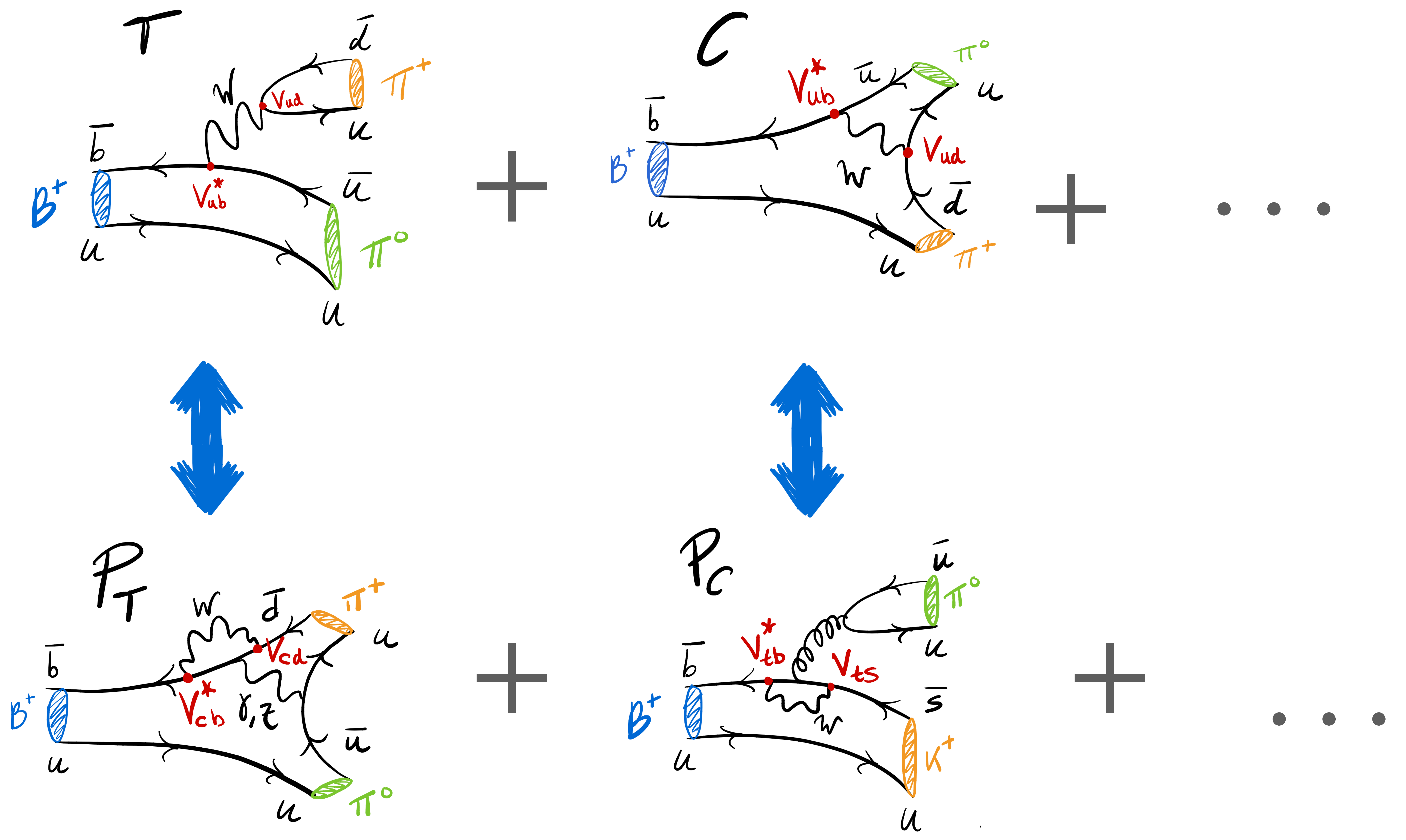
For every **tree** topology contributing to a decay we have its **penguin** counterpart:

$$A^{TDA}(B \to PP) = \lambda_u^{(q)} T^{TDA} + \lambda_c^{(q)} P^{TDA}$$

2. Topological parameterization

Tree amplitude : $T^{TDA} \sim V^*_{ub} V_{uq}$

Penguin amplitude : $P^{TDA} \sim V^*_{cb} V_{cq}$



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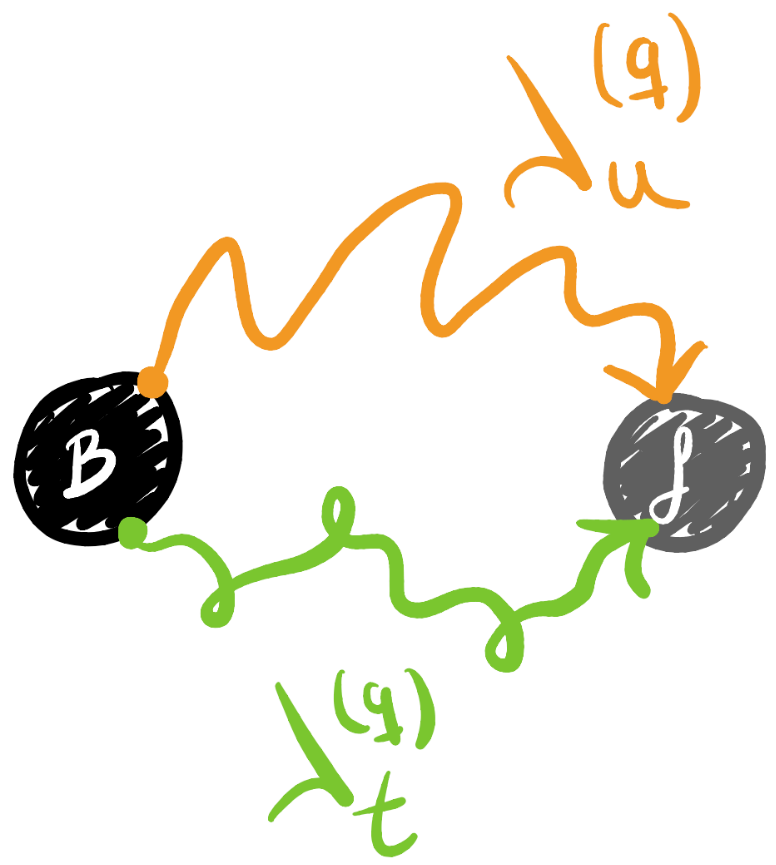
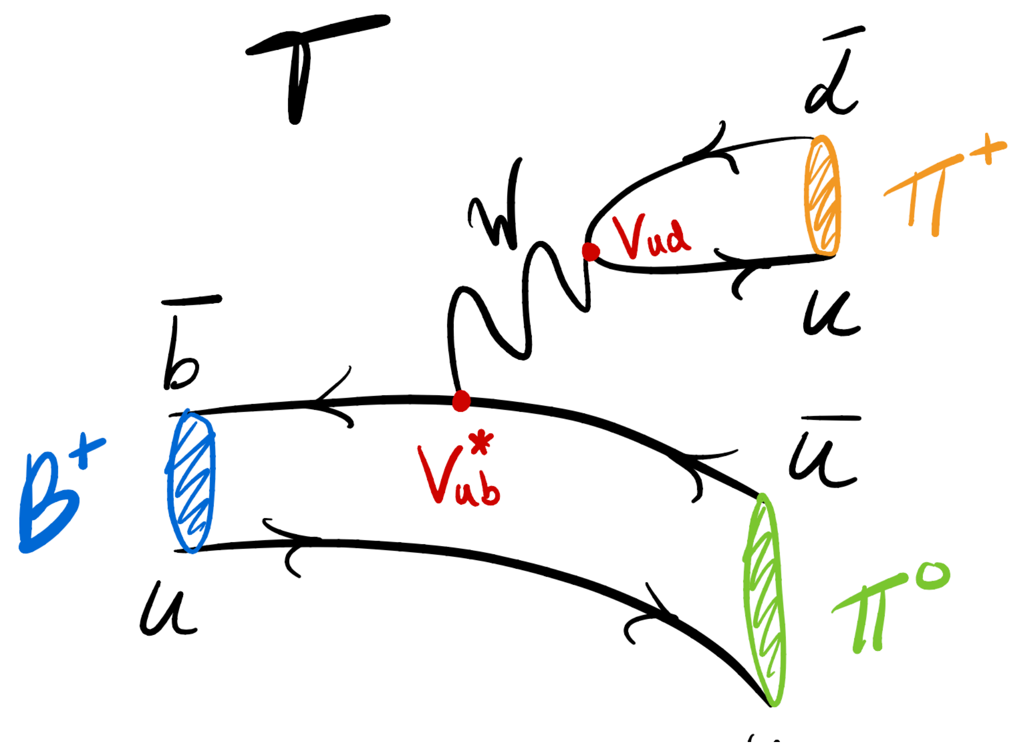
Tree amplitude
$T^{TDA} \sim V^*_{ub} V_{uq}$



Penguin amplitude
$P^{TDA} \sim V^*_{cb} V_{cq}$

- 

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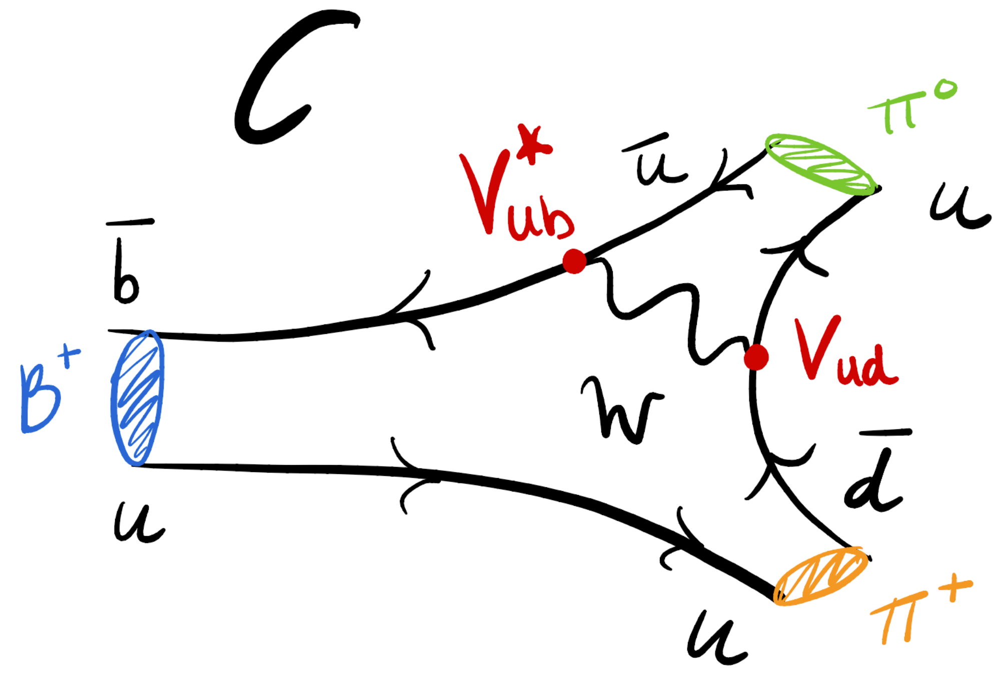

