Path integral

$$\langle A^{a_1}_{\mu_1} \ldots A^{a_n}_{\mu_n} \rangle = \int \mathcal{D}A^a_\mu \left(A^{a_1}_{\mu_1} \ldots A^{a_n}_{\mu_n} \right) e^{\frac{i}{\hbar} S_{\mathrm{classical}}}$$

$$S_{\mathrm{classical}} = -\frac{1}{4} \int d^4x \; F^a_{\mu\nu} F^{\mu\nu,a}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + ig\epsilon^{abc} A^b_\mu A^c_\nu$$

rescale to clarify coupling dependence

$$\hat{A}_\mu = g A_\mu \qquad \hat{F}^a_{\mu\nu} = g F^a_{\mu\nu}$$

(drop the hats in the following)

Usually the exponential gives rise to a suppression factor

$$e^{-\frac{1}{g^2} S_{Eucl.}}$$

(in Euclidean space, with $\hbar$=1)

Gauge transformations

$$D_\mu \equiv \partial_\mu + \mathcal{A}_\mu \to U^{-1} D_\mu U$$

$$\mathcal{A}_\mu \to U^{-1}\mathcal{A}_\mu U + U^{-1}\partial_\mu U$$

Abelian

$$\mathcal{A}_\mu = ieqA_\mu$$

$$U = e^{ieq\Lambda}$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

Non-abelian

$$\mathcal{A}_\mu = ig\sum_a A_\mu^a \left(\tfrac{1}{2}\sigma^a\right)$$

Pure gauge

$$\mathcal{A}_\mu = U^{-1}\partial_\mu U$$

Tye and Wong, arXiv 1505.03690

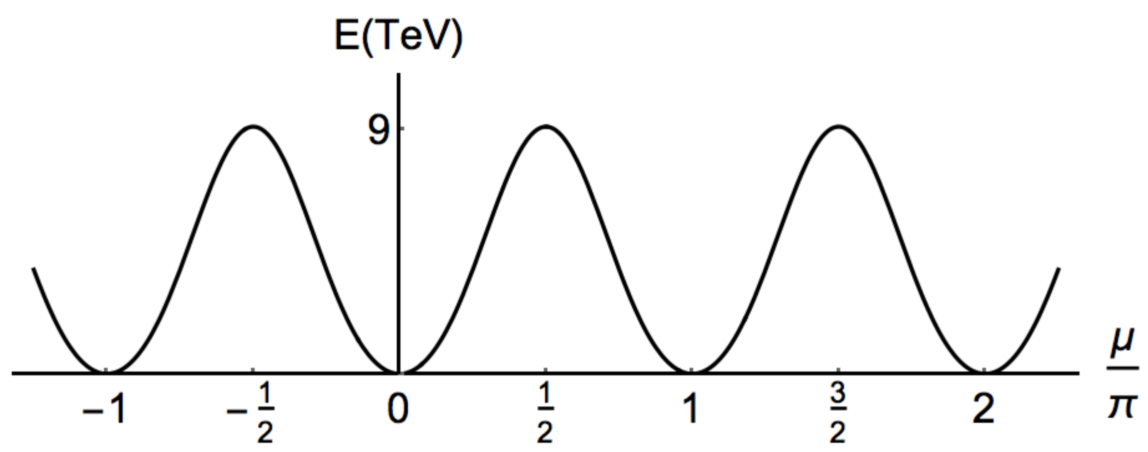
Introducing $Q = \mu/m_W$ (so Q has the dimension of a coordinate) we obtain a constant mass m and

$$\left(-\frac{1}{2m}\frac{\partial^2}{\partial Q^2} + V(Q)\right)\Psi(Q) = E\Psi(Q). \tag{1.1}$$

