Kaon Physics

Antonio Pich
IFIC, Univ. Valencia – CSIC

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Outline

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84 (2012) 399

1. **Theoretical Framework**

   Short and long-distance physics

2. **Leptonic and Semileptonic Decays**

   Lepton Universality. CKM determinations

3. **Nonleptonic Decays**

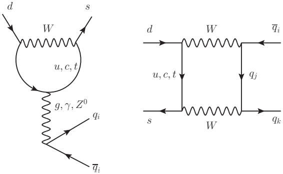
   Octet Enhancement. ε'/ε

4. **Rare and Radiative Decays**

   $K \to \pi\nu\bar{\nu}\,,\; K \to \pi\ell^+\ell^-\,,\; K \to \pi\gamma\gamma\; \ldots$

1. Theoretical Framework

- **Sensitivity to Short-Distance Scales:**



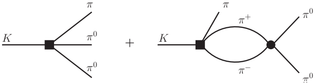
Charm mass prediction

Top quark

GIM cancellation

New Physics ?

- **Long-Distance Physics:**



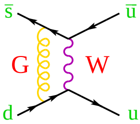
Chiral Dynamics

- **Multi-Scale Problem:**

$\log(M/\mu)$ **(OPE)** , $\log(\mu/m_\pi)$ **(χPT)**

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	⬇ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\rm QCD}^{(n_f=3)},\ \mathcal{L}_{\rm eff}^{\Delta S=1,2}$
	⬇ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

ΔS = 1 Transitions

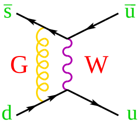


$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A}\,(\bar{u}_\beta d_\alpha)_{V-A}$$

$$Q_2 = (\bar{s}u)_{V-A}(\bar{u}d)_{V-A}$$

$$\mathcal{L}_{\mathrm{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}}\; V_{ud}\, V_{us}^* \sum_i C_i(\mu)\; Q_i(\mu)$$

$\Delta S = 1$ Transitions



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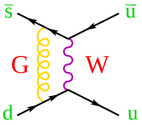
- **Experiment:** $A(K \to \pi\pi)_{\Delta I=\frac{1}{2}} / A(K \to \pi\pi)_{\Delta I=\frac{3}{2}} \approx 22$

$\Delta S = 1$ Transitions



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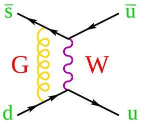
- $Q_- \equiv Q_2 - Q_1$ transforms as $(8_L, 1_R)\,,\ \Delta I = \frac{1}{2}$
- $Q_+ \equiv Q_2 + Q_1$ contains a piece $(27_L, 1_R)\,,\ \Delta I = \frac{3}{2}$
- **Electroweak SM ($\alpha_s = 0$):** $C_1 = 0\,,\ C_2 = 1$ ➡ $C_+ = C_- = \frac{1}{2}$

$\Delta S = 1$ Transitions



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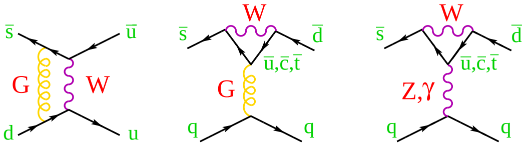
- **Short-distance QCD enhancement:** Altarelli-Maiani, Gaillard-Lee

$$C_\pm(\mu) \approx \frac{1}{2}\left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right)^{a_\pm} \quad , \qquad a_\pm = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \frac{6}{33 - 2N_f}$$

$\Delta S = 1$ Transitions



$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V_{us}^* \sum_i C_i(\mu)\; Q_i(\mu)$$

$$\begin{aligned}
Q_1 &= (\bar{s}_\alpha u_\beta)_{\rm V-A}\,(\bar{u}_\beta d_\alpha)_{\rm V-A} & Q_2 &= (\bar{s}u)_{\rm V-A}\,(\bar{u}d)_{\rm V-A} \\
Q_{3,5} &= (\bar{s}d)_{\rm V-A}\,\textstyle\sum_q (\bar{q}q)_{\rm V\mp A} & Q_4 &= (\bar{s}_\alpha d_\beta)_{\rm V-A}\,\textstyle\sum_q (\bar{q}_\beta q_\alpha)_{\rm V-A} \\
Q_{7,9} &= \tfrac{3}{2}(\bar{s}d)_{\rm V-A}\,\textstyle\sum_q e_q(\bar{q}q)_{\rm V\pm A} & Q_{10} &= \tfrac{3}{2}(\bar{s}_\alpha d_\beta)_{\rm V-A}\,\textstyle\sum_q e_q\,(\bar{q}_\beta q_\alpha)_{\rm V-A} \\
Q_6 &= -8\,\textstyle\sum_q (\bar{s}_L q_R)(\bar{q}_R d_L) & Q_8 &= -12\,\textstyle\sum_q e_q\,(\bar{s}_L q_R)(\bar{q}_R d_L) \\
Q_{11,12} &= (\bar{s}d)_{\rm V-A}\,\textstyle\sum_\ell (\bar{\ell}\ell)_{\rm V,A} & Q_{13} &= (\bar{s}d)_{\rm V-A}\,\textstyle\sum_\nu (\bar{\nu}\nu)_{\rm V-A}
\end{aligned}$$

- $q > \mu$: $\quad C_i(\mu) = z_i(\mu) - y_i(\mu)\,(V_{td}\, V_{ts}^* / V_{ud}\, V_{us}^*)$

  $O(\alpha_s^n t^n)\ ,\ O(\alpha_s^{n+1} t^n)$ $\qquad [t \equiv \log(M/m)]$ $\qquad$ Munich / Rome

- $q < \mu$: $\quad \langle \pi\pi | Q_i(\mu) | K \rangle$? $\qquad$ **Physics does not depend on μ**

Chiral Symmetry

$\mathbf{q} \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}$; $\mathbf{m_q = 0}$ (Chiral Limit)

$$\mathcal{L}^0_{QCD} = -\frac{1}{4} G^{\mu\nu}_a G^a_{\mu\nu} + \bar{\mathbf{q}}_L \, i\gamma^\mu D_\mu \mathbf{q}_L + \bar{\mathbf{q}}_R \, i\gamma^\mu D_\mu \mathbf{q}_R$$

- **Invariant under** $\mathbf{G \equiv SU(3)_L \otimes SU(3)_R}$ **:** $\mathbf{q}_{L,R} \to g_{L,R}\, \mathbf{q}_{L,R}$, $(g_L, g_R) \in \mathbf{G}$
- **Vacuum only invariant under** $\mathbf{SU(3)_V}$ **:** $\langle 0 | (\bar{\mathbf{q}}_L \mathbf{q}_R + \bar{\mathbf{q}}_R \mathbf{q}_L) | 0 \rangle \neq 0$

8 Massless 0^- Goldstone Bosons

$$\langle 0 | \bar{\mathbf{q}}^j_L \, \mathbf{q}^i_R | 0 \rangle \quad \Rightarrow \quad \mathbf{U}_{ij}(\phi) = \left\{ \exp\left(i\sqrt{2}\, \mathbf{\Phi}/F \right) \right\}_{ij} \longrightarrow g_R \; \mathbf{U}(\phi) \; g_L^\dagger$$

$$\mathbf{\Phi} \equiv \frac{\vec{\lambda}}{\sqrt{2}} \, \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix} \quad , \quad F_\pi = F \left\{ 1 + \mathcal{O}(m_\pi^2) \right\}$$

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	$\Downarrow$ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\Downarrow$ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

Chiral Perturbation Theory (χPT)

