

# Nonlocality and entanglement in quantum information

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# Formalism of quantum mechanics

## States

Contains the whole description of the system:  $|\psi\rangle \in H$

Example:  $|\psi\rangle = \frac{1}{\sqrt{2}}|j\rangle + \frac{1}{\sqrt{2}}|k\rangle \in H = \mathbb{C}^2$

## Multipartite system

$|\psi\rangle \in H_1 \otimes H_2 \otimes \dots$  (Notation:  $|\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_1\psi_2\rangle$ )

Example:  $|\Psi^+\rangle = \frac{1}{\sqrt{2}}|j\rangle + \frac{1}{\sqrt{2}}|k\rangle \in H \otimes H = \mathbb{C}^4$

## Measurement

Set of projective operators  $\{P_a\}$

$$|\psi_{\text{post-meas}}\rangle = \frac{P_a|\psi\rangle}{\sqrt{\langle\psi|P_a|\psi\rangle}} \quad , \quad P(a|\psi) = \langle\psi|P_a|\psi\rangle$$

# Formalism of quantum information

## States

Description of what is accessible (stat.mix.):  $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

Example:  $\rho = \frac{1}{2} |j\rangle\langle j| + \frac{1}{2} |j^\# \rangle\langle j^\#|$

## Multipartite system

$\rho_{ABC} \dots \in \text{Endo}(H_A \otimes H_B \otimes \dots)$

Example:  $\rho = \frac{1}{2} |j\rangle\langle j| + \frac{1}{2} |j^\# \rangle\langle j^\#|$

## Measurement

Set of **positive** operators  $F_a$  satisfying  $\sum_a F_a = I$  (POVM)

$$\rho_{\text{post-meas}} = \frac{U^\dagger F_a \rho F_a U}{P(a|\rho)}, \quad P(a|\rho) = \text{tr}(F_a \rho)$$

where  $U$  is a unitary operator.

# Formalism of quantum information - Remarks

## Superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}|j''\rangle + \frac{1}{\sqrt{2}}|j'\rangle$$

## Mixed state

$$\rho = \frac{1}{2}|j''\rangle\langle j''| + \frac{1}{2}|j'\rangle\langle j'|$$

$|\psi\rangle$  and  $\rho$  don't represent the same system!

$$|\psi\rangle\langle\psi| = \frac{1}{2}(|j''\rangle\langle j''| + |j''\rangle\langle j'| + |j'\rangle\langle j''| + |j'\rangle\langle j'|) \notin \rho$$

# Multipartite states

## Separable state

Use only local operations and classical communication (LOCC)

$$\rho_{AB\dots} = \sum_i p_i \rho_A^{(i)} \rho_B^{(i)} \dots$$

Example:  $\rho_{AB} = \frac{1}{2} |j\rangle\langle j| \otimes |j\rangle\langle j| + \frac{1}{2} |j\rangle\langle j| \otimes |j\rangle\langle j|$

## Entangled state

$\rho_{AB}$  is not separable

Example:  $\rho_{AB} = \frac{1}{2} \frac{|i\rangle\langle i| + |j\rangle\langle j|}{2} + \frac{1}{2} |\Psi^+\rangle\langle\Psi^+|$ , with  $|\Psi^+\rangle := \frac{|i\rangle\langle i| + |j\rangle\langle j|}{2}$

# Detect entanglement

Positive partial transpose (PPT) criterion<sup>1,2</sup>

$$\rho_{AB} \text{ separable } \Rightarrow \rho_{AB}^{T_B} = \sum_i p_i \rho_A^{(i)} \rho_B^{(i)T} \geq 0$$

K-symmetric extensions<sup>3,4</sup>

$$\rho_{AB} \text{ separable, } \exists k, \exists \rho_{AB_1 B_2 \dots B_k} \text{ such that } \rho_{AB_i} = \rho_{AB}, \forall i$$

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<sup>1</sup>Peres, Asher. "Separability Criterion for Density Matrices." *Physical Review Letters* 77

<sup>2</sup>Horodecki, Michał, Paweł Horodecki, and Ryszard Horodecki. "Separability of Mixed States: Necessary and Sufficient Conditions." *Physics Letters A* 223, no. 1

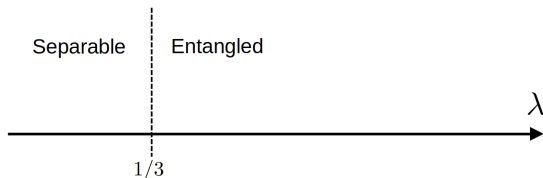
<sup>3</sup>Fannes, M., J. T. Lewis, and A. Verbeure. "Symmetric States of Composite Systems." *Letters in Mathematical Physics* 15, no. 3

<sup>4</sup>Raggio, G. A., and R. F. Werner. "Quantum Statistical Mechanics of General Mean Field Systems" ▶

