Nuclear Physics B330 (1990) 1–18
North-Holland

HIGH ENERGY BREAKDOWN OF PERTURBATION THEORY IN THE ELECTROWEAK INSTANTON SECTOR

A. RINGWALD

Deutsches Elektronen Synchrotron-DESY, Hamburg, FRG

Received 30 June 1989

We consider baryon and lepton number violating processes induced by instantons in an electroweak-type model. We show semi-quantitatively that processes of the type $q + q \to 7\bar{q} + 3\bar{\ell}$ are very much suppressed even at large energies. The inclusive cross sections for $q + q \to 7\bar{q} + 3\bar{\ell} + n_w W + n_h \varphi$, where W stands for $W^{\pm}$ and Z bosons and φ for Higgs bosons, are much larger at high energies. They increase with the energy and reach 1 pb for a parton center of mass energy of the order of tens of TeV. The reason for this behaviour is that the leading-order S-matrix elements for above processes are *local* and of order $n!\exp(-2\pi/\alpha_w)$, where $n = n_w + n_h$. We argue that before these energies are reached, perturbation theory in the instanton sector breaks down. We comment on how this fits onto the sphaleron picture of the anomalous baryon and lepton number violation at high energies.

1. Introduction

It is well known that baryon number (B) and lepton number (L) are not strictly conserved in the standard model. Indeed, as 't Hooft [1] observed, gauge field configurations with nonvanishing topological charge can cause explicit violation of B and L in the standard electroweak theory. In the vacuum sector this phenomenon is associated with instantons [2], describing tunneling transitions between topologically inequivalent vacua which are separated by an energy barrier of height $\sim m_w/\alpha_w$ [1, 3–5]. In weakly coupled theories the probabilities of these transitions are exponentially suppressed; in particular, the corresponding suppression factor in the standard electroweak theory is $\exp(-2S_{\text{inst}})$, where $S_{\text{inst}} = 2\pi/\alpha_w$ is the classical euclidean action of the instanton. Various authors [6–9] have suggested that, if the real energy of a system is large enough ($E \geqslant m_w/\alpha_w$), the system can pass over the energy barrier between different vacua rather than penetrate through the barrier, in which case the rate of the anomalous nonconservation of the fermion number can be unsuppressed. Progress in this direction was possible through the development [10–14] of a static saddle-point solution in the electroweak theory, which corresponds just to the barrier configuration between topological inequivalent vacua and provides a nontrivial source for B and L violation. This solution, dubbed a "sphaleron" [14], has been examined for its role in generating and destroying the

0550-3213/90/$03.50

baryon asymmetry of the universe at *high temperatures* in a cosmological context [15–21]. Another possibility mentioned in refs. [14, 22] is the B and L violation in *high energy* collisions.

The key question is whether anomalous B and L violating processes in the standard model are indeed unsuppressed at large energies or large temperatures. This question is not fully answered by previous calculations for the following reason [23, 24]. From simple energy considerations we expect that for energies larger than the barrier height which is of the order of $E_{\mathrm{sp}} \sim m_{\mathrm{w}}/\alpha_{\mathrm{w}}$ (the subscript "sp" stands for sphaleron), the anomalous processes are no more associated with tunneling and therefore should be essentially unsuppressed. But this seems to be in conflict with instanton based estimates. For B and L violating Green functions such as $\langle (\mathrm{qqq}\ell)^{n_{\mathrm{g}}} \rangle$, where n_{g} denotes the number of generations, to be non-zero one needs gauge fields with topological number one in order to provide for the necessary number of zero modes of the fermions. The euclidean action of the configurations with $Q = 1$, where Q is the topological number, is greater or equal to $8\pi^2/g^2$. This is true at zero as well as at non-zero temperature [9, 25]. Therefore B and L violating Green functions always encounter at least one factor of $\exp(-8\pi^2/g^2)$ [23, 24]. Arnold and McLerran [17] proposed a solution to this conflict. They argue that Green functions of the form $\langle (\mathrm{qqq}\ell)^{n_{\mathrm{g}}} \mathrm{W}^n \varphi^m \rangle$, where W denotes generically $\mathrm{W}^{\pm}$ and Z bosons and φ stands for the physical Higgs scalar, can lead to an effective vertex which is not exponentially suppressed in the coupling constant for n and m of the order of $1/\alpha_{\mathrm{w}}$. Their argument is based on the following simple physical picture (see also ref. [14]). Since the process mediated by the sphaleron is a classical one, it involves a large number of quanta. To cross the barrier, the fields must configure themselves into a physical sphaleron with energy $\sim m_{\mathrm{w}}/\alpha_{\mathrm{w}}$ and radius $\sim 1/m_{\mathrm{w}}$. When the sphaleron decays, the momenta of particles in the final state will be typically m_{w}. Therefore the sphaleron will decay into $\sim 1/\alpha_{\mathrm{w}}\, \mathrm{W}^{\pm}$ and Z bosons, producing the quarks and leptons as a side-effect due to the anomaly. The relevant B and L violating Green functions should therefore involve a large number of W, Z and/or Higgs bosons. Arnold and McLerran [17] argue that instanton estimates to amplitudes break down in the classical, many-quanta limit where the sphaleron estimates are made.

We want to check if this reasoning is correct, at least semi-quantitatively. To this end we calculate instanton induced B and L violating vertices in an electroweak-type model. This is done by calculating B and L violating Green functions with and without many W, Z or Higgs bosons by expanding the path integral around constrained instantons [26]. The corresponding B and L violating S-matrix elements are obtained from the amputated Green functions according to the LSZ reduction formula [27]. It is shown that the leading-order S–matrix elements are *local* and of order $n!\exp(-S_{\mathrm{inst}})$, where n is the number of external bosons.

We observe that perturbation theory in the instanton sector of the electroweak theory breaks down for $n \sim 1/\alpha_{\mathrm{w}}$ and energies $E \sim m_{\mathrm{w}}/\alpha_{\mathrm{w}}$. Note that these numbers nicely fit onto the estimates from the sphaleron analysis [13,14,17]. The energy just corresponds to the barrier height between topologically inequivalent vacua, $E_{\mathrm{sp}} \sim 10$ TeV. From naive considerations, Manton [13] suspected that at this energy scale or higher conventional perturbation theory is completely unreliable. We make this explicit in the instanton sector of the electroweak theory.