- **Expansion in powers of p^2/Λ_χ^2 :** $\mathcal{A} = \sum_n \mathcal{A}^{(n)}$ $(\Lambda_\chi \sim 4\pi F_\pi \sim 1.2\ \mathrm{GeV})$
- **Amplitude structure fixed by chiral symmetry**

$$\mathbf{SU(3)_L \otimes SU(3)_R \to SU(3)_V}$$

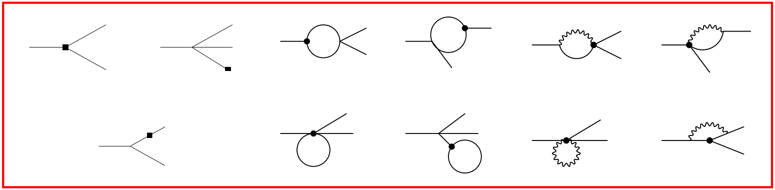
- **Short-distance dynamics encoded in Low-Energy Couplings (LECs)**
- **O(p^2) χPT: Goldstone interactions (π, K, η)** $\Phi \equiv \frac{1}{\sqrt{2}} \vec{\lambda}\, \vec{\varphi}$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \mathrm{Tr}(\lambda L_\mu L^\mu) + G_{27}\, F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3}\, L_{\mu 21} L^\mu_{13} \right)$$

$$G_R \equiv -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^*\, g_R \quad ; \quad L_\mu = -i U^\dagger D_\mu U \quad ; \quad \lambda \equiv \frac{1}{2}\lambda_{6-i7} \quad ; \quad U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$$

- **Loop corrections (χPT logarithms) unambiguously predicted**
- **LECs can be determined at $N_C \to \infty$ (matching)**
- **O(p^2) LECs (G_8, G_{27}) can be phenomenologically determined**

$\mathbf{O}\left[\mathbf{p^4}, (\mathbf{m_u} - \mathbf{m_d})\,\mathbf{p^2}, \mathbf{e^2 p^0}, \mathbf{e^2 p^2}\right]$ χPT



- **Nonleptonic weak Lagrangian:** $\mathcal{O}(G_F\, p^4)$

$$\mathcal{L}_{\rm weak}^{(4)} = \sum_i G_8\, N_i\, F^2\, O_i^8 + \sum_i G_{27}\, D_i\, F^2\, O_i^{27} + \text{h.c.}$$

- **Electroweak Lagrangian:** $\mathcal{O}(G_F\, e^2 p^{0,2})$ $\qquad \mathcal{Q} = \text{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$

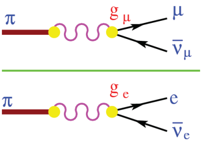
$$\mathcal{L}_{\rm EW} = e^2 F^6 G_8\, g_{ew}\, \text{Tr}(\lambda U^\dagger \mathcal{Q} U) + e^2 \sum_i G_8\, Z_i\, F^4\, O_i^{EW} + \text{h.c.}$$

- $\mathcal{O}(e^2 p^{0,2})$ **Electromagnetic** $+$ $\mathcal{O}(p^4)$ **Strong:** $\qquad Z\,,\ K_i\,,\ L_i$

2. (Semi) Leptonic Decays

Lepton Universality:

$$R_{e/\mu}^{(P)} \equiv \frac{\Gamma(P^- \to e^-\bar{\nu}_e)}{\Gamma(P^- \to \mu^-\bar{\nu}_\mu)}$$



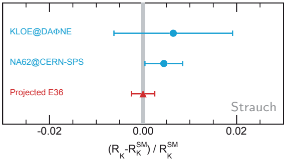
$$R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \cdot 10^{-4}$$

$$R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \cdot 10^{-5}$$

Cirigliano-Rosell '07

$$R_{e/\mu}^{(\pi)}\Big|_{\mathrm{exp}} = (1.230 \pm 0.004) \cdot 10^{-4}$$

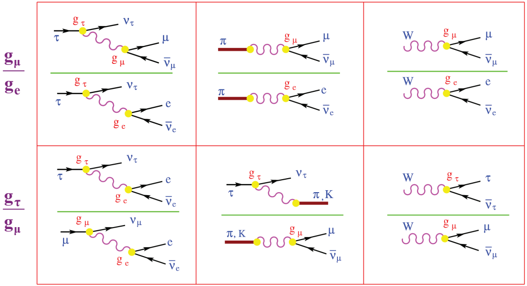
$$R_{e/\mu}^{(K)}\Big|_{\mathrm{exp}} = (2.488 \pm 0.010) \cdot 10^{-5}$$



$$\frac{|g_\mu|}{|g_e|} = \begin{cases} 1.0021 \pm 0.0016 & \pi \to \mu/e \\ 0.9978 \pm 0.0020 & K \to \mu/e \\ 1.0010 \pm 0.0025 & K \to \pi\mu/e \\ 1.0018 \pm 0.0014 & \tau \to \mu/e \end{cases}$$

A.P., PPNP 75 (2014) 41

Charged Current Universality



$|g_\mu / g_e|$

$B_{\tau\to\mu}/B_{\tau\to e}$	1.0018 ± 0.0014
$B_{\pi\to\mu}/B_{\pi\to e}$	1.0021 ± 0.0016
$B_{K\to\mu}/B_{K\to e}$	0.9978 ± 0.0020
$B_{K\to\pi\mu}/B_{K\to\pi e}$	1.0010 ± 0.0025
$B_{W\to\mu}/B_{W\to e}$	0.996 ± 0.010

$|g_\tau / g_\mu|$

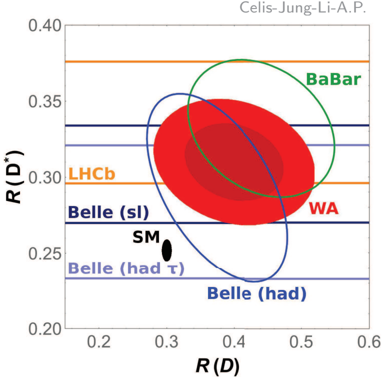
$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	1.0011 ± 0.0015
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	0.9962 ± 0.0027
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	0.9858 ± 0.0070
$B_{W\to\tau}/B_{W\to\mu}$	1.034 ± 0.013

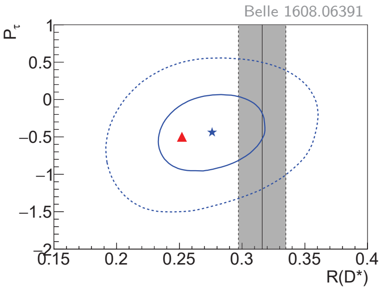
$|g_\tau / g_e|$

$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0030 ± 0.0015
$B_{W\to\tau}/B_{W\to e}$	1.031 ± 0.013

A.P., PPNP 75 (2014) 41

$$R(D^{(*)}) \equiv \frac{\mathrm{Br}(\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau)}{\mathrm{Br}(\bar{B} \to D^{(*)} \ell^- \bar{\nu}_\ell)}$$





LHCb: $(q^2 \in [1,6]\ \mathrm{GeV}^2)$

$$\frac{\mathrm{Br}(B^+ \to K^+ \mu^+ \mu^-)}{\mathrm{Br}(B^+ \to K^+ e^+ e^-)} = 0.745\,^{+0.090}_{-0.074} \pm 0.036$$

