Hough Transform and Bivariate Normal Distribution

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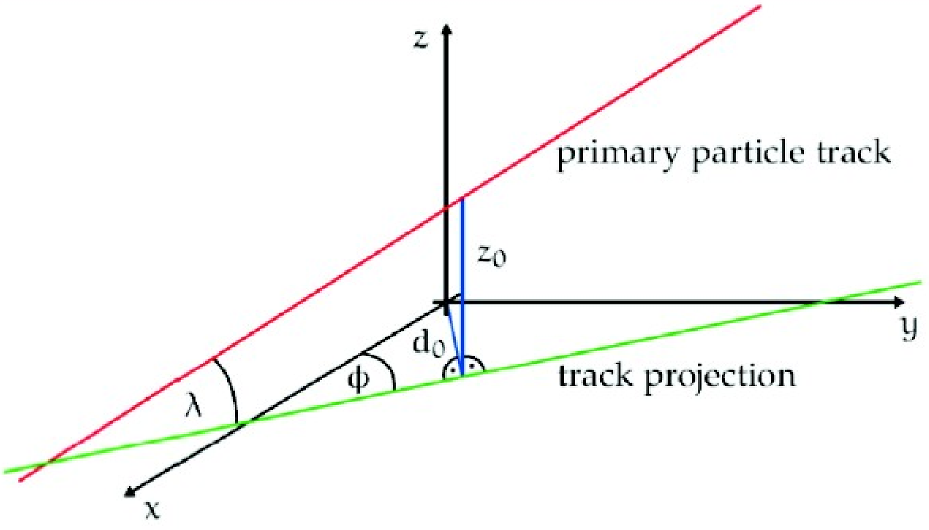
Track parameters

d_0
φ
Ω
z_0
$\tan(\lambda)$

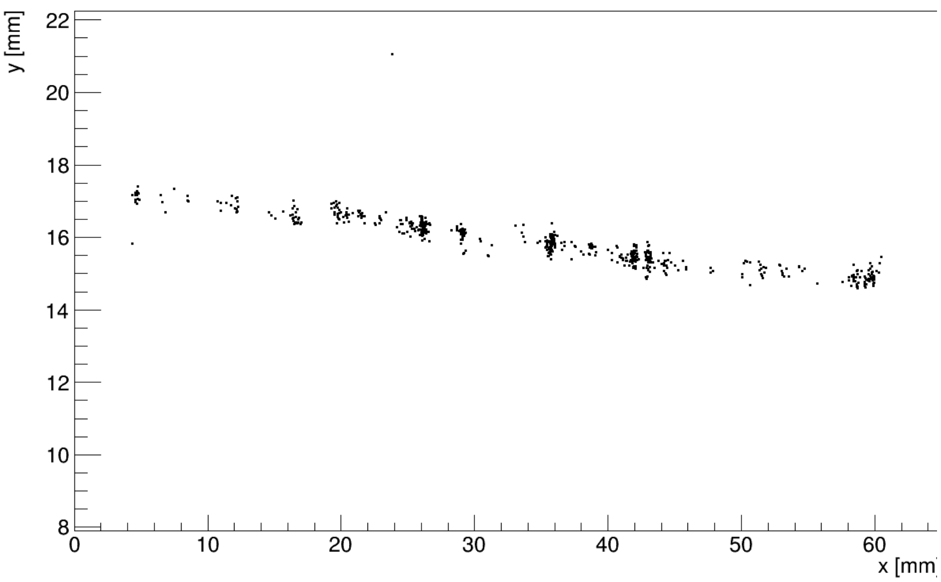


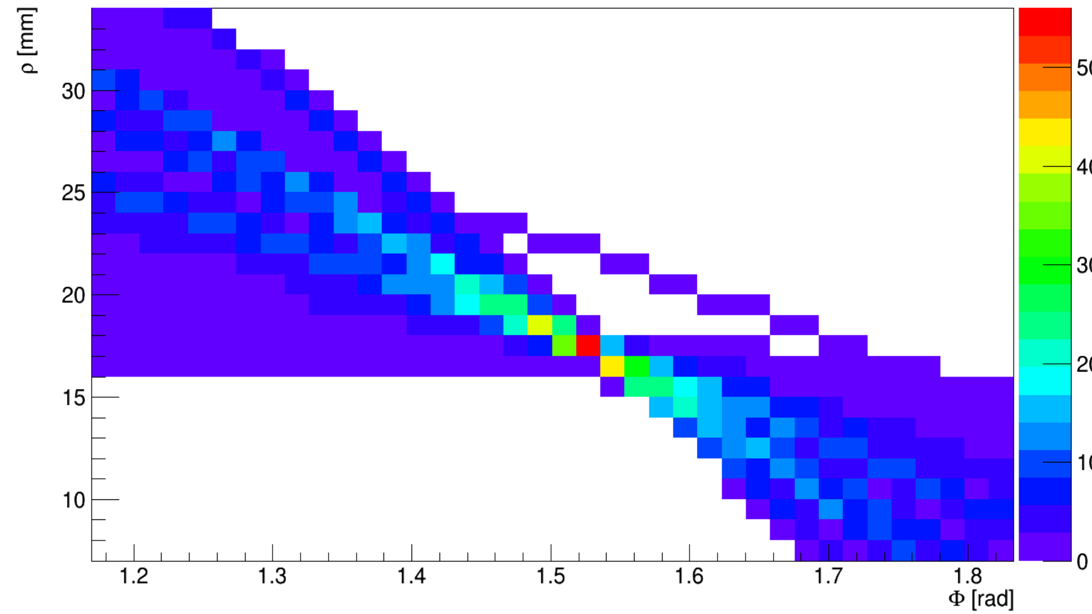
- For straight line Ω is zero.
- Finding 4 other parameters:
  - XY-plane => Finding (d_0, φ) => Collecting inliers
  - ZS-plane => Scanning Z direction for inliers => Finding($Z_0, \tan(\lambda)$)

XY_Plane

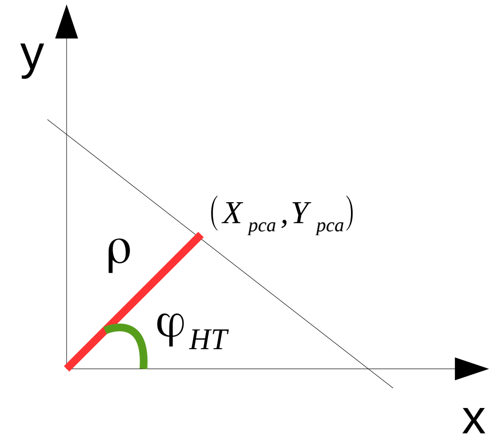
Hough Transform

$$\rho_{HT} = x\cos(\varphi_{HT}) + y\sin(\varphi_{HT})$$





- Bin with maximum returns φ_{HT} and ρ_{HT}
- $d_0 = \rho_{HT}$
- $\varphi_{track} = \varphi_{HT} - \frac{\pi}{2}$
- **Using Bivariate Normal Distribution for vicinity of the maximum**



XY_Plane

Diffusion and covariance matrix:

- Spacial Transverse resolution

$$\sigma_D^2 = Z D_T^2$$

- Covariance matrix: image space

$$\begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_D^2 \end{bmatrix}$$

- Covariance matrix: Hough space

$$\begin{bmatrix} \sigma_\rho^2 & \sigma_{\rho\varphi} \\ \sigma_{\varphi\rho} & \sigma_\varphi^2 \end{bmatrix}$$

First -order error propagation:

$$\rho = x\cos(\varphi) + y\sin(\varphi)$$

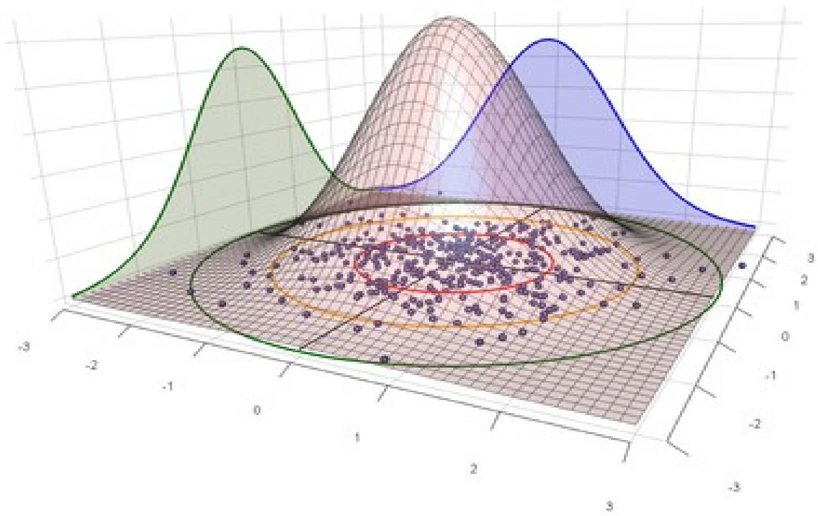
$$\begin{bmatrix} \sigma_\rho^2 & \sigma_{\rho\varphi} \\ \sigma_{\varphi\rho} & \sigma_\varphi^2 \end{bmatrix} = \nabla J \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_D^2 \end{bmatrix} \nabla J^T$$

$$\nabla J = \begin{bmatrix} \frac{\partial \rho}{\partial x} & \frac{\partial \rho}{\partial y} \\ \frac{\partial \varphi}{\partial x} & \frac{\partial \varphi}{\partial y} \end{bmatrix}$$

Bivariate Normal Distribution (BND):

$$G(\varphi,\rho)=\frac{1}{2\pi\sigma_\rho\sigma_\varphi\sqrt{1-r^2}}\exp\left[\frac{-c}{2(1-r^2)}\right]$$

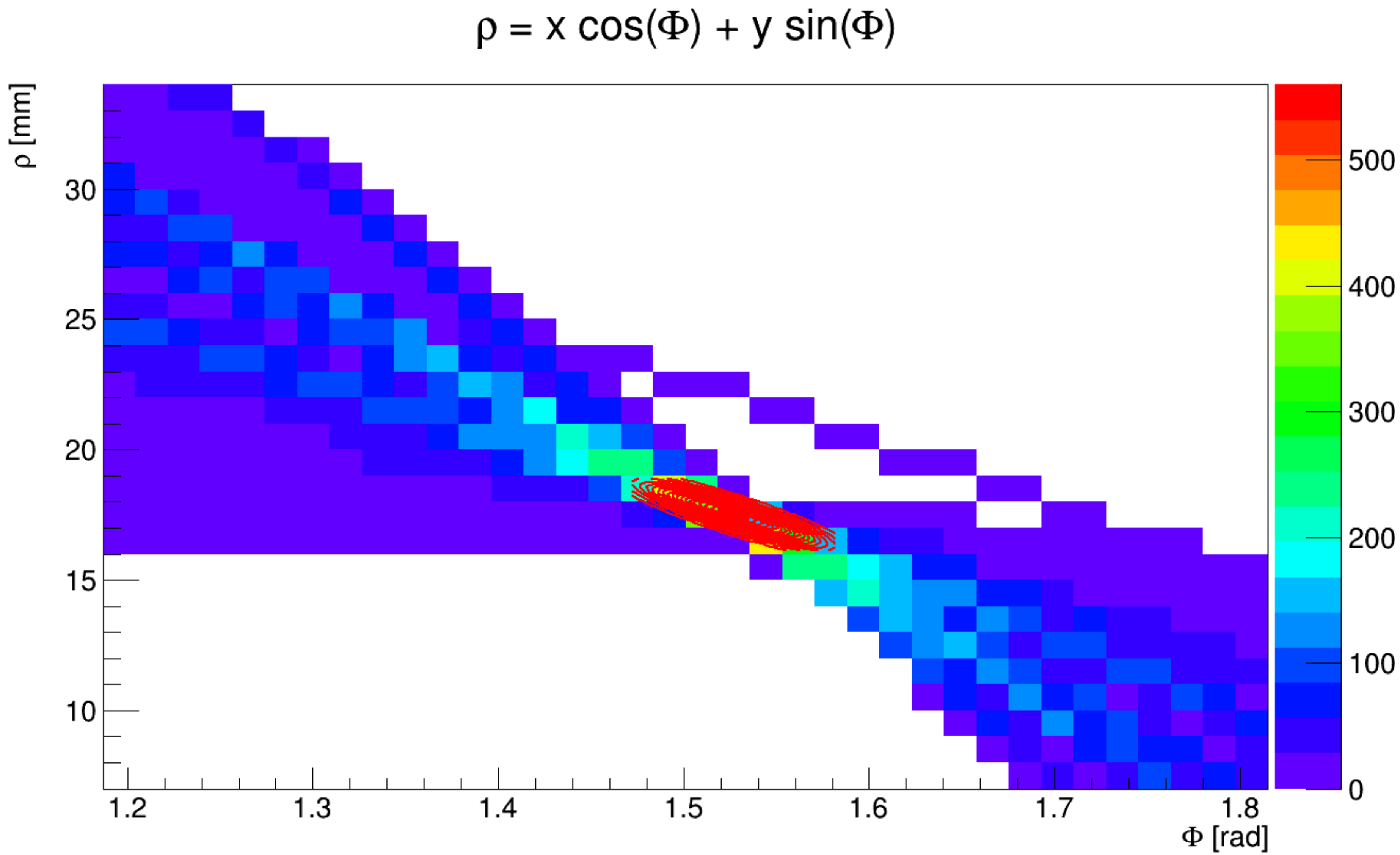
$$c\equiv\frac{(\varphi-\mu_\varphi)^2}{\sigma_\varphi^2}+\frac{(\rho-\mu_\rho)^2}{\sigma_\rho^2}-\frac{2r(\varphi-\mu_\varphi)(\rho-\mu_\rho)}{\sigma_\varphi\sigma_\rho}$$



http://ballistipedia.com/index.php?title=Closed_Form_Precision

- Correlation : $r\equiv\frac{\sigma_{\rho\varphi}}{\sigma_\rho\sigma_\varphi}$

XY_Plane



ρ = x cos(Φ) + y sin(Φ)
ρ [mm]
30
25
20
15
10
1.2
1.3
1.4
1.5
1.6
1.7
1.8
Φ [rad]
500
400
300
200
100
0

XY_Plane

Ellipse equation:

- General ellipse equation: $$1 = Ax^2 + Bxy + Cy^2$$
- **a** and **b** are semi_minor and semi_major axis $$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$
- Finding ellipse equation from BND:

$$t = \frac{1}{2\pi\sigma_\rho\sigma_\varphi\sqrt{1-r^2}} \exp\left[\frac{-c}{2(1-r^2)}\right]$$

$$k = 2\pi\sigma_\rho\sigma_\varphi\sqrt{1-r^2}$$ and $$q = 2(1-r^2)\ln\left(\frac{1}{kt}\right)$$

$$1 = \frac{(\varphi-\mu_\varphi)^2}{q\sigma_\varphi^2} + \frac{(\rho-\mu_\rho)^2}{q\sigma_\rho^2} - \frac{2r(\varphi-\mu_\varphi)(\rho-\mu_\rho)}{q\sigma_\varphi\sigma_\rho} \quad (1)$$

Ellipse equation:

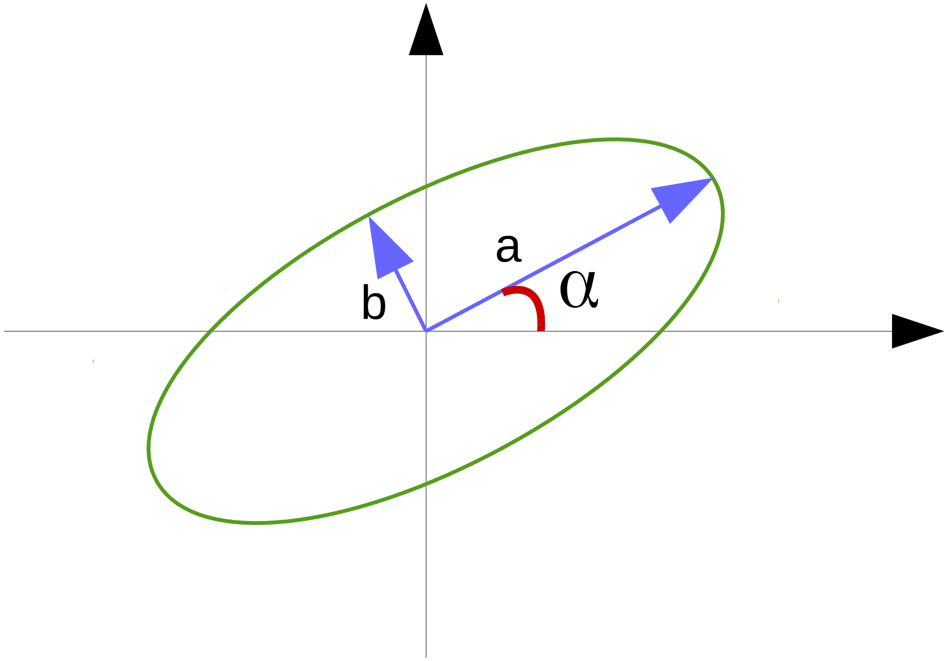
- Rotated ellipse:

$$1=\left(\frac{\cos^2(\alpha)}{a^2}+\frac{\sin^2(\alpha)}{b^2}\right)x^2-2\cos(\alpha)\sin(\alpha)\left(\frac{1}{a^2}-\frac{1}{b^2}\right)xy+\left(\frac{\sin^2(\alpha)}{a^2}+\frac{\cos^2(\alpha)}{b^2}\right)y^2 \quad (2)$$

- From (1) and (2):

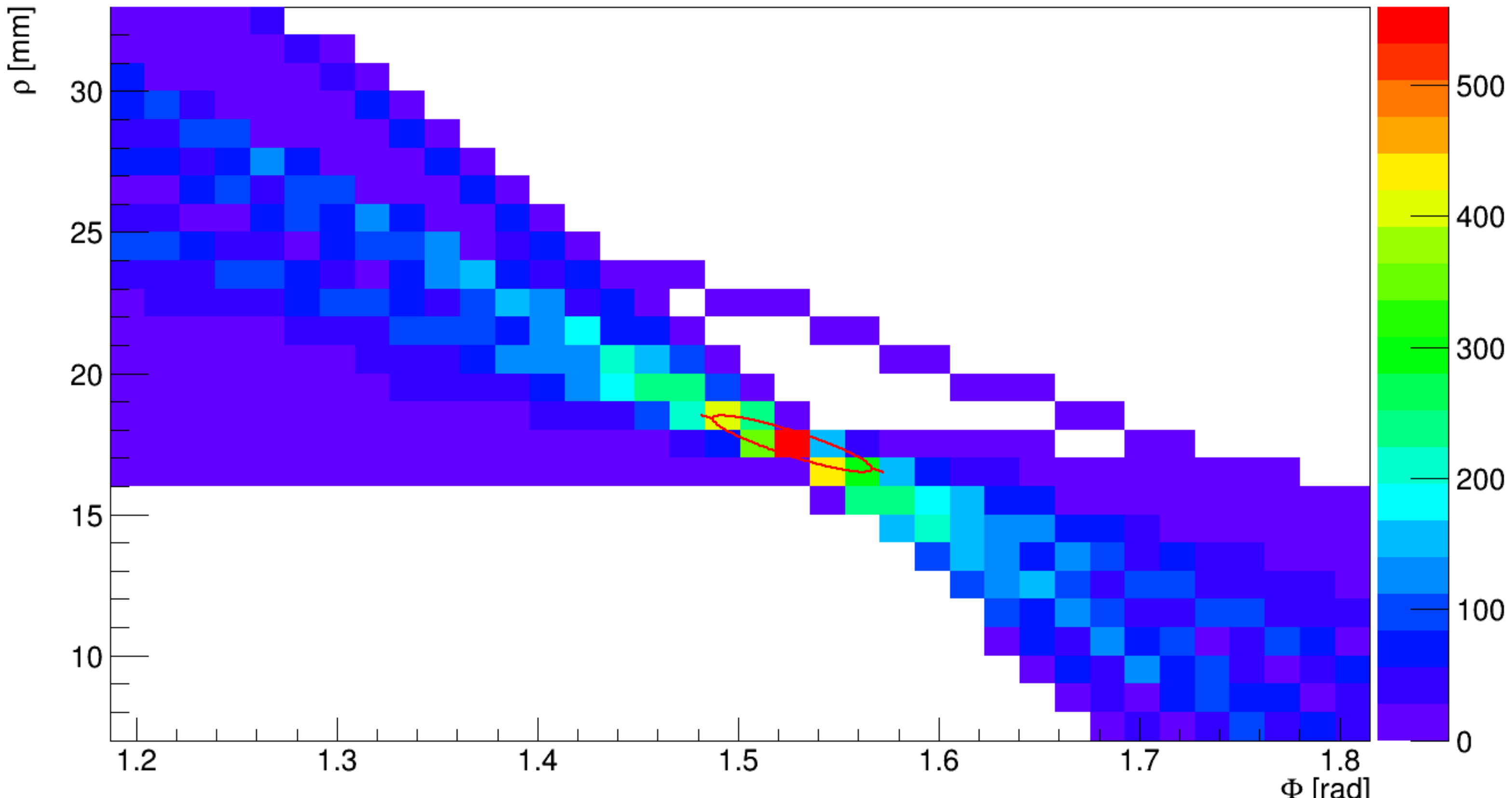
$$a^2=\frac{q\,\sigma_\varphi^2\sigma_\rho^2\cos(2\alpha)}{\sigma_\rho^2\cos^2(\alpha)-\sigma_\varphi^2\sin^2(\alpha)}$$

$$b^2=\frac{-q\,\sigma_\varphi^2\sigma_\rho^2\cos(2\alpha)}{\sigma_\rho^2\sin^2(\alpha)-\sigma_\varphi^2\cos^2(\alpha)}$$



