Nonlocality and entanglement in quantum information

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Theory Meets Experiment - Quantum Observables
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Formalism of quantum mechanics

States

Contains the whole description of the system: $\boxed{|\psi\rangle \in \mathcal{H}}$

Example: $|\psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle \in \mathcal{H} \cong \mathbb{C}^2$

Multipartite system

$\boxed{|\psi\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \dots}$ (Notation: $|\psi_1\rangle \otimes |\psi_2\rangle \equiv |\psi_1\psi_2\rangle$)

Example: $|\Psi^+\rangle = \frac{1}{\sqrt{2}}|\uparrow\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\downarrow\rangle \in \mathcal{H} \otimes \mathcal{H} \cong \mathbb{C}^4$

Measurement

Set of projective operators $\{|a\rangle\langle a|\}_a$

$$\boxed{|\psi_{\text{post-meas}}\rangle = \frac{\langle a|\psi\rangle}{\sqrt{P(a|\psi)}}|a\rangle \quad , \quad P(a|\psi) = |\langle a|\psi\rangle|^2}$$

Formalism of quantum information

States

Description of what is accessible (stat.mix.): $\boxed{\rho = \sum_i p_i \, |\psi_i\rangle\langle\psi_i|}$

Example: $\rho = \frac{1}{2}\,|\uparrow\rangle\langle\uparrow| + \frac{1}{2}\,|\downarrow\rangle\langle\downarrow|$

Multipartite system

$\boxed{\rho_{ABC\ldots} \in \mathrm{Endo}(\mathcal{H}_A \otimes \mathcal{H}_B \otimes \ldots)}$

Example: $\rho = \frac{1}{2}\,|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \frac{1}{2}\,|\downarrow\downarrow\rangle\langle\downarrow\downarrow|$

Measurement

Set of **positive** operators $\{F_a\}_a$ satisfying $\sum_a F_a = \mathbb{I}$ (POVM)

$$\boxed{\rho_{\text{post-meas}} = \frac{U\sqrt{F_a}\rho\sqrt{F_a}^{\dagger}U^{\dagger}}{P(a|\rho)} \quad , \quad P(a|\rho) = \mathrm{tr}(F_a\rho)}$$

where U is a unitary operator.

Formalism of quantum information - Remarks

Superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}\,|\uparrow\rangle + \frac{1}{\sqrt{2}}\,|\downarrow\rangle$$

Mixed state

$$\rho = \frac{1}{2}\,|\uparrow\rangle\langle\uparrow| + \frac{1}{2}\,|\downarrow\rangle\langle\downarrow|$$

$|\psi\rangle$ and ρ don't represent the same system!

$$|\psi\rangle\langle\psi| = \frac{1}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|) \neq \rho$$

Multipartite states

Separable state

Use only local operations and classical communication (LOCC)

$$\rho_{AB\ldots} = \sum_i p_i \, \rho_A^{(i)} \otimes \rho_B^{(i)} \otimes \ldots$$

Example: $\rho_{AB} = \frac{1}{2} |\uparrow\rangle\langle\uparrow| \otimes |\uparrow\rangle\langle\uparrow| + \frac{1}{2} |\downarrow\rangle\langle\downarrow| \otimes |\downarrow\rangle\langle\downarrow|$

Entangled state

ρ_{AB} is not separable

Example: $\rho_{AB} = \frac{1}{2} \frac{\mathbb{I}_4}{4} + \frac{1}{2} |\Psi^+\rangle\langle\Psi^+|$, with $|\Psi^+\rangle := \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}$

Detect entanglement

Positive partial transpose (PPT) criterion[1,2]

$$\rho_{AB} \text{ separable} \Rightarrow \rho_{AB}^{T_B} = \sum_i p_i\, \rho_A^{(i)} \otimes \rho_B^{(i)\,T} \succeq 0$$

K-symmetric extensions [3,4]

$$\rho_{AB} \text{ separable} \Leftrightarrow \forall k, \exists \rho_{AB_1 B_2 \ldots B_k} \text{ such that } \rho_{AB_i} = \rho_{AB}, \forall i$$

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[1] Peres, Asher. "Separability Criterion for Density Matrices." Physical Review Letters 77

[2] Horodecki, Michał, Paweł Horodecki, and Ryszard Horodecki. "Separability of Mixed States: Necessary and Sufficient Conditions." Physics Letters A 223, no. 1

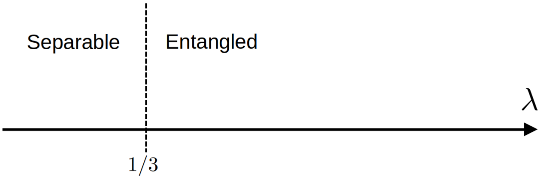
[3] Fannes, M., J. T. Lewis, and A. Verbeure. "Symmetric States of Composite Systems." Letters in Mathematical Physics 15, no. 3

