Transverse Momentum Distributions of Heavy Hadrons and Polarized Heavy Quarks.

Rebecca von Kuk (DESY), Johannes K. L. Michel (Nikhef/UvA), Zhiquan Sun (MIT)

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Nikhef Theory Seminar

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DESY.

Outline.

1. Motivation and introduction

- short introduction to **t**ransverse **m**omentum **d**ependent distributions (TMDs)
- why are heavy quarks interesting?

2. Heavy TMD fragmentation functions (FFs)

- discuss heavy TMD FFs in two regimes: $\Lambda_{\mathrm{QCD}} \sim k_T \ll m$ and $\Lambda_{\mathrm{QCD}} \ll k_T \sim m$

3. Heavy TMD parton distribution functions (PDFs)

4. Towards phenomenology

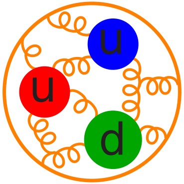
- $e^+e^- \to HHX$
- Semi-inclusive deep inelastic scattering (SIDIS)

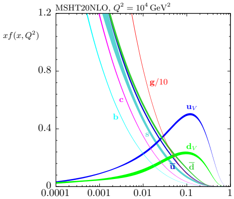
5. Outlook

Intro.

Collinear parton distribution functions (PDFs)

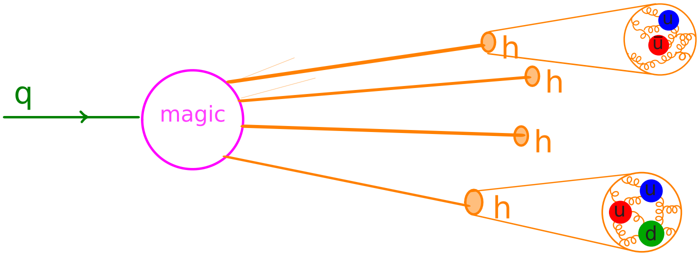
- collinear factorization [Collins, Soper, Sterman '89]: separation of physics at different scales, e.g. $m, k_T \ll Q$
- calculate a hard scattering process, but in hadron collisions the specific initial state is not known
- **Use PDFs:**
  - probability density for finding particle with certain longitudinal momentum fraction x at scale Q
  - non-perturbative: PDFs can not be calculated using perturbative QCD



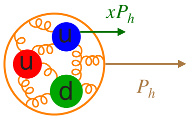


Collinear fragmentation functions (FFs)

- what about the final state?
- in QCD: can't observe free partons → parton fragments into a final state hadron
- Use **FFs**: describe probability of a final state hadron to originate from a given quark or gluon

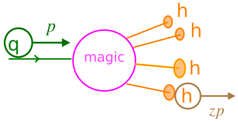


Introductions to Transverse Momentum Dependent Distributions.



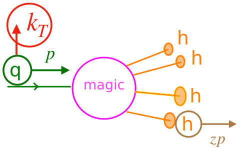
Collinear factorization theorem

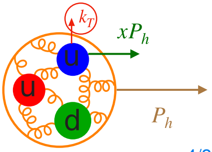
- cross section factorizes into PDFs, hard scattering process and FFs
- **But**: so far only at a 1D snapshot of longitudinal momentum distributions!



TMD factorization theorem

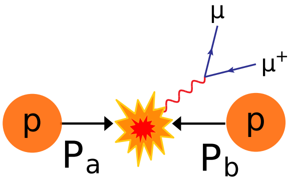
- allows for extraction of **3D** structure of the hadronization cascade
- TMDs are universal across processes [Collins, Metz '04]
- natural power-counting with $\boldsymbol{k_T \ll Q}$





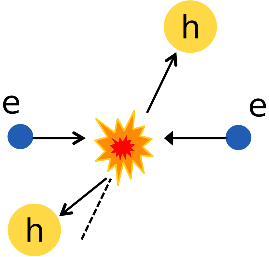
TMD factorization

Drell-Yan



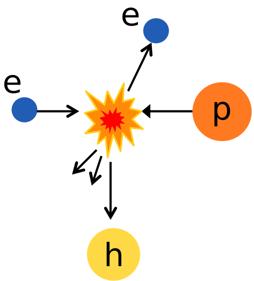
$$\frac{\mathrm{d}\sigma}{\mathrm{d}k_T} \sim f_{q/P}(x,k_T) \otimes f_{q/P}(x,k_T)$$

Dihadron in e+e−



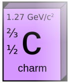
$$\frac{\mathrm{d}\sigma}{\mathrm{d}k_T} \sim D_{h/q}(z,k_T) \otimes D_{h/q}(z,k_T)$$

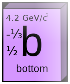
Semi-inclusive DIS



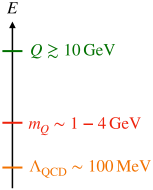
$$\frac{\mathrm{d}\sigma}{\mathrm{d}k_T} \sim f_{q/P}(x,k_T) \otimes D_{h/q}(z,k_T)$$

Why are heavy quarks interesting?

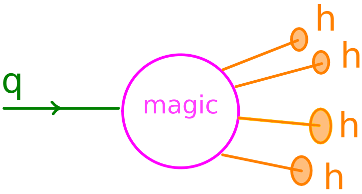


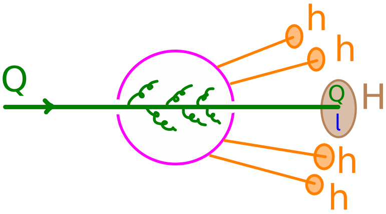


- bottom and charm quarks have $m_b, m_c \gg \Lambda_{\mathrm{QCD}}$
  - provides perturbative scale on otherwise non-perturbative dynamics of hadronization
- serve as static color source coupling to light degrees of freedom
- model independent prediction e.g. to improve heavy flavor modeling in MC



- **Ideal to study hadronization process**





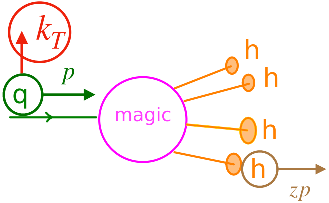
Heavy TMD FFs.

Basics of TMD FFs.