# Mixture of entangled and noisy state

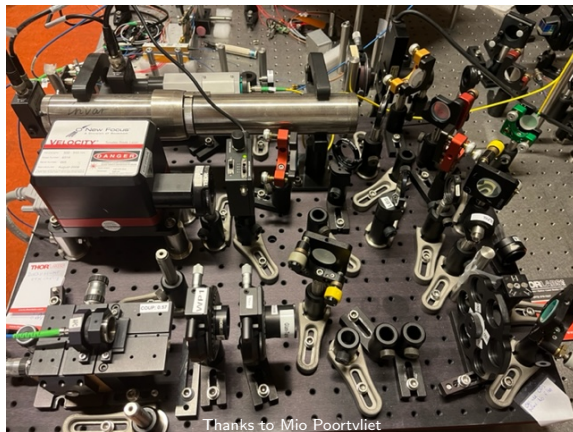
Werner state<sup>5</sup>

$$\rho_{AB} = \lambda |\Psi^+\rangle\langle\Psi^+| + (1 - \lambda) \frac{I}{4}$$



<sup>5</sup>Werner, Reinhard F. "Quantum States with Einstein-Podolsky-Rosen Correlations Admitting a Hidden-Variable Model." *Physical Review A* 40, no. 8 (October 1, 1989)

# Limitations of the model



Thanks to Mio Poortvliet

What is  $\rho$ ? What is the Hilbert space?



# Limitation of the model

## Problem

Models assume ideal/simplistic setup and conditions

) state we have is not the one we expect

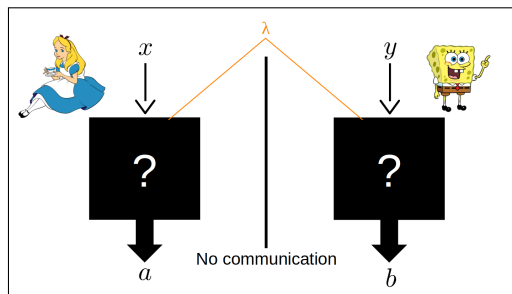
## Solution

Look directly at the correlations of the device

) Bell tests

# Bell test

Device:  $x, y \rightarrow a, b$



Repeat experiment  $\rightarrow P(a, b|x, y)$

# Correlation types

Local hidden variable (LHV) correlations<sup>6</sup>

$$P(a, b|x, y) = \int_{\Lambda} d\lambda q(\lambda) P(a|x, \lambda) P(b|y, \lambda)$$

Quantum correlations

$$P(a, b|x, y) = \text{Tr}[(M_x^{(a)} \otimes M_y^{(b)}) \rho_{AB}]$$

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<sup>6</sup>Bell, JS. "On the Einstein Podolsky Rosen Paradox" 1, no. 3 (1964)

# Bell inequality

## Bell expression

$$I[P] := \sum_{a,b;x,y} \alpha_{abxy} P(a, b|x, y)$$

For wisely chosen  $\alpha_{abxy}$ , we can find  $P \in \text{Quantum}$  s.t.

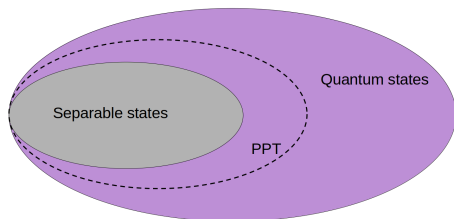
$$I[P] > \max_{P \in \text{Local}} I[P]$$

Example: CHSH inequality

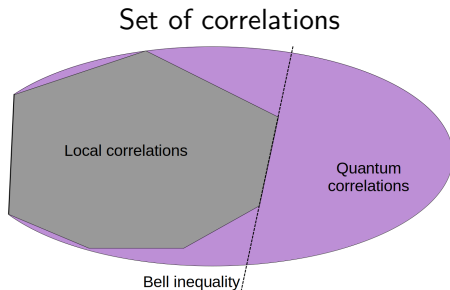
$$\sum_{a,b;x,y} (1)^{a+b+xy} P(a, b|x, y) \leq 2 \quad (\text{local bound})$$

Quantum bound:  $2\sqrt{2}$

# Sets of quantum states and correlations



Set of quantum states

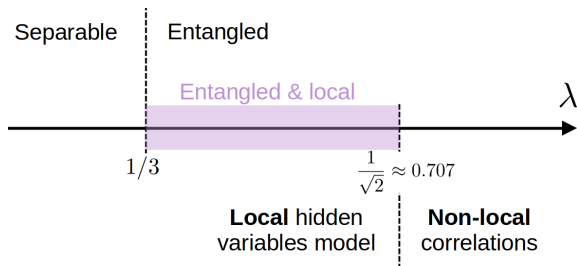


Set of correlations

# Entangled is not enough!

## Werner state

$$\rho_{AB} = \lambda |\Psi\rangle\langle\Psi| + (1 - \lambda) \frac{I}{4}$$



## Testing Bell inequalities in Higgs boson decays

Alan J. Barr

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(Dated: First submitted: 2 June 2021. This version: July 27, 2022)

Higgs boson decays produce pairs of  $W$  bosons in a maximally entangled state, the spins of which can be expected to violate Bell inequalities. We show that the spin density matrix of the  $W^\pm$  pair may be reconstructed experimentally from the directions of the charged lepton decay products, and from it the expectation values of various Bell operators determined. Numerical simulations of  $H \rightarrow WW^*$  decays indicate that violation of a generalised CHSH inequality is unlikely to be measurable, however the CGLMP inequality is near-maximally violated. Experimental Bell tests could be performed at a variety of colliders and in different production channels. If reconstruction effects and backgrounds can be controlled then statistically significant violations could be observable even with datasets comparable to those already collected at the LHC.