$2.6\,\sigma$ below the SM

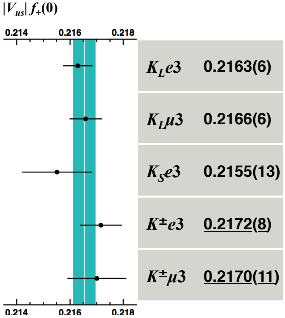
Violation of Lepton Flavour

$K \to \pi\, \ell\, \nu_\ell$

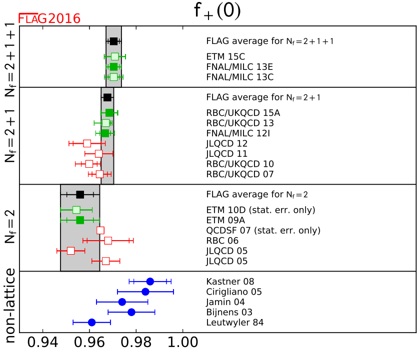
$$|\mathbf{V_{us}}\, \mathbf{f_+(0)}| \;=\; \mathbf{0.2165 \pm 0.0004}$$

Flavianet Kaon WG, arXiv:1005.2323 [hep-ph]

Moulson@CKM14, arXiv:1411.5252 [hep-ph]



$$\langle \pi^- | \bar{s}\gamma_\mu u | K^0 \rangle = (p_\pi + p_K)_\mu\; f_+(t) + (p_K - p_\pi)_\mu\; f_-(t)$$



$$f_+(0) = 0.9704\,(33)$$

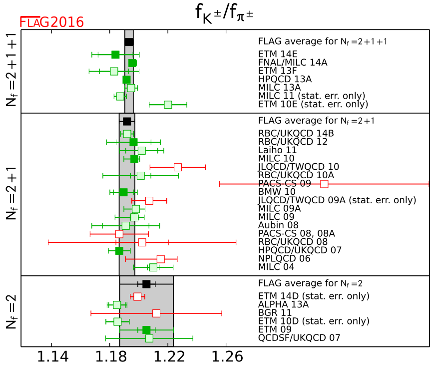
$$\longrightarrow \quad |V_{us}| = 0.2231\,(9)$$

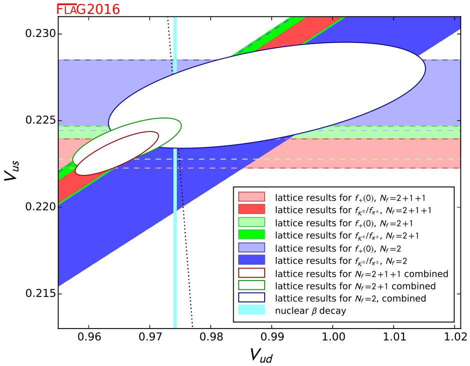
$$f_+(0) = 1 + f_2 + f_4 + \cdots$$

Large $\mathcal{O}(p^6)$ χPT correction

$\Gamma(K^+ \to \mu^+\nu_\mu)/\Gamma(\pi^+ \to \mu^+\nu_\mu)$

$$\frac{f_K}{f_\pi}\,\frac{|V_{us}|}{|V_{ud}|} = 0.2760\,(4) \quad \longrightarrow \quad \frac{|V_{us}|}{|V_{ud}|} = 0.2313\,(7)$$





$$f_K/f_\pi = 1.1933\,(29) \qquad\qquad \langle 0|\bar{d}_i\gamma^\mu\gamma_5 u_j|P(k)\rangle = i\,f_P\,k^\mu = i\,\sqrt{2}\,F_P\,k^\mu$$

3. Nonleptonic Decays

- **Octet Enhancement:** $\frac{A(K \to \pi\pi)_{I=0}}{A(K \to \pi\pi)_{I=2}} \approx 22$

  - **Short-distance:** **gluonic corrections, penguins**
  - **Long-distance:** **large χPT corrections (FSI)**
  - **Ongoing Lattice efforts**

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- **Direct CP Violation:** $\eta_{ij} \equiv \frac{A(K_L \to \pi^i \pi^j)}{A(K_S \to \pi^i \pi^j)}$

$$\mathrm{Re}\,(\epsilon'/\epsilon) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.8 \pm 1.4) \cdot 10^{-4}$$

NA31, E731, NA48, KTeV

$$\mathrm{Re}\,(\epsilon'/\epsilon)_{\mathrm{SM}} = \left(19 \pm 2\,^{+9}_{-6} \pm 6\right) \cdot 10^{-4}$$

Pallante-Pich-Scimemi

$K \to 2\pi$ Isospin Amplitudes

$$A[K^0 \to \pi^+\pi^-] \equiv A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2}$$

$$A[K^0 \to \pi^0\pi^0] \equiv A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2}$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2} A_2^+\, e^{i\chi_2^+}$$

1) $\Delta I = 1/2$ Rule: $\omega \equiv \frac{\mathrm{Re}\,(A_2)}{\mathrm{Re}\,(A_0)} \approx \frac{1}{22}$

2) Strong Final State Interactions: $\chi_0 - \chi_2 \approx \delta_0 - \delta_2 \approx 45^\circ$

$$\varepsilon'_K = \frac{-i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \omega \left\{ \frac{\mathrm{Im}\,(A_0)}{\mathrm{Re}\,(A_0)} - \frac{\mathrm{Im}\,(A_2)}{\mathrm{Re}\,(A_2)} \right\}$$

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$$\begin{aligned} A_0\, e^{i\chi_0} &= \mathcal{A}_{1/2} \\ A_2\, e^{i\chi_2} &= \mathcal{A}_{3/2} + \mathcal{A}_{5/2} \\ A_2^+\, e^{i\chi_2^+} &= \mathcal{A}_{3/2} - \frac{2}{3}\mathcal{A}_{5/2} \end{aligned}$$

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$\mathbf{O(p^2)}$ χPT

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle \; + \; G_{27}\, F^4 \left(L_{\mu 23}\, L^\mu_{11} + \frac{2}{3}\, L_{\mu 21}\, L^\mu_{13} \right)$$

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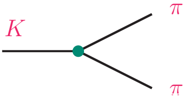


$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left(G_8 + \frac{1}{9}\, G_{27} \right) (m_K^2 - m_\pi^2)$$

$$\mathcal{A}_{3/2} = \frac{10}{9}\, F_\pi\, G_{27}\, (m_K^2 - m_\pi^2)$$

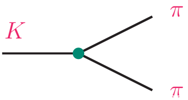
$$\mathcal{A}_{5/2} = 0 \qquad ; \qquad \delta_0 = \delta_2 = 0$$

$[\Gamma(K \to 2\pi) + \delta_I]_{\rm Exp}$ ➡ $|g_8| \approx 5.1$; $|g_{27}| \approx 0.29$

$O(p^2)$ χPT

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle \; + \; G_{27}\, F^4 \left(L_{\mu 23}\, L_{11}^\mu + \frac{2}{3}\, L_{\mu 21}\, L_{13}^\mu \right)$$

$G_R \equiv -\frac{G_F}{\sqrt{2}}\, V_{ud} V_{us}^*\, g_R$; $L_\mu = -iU^\dagger D_\mu U$; $\lambda \equiv \frac{1}{2}\lambda_{6-i7}$; $U \equiv \exp\left\{ i\sqrt{2}\,\Phi / F \right\}$



$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left(G_8 + \frac{1}{9}\, G_{27} \right) (m_K^2 - m_\pi^2)$$

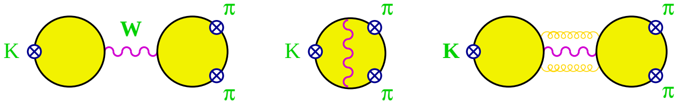
$$\mathcal{A}_{3/2} = \frac{10}{9}\, F_\pi\;\; G_{27}\, (m_K^2 - m_\pi^2)$$

$$\mathcal{A}_{5/2} = 0 \qquad ; \qquad \delta_0 = \delta_2 = 0$$

$[\Gamma(K \to 2\pi) + \delta_I]_{\mathrm{Exp}}$ ➡ $|g_8| \approx 5.1$; $|g_{27}| \approx 0.29$