We study semi-quantitatively the cross sections of the processes $\mathrm{q} + \mathrm{q} \to 7\bar{\mathrm{q}} + 3\ell + n_{\mathrm{w}}\mathrm{W} + n_{\mathrm{h}}\varphi$. For $n_{\mathrm{w}} = n_{\mathrm{h}} = 0$ the cross section is much too small to be ever observable. The cross section at high energies is larger if W, Z and Higgs bosons are in the final state. If we naively extrapolate the instanton-based results we observe that the cross sections grow with center of mass energy and reach, for $n \sim 1/\alpha_{\mathrm{w}}$, 1 pb at center of mass energies of the order of tens of TeV*, but we must stress that this occurs outside the range of validity of instanton based calculations. So the question whether anomalous electroweak B and L violating processes can be observed in future pp colliders is still open.

2. Fermion number violating Green functions

We consider the electroweak theory with n_{f} massless Weyl fermion doublets, $\psi^{(i)}$, $i = 1, \ldots, n_{\mathrm{f}}$, in the limit of vanishing Weinberg angle (with α_{w} fixed). For three generations, $n_{\mathrm{f}} = 12$. We are working in euclidean space-time. The euclidean action of the model reads

$$S = S_{\mathrm{g}} + S_{\mathrm{h}} + S_{\mathrm{f}}\,, \tag{1}$$

where

$$S_{\mathrm{g}} = \int \mathrm{d}^4x \tfrac{1}{2} \mathrm{tr}\left(F_{\mu\nu} F_{\mu\nu}\right), \tag{2}$$

$$S_{\mathrm{h}} = \int \mathrm{d}^4x \left\{ \left(D_\mu \Phi\right)^\dagger D_\mu \Phi + \lambda \left(\Phi^\dagger \Phi - \tfrac{1}{2} v^2\right)^2 \right\}, \tag{3}$$

$$S_{\mathrm{f}} = \int \mathrm{d}^4x \left\{ -i \sum_{i=1}^{n_{\mathrm{f}}} \psi^{(i)\dagger} \bar{\sigma}_\mu D_\mu \psi^{(i)} \right\}. \tag{4}$$

Here we have defined

$$F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig\left[W_\mu, W_\nu\right], \qquad W_\mu = (\sigma^a/2) W_\mu^a\,, \tag{5), (6}$$

$$D_\mu = \partial_\mu - igW_\mu, \qquad \bar{\sigma}_\mu = (i, -\boldsymbol{\sigma})\,. \tag{7), (8}$$

* The quoted numbers should not be taken too seriously. There are many uncertainties in the calculation which require a detailed analysis. This, however, is beyond the scope of the present paper, which concentrates on the qualitative aspects.

In this model the divergences of the fermion number currents are anomalous,

$$\partial_\mu J^\mu_{f^{(i)}} = -\frac{g^2}{16\pi^2}\operatorname{tr}\left(F_{\mu\nu}F^{*\mu\nu}\right), \tag{9}$$

where $F^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$ is the dual SU(2) field strength. For this reason one expects that the change of the fermion number $f^{(i)}$ in the external field is connected with the topological charge Q,

$$\Delta f^{(i)} = -Q, \tag{10}$$

where

$$Q = \frac{g^2}{16\pi^2}\int \mathrm{d}^4x \operatorname{tr}\left(F_{\mu\nu}F^*_{\mu\nu}\right). \tag{11}$$

We are interested in Green functions of the type

$$G(x_1,\dots,x_{n_f}) = \left\langle \prod_{i=1}^{n_f} \psi^{(i)}_{a_i\alpha_i}(x_i)\right\rangle. \tag{12}$$

Here the latin indices $a_i = 1,2$ stand for weak isospin, and the greek indices $\alpha_i = 1,2$ for spin, respectively. In the standard model with three generations a non-zero value of this Green function implies, according to the LSZ reduction formula [27], the existence of the process

$$\mathrm{q}_1 + \mathrm{q}_1 \to \bar{\mathrm{q}}_1 + 3\bar{\mathrm{q}}_2 + 3\bar{\mathrm{q}}_3 + \bar{\ell}_1 + \bar{\ell}_2 + \bar{\ell}_3\ , \tag{13}$$

or the corresponding CP conjugated process, where particles are replaced by anti-particles and vice versa. Here the subscripts label the generations, which we take to be three. The (anti-)quarks of the same generation should have different colors in (13). The fermion number of each "flavor" is changed by one unit in the process (13). So one expects that this process is induced by gauge fields with one unit of topological charge. The fact that the Green function (12) is non-zero was established by 't Hooft [1]. It is given by the euclidean path integral

$$G(x_1,\dots,x_{n_f}) = \int_{Q=1}[\mathrm{d}W][\mathrm{d}\Phi][\mathrm{d}\psi^\dagger][\mathrm{d}\psi]\exp\left\{-S\left[W,\Phi,\psi^\dagger,\psi\right]\right\}\prod_{i=1}^{n_f}\psi^{(i)}(x_i), \tag{14}$$

where the measure of the gauge fields in the unit-winding-number sector of the model is understood to contain also the gauge fixing and ψ stands collectively for all fermions in the model. We have dropped the indices in eq. (14) for notational simplicity. 't Hooft [1] considered the semiclassical calculation of (14) by expanding the integrand in eq. (14) around the unit-winding number instanton [2], which is given in the so-called singular gauge by

$$W_\mu^{a\,\mathrm{cl}}(x) = \frac{2}{g}\frac{\rho^2}{(x-z)^2}\frac{\eta_{a\mu\nu}(x-z)_\nu}{(x-z)^2+\rho^2}. \tag{15}$$

Here η are the 't Hooft symbols [1], and z_μ and ρ denote the instanton position and scale, respectively. In the absence of fermions and without a potential for the Higgs doublet he got for the vacuum to (gauge rotated) vacuum transition amplitude,

$$Z_0^{[1]} = \int_{Q=1}[\mathrm{d}W][\mathrm{d}\Phi]\exp\{-S_0[W,\Phi]\}, \tag{16}$$

where

$$S_0[W,\Phi] = \int \mathrm{d}^4x\left\{\tfrac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + |D_\mu\Phi|^2\right\}, \tag{17}$$

the answer

$$\mathrm{d}Z_0^{[1]} = \tilde{n}_0(\rho)\mathrm{d}^4z\,\mathrm{d}\rho. \tag{18}$$

Here $\tilde{n}_0(\rho)$ is the instanton density in the theory with action S_0 (17), given by

$$\tilde{n}_0(\rho) = \frac{1}{4\pi^2}\left(\frac{8\pi^2}{g^2}\right)^4 \exp\left(-\alpha(1)-\alpha\left(\tfrac{1}{2}\right)\right)\mu^{43/6}\exp\left(-\frac{8\pi^2}{g^2(\mu)}\right)\rho^{13/6}, \tag{19}$$

where [1]

$$\alpha(1) = 0.443307, \qquad \alpha\left(\tfrac{1}{2}\right) = 0.145873. \tag{20), (21}$$