## Quantum state tomography, entanglement detection and Bell violation prospects in weak decays of massive particles

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**ABSTRACT:** A rather general method for determining the spin density matrix of a multi-particle system from angular decay data is presented. The method is based on a Bloch parameterisation of the  $d$ -dimensional generalised Gell-Mann representation of  $\rho$  and exploits the associated Wigner- and Weyl-transforms on the sphere. Each parameter of a (possibly multipartite) spin density matrix can be measured from a simple average over an appropriate set of experimental angular decay distributions. The general procedures for both projective and non-projective decays are described, and the Wigner  $P$  and  $Q$  symbols calculated for the cases of spin-half, spin-one, and spin-3/2 systems. The methods are used to examine Monte Carlo simulations of  $pp$  collisions for bipartite systems:  $pp \rightarrow W^+W^-$ ,  $pp \rightarrow ZZ$ ,  $pp \rightarrow ZW^+$ ,  $pp \rightarrow W^+t$ ,  $t\bar{t}$ , and those from the Higgs boson decays  $H \rightarrow WW^*$  and  $H \rightarrow ZZ^*$ . Measurements are proposed for entanglement detection, exchange symmetry detection and Bell inequality violation in bipartite systems.

## Suggestion for Einstein–Podolsky–Rosen Experiments Using Reactions Like $e^+e^- \rightarrow \Lambda\bar{\Lambda} \rightarrow \pi^-\rho\pi^+\bar{\rho}$

Nils A. Törnqvist<sup>1</sup>

Received July 9, 1980

Since weakly decaying particles are their own polarimeters, reactions like  $\eta_c \rightarrow \Lambda\bar{\Lambda}$ ,  $\psi \rightarrow \Lambda\bar{\Lambda}$ ,  $e^+e^- \rightarrow \mu^+\mu^-$ , etc. are interesting for testing the non-locality of quantum mechanical predictions. Although such reactions, in principle, do not exclude all classes of hidden variable theories, they can be used to complement current experiments with external polarimeters. The reaction  $\eta_c \rightarrow \Lambda\bar{\Lambda} \rightarrow \pi^-\rho\pi^+\bar{\rho}$  is conceptually the simplest and most useful as a Gedanken experiment, although it has not yet been seen experimentally. The reaction  $e^+e^- \rightarrow \Lambda\bar{\Lambda} \rightarrow \pi^-\rho\pi^+\bar{\rho}$  near threshold or at the  $\psi$  resonance can be used for essentially the same test. This is feasible with presently available data and would be the first EPR experiment involving weak interactions.

## Quantum entanglement and Bell inequality violation at colliders

Alan J. Barr<sup>a,b</sup>, Marco Fabbricheschi<sup>a\*</sup>, Roberto Floresini<sup>a</sup>, Emidio Gabrielli<sup>a,c,e</sup>, Luca Marzola<sup>a</sup>

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## Abstract

The study of entanglement in particle physics has been gathering pace in the past few years. It is a new field that is providing important results about the possibility of detecting entanglement and testing Bell inequality at colliders for final states as diverse as top-quark or  $\tau$ -lepton pairs, massive gauge bosons and vector mesons. In this review, after presenting definitions, tools and basic results that are necessary for understanding these developments, we summarize the main findings—as published up to the end of year 2023. These investigations have been mostly theoretical since the experiments are only now catching up, with the notable exception of the observation of entanglement in top-quark pair production at the Large Hadron Collider. We include a detailed discussion of the results for both quark and quark systems, that is, final states containing spin one-half and spin one particles. Entanglement has also been proposed as a new tool to constrain new particles and fields beyond the Standard Model and we introduce the reader to this promising feature as well.

# Bell inequalities - Loopholes

## Locality loophole

Alice and Bob devices communicate

## Detection loophole

Set of detected events is an unfair sample

## Superdeterminism loophole

No free will. Everything (even the measurement choices) is governed by the same random variable.



# Conclusion

## Separable VS Entangled states

- ▶ PPT criterion, K-symmetric extension, ...
- ▶ Entangled but still admits LHV model

## Local VS Nonlocal correlations

- ▶ Bell test
- ▶ LHV model is not enough to describe QM
- ▶ Loopholes: locality, detection, "free will", ...

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# Bell inequality in HEP - example2

## Testing Bell inequalities in Higgs boson decays

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# Quantum state tomography for HEP

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## Key point: reconstructing the density matrix

Before doing anything, we need to get the density matrix.  
Is there a way to bypass the density matrix reconstruction ! to discuss

# Quantum state tomography for HEP

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