$O(p^4)$ χPT fit: $(\mathcal{A}_{5/2}\,,\, \delta_{0,2})$ $|g_8| \approx 3.6$; $|g_{27}| \approx 0.29$

Weak Currents Factorize at Large N_C



$O(N_C^2)$ $O(N_C)$ $O(1)$

$$A[K^0 \to \pi^0\pi^0] = 0 \quad \Longrightarrow \quad A_0 = \sqrt{2}\, A_2$$

No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

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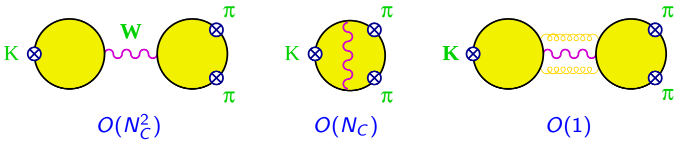

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No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

- **Multiscale problem:** **OPE** $\quad \frac{1}{N_C}\, \log\left(\frac{M_W}{\mu}\right) \sim \frac{1}{3} \times 4$

Short-distance logarithms must be summed

- **Large χPT logarithms:** **FSI** $\quad \frac{1}{N_C}\, \log\left(\frac{\mu}{m_\pi}\right) \sim \frac{1}{3} \times 2$

Infrared logarithms must also be included $\quad [\delta_I \sim O(1/N_C)\ ,\ \delta_0 - \delta_2 \approx 45^\circ]$

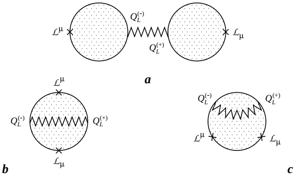
Dynamical understanding of the $\Delta I = 1/2$ rule

AP – E. de Rafael, PL B374 (1996) 186

$$\mathcal{L}_{\rm eff} \;=\; -\frac{G_F}{\sqrt{2}}\, F^4 \left[a\; \mathrm{Tr}(Q_L^{(-)} L_\mu)\, \mathrm{Tr}(Q_L^{(+)} L^\mu) \,+\, b\; \mathrm{Tr}(Q_L^{(-)} L_\mu Q_L^{(+)} L^\mu) \,+\, c\; \mathrm{Tr}(Q_L^{(-)} Q_L^{(+)} L_\mu L^\mu)\right]$$

$\mathcal{O}(N_C^2)$

$\mathcal{O}(N_C)$



$$Q_L^{(+)} = \begin{pmatrix} 0 & V_{\rm ud} & V_{\rm us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad Q_L^{(-)} = Q_L^{(+)\dagger}$$

$$g_8 \;=\; \frac{3}{5}\,(a+b) - b + c$$

$$g_{27} \;=\; \frac{3}{5}\,(a+b)$$

$$a \;=\; 1 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \qquad ; \qquad c \;=\; \mathrm{Re}\, C_4 - 16\, L_5\, \mathrm{Re}\, C_6(\mu^2) \left[\frac{\langle\bar\psi\psi\rangle}{f_\pi^3}\right]^2 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \;\simeq\; 0.3 \pm 0.2$$

$$|g_{27}| \;\simeq\; 0.29 \quad \Longrightarrow \quad b \simeq -0.52 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \quad \Longrightarrow \quad g_8 \;\simeq\; 1.1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

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$\mathcal{O}(N_C^2)$

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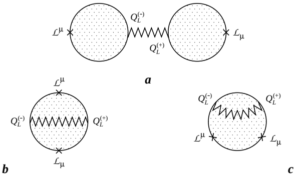

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$b < 0$ predicted through explicit calculations

AP–E. de Rafael, NP B358 (1991) 311
Bardeen-Buras-Gerard, Bijnens-Prades, Bertolini et al

Confirmed through inclusive QCD analysis

M. Jamin–AP, NP B425 (1994) 15

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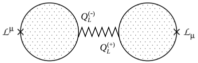
$\mathcal{O}(N_C^2)$



a

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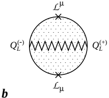
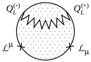
$\mathcal{O}(N_C)$



b



c

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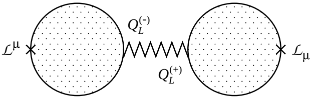
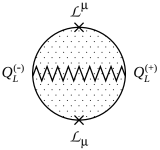
Confirmed recently by lattice calculations RBC-UKQCD, PRL 110 (2013) 15, 152001
PRD 91 (2015) 7, 074502

“A qualitative picture towards the understanding of the underlying physics begins to emerge”

AP – E. de Rafael, PL B374 (1996) 186





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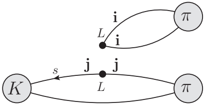
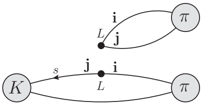
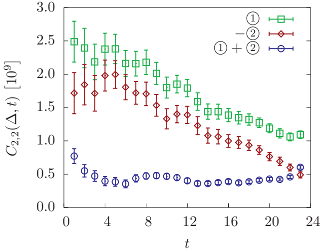
“Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD”

RBC-UKQCD, PRL 110 (2013) 15, 152001







$$b \approx -0.7\, a$$

Multi-Scale Problem: Summation of logarithms needed

A large $\log(M_1/M_2)$ compensates a $1/N_C$ suppression

❶ **Short-distance:** $\frac{1}{N_C}\,\log\left(M_W/\mu\right)$ — Bardeen-Buras-Gerard

$$\rightarrow \begin{cases} g_8^\infty = 1.13 \pm 0.05_\mu \pm 0.08_{L_5} \pm 0.05_{m_s} \\ g_{27}^\infty = 0.46 \pm 0.01_\mu \end{cases}$$

Cirigliano et al, Pallante et al

❷ **Long-distance (χPT):** $\frac{1}{N_C}\,\log\left(\mu/m_\pi\right)$ — Kambor et al, Pallante et al

$$g_8^{\mathrm{LO}} = 5.0 \rightarrow g_8^{\mathrm{NLO}} = 3.6$$

$$g_{27}^{\mathrm{LO}} = 0.285 \rightarrow g_{27}^{\mathrm{NLO}} = 0.286$$

Cirigliano et al

❸ **Isospin Violation:** $g_{27}^{\mathrm{NLO}} = 0.297$ — Cirigliano et al

$\mathbf{N_C \to \infty}$

$$g_8 \; = \; \left(\frac{3}{5}\, C_2 - \frac{2}{5}\, C_1 + C_4 \right) - 16 \;\; L_5 \; \left(\frac{\langle \bar q \, q \rangle (\mu)}{F_\pi^3} \right)^2 C_6(\mu)$$

$$g_{27} \; = \; \frac{3}{5}\, (C_2 + C_1)$$

$$e^2 \, g_8 \, g_{ew} \; = \; -3 \; \left(\frac{\langle \bar q \, q \rangle (\mu)}{F_\pi^3} \right)^2 \left[C_8(\mu) \; + \; \frac{16}{9}\; C_6(\mu)\; e^2 \; (K_9 - 2\, K_{10}) \right]$$

$$\frac{\langle \bar q \, q \rangle (\mu)}{F_\pi^3} \; = \; \frac{m_{K^0}^2}{(m_s + m_d)(\mu)\, F_\pi} \; \left\{ 1 \; - \; \frac{8\, m_{K^0}^2}{F_\pi^2}\, (2 L_8 - L_5) \; + \; \frac{4\, m_{\pi^0}^2}{F_\pi^2}\, L_5 \right\}$$

- **Equivalent to standard calculations of $\mathbf{B_i}$**
- μ **dependence only captured for $\mathbf{Q_{6,8}}$**