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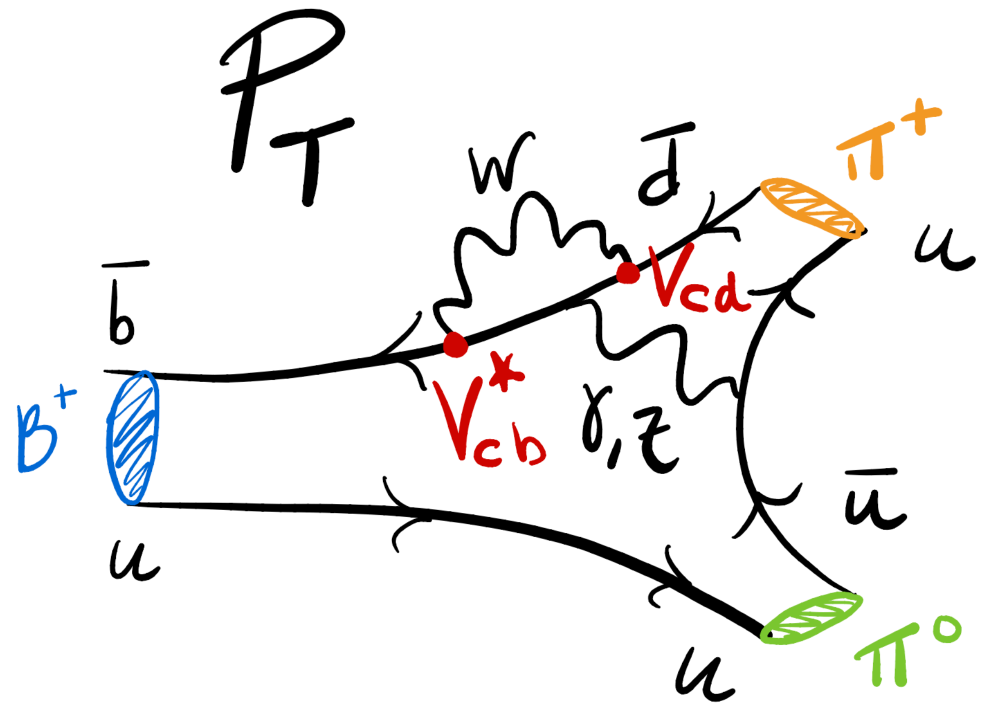
With same CKM structure
No CP Violation

- 

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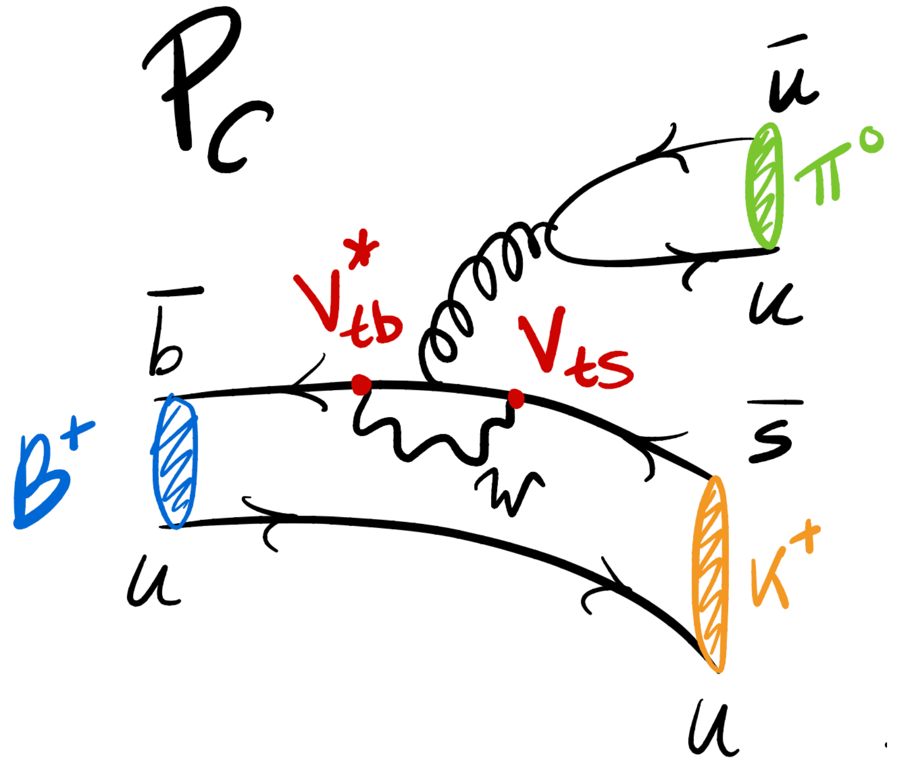

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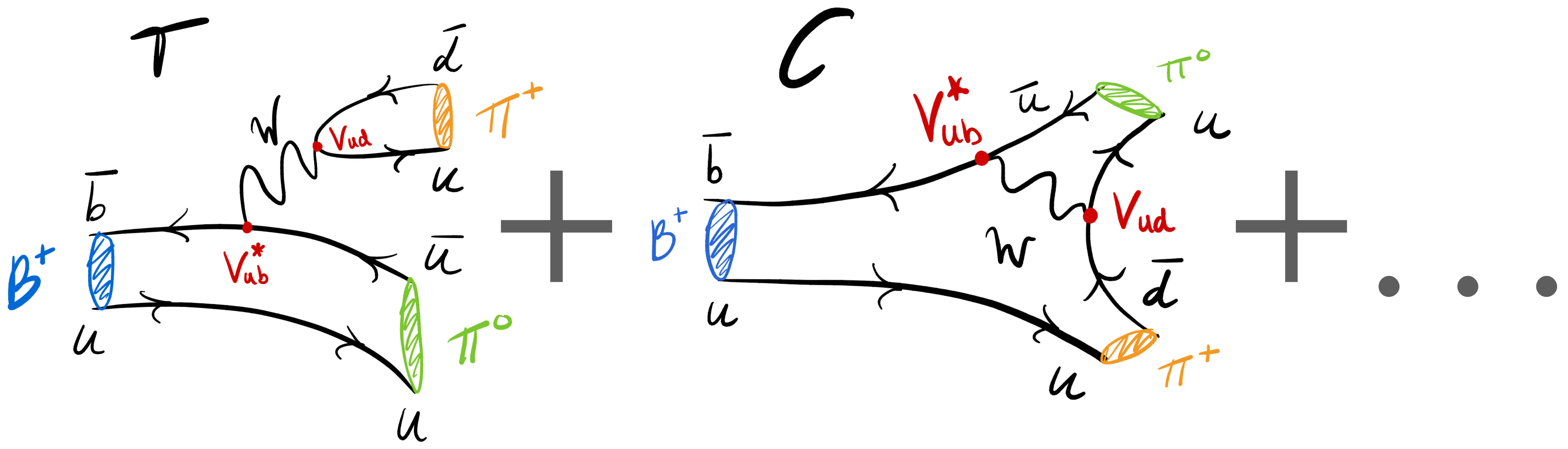
2. Topological parameterization

We can relate same coefficients in different decays:

$$B^+ \to \pi^0 \pi^+$$



$$A(B^+ \to \pi^0 \pi^+) = V_{ub}^* V_{ud} (T + C + \ldots)$$

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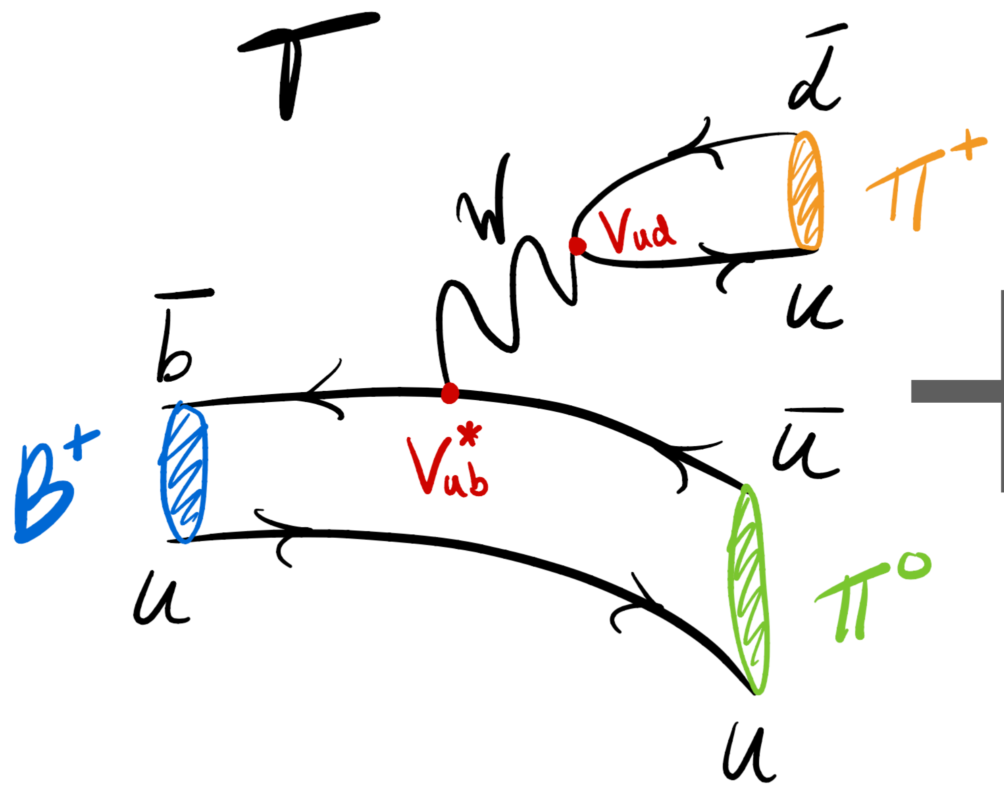

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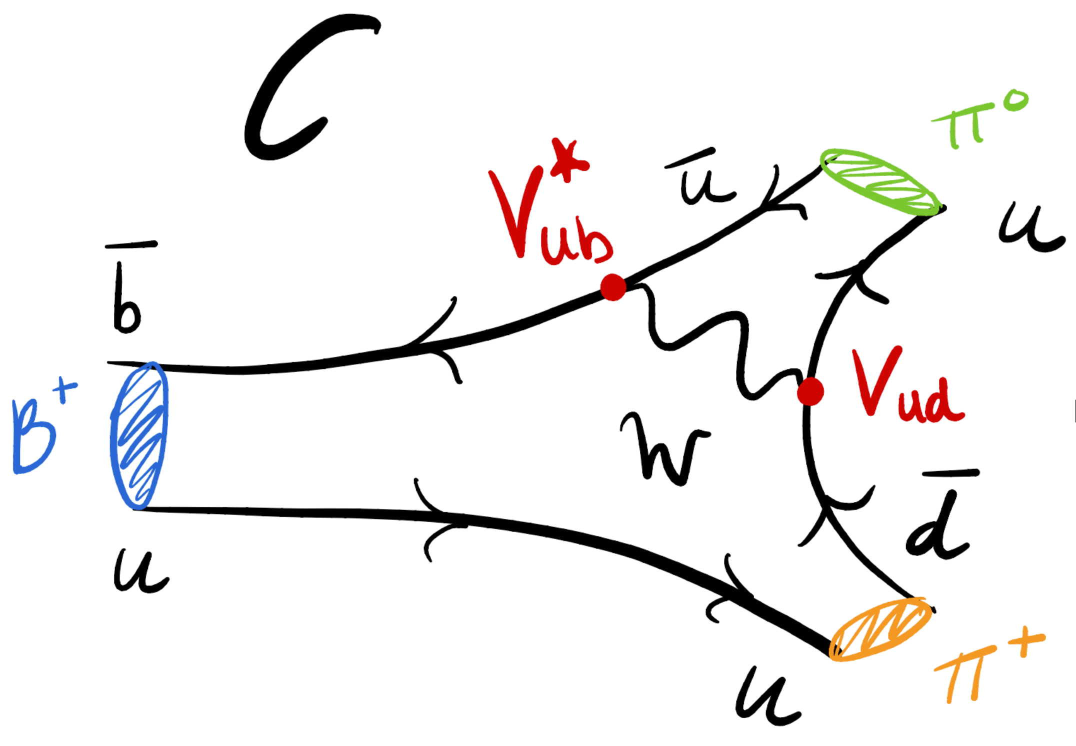

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$$A(B^+ \to \pi^0\pi^+) = V_{ub}^* V_{ud} (T + C + \dots)$$

