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# Oscillation systematics: Swim



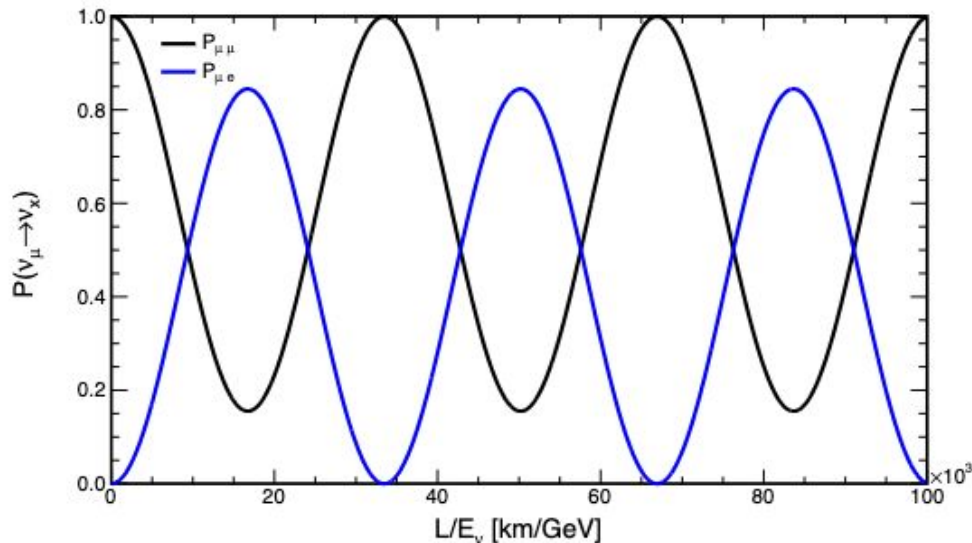
Víctor Carretero  
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Nikhef Group Outing



# Oscillation probabilities

Assuming neutrinos propagate as **plain waves** and that they are **ultrarelativistic**:

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu_\alpha \rangle|^2 = \sum_{ii} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* e^{-i \frac{\Delta m_{ij}^2 L}{2E}},$$



Assuming **two** flavour approximation:

- The mixing angle drives the **amplitude**.
- The mass splitting drives the **frequency**.

$$P(\nu_\mu \rightarrow \nu_e)(t) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E} \right)$$

$$P(\nu_\mu \rightarrow \nu_\mu)(t) = 1 - P(\nu_\mu \rightarrow \nu_e)(t) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E} \right)$$

# Matter effects

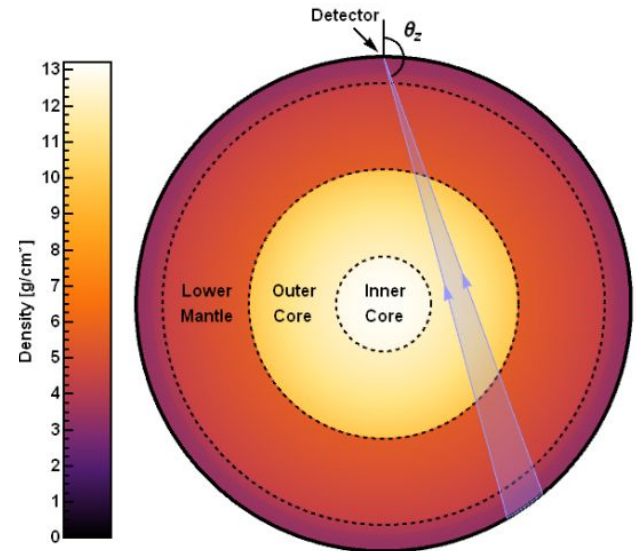
Neutrinos interact with **matter** via charged and neutral current interactions. Since ordinary matter contains **electrons** but not muons or taus, a difference is introduced in how electron neutrinos propagate that affect oscillations.