Anomalous Dimension Matrix

$$\gamma_s^{(0)} = \begin{pmatrix}
-\frac{3}{N_c^2} & \frac{3}{N_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{3}{N_c} & -\frac{3}{N_c^2} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{11}{3N_c^2} & \frac{11}{3N_c} & -\frac{2}{3N_c^2} & \frac{2}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{N_c} - \frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} - \frac{3}{N_c^2} & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & -\frac{n_f}{3N_c^2} & -3 + \frac{n_f}{3N_c} + \frac{3}{N_c^2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 \\
0 & 0 & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & 0 & -3 + \frac{3}{N_c^2} & 0 & 0 \\
0 & 0 & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & 0 & 0 & -\frac{3}{N_c^2} & 0 \\
0 & 0 & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & 0 & 0 & 0 & -\frac{3}{N_c^2}
\end{pmatrix}$$

Only γ_{66} and γ_{88} survive the large–N_C limit

Anatomy of ε'/ε calculation

$$\frac{\varepsilon'_K}{\varepsilon_K} \propto \left[\frac{105\,\mathrm{MeV}}{m_s(2\,\mathrm{GeV})}\right]^2 \left\{ B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.4\, B_8^{(3/2)} \right\}$$

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❶ **$O(p^4)$ χPT Loops:** **Large correction** (NLO in $1/N_C$) FSI

$$\mathcal{A}_n^{(X)} = a_n^{(X)} \left[1 + \Delta_L \mathcal{A}_n^{(X)} + \Delta_C \mathcal{A}_n^{(X)}\right]$$

Pallante-Pich-Scimemi

$$\Delta_L \mathcal{A}_{1/2}^{(8)} = 0.27 \pm 0.05 + 0.47\, i \quad ;$$

$$\Delta_L \mathcal{A}_{1/2}^{(27)} = 1.02 \pm 0.60 + 0.47\, i \quad ; \quad \Delta_L \mathcal{A}_{3/2}^{(27)} = -0.04 \pm 0.05 - 0.21\, i$$

$$\Delta_L \mathcal{A}_{1/2}^{(g)} = 0.27 \pm 0.05 + 0.47\, i \quad ; \quad \Delta_L \mathcal{A}_{3/2}^{(g)} = -0.50 \pm 0.20 - 0.21\, i$$

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❷ $O(p^4)$ **LECs fixed at** $N_C \to \infty$: **Small correction** $\Delta_C \mathcal{A}_n^{(X)}$

❸ **Isospin Breaking** $O\left[(m_u - m_d)\, p^2, e^2 p^2\right]$: **Sizeable correction**

$$\Omega_{\text{eff}} = 0.06 \pm 0.08$$

Cirigliano-Ecker-Neufeld-Pich

❹ $\mathrm{Re}(g_8)$, $\mathrm{Re}(g_{27})$, $\chi_0 - \chi_2$ **fitted to data**

$$\frac{\varepsilon'_K}{\varepsilon_K} \propto \left[\frac{105\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.4\ B_8^{(3/2)}\right\}$$

Delicate Cancellation. Strong Sensitivity to:

- m_s (quark condensate) $\qquad m_s(2\ \mathrm{GeV}) = 110 \pm 20$ MeV
- Isospin Breaking $(m_u \neq m_d\ ,\ \alpha)$ $\qquad \Omega_{\mathrm{eff}} = 0.06 \pm 0.08$

Cirigliano-Ecker-Neufeld-Pich

- Penguin Matrix Elements

χPT Loops (FSI): $\quad B_{6,\infty}^{(1/2)} \times (1.35 \pm 0.05) \quad ; \quad B_{8,\infty}^{(3/2)} \times (0.54 \pm 0.20)$

$$\frac{\varepsilon'_K}{\varepsilon_K} \propto \left[\frac{105\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{ B_6^{(1/2)} (1 - \Omega_{\mathrm{eff}}) - 0.4\ B_8^{(3/2)} \right\}$$

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Pallante–Pich–Scimemi '01: (updated '04)

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = \left(19 \pm 2_{\,\mu}\ {}^{+9}_{-6}{}_{\,m_s} \pm 6_{\,1/N_C}\right) \times 10^{-4}$$

Experimental world average: $\mathrm{Re}\,(\varepsilon'/\varepsilon) = (16.8 \pm 1.4) \times 10^{-4}$

Challenge: Control of subleading $1/N_C$ corrections to χPT couplings

Recent Lattice Results

Isospin limit: RBC-UKQCD 1505.07863, 1502.00263

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_2 = (1.50 \pm 0.04 \pm 0.14)\cdot 10^{-8}\ \mathrm{GeV} \qquad \mathrm{exp}: 1.482\,(2)\cdot 10^{-8}\ \mathrm{GeV}$$

0.1 σ

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_2 = -(6.99 \pm 0.20 \pm 0.84)\cdot 10^{-13}\ \mathrm{GeV}$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_0 = (4.66 \pm 1.00 \pm 1.26)\cdot 10^{-7}\ \mathrm{GeV} \qquad \mathrm{exp}: 3.112\,(1)\cdot 10^{-7}\ \mathrm{GeV}$$

1.0 σ

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_0 = -(1.90 \pm 1.23 \pm 1.08)\cdot 10^{-11}\ \mathrm{GeV}$$

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.38 \pm 5.15 \pm 4.59)\cdot 10^{-4} \qquad \mathrm{exp}: (16.8 \pm 1.4)\cdot 10^{-4}$$

2.2 σ

$$\delta_0 = (23.8 \pm 4.9 \pm 1.2)^\circ \qquad \mathrm{exp}: (39.2 \pm 1.5)^\circ$$

2.9 σ

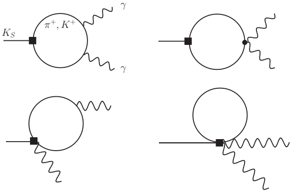
$$\delta_2 = -(11.6 \pm 2.5 \pm 1.2)^\circ \qquad \mathrm{exp}: -(8.5 \pm 1.5)^\circ$$

1.0 σ

4. Rare and Radiative Decays

$K^0 \to \gamma\gamma$

Long-distance dynamics



Finite loop: $\mathrm{Br}_{\mathrm{LO}} = 2.0 \cdot 10^{-6}$

D'Ambrosio-Espriu, Goity

$$\mathrm{Br}(K_S \to \gamma\gamma) = (2.63 \pm 0.17) \cdot 10^{-6}$$

Agreement at $O(p^6)$ (FSI)

$$K_S \to \pi\pi \to \pi^+\pi^- \to \gamma\gamma$$

Kambor-Holstein, Buchalla et al

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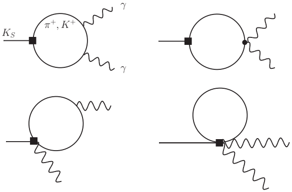

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Kambor-Holstein, Buchalla et al

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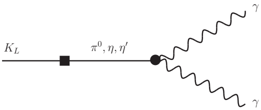


$$\mathrm{Br}(K_L \to \gamma\gamma) = (5.47 \pm 0.04) \cdot 10^{-4}$$

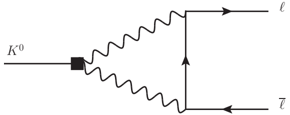
WZW Anomaly

$\mathbf{T}_{\mathrm{LO}} = \mathbf{0}$ [$\mathcal{O}(p^4)$, GMO cancel.]

$\mathcal{O}(p^6)$: SU(3) breaking, η–η' mixing

Well understood

$K^0 \to \ell^+\ell^-$



$K_S \to \ell^+\ell^-$

Long-distance dynamics

Finite 2-loop amplitude: Ecker-Pich

$$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{LO}} = 2.1 \cdot 10^{-14}$$

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{LO}} = 5.1 \cdot 10^{-12}$$

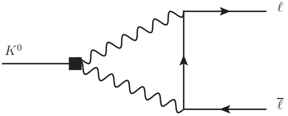
$$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{exp}} < 9 \cdot 10^{-9}$$

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{exp}} < 9 \cdot 10^{-9}$$ LHCb

$5.8 \cdot 10^{-9}$ LHCb prel.