[4] Raggio, G. A., and R. F. Werner. "Quantum Statistical Mechanics of General Mean Field Systems"

Mixture of entangled and noisy state

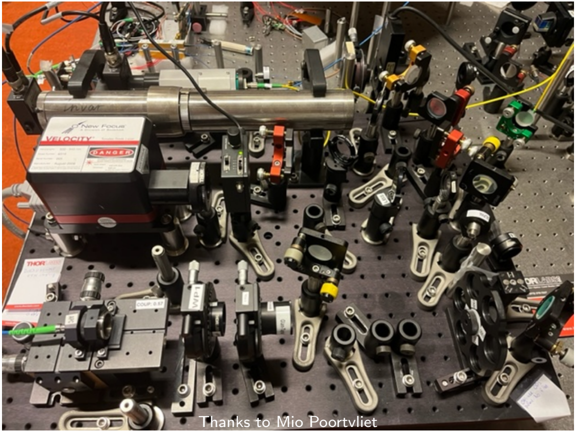
Werner state[5]

$$\rho_{AB} = \lambda \left|\Psi^+\right\rangle\left\langle\Psi^+\right| + (1-\lambda)\frac{\mathbb{I}_4}{4}$$



[5] Werner, Reinhard F. "Quantum States with Einstein-Podolsky-Rosen Correlations Admitting a Hidden-Variable Model." Physical Review A 40, no. 8 (October 1, 1989)

Limitations of the model



Thanks to Mio Poortvliet

What is ρ? What is the Hilbert space?

Limitation of the model

Problem

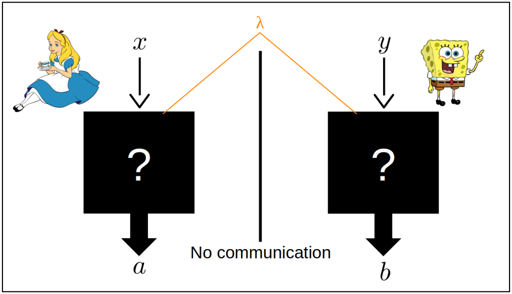
Models assume ideal/simplistic setup and conditions
$\Rightarrow$ state we have is not the one we expect

Solution

Look directly at the correlations of the device
$\Rightarrow$ Bell tests

Bell test

Device: $x, y \to a, b$



Repeat experiment $\Rightarrow P(a, b|x, y)$

Correlation types

Local hidden variable (LHV) correlations[6]

$$P(a,b|x,y) = \int_\Lambda d\lambda\, q(\lambda) P(a|x,\lambda) P(b|y,\lambda)$$

Quantum correlations

$$P(a,b|x,y) = \mathrm{Tr}[(M_x^{(a)} \otimes M_y^{(b)})\rho_{AB}]$$

[6] Bell, JS. "On the Einstein Podolsky Rosen Paradox" 1, no. 3 (1964)

Bell inequality

Bell expression

$$I[P] := \sum_{a,b,x,y} \alpha_{abxy} P(a, b|x, y)$$

For wisely chosen α_{abxy}, we can find $P \in$ Quantum s.t.

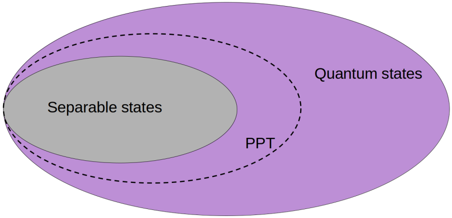
$$I[P] > \max_{P \in \text{Local}} I[P]$$

Example: CHSH inequality

$$\sum_{a,b,x,y=0}^{1} (-1)^{a+b+xy} P(a, b|x, y) \leq 2 \quad \text{(local bound)}$$

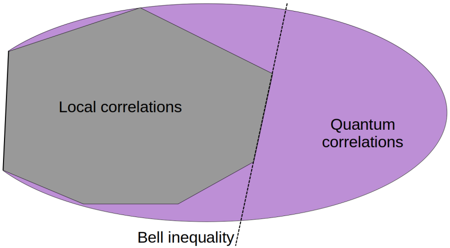
Quantum bound: $2\sqrt{2}$

Sets of quantum states and correlations



Set of quantum states

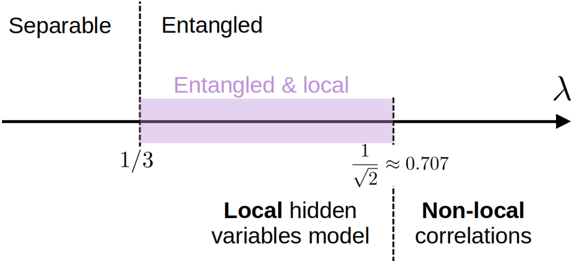
Set of correlations



Entangled is not enough!