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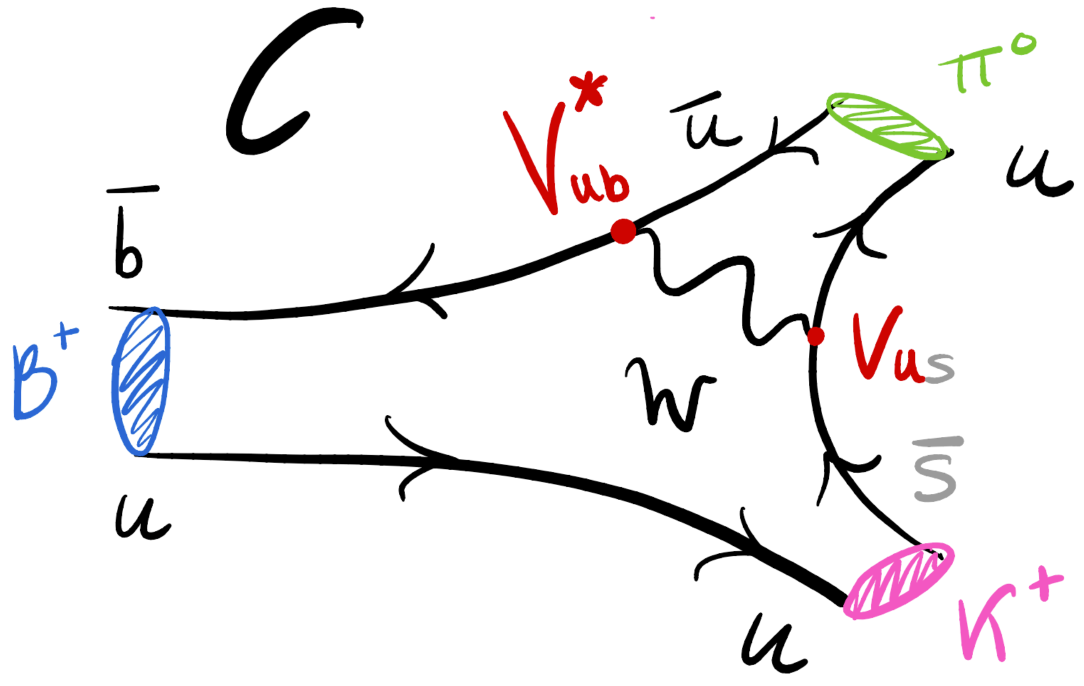

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Same C
Under $SU(3)$

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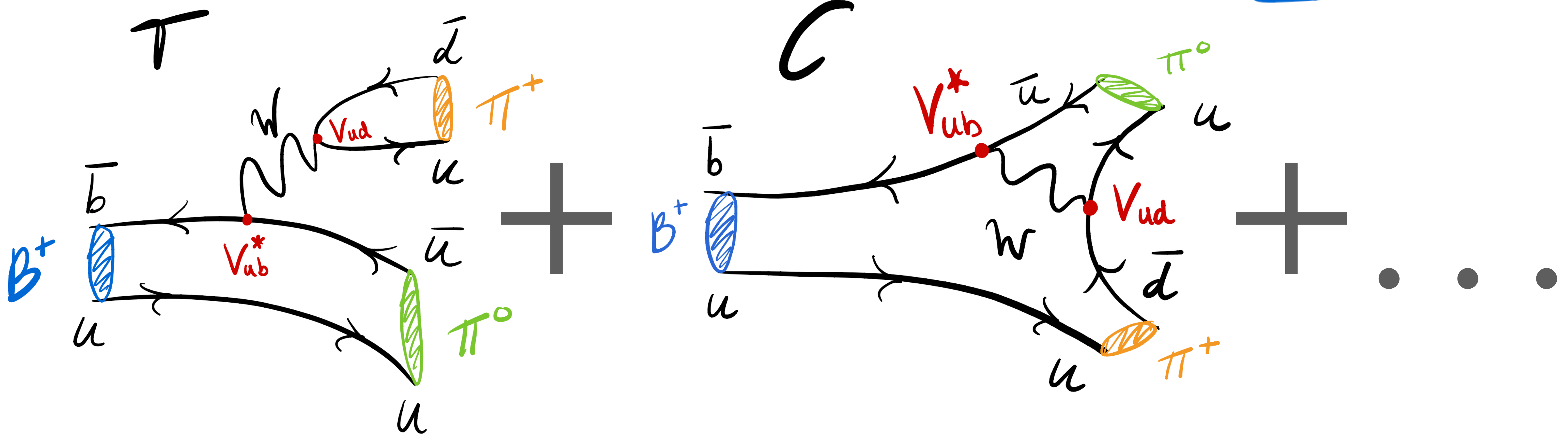

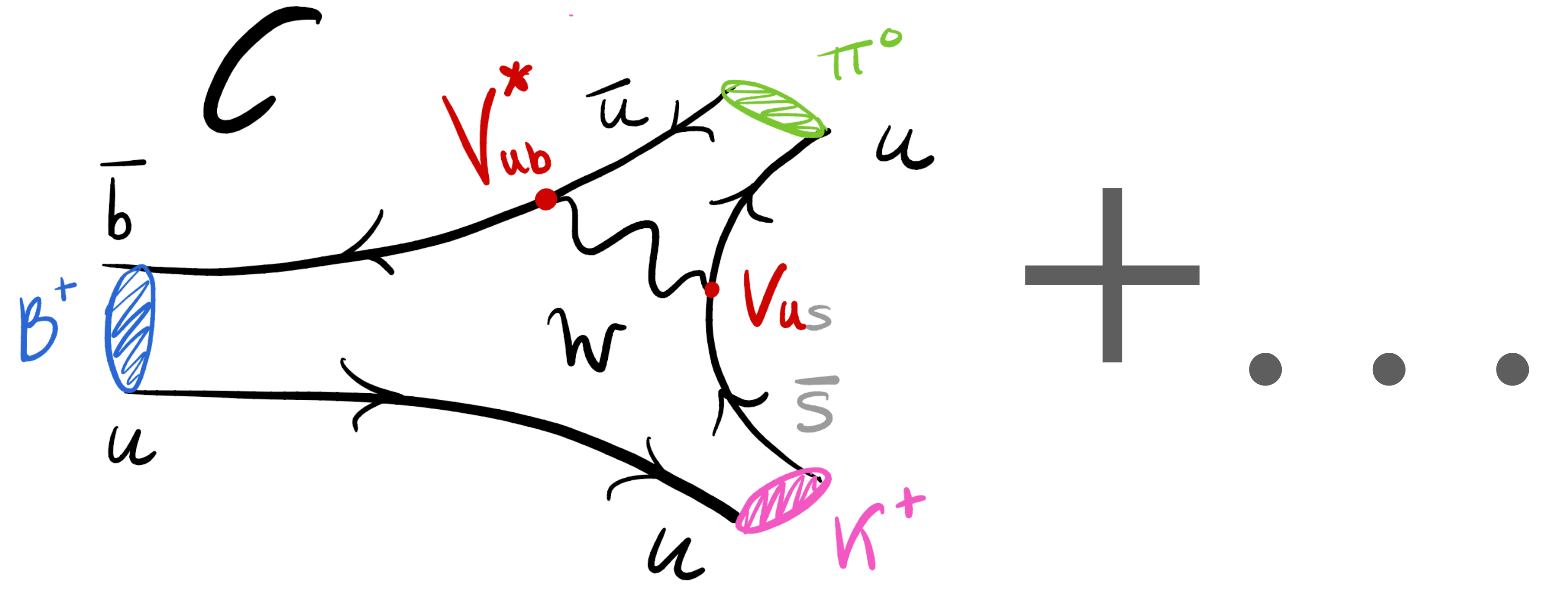

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Express **observables** in terms of the amplitudes under **topological parameterization**

+

Experimental results for Branching ratios and CP asymmetries

=

Fit the values for the **topological coefficients** in $SU(3)$ symmetry

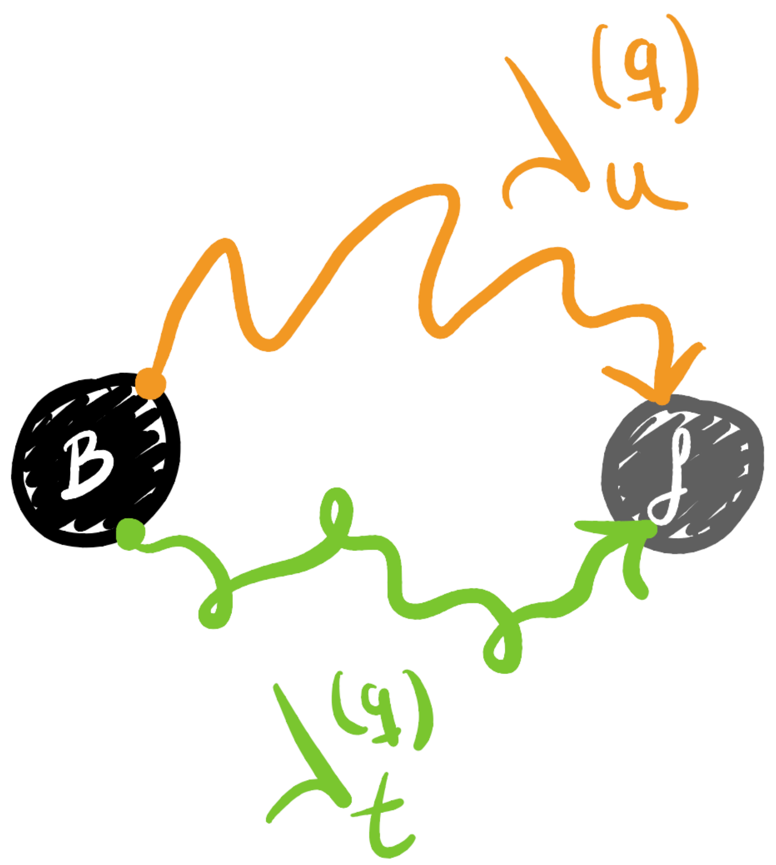
3. Extract the Coefficients from Experimental Data

Express **observables** in terms of the amplitudes under **topological parameterization** + **Experimental results** for Branching ratios and CP asymmetries = **Fit** the values for the **topological coefficients** in $SU(3)$ symmetry

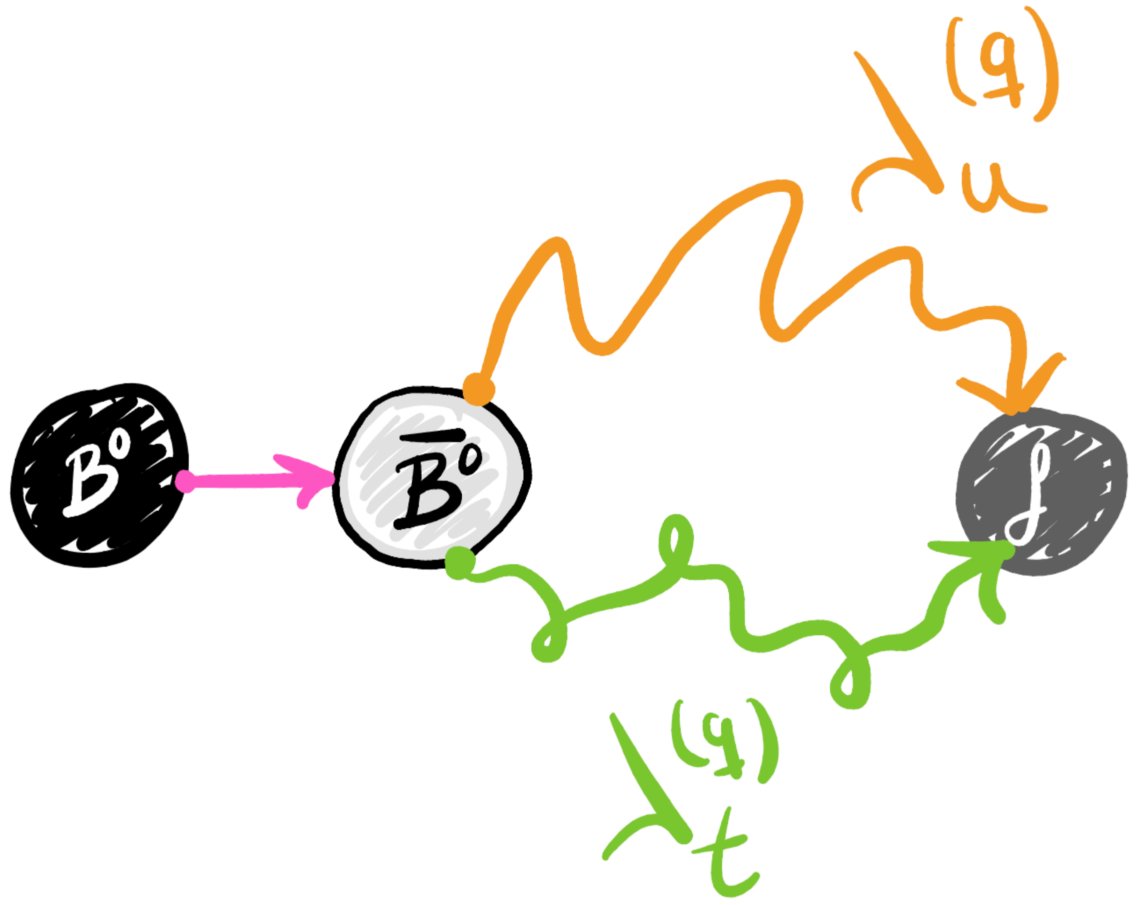
Observables

CP asymmetries $\mathscr{A}_{CP}(B \to P_1P_2) = \frac{\Gamma(B \to P_1P_2) - \Gamma(\bar{B} \to \bar{P}_1\bar{P}_2)}{\Gamma(B \to P_1P_2) + \Gamma(\bar{B} \to \bar{P}_1\bar{P}_2)}$