$$H(x) = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

$$V = \pm\sqrt{2}G_F n_e(x)$$

$$\sin^2 2\theta_{13}^M = \sin^2 2\theta_{13} \left( \frac{\Delta m_{31}^2}{\Delta^M} \right)^2,$$

$$\Delta^M = \sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - 2VE)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2}.$$

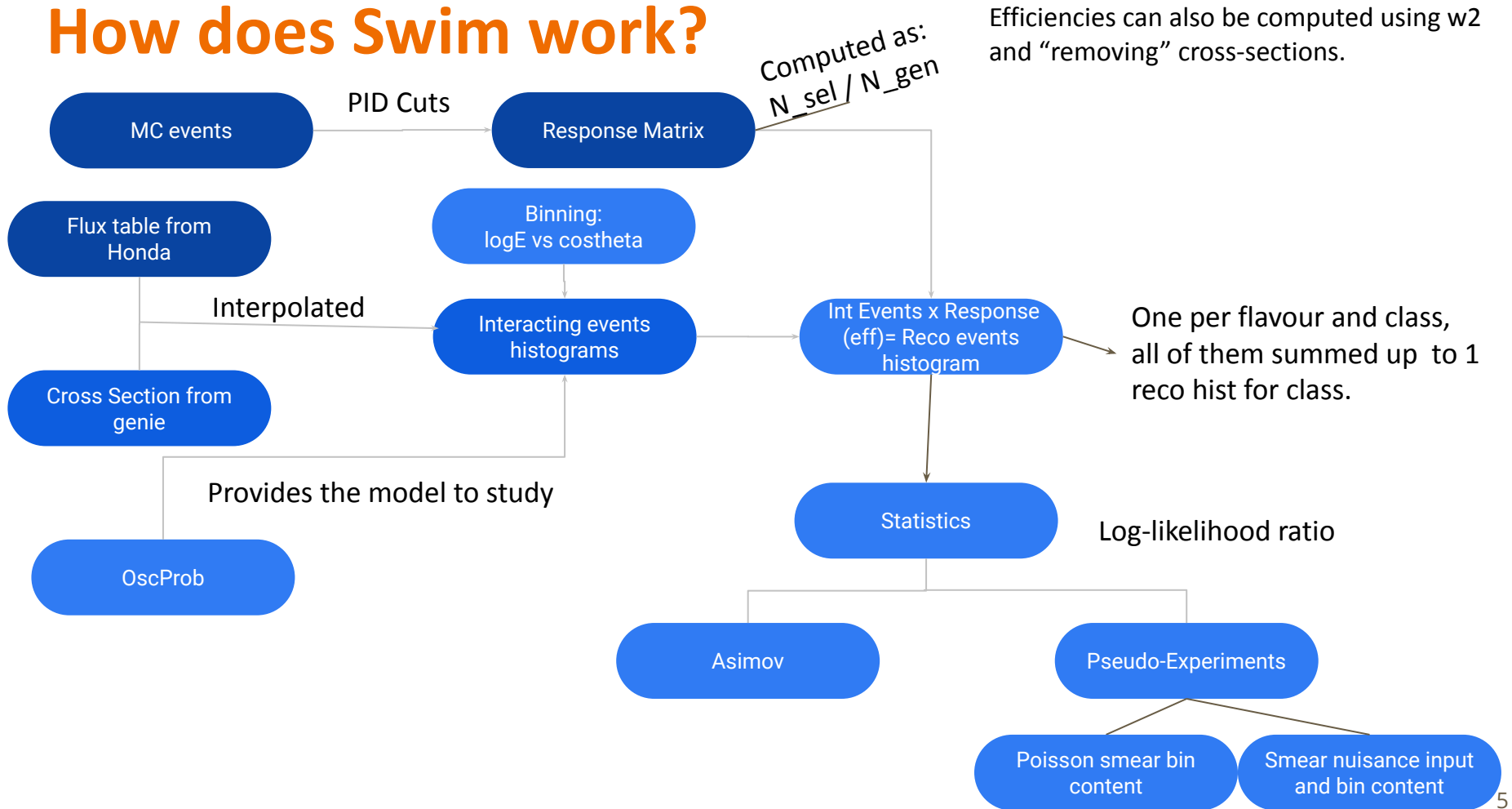


# Statistical method

- **2D binned** analysis in reconstructed energy and cosine of the zenith angle.
- **Log-likelihood ratio** coming from Poisson statistics with respect to the saturated model.
- Model is fitted **minimizing** the negative log-likelihood ratio by varying all its parameters ( $\theta$ ).
- **Systematics** (oscillation + nuisance) parameters ( $\mathbf{v}$ ) could be accounted for by a **prior**, that comes from external experiments or previous knowledge. **Gaussian distribution** assumed, adding the prior as a **penalty term**.

$$-2 \ln \lambda(\vec{\theta}, \vec{v}) = 2 \sum_{i=1}^N \left[ \mu_i(\vec{\theta}, \vec{v}) - n_i + n_i \ln \frac{n_i}{\mu_i(\theta)} \right] + \sum_l \left( \frac{v_l - \langle v_l \rangle}{\sigma_l} \right)^2$$