μ is the renormalization point. The infrared divergence in $Z_0^{[1]}$ is cured by the Higgs field expectation value which provides a cutoff in the ρ integral. Strictly speaking, for $v \neq 0$, there does not exist a finite action solution of the classical euclidean equations of motion. But in the limit $\rho v \ll 1$ both classical equations can be solved. The solutions are eq. (16) and [1]

$$\Phi^{\mathrm{cl}}(x) = \frac{v}{\sqrt{2}}\left[\frac{(x-z)^2}{(x-z)^2+\rho^2}\right]^{1/2}\begin{pmatrix}0\\1\end{pmatrix}. \tag{22}$$

Therefore, in addition to $\tilde{n}_0(\rho)$, there will be a factor $\exp(-S_{\mathrm{H}})$, where

$$S_{\mathrm{H}} = S_{\mathrm{h}}[\Phi^{\mathrm{cl}}, W^{\mathrm{cl}}] = \pi^2 v^2\rho^2 + \mathrm{O}(\lambda v^4\rho^4). \tag{23}$$

The integral over ρ now converges and is dominated by $\rho v \leqslant 1$. The source terms which have been dropped in the euclidean equations of motion are proportional to v^2 and are therefore indeed suppressed by $\rho^2 v^2 \ll 1$, justifying the approximation.

The reason, why we are concentrating on Green functions of the type (12), is the fact that we need only the knowledge of the fermion zero modes in order to calculate it semiclassically. This is true for Green functions in which each different fermion flavor appears exactly once. Collecting all results so far we obtain in the one-loop approximation

$$G(x_1,\ldots,x_{n_f}) = \int d^4 z \int_0^\infty d\rho\, \tilde{n}_0(\rho) \det{}'_{\mathrm{F}}(\rho) \exp(-\pi^2 v^2 \rho^2) \prod_{i=1}^{n_f} \psi^0_{a_i \alpha_i}(x_i - z)\,. \quad (24)$$

The zero modes of the euclidean Weyl equation in the presence of the instanton are given by [1]

$$\psi^0_{a\alpha}(x) = \left(\frac{2\rho^3}{\pi^2}\right)^{1/2} i \frac{(x_\mu \sigma_\mu)_{ab}}{|x|} \frac{\epsilon_{b\alpha}}{(x^2+\rho^2)^{3/2}}\,, \quad (25)$$

where $\sigma_\mu = (i, \boldsymbol{\sigma})$. The instanton is at the origin, $z = 0$, in eq. (25). The prime at the fermionic determinant means that zero modes should be omitted. For n_f Weyl doublets it reads [1]

$$\det{}'_{\mathrm{F}}(\rho) = \exp\left(-\tfrac{1}{3} n_f \ln \mu\rho + n_f \alpha(\tfrac{1}{2})\right). \quad (26)$$

To proceed it is useful to go to the Fourier transform of eq. (24). It is given by

$$\tilde{G}(p_1,\ldots,p_{n_f}) = (2\pi)^4 \delta^{(4)}\left(\sum_{i=1}^{n_f} p_i\right)(32\pi^2)^{n_f/2}$$
$$\times \frac{(\sigma_\mu p_1^\mu)_{a_1\kappa_1}}{(p_1)^2} \cdots \frac{(\sigma_\mu p_{n_f}^\mu)_{a_{n_f}\kappa_{n_f}}}{(p_{n_f})^2} \epsilon_{\kappa_1\alpha_1} \cdots \epsilon_{\kappa_{n_f}\alpha_{n_f}}$$
$$\times \int_0^\infty d\rho\, \rho^{3n_f/2} \tilde{n}(\rho) \prod_{i=1}^{n_f} f_i(\rho |p_i|)\,, \quad (27)$$

where

$$\tilde{n}(\rho) = \tilde{n}_0(\rho) \det{}'_{\mathrm{F}}(\rho) \exp(-\pi^2 v^2 \rho^2) \quad (28)$$

is the instanton density of our model. The functions f_i in eq. (27) are defined by

$$f_i(\rho|p_i|) = \frac{3}{\rho|p_i|}\int_0^\infty \mathrm{d}x\, x \frac{J_1(\rho|p_i|x)}{(1+x^2)^{5/2}}. \tag{29}$$

Note that $f_i(\rho|p_i|)$ is regular for $|p_i| \to 0$. It has $f_i(0) = 1$.

The amputated Green function is obtained from eq. (27) by removing the propagators for the external legs. It is proportional to

$$\tilde{\Gamma}(p_1,\ldots,p_{n_f}) = (2\pi)^4\delta^{(4)}\left(\sum_{i=1}^{n_f} p_i\right)(32\pi^2)^{n_f/2}\int_0^\infty \mathrm{d}\rho\,\rho^{3n_f/2}\tilde{n}(\rho)\prod_{i=1}^{n_f} f_i(\rho|p_i|). \tag{30}$$

The S-matrix element for the process (13) is proportional to (30), evaluated on-shell ($p_i^2 = 0$),

$$\tilde{\Gamma}(p_1,\ldots,p_{n_f})|_{p_i^2=0} = (2\pi)^4\delta^{(4)}\left(\sum_{i=1}^{n_f} p_i\right)\mathscr{G}. \tag{31}$$

Here we defined the effective coupling constant of the fermion number violating vertex by

$$\begin{aligned}\mathscr{G} &\equiv (32\pi^2)^{n_f/2}\int_0^\infty \mathrm{d}\rho\,\rho^{3n_f/2}\tilde{n}(\rho)\\ &= (32\pi^2)^{n_f/2} c\, \mathrm{e}^{-a}\mu^b \exp\left(-\frac{8\pi^2}{g^2(\mu)}\right)\frac{1}{2}(\pi^2 v^2)^{-(19+7n_f)/12}\,\Gamma\left(\frac{19+7n_f}{12}\right),\end{aligned} \tag{32}$$

where

$$a = \alpha(1) - (n_f - 1)\alpha(\tfrac{1}{2}), \qquad b = \frac{43}{6} - \frac{n_f}{3}, \qquad c = \frac{1}{4\pi^2}\left(\frac{8\pi^2}{g^2}\right)^4. \tag{33), (34), (35}$$

The on-shell amputated Green function (31) describes, according to the LSZ reduction formula [27], a fermion number violating n_f-fermion point-interaction. It has to be multiplied with the wave functions of the external fermions in order to obtain the S-matrix element for the process (13). It should be noted that our result is equivalent to 't Hooft's effective lagrangian [1]. The advantage of our approach is that it can be easily extended to calculate also the instanton contribution to fermion number violating Green functions involving many W, Z and Higgs bosons, as will be seen in sect. 3.