(90% CL)

$K^0 \to \ell^+\ell^-$



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(90% CL)

$K_L \to \ell^+\ell^-$

$$\mathrm{Br}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \cdot 10^{-9}$$

$$\mathrm{Br}(K_L \to e^+e^-) = (9\,^{+6}_{-4}) \cdot 10^{-12}$$

Saturated by absorptive contrib.

Local countertem ⟷ **SD**

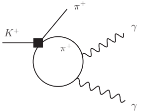
LD extracted from $\pi^0, \eta \to \ell^+\ell^-$

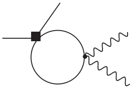
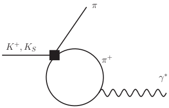
Gomez-Dumm, Pich

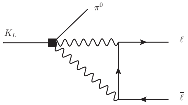
Fitted SD contrib. agrees with SM

Longitudinal Polarization: Ecker-Pich

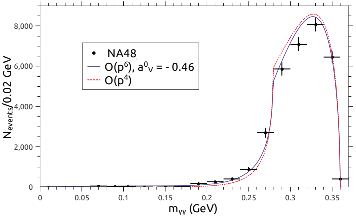
$$|\mathcal{P}_L| = (2.6 \pm 0.4) \cdot 10^{-3}$$





$$\mathrm{Br}(K_L \to \pi^0\gamma\gamma) = (1.27 \pm 0.03)\cdot 10^{-6}$$



Finite 1-loop amplitude $[\mathcal{O}(p^4)]$**:**

$$\mathrm{Br}(K_L \to \pi^0\gamma\gamma)_{\mathrm{LO}} = 6.8 \cdot 10^{-7}$$

Ecker-Pich-de Rafael, Cappiello-D'Ambrosio, Sehgal

$\mathcal{O}(p^6)$ unitarity corrections needed

Cohen et al, Cappiello et al, D'Ambrosio-Portolés

$$\mathrm{Br}(K_L \to \pi^0 e^+e^-) < 2.8 \cdot 10^{-10}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+\mu^-) < 3.8 \cdot 10^{-10}$$

(90% CL), KTeV

3 contributions:

Ecker-Pich-de Rafael

- **Direct** $\not{C}\not{P}$
- **Indirect** $\not{C}\not{P}$
- **CP conserving** (2γ)

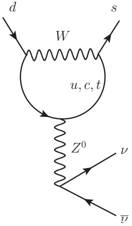
$\not{C}\not{P}$ **dominates for** e^+e^-**:**

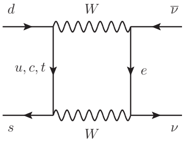
$$\mathrm{Br}(K_L \to \pi^0 e^+e^-) = 3.1\ (0.9)\cdot 10^{-11}$$

Buchalla et al

$K \to \pi\nu\bar{\nu}$

$$T \sim F\left(V_{is}^{*} V_{id}, \frac{m_i^2}{M_W^2}\right) \left(\bar{\nu}_L \gamma_\mu \nu_L\right) \langle \pi | \bar{s}_L \gamma^\mu d_L | K \rangle$$





Negligible long-distance contribution

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = (7.8 \pm 0.8) \cdot 10^{-11} \sim A^4 \left[\eta^2 + (1.4 - \rho)^2\right]$$

$$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) = (2.4 \pm 0.4) \cdot 10^{-11} \sim A^4 \eta^2$$

Buras et al

Brod et al

$\mathcal{A}(K_L \to \pi^0\nu\bar{\nu}) \neq 0$ ➡ **Direct** $\not{C}\not{P}$

BNL-E949: few events! ➡ $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = \left(1.73\,^{+1.15}_{-1.05}\right) \cdot 10^{-10}$

KEK-E391a: $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) < 2.6 \cdot 10^{-8}$ (90% CL)

Ongoing Experiments: NA62, K0TO

Summary

Kaons continue providing important physics information:

- **Interesting interplay of short and long-distances**
- **Sensitive to heavy mass scales. New Physics?**
- **Superb probe of flavour dynamics and $\not{\mathcal{C}}\not{\mathcal{P}}$**
- **Excellent testing ground of χPT dynamics**

Increased sensitivities at ongoing experiments $(K \to \pi\nu\bar{\nu})$

Theoretical challenge: precise control of QCD effects

Successful SM prediction for ϵ'/ϵ

$$\text{Re}\,(\epsilon'/\epsilon)_{\text{SM}} = \left(19 \pm 2\,{}^{+9}_{-6} \pm 6\right) \cdot 10^{-4}$$

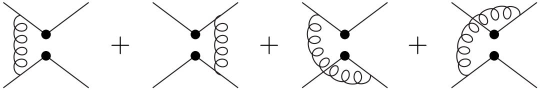
Pallante-Pich-Scimemi

Large uncertainty but no anomalies!

Backup

Wilson Coefficients in the Fermi EFT

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{12} V_{43}^* \, O_{\{1,2;3,4\}} \qquad ; \qquad O_{\{1,2;3,4\}} \equiv [\bar{q}_1 \gamma^\mu (1-\gamma_5) q_2]\,[\bar{q}_3 \gamma_\mu (1-\gamma_5) q_4]$$



Colour: $$\sum_a \lambda^a_{ij} \lambda^a_{kl} = -\frac{2}{N_C}\, \delta_{ij}\, \delta_{kl} + 2\, \delta_{il}\, \delta_{kj}$$

Fierz: $$[\gamma^\mu (1-\gamma_5)]_{\alpha\beta}\; [\gamma_\mu (1-\gamma_5)]_{\gamma\delta} = -\,[\gamma^\mu (1-\gamma_5)]_{\alpha\delta}\; [\gamma_\mu (1-\gamma_5)]_{\gamma\beta}$$

$$\mathcal{L}_{\rm eff} = \frac{G_F}{2\sqrt{2}} V_{12} V_{43}^* \, \{ c_+(\mu)\, Q_+ + c_-(\mu)\, Q_- \} \qquad ; \qquad Q_\pm \equiv O_{\{1,2;3,4\}} \pm O_{\{1,4;3,2\}}$$

$$\gamma_\pm = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \frac{\alpha_s}{\pi} \;\Longrightarrow\; \boxed{ c_\pm(\mu) \approx \left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)} \right)^{a_\pm} \quad , \quad a_\pm = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \frac{6}{33 - 2N_f} }$$

$\mathbf{O(p^2, e^2p^0)}\quad \chi\mathbf{PT}$

$\mathcal{Q} = \mathrm{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle + G_{27}\, F^4 \left(L_{\mu 23}\, L^\mu_{11} + \frac{2}{3}\, L_{\mu 21}\, L^\mu_{13} \right) + e^2\, F^6\, G_8\, g_{ew}\, \langle \lambda U^\dagger \mathcal{Q} U \rangle$$

$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left\{ G_8 \left[(m_K^2 - m_\pi^2) \left(1 - \frac{2}{3\sqrt{3}}\, \varepsilon^{(2)} \right) - \frac{2}{3}\, F_\pi^2\, e^2\, (g_{ew} + 2\, Z) \right] + \frac{1}{9}\, G_{27}\, (m_K^2 - m_\pi^2) \right\}$$