# How does Swim work?

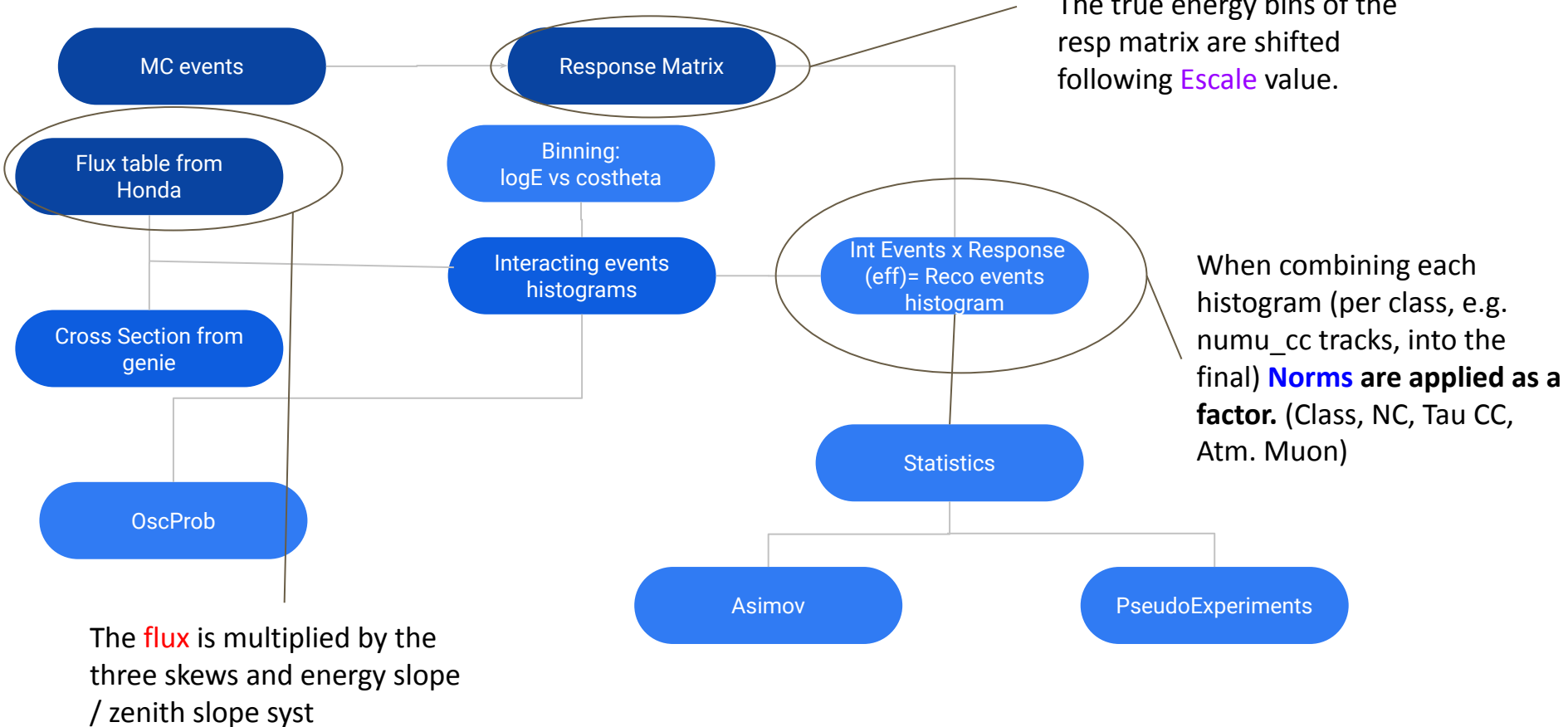


# Nuisance parameters

Norm, Flux (Shape) and Detector systematics.

- Norm Systematics: Applied as a scaling of a histogram.
  - Class Norms
  - Tau Norm (CC only or CC+NC)
  - NC Scale
  - Muon Norm.
- Shape Systematics:
  - Flavour ratio:  $\nu_{\mu}/\text{anti-}\nu_{\mu}$  skew,  $\nu_e/\text{anti-}\nu_e$  skew,  $\nu_{\mu}/\nu_e$  skew
  - Energy Slope.
  - Zenith Slope
- Detector effect systematics:
  - Energy Scale
  - High Energy Light Simulation

# Systematics implementation



# Shape systematics

- **Zenith Slope and Energy Slope**. They represent the uncertainties in energy and direction of our flux, applied as a power expansion:

$$\tilde{\phi}(E, \theta) = f_{\text{shape}} \phi(E, \theta) = E^{-s_{\text{energy}}} (1 + s_{\text{up/hor}} |\cos \theta|) \phi(E, \theta),$$

- The choice in the zenith slope is made to have an up/horizontal effect instead of an up/down effect.



# Shape flux systematics

- **Flux skews.** They represent the skew in the ratio of  $\text{numu}/\text{anumu}$ ,  $\text{nue}/\text{anue}$  and  $\text{numu}/\text{nue}$ .

$$f_e = (1 + s_{e\mu}) \cdot (1 + s_{e\bar{e}})$$