Inliers and 1/e of peak of BND:

$$t = \frac{1}{e}\left(\frac{1}{2\pi\sigma_\rho\sigma_\varphi\sqrt{1-r^2}}\right) \quad \text{and} \quad q = 2(1-r^2)$$

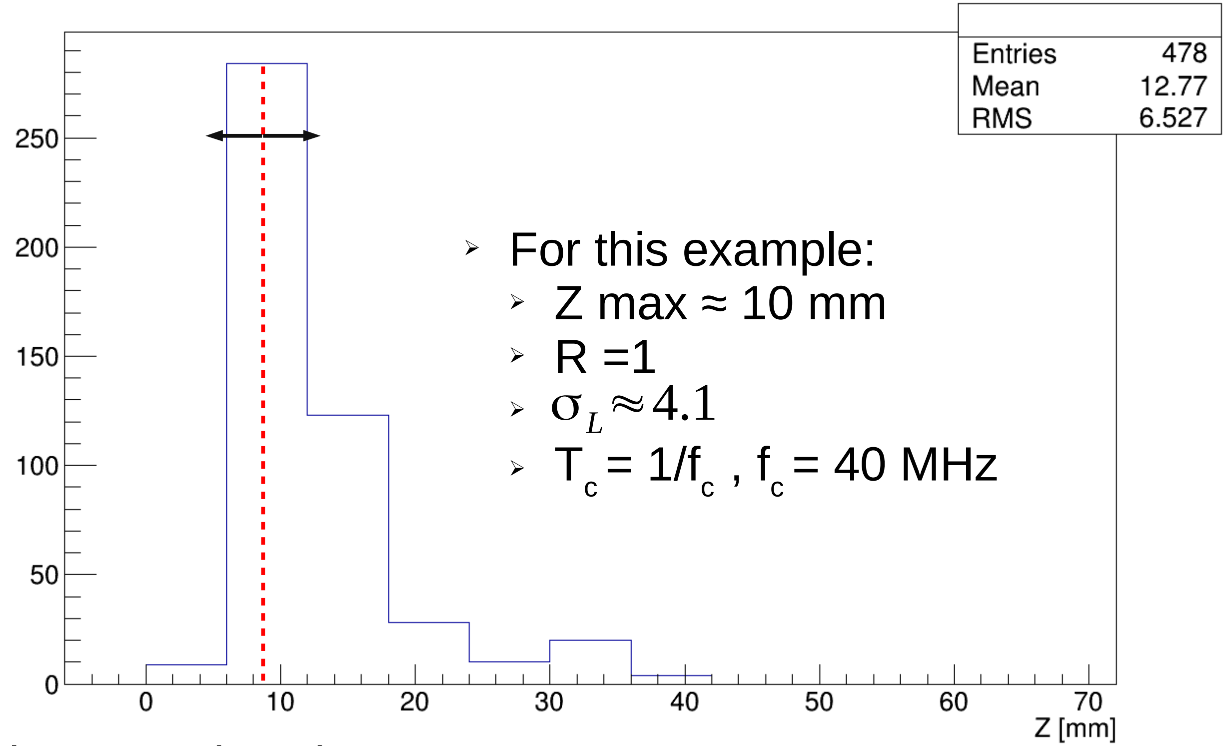


- Collecting all hits inside the ellipse => **Inliers**
- If number of inliers ≥ some % of all hits => Z-scanning
- Size of fitting is changeable => Number of inliers is changeable too

ZS_Plane

Z - scanning:

$$\sigma_L^2 = 16 + \frac{(T_c v_d)^2}{12} + Z D_L^2$$



- For this example:
  - Z max ≈ 10 mm
  - R =1
  - $\sigma_L \approx 4.1$
  - $T_c = 1/f_c$, $f_c = 40$ MHz

- Finding maximum of histogram (Z_{max})
- Collecting hits which has z distance less than of $|Z_{max} - R\sigma_L|$ => New inliers =>ZS-Hough Transform
- R =1,2,...
- If number of remaining hits ≥ some % of all inliers => ZS-Hough Transform

ZS Hough Transform:

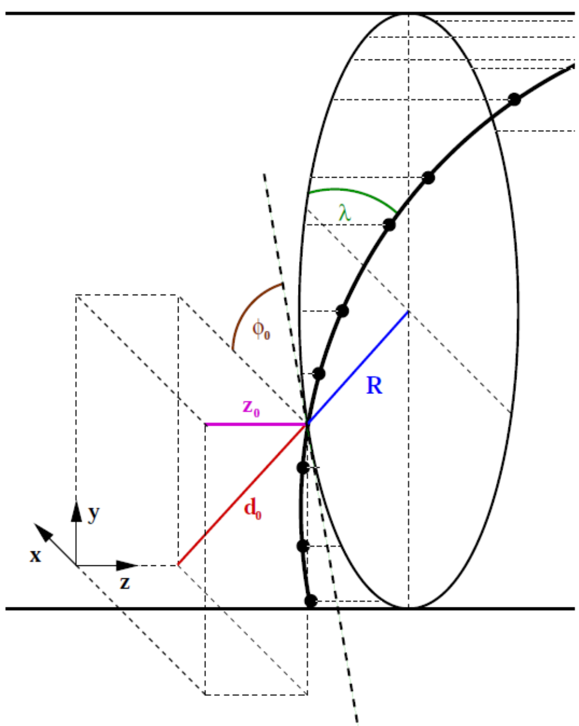
- **p**oint of **c**losest **a**pproach from XY -plane