- TMD quark-quark correlator describing fragmentation

$$\Delta_{h/q}(z_H, b_\perp) = \frac{1}{2z_H N_c} \int \frac{db^+}{4\pi} e^{ib^+(P_h^-/z_H)/2}$$
$$\times \mathrm{Tr} \sum_X \langle 0 | W^\dagger(b)\, \psi_q(b) | hX \rangle \langle hX | \bar{\psi}_q\, W | 0 \rangle \,,$$

- $k_\perp$: transverse momentum of the (heavy) quark
- $b = (0, b^+, b_\perp)$: Fourier conjugate of k
- z_H: fraction of quarks lightcone momentum retained by hadron
- P_h^-: large momentum component of the (heavy) hadron



$p_\perp^2 \equiv p_\perp \cdot p_\perp < 0$, and $p_T = \sqrt{-p_\perp^2}$, $\quad W(x) = \bar{P}\left[\exp\left(-ig \int_0^\infty d\bar{s}\, \bar{n} \cdot A(x + \bar{n}s)\right)\right]$

Basics of TMD FFs.

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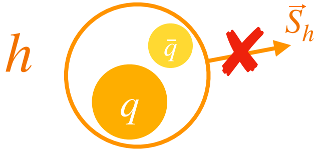
$$\times \operatorname{Tr} \sum\!\!\!\!\!\!\!\int_X \langle 0 | W^\dagger(b)\, \psi_q(b) | hX \rangle \langle hX | \bar{\psi}_q\, W | 0 \rangle\,,$$

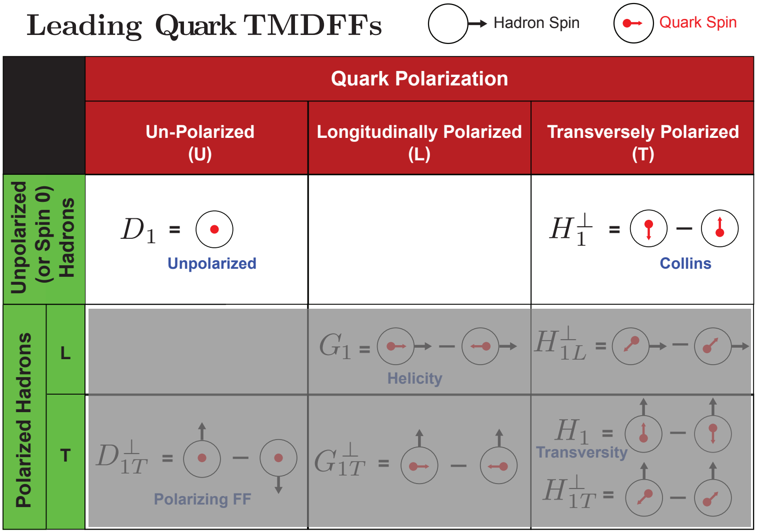
- sum over all possible hadron helicities

$$\sum\!\!\!\!\!\!\!\int_X |hX\rangle\langle hX| \equiv \sum\!\!\!\!\!\!\!\int_X \sum_{h_H} |h, h_H; X\rangle\langle h, h_H; X|$$

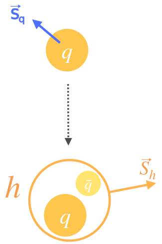
- **Here:** only allow unpolarized hadrons



Heavy TMD FFs.



[Figure taken from TMD Handbook '23]



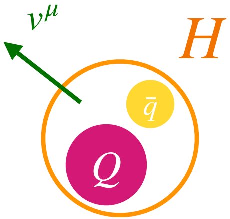
$$D_{1\,h/q}(z_H, b_T) = \mathrm{tr}\left[\frac{\not{\bar{n}}}{2}\Delta_{h/q}(z_H, b_\perp)\right]$$

$$H^{\perp(1)}_{1\,h/q}(z_H, b_T) = \mathrm{tr}\left[\frac{\not{\bar{n}}}{2}\frac{\not{b}_\perp}{M_H b_T^2}\Delta_{h/q}(z_H, b_\perp)\right]$$

bHQET

[Isgur, Wise '89 & '90] [Eichten, Hill '90] [Grinstein '90] [Georgi '90][Korner,Thompson '91], [Mannel, Roberts Ryzak '92], [Fleming, Hoang, Mantry, Stewart '08]

- Use bHQET to describe dynamics at non-perturbative scale
- HQET Lagrangian: $\mathcal{L} = \boxed{\bar{h}_v(iv \cdot D)h_v} + \boxed{\mathcal{L}_{\text{light}}} + \mathcal{O}\left(\frac{1}{m}\right)$ with $v^\mu = \frac{P_H^\mu}{M_H}$
- heavy quark spin symmetry: $[\mathcal{L}_{\text{HQET}}, S_Q] = 0$
- **1. step: tree-level matching:**
- QCD field: $\boxed{\psi_Q(x) = e^{-imv\cdot x} h_v(x)}$
- external state: $\boxed{|H, h_H; X\rangle = \sqrt{m}\,|H_v, h_H; X\rangle}$



HQET as a tool.

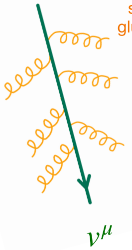
bHQET

- HQET Lagrangian: $\mathcal{L} = \bar{h}_v(iv \cdot D)h_v + \mathcal{L}_{\text{light}} + \mathcal{O}\left(\frac{1}{m}\right)$ with $v^\mu = \frac{P_H^\mu}{M_H}$
- **2. step: decouple light degrees of freedom**
- decouple spin degrees of freedom from light dynamics $h_v(x) = Y_v(x)\, h_v^{(0)}(x)$
  [Korchemsky and Radyushkin '92, Bauer, Pirjol and Stewart '02]
- $Y_v(x)$ takes the place of $h_v(x)$ in all external operators

$$h_v(x)\,|s_Q, h_Q; s_\ell, h_\ell, f_\ell; X\rangle = u(v, h_Q)\, Y_v(x)\, |s_\ell, h_\ell, f_\ell; X\rangle$$

- where $\underbrace{u(v, h_Q)}_{\text{HQET spinor}} = \underbrace{u(mv, h_Q)}_{\text{Dirac spinor}} / \sqrt{m}$

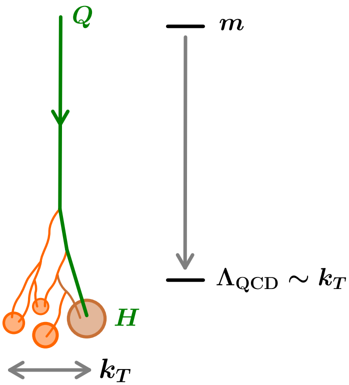
$$Y_v(x) = P\left[\exp\left(ig \int_0^\infty ds\, v \cdot A(x + vs)\right)\right]$$



Heavy TMD FFs.