Let us plug in numbers. We take $\sin^2\theta_{\mathrm{w}} = 0.23$ and $\mu = 100$ GeV for the renormalization point. Then

$$g^2(\mu) = g^2(100\ \mathrm{GeV}) = 0.406. \tag{36}$$

Using this and $v = 246$ GeV we get

$$\mathscr{G} = 1.6 \times 10^{-101}\ \mathrm{GeV}^{-14}, \tag{37}$$

for the case of three generations, $n_{\mathrm{f}} = 12$. The cross section of the reaction (13) is estimated by

$$\hat{\sigma}(\mathrm{qq} \to 7\bar{\mathrm{q}}3\bar{\ell}) \simeq C\mathscr{G}^2 \int \prod_{i=1}^{10} \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2|\boldsymbol{p}_i|} |\boldsymbol{p}_i| (2\pi)^4 \delta^{(4)}\left(P_{\mathrm{in}} - \sum_{i=1}^{10} p_i \right), \tag{38}$$

where the dimensionless constant C contains our ignorance about the results from averaging (summing) over initial (final) states of the fermions, projecting onto the color and charge singlet states, etc. We have not calculated it since its actual value is unimportant for our conclusions. The phase space integral for the fermions in (38) can be evaluated using the methods in ref. [28]. It yields

$$\tfrac{1}{2}(4\pi)^{-17} \frac{1}{13!14!} \hat{s}^{13}, \tag{39}$$

where $\sqrt{\hat{s}} = \sqrt{P_{\mathrm{in}}^2}$ is the center of mass energy of the colliding partons. We get therefore

$$\hat{\sigma}(\mathrm{qq} \to 7\bar{\mathrm{q}}3\bar{\ell}) \simeq 2 \times 10^{-40} C\mathscr{G}^2 \hat{s}^{13}, \tag{40}$$

The same behavior with energy was obtained from dimensional grounds by Ellis et al. [23]. Here we determined the effective coupling constant more accurately by integrating over all scale sizes which includes small scale sizes. Our naive formula has no form factor in it. We will soon argue that instanton based perturbation theory breaks down at $\sqrt{\hat{s}} \sim m_{\mathrm{w}}/\alpha_{\mathrm{w}}$. At this energy a form factor could arise which cuts off the rise in the cross section, but unfortunately we cannot rely on perturbation theory in the instanton sector in order to calculate it. The cross section (40) reaches the unitarity bound, $1/\hat{s}$, only at very large energies, $4 \times 10^8 C^{-1/28}$ GeV. Plugging in numbers we get at $\sqrt{\hat{s}} = 10$ TeV, for example,

$$\hat{\sigma} \simeq 5 \times 10^{-138} C\ \mathrm{GeV}^{-2} = 2 \times 10^{-129} C\ \mathrm{pb}. \tag{41}$$

This parton cross section has to be convoluted with parton distributions in order to get the observable cross section for the anomalous baryon and lepton number violating process (13) in a pp collision. But is clear already from (41) that the cross section for the process (13) is much too small in order to be observable in future pp colliders such as SSC, for example.

3. Many-particle Green functions

The fact that the processes (13) are very much suppressed does not necessarily mean that the total *inclusive* cross section for fermion number violating processes is unobservably small at large energies. As mentioned in sect. 1, the sphaleron picture suggests that reactions like

$$\mathrm{q}+\mathrm{q} \rightarrow 7\bar{\mathrm{q}}+3\bar{\ell}+n_{\mathrm{h}}\varphi+n_{\mathrm{w}}\mathrm{W}, \tag{42}$$

could occur more frequently.

Let us now consider fermion number violating Green functions with additional W, Z or Higgs bosons:

$$G^{a_1\dots a_n}_{\mu_1\dots\mu_n}(x_1,\dots,x_{n_\mathrm{f}},y_1,\dots,y_n)=\left\langle \prod_{i=1}^{n_\mathrm{f}}\psi^{(i)}_{a_i\alpha_i}(x_i)\prod_{j=1}^{n}W^{a_j}_{\mu_j}(y_j)\right\rangle, \tag{43}$$

$$G(x_1,\dots,x_{n_\mathrm{f}},y_1,\dots,y_n)=\left\langle \prod_{i=1}^{n_\mathrm{f}}\psi^{(i)}_{a_i\alpha_i}(x_i)\prod_{j=1}^{n}\varphi(y_j)\right\rangle, \tag{44}$$

or "mixed" Green functions involving both Higgs and W(Z) bosons, which we do not display explicitly for notational simplicity. These are the relevant Green functions in order to discuss the processes (42) (or the *CP* conjugated processes).

We want to consider the instanton contribution to the Green functions (43) and (44). However, in calculating (43) or (44) by steepest descent the following problem arises. If we were naively to calculate these Green functions using the pure Yang–Mills instanton (15) together with the expression for the physical excitation corresponding to the scalar configuration (22),

$$\varphi^{\mathrm{cl}}(x)=\frac{v}{\sqrt{2}}\left[\left(\frac{(x-z)^2}{(x-z)^2+\rho^2}\right)^{1/2}-1\right], \tag{45}$$

we would arrive at an expression in momentum space which has poles $\sim 1/k^2$ for the gauge and Higgs fields at zero momentum. To be definite, calculate the Fourier transforms of eqs. (15) and (45) for $z = 0$. These are given by

$$\tilde{W}_{\mu}^{a\,\mathrm{cl}}(k) = \frac{i}{g}(4\pi)^2 \frac{\eta_{a\mu\nu} k_\nu}{(k^2)^2} \left\{1 - \tfrac{1}{2} K_2(\rho|k|)\rho^2 k^2\right\}, \tag{46}$$

$$\tilde{\varphi}^{\mathrm{cl}}(k) = \frac{v}{\sqrt{2}} \frac{(2\pi)^2}{(k^2)^2} \int_0^\infty \mathrm{d}x\, x^2 J_1(x) \left[\frac{x}{\sqrt{x^2 + k^2\rho^2}} - 1\right]. \tag{47}$$

Note that the integral in eq. (47) behaves as $-\frac{1}{2}k^2\rho^2$ as $k^2 \to 0$. We observe explicitly that using eqs. (46) and (47) in the calculation of the Fourier transform of eqs. (43) and (44) will lead to the absurd conclusion that an exponentially small instanton effect is potent enough to prevent the Higgs phenomenon from occurring. The reason for that behavior is that (15) and (45) produce long range effects since they behave asymptotically like an inverse power of x.