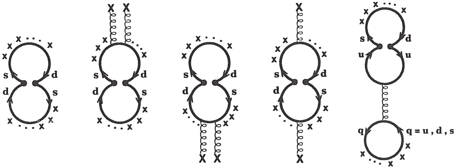
$$\mathcal{A}_{3/2} = \frac{2}{3}\, F_\pi \left\{ \left(\frac{5}{3}\, G_{27} + \frac{2}{\sqrt{3}}\, \varepsilon^{(2)}\, G_8 \right) (m_K^2 - m_\pi^2) - F_\pi^2\, e^2\, G_8\, (g_{ew} + 2\, Z) \right\}$$

$$\mathcal{A}_{5/2} = 0 \qquad ; \qquad \delta_0 = \delta_2 = 0$$

$\varepsilon^{(2)} = (\sqrt{3}/4)\, (m_d - m_u)/(m_s - \hat{m}) \approx 0.011$; $Z \approx (m_{\pi^\pm}^2 - m_{\pi^0}^2)/(2\, e^2 F_\pi^2) \approx 0.8$

Effective Action Model: Bosonization in Gluonic Background

AP–E. de Rafael, NP B358 (1991) 311



$$\Delta = \frac{1}{N_C}\left[1 - \frac{N_C}{2}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{16\pi^2 f_\pi^4} + \mathcal{O}(\alpha_s^2 N_C^2)\right] < 0$$

$$g_{27} \approx \frac{3}{5}\, C_+(\mu^2)\; \left\{1 + \Delta + \mathcal{O}(1/N_C^2)\right\}$$

$$g_8 \approx \frac{1}{2}\, C_-(\mu^2)\; \left\{1 - \Delta + \mathcal{O}(1/N_C^2)\right\} + \frac{1}{10}\, C_+(\mu^2)\; \left\{1 + \Delta + \mathcal{O}(1/N_C^2)\right\} + c$$

$$c = C_4(\mu^2) - 16\, C_6(\mu^2)\, L_5 \left[\frac{\langle\bar\psi\psi\rangle}{f_\pi^3}\right]^2 + \mathcal{O}(1/N_C^2)$$

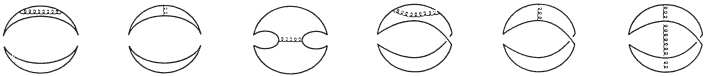
$$b = \frac{1}{2}\, C_+(\mu^2)\; \{1 + \Delta + \mathcal{O}(1/N_C^2)\} - \frac{1}{2}\, C_-(\mu^2)\; \{1 - \Delta + \mathcal{O}(1/N_C^2)\} < 0$$

$\mu \sim m_c\,, \quad \langle\frac{\alpha_s}{\pi}G^2\rangle \sim 330\ \mathrm{MeV}^4$ ➡ $b \sim -0.6 + \mathcal{O}(1/N_C^2)$

Two-point Functions

AP–E. de Rafael, NP B358 (1991) 311, PL B374 (1996) 186
M. Jamin–AP, NP B425 (1994) 15

$$\Psi^{\Delta S=1,2}(q^2) \equiv i\int d^4x\, e^{iq\cdot x}\,\langle 0|\, T\left(\mathcal{H}_{\rm eff}^{\Delta S=1,2}(x),\, \mathcal{H}_{\rm eff}^{\Delta S=1,2}(0)^\dagger\right)|0\rangle \;=\; \sum_{ij} C_i\, C_j^*\, \Psi_{ij}(q^2)$$



$$\frac{1}{\pi}\,{\rm Im}\Psi_{\pm\pm}(t) \;=\; \theta(t)\,\frac{2}{45}\,N_c^2\left(1\pm\frac{1}{N_c}\right)\frac{t^4}{(4\pi)^6}\,\alpha_s(t)^{-2a_\pm}\,C_\pm^2(M_W^2)\left[1+\frac{3}{4}\,\frac{\alpha_s(t)N_c}{\pi}\,\mathcal{K}_\pm\right]$$

$$a_\pm \;=\; \pm\frac{9}{11N_c}\,\frac{1\mp 1/N_c}{1-6/11N_c}$$

$$\mathcal{K}_+ \;=\; 1-\frac{30587}{3630}\,\frac{1}{N_c}+\frac{164936}{19965}\,\frac{1}{N_c^2}-\frac{51591}{14641}\,\frac{1}{N_c^3}+\frac{440193}{322102}\,\frac{1}{N_c^4}+\cdots = -\frac{3649}{3645}$$

$$\mathcal{K}_- \;=\; 1+\frac{30587}{3630}\,\frac{1}{N_c}+\frac{169706}{19965}\,\frac{1}{N_c^2}+\frac{70335}{14641}\,\frac{1}{N_c^3}+\frac{1810209}{322102}\,\frac{1}{N_c^4}+\cdots = +\frac{18278}{3645}$$

Phenomenological $K \to \pi\pi$ Fit

Cirigliano-Ecker-Neufeld-Pich

	LO-IC	LO-IB	NLO-IC	NLO-IB
Re g_8	4.96	4.99	3.62 ± 0.28	3.61 ± 0.28
Re g_{27}	0.285	0.253	0.286 ± 0.029	0.297 ± 0.029
$\chi_0 - \chi_2$	47.5°	47.8°	$(47.5 \pm 0.9)^\circ$	$(51.3 \pm 0.8)^\circ$

IC $\equiv$ $[m_u - m_d = \alpha = 0]$; IB $\equiv$ $[m_u - m_d \neq 0\,,\ \alpha \neq 0]$

Isospin Limit: $[\delta_0 - \delta_2]_{K\to\pi\pi} = (52.5 \pm 0.8_{exp} \pm 2.8_{th})^\circ$

$\pi\pi \to \pi\pi$: $\delta_0 - \delta_2 = (47.7 \pm 1.5)^\circ$ Colangelo–Gasser–Leutwyler '01

Isospin Breaking in ε'/ε

$$\epsilon'_K \sim \omega_+ \left\{ \frac{\text{Im}\, A_0^{(0)}}{\text{Re}\, A_0^{(0)}} \left(1 + \Delta_0 + f_{5/2}\right) - \frac{\text{Im}\, A_2}{\text{Re}\, A_2^{(0)}} \right\}$$

$$\sim \omega_+ \left\{ \frac{\text{Im}\, A_0^{(0)}}{\text{Re}\, A_0^{(0)}} \left(1 - \Omega_{\text{eff}}\right) - \frac{\text{Im}\, A_2^{\text{emp}}}{\text{Re}\, A_2^{(0)}} \right\}$$

$$\omega \equiv \frac{\text{Re}\, A_2}{\text{Re}\, A_0} = \omega_+ \left(1 + f_{5/2}\right) \quad ; \quad \omega_+ \equiv \frac{\text{Re}\, A_2^+}{\text{Re}\, A_0} \quad , \quad \Omega_{IB} = \frac{\text{Re}\, A_0^{(0)}}{\text{Re}\, A_2^{(0)}} \cdot \frac{\text{Im}\, A_2^{\text{non-emp}}}{\text{Im}\, A_0^{(0)}}$$

Cirigliano-Ecker-Neufeld-Pich

$\times 10^{-2}$	$\alpha = 0$		$\alpha \neq 0$	
	LO	NLO	LO	NLO
Ω_{IB}	11.7	15.9 ± 4.5	18.0 ± 6.5	22.7 ± 7.6
Δ_0	-0.004	-0.41 ± 0.05	8.7 ± 3.0	8.4 ± 3.6
$f_{5/2}$	0	0	0	8.3 ± 2.4
Ω_{eff}	11.7	16.3 ± 4.5	9.3 ± 5.8	6.0 ± 7.7

$$\Omega_{\text{eff}} = 0.06 \pm 0.08$$
$$\equiv \Omega_{IB} - \Delta_0 - f_{5/2}$$

$$\Omega_{\text{IB}}^{\pi^0\eta} = 0.16 \pm 0.03$$

Modelling (some) non-factorizable $1/N_C$ corrections

Buras-Gérard, 1507.06326

$$B_6^{(1/2)} = 1 - \frac{3}{2}\left[\frac{F_\pi}{F_K - F_\pi}\right]\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2}) = 1 - 0.66\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2})$$

$$B_8^{(1/2)} = 1 + \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2}) = 1 + 0.08\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2})$$

$$B_8^{(3/2)} = 1 - 2\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2}) = 1 - 0.17\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2})$$