Direct:



Mixing-induced:



With $\Gamma(B \to P_1P_2) = f(T, C, E, \ldots)$

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Branching Ratios $\mathscr{B}(B \to P_1 P_2) \propto \frac{\Gamma(B \to P_1 P_2)}{\Gamma(B \to all)}$

With $\Gamma(B \to P_1 P_2) = f(T, C, E, \ldots)$

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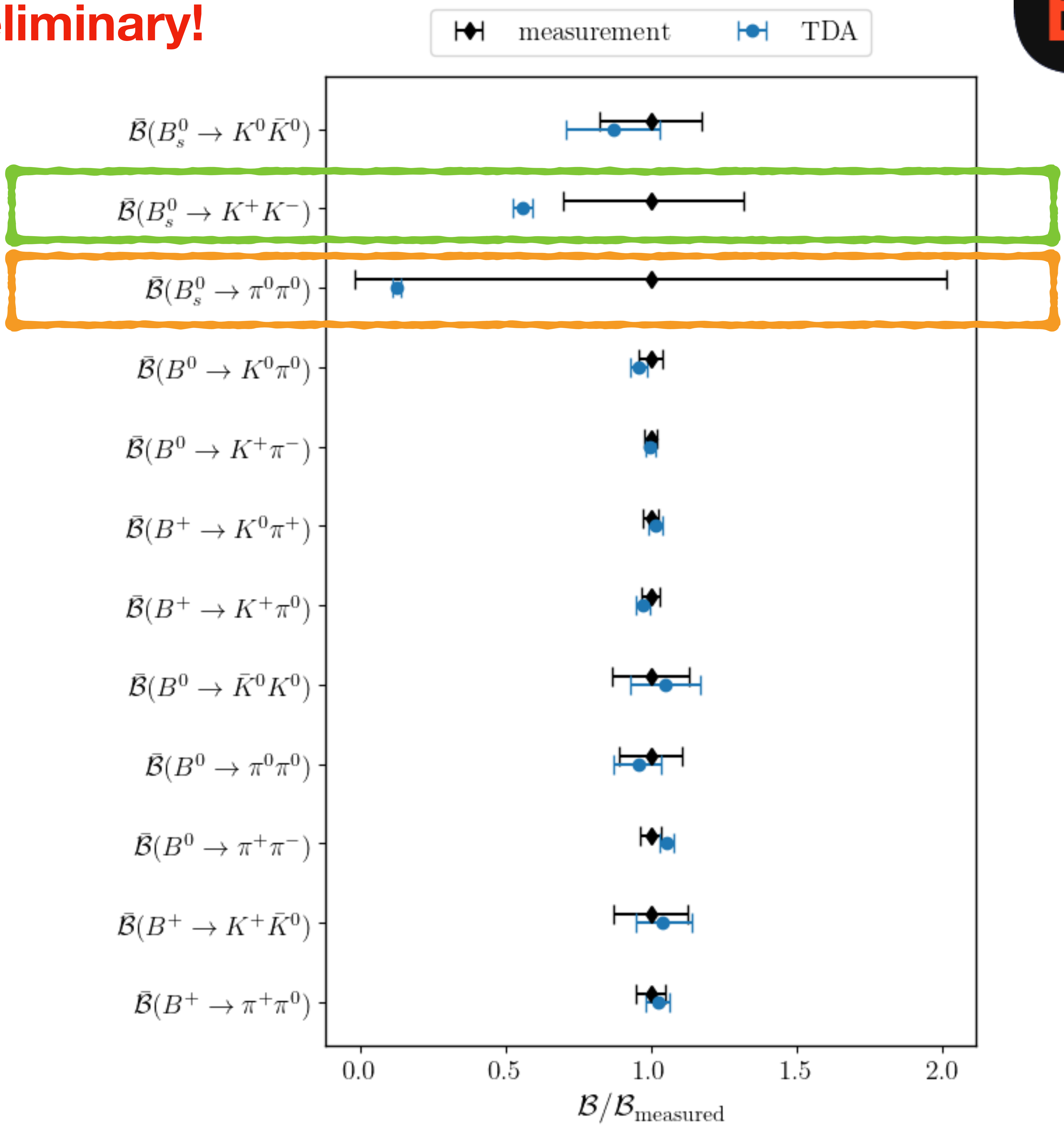
4. Predictions for the observables

Preliminary!



By constructing the observable predictions from the fitted coefficients we can:

- See if any decay deviates from the experimental result under the $SU(3)$ assumption
- Point out which measurements should be updated
- Obtain predictions for decays that have not been measured yet

Experimental data from LHCb, Belle and BaBar

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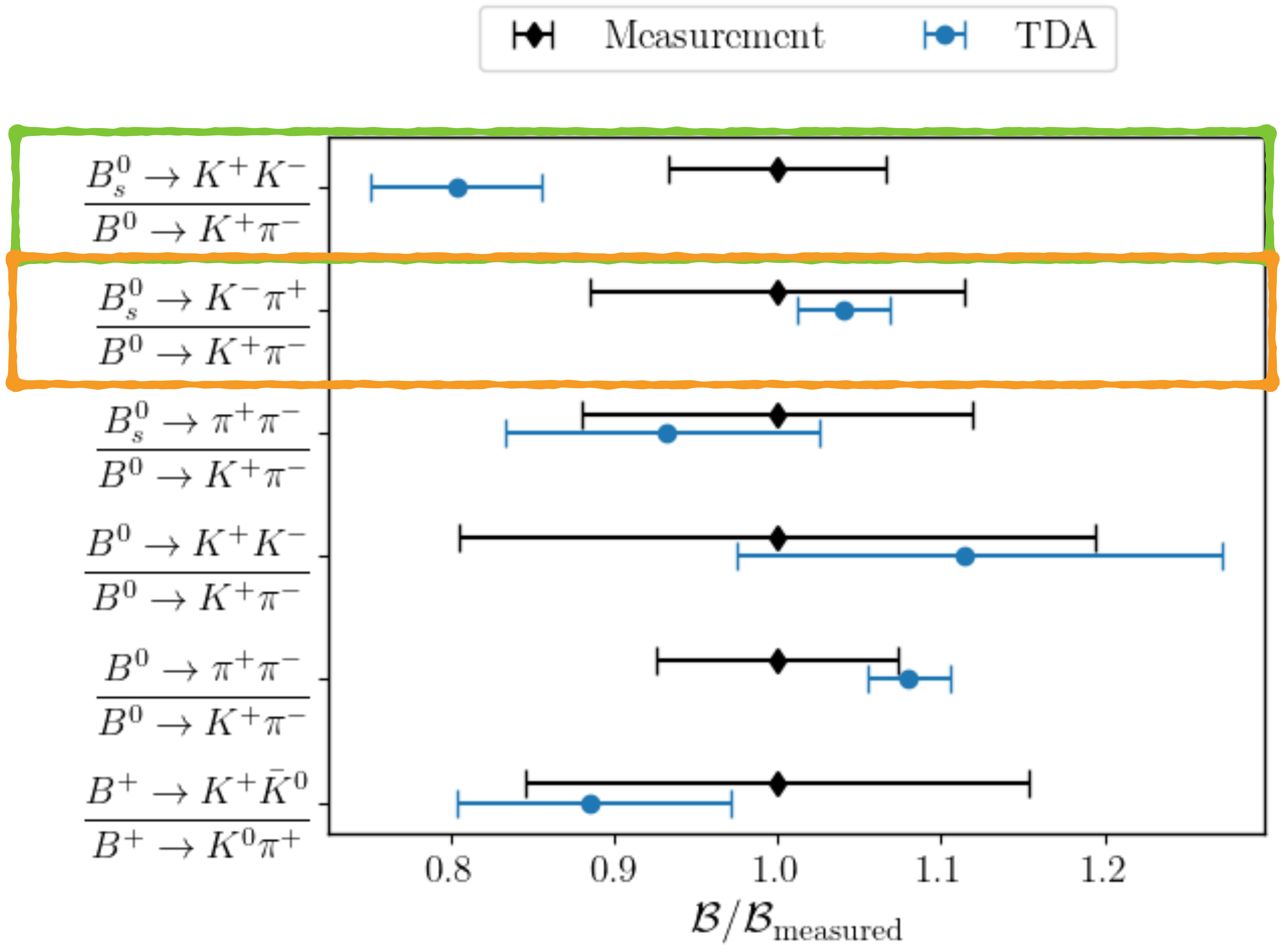

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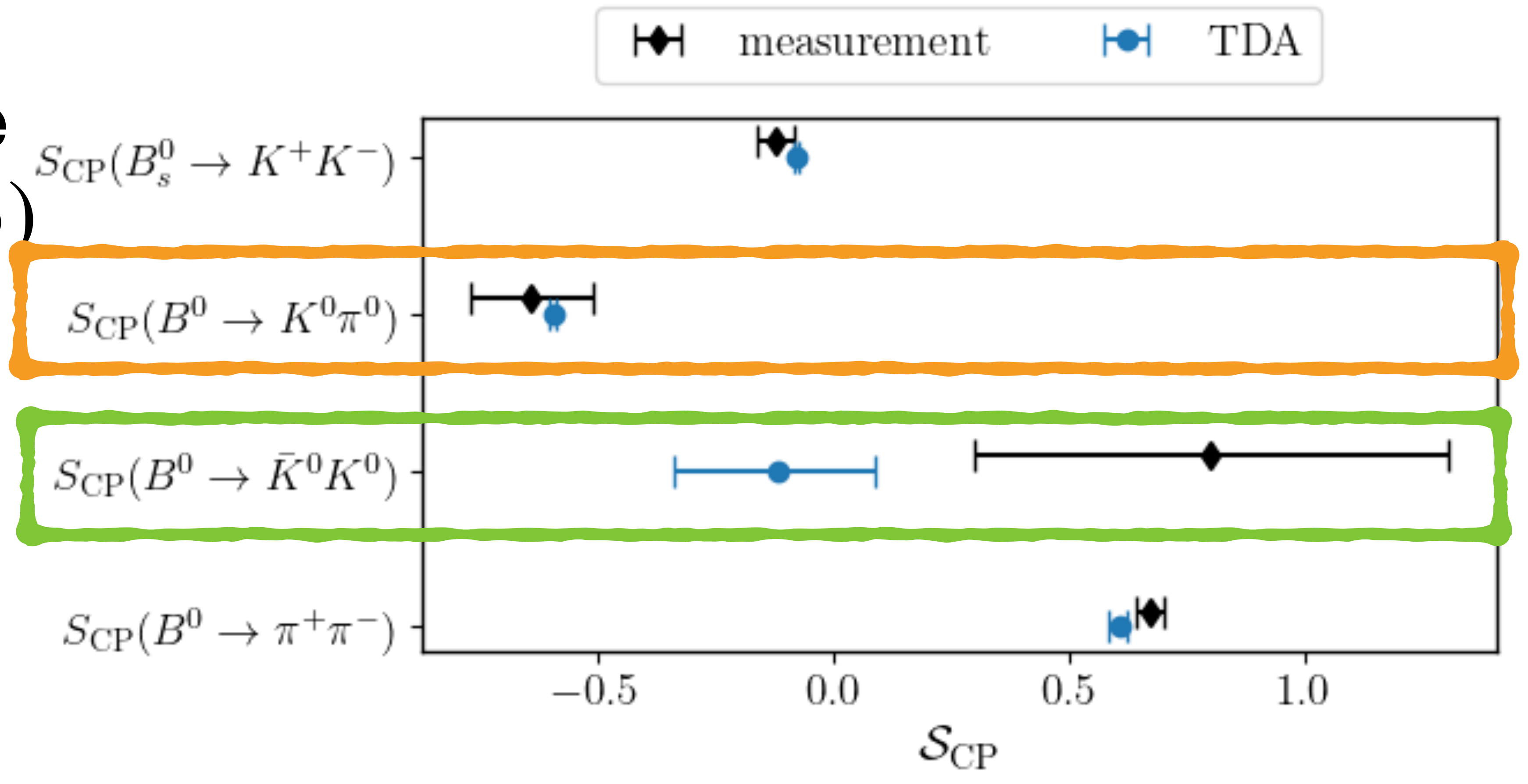