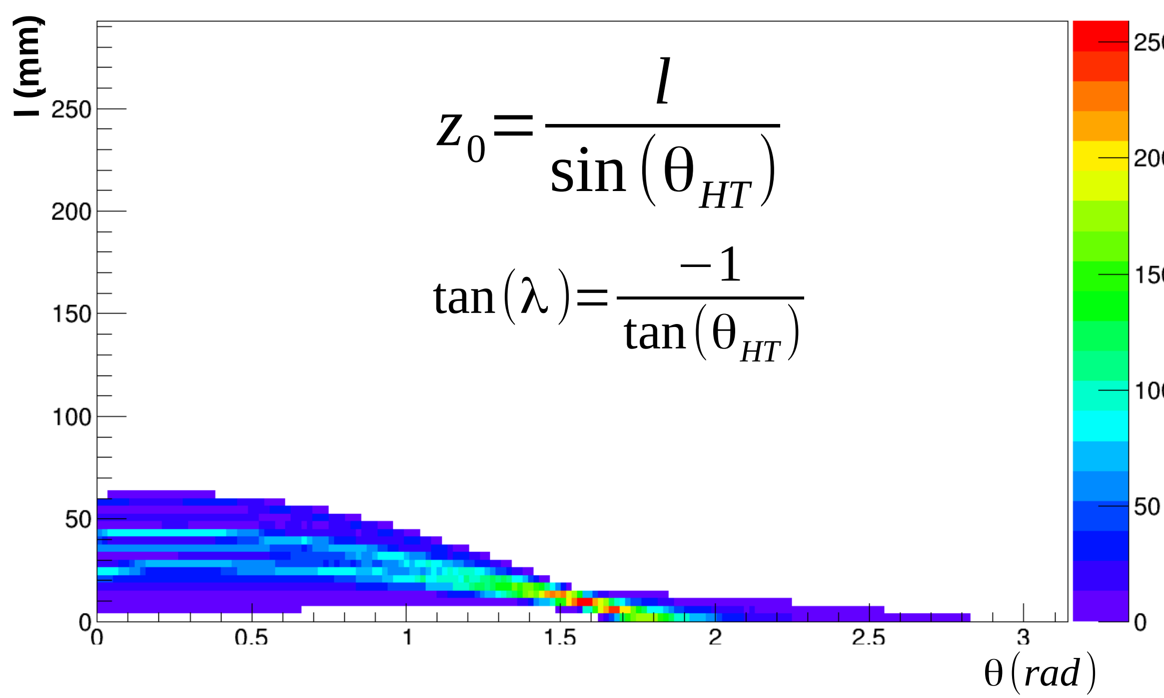
$$x_{pca} = -\rho \sin(\varphi) \qquad y_{pca} = \rho \cos(\varphi)$$

- Arc length (S): The shortest distance between the hit and PCA

$$s = \sqrt{(x_{hit} - x_{pca})^2 + (y_{hit} - y_{pca})^2}$$

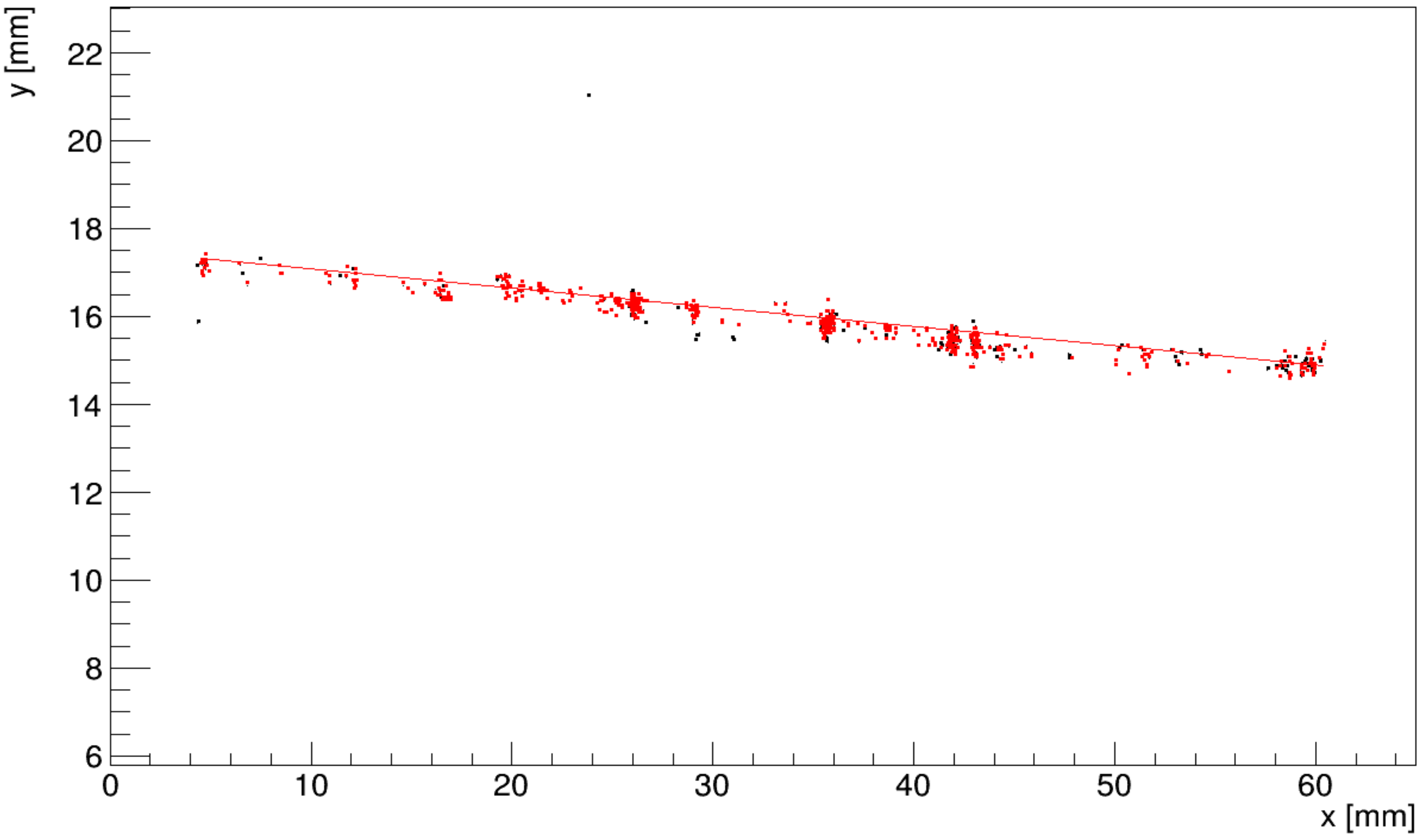
- In the ZS_plane a track is a straight line.