$\longrightarrow$ $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$

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$$\longrightarrow \qquad B_6^{(1/2)} \leq B_8^{(3/2)} < 1$$

- FSI $(1/N_C)$ not included
- Part of 1-loop χPT corrections (?)
- Difficult to account in a matching calculation

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$$B_8^{(3/2)} = 1 - 2\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1+\frac{\Lambda^2}{\tilde{m}_8^2}) = 1 - 0.17\,\ln(1+\frac{\Lambda^2}{\tilde{m}_8^2})$$

→ $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$

Not true in QCD

- FSI $(1/N_C)$ not included
- Part of 1-loop χPT corrections (?)
- Difficult to account in a matching calculation

BBG Model

Bardeen-Buras-Gerard

$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4} \left\{ \langle D_\mu U^\dagger D^\mu U \rangle + r \, \langle m \, (U + U^\dagger) \rangle - \frac{r}{\Lambda_\chi^2} \, \langle m \, (D^2 U + D^2 U^\dagger) \rangle \right\}$$

1. Equivalent to $\mathcal{O}(p^2)$ χPT + L_5 term $\quad (L_i = 0\,,\; i \neq 5)$
   Most L_i are leading in N_C ➔ $\mathcal{L}_{\rm eff}$ **does not represent large-N_C QCD**

2. **Cut-off loop regularization:** $\mathbf{M \sim (0.8 - 0.9)}$ **GeV**
   $f_\pi^2(M^2) = F_\pi^2 + 2\, I_2(m_\pi^2) + I_2(m_K^2) \quad , \quad I_2(m_i^2) = \frac{1}{16\pi^2} \left[M^2 - m_i^2 \, \log\left(1 + \frac{M^2}{m_i^2}\right)\right]$

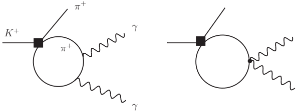
3. **Large-N_C factorization assumed to hold in the IR ($\mu = 0$):** $\langle J \cdot J \rangle = \langle J \rangle \langle J \rangle$

4. **M identified with SD renormalization scale μ: $C_i(\mu)$ running**
   Meson evolution ⟷ **Quark evolution**

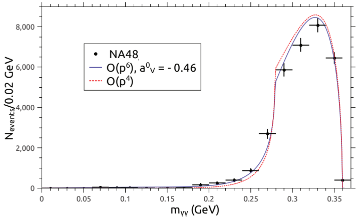
5. **Vector meson loops included through Hidden U(3) Gauge Symmetry**
   Could partially account for $L_{1,2,3,9,10}$
   L_8 still missing ➔ $\langle \bar{q} q \rangle$, $Q_{6,8}$ **not quite correct even at large-N_C**

6. $\pi\pi$ **re-scattering completely missing** ➔ $\delta_{0,2} = 0$, **FSI absent**

$K \to \pi\gamma\gamma$



$\mathrm{Br}(K_L \to \pi^0\gamma\gamma) = (1.27 \pm 0.03)\cdot 10^{-6}$



Finite 1-loop amplitude $[\mathcal{O}(p^4)]$:

$$\mathrm{Br}(K_L \to \pi^0\gamma\gamma)_{\mathrm{LO}} = 6.8 \cdot 10^{-7}$$

Ecker-Pich-de Rafael, Cappiello-D'Ambrosio, Sehgal

$\mathcal{O}(p^6)$ unitarity corrections needed

Cohen et al, Cappiello et al, D'Ambrosio-Portolés

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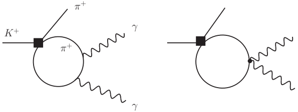

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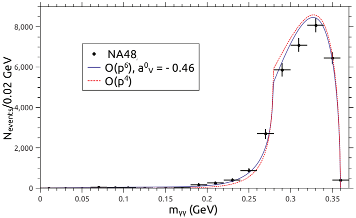

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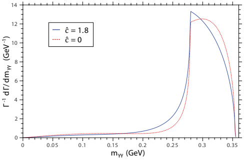
Ecker-Pich-de Rafael, Cappiello-D'Ambrosio, Sehgal

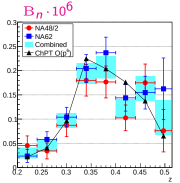
$\mathcal{O}(p^6)$ unitarity corrections needed

Cohen et al, Cappiello et al, D'Ambrosio-Portolés

$\mathrm{Br}(K^+ \to \pi^+\gamma\gamma) = 1.003\ (56) \cdot 10^{-6}$

NA48/2–NA62





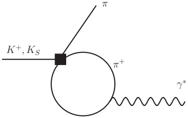
Local $\mathcal{O}(p^4)$ LEC: Ecker-Pich-de Rafael

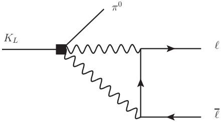
$$\hat{c} = \begin{cases} 1.72 \pm 0.21 & \mathcal{O}(p^4) \\ 1.86 \pm 0.25 & \mathcal{O}(p^6) \end{cases}$$

Small higher-order corrections

D'Ambrosio-Portolés

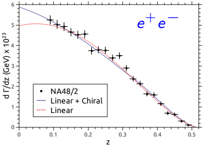
$K \to \pi \ell^+ \ell^-$

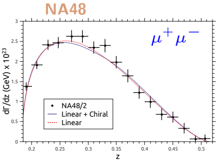




$$\mathrm{Br}(K^\pm \to \pi^\pm e^+ e^-) = 3.14\,(10) \cdot 10^{-7}$$

$$\mathrm{Br}(K^\pm \to \pi^\pm \mu^+ \mu^-) = 9.62\,(25) \cdot 10^{-8}$$



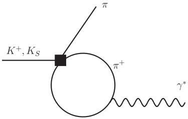


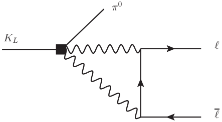
Local $\mathcal{O}(p^4)$ LECs Ecker-Pich-de Rafael

Electromagn. transition form factor

$\mathcal{O}(p^6)$ corrections D'Ambrosio et al

$K \to \pi\, \ell^+ \ell^-$





$$\mathrm{Br}(K^\pm \to \pi^\pm e^+ e^-) = 3.14\ (10) \cdot 10^{-7}$$

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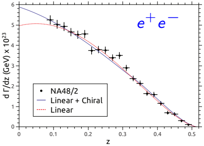

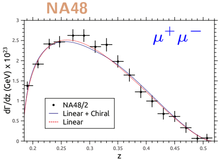

Local $\mathcal{O}(p^4)$ LECs Ecker-Pich-de Rafael

Electromagn. transition form factor

$\mathcal{O}(p^6)$ corrections D'Ambrosio et al

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \cdot 10^{-10}$$

(90% CL), KTeV

3 contributions: Ecker-Pich-de Rafael

- **Direct** $\not{\mathcal{CP}}$
- **Indirect** $\not{\mathcal{CP}}$
- **CP conserving** (2γ)

$\not{\mathcal{CP}}$ **dominates for e^+e^-:**

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) = 3.1\ (0.9) \cdot 10^{-11}$$

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