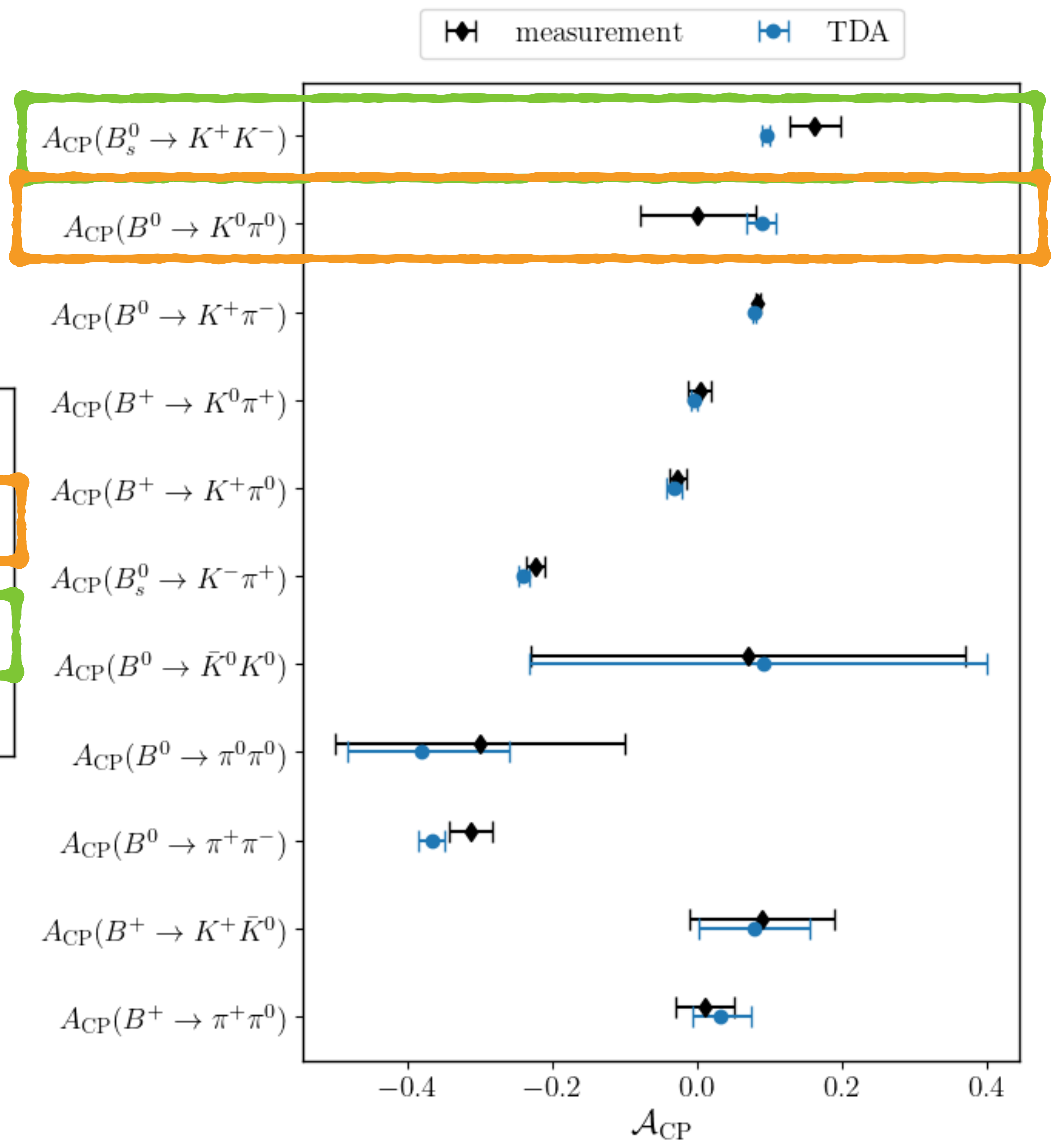

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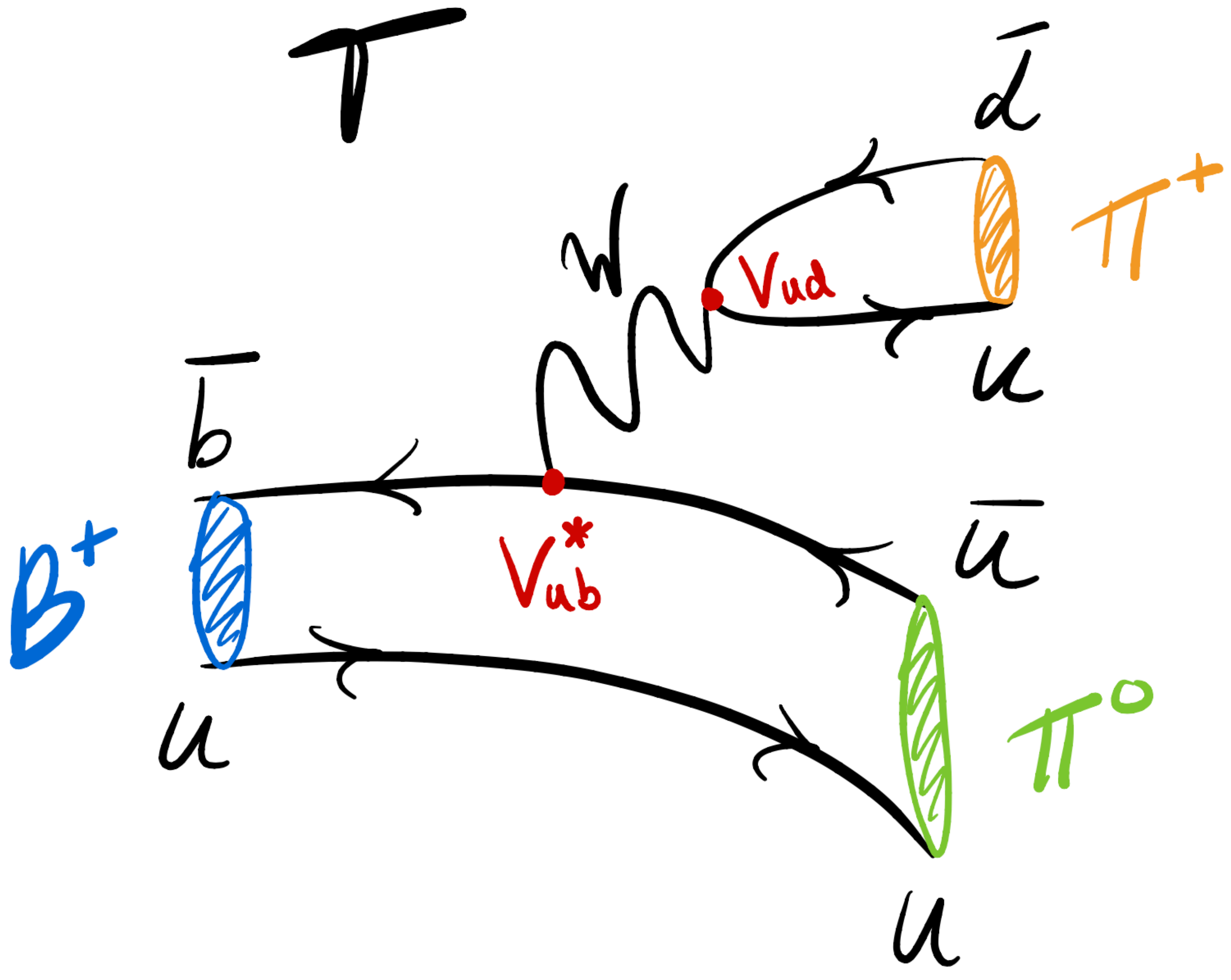
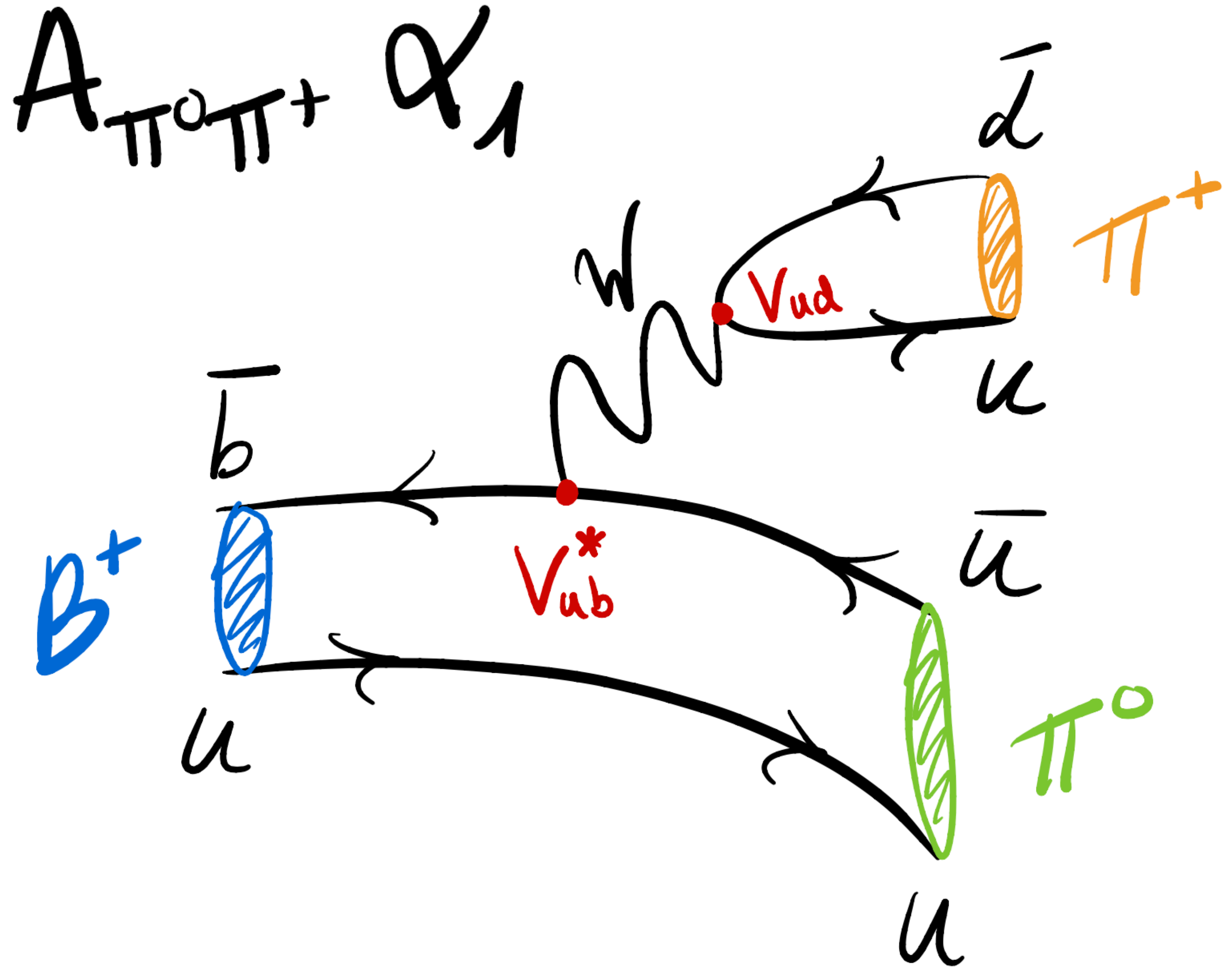
Factorizable $SU(3)$ breaking allows us to account for the different masses of the mesons without adding (almost) any new coefficient