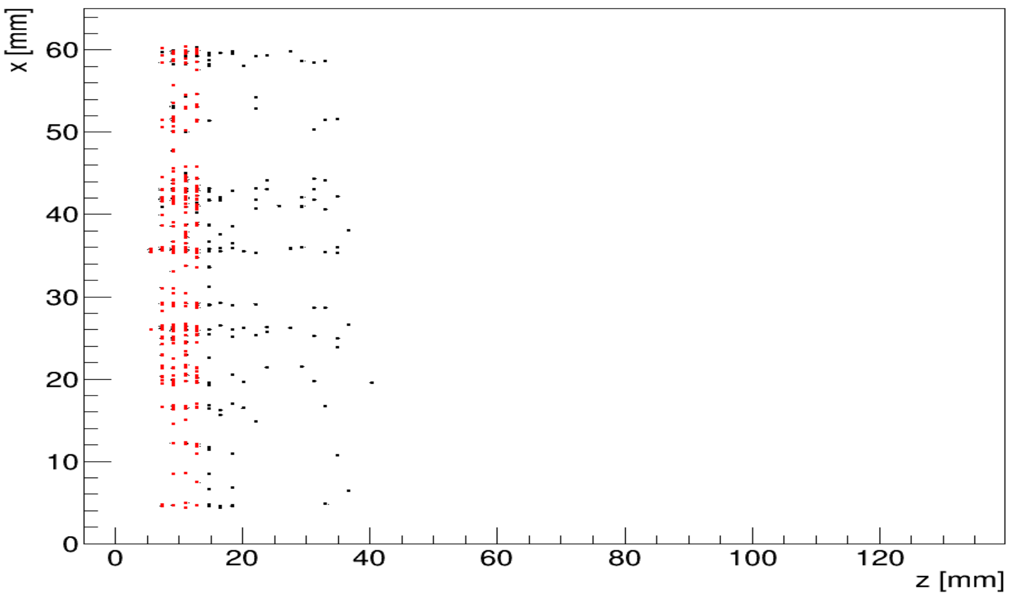
XY_Plane

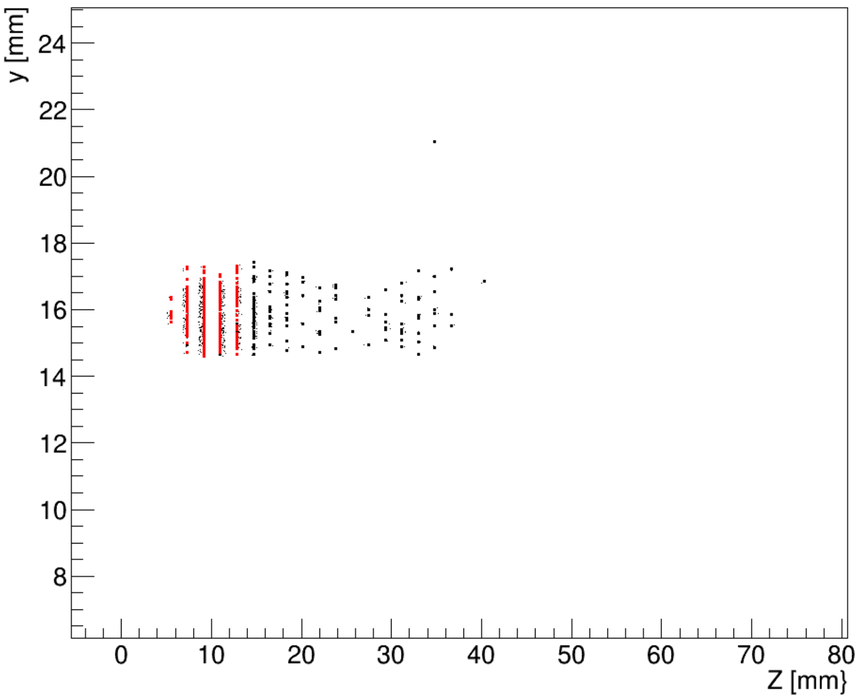
Track XY_Plane:



ZX_Plane and ZY_Plane

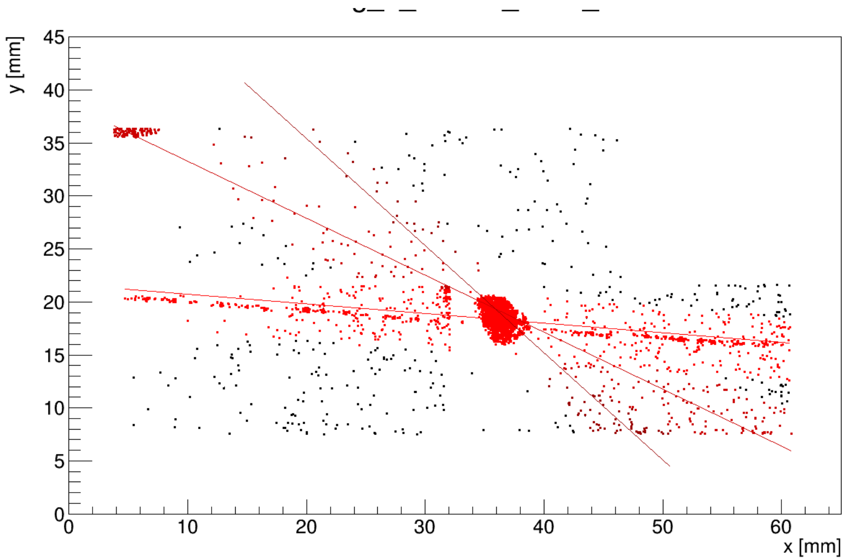
Track ZX and ZY Plane:



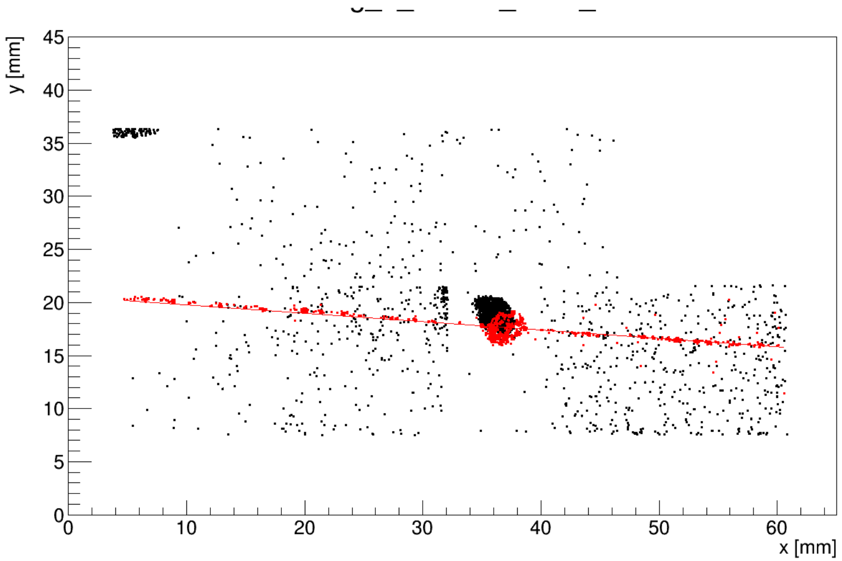


XY_Plane

Normal Hough Transform

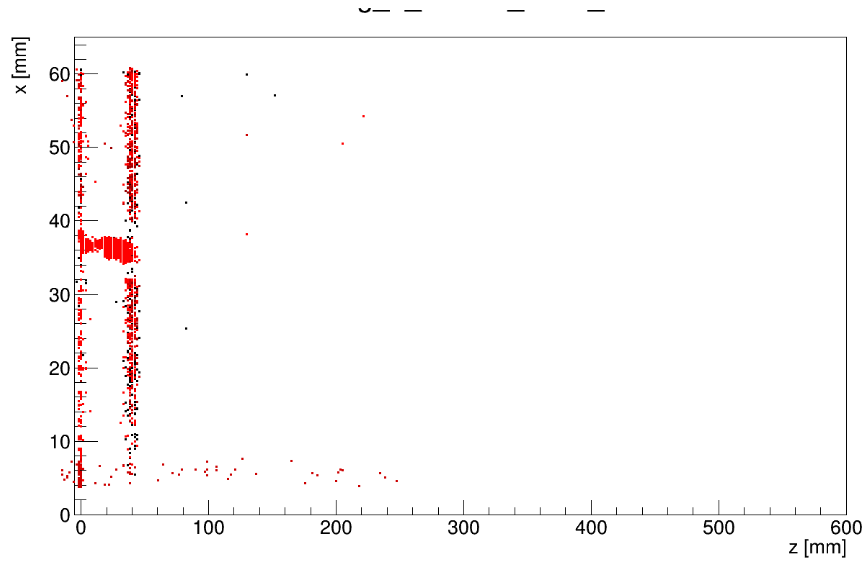


BND Hough Transform

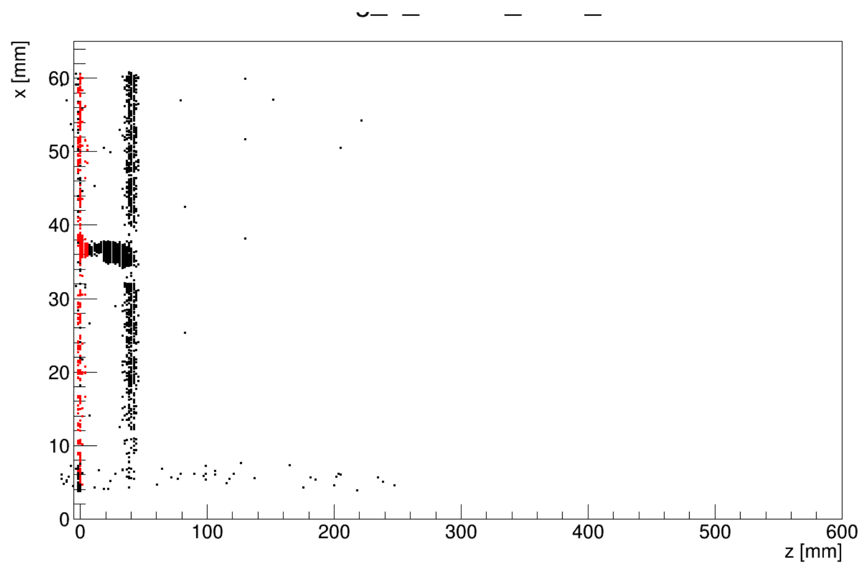


ZX_Plane

Normal Hough Transform



BND Hough Transform

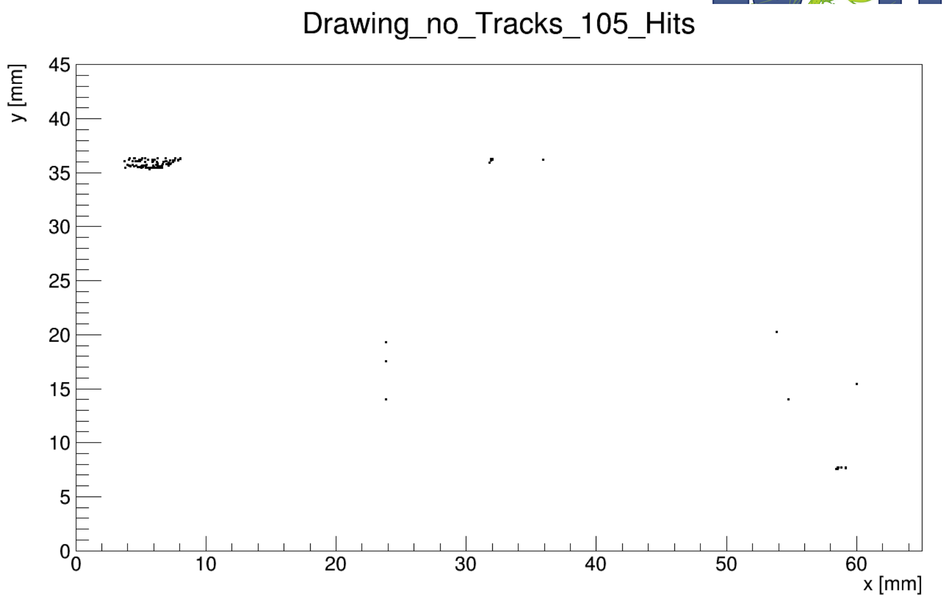


Preliminary Analysis

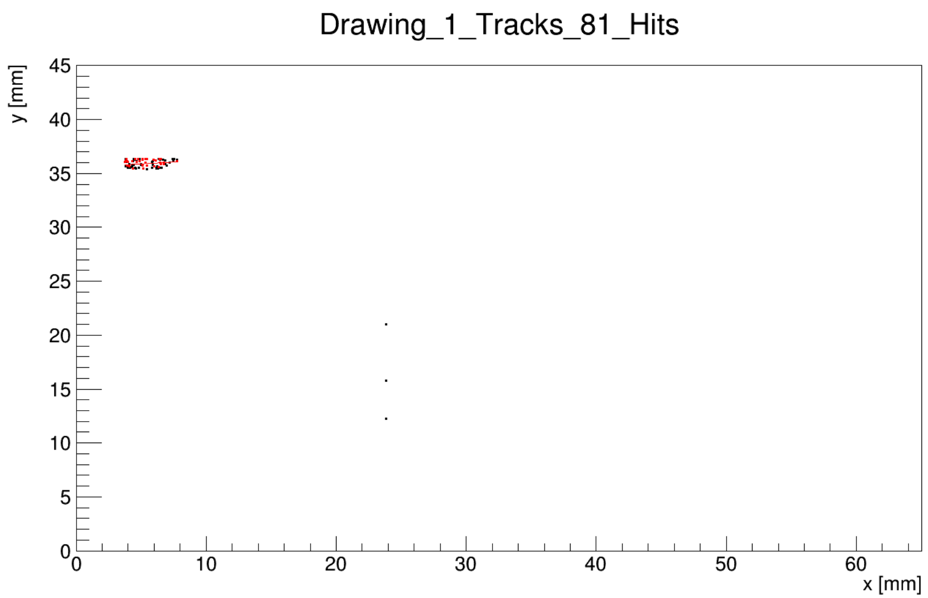
Definition:

- **Reconstruction Efficiency** (RE) = N_{reco} / N_{Event}
- **Selection Efficiency** (SE) = $N_{correct\ track}$ / N_{reco}
- **Ghost Track:** Tracks from noise or hits from different particles
- Ghost Rate (GR) = N_{ghost} / N_{Event}
- **Track Clones:** Two or more tracks for a particle => N_{clone} = N (reco a particle) -1
- Clone Rate (CR) = N_{clone} / N_{Event}
- **Total Efficiency** = RE x SE