A systematic method to estimate Green functions of the type (43) and (44) has been described by Affleck [26] and has been used in supersymmetric QCD [29, 30]. This method deals with the problem that no finite action solution exists with nonvanishing boundary conditions on the scalar fields (as mentioned above). For this reason one introduces a constraint in the path integral,

$$1 = \int \mathrm{d}\rho\, \Delta(\rho)\, \delta\left(\int \mathrm{d}^4x\, \mathcal{O} - \rho^{4-d}\right), \tag{48}$$

where $\mathcal{O}$ is a local operator of dimension $d > 4$ and Δ is a jacobian. Next one Fourier transforms the δ function,

$$\delta\left(\int \mathrm{d}^4x\, \mathcal{O} - \rho^{4-d}\right) = \int \frac{\mathrm{d}\omega}{2\pi} \exp\left[-i\omega\left(\int \mathrm{d}^4x\, \mathcal{O} - \rho^{4-d}\right)\right]. \tag{49}$$

One next performs the path integral (and the ω integral) with ρ held fixed. This amounts to calculating with a modified action (which is referred to as the constrained action)

$$S_c = S + i\omega \int \mathrm{d}^4x\, \mathcal{O}. \tag{50}$$

The path integral (and the ω integral) may now be performed by steepest descent. The ω integral is deformed into the complex plane so that the coupling constant, $i\omega$, is real at the saddle point. For appropriately chosen operators $\mathcal{O}$, such as $\mathrm{tr}\, F^3$ and $(\Phi^\dagger\Phi - v^2/2)^3$, classical solutions will exist. Since the operators $\mathcal{O}$ are constructed entirely out of boson fields, the lowest order perturbation of the fermion zero modes are independent of the constraint [29]. For $|x| \leqslant \rho \ll 1/v$ the equations of motion can again be approximated by neglecting the source terms, i.e. eqs. (15) and (22) are approximate solutions in this region. However, at large distances the source terms become important. They produce mass terms for the gauge and Higgs fields which cause exponential decay of W_μ (to a pure gauge) and Φ (to the expectation value),

$$W_\mu^{a\,\mathrm{cl}}(x) = -\frac{4\pi^2\rho^2}{g}\eta_{a\mu\nu}\,\partial_\nu G_{m_\mathrm{w}}(x)\,, \tag{51}$$

$$\Phi^{\mathrm{cl}}(x) = \frac{v}{\sqrt{2}}\left[1-2\pi^2\rho^2 G_{m_\mathrm{h}}(x)\right]\begin{pmatrix}0\\1\end{pmatrix}, \tag{52}$$

where $m_\mathrm{w} = \frac{1}{2}gv$ and $m_\mathrm{h} = \sqrt{2\lambda}\,v$ denote the W boson and Higgs boson masses, respectively. Here G_m denotes the solution of

$$(-\partial^2 + m^2)G_m(x) = \delta^{(4)}(x)\,. \tag{53}$$

This function decays exponentially, $G_m \sim \mathrm{e}^{-m|x|}$, at large $|x|$. The approximate solutions, eqs. (15) and (22), represent the first terms in an expansion in v, valid at $|x| \ll 1/v$. The solutions of the linearized equations (with the mass term), eqs. (51) and (52), are the first terms in an expansion in ρ, valid at $|x| \gg \rho$. The two expansions can be matched in the intermediate region $\rho \ll |x| \ll 1/v$. To lowest order $\mathcal{O}$ can be neglected entirely. Since the constrainted instanton decays exponentially at large $|x|$ it does not affect the long range behavior of the Higgs theory. By working with the constrained instanton one therefore should recover the poles at $k_i^2 = -m^2$ in the Fourier transform of eqs. (43) and (44) (the minus sign comes from the fact that we are working in euclidean space-time). From eqs. (51) and (52) we obtain the Fourier transforms near the $W(Z)$ and Higgs boson mass shell, i.e. for $k^2 + m^2_{\mathrm{w(h)}} \to 0$,

$$\tilde{W}_\mu^{a\,\mathrm{cl}}(k) = i\frac{4\pi^2}{g}\eta_{a\mu\nu}\frac{k_\nu}{k^2+m_\mathrm{w}^2}\rho^2, \qquad \tilde{\varphi}^{\mathrm{cl}}(k) = -\frac{v}{\sqrt{2}}\frac{2\pi^2\rho^2}{k^2+m_\mathrm{h}^2}, \tag{54), (55}$$

which makes this explicit.

Now we turn to the actual computation of the Fourier transform of eq. (44) (a similar formula holds for eq. (43)),

$$\begin{aligned}\tilde{G}(p_1,\dots,p_{n_\mathrm{f}},k_1,\dots,k_n) &= (2\pi)^4\delta^{(4)}\left(\sum_{i=1}^{n_\mathrm{f}} p_i + \sum_{j=1}^{n} k_j\right)(32\pi^2)^{n_\mathrm{f}/2}\frac{(\sigma_\mu p_1^\mu)_{a_1\kappa_1}}{(p_1)^2}\cdots\frac{(\sigma_\mu p_{n_\mathrm{f}}^\mu)_{a_{n_\mathrm{f}}\kappa_{n_\mathrm{f}}}}{(p_{n_\mathrm{f}})^2}\epsilon_{\kappa_1\alpha_1}\cdots\epsilon_{\kappa_{n_\mathrm{f}}\alpha_{n_\mathrm{f}}}\\ &\times\int_0^\infty \mathrm{d}\rho\,\rho^{3n_\mathrm{f}/2}\tilde{n}(\rho)\prod_{i=1}^{n_\mathrm{f}} f_i(\rho|p_i|)\prod_{j=1}^{n}\tilde{\varphi}^{\mathrm{cl}}(k_j)\,.\end{aligned} \tag{56}$$

This formula holds as long as the ρ integral gets cut off at small $\rho \leqslant 1/v$ [26]. Using eq. (55) we obtain, near the Higgs bosons' mass shell, $k_j^2 + m_h^2 \to 0$,

$$\tilde{G}(p_1, \ldots, p_{n_f}, k_1, \ldots, k_n)|_{k_j^2+m_h^2 \to 0}$$

$$= (2\pi)^4 \delta^{(4)}\left(\sum_{i=1}^{n_f} p_i + \sum_{j=1}^{n} k_j\right)(32\pi^2)^{n_f/2} \frac{(\sigma_\mu p_1^\mu)_{a_1\kappa_1}}{(p_1)^2} \cdots \frac{(\sigma_\mu p_{n_f}^\mu)_{a_{n_f}\kappa_{n_f}}}{(p_{n_f})^2} \epsilon_{\kappa_1\alpha_1} \cdots \epsilon_{\kappa_{n_f}\alpha_{n_f}}$$

$$\times(-)^n \left(\frac{v}{\sqrt{2}}\right)^n (2\pi^2)^n \prod_{j=1}^{n} \frac{1}{(k_j)^2 + m_h^2} \int_0^\infty \mathrm{d}\rho\, \rho^{3n_f/2+2n} \tilde{n}(\rho) \prod_{i=1}^{n_f} f_i(\rho|p_i|), \tag{57}$$