$$f_{\bar{e}} = (1 + s_{e\mu}) \cdot \left(1 - \frac{I_e}{I_{\bar{e}}} s_{e\bar{e}}\right)$$

$$f_{\mu} = \left(1 - \frac{I_e + I_{\bar{e}}}{I_{\mu} + I_{\bar{\mu}}} s_{e\mu}\right) \cdot (1 + s_{\mu\bar{\mu}})$$

$$f_{\bar{\mu}} = \left(1 - \frac{I_e + I_{\bar{e}}}{I_{\mu} + I_{\bar{\mu}}} s_{e\mu}\right) \cdot \left(1 - \frac{I_{\mu}}{I_{\bar{\mu}}} s_{\mu\bar{\mu}}\right)$$

# Systematic implementation (Shape)

- They are all applied assuming a **conservation of the total incident flux.**  
(Renormalisation)

$$I_x = \int \int \phi_x(E, \theta) dE d\theta.$$

$$I_e + I_{\bar{e}} + I_{\mu} + I_{\bar{\mu}} = \check{I}_e + \check{I}_{\bar{e}} + \check{I}_{\mu} + \check{I}_{\bar{\mu}}$$

$$\int \int \bar{\phi}(E, \theta) \equiv \int \int \phi(E, \theta) E^{-S_{\text{energy}}} (1 + s_{\text{up/hor}} |\cos \theta|) = \int \int \phi(E, \theta)$$

# Response Matrix

- Response matrix first computed as a generation weights matrix. Then the “efficiency” is computed by “removing” cross-sections.

$$R_{v_x}^c = \frac{m_p}{2\pi\Delta E\Delta \cos\theta\Delta yM} \frac{\sum_i w_{\text{gen},i,v_x}^c \frac{t_i}{\sum_j t_j}}{\frac{d\sigma_{v_x}}{dy}}.$$

- Energy scale can be applied directly to this matrix.