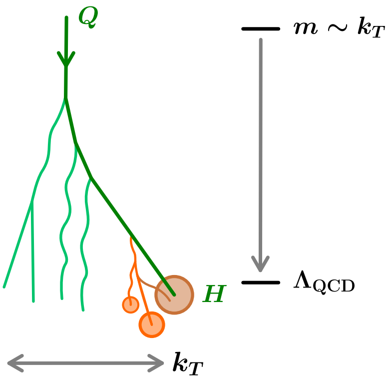
Two different regimes

Regime 1:



k_T set by soft emissions

Regime 2:



k_T set by perturbative emissions

Regime 1 $\Lambda_{\text{QCD}} \sim k_T \ll m$

- recall TMD quark-quark correlator:

$$\Delta_{H/Q}(z_H, b_\perp) = \frac{1}{2z_H N_c} \int \frac{\mathrm{d}b^+}{4\pi} e^{ib^+(P_H^-/z_H)/2} \,\mathrm{Tr} \sum_X \langle 0|W^\dagger(b)\,\psi_Q(b)|H\,X\rangle\langle H\,X|\bar{\psi}_Q\,W|0\rangle$$

$\Downarrow$ 1. step: tree-level matching

$$\Delta_{H/Q}(z_H, b_\perp) = \boxed{\frac{\delta(1-z_H)}{\bar{n}\cdot v} C_m(m)} \underbrace{\boxed{\frac{1}{2N_c} \,\mathrm{Tr} \sum_X \langle 0|W^\dagger(b_\perp)\,h_v(b_\perp)|H_v X\rangle\langle H_v X|\bar{h}_v\,W|0\rangle}}_{\text{HQET element}}$$

$$= \boxed{\frac{\delta(1-z_H)}{\bar{n}\cdot v} C_m(m)}\, F_H(b_\perp)$$

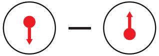
$$\psi_Q(x) = e^{-imv\cdot x} h_v(x)$$
$$|H, h_H; X\rangle = \sqrt{m}\,|H_v, h_H; X\rangle$$

Regime 1 $\Lambda_{\text{QCD}} \sim k_T \ll m$

- evaluate TMD FFs:

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \boxed{\delta(1-z_H)\, C_m(m,\mu,\zeta)}\ \boxed{\chi_{1,H}\Big(b_T, \mu, \frac{\sqrt{\zeta}}{m}\Big)}$$

$$b_T M_H\, H^{\perp(1)}_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \boxed{\delta(1-z_H)\, C_m(m,\mu,\zeta)}\ \boxed{\chi^{\perp}_{1,H}\Big(b_T, \mu, \frac{\sqrt{\zeta}}{m}\Big)}$$



- new scalar bHQET TMD fragmentation factors

$$\boxed{\chi_{1,H}(b_T) = \frac{1}{2}\,\mathrm{tr}\, F_H(b_\perp)}\,, \qquad \boxed{\chi^{\perp}_{1,H}(b_T) = \frac{1}{2}\,\mathrm{tr}\Big[\frac{\not{b}_\perp}{b_T}\not{z}\, F_H(b_\perp)\Big]}$$

- $\chi_{1,H}$ conditional probability to produce H given k_T
- $\chi^{\perp}_{1,H}$ conditional density of quark spin w.r.t magnetization axis given by Q and k_T

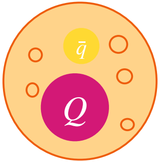
Regime 1: $\Lambda_{\rm QCD} \sim k_T \ll m$

- **2. step**: decouple spin d.o.f. from light dynamics

$$h_v(x) = Y_v(x)\, h_v^{(0)}(x)$$

$$F_H(b_\perp) = \frac{1}{2} \sum_{h_H} \sum_{h_Q, h'_Q} \sum_{h_\ell, h'_\ell} u(v, h_Q)\, \bar{u}(v, h'_Q) \overbrace{\langle s_Q, h_Q; s_\ell, h_\ell | s_H, h_H \rangle \langle s_H, h_H | s_Q, h'_Q; s_\ell, h'_\ell \rangle}^{\text{Clebsch-Gordan-Coefficients}}$$
$$\underbrace{\times \frac{1}{N_c} \mathrm{Tr} \sum\!\!\!\!\!\!\!\int_X \langle 0 | W^\dagger(b_\perp)\, Y_v(b_\perp) | s_\ell, h_\ell, f_\ell; X \rangle \langle s_\ell, h'_\ell, f_\ell; X | Y_v^\dagger(0)\, W(0) | 0 \rangle}_{\rho_{\ell, h_\ell h'_\ell}(b_\perp)}$$

- $\rho_{\ell, h_\ell h'_\ell}(b_\perp)$: light spin density matrix, encodes all non-perturbative physics



Regime 1.

Results for the unpolarized TMD FF

- performing trace sets $h_Q = h'_Q$ and $h_\ell = h'_\ell$

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) \propto \chi_{1,H}(b_T) = \frac{1}{2}\sum_{h_H}\sum_{h_Q}\sum_{h_\ell} \underbrace{|\langle s_Q, h_Q; s_\ell, h_\ell | s_H, h_H\rangle|^2}_{\text{Clebsch-Gordan-Coefficients}} \rho_{\ell, h_\ell h_\ell}(b_\perp)$$

- example:

D ($s_\ell = 1/2\,,\ s_H = 0$)

$$\chi_{1,D}(b_T) = \frac{1}{4}\left[\rho_{\ell,++}(b_\perp) + \rho_{\ell,--}(b_\perp)\right]$$

vs.

D^* meson ($s_\ell = 1/2\,,\ s_H = 1$)

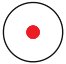
$$\chi_{1,D^*}(b_T) = \frac{3}{4}\left[\rho_{\ell,++}(b_\perp) + \rho_{\ell,--}(b_\perp)\right]$$

- **three times as likely to produce D^* than D!**
- **for the first time: show that relations hold point by point in k_T**

[proven for inclusive fragmentation Falk and Peskin ’94, Manohar and Wise ’00]

Regime 1.

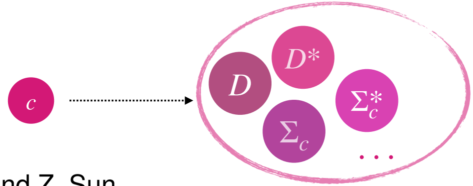
Results for the unpolarized TMD FF



- complete sum of states

$$\chi_1(b_T) \equiv \sum_H \chi_{1,H}(b_T) = \frac{1}{N_c} \mathrm{Tr} \left\langle 0 \middle| W^\dagger(b_\perp)\, Y_v(b_\perp)\, Y_v^\dagger(0)\, W(0) \middle| 0 \right\rangle$$

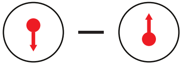
- **vacuum element of Wilson lines!**
- simplest TMD observable possible!