for the Fourier transform of eq. (44). Using eq. (54) we obtain in a similar way the Fourier transform of eq. (43) near the mass shell of the $W(Z)$ bosons, $k_j^2 + m_w^2 \to 0$,

$$\tilde{G}^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}(p_1, \ldots, p_{n_f}, k_1, \ldots, k_n)|_{k_j^2+m_w^2 \to 0}$$

$$= (2\pi)^4 \delta^{(4)}\left(\sum_{i=1}^{n_f} p_i + \sum_{j=1}^{n} k_j\right)(32\pi^2)^{n_f/2} \frac{(\sigma_\mu p_1^\mu)_{a_1\kappa_1}}{(p_1)^2} \cdots \frac{(\sigma_\mu p_{n_f}^\mu)_{a_{n_f}\kappa_{n_f}}}{(p_{n_f})^2} \epsilon_{\kappa_1\alpha_1} \cdots \epsilon_{\kappa_{n_f}\alpha_{n_f}}$$

$$\times i^n \left(\frac{4\pi^2}{g}\right)^n \prod_{j=1}^{n} \frac{\eta_{a_j\mu_j\nu} k_j^\nu}{(k_j)^2 + m_w^2} \int_0^\infty \mathrm{d}\rho\, \rho^{3n_f/2+2n} \tilde{n}(\rho) \prod_{i=1}^{n_f} f_i(\rho|p_i|). \tag{58}$$

The corresponding amputated Green functions will be proportional to

$$\tilde{\Gamma}_{n\varphi}(p_1, \ldots, p_{n_f}, k_1, \ldots, k_n)|_{k_j^2+m_h^2 \to 0}$$

$$= (2\pi)^4 \delta^{(4)}\left(\sum_{i=1}^{n_f} p_i + \sum_{j=1}^{n} k_j\right)(-1)^n (32\pi^2)^{n_f/2} 2^{n/2} \pi^{2n} v^n$$

$$\times \int_0^\infty \mathrm{d}\rho\, \rho^{3n_f/2+2n} \tilde{n}(\rho) \prod_{i=1}^{n_f} f_i(\rho|p_i|), \tag{59}$$

for the fermion number violating Higgs boson vertex, and

$$\tilde{\Gamma}^{a_1,\dots,a_n}_{nW;\mu_1\dots\mu_n}(p_1,\dots,p_{n_f},k_1,\dots,k_n)|_{k_j^2+m_w^2\to 0}$$

$$= (2\pi)^4\delta^{(4)}\left(\sum_{i=1}^{n_f} p_i + \sum_{j=1}^{n} k_j\right) i^n (32\pi^2)^{n_f/2}\left(\frac{4\pi^2}{g}\right)^n \prod_{j=1}^{n} \eta_{a_j\mu_j\nu} k_j^\nu$$

$$\times \int_0^\infty d\rho\, \rho^{3n_f/2+2n}\tilde{n}(\rho) \prod_{i=1}^{n_f} f_i(\rho|p_i|), \tag{60}$$

for the fermion number violating W(Z) boson vertex, respectively. Evaluation of the ρ integral for $p_i^2 = 0$ gives

$$c\, e^{-a}\mu^b \exp\left(-\frac{8\pi^2}{g^2(\mu)}\right)\frac{1}{2}(\pi^2 v^2)^{-n-(19+7n_f)/12}\Gamma\left(n+\frac{19+7n_f}{12}\right). \tag{61}$$

Using eqs. (32) and (61) we can write eqs. (59) and (60) as

$$\tilde{\Gamma}_{n\varphi}(p_1,\dots,p_{n_f},k_1,\dots,k_n)|_{p_i^2=0;\,k_j^2+m_h^2\to 0}$$

$$= (2\pi)^4\delta^{(4)}\left(\sum_{i=1}^{n_f} p_i + \sum_{j=1}^{n} k_j\right)(-1)^n \mathscr{G} 2^{n/2} v^{-n} \frac{\Gamma(n+(19+7n_f)/12)}{\Gamma((19+7n_f)/12)}, \tag{62}$$

and

$$\tilde{\Gamma}^{a_1\dots a_n}_{nW;\mu_1,\dots,\mu_n}(p_1,\dots,p_{n_f},k_1,\dots,k_n)|_{p_i^2=0;\,k_j^2+m_w^2\to 0}$$

$$= (2\pi)^4\delta^{(4)}\left(\sum_{i=1}^{n_f} p_i + \sum_{j=1}^{n} k_j\right)$$

$$\times i^n \mathscr{G}\left(\frac{4\pi^2}{g}\right)^n (\pi^2 v^2)^{-n} \frac{\Gamma(n+(19+7n_f)/12)}{\Gamma((19+7n_f)/12)} \prod_{j=1}^{n} \eta_{a_j\mu_j\nu} k_j^\nu, \tag{63}$$

respectively. The extension of (62) and (63) to "mixed" vertices involving both Higgs and W(Z) bosons is straightforward. In order to save place we omit the explicit expression.

We again arrived at *local* $(12+n)$-point vertices for the processes (42). These effective vertices are only valid for $n < 1/\alpha_w$ or $n < \pi^2/\lambda$. This is for the following reason. The ρ integral is dominated at $\rho_0 \simeq \sqrt{n}/(\pi v)$. However, in order that the formula for the instanton density (28) be true, we must require $\rho_o \ll 1/m_w(m_h)$, which leads to $n \ll 1/\alpha_w$ or $n \ll \pi^2/\lambda$. The maximal number of W(Z) bosons for which our effective vertex is valid is therefore of the order of 30. Since the quartic Higgs coupling λ or, equivalently, the Higgs mass are unknown, the maximal number of Higgs particles is uncertain. The highest value one could achieve is for the Higgs coupling of the order of g^4. In this case one gets $n_{\max} \sim 60$.

In addition, $n < 1/\alpha_w, \pi^2/\lambda$ is also a general consequence of unitarity. When $n \geqslant 1/\alpha_w$, perturbation theory breaks down. We can see this by looking at the dependence of a multi-particle scattering amplitude upon one of its momenta (see fig. 1). If we make a radiative exchange of a W(Z) boson we get a factor of α_w. It can tie to n legs. The correction is therefore of order $n\alpha_w$. That means that $n \ll 1/\alpha_w$ is necessary so that a weak coupling analysis is reliable. Similar arguments apply also for the Higgs field.