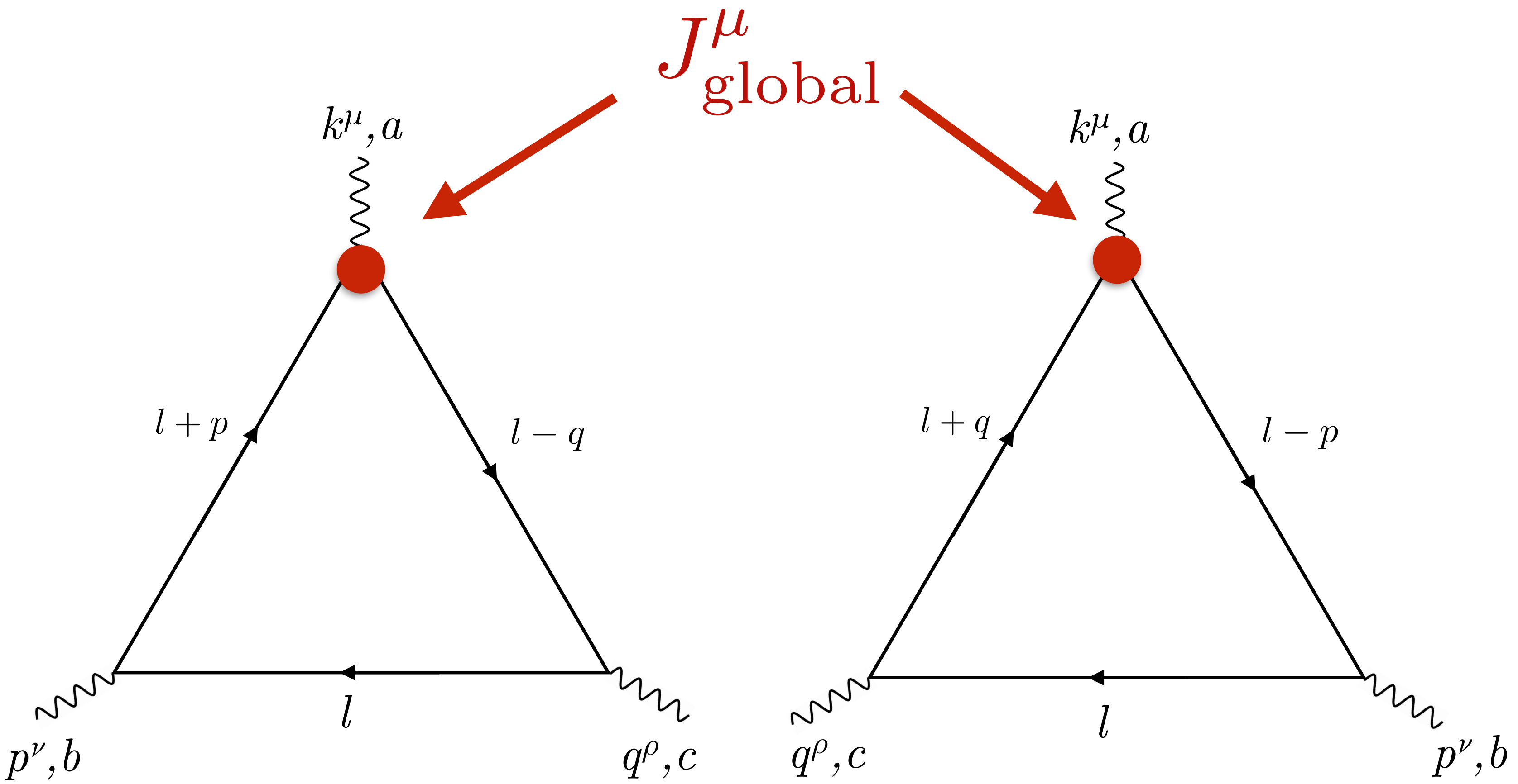
Using the known Higgs vacuum expectation value $v = 246$ GeV, W Boson mass $m_W = 80$ GeV and the Higgs Boson mass $m_H = 125$ GeV, we obtain

$$\begin{aligned} V(Q) &\simeq 4.75 \text{ TeV} \left(1.31 \sin^2(m_W Q) + 0.60 \sin^4(m_W Q)\right), \\ E_{sph} &= \max[V(Q)] = V\left(\frac{\pi}{2m_W}\right) = 9.11 \text{ TeV}, \\ m &= 17.1 \text{ TeV}, \end{aligned} \tag{1.2}$$

where the potential $V(Q)$ was obtained by Manton (see Fig. 1). Determining the value of this mass m is a main result of this paper. Note that a rescaling of Q rescales m, though the physics is unchanged.



Link between Chern Simons number and anomalies



Traces with γ_5 lead to current non-conservation

(γ_5 may originate from gauge boson coupling or current)

$$\partial_\mu J^\mu_{\rm global} = N_F \partial_\mu K^\mu_{\rm CS} \qquad K^\mu_{\rm CS} = -\frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}{\rm Tr}\left[A_\nu\partial_\rho A_\sigma + \frac{2}{3}A_\nu A_\rho A_\sigma\right]$$

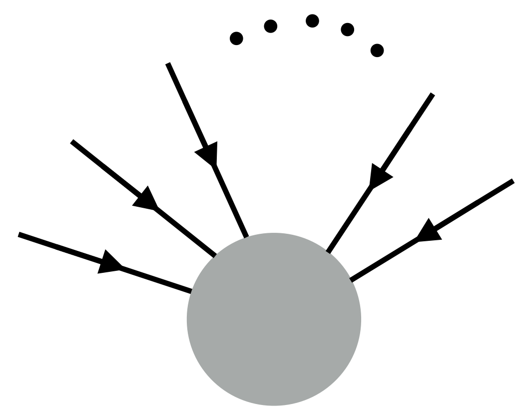
$$Q_{\rm global} = \int d^3x J^0_{\rm global}$$

$$\frac{d}{dt}(Q_{\rm global} - N_{\rm F}N_{\rm CS}) = 0$$

N_F is the number of Weyl multiplets

(taken to be left-handed)