- NLO result from ongoing work by RvK, J.K.L. Michel and Z. Sun

$$\chi_1(b_T, \mu, \rho) = 1 + \frac{\alpha_s(\mu)}{4\pi} C_F(-L_b)\big(4\ln\rho - 2\big) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\lambda_{\mathrm{QCD}}^2 b_T^2)$$

Results for the Collins TMD FF

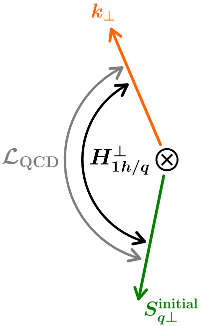


$$H^{(1),\perp}_{1,H/Q}(z_H, b_T) \propto \chi^{\perp}_{1,H}(b_T) = \frac{1}{2}\,\mathrm{tr}\left[\frac{\not{b}_\perp}{b_T}\not{z}\,F_H(b_\perp)\right]$$

- encodes correlation between $S^{\text{initial}}_{q\perp}$ and $k_\perp$
- naive expectation from HQ symmetry: Collins TMD FF should be suppressed by $1/m$

Light quarks:

- correlation directly from non-perturbative dynamics of $\mathcal{L}_{\text{QCD}}$



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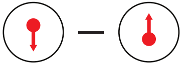

$$H_{1,H/Q}^{(1),\perp}(z_H, b_T) \propto \chi_{1,H}^{\perp}(b_T) = \frac{1}{2}\,\mathrm{tr}\left[\frac{\not{b}_\perp}{b_T}\not{z}\,F_H(b_\perp)\right]$$

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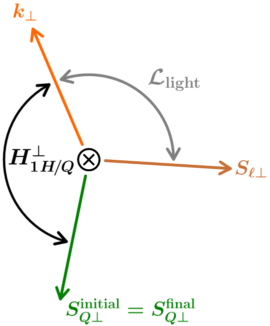

Light quarks:

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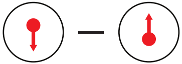
Heavy quarks:

- measuring D vs. D^* induces correlation between $S_{Q\perp}$ and $S_{\ell\perp}$

Regime 1.

Results for the Collins TMD FF



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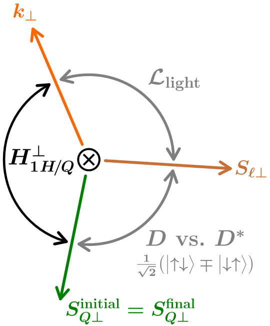

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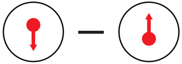

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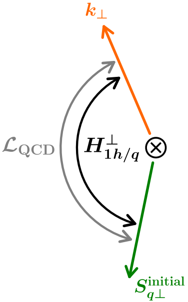
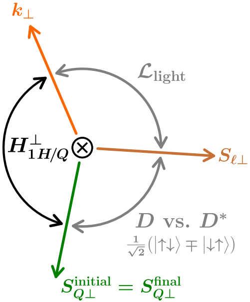
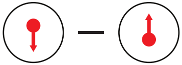
- naive expectation from HQ symmetry: Collins TMD FF should be suppressed by $1/m$



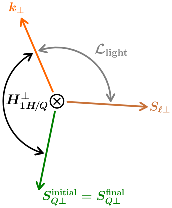


▶ **Collins effect is not suppressed by $1/m$!**

Results for the Collins TMD FF



$$H_{1,H/Q}^{(1),\perp}(z_H, b_T) \propto \chi_{1,H}^{\perp}(b_T) = \frac{1}{2}\,\mathrm{tr}\left[\frac{\not{b}_\perp}{b_T}\not{z}\,F_H(b_\perp)\right]$$



- example $s_\ell = 1/2\,,\ s_H = 0$

$$\boxed{\chi_{1,H}^{\perp}(b_T) = \frac{1}{4}\left[\rho_{\ell,-+}(b_\perp) - \rho_{\ell,+-}(b_\perp)\right]}$$

D

- sum over all hadrons within same spin multiplet M_ℓ (identical light spin and falvor state ℓ)

$$\sum_{H\in M_\ell} \chi_{1,H}^{\perp}(b_T) = 0 \quad \rightarrow \quad \boxed{H_{1,D/c}^{\perp}} + \boxed{H_{1,D^*/c}^{\perp}} = 0$$

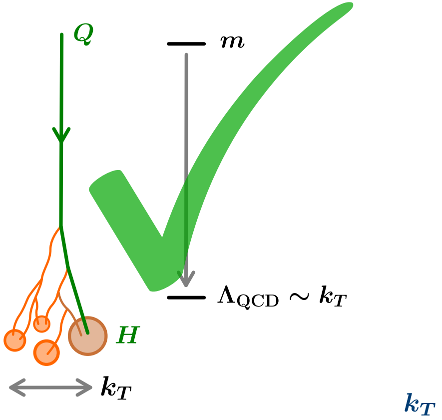
D D*

▶ **Heavy quark limit lets us sum rule point by point in q_T!**

[Previously shown in bare case. Requires sum over all hadrons and integration of z_H and k_T. Schäfer and Teryaev '00, Meissner, Metz and Pitonyak '10]

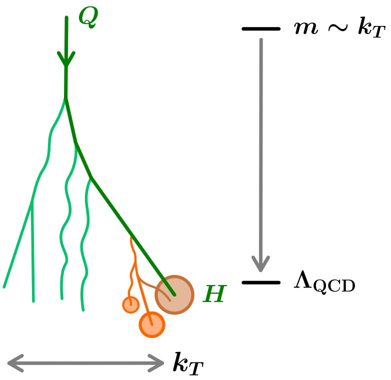
Two different regimes

Regime 1:



set by soft emissions

Regime 2:



k_T set by perturbative emissions

Regime 2.