The restriction on n means also a restriction on the energy where our effective vertices are valid. If E is the center of mass energy than by energy conservation the typical multiplicity will be $n \leqslant E/m_w(m_h)$. From these considerations we see that for $E \geqslant m_w/\alpha_w$ a weak coupling analysis in the instanton sector is unreliable and we can only extrapolate and guess. Note that the energy corresponds to the sphaleron energy, E_{sp}, the height of the barrier between topologically inequivalent vacua in the electroweak theory [13,14]. For energies larger than E_{sp} one would naively suspect instanton based calculations to break down [13,17]. Unfortunately, this is just the region where our results are most interesting, as we will see now.

Let us estimate the cross section for the processes (42). Using the effective vertex (62) we obtain

$$|\mathscr{M}(\mathrm{qq} \to 7\bar{\mathrm{q}}3\bar{\ell}n_h\varphi)|^2 \propto \mathscr{G}^2 2^{n_h} v^{-2n_h} \left[\frac{\Gamma(n_h + 103/12)}{\Gamma(103/12)}\right]^2. \tag{64}$$

From the properties of the η symbols [1] it follows

$$\sum_{\epsilon} \eta_{a\mu\nu} k^{\nu} \epsilon^{\mu} \eta_{a\kappa\lambda} k^{\lambda} \epsilon^{*\kappa} = -\delta_{aa} k^2 = \delta_{aa} m_w^2, \tag{65}$$

(no sum over a), where ϵ are the polarization vectors of the W(Z) bosons. That means that we get from the effective vertex (63), if we inclusively sum over $W^{\pm}$ and

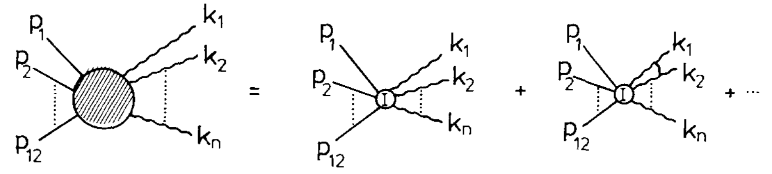

Fig. 1. Radiative correction to the B and L violating amplitude through W(Z) exchange.

Z bosons,

$$\sum_{\{a's, \epsilon's\}} |\mathcal{M}(\mathrm{qq} \to 7\bar{\mathrm{q}}3\bar{\ell} n_{\mathrm{w}}\mathrm{W})|^2 \propto \mathcal{G}^2 2^{n_{\mathrm{w}}} v^{-2n_{\mathrm{w}}} \left[\frac{\Gamma(n_{\mathrm{w}} + 103/12)}{\Gamma(103/12)} \right]^2 3^{n_{\mathrm{w}}}. \quad (66)$$

Again it is straightforward to write down the matrix elements also for the "mixed" channels. From these we estimate the cross sections of the reactions (42),

$$\hat{\sigma}(\mathrm{qq} \to 7\bar{\mathrm{q}}3\bar{\ell} n_{\mathrm{w}}\mathrm{W} n_{\mathrm{h}}\varphi)$$
$$\simeq \tilde{C}\mathcal{G}^2 2^n v^{-2n} \left[\frac{\Gamma(n + 103/12)}{\Gamma(103/12)} \right]^2 3^{n_{\mathrm{w}}}$$
$$\times \int \prod_{i=1}^{10} \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2|\boldsymbol{p}_i|} |\boldsymbol{p}_i| \prod_{j=1}^{n} \frac{\mathrm{d}^3 k_j}{(2\pi)^3 2E_j} (2\pi)^4 \delta^{(4)}\left(P_{\mathrm{in}} - \sum_{i=1}^{10} p_i - \sum_{j=1}^{n} k_j \right) \mathcal{S}, \quad (67)$$

where $n = n_{\mathrm{w}} + n_{\mathrm{h}}$. The statistical factor $\mathcal{S}$ is given by

$$\mathcal{S} = \frac{1}{n_{\mathrm{h}}! n_{\mathrm{w}}!}. \quad (68)$$

$\tilde{C}$ contains the result from averaging (summing) over initial (final) states of the fermions, from the projection into color singlet and charge neutral states etc. The phase space integral in eq. (67) gives

$$\tfrac{1}{2}(4\pi)^{-17-2n} \frac{1}{(13+n)!(14+n)!} \hat{s}^{13+n}, \quad (69)$$

in the extremely relativistic case, $E_i \gg m_{\mathrm{w}}(m_{\mathrm{H}})$ [28].

Fig. 2 shows the parton cross section (67) for $\tilde{C} = 1$ as a function of the parton center of mass energy for different values of $n = n_{\mathrm{w}} + n_{\mathrm{h}}$, where we took $n_{\mathrm{h}} = n_{\mathrm{w}}/3$. This figure shows also, for comparison, the s-wave unitarity bound, $1/\hat{s}$. It is reached for $n = 50$ already at around 60 TeV, whereas for the exclusive channel, $n = 0$, it is reached only at $\sim 10^5$ TeV. In fig. 3 we display the energies at which the unitarity limit is achieved as a function of the number of bosons produced in association. At least at these energies the leading-order calculation breaks down and one has to take into account higher-order corrections. From fig. 3 we infer that the leading-order amplitudes become strong, of the order of 1 pb, already at around 30 TeV. The relevent processes are those with a large number of external bosons,

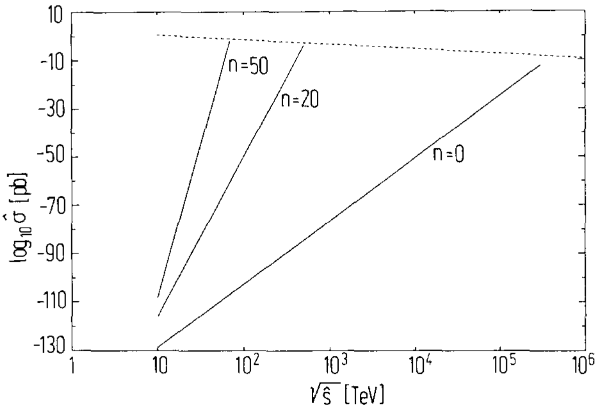

Fig. 2. Parton cross sections for the reactions $q + q \to 7\bar{q} + 3\bar{\ell} + nX$, where X stands collectively for W, Z, and Higgs bosons, for different numbers of n. The dashed line gives the s-wave unitarity bound, $\sim 1/\hat{s}$.

$n \sim \sqrt{\hat{s}}/m_w$, such that the emitted bosons are non-relativistic. We see that our results are most interesting when $n \gg 1/\alpha_w$ and $\sqrt{\hat{s}} \sim 10$ TeV. Unfortunately these values are beyond the validity of the instanton-based calculations and eq. (67) and fig. 2 give only a guess of the expected cross sections. The constant $\tilde{C}$ can of course change these values, but the qualitative behaviour of the cross sections remains the same.