Werner state

$$\rho_{AB} = \lambda \,|\Psi\rangle\langle\Psi| + (1-\lambda)\frac{\mathbb{I}_4}{4}$$



Quantum information in HEP

Testing Bell inequalities in Higgs boson decays

Alan J. Barr
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(Dated: First submitted: 2 June 2021. This version: July 27, 2022)

Higgs boson decays produce pairs of W bosons in a maximally entangled state, the spins of which can be expected to violate Bell inequalities. We show that the spin density matrix of the $W^{\pm}$ pair may be reconstructed experimentally from the directions of the charged lepton decay products, and from it the expectation values of various Bell operators determined. Numerical simulations of $H \to WW^*$ decays indicate that violation of a generalised CHSH inequality is unlikely to be measurable, however the CGLMP inequality is near-maximally violated. Experimental Bell tests could be performed at a variety of colliders and in different production channels. If reconstruction effects and backgrounds can be controlled then statistically significant violations could be observable even with datasets comparable to those already collected at the LHC.

Suggestion for Einstein–Podolsky–Rosen Experiments Using Reactions Like $e^+e^- \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$

Nils A. Törnqvist[1]

Received July 9, 1980

Since weakly decaying particles are their own polarimeters, reactions like $\eta_c \to \Lambda\bar{\Lambda}$, $\psi \to \Lambda\bar{\Lambda}$, $e^+e^- \to \mu^+\mu^-$, etc. are interesting for testing the non-locality of quantum mechanical predictions. Although such reactions, in principle, do not exclude all classes of hidden variable theories, they can be used to complement current experiments with external polarimeters. The reaction $\eta_c \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$ is conceptually the simplest and most useful as a gedanken experiment, although it has not yet been seen experimentally. The reaction $e^+e^- \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$ near threshold or at the ψ resonance can be used for essentially the same test. This is feasible with presently available data and would be the first EPR experiment involving weak interactions.

Quantum state tomography, entanglement detection and Bell violation prospects in weak decays of massive particles

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ABSTRACT: A rather general method for determining the spin density matrix of a multi-particle system from angular decay data is presented. The method is based on a Bloch parameterisation of the d-dimensional generalised Gell-Mann representation of ρ and exploits the associated Wigner- and Weyl-transforms on the sphere. Each parameter of a (possibly multipartite) spin density matrix can be measured from a simple average over an appropriate set of experimental angular decay distributions. The general procedures for both projective and non-projective decays are described, and the Wigner P and Q symbols calculated for the cases of spin-half, spin-one, and spin-3/2 systems. The methods are used to examine Monte Carlo simulations of pp collisions for bipartite systems: $pp \to W^+W^-$, $pp \to ZZ$, $pp \to ZW^+$, $pp \to W^+\bar{t}$, $t\bar{t}$, and those from the Higgs boson decays $H \to WW^*$ and $H \to ZZ^*$. Measurements are proposed for entanglement detection, exchange symmetry detection and Bell inequality violation in bipartite systems.

Quantum entanglement and Bell inequality violation at colliders

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Abstract

The study of entanglement in particle physics has been gathering pace in the past few years. It is a new field that is providing important results about the possibility of detecting entanglement and testing Bell inequality at colliders for final states as diverse as top-quark or τ-lepton pairs, massive gauge bosons and vector mesons. In this review, after presenting definitions, tools and basic results that are necessary for understanding these developments, we summarize the main findings—as published up to the end of year 2023. These investigations have been mostly theoretical since the experiments are only now catching up, with the notable exception of the observation of entanglement in top-quark pair production at the Large Hadron Collider. We include a detailed discussion of the results for both qubit and qutrits systems, that is, final states containing spin one-half and spin one particles. Entanglement has also been proposed as a new tool to constrain new particles and fields beyond the Standard Model and we introduce the reader to this promising feature as well.

Bell inequalities - Loopholes

Locality loophole

Alice and Bob devices communicate

Detection loophole

Set of detected events is an unfair sample

Superdeterminism loophole

No free will. Everything (even the measurement choices) is governed by the same random variable.

Conclusion

Separable VS Entangled states

- PPT criterion, K-symmetric extension, ...
- Entangled but still admits LHV model

Local VS Nonlocal correlations

- Bell test
- LHV model is not enough to describe QM
- Loopholes: locality, detection, "free will",...

Bell inequality in HEP - example 1

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Key point: reconstructing the density matrix

Before doing anything, we need to get the density matrix.
Is there a way to bypass the density matrix reconstruction $\rightarrow$ to discuss

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