Unpolarized for $\Lambda_{\text{QCD}} \ll k_T \sim m$

- match onto bHQET at $\mu \sim k_T \sim m$ [Nadolsky, Kidonakis, Olness and Yuan '03]

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \boxed{d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)}\;\boxed{\chi_H} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m}\right) + \mathcal{O}(\Lambda_{\text{QCD}} b_T)$$

- ► new perturbative matching coefficient:

$$\boxed{d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)} = \text{tr}\left[\frac{\not{\bar{n}}}{2}\,\Delta_{Q/Q}(z_H, b_\perp)\right] = \delta(1 - z_H) + \mathcal{O}(\alpha_s)$$

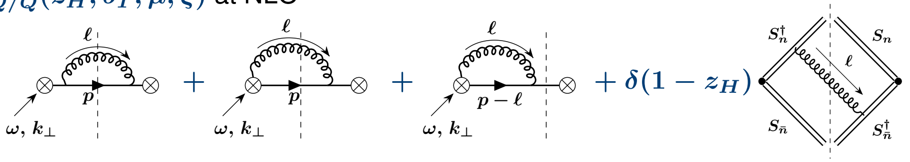
- $\boxed{\chi_H}$ total probability of Q to fragment into H

- ► **symmetry relations hold for all values of k_T and to all orders in α_s!**

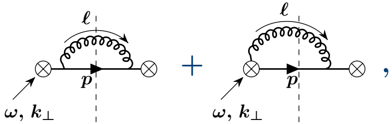
D $\quad D_{1\,H/Q} = \frac{1}{3} D_{1\,H^*/Q} \quad$ D*

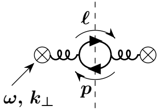
Unpolarized TMD FF: work in progress

- calculate $d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)$ at NLO

$d_{1Q/Q} =$ 

- also consider $d_{1\,g/Q}(z_H, b_T, \mu, \zeta)$ and $d_{1\,Q/g}(z_H, b_T, \mu, \zeta)$

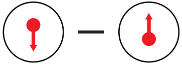
$d_{1g/Q} =$ 

$d_{1Q/g} =$ 

- check against:
  - heavy quark limit ✓
  - light quark limit✓
  - joint k_T & threshold resum. for $z \to 1$ with full mass dependence✓

Regime 2.

Collins for $\Lambda_{\text{QCD}} \ll k_T \sim m$



$$b_T M_H \, H_{1\,H/Q}^{\perp(1)}(z_H, b_T) = \boxed{\delta(1-z_H)\, b_T} \; \boxed{\chi_{H,G}} + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\text{QCD}}^2)$$

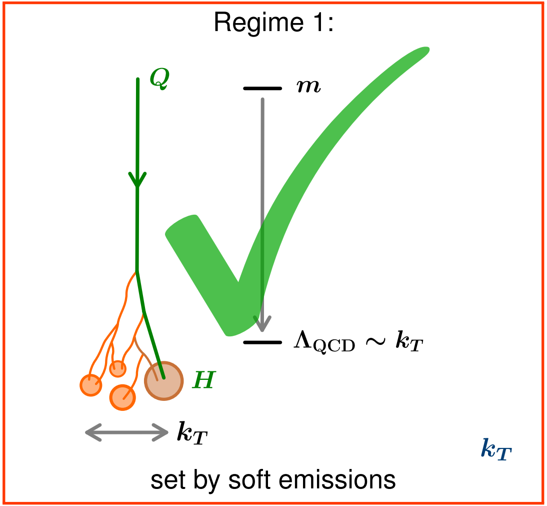
- $\boxed{\chi_{H,G}}$ correlation between gluon field polarization and transverse polarization

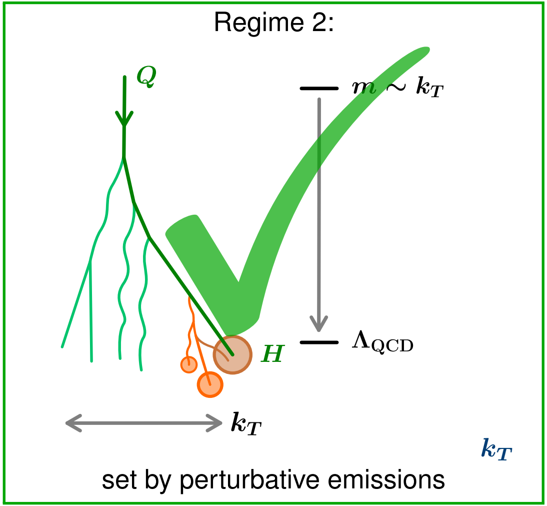
$$\boxed{\chi_{H,G}} \equiv \frac{1}{2N_c} \,\text{Tr}\,\text{tr} \sum\!\!\!\!\!\!\int_X \Big\{ \langle 0 | W^\dagger \, \sigma_{\beta\alpha} z^\beta \, [\mathrm{i}D_\perp^\alpha + g\mathcal{G}_\perp^\alpha] h_v | H_v X \rangle \langle H_v X | \bar{h}_v W | 0 \rangle + \text{h.c.} \Big\} \,.$$

- can easily show $\sum_{H \in M_\ell} \boxed{\chi_{H,G}} = 0 \rightarrow$ **new symmetry relation still holds!**

Heavy TMD FFs.

Two different regimes





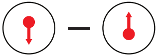
Heavy TMD FFs.

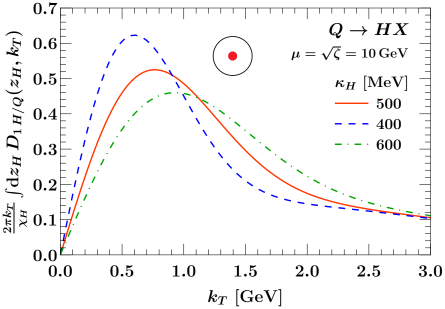
numerical model

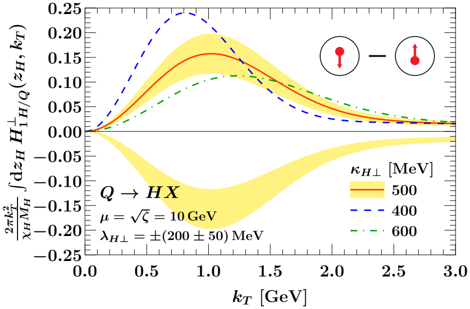
- at LL: TMD FFs completely specified by $\chi_{1,H}$, $\chi_{1,H}^{\perp}$ and TMD evolution
- assume simple Gaussian models

$$\chi_{1,H}(b_T, \mu_0, \rho_0) = \chi_H \exp\left(-\kappa_H^2 b_T^2\right)$$

$$\chi_{1,H}^{\perp}(b_T, \mu_0, \rho_0)\,, = \chi_H\, \lambda_{H\perp} b_T\, \exp\left(-\kappa_{H\perp}^2 b_T^2\right)$$