The actual observable cross section for baryon and lepton number violating processes in the collision of protons is obtained from eq. (67) by convolution with

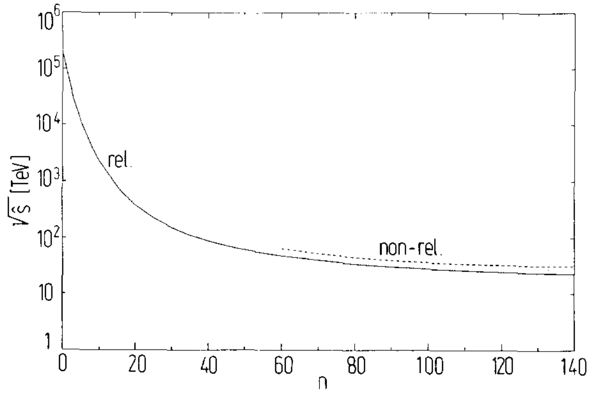

Fig. 3. The center of mass energies, $\sqrt{\hat{s}}$, at which the unitarity limit, $\sim 1/\hat{s}$, is reached, as a function of the number of associated bosons, n. The solid/dashed line corresponds to the use of the relativistic/non-relativisic phase space formula from ref. [28].

parton distribution functions (projected into the color singlet state). This should be done in a future work.

4. Conclusions

We considered baryon and lepton number violating processes induced by instantons in an electroweak-type model. We showed that exclusive processes like $\mathrm{q}+\mathrm{q} \to 7\bar{\mathrm{q}} + 3\bar{\ell}$ are too much suppressed by the exponential of the instanton action in order to be observable. We observed that the parton cross sections for the inclusive processes $\mathrm{q}+\mathrm{q} \to 7\bar{\mathrm{q}} + 3\bar{\ell} + n\mathrm{X}$, where X stands generically for $\mathrm{W}^{\pm}$, Z and Higgs bosons, can be much larger at high energies, but we showed also that perturbation theory in the instanton sector of the electroweak theory breaks down for $n \geqslant 1/\alpha_{\mathrm{w}}$ and $E \geqslant m_{\mathrm{w}}/\alpha_{\mathrm{w}}$. That this happens at these values of n and E was suspected before by Manton [13] and Arnold and McLerran [17]. We have made it explicit through our calculations. Our work puts the arguments by Arnold and McLerran [17], that there is no contradiction between instanton estimates and sphaleron estimates, on much firmer ground.

If we naively extrapolate our parton cross sections to energies of the order of tens of TeV and n of the order of $1/\alpha_{\mathrm{w}}$ (these numbers can change by a detailed calculation of the constant $\tilde{C}$, but the qualitative aspects, in particular the rising of the cross sections, remain unaltered), we obtain a cross section for $\mathrm{q}+\mathrm{q} \to 7\bar{\mathrm{q}} + 3\bar{\ell} + n\mathrm{X}$ of the order of 1 pb, but unfortunately we are then outside the range of validity of the weak coupling analysis in the instanton sector and our result only gives a guess for the expected cross section. Somewhere the rise in the cross sections has to be cut off by nonperturbative physics. For this reason we cannot definitively decide if the anomalous electroweak B and L violating processes can be observed in future pp colliders. Nevertheless, the results obtained are suggestive and justify a more detailed analysis.

I would like to thank A. Ali, J. Fuchs, J. Kripfganz, M. Lüscher, M.G. Schmidt, C. Wetterich and A. Wipf for helpful discussions. I would especially like to thank L. McLerran for encouragement, valuable discussions and useful hints.

References

[1] G. 't Hooft, Phys. Rev. Lett. 37 (1976) 8; Phys. Rev. D14 (1976) 3432; Phys. Rev. D18 (1978) 2199
[2] A. Belavin, A. Polyakov, A. Schwartz and Y. Tyupkin, Phys. Lett. B59 (1975) 85
[3] R. Jackiw and C. Rebbi, Phys. Rev. Lett. 37 (1976) 172
[4] C. Callan, R. Dashen and D. Gross, Phys. Lett. B63 (1976) 334
[5] K. Bitar and S.-J. Chang, Phys. Rev. D17 (1978) 486; Phys. Rev. D18 (1978) 435
[6] A. Linde, Phys. Lett. B70 (1977) 306; B100 (1981) 37
[7] S. Dimopoulos and L. Susskind, Phys. Rev. D18 (1978) 4500
[8] N. Christ, Phys. Rev. D21 (1980) 1591

[9] R. Noble, Phys. Rev. D25 (1982) 825
[10] R. Dashen, B. Hasslacher and A. Neveu, Phys. Rev. D10 (1974) 4138
[11] J.Boguta, Phys. Rev. Lett. 50 (1983) 148
[12] J.Burzlaff, Nucl. Phys. B233 (1984) 262
[13] N. Manton, Phys. Rev. D28 (1983) 2019
[14] F. Klinkhamer and N. Manton, Phys. Rev. D30 (1984) 2212
[15] V. Kuzmin, V. Rubakov and M. Shaposhnikov, Phys. Lett. B155 (1985) 36
[16] P. Arnold and L. McLerran, Phys. Rev. D36 (1987) 581
[17] P. Arnold and L. McLerran, Phys. Rev. D37 (1988) 1020
[18] A. Ringwald, Phys. Lett. B201 (1988) 510; Phys. Lett. B213 (1988) 61
[19] H. Aoyama, H. Goldberg and Z. Ryzak, Phys. Rev. Lett. 60 (1988) 1902
[20] T. Akiba, H. Kikuchi and T. Yanagida, Phys. Rev. D38 (1988) 1937; Phys. Rev. D40 (1989) 588
[21] J. Kripfganz and A. Ringwald, Z. Phys. C44 (1989) 213
[22] H. Aoyama and H. Goldberg, Phys. Lett. B188 (1987) 506
[23] J. Ellis, R. Flores, S. Rudaz and D. Seckel, Phys. Lett. B194 (1987) 241
[24] K. Freese, E. Mottola, V. Nair and F. Wilczek, unpublished
[25] D. Gross, R. Pisarski and L. Yaffe, Rev. Mod. Phys. 53 (1981) 43
[26] I. Affleck, Nucl. Phys. B191 (1981) 445
[27] H. Lehmann, K. Symanzik and W. Zimmerman, Nuovo Cimento 1 (1955) 1425
[28] E. Byckling and K. Kajantie, Particle Kinematics (Wiley, London, 1973) p. 281
[29] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B241 (1984) 493
[30] J. Fuchs and M.G. Schmidt, Z. Phys. C30 (1986) 161