$Q_{\rm global}$ is equal to 1 for each left-handed Weyl multiplet coupling to the gauge group, 0 for all other matter 't Hooft vertex



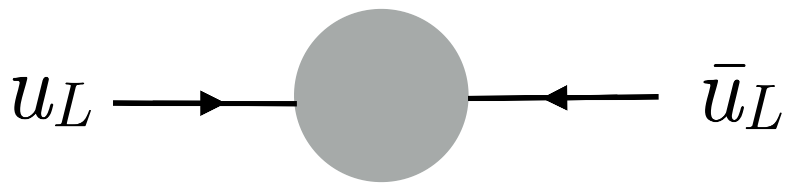
One incoming line for each left-handed Weyl fermion

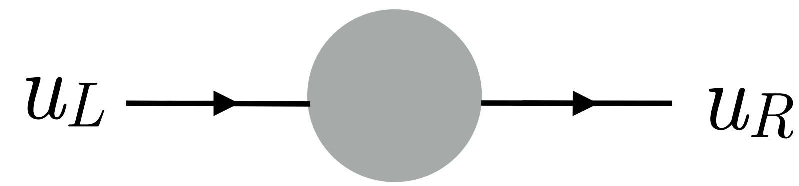
For example in QCD:

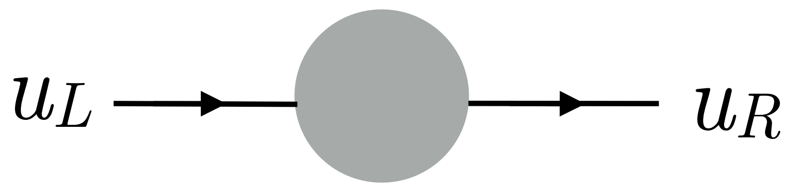
Up quark: One Dirac fermion ψ_u

Two Weyl components $\psi_{u,L}$ and $\psi_{u,R}$

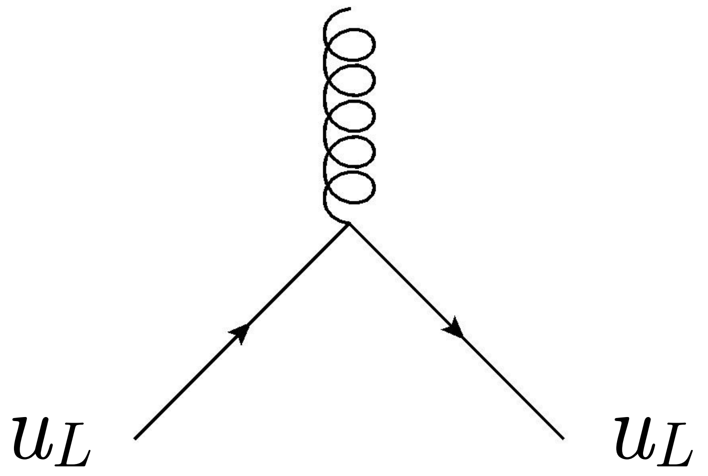
Equivalent to two left-handed Weyl components $\psi_{u,L}$ and $\psi_{\bar{u},L}$







Non-perturbative



perturbative

For three families

$$u_L \bar{u}_L d_L \bar{d}_L s_L \bar{s}_L c_L \bar{c}_L b_L \bar{b}_L t_L \bar{t}_L$$

$Q_{\text{global}} = Q_5$ ("axial charge")

$$\Delta Q_5 = 12$$

Weak interactions

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}\begin{pmatrix}u_r\\ d_r\end{pmatrix}\begin{pmatrix}u_g\\ d_g\end{pmatrix}\begin{pmatrix}u_b\\ d_b\end{pmatrix}\begin{pmatrix}c_r\\ s_r\end{pmatrix}\begin{pmatrix}c_g\\ s_g\end{pmatrix}\begin{pmatrix}c_b\\ s_b\end{pmatrix}\begin{pmatrix}t_r\\ b_r\end{pmatrix}\begin{pmatrix}t_g\\ b_g\end{pmatrix}\begin{pmatrix}t_b\\ b_b\end{pmatrix}$$

$SU(2)$ invariance: take six from row 1 and six from row 2;
(use crossing as needed)

For example

r=red
g=green
b=blue

$$u_r u_g \to e^+ \mu^+ \tau^+ \bar{u}_b \bar{c}_r \bar{c}_g \bar{c}_b \bar{b}_r \bar{b}_g \bar{b}_b$$

$$\Delta B = \Delta L = 3$$

$$\Delta(B - L) = 0$$
$$\Delta(B + L) = 6$$