Heavy TMD PDFs.

setup

- consider production of a heavy quark Q with from light partons within a (polarized) nucleon N
- heavy quarks is pair produced in initial state gluon splitting at $\mu \sim m$

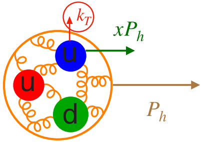
▶ **TMD PDFs can be calculated by matching them onto leading collinear PDFs**

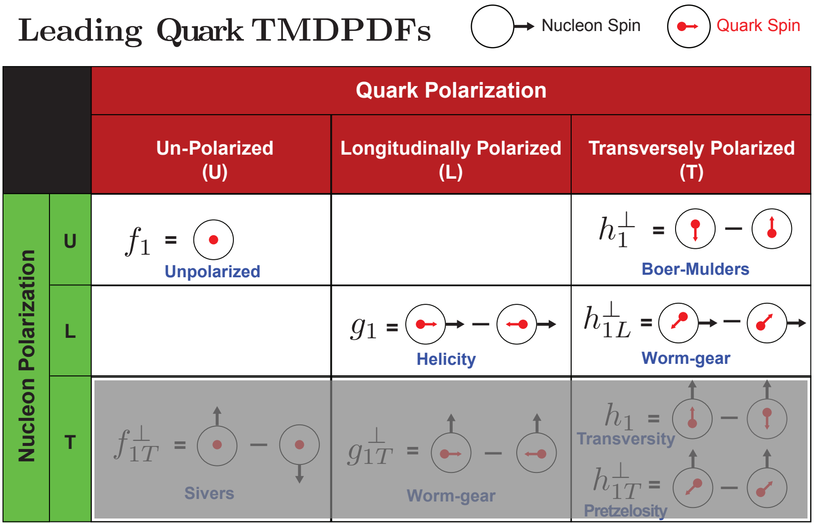
- TMD PDF decomposition: [Bacchetta, Diehl, Goeke, Metz, Mulders and Schlegel '07, Ebert, Gao and Stewart '22]

$$\Phi_{Q/N}(x, k_\perp) =$$

$$\left\{ f_{1\,Q/N} + g_{1L\,Q/N} S_L \gamma_5 + h^\perp_{1L\,Q/N} S_L \gamma_5 \frac{\not{k}_\perp}{M_N} + i h^\perp_{1\,Q/N} \frac{\not{k}_\perp}{M_N} + (\text{terms} \propto S_\perp) \right\} \frac{\not{n}}{4}$$

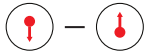
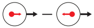
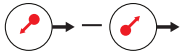
- S_L: longitudinal nucleon polarization, M_N: the nucleon mass
- all terms $\propto S_\perp$ vanish at order α_s





[Figure taken from TMD Handbook '23]

Heavy TMD PDFs.

	Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 = 		$h_1^{\perp}$ = 
L		g_1 = 	$h_{1L}^{\perp}$ = 

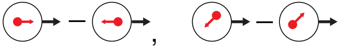
setup

- TMD PDF decomposition:

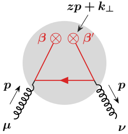
$$\Phi_{Q/N}(x, k_\perp) = \left\{ f_{1\,Q/N} + g_{1L\,Q/N}\, \boxed{S_L}\, \gamma_5 + h_{1L\,Q/N}^{\perp}\, \boxed{S_L}\, \gamma_5 \frac{\not{k}_\perp}{M_N} + \mathrm{i} h_{1\,Q/N}^{\perp} \frac{\not{k}_\perp}{M_N}) \right\} \frac{\not{n}}{4}$$

- collinear PDF decomposition [Collins '11]

$$\Phi_{g/N}^{\mu\nu}(x) = -\frac{g_\perp^{\mu\nu}}{2} f_{g/N}(x) + \frac{\mathrm{i}\epsilon_\perp^{\mu\nu}}{2} g_{g/N}(x)\, \boxed{S_L}$$

- unpolarized f_1 and Boer-Mulders $h_1^{\perp}$ match onto unpolarized PDF f_g 
- helicity $g_{1,L}$ and Worm-gear L $h_{1,L}^{\perp}$ match onto helicity PDF g_g 

Heavy TMD PDFs.



results

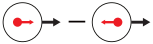
- unpolarized quark from unpolarized gluon:

$$C^{(1)}_{Q/g}(z, k_T, m) = T_F\, \Theta(z)\Theta(1-z)\, \frac{2}{\pi}\, \frac{k_T^2(1 - 2z + 2z^2) + m^2}{(k_T^2 + m^2)^2}$$

[Nadolsky, Kidonakis, Olness, Yuan '02], [Pietrulewicz, Samitz, Spiering, Tackmann '17]

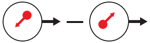
- **NEW:** longitudinally polarized quark from longitudinally polarized gluon:

$$C^{(1)}_{Q_\parallel/g_\parallel}(z, k_T, m) = T_F\, \Theta(z)\Theta(1-z)\, \frac{2}{\pi}\, \frac{k_T^2(2z - 1) + m^2}{(k_T^2 + m^2)^2}$$



- **NEW:** transversely polarized quark from longitudinally polarized gluon:

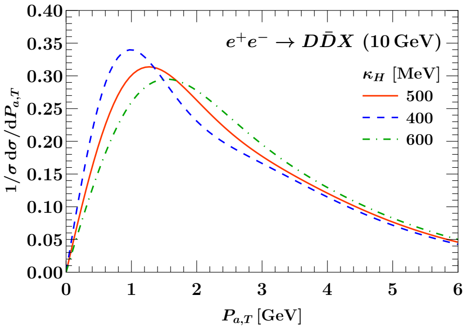
$$C^{(1)}_{Q_\perp/g_\parallel}(z, k_T, m) = T_F\, \Theta(z)\Theta(1-z)\, \frac{4}{\pi}\, \frac{m k_T(z - 1)}{(k_T^2 + m^2)^2}$$

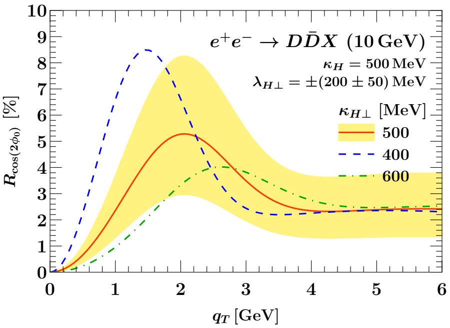


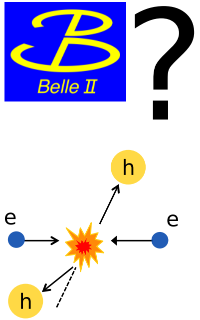
Towards Phenomenology.