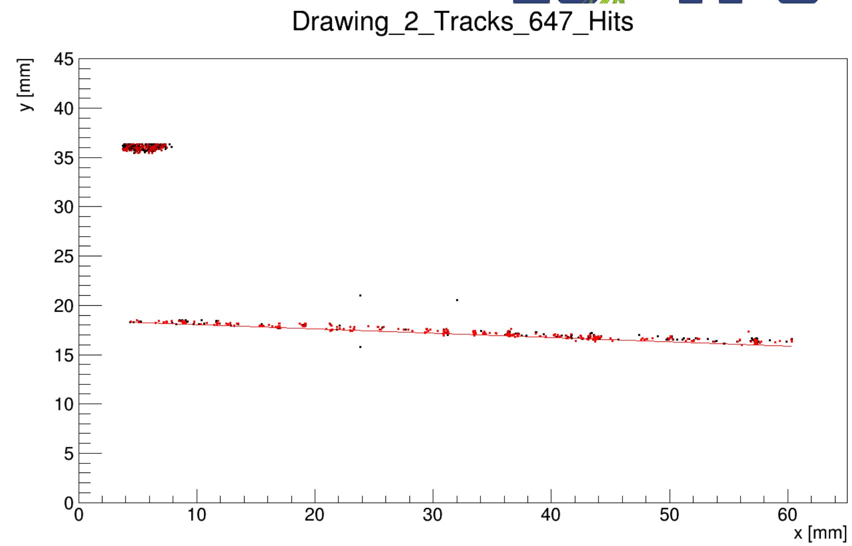
- Events without track



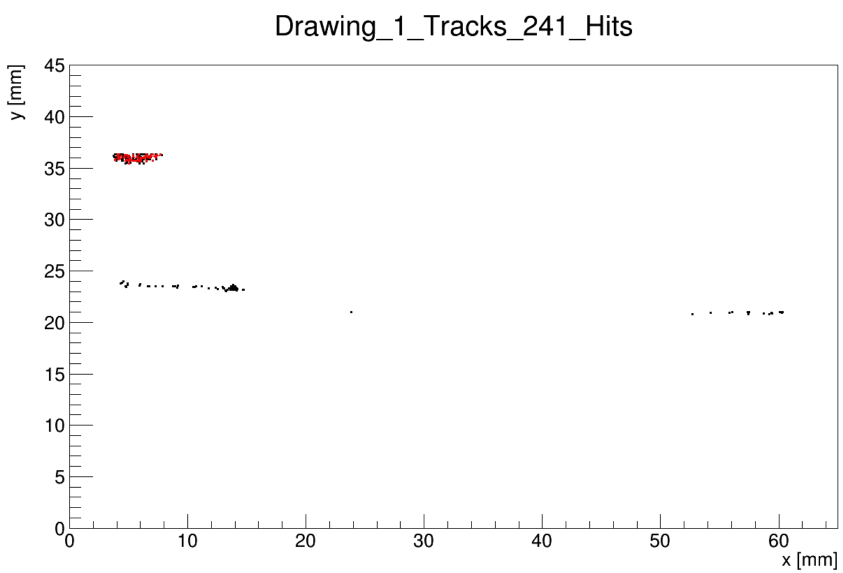
- Ghost track from pure noise events



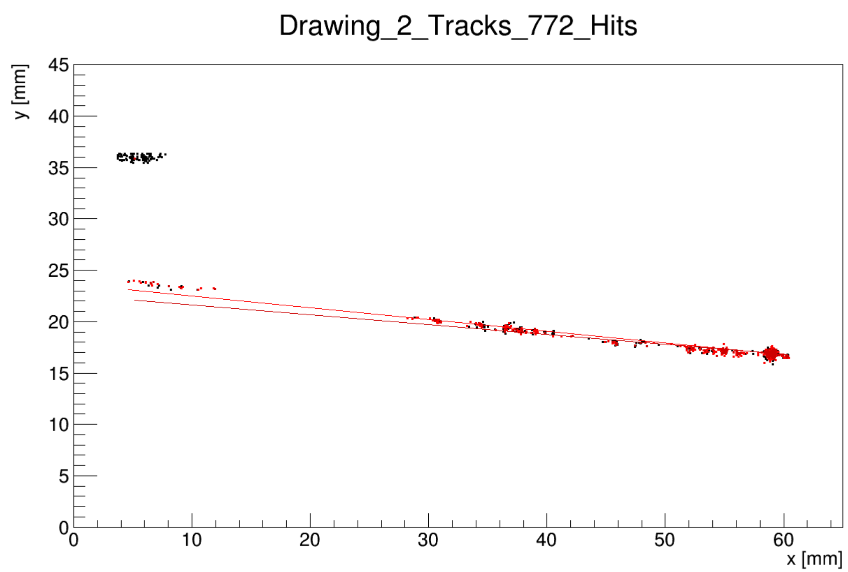
- Multi tracks (ghost and true track)



- Ghost track from noise + track events



- Clone track



InGridHoughTransformProcessor:

- _RXY: Scale factor of the range of BND fitting
- _RZ : Scale factor of the σ_z
- _RE: Scale factor of 1/e ellipse
- _PAH: Percentage of all hits
- _PIH: Percentage of inliers hits
- _Mrho: Maximum rho
- _Bn: Number of bins for rho
- _BZ: Number of bins for z scanning
- _MinPhi and _MaxPhi : the range of phi
- _gas: Drift gas for TPC
- _Efield: Drift Field

20130330-0330_13-16-55:

- Number of total event = 100
- Number of single track or without track (N_{event}) = 98
- Number of single track (N_{Strack})= 61
- 2 GeV

Name	scale 1/e	MinPhi	Maxphi	#bin rho	%allhit _PAH	%inliers _PIH	width BND	width sigZ	max Rho
BND_01	1	0	π	160	70	80	1	1	80
BND_02	1	0	π	160	30	80	1	1	80
BND_03	1	0	π	160	50	80	1	1	80
BND_04	1	0	π	160	50	80	2	1	80
BND_05	1	0	π	160	50	80	3	1	80

Name	track	N_{reco}	$N_{correct\ track}$	N_{ghost}	Num of Event (Pure noise)	$N_{mult.track}$	N_{clone}
BND_01	61	79	54	25	21	0	0
BND_02	61	81	60	40	21	15	2
BND_03	61	80	56	28	21	2	0
BND_04	61	80	56	29	21	3	1
BND_05	61	81	56	30	22	3	0

Reconstruction Efficiency (RE) = N_{reco} / N_{Event}

Selection Efficiency (SE) = $N_{correct\ track}$ / N_{reco}

Total Efficiency = RE x SE

Name	reconstruction efficiency	selection efficiency	Total Efficiency
BND_01	80.61%	68.35%	55.10%
BND_02	82.65%	74.07%	61.22%
BND_03	81.63%	70.00%	57.14%
BND_04	81.63%	70.00%	57.14%
BND_05	82.65%	69.14%	57.14%

20130401-0401_15-18-22:

- Number of total event = 100
- Number of single track or without track (N_{event}) = 98
- Number of single track (N_{Strack})= 79
- 6 GeV

Name	scale 1/e	MinPhi	Maxphi	#bin rho	%allhit _PAH	%inliers _PIH	width BND	width sigZ	max Rho
BND_01	1	0	π	160	70	80	1	1	80
BND_02	1	0	π	160	30	80	1	1	80

Name	track	N_{reco}	$N_{correct\ track}$	N_{ghost}	Num of Event (Pure noise)	N_{ghost}t+$N_{correct\ track}$	N_{clone}
BND_01	79	78	75	3	1	0	0
BND_02	79	79	77	6	1	0	0

Reconstruction Efficiency
(RE) = N_{reco} / N_{Event}

Selection Efficiency
(SE) = $N_{correct\ track}$ / N_{reco}

Total Efficiency = RE x SE

Name	reconstruction efficiency	selection efficiency	Total Efficiency
BND_01	79.59%	96.15%	76.53%
BND_02	80.61%	97.47%	78.57%

Summary and Outlook:

- Analysis of BND Hough Transform for One Octoboard is ongoing.
- For 160 chips with B =0 for most events works well but still is ongoing.
- Finding Tracklet based on the Octoboard unit
- Using 1 sigma of BND as a first approximation of the collecting hits
- Connecting tracklet in order to have full track
- Recollecting hits with 1,2 or 3 sigma of the tracks