$SU(3)$ symmetry: T → **Factorizable $SU(3)$ breaking: $A_{M_1M_2}\ \alpha_1$**





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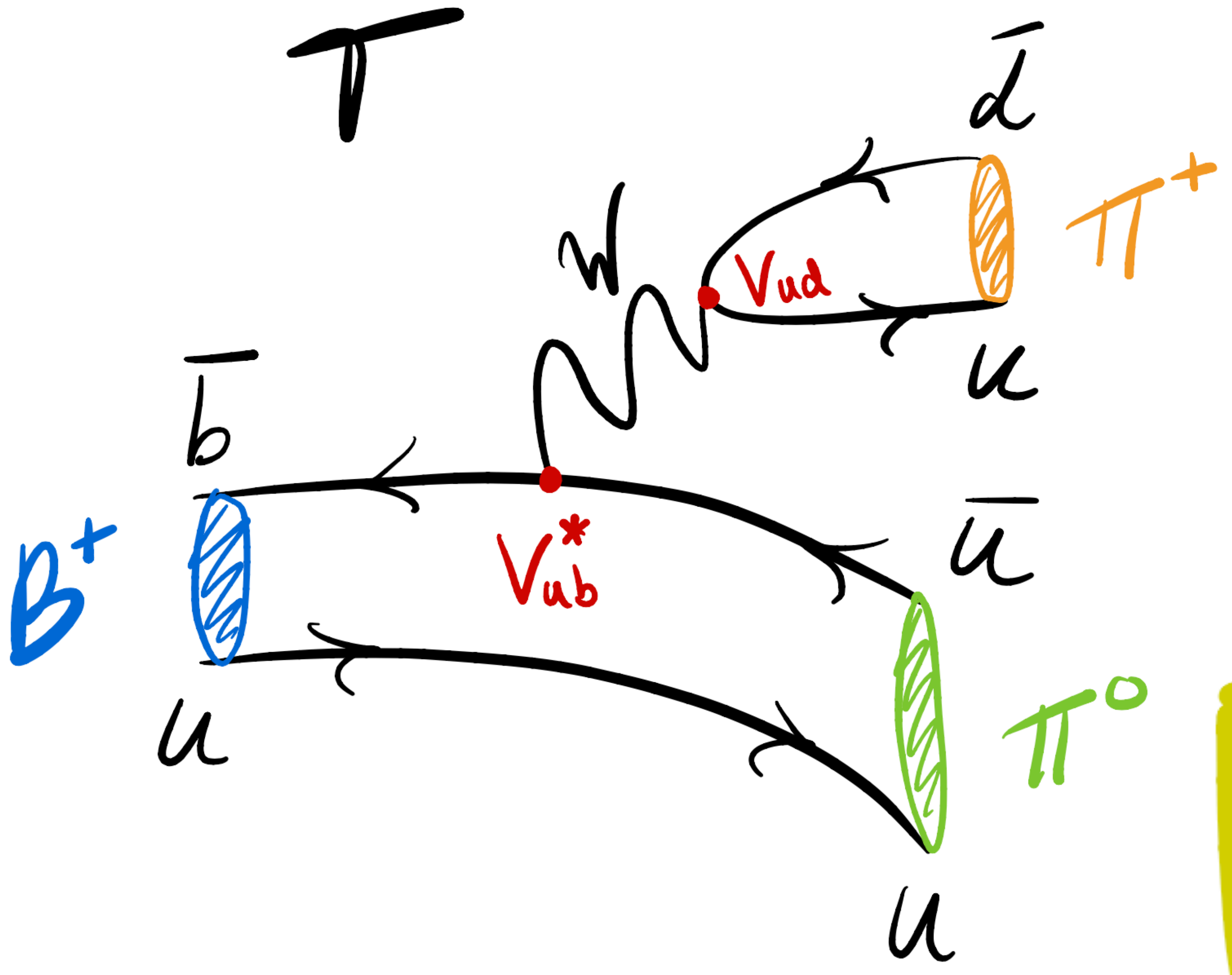
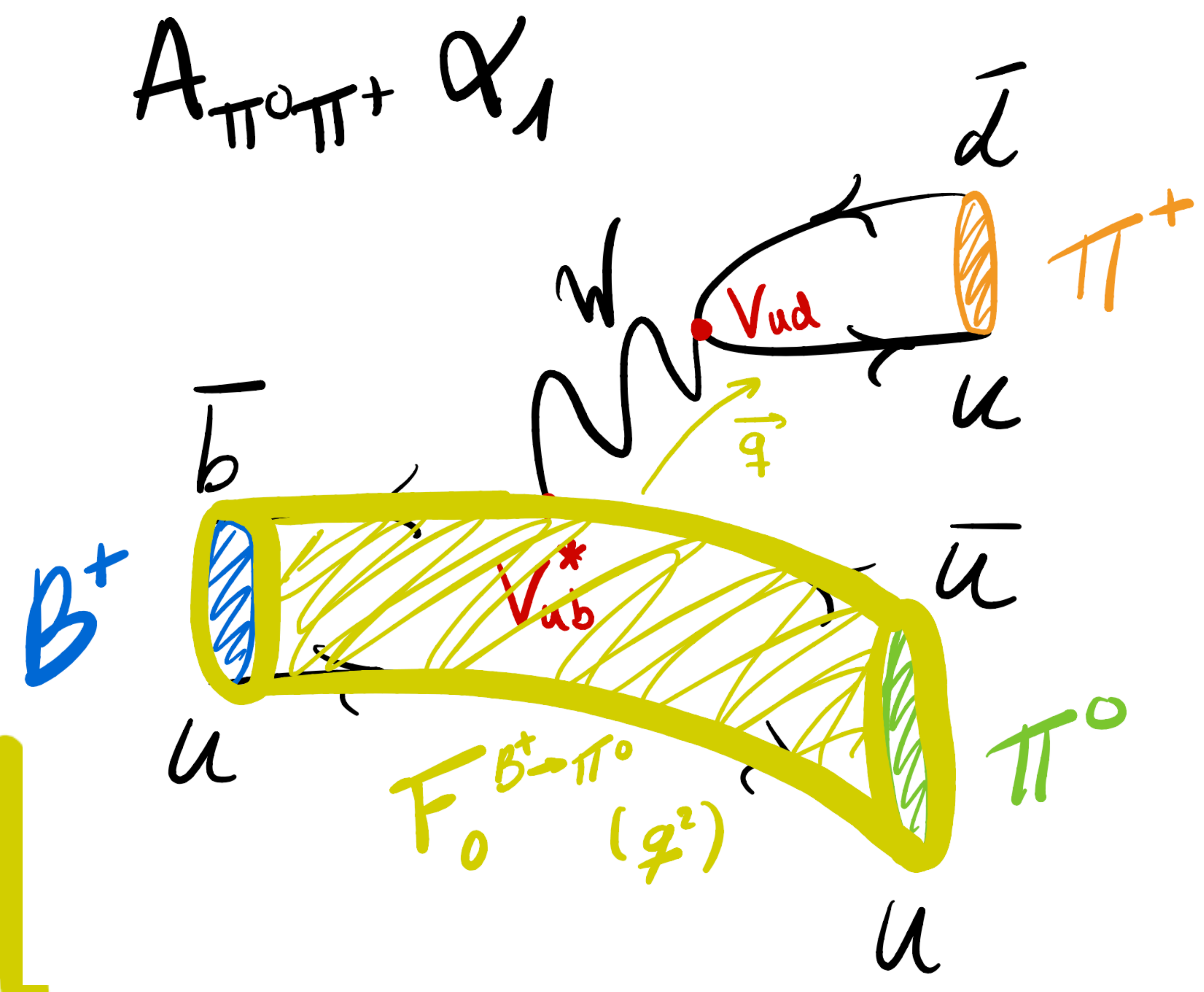


Factorizable $SU(3)$ breaking: $A_{M_1 M_2}\,\alpha_1$



Form factor: parameterizes B decays into mesons depending on the transferred momentum

$$A_{\pi^0\pi^+} = M_{B^+}^2\, F_0^{B^+\to\pi^0}(m_{\pi^+}^2)\, f_{\pi^+}$$

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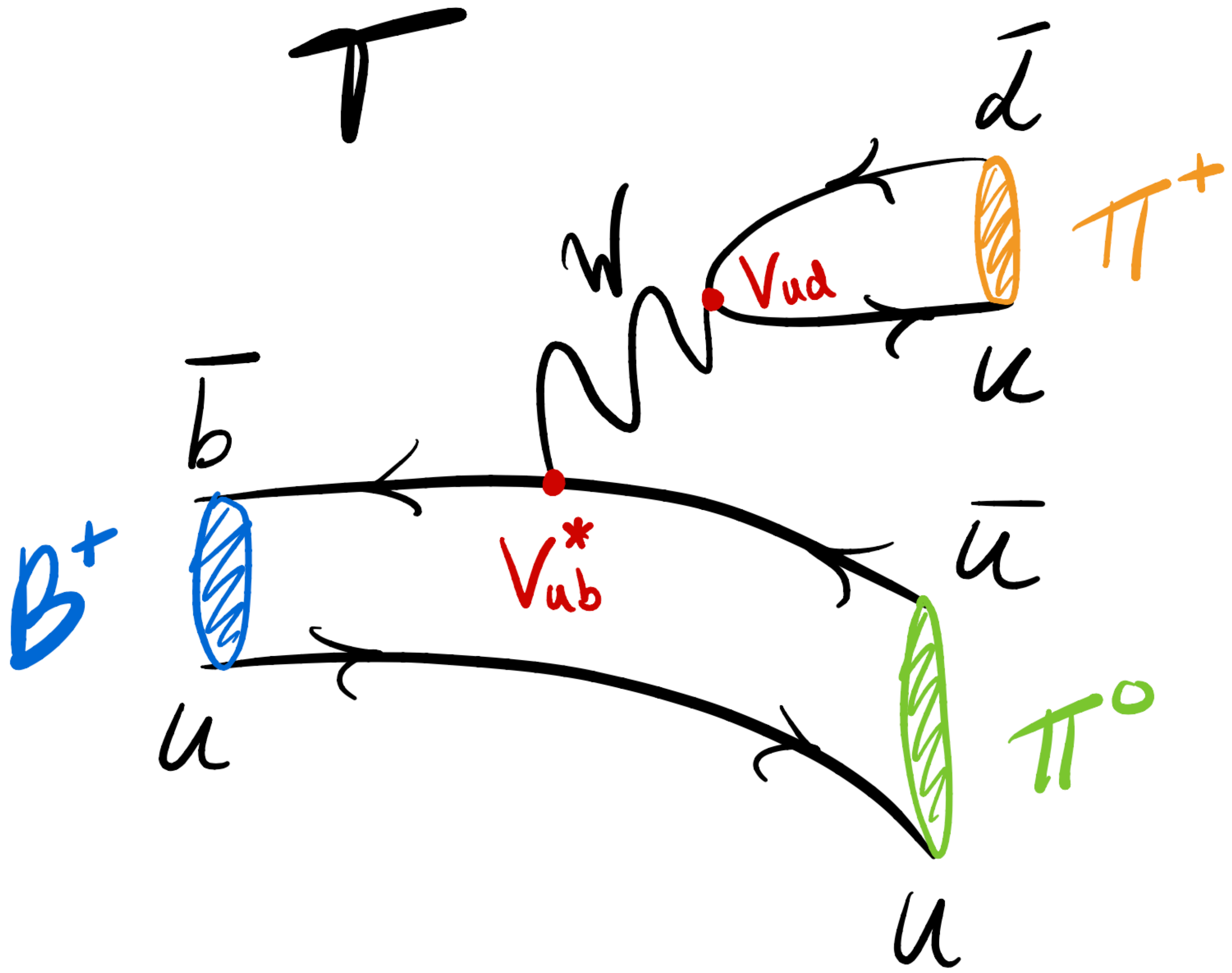