Charm continuum production

- cross section depends on transverse momentum of the hadron $P_{a,T}$ and partonic transverse momentum $q_T = P_{a,T}/z_a$
- Collins effect strength: $R_{\cos(2\phi_0)}(Q^2, q_T) = H_1^{\perp(1)} \otimes H_1^{\perp(1)} / D_1 \otimes D_1$



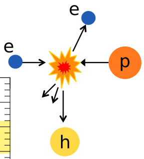


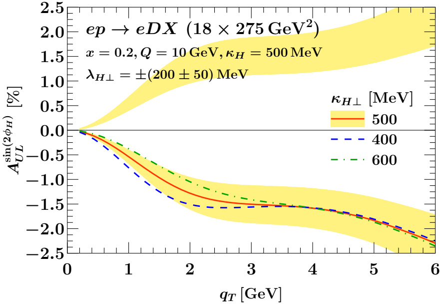


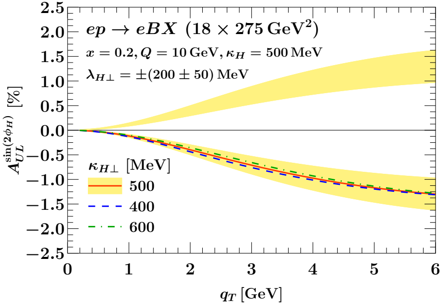
▶ absolute sign of the Collins functions lost!

Towards Phenomenology: SIDIS at future EIC.

Collins effect



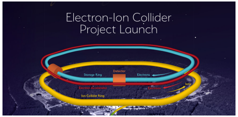




- get access to the sign of the Collins via spin asymmetry

$$A_{UL}^{\sin(2\phi_H)}(Q^2, x, q_T) = h_{1L}^{\perp} \otimes H_1^{\perp} / f_1 \otimes D_1$$

- resolving the sign should be possible within expected statistics at EIC!

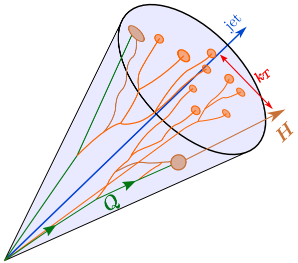


Outlook.

Outlook.

TMD FFs within jets

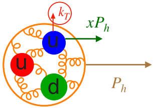
- consider heavy-quark TMD fragmentation within jets
- **straightforward extension: our results are independent of the factorization theorem!**
- polarization of inital state quark washed out



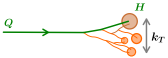
Polarized Hadrons

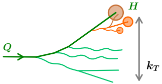
- also consider polarized hadrons $\rightarrow$ study all eight TMD FFs for heavy quarks
  - relevant for LHC(b): polarized hadrons can be reconstructed from angular distribution of decay products $\rightarrow$ access to same ρ_ℓ

Summary

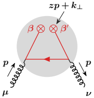


- heavy TMD FFs:
  - $\Lambda_{\mathrm{QCD}} \sim k_T \ll m$: new bHQET matrix elements $\chi_{1,H}$, $\chi_{1,H}^{\perp}$
  - $\Lambda_{\mathrm{QCD}} \ll m \sim k_T$: new inclusive bHQET matirx elements χ_1, $\chi_1^{\perp}$ and new perturbative matching coefficient $d_{1Q/Q}$

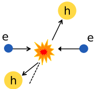


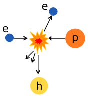


- heavy TMD PDFs
  - calculated matching coefficients for worm-gear-L and helicity TMD PDFs for the first time

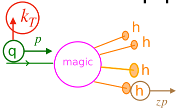


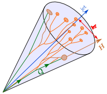
- towards phenomenology
  - measurement of Collins effect within reach for existing B factories
  - future EIC: provides possibility to measure the absolute sign of the Collins TMD FF





- Outlook: numerous follow up projects $\rightarrow$ **Stay tuned!**





Thank you!

Acknowledgements.

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erc

European Research Council

Established by the European Commission

Back up.

Regime 2.

Collins for $\Lambda_{\rm QCD} \ll k_T \sim m$

- two step matching to integrate out $\mu \sim k_T, m$
- use known matching for light quarks to integrate out k_T
  [Mulders and Tangerman '96, Boer and Mulders '98, Yuan and Zhou '09]

$$b_T M_h\, H_{1\,h/q}^{\perp(1)}(z_h, b_T) = \boxed{b_T}\, \boxed{\hat{H}_{h/q}(z_h)} + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\rm QCD}^2 b_T^2)\,,$$

- $\boxed{\hat{H}_{h/q}(z_h)}$ is twist-3 collinear fragmentation matrix element at the scale $\mu \sim \Lambda_{\rm QCD}$:

$$\hat{H}_{h/q}(z_h) \equiv \frac{z_h^2}{2N_c} \int \frac{\mathrm{d}x^+}{4\pi}\, e^{\mathrm{i}x^+(P_h^-/z_h)/2}\, \mathrm{Tr}\,\mathrm{tr} \sum_X \Big\{ \langle 0 | W^\dagger(x) \times \sigma_{\alpha -}\big[\mathrm{i}D_\perp^\alpha(x) + g\mathcal{G}_\perp^\alpha(x)\big]\psi_q(x) \big| hX \big\rangle \big\langle hX \big| \bar{\psi}_q(0) W(0) \big| 0 \big\rangle + \text{h.c.} \Big\}$$

Regime 2.