# Energy Scale

- In reality, there are two response matrices:
  - $P_{TV}$  gives the probability of having some visible energy  $E_V$ , given some true neutrino energy  $E_T$
  - $P_{VR}$  gives the probability of having some reco energy  $E_R$ , given some visible energy  $E_V$
- But we only have access in principle to a combination of the two:
  - $P_{TR}$  gives the probability of having some reco energy  $E_R$ , given some true neutrino energy  $E_T$
- We shift the bins of **true energy in the Response Matrix** to emulate the effect of the uncertainty in water properties and quantum efficiencies.

$$N(E_R) = \int \int P_{VR}(E_R, E_V) \times P_{TV}(E_V + \delta E, E_T) \times \sigma(E_T) \times \phi(E_T) \times P_{osc}(E_T) dE_V dE_T$$

$$P_{TR}(E_R, E_T, \delta E) = \int P_{VR}(E_R, E_V) P_{TV}(E_V + \delta E, E_T) dE_V$$

assuming:  $P_{TV}(E_V + \delta E, E_T) = P_{TV}(E_V, E_T - \delta E)$

$$N(E_R) = \int P_{TR}(E_R, E_T - \delta E) \times \sigma(E_T) \times \phi(E_T) \times P_{osc}(E_T) dE_T$$

# Compare with IceCube

- E scale: **9%**
- E slope: **0.3**
- $\nu_e/\nu_\mu$ : **2%**
- up/horiz: **2%**
- $\nu_e / \text{anti-}\nu_e$  **7%**
- $\nu_\mu / \text{anti-}\nu_\mu$  : **5%**
- NC norm: **20%**
- Tau norm: **20%**
- Atm muons free
- PID class norms: **free**
- Jsirene-norm : **50%**

Parameter	Prior	Fit value	Pull ( $\sigma$ )
<b>Detector</b> Baseline from calibration data			
DOM efficiency	10%	+6%	0.63
Rel. eff. $p_0$	Unconstrained	-0.27	-
Rel. eff. $p_1$	Unconstrained	-0.04	-
Ice absorption	Unconstrained	-3%	-
Ice scattering	Unconstrained	-1%	-
<b>Flux</b> Baseline from Honda et al.			
$\Delta\gamma_\nu$	0.1	+0.07	0.7
$\Delta\pi^\pm$ yields [A-F]	30%	+10%	0.35
$\Delta\pi^\pm$ yields G	30%	-6%	-0.18
$\Delta\pi^\pm$ yields H	15%	-2%	-0.12
$\Delta K^+$ yields W	40%	+8%	0.21
$\Delta K^-$ yields W	40%	-1%	-0.02
$\Delta K^+$ yields Y	30%	+11%	0.35
<b>Cross-section</b> Baseline from GENIE			
$M_A^{CCQE}$	0.99 GeV $^{+25\%}_{-15\%}$	+1%	0.03
$M_A^{CCRES}$	1.12 GeV $\pm 20\%$	+11%	0.57
$\sigma_{NC}/\sigma_{CC}$	20%	+13%	0.63
DIS CSMS	1.0	0.04	0.04
<b>Atm. muons</b> Baseline from Gaisser et al.+Sibyll2.1			
Atm. $\mu$ scale	Unconstrained	+39%	-
<b>Normalisation</b> Baseline from calibration+flux models			
$A_{eff}$ scale	Unconstrained	-18%	-

Decoupled effects

Change in particles from CR

Appart from NC scale they apply GENIE changes.

Single Norm

# Conclusions

- Our weakest point is the energy response of the detector.
- Implement a template approach as IceCube is a priority.
- Simulate absorption and QE of PMTs and redo MC, then extract the template and use it for the fits.
- Different approach for flux than them, no necessarily wrong.
- We use too many free normalisations.