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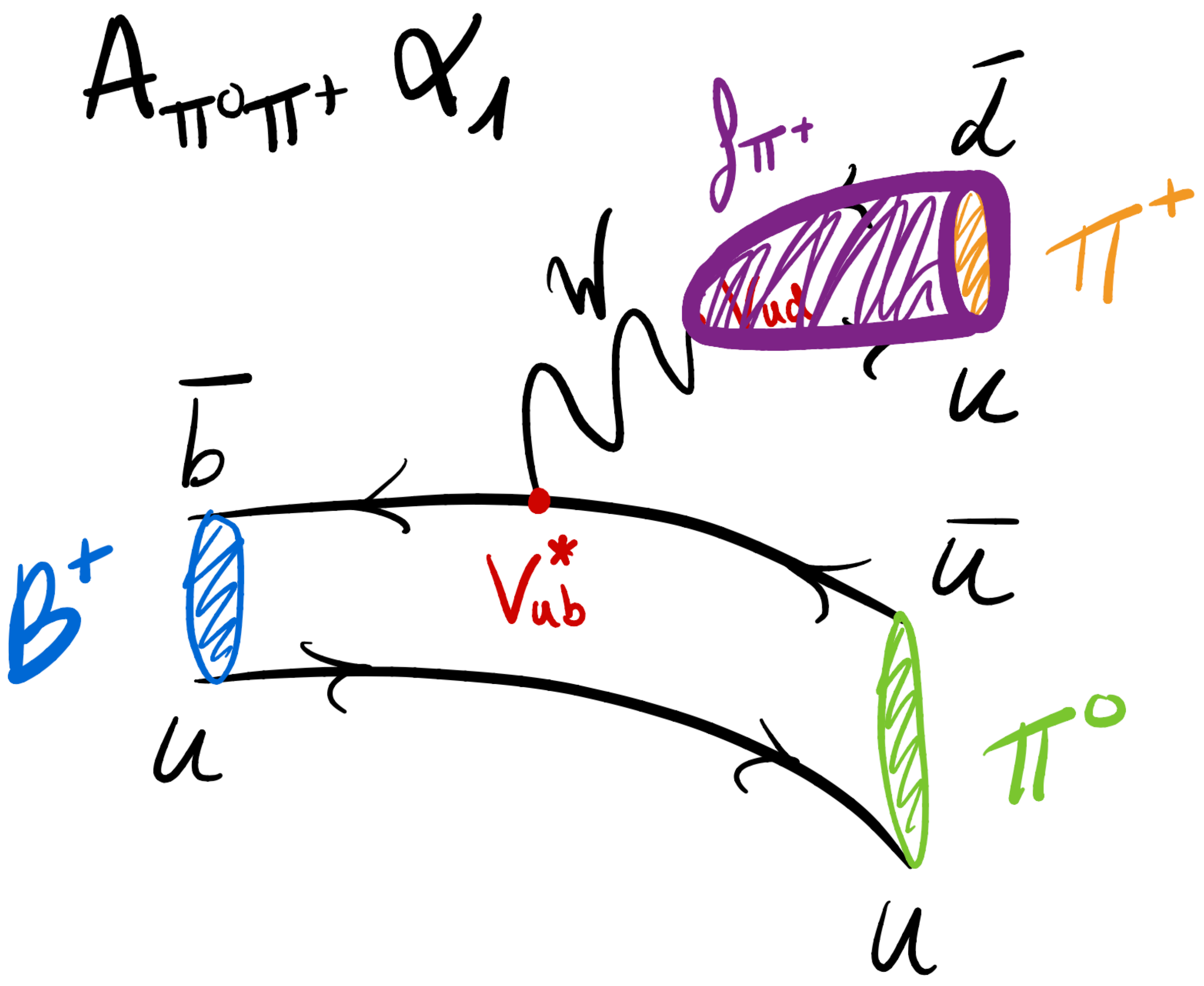

Decay constant:
amplitude for a meson to form/decay

$$A_{\pi^0\pi^+} = M^2_{B^+} F_0^{B^+\to\pi^0}(m^2_{\pi^+})\ f_{\pi^+}$$

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$SU(3)$ **symmetry:** T ➡ **Factorizable** $SU(3)$ **breaking:** $A_{M_1M_2}\ \alpha_1$

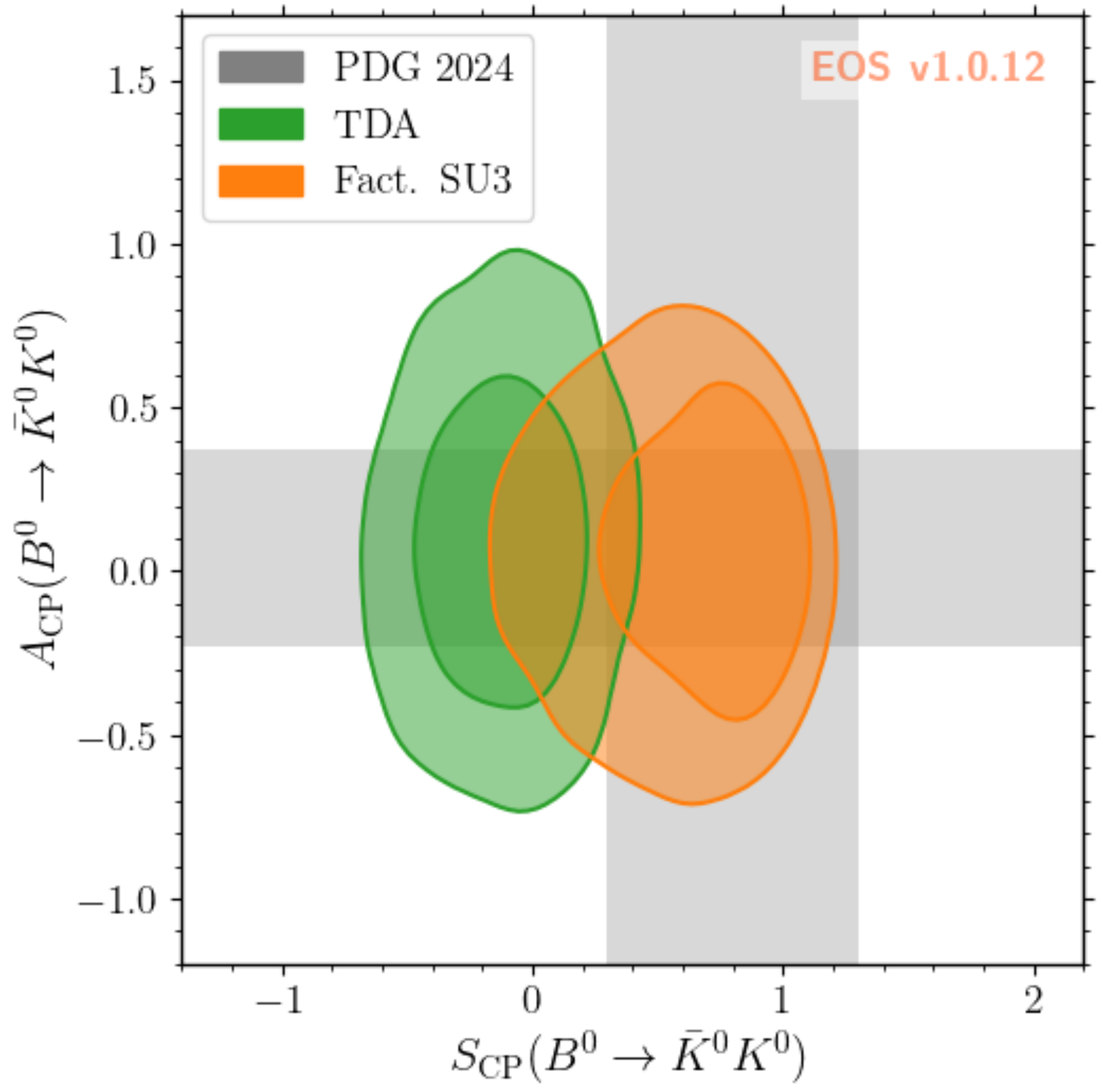
$A_{M_1M_2}$: $SU(3)$ **breaking**, but **known** from experiments! (No new coefficients)

α_1 : $SU(3)$ **symmetric**, needs to be fitted

Results: Factorizable $SU(3)$ breaking

Preliminary!

- Factorizable $SU(3)$ breaking describes data almost perfectly, with $\chi^2 \simeq 10$ for 10 degrees of freedom



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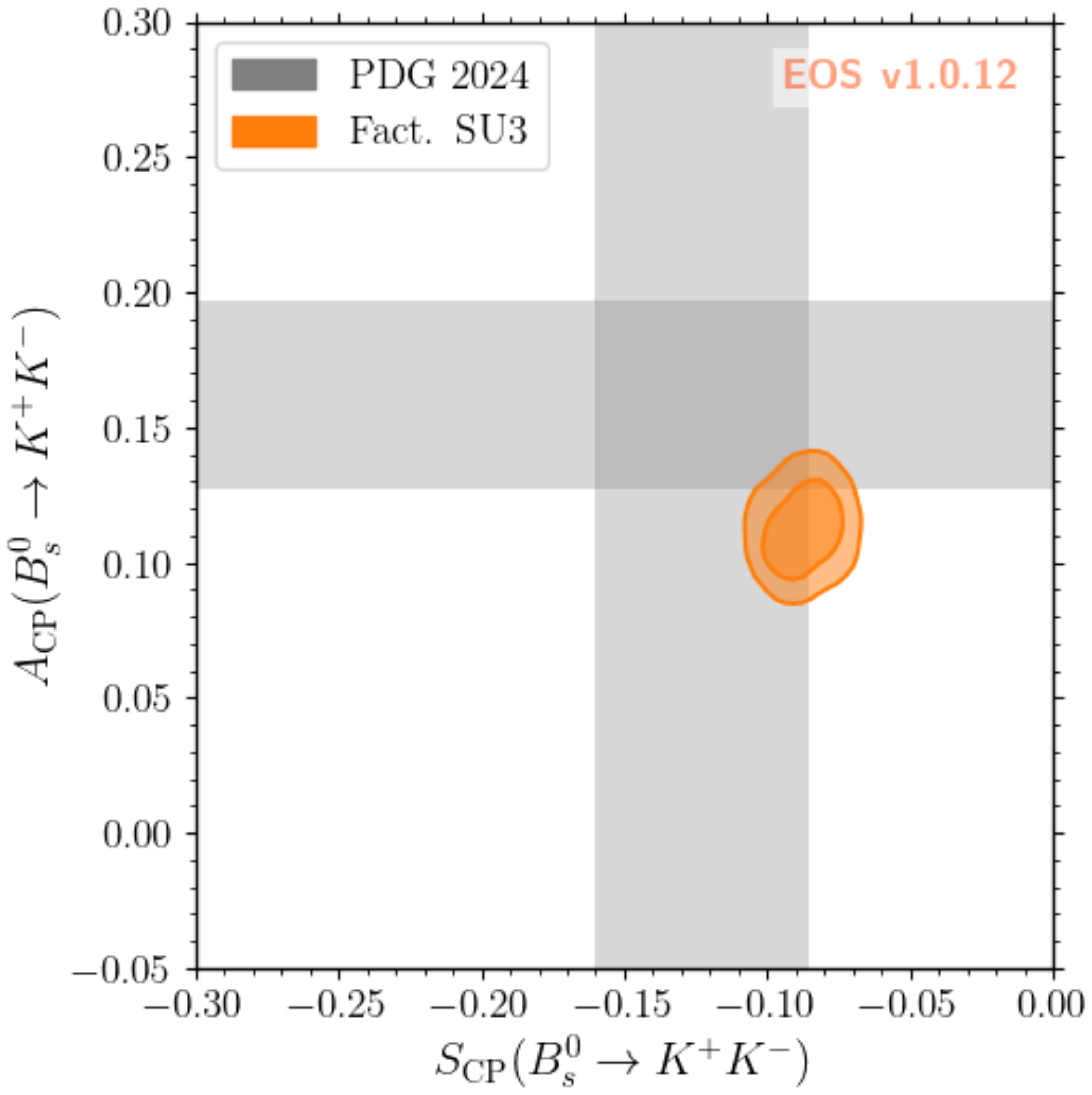
- Certain experimental measurements could be updated to test the validity of this approach



Conclusions

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- $SU(3)$ **flavor symmetry** is a useful approximation that allows us to make predictions on **hadronic decays**, for which we do not have a strict theoretical framework
- These predictions can be done by **relating similar topological** structure from different decays
- However, full $SU(3)$ cannot describe experimental data successfully

- Factorizable $SU(3)$ breaking allows to account for **factorizable dependence on the masses** without including (almost) **any new coefficients**
- Under this approach, data can be explained **almost perfectly**, although certain observables could be updated to further test its validity

Backup

$\eta - \eta'$: Flavor vs. Mass basis

Under $SU(3)$ flavor symmetry, in addition to pions and kaons, the pseudo scalar meson spectrum also includes η_8 and η_1

$$\eta_8 = \frac{uu + dd - 2 \cdot ss}{\sqrt{6}} \qquad \eta_1 = \frac{uu + dd + ss}{\sqrt{3}}$$

However, the mesons observed in experiments, η and η', are a mixture between these two

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \end{pmatrix}$$

$SU(3)$ limit in the $\eta - \eta'$ system

- In full $SU(3)$ flavor symmetry, $\theta = 0$ (therefore $\eta = \eta_8$ and $\eta' = \eta_1$) and η_8 meson is massless (like the pions and kaons.