Collins for $\Lambda_{\rm QCD} \ll k_T \sim m$

- match $\boxed{\hat{H}_{h/q}(z_h)}$ onto bHQET to integrate out m

$$\boxed{\hat{H}_{h/q}(z_h)} = \boxed{\delta(1 - z_H)}\,\boxed{\chi_{H,G}} + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{1}{m}\right),$$

- $\boxed{\chi_{H,G}}$ correlation between gluon field polarization and transverse polarization of light components in hadron
- matching coefficient from bHQET:

$$\boxed{\chi_{H,G}} \equiv \frac{1}{2N_c} \mathrm{Tr}\,\mathrm{tr} \sum\!\!\!\!\!\!\int_X \left\{ \langle 0 | W^\dagger \sigma_{\beta\alpha} z^\beta \left[\mathrm{i} D_\perp^\alpha + g \mathcal{G}_\perp^\alpha \right] h_v | H_v X \rangle \langle H_v X | \bar{h}_v W | 0 \rangle + \text{h.c.} \right\}$$

- final result for the tree-level matching of the heavy-quark Collins TMD FF onto bHQET

$$b_T M_H\, H_{1\,H/Q}^{\perp(1)}(z_H, b_T) = \boxed{\delta(1 - z_H)\, b_T}\,\boxed{\chi_{H,G}} + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\rm QCD}^2)$$

► can easily show $\sum_{H \in M_\ell} \boxed{\chi_{H,G}} = 0 \to$ **new symmetry relation still holds!**

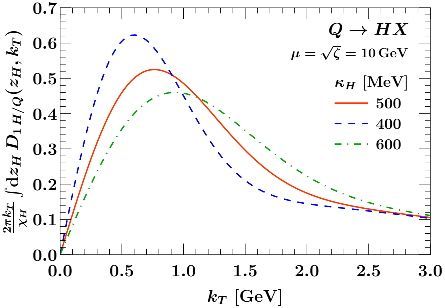
Heavy TMD FFs.

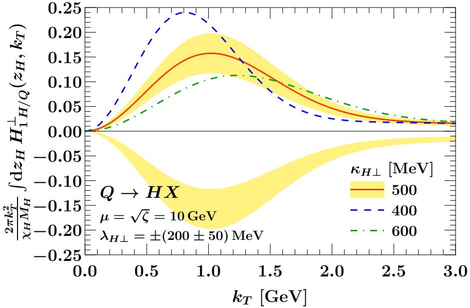
numerical model

- at LL: TMD FFs completely specified by $\chi_{1,H}$, $\chi_{1,H}^{\perp}$ and TMD evolution
- assume simple Gaussian models

$$\chi_{1,H}(b_T, \mu_0, \rho_0) = \chi_H \exp\left(-\kappa_H^2 b_T^2\right)$$

$$\chi_{1,H}^{\perp}(b_T, \mu_0, \rho_0)\,, = \chi_H\, \lambda_{H\perp} b_T\, \exp\left(-\kappa_{H\perp}^2 b_T^2\right)$$





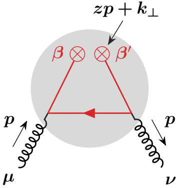
Heavy TMD PDFs.

matching calculation

- matching coefficient $C_{Q/g}$ is be defined by

$$\Phi^{\beta\beta'}_{Q/N}(x,k_\perp) = \int \frac{\mathrm{d}p^-}{p^-} C^{\beta\beta'}_{Q/g,\mu\nu}(xP_N^-, p^-, k_\perp, m)\, \Phi^{\mu\nu}_{g/N}\left(\frac{p^-}{P_N^-}\right)$$

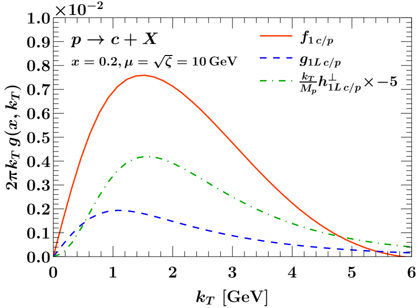
$$C^{\beta\beta'}_{Q/g,\mu\nu}(zp^-, p^-, k_\perp, m) =$$

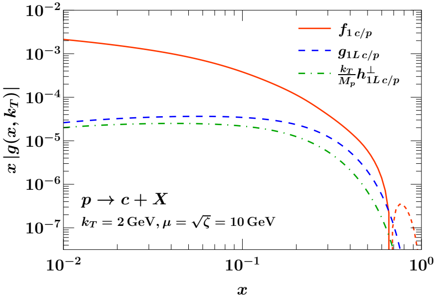


- where $z = xP_N^-/p^-$ is the fraction of p^- injected into the hard scattering process
- insert $\Phi_{g/N}$ and trace resulting Dirac spinors $(\ldots)^{\beta\beta'}$ against relevant Dirac structures

Heavy TMD PDFs.

numerical model

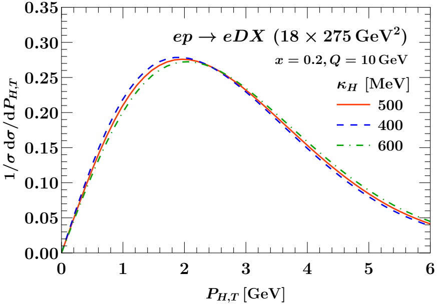


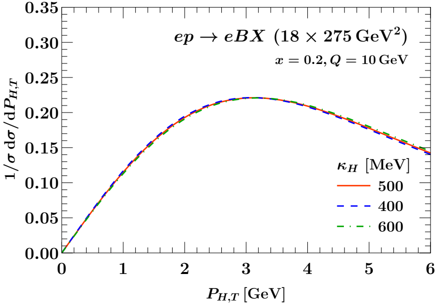


- unpolarized and helicity are linear in small k_T region, worm-gear L quadratic (after including Jacobian)
- unpolarized rises much more rapidly for small x
  - expected from smaller gluon polarized fraction at small x
- need for resummation of subleading-power threshold logs of $1 - x$

Towards Phenomenology: SIDIS at future EIC.

cross section





- compare cross sections D and B mesons
- broader peak for B because of different size of phase space
- very small dependence on parameter κ_H

Towards Phenomenology: SIDIS at future EIC.

SIDIS kinematics

- electron-nucleon collisions $e^-(\ell) + N(P) \to e^-(\ell') + H(P_H) + X$
- scattering mediated by an off-shell photon with $q = \ell - \ell'$ and $Q^2 \equiv -q^2 > 0$
- SIDIS cross section depends on: $x = Q^2/(2P \cdot q)$, $y = (P \cdot q)/(P \cdot \ell)$, $z_H = (P \cdot P_H)/(P \cdot q)$
- estimate sample at EIC:

$\sigma(eN \to eHX)$ [pb]	c, $x > 0.01$	c, $x > 0.1$	b, $x > 0.01$	b, $x > 0.1$
$q_T < 2\,\mathrm{GeV}, Q > 4\,\mathrm{GeV}$	84	3.47	18	0.65
$q_T < 4\,\mathrm{GeV}, Q > 10\,\mathrm{GeV}$	16	1.45	4.9	0.42

acceptance cuts: $0.01 < y < 0.95\,, \quad W^2 = \left(\frac{1}{x} - 1\right)Q^2 > 100\,\mathrm{GeV}^2$