- This is **not** the case for η_1, which remains massive even if the masses of the u, d and s are zero.

- In experiments, it is observed non-negligible mixing, $\theta \simeq -19°$: η receives contributions form η_1

- $SU(3)$ breaking is needed to describe the $\eta - \eta'$ system with a certain level of accuracy

Topological parameterization formula

Parameterize all $B \to PP$ decays in terms of topological coefficients

Each topological coefficient represents a different Feynman diagram

$$A^{TDA}(B \to PP) = i\frac{G_F}{\sqrt{2}}\left[\mathscr{T}^{TDA} + \mathscr{P}^{TDA}\right]$$

Tree amplitude

$$\begin{aligned}\mathscr{T}^{TDA} = &\ \mathbf{T}\ B_i(M)^i_j\bar{H}^{jl}_k(M)^k_l + \mathbf{C}\ B_i(M)^i_j\bar{H}^{lj}_k(M)^k_l + \mathbf{A}\ B_i\bar{H}^{il}_j(M)^j_k(M)^k_l + \mathbf{E}\ B_i\bar{H}^{li}_j(M)^j_k(M)^k_l \\ &+\mathbf{T_{ES}}\ B_i\bar{H}^{ij}_l(M)^l_j(M)^k_k + \mathbf{T_{AS}}\ B_i\bar{H}^{ji}_l(M)^l_j(M)^k_k + \mathbf{T_S}B_i(M)^i_j\bar{H}^{lj}_l(M)^k_k + \mathbf{T_{PA}}B_i\bar{H}^{li}_l(M)^j_k(M)^k_j \\ &+\mathbf{T_P}B_i(M)^i_j(M)^j_k\bar{H}^{lk}_l + \mathbf{T_{SS}}\ B_i\bar{H}^{li}_l(M)^j_j(M)^k_k\end{aligned}$$

Penguin amplitude

$$\begin{aligned}\mathscr{P}^{TDA} = &\ \mathbf{P}\ B_i(M)^i_j(M)^j_k\tilde{H}^k + \mathbf{P_T}\ B_i(M)^i_j\tilde{H}^{jl}_k(M)^k_l + \mathbf{S}\ B_i(M)^i_j\tilde{H}^{lj}_l(M)^k_k + \mathbf{P_C}\ B_i(M)^i_j\tilde{H}^{lj}_k(M)^k_l \\ &+\mathbf{P_{TA}}\ B_i\tilde{H}^{il}_j(M)^j_k(M)^k_l + \mathbf{P_A}\ B_i\tilde{H}^{li}_l(M)^j_k(M)^k_j + \mathbf{P_{TE}}\ B_i\tilde{H}^{ji}_k(M)^k_l(M)^l_j + \mathbf{P_{AS}}\ B_i\tilde{H}^{ji}_l(M)^l_j(M)^k_k \\ &+\mathbf{P_{SS}}\ B_i\tilde{H}^{li}_l(M)^j_j(M)^k_k + \mathbf{P_{ES}}\ B_i\tilde{H}^{ij}_l(M)^l_j(M)^k_k\end{aligned}$$

B-meson vector

$$B_i = (B^+, B^0, B^0_s)$$

Pseudo-scalar meson octet + singlet

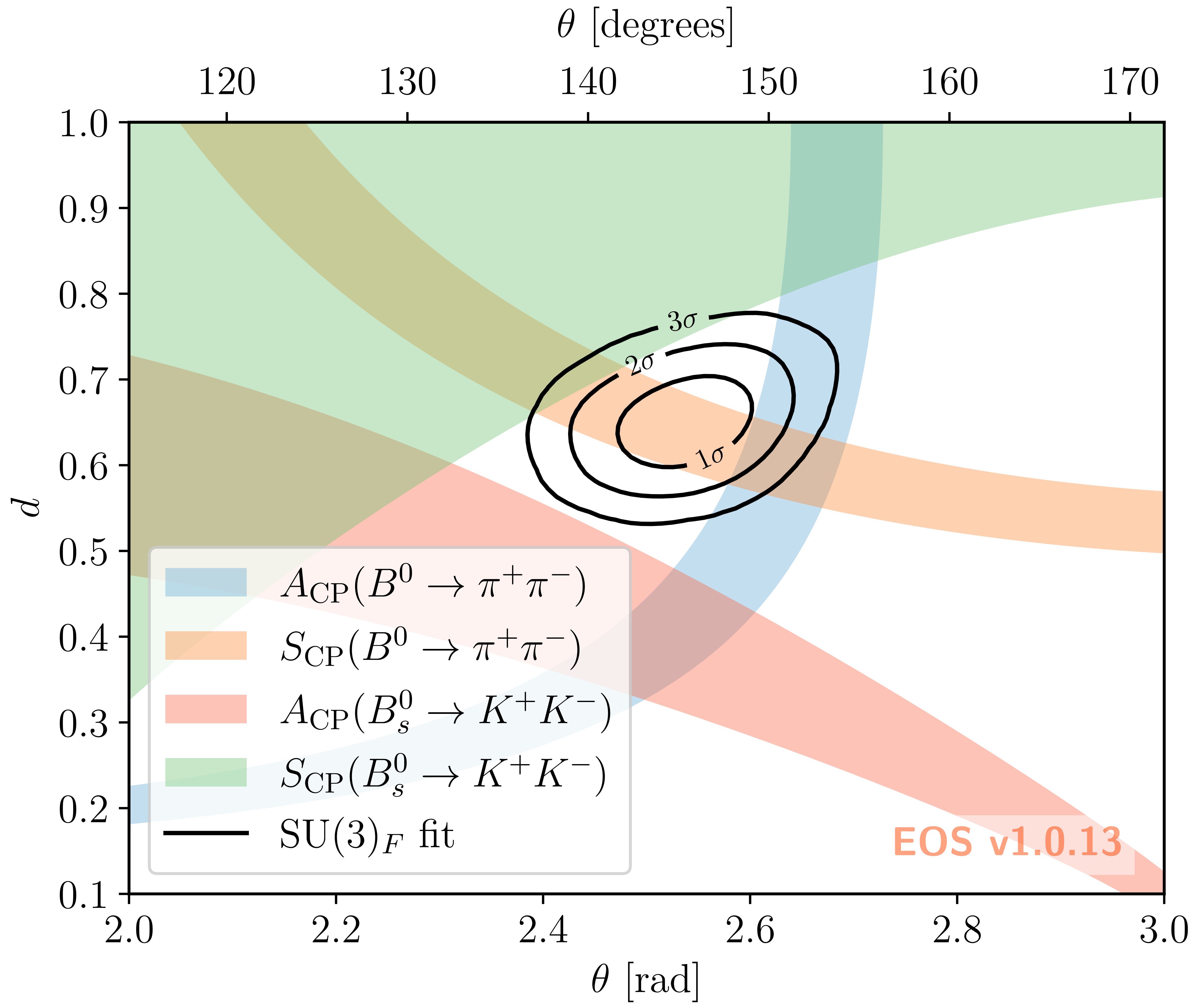
$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \bar{K}^0 \\ K^+ & K^0 & -2\frac{\eta_8}{\sqrt{6}} \end{pmatrix} + \begin{pmatrix} \frac{\eta_0}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{\eta_0}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{\eta_0}{\sqrt{3}} \end{pmatrix}$$

Flavor tensor (CKM elements)

$$H^{j,k}_i$$

U-Spin partners

Preliminary!



$SU(3)$ symmetry is not preserved in the experimental results, therefore the fit will not be able to accommodate all four observables simultaneously

Experimental results from $B^0 \to \pi^+\pi^-$ are more precise, therefore the fit will adjust to better align with the observables from this decay

Role of interference in CP violation

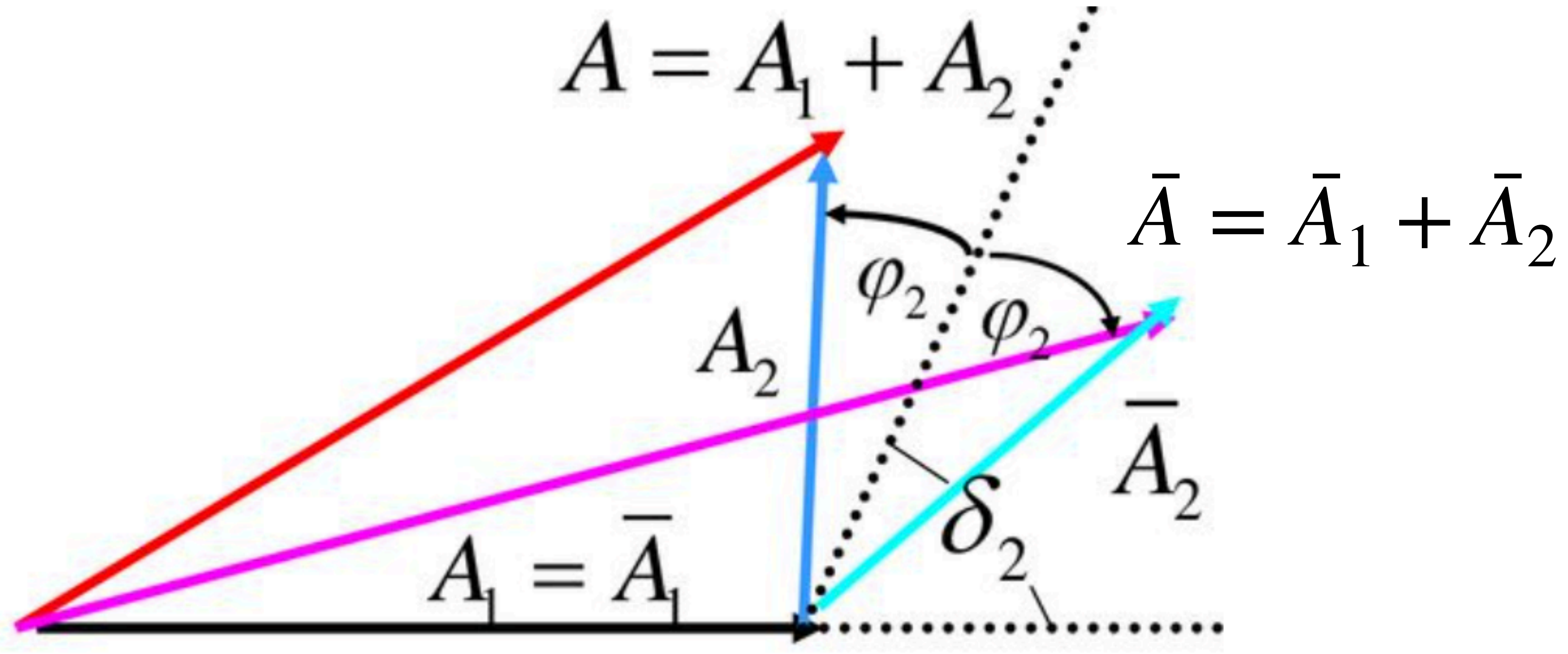
Consider the amplitude of a certain process $B \to f$. CP violation arises if for a certain observable $\mathcal{O}$ we have $\mathcal{O}(B \to f) \neq \mathcal{O}(\bar{B} \to \bar{f})$

Observables depend only on the **modulus** of the amplitude

Only weak phases are shifted when they are CP conjugated

We can define de amplitude of the process as $A = A_1 + A_2$

CP violation only appears if A_1 and A_2 have a relative weak phase, ϕ_2, and strong phase, δ_2



$A_1 = |A_1|$

$A_2 = e^{i\delta_2} e^{i\phi} |A_2|$